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ON THE PROBLEM OF THREE-BODY-POINTS WITH VARIABLE MASSES

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The problem of three body-points under Newton interaction is considered. Masses of the points change isotropically by the different laws $m_0 = m_0(t)$, $m_1 = m_1(t)$, $m_2 = m_2(t)$. Equations of motion in the Jacobi coordinates are given by

$$\mu_1 \ddot{\vec{r}}_1 = \mathrm{grad}_{\vec{r}_1} U, \quad \mu_2 \ddot{\vec{r}}_2 = \mathrm{grad}_{\vec{r}_2} U - \mu_2 (2\dot{\nu}_1 \dot{\vec{r}}_1 + \ddot{\nu}_1 \vec{r}_1), \tag{1}$$

$$\frac{\dot{m}_i}{m_i} \neq \frac{\dot{m}_j}{m_j}, \ \ i \neq j, \quad \mu_1 = \frac{m_1 m_0}{m_0 + m_1} \neq const, \quad \mu_2 = \frac{m_2(m_1 + m_0)}{m_0 + m_1 + m_2} \neq const,$$

where μ_i are the reduced masses, $\nu_1 = m_1/(m_0 + m_1)$, U is the power function [1]. In general, in contrast to the classical problem of three bodies with constant masses, non-autonomous differential equations (1) do not have any integral. The problem is investigated by the perturbation methods with use of the system of analytical calculations MATHEMATICA [2].

A periodical motion on quasionic section is used as an initial unperturped intermediate motion. Equations of secular perturbations in the analogues of the second system of the Poincare elements [1, 3] have the form

$$\begin{aligned} \dot{\xi}_i &= \frac{\partial R_{i\,sec}}{\partial \eta_i}, \quad \dot{\eta}_i = -\frac{\partial R_{i\,sec}}{\partial \xi_i}, \\ \dot{p}_i &= \frac{\partial R_{i\,sec}}{\partial q_i}, \quad \dot{q}_i = -\frac{\partial R_{i\,sec}}{\partial p_i}, \ \ i = 1,2 \end{aligned} \tag{2}$$

here $R_{i\,sec}$ are the corresponding expressions of the secular disturbing functions.

Masses of bodies are assumed to be comparable, but the laws of mass changing are arbitrary. Eccentricities and inclinations of the orbits of bodies are the small quantities. In expression of the perturbing function the members till the second degree inclusively concerning small quantities are preserved. And even in this case analytical calculations are very cumbersome and difficult foreseeable.

Under these assumptions the expression of the disturbing function in analogues of the second system of the Poincare elements is obtained. Equations of secular perturbations are obtained and solutions in the first approximation by the Picard method are analyzed.

References

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