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Intelligent System for Assessing the Socio-economic Situation in the Region

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Abstract. This article analyzes the problems of monitoring and managing the socio-economic situation. The analysis of the socio-economic situation involves the determination of the quantitative characteristics of the dynamic series, the trend of growth, decline or stabilization, the identification of causal factors, in specific territories and for different groups. The criterion of fuzzy controllability was obtained to solve the problem of forecasting and controlling the socio-economic situation. A mathematical model and algorithm for solving the task of monitoring and managing the socio-economic situation based on interval mathematics and their software implementation are described. The social effect will be expressed in improving the safety of people's lives. As a result, it will be possible to carry out preventive measures in the necessary territories.

Keywords: Social tension · Manageability · Interval mathematics · Linguistic variable · Fuzzy · Interval analysis

1 Introduction

The methodology of mathematical modeling has gained a strong position in the technological and natural science fields, and its progress is also significantly noticeable in applications to economic systems. If we talk about processes involving the «human factor» (first of all, about social processes), then the progress in this area is much more modest [1].

Therefore, the relevance of mathematical modeling of social processes and the development of intelligent information and analytical systems (IIAS) that allow monitoring indicators that characterize the socio-economic situation in the context of the country's regions is undeniable.

Mathematical modeling of socio-economic processes can be used by specialists to solve a number of applied issues, such as planning various preventive measures [2,3].

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2 Problem Statement

To develop a real system of automated forecasting of the socio-economic situation, a list of economic, demographic and social parameters is defined. Socio-economic indicators include, for example, gross domestic product (GDP) per capita, the index of the physical volume of GDP per capita, as a percentage of the previous year, the volume of industrial production in billion tenge, the index of the physical volume of industrial production, as a percentage of the previous year, the number of industrial enterprises and industries, etc. The following demographic indicators were taken as demographic indicators: the population, the number of people of working age, the number of births, the number of deaths, the number of emigrated citizens, the number of immigrated citizens, etc. The indicators of the crime rate in the region included, for example, the number of all registered crimes, the number of registered murders and attempted murders, the number of registered thefts, etc.

To provide automated monitoring of the socio-economic situation, the following indices were determined based on the results of the analysis and the possibility of obtaining specific data: the cost of living index; the human development index; the unemployment rate; the level of criminality; the level of medical support, etc.

As every one knows, social tension is determined not only by socio-economic factors, but also by the political activity of the population. To assess the political activity of the population, data from the media and the Internet were used. For this purpose, the following indicators of political activity have been introduced: the number of publications in regional and republican mass media with calls for acts of disobedience; the number of demonstrations, rallies and other mass protests against the leadership of the region and the country, which are of an economic nature; the number of demonstrations, rallies and other mass protests against the leadership of the region and the country that are of a political nature; the total number of participants in demonstrations, rallies and other mass demonstrations; the total number of participants in hunger strikes that are of an economic nature; the total number of participants in hunger strikes that are of a political nature; the level of activity of opposition parties. Based on the entered indicators and socio-economic indicators, the index of social tension is calculated.

Directives and measures at the national and regional levels are defined as the control parameters $u = (u_1, ..., u_l)$. In turn, the control parameters are divided into socio-economic and political ones.

3 Main Results

Using the features of the problem under study, a model is constructed in the form of a system of ordinary differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x, u, p, t) \tag{1}$$

where u - external factors, p - parameters of binding (setting) of the mathematical model to real data.

Management restrictions are given

$$u(t) \in U = \{u(t) : -L_i \leq u_i(t) \leq L_i, i = 1, m, t \in [t_0, t_1]\} \quad (2)$$

Using the methods of the mathematical theory of identification, based on the retrospective knowledge of the values of the parameters $x = (x_1, ..., x_n)$ and $u = (u_1, ..., u_l)$, the values of the parameters p are calculated. In this case, the form of functions *f(x,u,p,t)* (up to unknown constants p) are constructed taking into account the «physics» of the studied indicators.

Thus, a mathematical model is defined that characterizes the relationship between the input and control parameters. The IAS allows you to predict the behavior (dynamics) of the input parameters under various (specially specified) specified control actions.

Knowing the values of the parameters x at the present time, which is denoted by t_0:

$$x(t_0) = x_0 \quad (3)$$

solving the obtained Cauchy problem (1)–(3) by numerical methods (in particular, the Runge-Kutta method) for given external influences u, the values of the parameters *x, y* at time t_1, are found, i.e., the problem of forecasting for the period $[t_0, t_1]$ is solved.

By solving the sequence of Cauchy problems (1)–(3) for various (specially defined) given external influences $u_1, ..., u_k$, it is possible to predict the corresponding behavior of the parameters x at the time t_1, i.e. to give an opportunity to get an answer to the question (lose the situation), what can happen if the strategy u_1 is chosen as opposed to the strategy u_2, etc.

In classical control theory, we usually study the controllability problem (Problem 1) [4]: is there a control that satisfies the constraint (2) and transfers the system (1) from the initial state (3) to the final given state

$$x(t_1) = x_1. \quad (4)$$

for a fixed time $t_1 - t_0$.

The initial values of the state vector x_0 in formula (3) are set from the actual measurements. At the same time, for the task of monitoring the socio-economic situation, it is not the fixed value of x_1 at the final time t_1 in formula (4) that is relevant, but the translation of the system into a certain set that allows for a convenient interpretation.

In this connection, based on the theory of fuzzy sets, we introduce the corresponding linguistic variables for the state variables x of the system (1) as follows [6].

Each state variable x_i corresponds to the linguistic variable $x_{ling}, i = \overline{1, n}$. Since in the model (1)–(4) the variables of the state of the system have a quantitative character and their greater significance increases the degree of occurrence of socio-economic danger, the following values of linguistic variables are proposed:

TermLin[1]= «optimal level»,
TermLin[2]= «moderate level»,
TermLin[3]= «acceptable level»,
TermLin[4]= «critical level» .

Each j-th value of the i-th linguistic variable $x_{lingi,j}$ corresponds to a numeric interval $(x_{min,i,j}, x_{max,i,j})$ and many $\bigcup_{j=1}^{4}(x_{min,i,j,}, x_{max,i,j})$ must cover all possible values of the variable $(x_{min,i,j}, x_{max,i,j})$. In particular, it is acceptable to $\bigcup_{j=1}^{4}(x_{min,i,j,}, x_{max,i,j}) = (-\infty, +\infty)$.

We introduce a set of indices $I_{kr} \subseteq [1, .., n]$ that defines a list of state variables that are subject to terminal constraints. For example, if for the model (1)–(4) the terminal constraints are imposed only on the variable x_2, then the set of indices $I_{kr} = [2]$ consists of a single element.

Next, we consider the following fuzzy controllability problem (Problem 2): is there a control that satisfies the constraint (2) and transfers the system (1) from the initial state (11) to the final state

$$x_{lingi}(t_1) = TermLin[i_j], i \in I_{kr} \tag{5}$$

for a fixed time $t_1 - t_0$.

In (5), the index i_j corresponds to the selected j-th linguistic fuzzy value for the i–th state variable.

Problem 1 is a special case of problem 2.

Due to the properties imposed on the right side of the system of equations of the Cauchy problem (1), (3) with a fixed control $u(t) \in U$, the conditions of the theorem of the existence and uniqueness of the solution $x(t), t \in [t_0, t_1]$ are fulfilled [5].

We rewrite the Cauchy problem (1), (3) in integral recurrent form

$$x_{k+1}(t) = x_0 + \int_{t_0}^{t} f(x_k(\tau), u(\tau), \tau)d\tau. \tag{6}$$

Due to the properties imposed on the right side of Eq. (1) and restrictions on the function *u(t)* it is proved in [7] that the method of successive approximations (5) converges to the solution to the solution absolutely and uniformly for any fixed control.

Then the controllability problem is reduced to the study of the following problem: is there at least one control $u(t) \in U$, in which the solution of the integral Eq. (6) at time t_1 satisfies the condition (5).

To solve this problem, we apply the results of the interval analysis [8,9]. Denote by $\overline{v}$ the interval from $-L$ to L, by $\overline{f}$ the interval-valued function obtained from the function $f(x_k(t), u(t), t)$.

Substituting the interval $\overline{v}$ in Eq. (6) instead of the function *u(t)*, we obtain the interval integral equation

$$\overline{x}(t) = x_0 + \int_{t_0}^{t} \overline{f}_(\overline{x}(\tau), \overline{\nu}, \tau)d\tau. \tag{7}$$

The solution to the integral Eq. (7) can be found by the method of successive approximations

$$\overline{x}_{k+1}(t) = x_0 + \int_{t_0}^{t} \overline{f}_{(}\overline{x}_k(\tau), \overline{\nu}, \tau) d\tau. \tag{8}$$

Theorem 1. *In order for the system under study was managed is necessary and sufficient that overline*$x_i(t_1)$ *for all* $i \in I_{kr}$ *had a non-empty intersection with the set* $(x_{min,i,j}, x_{max,i,j})$.

Proof. Lots of

$$X(t_1) = \left\{ x_0 + \int_{t_0}^{t} f(x_k(\tau, u(\tau), \tau) d\tau | u(t) : -L_i \le u_i(t) \le L_i, \right.$$
$$\left. i = 1, m, t \in [t_0, t_1] \right\}$$

coincides with the interval solution of the integral Eq. (7), where all arithmetic operations are performed using interval calculations [9]. Hence, it is obvious that the original system (1)–(4) is controllable, it is necessary and sufficient that the solution of the integral interval Eq. (7) at time t_1 for all $i \in I_{kr}$ has a non-empty intersection with the set $(x_{min,i,j}, x_{max,i,j})$, i.e.

$$\{\bar{x}_i(t_1) \cap (x_{min,i,j}, x_{max,i,j}) \neq | i \in I_{kr}\} .$$

If the system under study is controllable (i.e., there is at least one control $u \in U$ that ensures the transfer of the system (1) from state (3) to state (5), then it is advisable to choose a control that, in addition to solving the problem, would deliver a minimum to a certain criterion (this may be energy consumption, speed, etc.).

The optimal control problem under constraints (5)–(6) is a problem with a movable right end.

The problem is solved by the methods of the mathematical theory of optimal control using the method of penalty functions.

In the case of several criteria for selecting optimal control actions, the multi-criteria problem with $J_i, i = \overline{1, n}$ functionals was reduced to a single-criteria optimal control problem with the functional $J = \sum_{i=1}^{n} \alpha_i J_i$. Here $\alpha_i, i = \overline{1, n}$ - global weight coefficients, are determined based on the hierarchy analysis method [10].

Next, we study the dynamical system (1) under the following assumptions: the right – hand side of the system of Eqs. (1) has the form, $f(x, u, t) = g(x, t) + Bu$. B is a constant *(n*m)*-matrix, *g(x,t)*-*n*-vector, the elements of which are continuously differentiable functions in their arguments.

Let's rewrite system (1) in the following form

$$\frac{dx}{dt} = g(x, t) + Bu. \tag{9}$$

The state of the system at the initial time t_0 is considered known (the initial state)

$$x(t_0) = x_0 \tag{10}$$

The desired state at a finite time T can be described as fixed

$$x(T) = x_T \tag{11}$$

or mobile (satisfying certain conditions)

$$\sum_{j=1}^{n} c_{ij} x_j(T) \leq d_i, i = \overline{1, k} \tag{12}$$

at the same time, the time point T can be set (fixed) or be based on certain requirements.

There are natural constraints on quantitative data

$$x_i(t) \geq 0, i = \overline{1, n}, t \in [t_0, T]. \tag{13}$$

To assess the quality of the system, the following criteria can be selected:

$$J = \int_{t_0}^{T} [u^*(t) R_0 u(t) + x(t)^* R_1 x(t)]\, dt \tag{14}$$

or

$$J = T - t_0 \tag{15}$$

In functional (14), R_0 is a positive – definite mxm-matrix, and R_1 is a non-negative-definite nxn matrix.

The problem of optimal control with phase constraints (13), control constraints (2) with fixed (10), (11) or movable ends (10), (12) is considered. Currently, the solution of such problems contains a number of mathematical difficulties. In this connection, we consider a number of statements of optimal control problems.

1 Optimal control problem with fixed right end and fixed time

The problem of minimizing the functional (14) under constraints is considered (9), (2), (10), (11). The time point T is considered to be set (fixed).

For the given optimal control problem, we compose the Hamilton function

$$H\left(x(t), u, \psi(t), \psi_0\right) = u^*(t) R_0 u(t) + x(t)^* R_1 x(t) + (g(x,t) + Bu(t))^* \psi. \tag{16}$$

Let's make a conjugate system of differential equations:

$$\frac{d\psi}{dt} = -(\frac{\partial g(t)}{\partial t})^*(t)\psi(t) - 2R_1 x(t), t \in [t_0, T]. \tag{17}$$

We determine the optimal control from condition (2) and the maximum of the Hamiltonian:

$$u = \begin{cases} -L & if \quad R_0^{-1} B\psi < 0 \\ R_0^{-1} B\psi & if\ 0 \leq R_0^{-1} B\psi \leq u_{max} \\ L & if \quad R_0^{-1} B\psi > u_{max} \end{cases} \tag{18}$$

Theorem 2. *Let the pair* $(u(t), x(t)), t \in [t_0, T]$ *- be the solution of the above problem. Then there must exist a vector-function* $\psi(t), t \in [t_0, T]$ *and a parameter* ψ_0 *such that*

$$\psi_0 \le 0, |\psi_0| + |\psi(t)| \ne 0, t \in [t_0, T]$$

in this case, $x(t), \psi(t), t \in [t_0, T]$ *is the solution of the boundary value problem for the system of differential Eqs. (9) and the corresponding conjugate system of differential Eqs. (17) under boundary conditions (10) and (11) and control (18).*

Proof. Since all the conditions of the Pontryagin maximum principle [11] are met for the formulated optimal control problem, the validity of the theorem follows from this.

2 Optimal control problem with a movable right end

We consider the problem of minimizing the functional (14), under the constraints (9), (2), (10), (12). The time point T is considered to be set (fixed).

The optimal control is found by the formula (18).

Theorem 3. *Let the* $(u(t), x(t)), t \in [t_0, T]$ *- pair be the solution of the above problem. Then there must exist a vector-function* $\psi(t), t \in [t_0, T]$ *and a parameter* ψ_0 *such that*

1) $\psi_0 \le 0, |\psi_0| + |\psi(t)| \ne 0, t \in [t_0, T]$

2) $\psi(t), t \in [t_0, T]$ *is the solution of the conjugate system of differential Eqs. (17) that satisfies the condition: there are numbers* $\beta_1, ..., \beta_k$ *such that*

$$\Psi_i(T) = \sum_{j=1}^{k} \beta_j c_{ji}, i = \overline{1,n}, \quad \beta_i \left(\sum_{j=1}^{n} c_{ij} x_j(T) - d_i \right) = 0,\ \beta_i \ge 0,\ i = \overline{1,k}$$

for each $t \in [t_0, T]$*, the function* $H(x(t), u, \psi(t), \psi_0)$ *(16) with respect to the variable* u *reaches its upper edge on the set* U *at* $u = u(t)$*, i.e.*

$$\sup_{u \in U} H(x(t), u, \Psi(t),\ \Psi_0) = H(x(t), u(t), \Psi(t),\ \Psi_0).$$

Proof. Since all the conditions of the Pontryagin maximum principle [11] are met for the formulated optimal control problem, the validity of the theorem follows from this.

3 Optimal performance problem with fixed right end

The problem of minimizing the functional (15) under constraints is considered (9), (2), (10), (11). The time point T is not specified and must be determined.

For the given optimal control problem, we compose the Hamilton function

$$H(x(t), u, \psi(t), \psi_0) = 1 + (g(x,t) + Bu(t))^* \psi. \tag{19}$$

Let's make a conjugate system of differential equations:

$$\frac{d\psi}{dt} = -(\frac{\partial g(t)}{\partial t})^*(t)\psi(t), t \in [t_0, T]. \tag{20}$$

Theorem 4. *Let the* $(u(t), x(t)), t \in [t_0, T]$ *- pair be the solution of the above problem. Then there must exist a vector-function* $\psi(t), t \in [t_0, T]$ *and a parameter* ψ_0 *such that*

1) $\psi_0 \leq 0, |\psi_0| + |\psi(t)| \neq 0, t \in [t_0, T]$

2) $\psi(t), t \in [t_0, T]$ *- solution of the conjugate system of differential Eqs. (20), which together with system (9) satisfies the boundary conditions (10) and (11)*

3) for each tu, the function Hx with respect to the variable u reaches its upper face on the set U at u = t, i.e.

$$\sup_{u \in U} H(x(t), u, \Psi(t), \Psi_0) = H(x(t), u(t), \Psi(t), \Psi_0).$$

Proof. Since all the conditions of the Pontryagin maximum principle [11] are met for the formulated optimal control problem, the validity of the theorem follows from this.

4 Numerical algorithm for solving the optimal control problem with fixed ends and phase constraints

The problem of optimal control with phase constraints (13), with fixed ends (10)–(11) and control constraints (2) is considered. At present, the solution of such problems contains a number of mathematical difficulties.

In this regard, for the practical solution of the optimal control problem, the method of penalty functions and the gradient method are used.

To account for the phase constraints (13) and the constraints on the end of the trajectory (11), we introduce the penalty functions $\Phi_{1k} = M_{k1}\sum_{i=1}^{n}\int_{t_0}^{T}[max\{x_i(t);0\}]^2dt$ and $\Phi_{2k} = M_{k2}\sum_{i=1}^{n}[x(T) - x_T)]^2$, where M_{k1}, M_{k2} are some given positive sequences tending to infinity.

Let's build a new functionality

$$J_k = \int_{t_0}^{T} \left\{u^*(t)R_0u(t) + x(t)^*R_1x(t) + M_{k1}[max\{x_i(t);0\}]^2\right\}dt + M_{k2}\sum_{i=1}^{n}[x(T) - x_T)]^2$$

Replace the original problem with the following: for a given k, find the optimal control that minimizes the functional J_k under constraints (10), (2), and (11). The resulting problem is an optimal control problem with a free right end and a constraint on the controls. For it, we will make a Hamilton function

$$H_k = u^*(t)R_0u(t) + x(t)^*R_1x(t) + M_{k1}[max\{x_i(t);0\}]^2 + (g(x,t) + Bu(t)) * \psi_k$$

The following solution algorithm is proposed.

Step 1. Let k = 0.

Step 2. Calculate the optimal control for the k-th iteration

$$u_k = \begin{cases} -L \quad if \quad R_0^{-1}B\psi_k < 0 \\ R_0^{-1}B\psi_k \; if \; 0 \leq R_0^{-1}B\psi_k \leq u_{max} \\ L \quad if \quad R_0^{-1}B\psi_k > u_{max} \end{cases} \quad (21)$$

where ψ_k is the solution of the conjugate system of differential equations

$$\frac{d\psi_k}{dt} = -(\frac{\partial g}{\partial x})^* \psi_k - 2R_1 x_k(t) + M_{k1}[max\,\{x_{ki}(t);0\}] \tag{22}$$

with a condition at the end

$$\Psi_k\,(T) = 2M_{k2}\sum_{i=1}^{n}[x_k(T) - x_T] \tag{23}$$

and x_k -the solution of the original system (9) under the initial conditions (10).

Step 3. When x_k and u_k are found, the value of the functional j_k is calculated.

Step 4. If $|J_k - J_{k-1}| \leq \varepsilon$ then go to step 5, otherwise k = k + 1 and go to step 2. (Here $\varepsilon > 0$ is the required calculation accuracy).

Step 5. The found pair (x_k, u_k) is the optimal solution for.

Numerical experiments on the model problem showed the convergence of the proposed algorithm already at the 6th iteration.

IIAS is developed on the MySQL DBMS [11] using the PHP WEB programming language. Currently, the IAS is in trial operation.

4 Discussion

Using the methods of correlation analysis, the IAS allows you to find the degree of dependence (correlation) between the parameters (while it is possible to take into account the lag effect). For example, you can find a correlation between: the unemployment rate and the number of crimes, the average monthly salary and the number of crimes.

To assess a number of parameters that cannot be measured in nature (for example, the social tension index), it is possible to conduct an expert study, which consists in the fact that each expert independently of the others gives an assessment of the selected parameter in a simulated or real situation, which is quantitatively characterized by the values of other parameters. For any selected parameter, a regression equation is constructed that depends on the other parameters.

The results obtained are applicable to the study of any dynamical system described by ordinary differential equations. Therefore, the application of the results of the article has great prospects for automating the solution of many problems of mathematical modeling.

5 Conclusion

The article discusses a dynamic model with a restriction on the right end based on linguistic variables, described by ordinary differential equations.

For forecasting and managing the socio-economic situation on the basis of interval mathematics, a criterion of fuzzy controllability was obtained.

Using the Pontryagin maximum principle, the optimal control and the optimal trajectory for the quadratic functional and the speed criterion are determined.

On the basis of the penalty function method and the gradient method, the optimal control problem with bounded controls and fixed ends is solved.

On the basis of the library of interval procedures [9], software has been developed for determining the controllability of a dynamic system described by ordinary differential equations.

The materials of the article are of practical value for designers of various socio-technical systems.

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For citations of references, we prefer the use of square brackets and consecutive numbers. Citations using labels or the author/year convention are also acceptable. The following bibliography provides a sample reference list with entries for journal articles [1], an LNCS chapter [2], a book [3], proceedings without editors [4], as well as a URL [5].

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