Multiscale (time and mass) dynamics of space objects

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Al. I. Cuza University of Iași, Romania

University of Rome Tor Vergata, Italy

Comitetul Național Român de Astronomie

Program and Abstract Book

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About

Scientific Rationale

The last decades have shown a continuous development in dynamics modelling of celestial bodies at various time scales, from days to periods of time comparable with the age of the Solar system, and length scales, from several kilometers to hundreds of astronomical units. New and more refined models are requested by the enormous amount of highly accurate observational data, collected from ground and space, as well as by the current and future space missions. The range of phenomena that manifest at all different time and length scales and the wide range of sizes of space objects, from minor bodies in the Solar system to exoplanets, from dust particles to Jupiter-size bodies, has required the development of modelling and analysis tools that can handle these different scales. The understanding of the dynamics of these space objects is a key to the advancement of space science and technology, with considerable benefits to society and economy. The emergence of new open problems in space science, such as the formation, habitability and long-term evolution of planetary systems, the complex dynamical behavior of minor bodies in the Solar system, the increased traffic in Earth orbit, the exploration and exploitation of space objects, has stimulated the birth of new lines of investigation, the development of new scientific methods and techniques as well as the development of technologies with a potential big impact on our everyday life.

This Symposium will cover the recent advances in the multi-scale dynamics of natural and artificial space objects from various perspectives: modelling, development of new methods and tools to analyze the dynamics, stability analysis, exploration and exploitation of minor bodies. The Symposium will serve a wide international community working in various fields: physics, celestial mechanics, astrodynamics, planetary sciences, space engineering, applied mathematics, dynamical systems. The Symposium will provide an ideal venue for interdisciplinary discussions, exchanging ideas, making future plans and developing new collaborations.

Key topics

The IAU Symposium will cover the following topics:

1. Large-scale body dynamics: planets and exoplanets;
2. Medium-scale body dynamics: asteroids, comets, NEOs, natural satellites;
3. Small-scale body dynamics: dust particles, rings and space debris;
4. Perturbation methods and long-term evolution of space objects;
5. Numerical and analytical methods for resonances and chaos;
6. Exploration and exploitation of space objects.

Invited Speakers

- **Jérémy Couturier**, Observatoire de Paris, France
- **Stanley Dermott**, University of Florida, USA
- **Christos Efthymiopoulos**, University of Padua, Italy
- **Sylvio Ferraz-Mello**, Universidade de Sao Paulo, Brazil
- **Agnès Fienga**, Observatoire de la Côte d’Azur, France
- **Giovanni F. Gronchi**, University of Pisa, Italy
- **Douglas P. Hamilton**, University of Maryland, USA
- **Kathleen Howell**, Purdue University, USA
- **Eiichiro Kokubo**, National Astronomical Observatory of Japan, Japan
- **Jacques Laskar**, Observatoire de Paris, France
- **Christoph Lhotka**, University of Rome Tor Vergata, Italy
- **Anne-Sophie Libert**, University of Namur, Belgium
- **Ugo Locatelli**, University of Rome Tor Vergata, Italy
- **Renu Malhotra**, University of Arizona, USA
- **Alessandro Morbidelli**, Observatoire de la Côte d’Azur, France
- **Aaron J. Rosengren**, The University of Arizona, USA
- **Alessandro Rossi**, IFAC-CNR Florence, Italy
- **Daniel J. Scheeres**, University of Colorado, USA
- **Kleomenis Tsiganis**, Aristotle University of Thessaloniki, Greece
- **Massimiliano Vasile**, University of Strathclyde, UK

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- **Massimiliano Vasile**, University of Strathclyde, UK
- **Marie Yseboodt**, Royal Observatory of Belgium, Belgium

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- **Roberto Paoli**, University Al. I. Cuza Iaşi, Romania
- **Vlad Turcu**, Astronomical Institute of the Romanian Academy, Romania
- **Tudor Vartolomei**, University of Rome Tor Vergata, Italy

Timetable

Monday, 18 October

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Wednesday, 20 October

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11:30–11:50	**Yoko FUNATO** University of Tokyo, Japan	Orbital Evolution of TNOs in the Galactic Potential
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Thursday, 21 October

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Friday, 22 October

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11:30–11:50	**Jérôme DAQUIN** University of Namur, Belgium	Intermittency phenomena in the semi-major axis of Molniya spacecraft
11:50–12:10	**Mahdi JAFARI NADOUSHAN** K N Toosi University of Technology, Iran	Intermittency phenomena in the semi-major axis of Molniya spacecraft
12:10–12:30	**Tudor VARTOLOMEI** University of Rome Tor Vergata, Italy	Back-tracing space debris using proper elements
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13:30–13:50	**Mauricio MISQUERO** University of Rome Tor Vergata, Italy	Accuracy and efficiency in the propagation of highly eccentric orbits
13:50–14:10	**Ioana-Lucia BOACA** Astronomical Institute of the Romanian Academy, Romania	Trajectory reconstruction of meteors detected by the MOROI network
14:10–14:30	**Simon ANGHEL** Astronomical Institute of the Romanian Academy, Romania	Energy estimation of decimeter scale meteoroids impacting the atmosphere
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20:00–20:10	**Closing**	

List of Abstracts – Talks

Energy estimation of decimeter scale meteoroids impacting the atmosphere

Simon ANGHEL (Astronomical Institute of the Romanian Academy / Faculty of Physics, University of Bucharest / IMCCE, Observatoire de Paris Bucharest Romania)

Co-Authors: Mirel Bîrlan, Dan-Alin Nedelcu

Recent developments of detector sensitivity has allowed us to gather better data from a meteoroid impact into the Earth's atmosphere. These recording tools range from optical and radio antennas, up to detections of meteoroid generated shock using infrasound arrays, and seismic detectors for larger events. When combining multiple methods to study the same event, we derive knowledge of the meteoroid mass, which can be further used to estimate the size-frequency distribution of impacts. In this study we explore several techniques presented in the literature and propose an accurate method of estimating fireball energies around the ton-TNT scale, and as a consequence, better constrain the meteoroid mass.

Chaotic orbit determination: some analytical results for Anosov diffeomorphisms

Giulio BAU' (University of Pisa, Italy)

Co-Authors: Milani Andrea

The classical orbit determination problem (Gauss 1809) becomes difficult, both conceptually and numerically, when applied to a chaotic orbit, if the time span of the observations being used in a least squares fit is much longer than the Lyapounov time. Nevertheless, such problems practically occur, both in Solar System Dynamics (e.g., in long term impact monitoring for NEO) and in Astrodynamics (e.g., in long satellite tours around the giant planets). Spoto and Milani (2016), and Serra, Spoto and Milani (2018) have tried to tackle such a complex problem by starting from the discrete dynamical system defined by the Chirikov standard map: they have provided numerical evidence that the asymptotic behavior (as the observed time span goes to infinity) changes radically whenever dynamical parameters are included in the fit parameters, besides the initial conditions, and pointed out the relationship between these results and a finite time version of the shadowing lemma. In this work we give analytical, rigorous proofs of theorems on Anosov diffeomorphisms on the 2-torus. The main result that we found is that when a dynamical parameter is included in the fit, the uncertainty of the solution for the parameter decreases as in the standard Gaussian statistics (e.g., in the central limit theorem) and not exponentially with the number of observations. The case of the dynamical systems, such as the three-body problem, in which there are chaotic orbits (on invariant hyperbolic sets) and ordered orbits (on invariant KAM tori), plus other less known behaviors, is necessarily more difficult: indeed the numerical evidence points to a less uniform behavior. Still some of the ideas developed in our research on model problems can already be applied to difficult practical problems.

Trajectory reconstruction of meteors detected by the MOROI network

Ioana-Lucia BOACA (Astronomical Institute of the Romanian Academy, Romania)

Co-Authors: Alin Nedelcu, Mirel Birlan, Tudor Boaca, Simon Anghel

We determine the atmospheric trajectory of the multistation events (captured by 3 or more cameras) detected by the Romanian allsky cameras network called MOROI. The falling ellipse for meteorite fragments will be presented, taking into account the influence of various atmospheric conditions.

This work was supported by a grant of the Romanian Ministry of Education and Research, CNCS-UEFISCDI, project number PN-III-P1-1.1-PD-2019-0784, within PNCDI III.

The influence of a very close companion star on the planets stability

Barbara Celi BRAGA CAMARGO (UNESP, Brazil)

Co-Authors: Othon Cabo Winter

In this work we present a study on the limits of the parameters of the secondary star that enable the formation and stability of a planet orbiting the primary star in a system type-S. There are a variety of multiple systems already discovered. In the case of binary systems, 98 have already been confirmed and on these, about 144 planets were detected. It is notorious that the formation of planets in compact binary systems is strongly influenced by stellar parameters, since the protoplanetary disk and the internal particles are exposed to the gravitational influence of the secondary star. Thus, not only gas drag can alter the evolution of immersed protoplanets, but happens in systems with a single star or with distant companions, but the gravitational influence of the disk and the secondary star also influence the formation and stability of planets in the system. To study the influence of the secondary star on the formation of a protoplanet around of the primary star, we explore a range of values for the semi-major axis and mass of the secondary star. From this, we evaluate the evolution of the gas disk and a protoplanet immersed in this disk. In our results we find that for a system where the secondary star is less from 10 au of the primary star the growth of a planet to 3 au using our model is unlikely unless the minor star's mass is approximately $0.1M_{Sun}$. It was also possible to trace to 30 au a limit for the secondary star's influence on the formation of the system's protoplanets. The influence of the protoplanet on the protoplanetary disks is another point that we can study in the future, as gas material is sometimes transported due to the planet, which may interfere in the formation of more planets in the system.

Orbital drift of satellites in a co-orbital formation flying due to Earth's gravity

Josué CARDOSO DOS SANTOS (Aeronautics Institute of Technology, Brazil and Technion - Israel Institute of Technology, Israel)

Co-Authors: Willer Gomes dos Santos, Artur Robson Cutolo

The guidance and control of satellites in formation flying is a key aspect in such a type of space mission. A precise determination of the position of each satellite is essential for the success of these missions since the relative drifts between the satellites can cause the loss of synchronisation in their relative motion. It especially can affect the precision of observations and measurements during experiments, potentially impacting the goals and success of the mission. Thus, this work intends to verify the effects of several terms of the Earth's gravitational field over the relative orbits of a co-orbital formation flying mission. The focus on applications is given to the ITSAT-2 mission, composed of three nanosatellites, and which is an ITA's joint project with NASA, American universities and the Technion.

Chaos identification through the autocorrelation function indicator (ACFI)

Valerio CARRUBA (UNESP, Brazil)

Co-Authors: Safwan Aljbaae, Rita Cassia Domingos, Mariela Huaman, William Barletta

Chaotic motion affecting small bodies in the Solar System can be caused by close encounters or collisions, or by resonance overlapping. Chaotic motion can be detected using approaches that measure the separation rate of trajectories that starts infinitesimally close, or changes in the frequency power spectrum of time series, among others. In this work, we introduce an approach based on the auto-correlation function of time series, the ACF index ($ACFI$). Auto-correlation coefficients measure the correlation of a time-series with a lagged copy of itself. By measuring the fraction of auto-correlation coefficients obtained after a given time-lag that are higher than the 5% null hypothesis threshold, we can determine how the time-series auto-correlates with itself. This allows identifying unpredictable time-series, characterized by low values of $ACFI$. Applications of $ACFI$ to orbital regions affected by both types of chaos show that this method can correctly identify chaotic motion caused by resonance overlapping, but it is mostly blind to close-encounters induced chaos. $ACFI$ could be used in these regions to select the effects of resonance overlapping. Accepted for publication in Celestial Mechanics and Dynamical Astronomy. This is a publication from the MASB (Machine-learning applied to small bodies, https://valeriocarruba.github.io/Site-MASB/) research group.

Closed-form perturbation theory in the restricted three body problem without relegation

Irene CAVALLARI (Universita' di Pisa, Italy)

Co-Authors: Christos Efthymiopoulos

We present a closed-form normalization method suitable for the study of the secular dynamics of small bodies inside the trajectory of Jupiter. The method makes no use of relegation, thus, circumventing all convergence issues related to that technique. The method is based on a convenient use of the book keeping parameter introduced not only in the Lie series organization but also in the Poisson bracket structure employed in all the perturbative steps. In particular, we show how the above scheme leads to a redefinition of the remainder of the normal form at every step of the formal solution of the homological equation. Applications for the computation of proper elements of asteroids in highly eccentric orbits ($e \sim 0.5$) are presented.

Study of the proximity of exoplanets to first-order MMRs

Carolina CHARALAMBOUS (UNamur, Belgium)

Co-Authors: Anne-Sophie Libert, Jean Teyssandier

As of July 2021, more than 4700 exoplanets have been detected. Planetary formation theories and, more specifically, migration models predict that planets are captured in mean motion resonance during the disc phase. However, from the statistical analysis of the detected population, a peak is observed near the nominal position of the mean motion resonances and not precisely in their nominal locations. Several works seek to understand the observed offsets through disc effects, since the disc parameters (which influence the migration speed and final system architectures) still have significant uncertainties. Here, we propose a different approach, by also considering different masses and initial separations for the planets during the protoplanetary disc phase. The dynamical study is carried out for several first-order resonances and for planets in the low-mass regime, which undergo type-I classical migration. We show how the planetary parameters can yield significant offsets from exact resonance and compare our results with the observational data.

Asynchronous and chaotic rotation for compact planetary systems

Alexandre CORREIA (University of Coimbra, Portugal)

Co-Authors: Jean-Baptiste Delisle

We study the spin evolution of close-in planets in compact multi-planetary systems. The rotation period of these planets is often assumed to be synchronous with the orbital period due to tidal dissipation. Here we show that planet-planet perturbations can drive the spin of these planets into non-synchronous or even chaotic states. These asynchronous configurations are possible even for nearly circular orbits and will impact the habitability of these planets. We also present a very simple method to probe the spin dynamics from the orbital perturbations.

An analytical model for tidal evolution in co-orbital systems

Jérémy COUTURIER (IMCCE, Observatoire de Paris, France)

Co-Authors: Philippe Robutel, Alexandre C. M. Correia

Close-in co-orbital planets (in a 1:1 mean-motion resonance) can experience strong tidal interactions with the central star. Here, we develop an analytical model adapted to the study of the tidal evolution of those systems. We use a Hamiltonian version of the constant time-lag tidal model, which extends the Hamiltonian formalism developed for the point-mass case. We show that co-orbital systems undergoing tidal dissipation favour either the Lagrange or the anti-Lagrange configurations, depending on the system parameters. However, for all range of parameters and initial conditions, both configurations become unstable, although the timescale for the destruction of the system can be larger than the lifetime of the star. We provide an easy-to-use criterion to determine whether an already known close-in exoplanet may have an undetected co-orbital companion.

Intermittency phenomena in the semi-major axis of Molniya spacecraft

Jérôme DAQUIN (University of Namur, Belgium)

Co-Authors: E.M.,Alessi, J. O'Leary, A.,Lemaître, A.,Buzzoni.

We describe the phase space structures related to the semi-major axis of Molniya-like artificial satellites subject to tesseral and lunisolar resonances. In particular, we dissect the indirect interplay of the critical inclination resonance onthe semi-geosynchronous resonance using a hierarchy of more realistic dynamical systems, thus discussing the dynamics beyond the integrable approximation. By introducing secular (averaged over fast angles of systems) models, we numerically demarcate the hyperbolic structures organising the phase space. Based on the publicly available two-line elements space orbital data, we identify two satellites, namely M1-69 and M1-87, displaying fingerprints consistent with the dynamics associated to the hyperbolic set. The computations of the associated dynamical maps highlight that the spacecraft are trapped within the hyperbolic tangle. The research therefore reports evidence of actual artificial satellites in the near-Earth environment whose dynamics are ruled by hyperbolic manifolds and resonant mechanisms.

Satellites' orbital stability through normal forms

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Normal form stability estimates are a basic tool of Celestial Mechanics for characterizing the long-term stability of the orbits of natural and artificial bodies. Using high-order normal form constructions, different estimates for the orbital stability of point-mass satellites orbiting around the Earth are provided. In particular, in the framework of the J_2 problem, a normal form construction eliminating the fast angle in the corresponding Hamiltonian allows to prove the long term stability for the semimajor axis; on the other hand, by considering a secular Hamiltonian model including also lunisolar perturbations, it is possible to demonstrate the stability of the eccentricity and inclination for the "geolunisolar" model after a suitable reduction of the Hamiltonian to the Laplace plane. With a view to the Nekhoroshev's theorem on the exponential stability of the orbits, the convexity and steepness properties of the two models are investigated, leading to the conclusion that the J_2 model satisfies a "three-jet" non-degeneracy condition, while the geolunisolar model is quasi-convex. Furthermore, the stability of the orbits in the secular geolunisolar model is studied by means of a non-resonant version of the Nekhoroshev's theorem: the domain of applicability of the theorem is physically relevant for relatively low altitudes, say up to 20 000 km. Within such domain, stability times of the order of several thousands years are obtained for a large set of inclinations and eccentricities for altitudes of the order of 11 000 km. As the altitude increases and approaches the value of 20 000 km, the stability time decreases to a few hundred years and the theorem holds for a small set of eccentricities and inclinations. In this framework, the presence of resonances is a crucial cause of stability loss.

Constraints on the dynamical evolution of the asteroid belt

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Uncertainties in our current understanding of the dynamical evolution of the asteroid belt include: (1) the timescales of the asteroid loss mechanisms and (2) the total number of major asteroid families, including ghost families. We argue that there is a need for more observational constraints and that some insight into the dynamical evolution of the belt is gained from (1) an analysis of the observed variations of the mean orbital eccentricities and inclinations with asteroid size, and (2) an analysis of the size-frequency distributions (SFDs) of the small asteroids in the major families (Dermott et al., 2018). Small asteroids are lost from the belt by collisional and rotational disruption, and by Yarkovsky radiation forces that transport the asteroids to mean motion and secular resonances. We show here that in the inner belt the loss of asteroids from the ν_6 secular resonance and the 3:1 Jovian mean motion resonance may account for the observation that small asteroids have been preferentially lost from high inclination orbits. We also show (1) that the timescales of the small asteroid loss mechanisms are constrained by the observed distributions of their mean orbital inclinations, and (2) that the SFDs of the small asteroids in the major families can be used to estimate the family ages. Finally, the distributions of the sizes, eccentricities and inclinations of the thousands of small asteroids trapped in the 1:2 mean motion with Mars are determined by the competing actions of the known gravitational forces and the unknown Yarkovsky forces. An analysis of these distributions could lead to a separate calibration of the Yarkovsky forces and their dependence on asteroid size and type.

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Chaos and discreteness effects in gravitational N-body problems

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We revisit the rôle of discreteness effects and chaos in the dynamics of self-gravitating systems by means of N-body simulations with spherically symmetric isotropic or anisotropic initial conditions. We study the Lyapunov exponents of single particles in a frozen N-body potential as well as the orbits of the system in the full $6N$-dimensional phase space. Moreover we also investigate the intermediate case where a test particle moves in the field generated by N non-interacting particles, which in turn move in a static smooth potential. For the full N-body system we observe that the dependence on N of the largest Lyapunov exponent is consistent with the $N^{-1/3}$ trend predicted by Gurzdayan & Savidy (1984,1986) with differential geometry arguments and associated to a "collective relaxation" channel die to discreteness effects. For single tracer particle orbits we also study the dependence on their Lyapunov spectra on angular momentum and on the energy. Our results confirm the expectation that orbital properties of single orbits in finite-N systems approach those of orbits in smooth potentials in the continuum limit $N \to \infty$. However individual orbits in frozen models and active self-consistent models have different largest Lyapunov exponents and their N-dependence is non-trivial, so that the use of frozen N-body potentials to gain information on large-N systems or on the continuum limit may be misleading in certain cases.

Analysis of Euler integral in the three-body problem

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We present a numerical study of the three–body problem in the planar case through the analysis of the variation of the Euler integral. We reduce the Hamiltonian from 3 to 2 degrees of freedom and discuss in detail the 3–dimensional phase space of two concrete orbits. In particular, we study the variation of the Euler integral function around an orbit which spends much time closely to the saddle point of a manifold where the Euler integral is constant. Moreover we show the existence of chaos, via symbolic dynamics and using covering relations, in the region close to the saddle point.

Role of inner giant planets in the secular dynamics of plutinos: a $1997\,QJ_4$, $2002\,VR_{128}$ case study

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A significant fraction of Kuiper belt objects exists in mean-motion resonances (MMR) with Neptune. Plutinos (MMR $2:3$) being one of the most noteworthy populations. We study the secular evolution of the orbits of plutinos, utilizing the approach for the analysis of the resonant dynamics developed by Wisdom (1985). Using the plutinos $1997QJ_4$ and $2002VR_{128}$ as a principal example we show that although the influence of inner giant planets - Jupiter, Saturn, and Uranus - is very small in magnitude, it still affects secular evolution in a substantial way. By creating Lidov-Kozai resonance islands it puts those objects on secular trajectories with the librating argument of pericenter ω, which otherwise would circulate. The reason for such sensitivity to additional arguments is the fact that the bottom of the effective potential function defined by Neptune's gravity is very flat. Therefore objects deep enough inside the MMR, as the ones being considered, are in a position to be easily influenced by very slight changes in the potential. In contrast to the non-MMR case, where the influence of the inner giant planets only noticeable for objects with the eccentricity large enough to cross the planets' orbits (Thomas and Morbidelli, 1996), inside MMR it happens at relatively low eccentricities ($e \sim 0.2$). It is also important, that perturbations by all three inner giant planets turn out to be very close to one another in terms of magnitude. This means that the dynamics of certain KBOs can be adequately described only by no less than 6-body models and therefore calls for the revision of the conclusions of Wan and Huang (2007).

Closed-form perturbation theory for small bodies in the solar system

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Closed form perturbation theory aims at computing a secular model for nearly-keplerian gravitational dynamics without expansions in the orbital eccentricity. Starting from Brouwer and Deprit's works, algorithms for closed-form expansions have been so far discussed mostly in the framework of the main problem of satellite dynamics (J2 + tidal perturbations). In the case of the restricted three body problem, instead, closed form algorithms can be arrived at using a relegation technique to deal with non-inertial terms in the rotating frame of motion. We will review some main advances in closed-form perturbation theory in the last two decades which have resulted in a substantial improvement of the possibility to represent semi-analytically highly eccentric orbits of small bodies (e.g. satellites or asteroids). We will explore the usefulness in such techniques also in the determination of approximate integrals of motion (mean or proper elements). We will finally refer to the intricacies of the relegation approach and outline an alternative approach allowing to overcome these intricacies.

Proper elements for resonant planet-crossing orbits

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The gravitational N-body problem is non-integrable for $N > 2$, in the sense that there are not enough integrals of motion necessary to find a complete set of action-angle coordinates. In the context of our Solar System, proper elements are quasi-integrals of motion, meaning that they vary much more slowly than the osculating elements and therefore they can be assumed constant over a certain period of time. These quantities are important because they permit the description of the long-term motion with only a few parameters. Proper elements were computed first for main-belt asteroids (MBAs), and they are a fundamental tool for studying the dynamical structure of the main belt, for the identification and the age determination of asteroid families. Differently from MBAs, the orbit of near-Earth objects (NEOs) can cross that of a planet, and this prevents to use the techniques used for the computation of proper elements of MBAs because a singularity appears in the secular Hamiltonian. To this purpose, Gronchi and Milani 2001 introduced a semi-analytical technique for the computation of proper elements of non-resonant planet-crossing asteroids. In this talk, we extend the work by Gronchi and Milani 2001 to the case of resonant planet-crossing asteroids. Our technique is based on a semi-secular Hamiltonian model, obtained by averaging over all the fast angles, that is used to propagate the long-term dynamics. Then, we compute the proper elements by removing short periodic perturbations present in the solution, performed by using a digital filtering technique. Finally, we present some comparisons with the results obtained by the non-resonant model by Gronchi and Milani 2001, showing that taking into account the resonance is needed for the computation of appropriate proper elements.

Tides and Exoplanets

Sylvio FERRAZ-MELLO (University of São Paulo, Brazil)

Review of the basic equations used in the study of the tidal variations of the orbital and rotational elements of a systems formed by one star and one close-in planet as given by the creep tide and Darwin’s constant time lag CTL) theories Discussion of the variations of the stellar rotation due to the leakage of angular momentum associated with the activity of the star (stellar winds). Review and discussion of the determinations of the relaxation factors of stars and hot Jupiters from actual observations of orbital decay, stellar rotation and age, etc..

Evolution of INPOP planetary ephemerides and Bepi-Colombo simulations

Agnès FIENGA (Observatoire de la Côte d'Azur, France)

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We used the new INPOP planetary ephemerides, INPOP21a, including Sun induced Lense-Thirring effect and an updated modeling of Kuiper Belt Object acceleration, fitted over the latest juno perojove data to infer new constraints on PPN parameters β and γ and $\dot{G}/G$. We also consider range simulations of the Bepi-Colombo MORE experiment. A new approach based on the analysis of the χ^2 distribution is used for constraining the PPN parameters and an analysis of the impact of the Sun core rotation hypothesis is presented.

Orbital Evolution of TNOs in the Galactic Potential

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Co-Authors: Junichiro Makino

Recently many distant Trans Neptunian Objects (TNOs) have been discovered. The range of their semi-major axes is in 100 AU to near 1000 AU. Owing to the increasing number of the distant TNOs, the statistical properties of their orbits become clear. For an example, one of their characteristic properties is reported by Batygin and Brown (2016, AJ, 151). They reported a clustering of orbital elements of distant and eccentric TNOs. They argue that the clustering is explained by the 9th planet. But the 9th planet has not been discovered yet. There may be another reason for the clustering. The Galaxy is one of candidates. The Solar System is in the Galaxy. The Galactic gravitational field may affect the dynamical evolution of the Solar System. Usually the effect of the Galactic Potential is considered to drive the Lidov-Kozai mechanism to the orbits of the TNOs and Oort Cloud Objects. Their orbital evolution is predicted by using the orbit averaging technique. This procedure is equivalent to an simplification that the Galactic Field is a constant disk. In the present study, we numerically investigated the long term orbital evolution of the TNOs and the Oort Cloud Objects in the axisymmetric Galactic Field. We adopted the Miyamoto-Nagai Potential as the Galactic Potential. In our simulation, an Outer Solar Object is modeled as a test particle. It is a restricted three-body problem. We integrate its motion which is driven by the force from the Sun and that from the Galaxy. We compared our numerical results with the prediction from the Lidov-Kozai Mechanism (analytical prediction). We found that there is difference between numerical results and the prediction from the Lidov-Kozai mechanism when an object has a large semi-major axis from the Sun. The difference seems to be caused by a treatment of the Galactic field when using the analytical prediction. Our result suggests that a precise numerical integration is required in order to investigate the orbital evolution of the Outer Solar Objects.

Semianalytical model for planetary resonances

Tabare GALLARDO (Facultad de Ciencias, Udelar, Uruguay)

Co-Authors: Cristian Beauge, Cristian Giuppone

We present a semianalytical model that describes the resonance strength, width in au, location and stability of fixed points, and periods of small-amplitude librations [1]. The model is valid for any two gravitationally interacting massive bodies, and is thus applicable to planets around single or binary stars with arbitrary eccentricities and inclinations. The model assumes some simplifications that limit its validity in some specific cases, mainly for first order resonances at very small eccentricities. In spite of these simplifications the model is very helpful to understand the properties of the planetary resonances and its potential is revealed at high eccentricities and/or high inclinations. In particular, it is very simple to generate atlas of resonances for a given planetary system, that means, the domains of the resonances in the plane (a,e) or (a,i).

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On the dynamics of the planetary system of Kepler-90

Daniel Martin GASLAC GALLARDO (Sao Paulo State University UNESP, Brazil)

Co-Authors: Silvia Giuliatti Winter, Othon Winter

The planetary system Kepler-90 has eight planets b, c, i, d, e, f, g, and h, in increasing distance from the star. Planets g and h are similar to gas giants, while planets d, e, and f are similar to super-Earths. They are similar to our Solar System, small planets are closer and the larger ones are distant from the star, although the outer planet has an orbital distance equals to 1AU. Through frequency analysis and long-term evolution, we analyse their stability for a sample of parameters of the planets, such as their masses, semi-major axes and eccentricities. We performed simulations numerical to analyze three different intervals of eccentricity: the first interval from 0 to 0.001, the second interval from 0.001 to 0.01 and the third interval from 0.01 to 0.1. The values of eccentricity, argument of pericentre, longitude of the ascending node, and mean longitude were randomly chosen in each eccentricity interval. Our results showed that the planets which eccentricities belong to the first and second intervals are stable, while most of the planets with large eccentricity, 0.01 to 0.1, are ejected from the system. The variation of the eccentricity of the planets in the two first intervals indicate that the planet h is dominant in the nominal systems being important for the stability of the system Kepler-90. The first interval of eccentricity has two nominal systems where the MMRs appear among the planets, b and c are in 4:5 MMR, and the planets g and h are near 2:3 MMR, corroborating the results obtained by Granados et al (2018). A study of a sample of particles located in this system was performed through the frequency map analysis. It was identified four stable regions between the orbits of the planets c-i, i-d, d-e, and beyond the orbit of planet h that were identified as regions 1, 2, 3, and region 4, respectively. The islands of resonance are identified with the planet i and planet h. Numerical simulations showed that some test particles are close to 2:3, 7:8, and 9:10 MMR with the planet i, and are close to 1:2, 3:4, and 3:8 MMR with planet h.

Stratified Rheological Models For Icy Worlds With Liquid Layers

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Co-Authors: Gwenaël Boué, Clodoaldo Ragazzo

We work on proposing and analyzing multilayered rheological models for the dynamics of extended deformable bodies evolving under the influence of gravitational forces. A typical case is that of a satellite with icy crusts, subsurface oceans, molten mantles and solid cores orbiting a giant planet in either our solar system or in exoplanetary systems. Important questions that we intend to address with the proposed models are, among others, whether the tidal dissipation in the liquid layer caused by the forced librations and the obliquity of the orbit of the satellites is large enough compared with the dissipation in the solid layers and the radiogenic heating and whether a possible mechanical decoupling between the dynamics of the solid and liquid layers can lead to anomalous spin states in stratified bodies, as it seems to be the case of Titan.

The fate of Pallene's diffuse ring

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Co-Authors: Gustavo Madeira, Marco Muñoz-Gutiérrez, Joseph A'Hearn, Paula Granados Contreras

Pallene is one of the small satellites of Saturn discovered by Cassini mission. Along with these small satellites, Aegaeon, Anthe and Methone, Pallene also has a ring composed of large and small particles. In this work we present a study on the dynamical evolution of mu m-sized particles forming the ring, considering gravitational and non-gravitational forces (solar radiation force, electromagnetic force and plasma drag). Non-gravitational forces are responsible for particles' vertical excursions and outward migration related to the satellite's orbit. By estimating Pallene's mass production rate, we found that it could be responsible for keeping the ring in steady-state, only if it is mainly composed of large particles. If the ring is mainly composed of particles with a few micrometres in radius, it will spread out, radially and vertically, until it finally disappears.

On the geometry of two Keplerian orbits with a common focus

Giovanni Federico GRONCHI (University of Pisa, Italy)

The computation of the distance d_{min} between two Keplerian trajectories A, A' with a common focus, also called MOID or orbit distance, is relevant for different purposes in Celestial Mechanics. Several authors introduced efficient methods to compute d_{min}. Small values of d_{min} are relevant for the assessment of the hazard of near-Earth asteroids (NEAs) with the Earth or for the detection of conjunctions between satellites of the Earth. On the other hand, we may wish to check whether d_{min} can assume large values, because in this case it is more difficult to observe a small celestial body moving along A from a point following A' and the probability to miss its discovery is higher. Therefore, understanding the occurrence of large values of d_{min} is relevant to detect observational biases in the known population of faint NEAs. In this talk we shall review some results concerning the computation of d_{min} and of its uncertainty, making an analysis of the singularities of the orbit distance. Moreover, we shall state upper bounds for its value (which are optimal if A' is circular) as functions of selected pairs of orbital elements of A, when the other elements can vary. We also present some questions that are still open in this context.

On the Formation of Multi-planet Resonant Chains and the Diversity of Their Resonances

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The discovery of multi-planet resonant chains such as those in TRAPPIST-1 and Kepler-90, where adjacent planets are in different resonances, has raised questions on the formation of these systems. It is accepted that these systems formed through the combination of migration and resonance-capture where migrating planets capture each other in resonances. That, however, raises an issue because migrating planets tend to capture each other in the same resonance. It has been suggested that tidal forces are the reason that resonant-chain planets are in different commensurabilities. The latter motivated us to examine the validity of this statement. We carried out extensive simulations of planet migration, and determined the probability of capture for different resonances. Results demonstrated that migrating planets can in fact be captured in different resonances confirming that the diversity of resonances observed in resonant chains is the natural consequence of the resonance capture mechanism and does not require a secondary process. Results also showed that the probability of capture (and, therefore, the final commensurabilities) is highly depended on the characteristics of the systems, especially the planets' mass-ratio and migration speed. Finally, our simulations indicated that capture in a resonance never occurs at the resonance's exact commensurability and there is always some deviation. The extent of this deviation also depends on the mass-ratio and orbital characteristics of the planets and the mechanism through which migrating planets lose energy. This also confirms that unlike previous studies, no post-capture mechanism is needed to explain the deviation from exact resonances observed in Kepler (and RV) planet pairs. We present the details of our study and discuss their implications for the formation and orbital architecture of resonant, multi-planet systems.

Circumplanetary Dust: Spokes in Saturn’s B Ring

Douglas P. HAMILTON (University of Maryland, USA)

The origin of the ghostly spokes that appear in Saturn’s main rings is an enduring mystery that has eluded planetary scientists for over forty years. Spokes are dark and roughly radial features that are usually seen on the lit side of Saturn’s rings. They are thought to be composed of micron sized dusty material. Spokes are seen to begin as purely radial features which gradually become triangular in shape as particles with different Kepler periods shear apart. For the first few hours, however, the trailing edge of a triangular spoke typically remains vertical, indicating active generation of new dusty material along that edge. We are developing a physical model for the origin of spokes that begins with an interplanetary impact onto a large ring particle that creates a swarm of tiny dust grains. Submicron grains are strongly affected by Saturn’s magnetic field which causes them to gyrate around field lines just as ions do. After being created, these particles leave the ring plane, are accelerated to high a relative speed by the planet’s magnetic field, and then return to a different radial location. When they strike ring particles at high speed, they generate further submicron grains that can continue the process. In this way, an initially localized disturbance is propagated radially in the rings. With each submicron impact, some visible micron-sized material is also generated which becomes visible as spokes. We identify the length of time that an edge is active with the continued generation of submicron particles. To test our Collisional Cascade model, we are currently analyzing 10,000 Cassini images of Saturn’s rings. We are developing a pipeline to identify and catalog spokes in each of the images and ultimately to critically test our theoretical model.

Chaotic diffusion of the fundamental frequencies in the Solar System

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The long-term variations of the orbit of the Earth govern the insolation on its surface and hence its climate. The use of the astronomical signal, whose imprint has been recovered in the geological records, has revolutionized the determination of the geological time scales(e.g. Gradstein and Ogg 2020). However, the orbital variations beyond 60 Myr cannot be reliably predicted because of the chaotic dynamics of the planetary orbits in the Solar System (Laskar 1989). Taking into account this dynamical uncertainty is necessary for a complete astronomical calibration of geological records. Our work addresses this problem with a statistical analysis of 120 000 orbital solutions of the secular model of the Solar System ranging from 500 Myr to 5 Gyr. We obtain the marginal probability density functions of the fundamental secular frequencies using kernel density estimation. The uncertainty of the density estimation is also obtained here in the form of confidence intervals determined by the moving block bootstrap method. The results of the secular model are shown to be in good agreement with those of the direct integrations of a comprehensive model of the Solar System. Application of our work is illustrated on two geological data: the Newark-Hartford records and the Libsack core.

Low-Energy Transfers from Cislunar to Heliocentric Space to Support Sun-Earth L1 and L2 Space Telescopes and Observatories

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Transit between cislunar space and the regions near the Sun-Earth L1 and L2 libration points offer many interesting opportunities. For example, the planned NASA Gateway facility in cislunar space is expected to serve as a stepping stone for missions to various other destinations in the solar system. Many space telescopes, including NASA's James Webb Space Telescope (JWST) and the Nancy Roman Space Telescope (NRST), as well as ESA's Euclid and PLATO telescopes, are planned to launch into Sun-Earth L2 libration point orbits in the next decade. Additionally, space observatory missions are also scheduled for the 2020s toward orbits associated with the Sun-Earth L1 libration point. All may eventually require servicing capabilities. The Earth-Moon-Sun transit dynamics are complex, yet natural dynamical pathways are sought for low-energy transfers originating in cislunar space for delivery to and from the heliocentric orbits near the Sun-Earth L1 and L2 regions.

Perturbation theory for non-stationary problems of celestial mechanics and its applications in dynamics of gravity systems with variable masses

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The classical two-body problem in celestial mechanics is important, while the masses of the bodies are constant, and in gravitationally coupled systems, the motion is Keplerian. Perturbation theory based on solving the classical two-body problem in celestial mechanics is well developed. This theory has been successfully applied in solving a number of problems in the dynamics of gravitating systems. If we had a fairly simple general solution of the two-body problem with variable masses, then we could construct a similar perturbation theory. However, now, such solution of the problem with variable masses is unknown. In this regard, a perturbation theory is proposed for non-stationary gravitating systems based on aperiodic motion over a quasiconic section. Aperiodic motion over a quasiconic section represents a class of unperturbed motion with one arbitrary dimensionless function. In particular, the specific choice of this arbitrary function determines the well-known Meshchersky-Vinti solution of the two-body problem with variable masses. If this arbitrary function is constant and equal to unity, then we get an aperiodic motion over the conical section represented by Omarov and Hadjidemetriou. In work based on an aperiodic motion of quasi-conical cross-section developed perturbation theory, including the canonical perturbation theory. Analogs of Keplerian osculating elements, analogs of the canonical elements of Jacobi, Delaunay and Poincare are introduced. Various new forms of the perturbed motion equation are obtained. In particular, when the body masses are constant introduced osculating elements will turn into classical Keplerian elements. As applications, the equations of the perturbed motion of the two-planet three-body problem with isotropically varying masses and the restricted three-body problem with non-isotropically varying masses in the presence of reactive forces are given.

Migration of planetesimals to forming terrestrial planets

Sergei IPATOV (Vernadsky Institute of Geochemistry and Analytical Chemistry of Russian Academy of Sciences, Moscow,Russia)

Migration of planetesimals under the gravitational influence of planets or their embryos was calculated. In some calculations all planets were considered, in other calculations Uranus and Neptune were excluded. Masses of the embryos of the terrestrial planets in some calculations equaled to 0.1 or 0.3 of masses of the planets. Based on the arrays of orbital elements of migrated planetesimals, for several considered time intervals, the probabilities of collisions of planetesimals with planets, the Moon, and with their embryos were calculated. In each calculation, initial semi-major axes of planetesimals varied inside an interval with a width equaled to 0.2, 2.5, and 0.1 AU for axes between 0.3 and 1.5 AU [1], between 2.5 and 42.5 AU [2], and between 3 and 5 AU, respectively. Initial eccentricities of planetesimals were 0.05 or 0.3, and initial inclinations in radians were half of initial eccentricities. Analysis of the probabilities showed that the Earth and Venus could acquire more than a half of their masses in 5 Myr. Upper layers of the Earth and Venus accumulated similar planetesimals. It was supposed that embryos of Mars and Mercury with masses that were several times smaller than those of these planets could be formed as a result of the compression of rarefied condensations. The features of the formation of the terrestrial planets can be explained with a relatively smooth decrease of the semi-major axis of Jupiter caused by its ejection of planetesimals into hyperbolic orbits. The ratio of probabilities of planetesimals collided with the embryos of the Earth and Moon was less than the ratio of masses of the embryos. While considering thousands of planetesimals, the probability of a collision of a planetesimal with the Earth for the region between 5 and 10 AU from the Sun could exceed $2 \cdot 10^{-6}$ by at least a factor of several. On average, for the region between 20 and 40 AU the probability could be about 10^{-6}. The mass of the material delivered from beyond the orbit of Jupiter to a planet to the mass of the planet for Mars was about two times greater than that for the Earth, and such ratios for Mercury and Venus were a little greater than that for the Earth. The work was carried out as a part of the state assignments of the Vernadsky Institute of RAS.

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A cartographic study of spin-orbit coupling in binary asteroids

Mahdi JAFARI NADOUSHAN (K N Toosi University of Technology, Iran)

In the spin-orbit resonances, we assume that the orbit of secondary asteroid around primary is invariant, which is the reasonable assumption at first glance. Owing to the irregularity of asteroids' geometry and their effect on the mutual orbit, this assumption should be revised. Therefore, we focus on a binary asteroid with a spherical primary and a secondary with irregular shapes. When the geometry of the asteroids is not sphere, the gravitational interaction will be important and we should simultaneously consider the interaction of orbit and spin. We generate FLI map for both spin-orbit resonance and spin-orbit coupling problems and investigate considering orbit alternation on the phase space.

Evolution and stability of Laplace-like resonances under tidal dissipation

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The Laplace resonance is a configuration that involves the commensurability between the mean motions of three small bodies revolving around a massive central one. This resonance was first observed in the case of the three inner Galilean satellites, Io, Europa and Ganymede. In this work the Laplace resonance is generalised by considering a system of three satellites orbiting a planet that are involved in mean motion resonances. These Laplace-like resonances are classified in three categories: first-order ($2:1\,\&\,2:1$, $3:2\,\&\,3:2$, $2:1\,\&\,3:2$), second-order ($3:1\,\&\,3:1$) and mixed-order resonances ($2:1\,\&\,3:1$). In order to study the dynamics of the system we implement a model that includes the gravitational interaction with the central body, the mutual gravitational interactions of the satllites, the effects due to the oblateness of the central body and the secular interaction of a fourth satellite and a distant star, such as the Sun in the case of the Laplace resonance. Along with these contributions we include the tidal interaction between the central body and the innermost satellites. We study the survival of the Laplace-like resonances and the evolution of the orbital elements of the satellites under the tidal effects. Moreover, we investigate the dependence of the dynamical evolution on the oblateness coefficient and the inital values of the semi-major axis and the eccentricity of the innermost satellite. Furthermore, we study the possibility of capture into resonance of the fourth satellite.

The cycle slip problem in the computation of synthetic secular frequencies

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Current procedure to compute synthetic secular frequencies for the high inclination asteroids suffers from the problem with correct counting of the number of full revolutions of secular angles over the integration time spans much longer than the revolution periods. The "cycle slips", i.e. backward jumps by 360 degrees, give rise to large errors in the linear fit to the continuous time series of the longitudes of perihelia when the orbit is passing very close to the origin in projection to the plane of equinoctial elements (k,h) (that is, when the eccentricity is very small). The slips are found to be caused by the looping around the origin due to the unfiltered low amplitude secular perturbations, and not, as previously thought, to the small eccentricity that makes longitude of perihelion undetermined. To eliminate the cycle slips for nearly all the affected asteroids, we introduced a double filtering of the longitude of perihelion time series, consisting in an additional smoothing of the already filtered output of numerical integration of orbits. With appropriately selected filters this procedure results in almost complete removal of the slips and in the significantly more accurate frequencies.

Orbital Architecture of Self-Organized Planetary Systems

Eiichiro KOKUBO (National Astronomical Observatory of Japan, Japan)

In the standard model of terrestrial planet formation in the solar system, terrestrial planets are spontaneously formed by giant impacts of protoplanets or planetary embryos after the dispersal of protoplanetary disk gas. A similar model is also proposed for the formation of close-in super-Earths discovered by the Kepler transit survey. In this giant impact stage protoplanets gravitationally perturb and collide with each other to complete planets. We investigate the formation of planetary systems from protoplanet systems and their resultant orbital architecture using *N*-body simulations. The goal of this study is to obtain the basic scaling laws for the orbital architecture of planetary systems formed by gravitational scattering and collision among protoplanets, in other words, the final state of accretionary evolution of self-gravitating bodies. We systematically change the system parameters of initial protoplanet systems such as the total mass, mean semimajor axis and angular momentum deficit and investigate their effects on final planetary systems. We find that the orbital architecture can be scaled by the Hill radius of planets and the ratio of the physical radius to the Hill radius. The mean eccentricity increases with the mean orbital separation of adjacent pairs and the final state is determined by the physical to Hill radius ratio.

Tori and Manifolds of Jupiter-Europa and Jupiter-Ganymede Resonances in the Planar Concentric Circular Restricted 4-Body Problem

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Many unstable periodic orbits of the planar circular restricted 3-body problem (PCRTBP) persist as 2D unstable invariant tori when a periodic forcing is added to the equations of motion. Among these persistent PCRTBP orbits are the unstable resonant periodic orbits, which have stable and unstable manifolds that are known to be useful for mission design in multi-moon systems. In this study, we compute the tori corresponding to some Jupiter-Europa and Jupiter-Ganymede resonances in a planar concentric circular restricted 4-body problem (CCR4BP) model of the Jupiter-Europa-Ganymede system. We then develop representations of their manifolds as well, globalizing them and visualizing the results. We discuss the potential for using these manifolds for transfers between Jupiter-Ganymede and Jupiter-Europa resonant quasi-periodic orbits in the CCR4BP.

Probabilistic evolution of pairs of trans-Neptunian objects in close orbits

Eduard KUZNETSOV (Ural Federal University, Russia)

Co-Authors: E.D. Kuznetsov, O.M. Al-Shiblawi, V.D. Gusev

We perform a search for statistically significant pairs of dynamically correlated objects through those with a semimajor axis greater than 30 au, applying a novel technique that uses Kholshevnikov metrics in the space of Keplerian orbits. The metric ϱ_2 defines the distance between two orbits in the five-dimensional space of Keplerian orbits. The metric ϱ_5 defines the distance in the three-dimensional factor-space of the positional elements. It gives the minimum distance between the orbits among all possible positions of the nodes and the pericenters. If the metrics ϱ_2 and ϱ_5 are small (for TNO, one can limit ourselves to 0.07 au$^{1/2}$) and have similar values (e.g., $\varrho_2\breve{}\varrho_5 < 0.015$ au$^{1/2}$), then such a pair of TNOs we considered as a candidate for young pair. We have used the orbital elements from the AstDyS database. We found 26 pairs with metric $\varrho_2 < 0.07$ au$^{1/2}$ (e.g., 1999 HV11 – 2015 VF172), 22 pairs in which one of the components is binary, for metric $\varrho_2 < 0.12$ au$^{1/2}$ (e.g., 2001 OG109 – 2005 GD187), and 11 pairs of binary trans-Neptunian objects with metric $\varrho_2 < 0.3$ au$^{1/2}$ (e.g., 2003 QY90 – 2005 CE81). All pairs belong to cold classical Kuiper Belt Objects. We have studied the probabilistic evolution of two pairs: 1999 HV11 – 2015 VF172 and 2003 QL91 – 2015 VA173. To estimate the age of the pairs, we considered 1000 clones, each TNO in pair. We took covariance matrix values and element errors from the AstDyS database. Based on this data, 1000 clones with a $\pm 3\sigma$ dispersion were generated for each nominal orbit. We have performed numerical integrations of the orbits of TNOs in pairs backward in a span of 10 Myr with the code Orbit9. We searched for low relative-velocity close encounters between TNOs in pairs. The work was supported by the Ministry of Science and Higher Education of the Russian Federation via the State Assignment Project FEUZ-2020-0038.

The AstroGeo project. Retrieving the solar system orbital history through geological data.

Jacques LASKAR (Observatoire de Paris, France)

Due to chaotic behaviour of the orbital motion of the planets in the solar system, it is not possible to predict the planetary motion beyond their horizon of predictability of about 60 Myr (Laskar, 1989, 1990, Laskar et al, 2011). This has special importance for geological studies and the construction of geological time scales. Indeed, according to Milankovitch's theory (1941), part of the large climatic changes of the past is due to the variations of the insolation on the surface of the Earth resulting from the deformation of its orbit resulting from the gravitational disturbances of the other planets. These variations can be extracted from the stratigraphic records accumulated over several Myr. The correlation between the geological data and the calculations of celestial mechanics is now sufficiently established so that the geological time scales of the most recent periods are constructed using the astronomical solutions (Laskar et al, 2004). However, extending this work beyond 60 Myr is difficult because of the chaotic nature of the movement of the planets. The AstroGeo project aims to overcome this predictability horizon, imposed by the laws of gravitation. This will be achieved by considering statistical methods and by using ancient geological data as an additional constraint in obtaining astronomical solutions. The feasibility of such approach was demonstrated by the recent analysis of the Newark sedimentary data record showing the possibility to recover the state of the solar system 200 Ma ago beyond the horizon of predictibility imposed by the laws of Celestial Mechanics (Olsen et al., 2019). AstroGeo will more generally aim to bring astronomical solutions to a next level of accuracy and time validity.

Multiple bifurcations around 433 Eros using Harmonic Balance Method

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The objective of this paper is to carry out orbital propagation around asteroid 433 Eros. Specifically, we propose to exploit a frequency-domain method, the harmonic balance method, as an efficient alternative to the usual time integration. The stability and bifurcations of the periodic orbits are also assessed thanks to the Floquet exponents. Numerous periodic orbits are found with various periods and shapes. Different bifurcations, including period doubling, tangent, real saddle and Neimark-Sacker bifurcations, are encountered during the continuation process. Resonance phenomena are highlighted as well.

Analytic Theory for Secular Lunisolar Resonances

Edoardo LEGNARO (Academy of Athens, Italy)

Co-Authors: Christos Efthymiopoulos

Inclination dependent lunisolar resonances occur whenever there is a commensurability relationship between the argument of perigee and the longitude of the nodes of a space debris and the perturbing bodies. They shape the dynamics of a MEO object (navigation satellite or space debris) over secular timescales (i. e. several decades). Exploiting such resonances has also been proposed as an efficient and cost effective strategy for the End-of-Life (EoL) disposal of satellites. This is based on the eccentricity growth along the hyperbolic directions of the phase space, leading to fast re-entry. Our approach improves on the heuristic estimates of previous works. by providing a precise analytical calculation of the borders of the separatrices of the resonances. These results are then compared with a numerical cartography obtained by the FLI. We find that our analytical approach provides an excellent approximation for a wide range of values of the semi-major axis. However, this picture is disturbed when major crossings take place between the inclination-only dependent resonance considered and one of the two resonances $\Omega - \Omega_L$ and $2\Omega - \Omega_L$ that contain the precession of the lunar node. In particular, as these two resonances sweep the phase space (for increasing values of a) they cause large chaotic domains where our analytical estimates are no longer applicable. In these domains instead the secular evolution is governed by manifold dynamics. This is the dynamics of the stable and unstable manifolds emanating from the center manifold of circular orbits ($e = 0$). In fact, the latter locally satisfies the properties of a normally hyperbolic invariant manifold (NHIM). Applications to the problem of EoL disposal are finally discussed.

Dynamics of co-orbital charged dust in the solar system

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We report on the role of the interplanetary magnetic field on the celestial mechanics of charged, co-orbital dust in the inner and outer solar system. We derive a secular model and investigate the structure of the resonance close to the Lagrange points of Venus and planet Jupiter. We find asymmetry between L4 and L5 as well as different times of temporary capture. We conclude with a detailed comparison between co-orbital motions of charged dust in the inner and outer solar system.

The Lidov-Kozai resonance at different scales

Anne-sophie LIBERT (naXys, University of Namur, Belgium)

The Lidov-Kozai resonance is one of the most widely discussed resonances since the discovery of exoplanets on eccentric orbit. It constitutes a secular protection mechanism for systems with high mutual inclination, although large variations in eccentricity and inclination are observed. Here I will show how the Lidov-Kozai resonance influences the dynamics of the three-body problem at different scales, namely i) for two-planet extrasolar systems where the orbital variations occur in a coherent way such that the system remains stable, ii) for inclined planets in protoplanetary discs where the Kozai cycles are produced by the gravitational force exerted by the disc on the planet, iii) for migrating planets in binary star systems, whose dynamical evolution is strongly affected by the Lidov-Kozai resonance even without experiencing a resonance capture, and iv) for triple-star systems for which the migration through Lidov-Kozai cycles combined with tidal friction is a possible explanation for the short-period pile-up observed in the distribution of multiple stars.

A numerical criterion evaluating the robustness of planetary architectures. Applications to the Upsilon Andromedae system

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Among the different detection techniques allowing to observe exoplanets, the radial velocity (RV) method better highlights the skeleton of a multi-planetary system, as it is particularly sensitive to more massive bodies. Nevertheless, it is well known that the RV method is unable to determine some of the orbital elements (mean anomalies, inclinations, longitudes of the nodes) and can provide just minimal values of the planetary masses. The dynamical stability of the extrasolar systems crucially depends also on all these parameters. The so called “apsidal locking”, i.e., the libration regime of the difference of the pericenter longitudes, is expected to play a stabilizing role for multi-planet systems with rather eccentric orbits. In a recent work of ours, we showed that when such a phenomenon occurs we can prove the existence of KAM tori that provide upper bounds on the eccentricities values, in such a way to ensure the stability of an exoplanetary system. The convergence of our computational algorithm get faster, when the area of a specific region is decreasing. Therefore, we define a numerical criterion minimizing that area in order to select the most robust orbits. With the aim of determining configurations that are “a priori stable” in the sense of KAM theory, we apply such a criterion in such a way to complete the unknown orbital elements of υ And c and υ And d (that are the exoplanets that are expected to be the most massive ones among those orbiting around υ Andromedæ star). This system is particularly interesting also because previous astrometric observations have allowed to determine ranges of possible values for inclinations, masses and longitudes of the nodes. The results given by the application of our criterion are partially surprising: the most robust configuration (among those compatible with the observations) is that corresponding to the largest possible value of the mass of exoplanet υ And c.

The effect of the passage of Gliese 710 on Oort cloud comets

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Based on observations by [1] who propose a close flyby of the K-type star Gliese 710 in approximately 1.36 Myr we investigate the influence of the stellar passage on trajectories of Oort cloud objects. Using a newly developed GPU-based N-body code [3] we study the motion of 3.6 million testparticles in the outer Solar system where the comets are distributed in three different "layers" around the Sun and the 4 giant planets: A flat disk from 50 to 5000 au ($i < 1$ deg), a flared disk from 5000 to 10 000 au ($i < 45$ deg), and a spherical cloud between 10 000 and 100 000 au (0 deg $< i < 180$ deg). Initial eccentricities of the testparticles are always less than 0.1. We study the influence of Gliese 710 at three passage distances of 1200, 4800, and 12000 au [1]. Additionally, different inclinations of the approaching star are considered. The velocity of the passing star is taken from [2] and computations are run for a time of 30 000 yr. Depending on the passage distance a small number of comets (mainly from the disk and flared disk) is scattered into the observable region (< 5 au) around the Sun. In addition, a huge number of comets (mainly the ones directly in the path of the passing star) shows significant changes of their perihelia. But, they will enter the inner Solar system a long time after the stellar flyby depending on their dynamical evolution.

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Weak stability transition region near the orbit of the Moon

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Co-Authors: Julia Salamon

The weak capture represents the event where the Keplerian energy of the massless particle relative to one of the primaries changes its sign from positive to negative in the context of the restricted three-body problem. Belbruno (1987) introduced the notion of weak stability boundary (WSB) by designing transfer orbits from Earth to Moon. In this paper, Belbruno also proposed an algorithmic definition of the WSB, where the initial conditions are classified to be stable or unstable according to weak stability criteria. The weak stability transition region (WSTR) is the transition zone from the connected part of weak the stable region to the connected part of the weak unstable region in the fixed reference frame. This paper provides a study on the WSTR in the framework of the planar elliptic restricted three-body problem. We define the boundary curves of the WSTR and as a particular case, we determine these curves in the Sun-Earth system. The locations of the boundary curves are compared to the Earth-Moon mean distance. The analysis shows that the lower boundary curve is near to the Moon orbit and a part of the Moon's orbit is within the weak stability transition region.

New results on orbital resonances

Renu MALHOTRA (The University of Arizona, USA)

Orbital resonances have a decisive influence on the dynamical architecture and evolution of planetary systems. Previous analyses predict singularities and/or divergences of resonance widths at low eccentricities and at planet-grazing eccentricities. New studies using non-perturbative numerical approaches find that the apparent singularities and divergences were artifacts of the single resonance approximation and of the choice of dynamical variables common to many previous studies. At low eccentricity, first order resonances do not have diverging widths but have two asymmetric branches leading away from the nominal resonance location. A sequence of structures called "low-eccentricity resonant bridges" connecting neighboring resonances is revealed. At planet-grazing eccentricity, the true resonance width is non-divergent. At higher eccentricities, the new results reveal hitherto unknown structures and show that these parameter regions have a loss of some – though not necessarily entire – resonance libration zones to chaos. The chaos at high eccentricities was previously attributed to the overlap of neighboring resonances. The new results reveal the additional role of bifurcations of the stable and unstable periodic orbits at higher eccentricities. A geometric view of the symmetries and self-intersections of resonant orbits in the rotating frame aids in understanding resonance structures at higher eccentricities. We outline some directions for future research to advance understanding of the dynamics of mean motion resonances.

Orbit determination: from order to chaos

Stefano MARO' (University of Pisa, Italy)

Orbit determination is a classical problem in celestial mechanics. Given a model describing the evolution of a system, e.g. the Kepler problem, it consists in determining the initial condition of an orbit and possibly some other parameter from the observations. A typical example is given by the determination of the orbit of an asteroid given the observations of the angular positions at some times. A solution to this problem goes back to Gauss and leads to the least squares method. Since the observations admit errors, the solution comes with a covariance matrix describing the uncertainty of the solution itself. The behavior of the uncertainties as the number of observations grows is of particular interest. Recent numerical results on a toy model (the standard map) suggest that this behavior depends on the dynamics. I will present some rigorous results describing the trend of the uncertainties for some classes of maps admitting chaotic zones and/or regular motions.

Secular dynamics in extrasolar systems with two planets in mutually inclined orbits

Rita MASTROIANNI (University of Padova, Italy)

Co-Authors: Christos Efthymiopoulos

We revisit the problem of the secular dynamics in two-planet systems in which the planetary orbits exhibit a high value of the mutual inclination. We propose a 'basic hamiltonian model' for secular dynamics, parameterized in terms of the system's Angular Momentum Deficit(AMD). The secular Hamiltonian can be obtained in closed form, using multipole expansions in powers of the distance ratio between the planets, or in the usual Laplace-Lagrange form. The main features of the phase space (number and stability of periodic orbits, bifurcations from the main apsidal corotation resonances, Kozai resonance etc.) can all be recovered by choosing the corresponding terms in the 'basic Hamiltonian'. Applications include the semi-analytical determination of the actual orbital state of the system using Hamiltonian normalization techniques. An example is discussed referring to the system of two outermost planets of the ν-Andromedae system.

The functional relation between the three-body mean motion resonances and the Yarkovsky drift speeds

Ivana MILIĆ ŽITNIK (Astronomical Observatory Belgrade, Serbia)

We examined asteroid's motion across the three-body mean motion resonances (MMRs) with Jupiter and Saturn and with the Yarkovsky drift speed in the semimajor axis of asteroids. The research was conducted using numerical integrations performed in the Orbit9 integrator with 84,000 test asteroids. We calculated time delays $\mathrm{d}tr$ caused by the 7 three-body MMRs on the mobility of test asteroids with 10 positive and 10 negative Yarkovsky drift speeds reliable for the Main Belt asteroids. Our final results considered only test asteroids that successfully crossed over the MMRs without close approaches to the planets. We devised two equations that accurately describe functional relation between the average time $\langle \mathrm{d}tr \rangle$ spent in the resonance, the strength of the resonance SR, and the semimajor axis drift speed $\mathrm{d}a/\mathrm{d}t$ (positive and negative) with asteroids' orbital eccentricities in the range $(0, 0.1)$. Comparing obtained values of $\langle \mathrm{d}tr \rangle$ by from the numerical integrations and by from the derived functional relations, we analysed average values of $\langle \mathrm{d}tr \rangle$ in all three-body MMRs for every $\mathrm{d}a/\mathrm{d}t$. The main conclusion is that the analytical and numerical estimates of the average time $\langle \mathrm{d}tr \rangle$ are in a very good accordance, for both positive and negative $\mathrm{d}a/\mathrm{d}t$. Finally, this study validated our previous old equations on the interaction between the two-body MMRs with Jupiter and the Yarkovsky drift speeds, thus confirming that the equations were correctly devised.

Accuracy and efficiency in the propagation of highly eccentric orbits

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Co-Authors: Catalin Galeş

In this work we compare the computational suitability of several models in the study of highly eccentric orbits of small bodies in the Earth's environment. We include the effects of the Earth's oblateness, the third body perturbations of the Sun and Moon and the solar radiation pressure in the cannonball approximation. From the comparison we conclude that the expansions of the perturbing functions given by a method by Kaufman and Dasenbrock provide very accurate and efficient results in the computation of highly eccentric orbits. We apply this method to an open competition, the Stardust-R "Andrea Milani" Challenge (Space debris: the origin), that consists on estimating the area to mass ratio of 100 pieces of debris and find out which satellite comes from (among 100 satellites).

Long-term dynamics of the solar system inner planets

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Although the discovery of the chaotic motion of the inner planets in the solar system dates back to more than thirty years ago, the secular chaos of their orbits still dares more analytical analyses. Apart from the high-dimensional structure of the motion, this is probably related to the lack of an adequately simple dynamical model. Here, we consider a new secular dynamics for the inner planets, with the aim of retaining a fundamental set of interactions responsible for their chaotic behaviour, while being consistent with the predictions of the most precise orbital solutions currently available. We exploit the regularity in the secular motion of the outer planets, to predetermine a quasi-periodic solution for their orbits. This reduces the secular phase space to the degrees of freedom dominated by the inner planets. On top of that, the smallness of the inner planet masses and the absence of strong mean-motion resonances permits to restrict ourselves to first-order secular averaging. The resulting dynamics can be integrated numerically in a very efficient way through Gauss's method, while computer algebra allows for analytical inspection of planet interactions, once the Hamiltonian is truncated at a given total degree in eccentricities and inclinations. The new model matches very satisfactorily reference orbital solutions of the solar system over timescales shorter than or comparable to the Lyapunov time. It correctly reproduces the maximum Lyapunov exponent of the inner system and the statistics of the high eccentricities of Mercury over the next five billion years. The destabilizing role of the g1-g5 secular resonance also arises. A numerical experiment, consisting of a thousand orbital solutions over one hundred billion years, reveals the essential properties of the stochastic process driving the destabilization of the inner solar system and clarifies its current metastable state. Pre-print: arXiv:2105.14976

The role of periodic orbits in retrograde resonances' capture

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(514107) Kaepaokaawela and (330759) 2008 SO218 are in the 1/1 and 1/2 retrograde resonances with Jupiter. Numerical simulations of asteroids slowly drifting towards a Jupiter mass planet show that retrograde resonances capture more efficiently than prograde resonances. In particular, circular orbits are captured with probability nearly one at the 1/1 retrograde resonance for inclination near 180 degrees, and at the 1/2 retrograde resonance in the inclination range 130 to 170 degrees. These numerical simulations also showed that capture in retrograde resonances involve transitions between different libration modes mediated by the Kozai mechanism. We computed the families of periodic orbits that occur near period ratios 1/1 and 1/2 in the planar circular restricted 3-body problem at mass ratio 0.001, as well as their bifurcations into 3-dimensional periodic orbits. For both 1/1 and 1/2 resonances there are 2 planar modes where the resonant angles librate around 0 or 180 degrees. The examples of these resonances in the Solar System exhibit libration around 0 degrees, which is only possible above an eccentricity threshold imposed by the planet collision condition. The circular outer families associated with each resonance have vertical critical orbits (vcos) where there are bifurcations into 3D symmetric periodic families. As drifting towards the planet occurs, these vcos are encountered before bifurcation into 2D periodic families associated with libration center 180 degrees thus preventing reaching the latter resonant mode. The 3D families connect to a weakly chaotic branch that bifurcates from vertical critical orbits on the resonant family with libration center 0 degrees. At the bifurcation point on the 3D family, connecting both branches that originate at the vcos, the Kozai mechanism acts by increasing eccentricity thus allowing capture into a quasi-periodic orbit with the resonant angle librating around 0.

The disruption of resonant chains of planets

Alessandro MORBIDELLI (CNRS/OCA, France)

The formation of resonant chains of planets seems to be an unavoidable outcome of planet migration in protoplanetary disks, whenever the planets exhibit convergent migration. This is the case, for instance, when they reach the inner edge of the disk (as in the case of close-in super-Earths) or are ranked in masses (as in the case of the giant planets of the solar system). Among extrasolar planetary systems there are some that are in a resonant chain, Trappist-1 being the most notable example, but they are rare. The observed orbital properties of extrasolar super-Earths can be explained if more than 90% of the original resonant chains became unstable after the disappearance of gas from the proto-planetary disk (Izidoro et al., 2017, 2021). Numerical simulations show that many resonant chains do become unstable without the need of external perturbations once the damping effect of the gas is removed. Matsumoto et al. (2021) conducted a complete numerical survey of the evolution of resonant chains of equal-mass planets and determined the number of planets beyond which the system becomes unstable as a function of the planets' separation in terms of mutual Hill radii. Yet, the origin of these instabilities are not well understood. In Pichierri and Morbidelli (2020) we have studied chains of three planets in a k:k-1 resonances and we have shown both analytically and numerically that the instability occurs because of the emergence of secondary resonances between a fraction of the synodic frequency $2\pi(1/P1-1/P2)$ and the libration frequencies in the resonance. We also obtained a generalization of this dynamical mechanism for an arbitrary value of k and of the number of planets in the chain. Finally, the destabilization of a resonant chain can also occur due to external perturbations. The role of planetesimal scattering, tidal migration, the migration of the inner edge of the disk at disk removal will be briefly discussed.

Invariant manifolds near L$_3$ and their role in the transport for the Earth-Moon Bicircular model

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Co-Authors: Àngel Jorba

In this presentation we discuss the dynamics near the point L$_3$ of the planar Earth-Moon system, including the direct gravitational effect due to the Sun. The Lyapunov family of invariant tori related to L$_3$ has been computed, along with their stable and unstable manifolds. We show that these dynamical structures are a skeleton for transport in the Earth-Moon system. For example, they facilitate the transport of Moon ejecta to the Earth, and they also act as a gateway for asteroids to enter and escape the Earth-Moon system. In particular, we will show how they can be used to capture a NEA near L$_3$.

Orbit propagation around small bodies using spherical harmonic coefficients obtained from polyhedron shape models

Pelayo PENARROYA and Roberto PAOLI (DEIMOS Spain and Al. I. Cuza University of Iaşi, Romania)

Missions to asteroids have been the trend in space exploration for the last years. They provide information about the formation and evolution of the Solar System, contribute to direct planetary defense tasks, and could be potentially exploited for resource mining. Be their purpose as it may, the factor that all these mission types have in common is the challenging dynamical environment they have to deal with. The gravitational environment of a certain asteroid is most of the times not accurately known until very late mission phases when the spacecraft has already orbited the body for some time. Shape models help to estimate the gravitational potential with a density distribution assumption (usually constant value) and some optical measurements of the body. These measurements, unlike the ones needed for harmonic coefficient estimation, can be taken from well before arriving at the asteroid's sphere of influence, which allows to obtain a better approximation of the gravitational dynamics much sooner. The disadvantage they pose is that obtaining acceleration values from these models implies a heavy computational burden on the on-board processing unit, which is often too time-consuming for the mission profile. In this paper, the technique developed on [1] is used to create a validated Python-based tool that obtains spherical harmonic coefficients from the shape model of the asteroid, given a certain density for the body. This software suite, called astroHarm, is used to analyse the accuracy of the models obtained and the improvements in computational efficiency in a simulated spacecraft orbiting an asteroid. A parametric analysis is also performed regarding different density distributions and shape model superpositions. The results obtained are shown offering a qualitative comparison between different order spherical harmonic models and the original shape model. Finally, the creation of a catalogue for harmonics is proposed together with some thoughts on complex modelling using this tool.

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Planetary and lunar ephemeris EPM2021 and its significance for Solar system research

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Co-Authors: Elena Pitjeva, Dan Aksim, Margarita Kan

An updated public version of EPM (Ephemerides of Planets and the Moon) will be presented. Since the last public version, EPM2017, many improvements were made in both the observational database and the mathematical model. Latest lunar laser ranging observations have been added, as well as radio ranges of Juno spacecraft and more recent ranges of Odyssey and Mars Reconnaissance Orbiter. EPM2021 uses a new improved way to calculate radio signal delays in solar plasma and has a major update in the method of determination of asteroid masses. Also, a delay-capable multistep numerical integrator was implemented for EPM in order to properly account for tide delay in the equations of the motion of the Moon. The improved processing accuracy has allowed to refine existing estimates of the mass of the Sun and its change rate, parameters of the Earth-Moon system, masses of the Main asteroid belt and the Kuiper belt; and also to raise important questions about the existing numerical models of solar wind.

The semi-analytical motion theory of the third order in planetary masses for the Sun - Jupiter - Saturn - Uranus - Neptune's system

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Co-Authors: Eduard Kuznetsov

This work is devoted to the problem of the orbital evolution of the Solar system's giant planets on long time intervals. The averaged semi-analytical motion theory of the four-planetary problem is constructed up to the third order in planetary masses. The second system of Poincare elements and Jacobi coordinate system are used for the construction of the Hamiltonian expansion. The averaged Hamiltonian in the third approximation is constructed by the Hori-Deprit method. The maximum degree of eccentric and oblique Poincare elements in terms of the first order by planetary masses is 6; the second-order terms are constructed up to 4th degree, the third-order terms are constructed up to 2nd degree. All analytical transformations are performed by using CAS Piranha (authored by F. Biscani). The constructed equations of motion in averaged elements are numerically integrated for the giant planets of the Solar system over time intervals up to 10 Gyr. The comparison of obtained results, such as amplitudes and periods of the change of the orbital elements, with numerical motion theories shows an excellent agreement with them. This agreement suggests that this motion theory is constructed correctly, and it can be used for the investigation of the dynamical evolution of various extrasolar planetary systems with moderate orbital eccentricities and inclinations.

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The path to instability in multiplanetary systems

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Numerical simulations have shown that the fate of planetary systems with more than three planets is qualitatively different than for the case of two planet systems. Indeed, while in the two planet case, a sharp stability boundary exists, the three and more planet systems experience instability on timescales up to billions of years. Moreover, there exists an exponential trend between the planet orbital separation and the instability time. While the result has been observed in numerous simulations, little is known about the actual mechanism leading to instability. Indeed, planetary systems seems to remain dynamically quiescent for most of their lifetime before a very short unstable phase. In this work, we show how the chaotic diffusion, due to overlap between three-body resonances, dominates the timescale of instability. We generalise the empirical trend obtained for equal mass and equally spaced planets to general systems and motivate analytically the empirical relation. The simplified mechanism studied reproduces very well the qualitative behaviour found in numerical simulations, in particular the very short phase involving planet-planet scattering. We also discuss the consequences of the instability on system's architecture and planet formation.

On the scattering and dynamical evolution of Oort cloud comets caused by a stellar fly-by

Elke PILAT-LOHINGER (Department of Astrophysics, University of Vienna, Austria)

Co-Authors: Sharleena Clees, Maximilian Zimmermann

Recent GAIA observations revealed that the K-type star Gliese 710 will cross the Oort cloud in a distance between approx. 4000 and 12000 au in about 1.3 Myrs. Therefore, we study the influence of this stellar encounter on comets in the outer region of the solar system where 10 to 20 mio objects were distributed in the region between 100 and 100000 au from the sun. First, we give a statistical overview of objects that are immediately scattered towards the sun for fly-bys at 4000, 8000 and 12000 au and afterwards, we show a long-term study of objects that were moved into highly eccentric motion by the stellar fly-by and analyze the probability that these objects might visit the terrestrial planets at later times.

Sample-return missions to asteroids: one step further for space exploration

Marcel POPESCU (Astronomical Institute of the Romanian Academy, Romania)

Co-Authors: N. G. N. G. Simion, J. Licandro, O. Vaduvescu, J. de Leon, R. M. Gherase

The space exploration represents the new adventure of mankind. In this context, the small bodies of the Solar System play a key role. Some of these can be easily reached by a spacecraft. As a consequence they are considered ideal targets for in-situ resource utilization and could become a source of materials for space activities in the near future. During the recent years two space missions had the objective to bring samples from different near-Earth asteroids. Namely, the JAXA/Hayabusa 2 spacecraft (which is the successor of the first Hayabusa mission) returned to Earth the sample retrieved from (162173) Ryugu, and the NASA/OSIRIS-REx is on its way back to deliver the material successfully acquired from (101955) Bennu.
In this talk I will present some the amazing results obtained by the two missions. These will include the detailed images and maps of these tiny worlds, information about their composition, a discussion of the processes that affect the surface of airless bodies, and the conclusions that can be drawn about asteroids origin and formation.

The network of periodic orbit families in co-orbital motion. Taking advantage of the averaged problem in order to compute solutions in the restricted three-body problem

Alexandre POUSSE (IMATI-CNR, Italy)

A classical approach of the restricted three-body problem is to analyze the dynamics of the massless body in the synodic reference frame: families of periodic orbits as well as the dynamics in their neighborhood are computed with the help of Poincaré maps and continuation methods. Perturbative treatments provide a different approach. In the specific case of mean-motion resonances, the averaged problem allows to investigate the long-term behavior of the dynamics through a suitable approximation that focuses on a particular region of the phase space of the restricted three-body problem. In the framework of the 1:1 mean-motion resonance, I will discuss on how to take advantage of the averaged problem in order to compute solutions in the synodic reference frame, especially periodic orbits. In the circular-planar case, I will present a « map » of the phase space that depicts the co-orbital dynamics and outline a method that allows to compute the structured network of periodic orbit families that fill the co-orbital region. This is a joint work with E.M. Alessi.

GM=tc^3 Lunar Orbit Anomaly and Cosmology

Louise RIOFRIO (International Lunar Observatory, USA)

The evolution of the lunar orbit allows tests of cosmology and Relativity. A cosmology where $GM = tc^3$ is indicated by the Lunar Laser Ranging Experiment, and may be further tested with the ESA Atomic Clock Ensemble in Space aboard ISS. Further experiments use the microscopic world of PLanck values and the scale of living cells.

Dynamical stability in the vicinity of Saturnian small moons: the casesof Aegaeon, Methone, Anthe and Pallene

Adrian RODRIGUEZ (Universidade Federal do Rio de Janeiro, Brazil)

Co-Authors: Nelson Callegari Jr.

In this work we analyze the orbital evolution and dynamical stability in the vicinity of the small Saturnian moons Aegaeon, Methone, Anthe and Pallene. We numerically resolve the exact equations of motions to investigate the orbital motion of thousands of test particles within and near the domain of the 7/6, 14/15, 10/11 mean-motion resonances of Aegaeon, Methone and Anthe with Mimas, respectively. We show that, for massless small moons, the orbits of particles initially restricted to the resonant domains remain stable for at least 10,000 yr. We also conduct numerical simulations considering Aegaeon, Methone, Anthe and Pallene as massive bodies. The results show that most particles undergo significant perturbations in their orbital motions, ultimately destabilizing on time-scales of a few hundreds of years or even less through collisions with the four small moons. In addition, we also simulate the orbital evolution of test particles initially distributed in the form of arcs around Aegaeon, Methone, and Anthe. We show that the initial arcs are dynamically eroded on time-scales of hundreds of years, allowing us to constrain the time-scales on which gravitational forces operate to remove particles from the observed arcs.

Ejection-collision Orbits in the RTBP

Óscar RODRíGUEZ DEL RíO (Universitat Politècnica de Catalunya and Università di Pisa, Italy)

Co-Authors: Mercè Ollé, Jaume Soler

In this talk, we discuss the advantages and disadvantages of Levi-Civita and McGehee regularizations. In particular, we use them to study ejection-collision orbits. As it is well known, for any value of the mass parameter μ and sufficiently restricted Hill regions, there are exactly four ejection-collision orbits. We check their existence and extend numerically these four orbits for any value of μ and for less restrictive values of the Jacobi constant. We introduce the concept of n-ejection-collision orbits and we present some analytical and numerical results in the existence and the evolution of this kind of orbits.

Dynamical evolution of extrasolar planets in binary star systems

Arnaud ROISIN (University of Namur, naXys, Belgium)

Co-Authors: Anne-Sophie Libert, Jean Teyssandier

About half of the Sun-like stars are part of multiple-star systems. Today more than 100 S-type planets (also called circumprimary planets) are known moving around one stellar component of a binary star. Most of them are found in wide binaries with separation larger than 500 AU and with very diverse eccentricities. These discoveries raise the question of the formation and long-term evolution of these planets because the stellar companion can strongly affect the planet formation process. In this work, by means of a symplectic integrator designed for binary star systems, we study the dynamical influence of a wide binary companion on the evolution of giant planets during their migration in the protoplanetary disk and their long-term evolution after the dispersal of the disk. In particular, we highlight the importance of the Lidov-Kozai resonance for highly inclined binary companions. We also study the mean motion resonance captures of the planets during the migration phase. We finally show how our work can explain several features of the detected circumprimary planets, such as their high eccentricities and spin-orbit misalignment.

Some most interesting cases of close asteroid pairs perturbed by resonance

Alexey ROSAEV (Research and Educational Center "Nonlinear Dynamics", Yaroslavl State University, Russia)

Co-Authors: Eva Plavalova

In our previous paper [1] we give the list of close asteroid pairs in vicinity of resonance but we have not dive the detailed study of the interesting cases. Some members of close pairs display the jumps in semimajor axis from one side of resonance to another. It is point of particular interest. First of all we bring attention to the two pairs of asteroids in 2-1J-1M three body resonance. Duddy et al. [2] pointed that pair (7343) Ockeghem and (154634) 2003 XX38 is in 2-1J-1M. Here we report another close asteroid pair 56232 (1999 JM31) and 115978 (2003 WQ56) in this resonance. The mentioned pair is even closer to core of resonance and more strongly perturbed. The small difference in the orbital elements of this two pairs allows us to study some features of interaction with this resonance. Additionally, we discuss the possibility of common origin of this two pair. Second important object of careful studying is (10123) Fideoja and (117306) 2004 VF21 pair. Pravec et al. [3] note that the orbits in pair are undergo irregular jumps over the 7:2 mean motion resonance with Jupiter. However it is not true. In this paper we have proved that this pair perturbed by 9-6J-4M three body resonance.

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Multiscale analysis for space situational awareness

Aaron Jay ROSENGREN (University of California San Diego, USA)

While the capability to maintain space situational awareness (SSA) in the confines of the traditional orbits out to the geosynchronous (GEO) belt is mature, the extreme range, difficult observing geometries, and unstable astrodynamics create particular challenges associated with conducting SSA in the cislunar environment. The ability to quantify, assess, and predict the behavior of objects in space is foundational to SSA, and the trustworthy detection and characterization of orbital events is essential to the assessment of potential threats in space. These remain pressing problems in the classical circumterrestrial regime, but present unique and diverse difficulties for the vast cislunar domain beyond GEO (xGEO). The nonlinear astrodynamics in xGEO, encompassing secular, resonant, chaotic, close-encounter, and manifold dynamics, is dramatically different than the weakly perturbed Keplerian approach used for over a half century for the detection and tracking of objects near Earth. In the SSA context, for example, both the modest stationkeeping for orbit maintenance in libration point trajectories, as demonstrated by the NASA ARTEMIS spacecrafts, and the intrinsic sensitivity of these trajectories on account of space-manifold dynamics can serve to complicate attempts to maintain custody of an object in this unstable orbital environment. It is precisely these distinctive dynamical features, including rapid uncertainty propagation, that frustrate SSA but enable novel space-mission concepts that are not simply predicated by Keplerian motion. In this talk, we will review the multiscale dynamics of cislunar space and discuss its importance for SSA and space sustainability.

Are we doing enough? Space debris in the New Space Economy era

Alessandro ROSSI (IFAC-CNR, Italy)

Space as we knew it is no more there. In the last few years the number of satellites injected in orbit has outgrown the average of the past 50 years by a factor of 6. This radical change is mostly related to the exploitation of the space resource by new private entrepreneurs. Large constellations of satellites are starting to populate the already overcrowded regions of the Low Earth Orbit. The miniaturization of satellites is allowing the launch of much smaller spacecraft sometimes with reduced maneuvering capabilities. In this changing environment the fragile regulatory system presently available for the space activities might be not suitable to guarantee a sustainable future for this fundamental common resource. By means of evolutionary models, the effectiveness of the current mitigation measures in the new space era is evaluated under different future scenarios. A combination of mitigation and remediation options (including active and passive deorbiting, collision avoidance, active debris removal, in-orbit refueling, etc.) to stabilize the future population of space objects is studied and proposed, along with some considerations about its applicability and costs.

Characterization of the stability for trajectories exterior to Jupiter in the restricted three-body problem via closed-form perturbation theory

Mattia ROSSI (Department of Mathematics - Università degli Studi di Padova, Italy)

Co-Authors: Christos Efthymiopoulos

We address the question of identifying the long-term (secular) stability regions in the semi-major axis-eccentricity projected phase space of the Sun-Jupiter planar circular restricted three-body problem in the domains i) below the curve of apsis equal to the planet's orbital radius (ensuring protection from collisions) and ii) above that curve. This last domain contains several Jupiter's crossing trajectories. We discuss the structure of the numerical stability map in the (a, e) plane in relation to manifold dynamics. We also present a closed-form perturbation theory for particles with non-crossing highly eccentric trajectories exterior to the planet's trajectory. Starting with a multipole expansion of the barycentric Hamiltonian, our method carries out a sequence of normalizations by Lie series in closed-form and without relegation. We discuss the applicability of the method as a criterion for estimating the boundary of the domain of regular motion.

The large obliquity of Saturn explained by the fast migration of Titan

Melaine SAILLENFEST (IMCCE, Paris Observatory, France)

Co-Authors: Giacomo Lari, Gwenaël Boué, Ariane Courtot

The large tilt of Saturn's spin axis (27°) as compared to Jupiter's (3°) requires a dynamical explanation. For decades, it was thought that Saturn had been tilted during the late planetary migration, more than four gigayears ago, because of a capture in secular spin-orbit resonance. However, this traditional scenario is at odds with the orbital expansion of Saturn's biggest moon, Titan, which has recently been measured. Instead, the resonance has probably been encountered recently, and Saturn may still be tilting today. I will explain how a migrating satellite can incline its host planet and discuss the most likely evolution pathway followed by Saturn and Titan.

Energy and Angular Momentum partitioning in the N-Body Problem

Daniel SCHEERES (University of Colorado Boulder, USA)

Co-Authors: Gavin Brown

This work studies how the initial energy and angular momentum of an N-body system is partitioned and redistributed between escaping components and bound multiple body systems. A generic initial distribution of N bodies will naturally disrupt due to multi-body dynamical interactions. The bodies in the system generally end up as ejected singles or pairs of bodies. If the bodies have finite density, they will also form condensed distributions. These types of self-disrupting systems are of particular interest in their applications to problems in the field of asteroid mechanics, such as in rubble-pile asteroid formation after a cataclysmic impact. A specific question of interest is the amount of angular momentum and energy that is lost from an original distribution of bodies due to gravitational ejection. This can be tracked using the amended potential / minimum energy function of a system. This function combines the total angular momentum, current mass distribution and potential energy of a system into one function that serves as a sharp lower bound on the system energy for a given level of angular momentum. The function also provides for precise computation of the necessary energy for a rubble pile system to disrupt and lose components. However, once a component or group of components undergo a mutual escape the function no longer serves a useful purpose, and it takes on an extreme, conservative lower bound value that provides no significant insight. Thus, in previous analysis it has not been useful for tracking components of a system after they undergo gravitational disruption. Here we develop and apply a form of the Jacobi equations for an N-rigid body system that provides a remedy for this situation. We show that if the system is written in Jacobi coordinates, there is a clear way in which the minimum energy function can be decomposed to maintain its utility across an arbitrary partition of a system and its associated disruption. To show this, we prove an inequality that leverages the Jacobi coordinate formulation of the N rigid body problem, and maintains its sharp lower limit behavior for all elements of a system after it has shed components.

More on properties of retrograde 1:1 mean motion resonance

Vladislav SIDORENKO (Keldysh Institute of Applied Mathematics, Moscow, Russia)

Most of objects in the Solar system move around the Sun in the anticlockwise manner when seen from above the north ecliptic pole. And only a small number of celestial bodies move in opposite direction [1,2]. Similar to prograde motion, the retrograde motion of a celestial body can also be in resonance with one of the major planets. For example, the asteroid 2015 BZ509 is in retrograde 1:1 mean motion resonance with Jupiter [3]. Theoretical studies demonstrated that such resonance can prevent collision of asteroid and planet in the case of co-orbital motion [4,5]. The aim of our investigation is to obtain more information about the properties of this resonance. Similar to other MMR three dynamical processes can be distinguished in the case under consideration: "fast" process corresponds to planet and asteroid motions in orbit, "semi-fast" process is variation of the resonance argument (which describes the relative position of the planet and the asteroid in their orbital motions), and, finally, "slow" process is the secular evolution of the orbit shape (characterized by the eccentricity) and orientation (which depends on the ascending node longitude, inclination and argument of pericenter). With the use of double numerical averaging we construct evolutionary equations that describe the long-term behavior of asteroid's orbital elements (the “slow” process). Special attention is paid to possible transitions between different types of orbits existing at retrograde 1:1 resonance.

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Central Configurations in the General Coplanar Four-Body Problem

Bonnie STEVES (Glasgow Caledonian University, Scotland, UK)

Co-Authors: Muhammad Shoaib, Brahim. Benhammouda, Winston. L. Sweatman

Central configurations play a very important role in understanding the dynamics of n-body problems. The general coplanar four-body gravitational dynamical problem considered in this paper has no symmetry restrictions. The masses of the four bodies are m_0 , m_1, m_2 and m_3. Three of the bodies are arranged at the vertices of a triangle with position coordinates $r_0 = (0, b),\ r_1 = (-1, 0),\ r_2 = (0, a)$. The fourth body, with position $r_3 = (c_1, c_2)$, is allowed to be anywhere in the plane forming either convex or concave central configurations. The necessary condition for the existence of central configurations in the general coplanar four-body problem is derived. Using both analytical and numerical techniques, regions of central configurations solutions are derived for positive masses in the general problem. To better visualize the central configurations and regions for positive masses, a wide range of special cases of four-body configurations are investigated. These include right quadrilaterals, the convex kite, the concave kite and the isosceles trapezoid. In most of these special cases, we were able to reduce the number of parameters of the problem from four to two and hence could graphically show the regions of central configuration solutions.

Four- and Five-body periodic Caledonian orbits

Winston SWEATMAN (Massey University, New Zealand)

Co-Authors: Valerie Chopovda

We consider four- and five-body problems with symmetrical masses (Caledonian problems). Families of periodic orbits originate from their collinear Schubart orbits. We present and discuss some of these periodic orbits.

Rotation of an oblate satellite: chaos control

Mariusz TARNOPOLSKI (Jagiellonian University, Poland)

A model of planar oscillations, described by the Beletskii equation, was investigated. The Hamiltonian formalism was utilized to employ a control method for suppressing chaos. An additive control term (an order of magnitude smaller than the potential) was constructed. This allows not only for significantly diminished diffusion of the trajectory in the phase space, but turns the purely chaotic motion into strictly periodic motion.

Solar system chaos ordered in arch-like structures

Nataša TODOROVIĆ (Astronomical Observatory in Belgrade, Serbia)

Co-Authors: Aaron J. Rosengren, Di Wu

We map and study the Solar System dynamics in extremely short-decadal timescales. Using sophisticated numerical methods and a realistic physical model in a domain covering the region from the main asteroid belt to Uranus and beyond, we observe an intricate and elegant ornamental structure of chaos, connected in a series of arches. Such structures, traces of space manifolds, represent a source of rapid and large-scale transport. We find that Jupiter, on account of such manifolds, has an exuberant and profound control even on very distant small bodies. Orbits along these structures close to Jupiter, can be kicked to over 100 astronomical units in less than a century reaching the distances of Uranus and Neptune in a mere decade. All planets generate similar chaotic structures that permeate space allowing fast transport throughout the Solar System. We will describe and discuss other various aspects of the manifolds we have observed.

TBA

Kleomenis TSIGANIS (Aristotle University of Thessaloniki, Greece)

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TBA

Secular resonances for satellite orbits: application to Phobos

Timothée VAILLANT (CFisUC, Universidade de Coimbra, Portugal)

Co-Authors: Alexandre C. M. Correia

Several phenomena are able to modify significantly the orbit of a satellite. For instance, some secular resonances between the precession frequencies of its orbit and the mean motion of the host planet around the star can lead to important variations of the orbital parameters of the satellite. The evection resonance, which corresponds to the interaction between the secular precession of the pericenter of the satellite and the mean motion of the satellite, can in particular induce an important increase of the eccentricity of the satellite in the case where the capture in the resonance occurs. In the same way, other resonances like the eviction resonance can also lead to variations of the inclination of the satellite orbit. We study here the secular resonances similar to the eviction resonance in order to estimate in what extent they can influence the evolution of the inclination of a satellite. We consider the case of Phobos, the largest satellite of Mars, that will encounter in the future some resonances of this type. For this case, we compute the capture probabilities in function of the eccentricity of the satellite and the obliquity of the planet. In the case where the capture does not occur, we estimate the effects due to the resonance crossings on the orbit of the satellite. We also study the interaction between different secular resonances with the method of the frequency map analysis, and show that the overlapping of resonances significantly modifies the capture probabilities and the dynamics of the satellite.

Capture of interstellar objects at close planetary encounters: analytical theory

Giovanni VALSECCHI (IAPS-INAF, Italy)

The recent discovery of the first two interstellar objects has renewed the interest in the possibility for some of them to be captured in elliptic heliocentric orbits as a consequence of close encounters with one of the giant planets. This problem can be treated in the framework of the analytical theory of close encounters, that has the advantage of allowing the exploration of the range of initial conditions for which this type of capture is possible, and of the main features of the resulting orbits.

Back-tracing space debris using proper elements

Tudor VARTOLOMEI (University of Rome Tor Vergata, Italy)

Co-Authors: Alessandra Celletti, Giuseppe Pucacco

Normal form methods allow one to compute quasi-invariants of a Hamiltonian system, which are referred to as proper elements. The computation of the proper elements turns out to be useful to associate dynamical properties that lead to identify families of space debris, as it was done in the past for families of asteroids. In particular, through proper elements we are able to group fragments generated by the same break-up event and we possibly associate them to a parent body. The results are corroborated by a statistical data analysis based on the check of the Kolmogorov-Smirnov test and the computation of the Pearson correlation coefficient. This talk refers to works in collaboration with Alessandra Celletti and Giuseppe Pucacco.

From Uncertainty Quantification to Stochastic Dynamic Indicators

Massimiliano VASILE (University of Strathclyde, UK)

Recently techniques for uncertainty quantification were introduced in astrodynamics to better capture the effect of uncertainty in initial conditions, dynamic model and observations. Many of these techniques yield some form of polynomial expansion of a quantity of interest with respect to the uncertain quantities. The way the coefficients of the polynomial are computed is what makes the difference. This talk starts from a gentle introduction to some uncertainty quantification techniques with particular focus on what is generally known as intrusive polynomial chaos expansions. Then, it presents some ideas to derive stochastic dynamic indicators that can capture the effect of uncertainty on the evolution of a dynamical system. In this talk the term uncertainty encompasses both stochastic processes and uncertain system parameters. It will be shown that these indicators assume different values in the case of regular and chaotic motion as well as diffusive, super-diffusive and sub-diffusive processes. The behaviour of the proposed stochastic dynamic indicators is illustrated with some examples of known dynamical systems.

The role of tidal forces in the long-term evolution of the Galilean system

Mara VOLPI (University of Rome Tor Vergata, Italy)

Co-Authors: Alessandra Celletti, Efsevia Karampotsiou, Christoph Lhotka, Giuseppe Pucacco

Jupiter's first three moons Io, Europa, and Ganymede are found in the so-called Laplace resonance, so that their orbits are locked in a $2:1$ resonant chain. We study the persistence of the resonance along the evolution of the system when considering the tidal interaction between Jupiter and Io. In order to constrain the computational cost of the task, we enhance this dissipative effect by means of a multiplying factor α_P. We develop a simplified model to study the propagation of the tidal effects from Io to the other moons, resulting in the outward migration of the satellites, and provide an analytical description of the phenomenon. A more elaborate Hamiltonian model allows us to study the long-term evolution of the system along few gigayears, including the possibility of the trapping into resonance of Callisto depending on its initial conditions. Finally, we analyse of the dependency of the results on the chosen value of α_P.

On the families of periodic orbits around irregular-shaped asteroids

George VOYATZIS (Aristotle University of Thessaloniki, Greece)

Co-Authors: Dionysios Karydis, Kleomenis Tsiganis

Periodic orbits are of special importance in orbital mechanics and find many applications for space missions. Their existence and stability type affects essentially the phase space dynamics and the evolution of the other orbits around them. In this work we study periodic orbits around minor celestial bodies which, in general, are of irregular shape and their gravitational field maybe quite complex. Such orbits should be strictly asymmetric and three dimensional and this leads to difficulties for their computation. They evolve in a six-dimensional phase space and are, in general, not isolated but forms monoparametric families. The characteristic curves of these families are quite complex compared with the families of simple symmetric models e.g. in an ellipsoid. In this study we compute and discuss dynamical properties of such families concerning bifurcations, stability and termination. Our model for computations is the 433 Eros asteroid. We consider various approaches for determining a single periodic orbit in the asteroid's gravitational field (genetic algorithm, particle swarm optimization, random local search and shape continuation). When such an orbit is found can be used as a seed for analytic continuation which is implemented with differential corrections providing the corresponding family.

Analytical representation for ephemeride with short time spans: Application to Titan

Xiaojin XI (National Time Service Cente, Chinese Academy of Sciences, China)

Co-Authors: Alain Vienne

The ephemerides of natural satellites resulting from numerical integration have a very good precision on the fitting to recent observations, in a limited interval. Meanwhile,synthetic ephemerides like the Theorie Analytique des Satellites de Saturne (TASS) by Vienne and Duriez describe in detail the dynamical system by a representation based on the combinations of the proper frequencies. Some theoretical studies need to have both advantages. For example, to study the rotation of Titan, one needs to know the representation of its longitude. We aim to use these two types of ephemerides in order to rebuild a long-lasting and high-precision ephemeris with proper frequencies based on the numerical integration ephemeris. The aim is to describe the numerical ephemerides with formulas similar to analytical ones. We used the representation of the orbital elements from the TASS ephemeris analysed over 10,000 years as a reference template. We obtained the proper frequencies with both numerical and the TASS ephemeris over 1,000 years only. A least-square procedure allowed us to get the analytical representation of an orbital element in this limited interval. We acquire the representation of the mean longitude of Titan from JPL ephemeris over 1,000 years. For almost all components, the corresponding amplitudes and phases are similar to the relative terms from TASS. The biggest difference between our representation and the mean longitude of Titan of JPL is less than 100 kilometres over 1,000 years, and the standard deviation is about 26 kilometres.

Creep Tide Model for the 3-Body Problem

Federico ZOPPETTI (Observatorio Astronómico de Córdoba, Argentina)

Co-Authors: H. Folonier, A. M. Leiva, C. Beaugé

We present a tidal model for treating the rotational evolution in the general three-body problem with arbitrary viscosities, in which all the masses are considered to be extended and all the tidal interactions between pairs are taken into account. Based on the creeptide theory, we present the set of differential equations that describes the rotational evolution of each body, in a formalism that is easily extensible to the N tidally-interacting body problem. We apply our model to the case of a circumbinary planet and use a Kepler-38 like binary system as a working example. We found that, in this low planetary eccentricity case, the most likely final stationary rotation state is the 1:1 spin-orbit resonance, considering an arbitrary planetary viscosity inside the estimated range for the solar system planets. We derive analytical expressions for the mean rotational stationary state, based on high-order elliptical expansions of the semi major axes ratio α and low-order expansions of the eccentricities. These are found to reproduce very accurately the mean behaviour of the low-eccentric numerical integrations for arbitrary planetary relaxation factors, and up to $\alpha \quad 0.4$. Our analytical model is used to predict the stationary rotation of the Kepler circumbinary planets and found that most of them are probably rotating in a sub-synchronous state, although the synchrony shift is much less important than the one estimated with another tidal model. We present a comparison of our results with those obtained with the Constant Time Lag and find that, unlike what we assumed in our previous works, the cross torques do not have a negligible net secular contribution, and must betaken into account when computing the tides over each body in an N-extended-body system from an arbitrary reference frame. These torques are naturally taken into account in the creep theory. In addition to this, the latter formalism considers a more realistic rheology that proved to reduce to the Constant Time Lag model in the gaseous limit and also allows to study several additional relevant physical phenomena.

List of Abstracts – Posters

On the Spatial Collinear Four-Body Problem With Non-Spherical Primaries

Rajiv AGGARWAL (Deshbandhu College, University of Delhi, India)

Co-Authors: Md Sanam Suraj, Amit Mittal, Md Chand Asique

In the present work a systematic study has been presented in the context of the existence as well as the stability of libration points and the fractal basins of convergence associated with these libration points in the spatial configuration of the collinear four-body problem with non-spherical primaries (i.e., the primaries are oblate or prolate spheroid). The parametric evolution of the position of the libration points as function of the oblateness and prolateness of the primaries and its stability are illustrated numerically. Moreover, the regions of possible motion are also depicted, where the infinitesimal mass are free to orbit, as function of Jacobian constant. It has been observed that the oblateness or prolateness of the primaries have significant influence on the topology of the domain of basins of convergence linked with in-plane and out-of-plane libration points.

Monte Carlo simulations for hazardous asteroids using new highly optimized algorithm

Ivan BALYAEV (Saint Petersburg State University, Russia)

To estimate impact probability, Monte Carlo method requires a large number of virtual asteroids. Program R^0 was created to improve numerical integration of asteroid motion equations including calculation of close approaches. This program is based on Everhart's method and uses the JPL ephemeris DE430. Creation from scratch allowed to reach 23 times higher performance compared to old program. For the first task, 200 asteroids were selected from the NASA website. Potential impacts were discovered for the Earth, the Moon, some other planets.

The evolution of asteroid movements in our Solar System

Afrodita Liliana BOLDEA (National Institute for Physics and Nuclear Engineering, Bucharest, University of Craiova, Romania)

Co-Authors: Magda Stavinschi

To study the evolution of asteroid movement and its impact on our planet we used astronomy data, namely images obtained through the Isaac Newton Group of Telescopes, from the island La Palma in the Canary Islands, for celestial bodies of the type Near Earth Objects – NEOs. They must be identified and included in the Minor Planet Center database through the EURONEAR consortium. We used, as softwares, ASTROMETRICA and NERBY. Thus, objects were found, asteroids, for which the trajectory can be determined. through calculation formulas and SAO Image DS9 software or, comparatively, accessing the specialized utilities, NEODyS, respectively JPL Small-Body Database Browser. Of all the groups of asteroids in our Solar System, the most interesting for us and the most investigated are those that are likely to approach Earth, the Near Earth Asteroids – NEA. In this paper we present the need to study the movements of asteroids near our planet, to expand research programs for students, researchers, amateur astronomers and to disseminate research results for academic education. In the last part of this paper we performed a comparative statistical study between the different techniques for obtaining the trajectories of asteroids using different utility software, depending on the students' options, which were thus coordinated in the educational process of teaching-learning-assessment.

Orbital Dynamics Study in the Process of Space Weathering of the Asteroid (162173) Ryugu

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Asteroids receive various classifications according to their orbital, chemical, physical, and mineralogical characteristics. The process that occurs on the surface of asteroids, changing their surface morphologically, under the effect of various exogenous factors, are called space weathering. In this work, the asteroid (162173) Ryugu, which was the target of the Japanese Aerospace Exploration Agency's Hayabusa 2 sample return mission, is studied. It is an asteroid belonging to the Near-Earth Asteroids group (NEAs), whose most likely origin is the main asteroid belt. Studies indicate that its most likely source is the region of the main belt that is under the action of $\upsilon 6$ secular resonance. According to data obtained by the Hayabusa 2 mission, it was observed that Ryugu must have suffered the phenomenon of space weathering, this aging is caused by the radiation received by the asteroid due to approximations with the Sun. As a result, the present work aims to study Ryugu's orbital evolution, verifying the possibility that it has suffered close encounters with the Sun, and to calculating the solar radiation due to this approximation with the Sun and the radiation accumulated on its surface since the moment in which it becomes a NEA, so that its degree of space weathering can be estimated. Thus, to analyze the orbital evolution of Ryugu, numerical integrations of the N-body gravitational problem were performed using the Mercury integrator package. Then, 7.000 clones were generated starting from the vicinity of the $\upsilon 6$ resonance, checking which ones arrived in the range of Ryugu's current orbit. The orbital evolution of the clones was analyzed, identifying those that were successful. And for these, the accumulated radiation overtime was calculated. Based on observations of Hayabusa 2, it was expected that Ryugu would obtain close encounters with the Sun our results corroborated the observations, showing that some clones arrived at a very close distance to the Sun, receiving a large amount of radiation at once, and such radiation may have modified its surface in a single punctual event. We also analyzed the flux of accumulated radiation throughout their life as NEA, in which this accumulation may have modified their surface over the years and not only in a single event.

On the Perturbed Restricted Three-Body Problem with Straight Segment

Harindri CHAUDHARY (Deshbandhu College, University of Delhi, India)

Co-Authors: Dinesh Kumar, Rajiv Aggarwal

In this paper, we study the effect of small perturbations in the Coriolis and centrifugal forces on the existence and linear stability of the equilibrium points in the restricted three-body problem. The less massive primary is considered as a straight segment. The equations of motion of the infinitesimal body are derived. The problem possesses five equilibrium points out of which three are collinear and two are non-collinear. The non-collinear equilibrium points form a scalene triangle with the centers of the primary bodies. The positions of all the equilibrium points are affected by the mass, length and small perturbations in the Coriolis and centrifugal forces parameters. The linear stability of the obtained equilibrium points is also discussed and it is observed that the collinear equilibrium points are always found to be unstable for all values of the parameters involved, whereas the non-collinear equilibrium points are stable.

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Perturbed Robe's restricted three-body problem with arbitrary density parameter

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The present paper deals with study of the effect of small perturbations ϵ and ϵ' respectively in the Coriolis and centrifugal forces on the existence and stability of the equilibrium points in the Robe's restricted three-body problem when the smaller primary is a finite straight segment. The effect of small perturbation in the centrifugal force has a substantial effect on the location of the equilibrium points, but a small perturbation in the Coriolis force has no impact on them. The present model has two collinear, two out-of-plane and infinite number of non-collinear equilibrium points. Further, we have discussed the stability of the equilibrium points analytically. The collinear equilibrium points L_1 and L_2 are found to be conditionally stable for the parameters μ, k, l, ϵ and ϵ'. It is perceived that for any ϵ with $\mid \epsilon \mid \ll 1$ and $\epsilon' \leq 0.2$, L_1 is stable and for $\epsilon' \geq 0.23$, it becomes unstable. Further, for any ϵ with $\mid \epsilon \mid \ll 1$ and $\epsilon' \leq -0.025$, L_2 is stable and for $\epsilon' \geq -0.024$, it becomes unstable. The non-collinear and out-of-plane equilibrium points are always unstable.

Cassini states of large triaxial icy satellites

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Such as the Moon, the large icy satellites of Jupiter and Saturn are in an equilibrium synchronous rotation state called a Cassini state. In that state, the spin axis, the normal to the orbit and the normal to the Laplace plane remain coplanar, and the precession rate of the rotation axis is nearly equal to the precession rate of the normal to the orbit. The obliquity is nearly constant although the spin axis presents small periodic variations called latitudinal librations. Up to 4 Cassini states are possible for biaxial bodies (Peale, 1969), with obliquity close to 0, $\pi/2$, $-\pi/2$ and π. For the Moon, only two Cassini states (close to 0 and π) are theoretically possible. For the icy satellites, two of them (obliquities close to $\pm\pi/2$) are unstable. We here study the Cassini states of the Moon and of the Galilean satellites and investigate how they are influenced by the triaxiality and the presence of an internal global liquid layer.

Influence of the Multipole Moments in the dynamics at strong and weak limits of a Black Hole Plus Halo system

Fredy DUBEIBE (Universidad de los Llanos, Colombia)

In this work, we study a Newtonian system whose relativistic equivalent describes a black hole superimposed with a halo. We aim to discover how the quadrupole and octupole moments influence the motion of a test particle moving in the close vicinity of the black hole in both regimes. The different types of trajectories for the test particle shall be classified as bounded, collisional, and escaping, using color-coded diagrams. Furthermore, the different types of bounded orbits will be classified as: regular, sticky, and chaotic). It is found that in both regimes in absence of quadrupole moment it is possible to get regions of chaotic motion, however, in the absence of the octupolar moment chaotic motion is only present in the Newtonian regime.

Orbit mapping around solar system moons

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Co-Authors: Antônio Fernando Bertachini de Almeida Prado, Rafael Sfair de Oliveira

The present has the goal of investigating the stability of orbits of a negligible mass probe around several natural satellites of the Solar System, considering the perturbative effects of the third body of their respective planets. Using a system with normalized masses, distance and time, this work integrated the restricted circular three-body problem (RTBP), using the REBOUND integration package, to obtain the temporal evolution of the orbits of the probes implemented around each natural satellite. We investigated natural satellites with mass ratios in the range of 10^{-1} to 10^{-7}, thus encompassing orbits around natural satellites of Jupiter, Saturn, Neptune, and Pluto. The studies were carried out considering intervals of initial conditions, for the probe's orbit, comprised within the region where the Kozai-Lidov effect is present, investigating, in each simulation, the possibility of the probe's escape from the region of interest, a likely collision of the probe with the natural satellite or survival of these orbits at an established total integration time. The results allowed obtaining useful lifetime maps for the probes' orbits, which show zones with acceptable orbital stability for accomplishing possible missions with durations of months or years. As well as it showed a general dependence for the growth of the lifetime of the orbits according to the choice of the initial inclination, eccentricity, and semi-major axis. However, the results also showed the existence of isolated regions on the maps where the recorded lifetimes were greater than their entire neighborhood of initial conditions, configuring island of orbital stability. The location of these regions, as a function of the inclination and, more strongly, the value of the initial semi-major axis, showed an important dependence of its location on the value of the mass ratio of the natural satellite, thus showing the effects due to disturbances third-body.

On the dynamics of the planetary system of Kepler-90

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Co-Authors: Silvia Giuliatti Winter, Othon Winter

The planetary system Kepler-90 has eight planets b, c, i, d, e, f, g, and h, in increasing distance from the star. Planets g and h are similar to gas giants, while planets d, e, and f are similar to super-Earths. They are similar to our Solar System, small planets are closer and the larger ones are distant from the star, although the outer planet has an orbital distance equals to 1AU. Through frequency analysis and long-term evolution, we analyse their stability for a sample of parameters of the planets, such as their masses, semi-major axes and eccentricities. We performed simulations numerical to analyze three different intervals of eccentricity: the first interval from 0 to 0.001, the second interval from 0.001 to 0.01 and the third interval from 0.01 to 0.1. The values of eccentricity, argument of pericentre, longitude of the ascending node, and mean longitude were randomly chosen in each eccentricity interval. Our results showed that the planets which eccentricities belong to the first and second intervals are stable, while most of the planets with large eccentricity, 0.01 to 0.1, are ejected from the system. The variation of the eccentricity of the planets in the two first intervals indicate that the planet h is dominant in the nominal systems being important for the stability of the system Kepler-90. The first interval of eccentricity has two nominal systems where the MMRs appear among the planets, b and c are in 4:5 MMR, and the planets g and h are near 2:3 MMR, corroborating the results obtained by Granados et al (2018). A study of a sample of particles located in this system was performed through the frequency map analysis. It was identified four stable regions between the orbits of the planets c-i, i-d, d-e, and beyond the orbit of planet h that were identified as regions 1, 2, 3, and region 4, respectively. The islands of resonance are identified with the planet i and planet h. Numerical simulations showed that some test particles are close to 2:3, 7:8, and 9:10 MMR with the planet i, and are close to 1:2, 3:4, and 3:8 MMR with planet h.

Robe's Restricted Three-Body Problem when one of the Primaries is a Finite Straight Segment

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Co-Authors: Dinesh Kumar, Shipra Chauhan, Vinay Kumar

In this paper we study the existence and linear stability of the equilibrium points in the Robe's restricted three-body problem when one of the primaries is a finite straight segment. We observe that the center of the first primary is always an equilibrium point for all values of the density parameter k, mass parameter μ and length parameter l. There is an additional equilibrium point lying on the line joining the center of the first primary and the second primary, provided $k > 1 + l^2$. When $k + \mu = 1 + l^2(1 - \mu)$, there are infinite number of equilibrium points lying on a circle with radius $1 - \frac{2}{3}l^2$ and center as the second primary, provided they lie within the spherical shell. When $k < 0$ and $k + \mu + 2\mu l^2 > 0$, there are two more equilibrium points lying in the xz-plane forming triangles with the center of the shell and the second primary. We analyze the linear stability of the equilibrium points.

Restricted Problem of 2+2 Bodies under the effect of Oblateness and Straight Segment

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The present study investigates the combined effects of the oblateness and straight segment on the positions and linear stability of the equilibrium points in the restricted 2+2 body problem. The present model holds fourteen equilibrium points, out of which six are collinear with the centers of the primaries and rest are non-collinear. It has been observed that the positions of all the equilibrium points are subsequently affected by the oblateness and length of the primary bodies. The linear stability of the equilibrium points is also presented by slightly perturbing the position of the equilibrium points. It is observed that for a considered set of parameters, all the fourteen equilibrium points are unstable. An application of the present model is also studied, for which the position and stability of the equilibrium points are investigated for Earth-22 Kalliope-dual satellite system. It has been observed that for this system, all the equilibrium points are unstable except two non-collinear equilibrium points that are found to be stable.

Stability analysis of circular Robe's R3BP with finite straight segment and viscosity

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In this paper, the effect of viscous force on the linear stability of equilibrium points of the circular Robe's restricted three-body problem (CRR3BP) with smaller primary as a finite straight segment has been studied. The present model comprises of a bigger primary m_1^* which is a rigid spherical shell filled with a homogeneous incompressible fluid of density ρ_1 and the smaller primary m_2 lies outside the shell. The infinitesimal mass m_3 is a small solid sphere of density ρ_3 moving inside m_1^*. The pertinent equations of motion of m_3 are derived and solved for the equilibrium points. Routh-Hurwitz criterion is used to detect the stability of the obtained equilibrium points. The stability of the collinear equilibrium points has been studied systematically in the different regions for the various values of the parameters involved. These points are found to be conditionally stable, whereas the non-collinear and out-of-plane equilibrium points are always unstable for all the values of the parameters. We observed that viscosity has no effect on the location of equilibrium points. However its effect along with the length parameter l is evident on the stability of equilibrium points.

Dynamics around the binary system (65803) Didymos

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Didymos and Dimorphos are primary and secondary, respectively, asteroids who compose a binary system that make up the set of Near Earth Asteroids (NEAs). They are targets of the Double Asteroid Redirection Test (DART), the first test mission dedicated to study of planetary defense, for which the main goal is to measure the changes caused after the secundary body is hit by a kinect impactor. The present work intends to do an analysis of the possible changes in the orbital elements of the system after the colission, and conduct a study, through numerical integrations, on the dynamics of massless particles distributed in the vicinity of the two bodies. An approximate shape for the primary body was considered as a model of mass concentrations (mascons) and the secondary was considered as a massive point. Furthermore, it was assumed a density of 2.170 g/cm^3 and the spin period of Didymos as 2.26 h. For Dimorphos was adopted an orbital period of 11.92 h, an eccentricity of 0.05 and semimajor axes of 1178 m. Our results show the location and size of stable regions, which are associated to some specific resonances that were identified.

Building the Galilean moons system via pebble accretion and migration: a primordial resonant chain

Gustavo MADEIRA (São Paulo State University, Brazil)

The origin of the Galilean satellites - namely Io, Europa, Ganymede, and Callisto – is still not fully understood and have been investigated by Canup and Ward (2002, 2006, 2009) and Mosqueira and Estrada (2003a,b), considering it in a circumplanetary disk composed of gas and a large number of satellitesimals. The formation of satellitesimals is only possible under special conditions (Shibaike et al., 2017), presenting an open point in their models and a challenge for the satellites formation models. Other challenging points are the 4:2:1 Laplacian resonance between Io, Europa, and Ganymede and the typical mass distribution of the system. Here, we study the formation of Galilean satellites through numerical simulations performed in the Mercury package (Chambers 1999). We assume that the satellites formed in a gaseous circumplanetary disc around Jupiter, and included in the simulations a family of satellitesimals, the effects of pebble accretion (Ormel and Liu, 2018), gas-driven migration (Adachi et al., 1976, Paardekooper et al., 2010), and gas tidal damping (Cresswell and Nelson, 2008). Satellitesimals first grow via pebble accretion and start to migrate inwards. When they reach the trap at the disc's inner edge or are trapped in mean-motion resonance with an inner satellite, migration ceases and further collisions increase the satellite's mass. At the end of 2 Myr, multi-resonant configurations anchored at the disc inner edge are formed. Our results show that integrated pebble fluxes of $\geq 2e-3\ M_J$ result in satellites with masses typically larger than those of the Galilean satellites, while our best mass results are obtained in simulations with integrated pebble fluxes of 1$e-3\ M_J$. In our best analogue systems, we get four satellites with masses similar to those of the Galileans and with adjacent pairs of satellites locked in 2:1 mean motion resonances. However, they are in excited orbits (0.1), unlike the real satellites. Invoking dynamical tides effects (Fuller et al. 2016) in the post-gas-dissipation phase, the eccentricities are damped to the order of the real satellites and Callisto leaves the resonance chain. We conclude that the Galilean satellites system is a primordial resonant chain, similar to exoplanet systems like TRAPPIST-1, Kepler-223, and TOI-178. We also conclude that Callisto was probably in resonance with Ganymede in the past, but left this configuration via divergent migration due to tidal planet–satellite interactions. Our results are presented in Madeira et al. 2021.

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Study of topographical and dynamical characteristics of Saturn's small moons

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Saturn has the largest number of satellites among all the planets in the solar system, and in this work we focus on its small moons: Daphnis and Pan. Both are located in Saturn's A ring and these bodies share the characteristic of being irregular with equatorial ridges, whereas Daphnis has a diameter of 7.6 km and Pan a diameter of 28.2 km. From the images of these satellites obtained by the Cassini spacecraft, we used the polyhedra method to reproduce the shape of the studied bodies and, knowing the density of each object, we explored some properties such as the gravitational potential, potential speed, slope, equilibrium points and accelerations on the surface of the body. And using the method of mascons, we analyzed the stable regions around the satellites. All this initial analysis was done so that we can, in a second step, study the influence of Saturn on the surface and shape of these small moons.

The analysis of the fractal basins of convergence in the (N + 1)-body ring problem

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The fractal basins of convergence (BoC) linked with the libration points are Investigated in the (N + 1)-body ring problem under the effect of small perturbations in the Coriolis and centrifugal forces when the primary bodies are sources of radiation. The evolution of the positions and the stability of the libration points and the possible regions of motion are determined as the function of mass parameter, the Coriolis, and centrifugal and radiation parameters. The multivariate version of the Newton–Raphson(NR) iterative method is used to analyze the effect of the radiation parameters and mass parameter on the topology of the BoC.

Orbits generated by mobile coordinates using the predictor-corrector method

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The present paper deals with the periodic orbits generated by Lagrangian solutions of the restricted three-body problem when both the primaries are oblate bodies. We have illustrated the periodic orbits for different values of μ, h, σ_1 and σ_2 (h is energy constant, μ is mass ratio of the two primaries, σ_1 and σ_2 are oblateness factors). These orbits have been determined by giving displacements along the tangent and normal to the mobile coordinates as defined by (Karimov and Sokolsky, 1989). We have applied the predictor-corrector algorithm to construct the periodic orbits in an attempt to unveil the effect of oblateness of the primaries by taking the fixed values of parameters μ, h, σ_1 and σ_2.

On the non-Existence of Coorbitals among the Galilean Satellites

Daniela MOURãO (UNESP - São Paulo State University, Brazil)

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If on one hand, Saturn has three coorbital satellites systems, on the other, Jupiter does not have any. The main goal of the present work is to investigate why there is no coorbital body related to Jupiter's Galilean satellites: Io, Europa, Ganymede and Callisto. The first three Galilean satellites, Io, Europa and Ganymede are in a 1:3:2 resonance, known as Laplacian resonance. Then, we first analysed the stability of the regions coorbital to each one of the four satellites performing long period simulations of massless particles in such regions. The results showed that for all four satellites, a significant part of the particles remained stably confined in coorbital trajectories (tadpole and horseshoe). Particles initially disposed in large angle horseshoe orbits, that approach up to 10 degrees from the satellite, were ejected or collided with the respective satellite. We found that the angular amplitude of the separatrix region, between the tadpole regions, around the L3 point, was more prominent for Europa (76 degrees), while it was much smaller for Callisto, only 4 degrees. In order to identify the possible influence of the current resonances, we repeated the simulation, but with Io and Ganymede displaced by 5 percent of their current semi-major axis. So that, they were closer than their current positions and not in the Laplacian resonance anymore. The results were very similar. Actually, the regions of coorbital stable orbits expanded in size. Finally, we explored other configurations that the satellites might have had in the past. All results will be presented.

Astronomy Education in Cameroon

Mbonteh Roland NDUNGE (Cameroon Astronomy and Space Research Organization, Cameroon)

Structure of education: Cameroon has two educational sub-systems (English and French) operating simultaneously, one based on its British colonial past and the other on its French colonial past. We have free state Primary and Subsidized Secondary Education. In government schools, primary education became free for all children in Cameroon in the year 2000 but parents pay minimal Parent-Teacher Association (PTA) levies. The government is the largest provider of primary education. In the cities, the number of privately-owned primary schools are rising but they charge very high fees for higher quality education, involving ICT. Secondary/high school education is equally highly subsidized by the government and some households can afford it. But there are many private and prestigious mission secondary/high schools which charge very high fees, families who can afford them or students who do not gain admission into government secondary/high schools resort to the mission/private institutions. The percentage of students advancing to the Ordinary Level and the number of students attending universities have increased as a result of the government's policy of creating schools in every neighborhood and at least a university in each region of the country. Cameroon's education can be described as follows: 3 years optional pre-school (1 year pre-nursery and 2 years nursery) from age 3 or 5, 6 years in basic primary education (for children aged 5-11), 5 years of secondary education (ages 11-16), 2 years of high school education (ages 16-20). The elementary level classes are known as primary, while the secondary ones are known as forms and the high school ones are known as lower and upper sixth forms.
Education facilities: Class sizes range from 10 to 500 per class. Most Cameroon schools have access to running water and average internet connections. In some smaller communities without secondary schools, students cover very long distances everyday to go to attend schools in different communities with schools, while a few that can afford rent in halls closer to Schools. Staying away from home is also common private schools. School buildings are well-managed.
Governance and organisation: The National Ministry of Basic Education develops the primary school curriculum while the Ministry of Secondary Education develops the secondary school curriculum. Public schools are run by the central government. Private schools are run as a social service provided by private partners with support from central, regional and local government.
Teacher Training: Primary and secondary school teachers study undergraduate degrees (three or four years) in education at a university, then write entrance exams into the training field, After one and two more years respectively, they become qualified for teaching. Or after obtaining the A/L, they can write entrance concours into the training field and spend two and three more years respectively. There is no specific way for university lecturers to be trained. After your PhD, you can start teaching as a private lecturer. In 2020, the president of the Republic recruited PhD holders by a presidential degree. The recruitment of 2000 lecturers through evaluation of documents will end in 2021.
Astronomy in the curriculum: There are no specialised school courses in astronomy in Cameroon. Some topics of Astronomy like the solar system, the Galaxy, planets, moon, sun, gravity, etc. are seen from the first year of secondary school Physics, Geography, Geology/SVTEEHB etc.
Astronomy education outside the classroom: There is only the Astronomy Club of Cameroon

carrying out Astronomy activities, created by Mbonteh Roland Ndunge. The only Astronomy materials we have are three Bresser telescopes and an SSV Itelescope. But we have formed a solid base since the NOC Committee members are all over the country. That would help us to easily disseminate Astronomy Education in the ten regions of the country if we have the resources. There is more to be done here for Astronomy Education.
The International Astronomical Union's National Astronomy Education Coordinator (NAEC) Team for Cameroon: Monte Roland Ndunge (Contact), Tchana Kamgne Duclair, Tchaptchet William Christian (Chair), Njiki Chatué Colette, Zeugue Donfack Lynda
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For specific information about astronomy education in Cameroon or on this document please contact the Office of Astronomy for Education (oae@astro4edu.org).

Analysis of the characteristics of surface and dynamics in the vicinity of the binary system (90) Antiope

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(90) Antiope is a binary asteroid system located in the main belt. Each component (alpha and beta) measures approximately 80 km in diameter. Through computer simulations using data obtained by radio telescopes it is possible to explore in detail the dynamics in the surface and in the vicinity of the binary system. The shape of these bodies can be represented by the method of polyhedra. Assuming the value of 1.67 g cm^{-3} for the density, and considering that they are homogeneous, it is possible to determine the gravitational potential through numerical integrations based on faces and vertices that make up the polyhedron of each body. We then determine dynamical features such as the zero velocity curves, the equilibrium points, and find stability regions around the system. Another method used to address the dynamics in the vicinity of the bodies is by representation in terms of mascons (mass concentration). From there, an irregularly shaped body is approximated by a set of mascons properly placed to produce the object mass distribution. In this way, simulations will be carried out to evidence the effects that each body has on the surface of the other. These results might be useful for space exploration around this asteroid.

Terrestrial Planet Formation constrained by Mercury collisions

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The planet Mercury is predominantly made up of an iron core covered by a thin layer of silicates, which has led to the idea that this configuration is the product of a giant impact. In particular, the impact called classic hit-and-run has been explored, in which a proto-Mercury with mass 0.1 $M_\oplus$ collides with a target with mass 1 $M_\oplus$, losing part of the material of your mantle. Simulations of the formation of terrestrial planets, using numerical algorithms of N-bodies, have been shown to be incapable of producing an object with the characteristics of Mercury. Part of this limitation is due to the fact that, in this type of simulation, collisions are always treated as inelastic, making the resulting body mass the sum of the masses of the two colliding bodies. Furthermore, we observe that configurations that seek to explain Mercury by a giant impact such as those required by the classic hit-and-run scenario are rare, even when there are a lot of collisions on the accretion disk. On the other hand, we found that hit-and-run collisions, different from the classical scenario, in which the target and projectile masses are similar, occur much more frequently in N-body simulations. In this work, we aim to investigate whether this last type of collision can favor the formation of Mercury using hydrodynamic numerical simulations (SPH). Our results indicate that considering targets with masses similar to that of proto-Mercury, it is possible to obtain the desired result as long as the impact energy is sufficiently high, but still compatible with the values observed in the N-body simulations.

Stady of stability of the Sailboat stable region for binaries system

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The *sailboat* is a stable region in the Pluto-Charon system discovered by Giuliatti Winter, et al. (2010), who showed this region is located at $a = (0.5d, 0.7d)$ and $e = (0.2, 0.9)$, where a and the initial values of semi-major axis and eccentricity of particles, respectively and d is the separation of the binary. The *sailboat* is associated with a family "BD" of periodic orbits derived from the planar, circular, restricted three-body problem. In this work, we analyzed through numerical simulations the structure and stability of *sailboat* in hypothetical systems with different values of mass ratio (μ) and for several orbital configurations. Our results show the sailboat is robust and it exists for $\mu = [0.01, 0.27]$ and for large intervals of argument of pericentre and inclination.

The evolution and fate of particles ejected from Distant Retrograde Orbits around the Moon

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Recently has grown the interest in placing natural objects in the neighborhood of the Moon and explore them commercially (Graps et al. 2016) or scientifically (Brophy et al. 2012; Condon $\&$ Williams 2014; Mazanek et al. 2016). Pires $\&$ Winter (2020) found Distant Retrograde Orbits (DRO) around the Moon with semi-major axis (a) values of the initial osculating orbits, varying between 110,000 and 185,000km and eccentricity (e), following the equation, $e = 2.25963 \times 10^{-6}a + 0.23845$ (standard error of 1$\%$), remaining stable for time enough to be useful for lunar missions, such as the ones that would require some decades staying around the satellite and use stable distant orbits. Near-Earth asteroids are identified as good alternative sources of useful materials, such as water, metals, and semiconductors (McKay et al. 1992; Lewis 1997). In this work, we investigate the evolution and fate of ejected particles, considering various initial speeds, from those stable DRO that we previously found. Asteroid mining can produce regolith in space, so it is important to understand their dynamics in the Earth-Moon system to reduce the risk of impacts with Earth. The dynamical evolution of particles leaving the stable DRO region is complex and the particles may reach the Earth's or the Moon's surface, or even escape the Earth-Moon domain.

Secular evolution of resonant asteroids: semianalytical approach for arbitrary eccentricities

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In this work we study the secular evolution of a body of mass m_1 perturbed by a body of mass m_2, both orbiting a central body of mass m_0, such that $m_0 >> m_2 >> m_1 \simeq 0$ (restricted problem). A common example would be an asteroid perturbed by a planet, both orbiting a star. We consider the planar problem but no restriction on the eccentricity of the particle nor the planet are imposed. We also assume that the particle is in mean motion resonance with the massive body. The methodology used is based on a semi-analytical model (Gallardo 2020) that consists on calculating the averaged resonant perturbing function numerically. We found a method to obtain the equilibrium points in the space (σ, e_1, ϖ_1) and also, in the cases where the center of libration of σ remains constant in large scales of time, we show that the secular evolution of e_1 and ϖ_1 is given by the contour levels maps of the function $R(e_1, \varpi_1)$. The first order resonances 2:1, 3:1 and 3:2 are used as examples of some results.

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Mapping the structure of the planetary 2/1 mean motion resonance. The TOI-216, K2-24, and HD27894 systems.

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Mean motion resonances (MMR) are a frequent phenomenon among extra-solar planetary systems. Current observations indicate that many systems have planets that are close to or inside the 2/1 MMR, when the orbital period of one of the planets is twice the other. Analytical models to describe this particular MMR have the limitation of the many degrees of freedom of the system, which can only be reduced to integrable approximations in a few specific cases. There have been several successful approaches using semi-analytical or semi-numerical methods, yet these are not enough to completely understand the resonant dynamics. In this work, we propose to apply a well-established numerical method to assess the global portrait of the resonant dynamics, which consists of constructing dynamical maps. These maps may help to understand the underlying resonant structure, helping to identify the different behaviours that can be expected in different regions of the phase space and for different values of the model parameters. We are able to validate previous results from semi-analytical models, that show that the family of stable resonant points bifurcate from symmetric to asymmetric librations, depending on the planets' masses and eccentricities. We construct dynamical maps for three extra-solar planetary systems, TOI-216, K2-24, and HD27894, and discuss their stability in the light of the many orbital parameters fits available in the literature.

The effect of small perturbations in the Coriolis and centrifugal forces in the axisymmetric restricted five-body problem

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In the framework of the axisymmetric problem of restricted five bodies, the existence and stability of the libration points, the regions of possible motion are illustrated and analyzed numerically, under the effect of small perturbations in the Coriolis and centrifugal forces. It is explored how the parameters influence substantially the positions of the libration points and the possible regions of motion. In an attempt to understand how the parameters involved due to the small perturbations in the Coriolis and centrifugal forces affect the stability of the libration points, we perform a systematic investigation and reveal that some of the collinear and non-collinear libration points are stable under these perturbations, whereas none of these libration points are stable for any combination of the angle parameters when the effects of these forces are neglected.

Constraining the nature of the possible extrasolar PDS110b ring system

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The star PDS110 underwent two similar eclipses in 2008 and 2011, which lasted around 25 days each. One plausible explanation for these events is that an unseen giant planet (PDS110b) eclipsed the star, and this planet is circled by a ring system that fills a significant fraction of its Hill sphere. Since several physical parameters of the planet could not be determined from the observational data, we propose to constrain the mass and eccentricity of this planet and the size and inclination of the ring through thousands of numerical simulations of the three-body problem. The simulations were performed with the *Rebound* package assuming a wide range of different orbital configurations of the planet and particles. Each simulation was carried out for 10^4 orbital periods of the particle, and a system was considered unstable if a collision or ejection happened. Although our results show that a stable ring system with a radial extent that matches the observed eclipse can be found both for prograde or retrograde configuration, the preferred solution is that the ring has an inclination lower than 60○ and a radius between 0.1 and 0.2 au and that the planet is more massive than 35MJup and has a low eccentricity (< 0.05).

Apse alignment in migrating dust generating crescents in planet-harbouring debris discs

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A circumsolar disc of dust has long been known to pervade the space inhabited by the planets, known as the zodiacal cloud. Constantly replenished by comets and asteroids, dust grains of a given size slowly migrate inward under Poynting-Robertson drag, until being destroyed in the vicinity of the Sun. As the dust traverses orbital regions, its spatial distribution is shaped by the gravitational interaction with the planets. In particular the so called resonant rings, a circumsolar toroidal structure generated by entrapment of dust in certain mean-motion resonances, have been subject of numerical as well as observational studies. Here we report on another feature generating effect that may occur in migrating dust discs of planetary systems. We show numerically, that moderately eccentric planets (such as Mars) induce a directed apsidal precession with preferred final apse line orientation in particles as they traverse the orbital region of the planet. The consequence are two mirrored crescent-shaped features in the disc's density distribution that remain stationary with respect to the apse line of the perturbing planet. We analyse the strength of this effect with respect to planetary, as well as particle parameters. The emergence of the apse alignment pattern even with planets incapable of producing a meaningful resonant enhancement — due to their high orbital eccentricity or low mass — is especially noteworthy. These findings may be particularly relevant to surveys of feature-bearing, and thus potentially exoplanet harbouring debris disks around distant stars.

The analysis of fractal basins of convergence in the photogravitational restricted five-body problem

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In the framework of photogravitational version of the restricted five-body problem, the existence and stability of the in-plane equilibrium points, the possible regions for motion are explored and analysed numerically, under the combined effect of small perturbations in the Coriolis and centrifugal forces. Moreover, the multivariate version of the Newton-Raphson iterative scheme is applied in an attempt to unveil the topology of the basins of convergence linked with the libration points as function of radiation parameters, and the parameters corresponding to Coriolis and centrifugal forces.

The generalized Levi-Civita and Kustaanheimo-Stiefel transformations

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It is known that the regularizing technique is very useful for n-body simulations to handle close encounters. The generailzed Levi-Civita and Kustaanheimo-Stiefel transformations are presented, which provide us a whole family of polynomial functions having the same topology. Even if the Levi-Civita and Kustaanheimo-Stiefel regularization methods are easier to use, it is interesting to encapsulate these in a family of methods, which all conserve the transformations' properties. Applying the generalized Levi-Civita and Kustaanheimo-Stiefel transformations in the study of regularization, it can give an overview of the orbit's behavior around the singularities.

Cascade disruption in Rampo family

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Pravec and Vokrouhlický (2009) discovered the Rampo family with only three members since that number of members belonging to this cluster has increased up to 7 (Pravec et al. 2018). Kuznetsov and Vasileva (2019) discovered six new members. Pravec et al. (2018) gave formal age estimation of this family as 780 ± 130 kyrs. However, they provide very different values of age for each pair in cluster. For this goal, we integrate orbits of Rampo cluster in Mercury6 package in 1800 kyrs. We use the convergence of orbital elements and velocity relative to the main member of family (10321) Rampo. We outline four groups of orbits converged about 250 kyrs – 2015 TA367; 500 kyrs – (294272) 2007 UM101; 750 kyrs – 2006 UA169, 2010 VO19, 2013 RL101, 2013 VC30, 2014 HS9, 2015 HT91, and 2016 TE87; and 900 kyrs – (451686) 2013BR67, 2009 HD95, 2009 SR371, 2013 JF69, 2015 TM372, and 2017 UH21. We can conclude that there is a very low possibility that the Rampo family was formed in a single breakup event. This conclusion lies in agreement with a novel idea about the cascade breakup of some young asteroid families (Fatka et al. 2020). Rampo family asteroids have very small but nonzero eccentricities. For this reason, perihelion argument sometimes has change precession to regression motion. This fact complicates the condition of the orbital elements convergence and required future studying.

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The algorithm for automatic identification of asymmetric transits in the TESS database

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Currently, the Transiting Exoplanet Survey Satellites (TESS) searches for Earth-size planets around nearby F, G, K - dwarf stars. Analysis of High-quality light curves collected by this space mission ALLOWs potentially identify some specific variations in the star. Sophisticated data processing methods and analysis of the brightness curve shapes should be developed to solve the problem because of the vast data sets and the very weak manifestation of this physical phenomenon. We report some preliminary results of the work in progress to identify the minima in the star brightness from the TESS pipeline data collected in the MAST database. We discuss the developed code based on the Python package "lightkurve" (Barensten et al., 2019) for the processing of the short-cadence (2-min) TESS light curves. The code allows identifying minima in the light curves taking into account the specific variation of the light curves and some artifacts that pollute the data. The proposed results are intermediate and will serve for the identification of exocometary transits applying machine learning methods.

Astrometry and photometry of asteroids from the UkrVO database of astroplates

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We present the developed methods of digitization, image processing, reduction, and scientific data mining with the latest reference catalogs, which allowed us to obtain a good positional and photometric accuracy in B-band of 6,500 asteroids down to 17.5m from database of old photographic astroplates. These values, distribution, and types of asteroids from the published two catalogs related to the FON-Kyiv and Fon-Kitab sky surveys are compared with current estimates for the FON-Dushanbe survey and astroplate archives of the Baldone and Tautenburg observatories. For some of asteroids, observations are either completely absent or not enough over the certain time interval to the moments of their official discoveries (about 300 such objects were found). Positional observations for this long-term period are highly useful for a more detailed study of the dynamics and orbital parameters of asteroids as well as the obtained photometric parameters are very complementary with present-day data for studying changes in brightness and light curves.

Families of Lyapunov periodic orbits with continuation in radiation and albedo parameter

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The goal of our study is to investigate the evolution of periodic orbits in the restricted three body problem under the influence of radiation pressure and albedo. Lyapunov periodic orbits about the collinear points are generated from continuation in radiation and albedo parameter, and found that periodic orbits remains unstable. Moreover, change in the structure of eigenvalues and stability indices with respect to radiation and albedo parameter are analyzed. Due to continuation in the radiation parameter, new family of periodic orbits are identified.

Martian Rotation and Orientation Angles: transformation between the Euler angles and the Earth equatorial coordinates

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Co-Authors: Rose-Marie Baland, Sebastien Le Maistre, Veronique Dehant

Mars rotation models are regularly updated based on the precise radioscience data coming from Martian orbiters and landers and new theoretical developments. Two precise radioscience experiments are ongoing or will arrive soon: the RISE experiment on board the InSight mission (Folkner et al., 2018) and the future LaRa experiment on ExoMars 2022 (Dehant et al., 2020). Two different sets of angles are commonly used in the rotation matrix transforming the coordinates from the body frame to an inertial frame: the Euler angles versus the IAU formulation with the Earth equatorial coordinates. We present the transformations between the 2 sets. We compute the initial values of the angles, their temporal drift (diurnal rate or precession rate) and their quadratic term, and their periodic variations (nutations and length-of-day LOD variations). The objective is to have a rotation matrix difference smaller than 1 mas. The improvements proposed in the transformations mostly affect the rotation angle (or LOD) variations and the quadratic terms.

The dynamical evolution of planar and inclined planetesimal disks in S-type motion in binary star systems

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We investigate the dynamical evolution of circumstellar protoplanetary disks which are inclined w.r.t. the binary stars orbital plane. In this study, we focus on the late stages of terrestrial planet formation when planetesimals and planetary embryos have formed and the gas has dissipated. For simplicity the secondary star is inclined w.r.t. the plane of the protoplanetary disk around the primary star. We use different binary star configruations where we vary the seperation of the binary stars between 30 and 60 au, the eccentricity between 0.0 and 0.2 and the inclination between 0 and 45°. Both stars have a mass of 1 M_{sun}. The protoplanetary disk contains some thousands planetesimals and a few tens planetary embryos. They are distributed between 1 and 4 au around the primary star and are initially dynamically cold. The gravitaional interaction of all objects has been simulated for 1 Myr in each binray configuration using a newly self-developed highly parallelized GPU n-body integrator [1] which uses the Bulirsch-Stoer integration method and the so-called "perfect merging" in case of collisions. The simulations show that the inclined binary star system leads to a vertical distribution of the disk objects. In all inclined cases the disk objects oscillate between 0 and $2 * i_b$ (i_b the inclination of the binary star system). Thus the planetary embryos tend to grow slower than in the planar case. However, a binary star system with an inclination of 45° shows a slight inward migration of the inner disk objects (independently of the binary stars seperation and eccentricity). While a planar binary configuration shows an outward migration of the outer planetesimals. The overall result of our study indicates different timescales for the growth of terrestrial planets in planar and inclined close binary star systems.

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