13th International ISAAC Congress
August 2–6, 2021 - Ghent, Belgium

Conference programme

Overview

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00–10:00	10	2,4,8,10 11,13	4,6,10,11,14 + JOINT (7,8,13)	3,6,7,8 12,14,16	1,3,6,7 8,14,16
10:00–10:30	Break	Break	Break	Break	Break
10:30–12:00	10,15	2,4,8,10 11,13,15	4,6,10,11,14,15 + JOINT (7,8,13)	3,6,7,8 12,14,16	1,3,6,7 8,14,16
12:15–13:00	**Opening**	Break	Break	Break	Break
13:00–14:00	de Hoop	Break	Break	Break	Break
14:00–15:00	De Philippis	Kaltenbacher	Kuijlaars	Jaffard	Seip
15:00–16:30	2,3,5,7,8,14	1,3,5,7 9,12,14	1,3,7,8,10,12,14	2,4,5,8,10 13,14,15	4,9,10,15
16:30–17:00	Break	Break	Break	Break	Break
17:00–19:00	2,3,5,7,8,14	1,5,7 9,12,14	1,3,7,8,10,12,14	2,4,5,8,10 13,14,15	4,10 18:30 **Closing**

All times are in time zone UTC+2.

Session 5 Constructive Methods in the Theory of Composite and Porous Media

Organizers: Vladimir Mityushev, Natalia Rylko and Piotr Drygaś

	Monday	Tuesday	Wednesday	Thursday	Friday
12:00	12:15: **Opening**				
13:00	**de Hoop**				
14:00	**De Philippis**	**Kaltenbacher**	**Kuijlaars**	**Jaffard**	**Seip**
15:00	Kolpakov	Andrianov		Zaccaron	
15:30	Natroshvili	Kurtyka		Wojnar	
16:00	Kakulashvili	Stawiarz		Bosiakov	
16:30	Break	Break		Break	
17:00	Gric	Necka		Ashimov	
17:30	Gulua	Nasser		Krzaczek	
18:00	Giorgadze	Drygas			
18:30	Paszkuta	Mityushev			**Closing**

Session 5: Constructive Methods in the Theory of Composite and Porous Media

Organizers: Vladimir Mityushev, Natalia Rylko and Piotr Drygaś

ESTIMATIONS OF THE EFFECTIVE HEAT CONDUCTIVITY OF A COMPOSITE WITH HEXAGONAL LATTICE OF PERFECTLY CONDUCTING CIRCULAR INCLUSIONS

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Joint work with G. A. Starushenko, S. A. Kvitka

The problem of thermal conductivity for a composite of a hexagonal structure with perfectly conducting circular inclusions is considered in non-dilute case. Namely, we suppose $a = r/l \leq 1/\sqrt{3}$, where r is the radius of inclusion and l is the distance between opposite sides of periodic cell of lattice. The use of the lubrication theory [2] (thin layer asymptotics [6]) and homogenization procedure [5] leads to the following expression of the effective thermal conductivity at $\lambda \to \infty$ (parameter λ characterizes the contrast of matrix and inclusions properties) [4]:

$$q = F_1(a) + F_2(a), \tag{1}$$

where

$$F_1(a) = 1 + \frac{2\sqrt{3}a^2}{\sqrt{1-a^2}}\left(\arctan\frac{\sqrt{3}}{3\sqrt{1-a^2}} + \frac{2}{3}\arctan\sqrt{\frac{1-a}{1+a}}\right) - \frac{\sqrt{3}\pi a^2}{6},$$

$$F_2(a) = \frac{\sqrt{3}a^2}{\sqrt{1-a^2}}\left(\frac{2}{3}\arctan\frac{a}{\sqrt{1-a^2}} - \arctan\frac{\sqrt{3}}{3\sqrt{1-a^2}}\right).$$

Approximation (1) does not give the correct asymptotics for $a \to 1,\ \lambda \to \infty$. Improving of it is achieved using the Padé approximation. Expanding the function F_2 into a power series and rearranging the first 12 terms into a fractionally rational function according to the Padé scheme one obtains

$$F_2(a) \approx q_{[2/10]} = F_3(a). \tag{2}$$

New expression for the effective thermal conductivity is:

$$q + 1 = F_1(a) + F_3(a) \tag{3}$$

gives a good approximation for any inclusion size $0 \leq a < 1$ and has the asymptotics

$$q_1 \to \frac{\sqrt{3}\pi}{\sqrt{1-a^2}} - 6.345 \text{ at } a \to 1,$$

whose main term coincides with the well-known expression [1].
The approximate solutions q [4] and give, respectively, the upper and lower estimates of the reduced parameter and the gap between them is small for any size of the inclusions $0 \leq a < 1,\ a \to 1.$

References

1. Berlyand L., Novikov A. (2002) Error of the network approximation for densely packed composites with irregular geometry. SIAM J. Math. Anal. 34(2), 385-408.
2.Christensen R.M. Mechanics of Composite Materials (Dover Publisher: Mineola, NY, 2005).
3.Gluzman S., Mityushev V., Nawalaniec W., Starushenko G. (2016) Effective conductivity and critical properties of a hexagonal array of superconducting cylinders. In: Contributions in Mathematics and Engineering. In Honor of Constantin Carathéodory. Eds. Pardalos P.M., Rassias T.M. Switzerland: Springer International Publishing, 255-297.
4.Kalamkarov A.L., Andrianov I.V., Pacheco P.M.C.L., Savi M.A., Starushenko G.A. (2016) Asymptotic analysis of fiber-reinforced composites of hexagonal structure. J. Multiscale Model. 7(3), 1650006-1-1650006-32.
5.Lions J.-L. (1982) On some homogenisation problem. ZAMM 62(5), 251-262.
6.Tayler A.B. Mathematical Models in Applied Mechanics (Oxford University Press, 1986).

Optimal design problem for three disks on torus

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Joint work with V. Mityushev, K. Dosmagulova, Zh. Zhunussova

The talk is devoted to recently established connection between the packing problem of disks on torus and the effective conductivity of composites with circular inclusions [1]. The packing problem is usually investigated by geometrical arguments, the conductivity problem by means of elliptic functions. An algorithm is developed in order to determine the optimal location of three disks on torus formed by the hexagonal lattice. The corresponding minimization function is constructed in terms of expressions consisting of elliptic functions with unknown arguments. The numerically found roots coincide with the previously established optimal points by a pure geometrical study.

[1] Mityushev, V.; Rylko, N. Optimal distribution of the non-overlapping conducting disks. Multiscale Model. Simul., 10, 180–190, 2012.

Fractional-differential model of the viscoelastic periodontal ligament

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The periodontal ligament is a complex component of the tooth root supporting apparatus. It's a thin layer of soft connective tissue located between the hard surfaces of the tooth root and the dental alveolar bone. One of the approaches to analytical modeling of the viscoelastic periodontal ligament behavior is the usage of a fractional kernel. The aim of this study is the assessment of the parameters of the Rabontov's fractional relaxation kernel during the tipping of the asymmetric tooth root.

The assessment of the relaxation kernel parameters was carried out on the basis of experimental dependences (R.J. Pryputniewicz and C.J. Burstone, 1979) of the tooth root displacements in the periodontal ligament under the action of a concentrated load. It was appeared that for any load, the simultaneous change of the parameter $\nu_\sigma = \frac{E_\infty - E_0}{E_0}$ and the fractional parameter γ enables to assign different time intervals for the phase transition of the periodontal ligament and the maximum magnitude of the tooth root displacements in the periodontal ligament. The maximal displacement magnitude and angle of rotation for different load values can be assign by changing the parameter ν_σ. An increase of the fractional parameter leads to an increase of the duration of the phase transition and the magnitude of the tooth root maximum displacement (for constant values of the parameters ν_σ and the retardation time τ_σ).

Study was supported by project Ph20R-083 of the Belarusian Republican Found for Fundamental Research.

Random elastic composites with circular inclusions

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Joint work with S. Gluzman, V. Mityushev, W. Nawalaniec

Consider 2D multi-phase random composites with different circular inclusions. A finite number n of inclusions on the infinite plane forms a cluster. The corresponding boundary value problem for Muskhelishvili's potentials is reduced to a system of functional equations. Solution to the functional equations can be obtained by a method of successive approximations or by the Taylor expansion of the unknown analytic functions. Next, the local stress-strain fields are calculated and the averaged elastic constants are obtained in symbolic form. An extension of Maxwell's approach and other various self-consisting cluster methods from single- to n- inclusions problems is developed. An uncertainty when the number of elements n in a cluster tends to infinity is analyzed by means of the conditionally convergent series. Application of the Eisenstein summation yields new analytical approximate formulas for the effective constants for macroscopically isotropic random 2D composites.

Monodromy of Pfaffian equations for group-valued functions on Riemann surfaces