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ТЕЗИСЫ ДОКЛАДОВ

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[3] Sadybekov M.A., Dukenbayeva A.A. On boundary value problem of the Samarskii-Ionkin type for the Laplace operator in a ball, *Kazakh Mathematical Journal*, **20**:1 (2020), 84–94.

[4] Sadybekov M.A., Dukenbayeva A.A. On boundary value problems of the SamarskiiIonkin type for the Laplace operator in a ball, *Complex Var. Elliptic Equ.*, (2020) DOI:10.1080/17476933. 2020. 1828377.

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COMPACTNESS RESULTS FOR VARIABLE EXPONENT SPACES

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Let Ω be an open subset of $\mathbb{R}^N$, and let $p,\ q : \Omega \to [1,\infty]$ be measurable functions. We give a necessary and sufficient condition for the embedding of the variable exponent space $L^{p(\cdot)}(\Omega)$ in $L^{q(\cdot)}(\Omega)$ to be almost compact. This leads to a condition on Ω, p and q sufficient to ensure that the Sobolev space $W^{1,p(\cdot)}(\Omega)$ based on $L^{p(\cdot)}(\Omega)$ is compactly embedded in $L^{q(\cdot)}(\Omega)$; compact embedding results of this type already in the literature are included as special cases.

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Keywords: almost-compact embeddings, Banach function spaces, variable Lebesgue spaces, variable Sobolev spaces.

2010 Mathematics Subject Classification: 46E30, 26D15

References

[1] Fiorenza A., Gogatishvili A., Nekvinda A., Rakotoson J. M. Remarks on compactness results for variable exponent spaces $L^{p(\cdot)}$, *J. Mathématiques Pures et Appliquées,* accepted 2021.

[2] Edmunds D.E., Gogatishvilli A., Nekvinda A. Almost-compact and compact embeddings of variable exponent spaces, *arXiv*, 2101.00182, 2021.

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ON A NUMERICAL METHOD FOR SOLVING A NONLINEAR BOUNDARY VALUE PROBLEM FOR DIFFERENTIAL EQUATION WITH DELAYED

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Due to applications in physics, biology, epidemiology, and so on, much of the literature on lagged differential equations has focused on the existence of a periodic solution, oscillation, and so on.

The importance and variety of applications served to increase interest in the theory of boundary value problems for differential equations with a delay argument, and the development of computer technology and its comprehensive application in applied problems presented new requirements for the developed methods, paying special attention to their constructibility and feasibility.

One of the constructive methods widely used for the study of boundary value problems for differential equations is the parametrization method of D.S.Dzhumabaev [1]. In [2], on the basis of one modification of the parametrization method algorithms, sufficient conditions were obtain for the existence of an isolated solution of a boundary value problem for a system of linear differential equations with a delay argument that satisfies essentially nonlinear boundary conditions.

We consider a nonlinear boundary value problem for a system of differential equations with delay argument

$$\frac{dx}{dt} = A(t)x(t) + B(t)x(t-\tau) + f(t), \quad x \in R^n, \quad t \in (0,T), \quad \tau > 0, \tag{1}$$

$$x(t) = \mathrm{diag}[x(0)] \cdot \varphi(t), \quad t \in [-\tau, 0], \tag{2}$$

$$g(x(0), x(T)) = 0, \tag{3}$$

where $(n \times n)$-matrices $A(t)$, $B(t)$ and the function $f(t)$ are continuous on $[0,T]$, $\varphi : [-\tau, 0] \to R^n$ is a continuously differentiable functions such that $\varphi_i(0) = 1$, $i = 1 : n$, τ is a constant delay such that $T = N\tau$ $(N = 1, 2, \ldots)$, $\|A(t)\| \leq \alpha$, $\|B(t)\| \leq \beta$, where α, β are constant.

The solution of the boundary value problem (1)-(3) is a continuous on $[-\tau, N\tau]$, continuously differentiable on $[-\tau, 0] \bigcup [0, N\tau]$ vector function $x(t)$ that satisfies the differential equation (1) and has values $x(0), x(T)$, for which equalities (2), (3) are valid.

In [3], in terms of input data, necessary and sufficient conditions for the existence of an isolated solution to a periodic boundary value problem for a system of nonlinear delay differential equations were established. In [2], on the basis of modified algorithms of the parametrization method, sufficient conditions were obtained for the existence of an isolated in some ball solution to problem (1) - (3) for a system of linear delay differential equations subject to essentially nonlinear boundary conditions. Following the standard scheme of the parametrization method, problem (1) - (3) was reduced to a multipoint boundary value problem with parameters. In the course of proving the conditions for the existence of a solution to the problem with parameters, a procedure for the sequential construction of its solution is shown, which includes the solution of a system of nonlinear algebraic equations of a special structure and the solution of Cauchy problems on the partition subintervals. Estimates were established for the difference between the exact solution and the solution found at a certain step.

The algorithms of the parametrization method are quite suitable for developing a numerical method for solving the problem under study. To construct the equations of the system in unknown parameters (the values of the desired solution at the partition subintervals $[-\tau, N\tau]$), methods of numerical solution of the Cauchy problem are used. To solve the constructed system of nonlinear algebraic equations, a sharper version of the local Hadamard theorem [4] is applied. Using the found parameters, we find the numerical solutions of the Cauchy problems associated with equation (1), thereby determining the numerical values of the desired solution at the points of the partition subintervals.

Thus, the numerical method proposed in the present paper is a balanced combination of known numerical methods. It should be noted that the results numerical experiments carried out on test problems are fairly good at the first steps of the algorithms of the parameterization method.

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Keywords: boundary value problems, equation with delay argument, isolated solution.

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References

[1] Dzhumabaev D.S. *Conditions the unique solvability of a linear boundary value problem for an ordinary differential equations*, *Computational Mathematics and Mathematical Physics*, **29**:1 (1989), 34–46.

[2] Iskakova N.B., Temesheva S.M., Abildayeva A.D. On conditions of solvability of a nonlinear boundary value problem for a system of differential equations with a delay argument, *Kazakh Mathematical Journal*, **20**:4 (2020), 58–73.

[3] Iskakova N.B. Correct solvability of a periodic boundary value problem for a system of differential equations with a delay argument, *J. of Math., Mech. and Comp. Sci.*, **45**:2 (2005), 35–46. (in Russian).

[4] Dzhumabaev D.S., Temesheva S.M. A parametrization method for solving nonlinear two-point boundary value problems, *Comput. Math. Math. Phys.*, **47**:1 (2007), 37–61.

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A PROBLEM FOR ESSENTIALLY LOADED DIFFERENTIAL EQUATIONS

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We consider the following linear boundary value problem for essentially loaded differential equations with multi-point conditions:

$$\frac{dx}{dt} = A(t)x + \sum_{j=1}^{m} M_j(t)\dot{x}(\theta_j) + \sum_{i=0}^{m+1} K_i(t)x(\theta_i) + f(t), \quad t \in (0,T), \tag{1}$$

$$\sum_{i=0}^{m+1} C_i x(\theta_i) = d, \quad d \in R^n, \quad x \in R^n, \tag{2}$$

where the $(n \times n)$ -matrices $A(t)$, $M_j(t)$ $(j = \overline{1,m})$, $K_i(t)$ $(i = \overline{0,m+1})$, and n-vector-function $f(t)$ are continuous on $[0,T]$, C_i $(i = \overline{0,m+1})$ are constant $(n \times n)$ - matrices, and $0 = \theta_0 < \theta_1 < \theta_2 < \ldots < \theta_{m-1} < \theta_m < \theta_{m+1} = T$; $\quad \|x\| = \max_{i=\overline{1,n}} |x_i|$.

A solution to problem (1), (2) is a continuous on $[0,T]$ and continuously differentiable on $(0,T)$ function $x(t)$ satisfying the essentially loaded differential equations (1) and the multi-point condition (2).

In recent years the theory of differential-boundary equations or loaded differential equations has been advanced. These equations describe problems in optimal control, regulation of the layer of soil water and ground moisture, underground fluid and gas dynamics [1]. A numerical method of solving systems of loaded linear non-autonomous ordinary differential equations with non-separated multi-point and integral conditions is proposed in work [2].

In the present paper, a linear multi-point boundary value problem for essentially loaded differential equations is investigated. The significance is that the loaded members of the equation appear in the form of derivatives of solutions at load points of the interval. The presence of derivatives of solutions at load points has a strong influence on the properties of equations. Using the properties of essentially loaded differential equation and assuming the invertibility of the matrix compiled through the coefficients at the values of the derivative of the desired function at load points, we reduce the considered problem to a multi-point boundary value problem for loaded differential equations. The parameterization method [3] is used for solving this problem. The problem under consideration is reduced to solving a system of linear algebraic equations. The coefficients and right-hand side of the system are calculated by solving the Cauchy problems for ordinary differential equations. A numerical algorithm is offered for solving the considering problem. Numerical experiments are performed to test the proposed approach on examples.

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