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ТЕЗИСЫ ДОКЛАДОВ

Алматы 2021

<i>Akhmet M., Tleubergenova M., Nugayeva Z.</i> Linear system with unpredictable impulses	67
<i>Aldai M., Karatayeva D.</i> Local integral relations of the coefficients of a disconjugate differential equation	68
<i>Ashyraliyev A., Ashyraliyev M., Ashyraliyeva M.A.</i> Stability of hyperbolic-parabolic differential and difference equations with involution	70
<i>Assanova A., Ermek A.</i> A multipoint problem for Fredholm integro-differential equations	70
<i>Assanova A., Sabalakhova A., Toleukhanova Z.</i> On the solvability of a family of problems for integro-differential equations of mixed type	72
<i>Assanova A., Shynarbek Ye.</i> A problem with parameter for differential equation of second order	73
<i>Auzerkhan G., Kanguzhin B., Kaiyrbek Zh.</i> Restoring two-point boundary conditions for a final set of own the values of the boundary problems for differential equations higher orders	75
<i>Beisenbay A.</i> Van der Corput lemmas with Bessel functions	76
<i>Biyarov B.</i> Similar transformation of one class of correct restrictions	77
<i>Bizhanova G.</i> Investigation of the Cauchy problems for parabolic equations in the weighted Hölder spaces	78
<i>Bliev N.K.</i> Multidimensional singular integrals and integral equations in fractional spaces	79
<i>Bokayev N., Onerbek Zh., Adilkhanov A.</i> Potential type operator in global Morrey-type spaces with variable exponent on unbounded sets	80
<i>Dosmagulova K.</i> Scientific investigations of differential operators on Riemannian manifolds	82
<i>Dukenbayeva A., Sadybekov M.</i> On boundary value problems of the Samarskii-Ionkin type for the Laplace operator in a ball	82
<i>Gogatishvili A.</i> Compactness results for variable exponent spaces	84
<i>Iskakova N.B., Temesheva S.M., Uteshova R.E.</i> On a numerical method for solving a nonlinear boundary value problem for differential equation with delayed	84
<i>Kadirbayeva Zh.</i> A problem for essentially loaded differential equations	86
<i>Kakharman N.</i> Spectral properties of regular boundary value problems for differential equations	87
<i>Kalidolday A., Nursultanov E.</i> Interpolation properties of Net spaces	88
<i>Karimov E.T., Sobirov Z.A., Khujakulov J.R.</i> One non-local problem for a time fractional equations with the Hilfer operator on metric graph	90
<i>Khompysh Kh., Shakir A.</i> Inverse problem for pseudoparabolic equations with p-Laplacian	91
<i>Koshanov B., Kuntuarova A.</i> On Fredholm property and on the index of the generalized Neumann problem	92
<i>Nurmukanbet Sh.</i> Unique solvability of problem for integro-differential equation with weakly singular kernels	93
<i>Ochilova N.K.</i> Nonlocal boundary value problem for the degenerating mixed type equation with fractional derivative	95
<i>Raikhan M., Uatayeva A.</i> Submajorisation inequalities for matrices of measurable operators	96
<i>Rasa G.H.A., Kanguzhin B., Kaiyrbek Zh.</i> D'Alembert's formula for the wave equation on a graph-star	97
<i>Restrepo J.E.</i> Recent developments on fractional differential equations with variable coefficients and applications	99

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COMPACTNESS RESULTS FOR VARIABLE EXPONENT SPACES

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Let Ω be an open subset of \mathbb{R}^N , and let $p, q : \Omega \rightarrow [1, \infty]$ be measurable functions. We give a necessary and sufficient condition for the embedding of the variable exponent space $L^{p(\cdot)}(\Omega)$ in $L^{q(\cdot)}(\Omega)$ to be almost compact. This leads to a condition on Ω , p and q sufficient to ensure that the Sobolev space $W^{1,p(\cdot)}(\Omega)$ based on $L^{p(\cdot)}(\Omega)$ is compactly embedded in $L^{q(\cdot)}(\Omega)$; compact embedding results of this type already in the literature are included as special cases.

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ON A NUMERICAL METHOD FOR SOLVING A NONLINEAR BOUNDARY VALUE PROBLEM FOR DIFFERENTIAL EQUATION WITH DELAYED

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Due to applications in physics, biology, epidemiology, and so on, much of the literature on lagged differential equations has focused on the existence of a periodic solution, oscillation, and so on.

The importance and variety of applications served to increase interest in the theory of boundary value problems for differential equations with a delay argument, and the development of computer technology and its comprehensive application in applied problems presented new requirements for the developed methods, paying special attention to their constructibility and feasibility.

One of the constructive methods widely used for the study of boundary value problems for differential equations is the parametrization method of D.S.Dzhumabaev [1]. In [2], on the basis of one modification of the parametrization method algorithms, sufficient conditions were obtained for the existence of an isolated solution of a boundary value problem for a system of linear differential equations with a delay argument that satisfies essentially nonlinear boundary conditions.

We consider a nonlinear boundary value problem for a system of differential equations with delay argument

$$\frac{dx}{dt} = A(t)x(t) + B(t)x(t - \tau) + f(t), \quad x \in R^n, \quad t \in (0, T), \quad \tau > 0, \quad (1)$$

$$x(t) = \text{diag}[x(0)] \cdot \varphi(t), \quad t \in [-\tau, 0], \quad (2)$$

$$g(x(0), x(T)) = 0, \quad (3)$$

where $(n \times n)$ -matrices $A(t)$, $B(t)$ and the function $f(t)$ are continuous on $[0, T]$, $\varphi : [-\tau, 0] \rightarrow R^n$ is a continuously differentiable functions such that $\varphi_i(0) = 1$, $i = 1 : n$, τ is a constant delay such that $T = N\tau$ ($N = 1, 2, \dots$), $\|A(t)\| \leq \alpha$, $\|B(t)\| \leq \beta$, where α, β are constant.

The solution of the boundary value problem (1)-(3) is a continuous on $[-\tau, N\tau]$, continuously differentiable on $[-\tau, 0] \cup [0, N\tau]$ vector function $x(t)$ that satisfies the differential equation (1) and has values $x(0), x(T)$, for which equalities (2), (3) are valid.

In [3], in terms of input data, necessary and sufficient conditions for the existence of an isolated solution to a periodic boundary value problem for a system of nonlinear delay differential equations were established. In [2], on the basis of modified algorithms of the parametrization method, sufficient conditions were obtained for the existence of an isolated in some ball solution to problem (1) - (3) for a system of linear delay differential equations subject to essentially nonlinear boundary conditions. Following the standard scheme of the parametrization method, problem (1) - (3) was reduced to a multipoint boundary value problem with parameters. In the course of proving the conditions for the existence of a solution to the problem with parameters, a procedure for the sequential construction of its solution is shown, which includes the solution of a system of nonlinear algebraic equations of a special structure and the solution of Cauchy problems on the partition subintervals. Estimates were established for the difference between the exact solution and the solution found at a certain step.

The algorithms of the parametrization method are quite suitable for developing a numerical method for solving the problem under study. To construct the equations of the system in unknown parameters (the values of the desired solution at the partition subintervals $[-\tau, N\tau]$), methods of numerical solution of the Cauchy problem are used. To solve the constructed system of nonlinear algebraic equations, a sharper version of the local Hadamard theorem [4] is applied. Using the found parameters, we find the numerical solutions of the Cauchy problems associated with equation (1), thereby determining the numerical values of the desired solution at the points of the partition subintervals.

Thus, the numerical method proposed in the present paper is a balanced combination of known numerical methods. It should be noted that the results numerical experiments carried out on test problems are fairly good at the first steps of the algorithms of the parameterization method.

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Keywords: boundary value problems, equation with delay argument, isolated solution.

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A PROBLEM FOR ESSENTIALLY LOADED DIFFERENTIAL EQUATIONS

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We consider the following linear boundary value problem for essentially loaded differential equations with multi-point conditions:

$$\frac{dx}{dt} = A(t)x + \sum_{j=1}^m M_j(t)\dot{x}(\theta_j) + \sum_{i=0}^{m+1} K_i(t)x(\theta_i) + f(t), \quad t \in (0, T), \quad (1)$$

$$\sum_{i=0}^{m+1} C_i x(\theta_i) = d, \quad d \in R^n, \quad x \in R^n, \quad (2)$$

where the $(n \times n)$ -matrices $A(t)$, $M_j(t)$ ($j = \overline{1, m}$), $K_i(t)$ ($i = \overline{0, m+1}$), and n -vector-function $f(t)$ are continuous on $[0, T]$, C_i ($i = \overline{0, m+1}$) are constant $(n \times n)$ -matrices, and $0 = \theta_0 < \theta_1 < \theta_2 < \dots < \theta_{m-1} < \theta_m < \theta_{m+1} = T$; $\|x\| = \max_{i=\overline{1, n}} |x_i|$.

A solution to problem (1), (2) is a continuous on $[0, T]$ and continuously differentiable on $(0, T)$ function $x(t)$ satisfying the essentially loaded differential equations (1) and the multi-point condition (2).

In recent years the theory of differential-boundary equations or loaded differential equations has been advanced. These equations describe problems in optimal control, regulation of the layer of soil water and ground moisture, underground fluid and gas dynamics [1]. A numerical method of solving systems of loaded linear non-autonomous ordinary differential equations with non-separated multi-point and integral conditions is proposed in work [2].

In the present paper, a linear multi-point boundary value problem for essentially loaded differential equations is investigated. The significance is that the loaded members of the equation appear in the form of derivatives of solutions at load points of the interval. The presence of derivatives of solutions at load points has a strong influence on the properties of equations. Using the properties of essentially loaded differential equation and assuming the invertibility of the matrix compiled through the coefficients at the values of the derivative of the desired function at load points, we reduce the considered problem to a multi-point boundary value problem for loaded differential equations. The parameterization method [3] is used for solving this problem. The problem under consideration is reduced to solving a system of linear algebraic equations. The coefficients and right-hand side of the system are calculated by solving the Cauchy problems for ordinary differential equations. A numerical algorithm is offered for solving the considering problem. Numerical experiments are performed to test the proposed approach on examples.

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