MOSCOW
M.V. Lomonosov
STATE UNIVERSITY

RUSSIAN ACADEMY of SCIENCES
V.A. Steklov
MATHEMATICAL INSTITUTE

INTERNATIONAL CONFERENCE

«Differential Equations and Related Topics»,

dedicated to

IVAN G. PETROVSKII

(1901-1973)

XXIII Joint Session of Petrovskii Seminar and Moscow Mathematical Society

BOOK of ABSTRACTS

СБОРНИК ТЕЗИСОВ

Moscow, May 30 - June 4

2011

http://www.math.msu.su/petrovskii-110

[5] Suslina T. A., Vasilevskaya E. S., *Homogenization of parabolic and elliptic periodic operators in $L_2(\mathbb{R}^d)$ with the first and second correctors taken into account*, in preparation.

[6] Vasilevskaya E. S., *Homogenization with a corrector for a parabolic Cauchy problem with periodic coefficients*, Algebra i Analiz 21 (2009), no. 1, 3–60. English transl., St. Petersburg Math. J. 21 (2010), no. 1, 1–41.

A parametrization method for solving nonlinear nonlocal boundary value problem for the system of hyperbolic equations

Temesheva S. M. (Institute of Mathematics, Almaty, Kazakhstan)

Consider the nonlinear boundary value problem

$$\frac{\partial^2 u}{\partial x \partial t} = f\Big(x, t, u, \frac{\partial u}{\partial x}\Big), \quad (x,t) \in \Omega = (0,\omega) \times (0,T), \quad u \in R^n, \tag{1}$$

$$u(0,t) = 0, \quad t \in [0,\omega], \tag{2}$$

$$g\Big(x, u(x,0), \frac{\partial u}{\partial x}\Big|_{t=0}, u(x,T), \frac{\partial u}{\partial x}\Big|_{t=T}\Big) = 0, \quad x \in [0,\omega], \tag{3}$$

where $f : \bar{\Omega} \times R^{2n} \to R^n$ and $g : [0,\omega] \times R^{4n} \to R^n$ are continuous functions.

In the report an algorithm of finding solution to problem (1)–(3) is proposed. Denote by $v(x,t)$ an unknown function $u_x(x,t)$, $(x,t) \in \bar{\Omega}$. The problem (1)–(3) is reduced to equivalent nonlinear boundary value problem for the system of integral-differential equations with partial derivatives. This problem is investigated by a parametrization method [**1**].

For the chosen step size $h > 0$, where $Nh = T$ and $N = 1, 2, \ldots$, we perform the partition

$$[0,\omega] \times [0,T) = \bigcup_{r=1}^{N} [0,\omega] \times [(r-1)h, rh).$$

The functional parameters is introduced as values unknown function $v(x,t)$ on the lines $t = (r-1)h$, $r = 1 : N$. Then the nonlinear value problem for the system of integral-differential equations with partial derivatives is reduced to equivalent multicharacteristics boundary value problem.

An algorithm for solving the problem with functional parameters is proposed. Each step of this algorithm consists of two stages:

1°. Implicit system of the nonlinear Volterra integral equations with respect to introducing functional parameters is solved.

2°. Cauchy problems for the system of integral-differential equations with partial derivatives are solved using the components of computed functional parameters at stage 1.

We find conditions on the functions f and g and the domain $\bar{\Omega}$ ensuring the existence of an isolated solution to problem (1)–(3) and the convergence of the parametrization method algorithm to this solution.

The definition of an "isolated" solution to nonlinear nonlocal boundary value problem (1)–(3) with continuously differentiable data is introduced. Necessary and sufficient conditions for the existence of "isolated" solution are derived in terms of the initial data of problem (1)–(3).

The talk is based on the joint paper with D. S. Dzhumabaev.

References

[1] Dzhumabaev D. S., Temesheva S. M., *A parametrization method for solving nonlinear two-point boundary value problems*, Computational Mathematics and Mathematical Physics. Vol. 47 (2007), no. 1, 37–61.

Frequency locking of modulated waves in system with symmetry

Tkachenko V. I. (Institute of Mathematics, Kiev, Ukraine)

We consider the following system which arises as mathematical model of optical laser

$$\frac{dx}{dt} = f(x) + \varepsilon g(x, \alpha t, \beta t), \tag{1}$$

where $x \in \mathbb{R}^n$, vector field $f : \mathbb{R}^n \to \mathbb{R}^n$ is smooth and equivariant with respect to an S^1-representation $e^{\gamma A}$ on $\mathbb{R}^n$, i. e.,

$$e^{\gamma A} f(x) = f(e^{\gamma A} x) \quad \text{for all } \gamma \in \mathbb{R} \text{ and } x \in \mathbb{R}^n,$$

where $A \neq 0$ is a skew-symmetric real $n \times n$-matrix such that $e^{2\pi A} = I$. Smooth function g is 2π-periodic in βt and αt and equivariant in some sense, α and β are positive parameters.

By $\varepsilon = 0$, unperturbed system $\dot{x} = f(x)$ has an exponentially orbitally stable quasi-periodic solution of modulated wave type

$$x(t) = e^{A\alpha_0 t} x_0(\beta_0 t), \tag{2}$$

where $x_0(.)$ is smooth 2π-periodic function, α_0 and β_0 are positive constants.

Using methods of perturbation theory we investigate the behavior of perturbed system (1) in the neighborhood of (2). By assumption $\beta \approx \beta_0$ and $\alpha \gg \alpha_0$, we obtain the parameter domain (with respect to parameters α, β and ε) where stable frequency locking occurs.

Special case of system (1) was investigated in [**1**].

References

[1] Recke L., Samoilenko A., Teplinsky O., Tkachenko V., Yanchuk S. *Frequency locking of modulated waves*, Discrete Cont. Dynam. Systems, Ser. A (to appear), arXiv:1006.5518.

Singularities of the Manakov top integrable system

Tonkonog D. I. (Moscow State University, Russia)

An *integrable Hamiltonian system* $(M, \omega, h_1, \ldots, h_n)$ is a symplectic $2n$-manifold (M, ω) with functionally independent commuting functions $h_1, \ldots, h_n : M \to \mathbb{R}$ traditionally called *integrals*. The *momentum map* $\mathcal{F} : M \to \mathbb{R}^n$ is given by $\mathcal{F}(x) := (h_1(x), \ldots, h_n(x))$. Level sets of $\mathcal{F}$ define singular *Liouville foliation* on M. A point $x \in M$ is called a *singular (critical) point of rank* r, $0 \leq r < n$, if $\operatorname{rk} d\mathcal{F}(x) = r$.

We study singularities of the Manakov top system (with two degrees of freedom), previously explored in [**11, 2**]. We describe topology of the Liouville foliation on preimages $\mathcal{F}^{-1}(U)$ where $U \subset \mathbb{R}^2$ is a neighborhood of the $\mathcal{F}$-image of a zero-rank singular *saddle-saddle* point. This is done with the help of theory developed by Fomenko and his collaborators [**4, 3, 9**].