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SPECTRAL THEORY AND ITS APPLICATIONS

Book of Abstracts

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Solution to a coefficient inverse problem for a system of linear ODEs

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We consider a problem of identifying the parameters of a dynamic system

$$\dot{x}(t) = A(t)x(t) + B(t)C + F(t), \quad t \in [t_0, T].$$

Here $x(t) \in R^n$ is the phase state of the system; $C \in R^l$ is the required parameter vector; $A(t)$, $B(t)$, $F(t)$ are given matrices of the dimensions $(n \times n)$, $(n \times l)$, $(n \times 1)$, respectively.

It is assumed that to identify the parameter vector C, we have N carried out experiments under different initial conditions:

$$x^j(t_0^j) = x_0^j\,, \ \ j = \overline{1, N},$$

During the experiments, observations over the state of the dynamic system are carried out. The following forms of the results of observations are possible:
a) Separated multipoint conditions:

$$x_\nu^j(t_i^j) = x_{\nu i}^j \quad , \quad \nu = \overline{1, m_j}\,, \ i = \overline{1, s_j}, \, , \quad j = \overline{1, N}. \qquad (1)$$

Here $\nu = \overline{1, m_j}$ are the indices of the observable phase coordinates $x(t)$ ate the points of time $t_1^j, ..., t_{s_j}^j$ under j^{th} experiment for $t \in [t_0^j, T^j]$; s_j is the number of the observation time in the j^{th} experiment; $L = \sum\limits_{j=1}^{N} m_j s_j$ is the total number of additional observations for the determination of the vector C.
b) Non-separated multipoint nonlocal conditions

$$\sum_{i=1}^{k} \tilde{\alpha}_i^j x(t_i^j) = \tilde{\beta}^j, \quad j = \overline{1, N}. \tag{2}$$

Here the given matrices $\tilde{\alpha}_i^j$, $\tilde{\beta}^j$, $j = \overline{1, N}$, have the dimensions $(L \times n)$ and $(L \times 1)$, respectively.

In the report, we will present the results of the numerical experiments which have been obtained when solving some test problems with observations of the form (1) and (2) for $L = l$ and $L > l$. We will present the results of the numerical experiments and analysis of the considered inverse problem with errors in the observations.

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Construction of fundamental solution of middle boundary problem for one integro-differential equation of 3D Bianchi

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In recent years, interest in three-dimensional local and non-local boundary value problems for the Bianchi equations has increased significantly. This is due to their appearance in various tasks of an applied nature. Three dimensional Bianchi equation is used in models of vibration processes, and also has an important significance in the theory of approximation.

Consider the integro-differential equation 3D (three dimensional) Bianchi:

$$(V_{1,1,1}u)(x,y,z) \equiv u_{xyz}(x,y,z) + \sum_{\substack{i,j,k=0 \\ i+j+k<3}}^{1} A_{i,j,k}(x,y,z)\times$$

$$\times D_x^i D_y^j D_z^k u(x,y,z)+$$

$$+\sum_{\substack{i,j,k=0 \\ i+j+k<3}}^{1} \int\limits_{\frac{x_0+x_1}{2}}^{x} \int\limits_{\frac{y_0+y_1}{2}}^{y} \int\limits_{\frac{z_0+z_1}{2}}^{z} K_{i,j,k}(\tau,\xi,\eta;x,y,z)\times$$

$$\times D_x^i D_y^j D_z^k u(\tau,\xi,\eta)d\tau d\xi d\eta = \varphi_{1,1,1}(x,y,z), (x,y,z)\in G, \quad (1)$$

with middle boundary conditions in the non-classical form

$$\left\{\begin{array}{c} V_{0,0,0}u \equiv u\left(\frac{x_0+x_1}{2}, \frac{y_0+y_1}{2}, \frac{z_0+z_1}{2}\right) = \varphi_{0,0,0}, \\ (V_{1,0,0}u)(x) \equiv u_x\left(x, \frac{y_0+y_1}{2}, \frac{z_0+z_1}{2}\right) = \varphi_{1,0,0}(x) \\ (V_{0,1,0}u)(y) \equiv u_y\left(\frac{x_0+x_1}{2}, y, \frac{z_0+z_1}{2}\right) = \varphi_{0,1,0}(y), \\ (V_{0,0,1}u)(z) \equiv u_z\left(\frac{x_0+x_1}{2}, \frac{y_0+y_1}{2}, z\right) = \varphi_{0,0,1}(z), \\ (V_{1,1,0}u)(x,y) \equiv u_{xy}\left(x, y, \frac{z_0+z_1}{2}\right) = \varphi_{1,1,0}(x,y), \\ (V_{0,1,1}u)(y,z) \equiv u_{yz}\left(\frac{x_0+x_1}{2}, y, z\right) = \varphi_{0,1,1}(y,z), \\ (V_{1,0,1}u)(x,z) \equiv u_{xz}\left(x, \frac{y_0+y_1}{2}, z\right) = \varphi_{1,0,1}(x,z). \end{array}\right. \quad (2)$$

Here $u = u(x,y,z)$ is the desired function defined on G; $A_{i,j,k} = A_{i,j,k}(x,y,z)$ the given measurable functions on $G = G_1 \times G_2 \times G_3$, $K_{i,j,k}(\tau,\xi,\eta;x,y,z) \in L_\infty(G \times G)$, where $G_1 = (x_0,x_1)$, $G_2 = (y_0,y_1)$, $G_3 = (z_0,z_1)$; $\varphi_{i,j,k}(x,y,z)$ the given measurable functions on G .

In this work the fundamental solution of middle boundary problem (1) and (2) for integro-differential equation of 3D Bianchi (1) with nonsmooth coefficients is constructed.

On a boundary value problem for operator-differential equation on finite segment

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Let H be a separable Hilbert space A be a positive -definite self-adjoint operator in H the with domain of definition $D(A)$.

The domain of definition of the operator A^γ beacons Hilbert space H_γ $\gamma \geq 0$ with respect to the scalar product $(x,y)_\gamma = (A^\gamma x, A^\gamma y), x, y \in D(A^\gamma)$. For $\gamma = 0$ we set $H_0 = H$.

In Hilbert space H we consider the following boundary value problem:

$$-\frac{d^2u(t)}{dt} + \rho(t)A^2u(t) + A_1\frac{du(t)}{dt} + A_2u(t) = f(t), t \in (0,1), \quad (1)$$

$$u(0) = \varphi_0, u(1) = \varphi_1, \quad (2)$$

where $\varphi_0, \varphi_1 \in H_{3/2}$, $f(t) \in L_2((0,1); H), u(t) \in W_2^2((0,1); H)$, and the operator coefficients satisfy the function such:

1) $\rho(t)$ is a bounded scalar function such that $0 < \alpha \leq \rho(t) \leq \beta < \infty$;

2) the operator A is a normal, with completely continuous inverse A^{-1} whose spectrum is in the sector

$$S_\varepsilon = \{\lambda : |\arg\lambda| \leq \varepsilon\}, 0 \leq \varepsilon < \pi/2;$$

3) the operator $B_1 = A_1A^{-1}, B_2 = A_2A^{-2}$ are bounded in H.

Theorem. *Let conditions 1)-3) be fulfilled and the following inequality hold*

$$q(\varepsilon) = c_1(\alpha,\varepsilon)\|B_1\| + c_2(\alpha,\varepsilon)\|B_2\| < 1,$$

$$c_1(\alpha,\varepsilon) = \frac{1}{2\sqrt{\alpha}\cos\varepsilon}, 0 \leq \varepsilon < \pi/2;$$

$$c_2(\alpha,\varepsilon) = \begin{cases} \frac{1}{\alpha}, 0 \leq \varepsilon < \frac{\pi}{4}, \\ \frac{1}{\sqrt{2}\alpha\cos\varepsilon}, \frac{\pi}{4} \leq \varepsilon < \frac{\pi}{2}. \end{cases}$$

Then problem (1) and (2) for any $f(t) \in L_2((0,1);H)$ has a unique solution from $W_2^2((0,1);H)$.

On the solution of a mixed problem for a fourth-order equation

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In the present paper, we consider the equation

$$\frac{\partial u(x,t)}{\partial t} = i\frac{\partial^4 u(x,t)}{\partial x^4} + q(x)\frac{\partial^2 u(x,t)}{\partial x^2},\ 0 < x < 1,\ t > 0, \quad (1)$$

with the initial condition

$$u(x,0) = \varphi(x)\ (0 < x < 1), \quad (2)$$

and the boundary conditions

$$\begin{array}{l} L_1(u) \equiv u(0,t) = 0, \\ L_2(u) \equiv u(1,t) = 0, \\ L_3(u) \equiv u_x(0,t) = 0, \\ L_4(u) \equiv u_x(1,t) = 0. \end{array} \quad (3)$$

For the solution of the mixed problem (1)-(3) the following theorem is valid.

Theorem. *Suppose that the conditions*

$$q(x) \in C[0,1],\ \varphi(x) \in C^2[0,1],\ \varphi(0) = \varphi(1) = \varphi'(0) = \varphi'(1) = 0,$$

and $\operatorname{Re} \int_0^1 q(\tau)d\tau > 0$ *hold.Then the solution of the mixed problem (1)-(3) has the form:*

$$u(x,t) = \sum_{k=1}^{4} \sum_{n=1}^{\infty} ie^{\lambda^4 t} \underset{\lambda=\lambda_{n,k}}{res} \int_0^1 G(x,\xi,\lambda)\varphi(\xi)d\xi,$$

where $G(x,\xi,\lambda)$ *is the Green function for spectral problem of (1)-(3) and*

$$\lambda_{n,k} = \left(\frac{\lambda}{\theta}\right)^4 \left(\frac{1+2n}{n}\right)^4 + \frac{b(1)}{\theta^3}\pi^2 \left(\frac{1+2n}{n}\right)^2 + O(n),$$

$$n \to \infty,\ k = \overline{1,4},\ b(1) = \frac{1}{-4i\theta} \int_0^1 q(\tau)d\tau,\ \theta = e^{-\frac{\pi}{8}i}.$$

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Solvability of the Dirichlet problem for the Laplace equation with boundary value from the Morrey space

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Non-tangential maximal function is considered and it is estimated from above a maximum operator, and the proof is carried out for the Poisson-Stieltjes integral, when the density belongs to the corresponding Morrey-Lebesgue space. The obtained results are applied to the solution of the Dirichlet problem for the Laplace equation with boundary value from Morrey-Lebesgue space.

The following theorem is proved.

Theorem. *Assume that the measure $\mu(\cdot)$ satisfies the conditions (I is an interval)*

$$\mu(I) \sim |I|, \forall I \subset R; \sup_{y>0;\, x\in R} \int_R P_y(s-|x|)\, d\mu(s) < +\infty.$$

Let

$$u_\mu(z) = u_\mu(x;\, y) =$$

$$= \int_R P_y(x-t)\, f(t)\, d\mu(t)\,,\, f \in L^{p,\,\alpha}(d\mu)\,,\; 0 \le 1-\alpha < 1,$$

where $L^{p,\,\alpha}(d\mu)$ *is a Morrey space equipped with the norm*

$$\|f\|_{p,\,\alpha;\,d\mu} = \sup_{I\subset R}\left\{\frac{1}{|I|^{1-\alpha}}\int_I |f(y)|^p\, d\mu(y)\right\}^{1/p}.$$

Then for $\forall\alpha_0 > 0$, $\exists A_{\alpha_0} > 0$*:*

$$\sup_{(x;\,y)\in\Gamma_{\alpha_0}(t)} |u_\mu(x;y)| \le A_{\alpha_0} M_\mu f(t)\,, \forall t \in R,$$

and $u_\mu^* \in h^{p,\,\alpha}(d\mu)$*:*

$$\left\|u_\mu^*\right\|_{h^{p,\,\alpha}(d\mu)} \le A_{\alpha_0}\, \|f\|_{p,\,\alpha;\,d\mu}\,,$$

where $u_\mu^*(\cdot)$ *is a nontangential maximal function for* u*:*

$$u_\mu^*(t) = \sup_{z\in\Gamma_{\alpha_0}(t)} |u_\mu(z)|\,, t \in R.$$

A priori estimate of solution for a class of hyperbolic equations system

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In the paper second order hyperbolic equations system

$$(Vz)(t,x) \equiv z_{tx}(t,x) + z(t,x)A_{0,0}(t,x) + z_t(t,x)A_{1,0}(t,x) +$$
$$+z_x(t,x)A_{0,1}(t,x) = \varphi(t,x), (t,x) \in G = G_0 \times G_1,$$
$$G_0 = (0,T) \times (0,\alpha), \quad G_1 = (0,T) \times (\alpha, l), \tag{1}$$

in considered under nonlocal boundary conditions,

$$(V_k z)(t) \equiv z(t,0)\beta_{0,k}(t) + z(t,\alpha-0)\beta_{1,k}(t) + z(t,\alpha+0)\beta_{2,k}(t)+$$
$$+z(t,l)\beta_{3,k}(t) + z_t(t,0)g_{0,k}(t) + z_t(t,\alpha-0)g_{1,k}(t) + z_t(t,\alpha+0)\times$$
$$\times g_{2,k}(t) + z_t(t,l)g_{3,k}(t) = \varphi_k(t), \ \ t \in (0,T), \ k = 1,2; \tag{2}$$

$$(V_3 z)(x) \equiv \sum_{j=1}^{m} [z_x(\tau_j, x)\gamma_j(x) + z(\tau_j, x)\mu_j(x)] = \varphi_3(x), \ x \in (0,l); \tag{3}$$

$$V_0 z \equiv z(0,0) = \varphi_0 \, . \tag{4}$$

Here: $z(t,x) \equiv (z_1(t,x), ..., z_n(t,x))$ characterize state of system and continuous on the sets $\bar{G}_0$ and $\bar{G}_1$; $A_{i,j}(t,x)$ $(i,j = 0,1)$ –

are given $n \times n$- matrices, $A_{0,0} \in L_{p,n\times n}(G)$; there exist such functions $A^0_{1,0} \in L_p(0,l)$, $A^0_{0,1} \in L_p(0,T)$ that $\left\|A^0_{1,0}(t,x)\right\| \leq A^0_{1,0}(x)$, $\left\|A^0_{0,1}(t,x)\right\| \leq A^0_{0,1}(t)$ a.e. on G; $\beta_{i,k} \in L_{p,n\times n}(0,T)$, $g_{i,k} \in L_{\infty,n\times n}(0,T)$; $\varphi \in L_{p,n}(G)$, $\varphi_k \in L_{p,n}(0,T)$; $\varphi_3 \in L_{p,n}(0,l)$; φ_0 –is given n- vector; $\gamma_j(x) \in L_{\infty,n}(0,l)$, $\mu_j(x) \in L_{p,n}(0,l)$; $\tau_j \in [0,T]$, $j = 1,...,m$ are given numbers; $\alpha \in (0,l)$.

Let $W_{p,n}(G_k) = \{\, z : \ z \in L_{p,n}(G_k)\, ; \ z_t, z_x, z_{tx} \in L_{p,n}(G_k) \,\}$, $k = 0,1,$ be Sobolev space and $\hat{W}_{p,n}(G) = \{z : \ z \in L_{p,n}(G)$, $z \in W_{p,n}(G_0)$, $z \in W_{p,n}(G_1)$, $z(0,\alpha - 0) = z(0,\alpha + 0) \,\}$.

Under abovementioned conditions on the data of the problem (1)-(4) we can assume its solution from space $\hat{W}_{p,n}(G)$. In other words, the operator $\hat{V} = (V_0, V_1, V_2, V_3, V)$ has been defined on $\hat{W}_{p,n}(G)$ and acts onto space $\hat{Q}_{p,n}(G) = R^n \times L_{p,n}(0,T) \times L_{p,n}(0,T) \times L_{p,n}(0,l) \times L_{p,n}(G)$.

In this work there has been used an isomorphism which is implemented by the operator $Nz \equiv (z(0,0), z_t(t,0), z_t(\alpha+0,0),$ $z_x(0,x)\, z_{tx}(t,x))$. Using this isomorphism there has been gotten representation of solution with independent elements. This representation allowed first to get a priori estimates for independent elements, after for the solution of the problem (1)-(4).

Recurrent sequences and their applications

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In this work, we study the behavior of the sequence a_n of complex numbers satisfying the relation $a_{n+k} = q_1 a_n + q_2 a_{n+1} + ::: + q_k a_{n+k-1}$; where q_n is a fixed sequence of complex numbers. Such kind of sequences arise in problems of analysis, fixed point theory, dynamical systems, theory of chaos, etc. [1]-[4]. Investigating the spectra of triple and more than triple band triangle operator-matrices, the behavior of such sequence required [5, 6]. From the point of application, the proved results and formulas in the literature for the spectra of the operator-matrices look like very complicated. In this work, we eliminate the indicated flaws and apply new approach where the formulas for the spectra describe circular domains. Also we apply receiving results to some natural processes.

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Integral representation of solution for a class of hyperbolic equations system

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In the paper second order hyperbolic equations system

$$(Vz)\,(t,x) \equiv z_{tx}\,(t,x) + z\,(t,x)\,A_{0,0}\,(t,x) + z_t\,(t,x)\,A_{1,0}\,(t,x) +$$

$$+ z_x\,(t,x)\,A_{0,1}\,(t,x) = \varphi\,(t,x)\,, (t,x) \in G = G_0 \times G_1,$$

$$G_0 = (0,T) \times (0,\alpha)\ ,\quad G_1 = (0,T) \times (\alpha,l)\ , \tag{1}$$

in considered under nonlocal boundary conditions,

$$(V_k z)(t) \equiv z(t,0)\beta_{0,k}(t) + z(t,\alpha-0)\beta_{1,k}(t) + z(t,\alpha+0)\beta_{2,k}(t) +$$

$$+ z(t,l)\beta_{3,k}(t) + z_t(t,0)g_{0,k}(t) + z_t(t,\alpha-0)g_{1,k}(t) + z_t(t,\alpha+0)g_{2,k}(t) +$$

$$+ z_t(t,l)g_{3,k}(t) = \varphi_k(t),\ \ t \in (0,T),\ \ k = 1,2; \tag{2}$$

$$(V_3 z)(x) \equiv \sum_{j=1}^{m} \left[z_x(\tau_j,x)\gamma_j(x) + z(\tau_j,x)\mu_j(x) \right] = \varphi_3(x),\ \ x \in (0,l); \tag{3}$$

$$V_0 z \equiv z\,(0,0) = \varphi_0\,. \tag{4}$$

Here: $z\,(t,x) \equiv (z_1\,(t,x)\,,...,z_n\,(t,x))$ characterize state of system and continuous on the sets $\bar{G}_0$ and $\bar{G}_1$; $A_{i,j}\,(t,x)$ $(i,j = 0,1)-$ are given $n \times n$-matrices, $A_{0,0} \in L_{p,n\times n}\,(G)$; there exist such functions $A^0_{1,0} \in L_p\,(0,l)$, $A^0_{0,1} \in L_p\,(0,T)$ that $\left\| A^0_{1,0}(t,x) \right\| \leq A^0_{1,0}(x)\,,\ \left\| A^0_{0,1}(t,x) \right\| \leq A^0_{0,1}(t)$ a.e. on G; $\beta_{i,k} \in L_{p,n\times n}\,(0,T)$, $g_{i,k} \in L_{\infty,n\times n}\,(0,T)$; $\varphi \in L_{p,n}\,(G)$, $\varphi_k \in L_{p,n}\,(0,T)$; $\varphi_3 \in L_{p,n}\,(0,l)$; φ_0 $-$is given n- vector; $\gamma_j\,(x) \in L_{\infty,n}\,(0,l)$, $\mu_j\,(x) \in L_{p,n}\,(0,l)$; $\tau_j \in [0,T]$, $j = 1,...,m$ are given numbers; $\alpha \in (0,l)$.

Let $W_{p,n}(G_k) = \{ z : \ z \in L_{p,n}(G_k) \ ; \ z_t, z_x, z_{tx} \in L_{p,n}(G_k) \}$, $k = 0, 1,$ be Sobolev space and

$$\hat{W}_{p,n}(G) = \{ z : \ z \in L_{p,n}(G) \ , \ z \in W_{p,n}(G_0) \ , \ z \in W_{p,n}(G_1) \ ,$$
$$z(0, \alpha - 0) = z(0, \alpha + 0) \} \ .$$

Basing on the data of the problem (1)-(4) we can assume its solution from the $\hat{W}_{p,n}(G)$. In this work there has been used an isomorphism which is implemented by the operator $Nz \equiv (z(0,0), z_t(t,0), z_t(\alpha+0,0), z_x(0,x), z_{tx}(t,x))$ from $\hat{W}_{p,n}(G)$ onto space $\hat{Q}_{p,n}(G) = R^n \times L_{p,n}(0,T) \times L_{p,n}(0,T) \times L_{p,n}(0,l) \times L_{p,n}(G)$. Using this isomorphism adjoint problem has been defined in form of integro-algebraic system and the concept of a fundamental solution is introduced. With help of the fundamental solution of the problem (1)-(4) there has been gotten integral representation of solution of the problem (1)-(4).

Finiteness of the spectrum of boundary value problems

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For the differential equation

$$y^{(n)} + a_1(x,\lambda)\, y^{n-1} + \cdots + a_{n-1}(x,\lambda)\, y' + a_n(x,\lambda)\, y = 0, \quad (1)$$

consider the boundary value problem

$$U_j(y) = \sum_{k=0}^{n} b_{jk}\, y^{(k)}(0) + \sum_{k=0}^{n} b_{j\,k+n}\; y^{(k)}(1) = 0, \qquad (2)$$

where $j = 1, 2, \ldots, n$, rank $||b_{jk}||_{n\times 2n} = n$, the functions $a_q(x, \lambda)$ $(q = 1, \ldots, n)$ are continuous in x on the interval [0,1] and polynomial in the parameter λ, the coefficients b_{jk} are complex.

In 2008, Locker [1] posed the following question for the equations

$$y^{(n)} + a_1(x)\, y^{n-1} + \cdots + a_{n-1}(x)\, y' + a_n(x)\, y = \lambda\, y(x), \quad (3)$$

Can the boundary value problem (3), (2) have finite spectrum? In the same year, Kal'menov and Suragan [2] proved that the spectrum of regular partial differential boundary value problems, including problems (3), (2) is either empty or infinite. The following assertion shows that this result also holds for one general class of problems (1), (2).

Theorem 1. *If the function $a_q(x, \lambda)$ have the form*

$$a_q(x,\lambda) = \lambda^q \textstyle\sum_{j=0}^{q} \lambda^{-j}\, a_{qj}(x), \quad a_{q0}(x) = a_{q0}\cdot r(x), \\ r(x) > 0, \quad q = 1, 2, \ldots, n,$$

and the polynomial $\pi(\lambda) = \lambda^n + a_{10}\,\lambda^{n-1} + \cdots + a_{q0}$ does not have multiple roots, then the spectrum of the boundary value problem (1), (2) is either empty or infinite.

The proof of Theorem 1 follows from the results obtained by Lidskii and Sadovnichii [3,4], who showed that the characteristic determinant $\Delta(\lambda)$ of problem (1), (2) satisfying the assumptions of Theorem 1 is an entire function of class K and the number of roots (if any) of this function is infinite. (Their asymptotic representations are given in [3,4] as well).

Now assume that the polynomial $\pi(\lambda)$ has multiple roots. The following question arises: Can the boundary value problem (1), (2) have finite spectrum in this case?

Theorem 2. *Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ given complex numbers. There exists a boundary value problem (1), (2) whose spectrum consists precisely of the numbers $\lambda_1, \lambda_2, \ldots, \lambda_n$.*

Proof. We denote the product $(\lambda - \lambda_1) \cdot \ldots \cdot (\lambda - \lambda_n)$ by $p(\lambda)$, $p(\lambda) - 1$ by d, and for the differential equation

$$y'' - 2\,d\,y' + d^2\,y = 0 \tag{4}$$

we consider the boundary value problem

$$U_1(y) = y(0) = 0, \quad U_2(y) = y'(1) = 0. \tag{5}$$

Then for the characteristic determinant $\Delta(\lambda)$ of problem (4), (5) we obtain

$$\Delta(\lambda) = p(\lambda)\, e^{d(\lambda)}.$$

Therefore, the roots of the characteristic determinant $\Delta(\lambda)$ are precisely the roots of the polynomial $p(\lambda)$. The proof of the theorem is complete.

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The inverse problem for parabolic equation in a domain with moving boundary

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The goal of this paper is to study the well-posedness of an inverse problem of determining the unknown coefficient on the right side of parabolic equation. The inverse problem in a domain with moving boundary in case of the Dirichlet boundary condition with additional integral information.

We consider an inverse problem of determining a pair of functions

$$\{f(t),\ u(x,t)\}$$

from conditions

$$u_t - u_{xx} = f(t)g(x),\ (x,t) \in D = \{s_1(t) < x < s_2(t), 0 < t \leq T\},$$

$$u(x,0) = \varphi(x), \quad x \in [s_1(0), \quad s_2(0)]$$

$$u(s_1(t),\ t) = \psi_1(t), \quad u(s_2(t),\ t) = \psi_2(t), \quad t \in [0,T]$$

$$\int\limits_0^{\gamma(t)} u(x,t)dx = h(t), \quad t \in [0,T]$$

where the data $g(x),\ \varphi(x),\ \psi_1(t),\ \psi_2(t),\ h(t)$ and $\gamma(t)$ are continuous functions whose degree of continuity we shall make precise later. Recall that we have assumed that $s_i(t) \in C^1([0,T]),\ i = 1,2;\ s_1(t) < \gamma(t) < s_2(t),\ t \in [0,T]$. Here, we shall also assume that

$$\inf_{0 \leq t \leq T} |s_1(t) - s_2(t)| > 0, \quad \sup_{0 \leq t \leq T} |s_1(t) - s_2(t)| < \infty$$

The additional condition represents the specification of a relative heat content of a portion of the conductor. For diffusion, the condition is equivalent to the specification of mass in a portion of the domain of diffusion.

The uniqueness theorem and the estimation of stability of the solution of inverse problems occupy a central place in investigation of their well-posedness. In the paper, the uniqueness of solution for considering problem is proved under more general assumptions and the estimation characterizing the "conditional" stability of the problem is established.

Attractor for nonlinear transmission acoustic problem

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Let Ω be a bounded domain in R^3 with smooth boundary Γ_1, $\Omega_2 \subset \Omega_1$ is a subdomain with smooth boundary Γ_2 and $\Omega_1 = \Omega \backslash \Omega_2$ is a subdomain with boundary $\Gamma = \Gamma_1 \cup \Gamma_2$, $\Gamma_1 \cap \Gamma_2 = \emptyset$. The nonlinear transmission acoustic problem considered here is

$$u_{tt} - \Delta u + \alpha_1 u_t + u + f_1(u) = 0 \ \text{ in } \ \Omega_1 \times (0, \infty), \qquad (1)$$

$$\vartheta_{tt} - \Delta\vartheta + \alpha_1\vartheta_t + \vartheta + f_2(\vartheta) = 0 \text{ in } \Omega_2 \times (0,\infty), \qquad (2)$$

$$\delta_{tt} + \beta\delta_t + \delta = -u_t \text{ on } \Gamma_2 \times (0,\infty), \qquad (3)$$

$$u = 0 \text{ on } \Gamma_1 \times (0,\infty), \qquad (4)$$

$$u = \vartheta, \delta_t = \frac{\partial u}{\partial \nu} - \frac{\partial \vartheta}{\partial \nu} \text{ on } \Gamma_2 \times (0,\infty), \qquad (5)$$

$$u(x,0) = u_0(x), u_t(x,0) = u_1(x), \; x \in \bar{\Omega}_1, \qquad (6)$$

$$\vartheta(x,0) = \vartheta_0(x), \vartheta_t(x,0) = \vartheta_1(x), \; x \in \bar{\Omega}_2, \qquad (7)$$

$$\delta(x,0) = \delta_0(x), \delta_t(x,0) = \frac{\partial u_0}{\partial \nu} - \frac{\partial \vartheta_0}{\partial \nu} \equiv \delta_1 \; x \in \bar{\Gamma}_2 \qquad (8)$$

where ν is the unit outward normal vector to Γ; $\alpha_i > 0 \; (i = 1,2)$ and $\beta > 0$; $f_i : R \to R \; (i = 1,2)$, $u_0, u_1 : \bar{\Omega}_1 \to R$, $\vartheta_0, \vartheta_1 : \bar{\Omega}_2 \to R$, $\delta_0 : \bar{\Gamma}_2 \to R$.

Assume that $f_i \in C^1(R), i = 1,2$ and there exist constants $c_{1i} \geq 0, i = 1,2$, such that

$$|f_i'(s)| \leq c_{1i}(1 + s^2), \liminf_{|s|\to\infty} \frac{f_i(s)}{s} > -1. \qquad (9)$$

The problem (1)-(8) can be formulated in the energy space

$$V = H^1_{\Gamma_1}(\Omega_1) \times L^2(\Omega_1) \times H^1(\Omega_2) \times L^2(\Omega_2) \times L^2(\Gamma_2) \times L^2(\Gamma_2).$$

We introduce the linear unbounded operator $A : D(A) \subset V \to V$:

$$Aw = (w_2, \Delta w_1 - w_1 - \alpha_1 w_2, w_4,$$

$$\Delta w_3 - w_3 - \alpha_2 w_4, w_6, -w_2 - w_5 - \beta w_6).$$

Furthermore, we introduce the nonlinear function $\Phi : V \to V$:

$$\Phi(w) = (0, -f_1(w_1), 0, -f_2(w_3), 0, 0)$$

for every $w \in V$.

Then the problem (1)-(8) can be put in the form

$$\begin{cases} w_t = Aw + \Phi(w), \\ w(0) = w_0, \end{cases} \tag{10}$$

where $w = (u, u_t, \vartheta, \vartheta_t, \delta, \delta_t)$ and $w_0 = (u_0, u_1, \vartheta_0, \vartheta_1, \delta_0, \delta_1) \in V$.

Theorem. *Let (9) hold. Then the problem (10) with $\alpha_1 = \alpha_2 = \alpha$ possesses a unique global attractor A in the energy phase space V.*

On the theory of a class of fourth-order polynomial operator pencils

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Let H be a separable Hilbert space with scalar product (x, y), $x, y \in H$, and A be a positive definite self-adjoint operator in H $(A = A^* \geq cE,\ c > 0,\ E$ is an identity operator). By $H_\gamma\,(\gamma \geq 0)$ we will mean the scale of Hilbert spaces generated by the operator A, i.e., $H_\gamma = D\,(A^\gamma)\,,\ (x,y)_\gamma = (A^\gamma x,\ A^\gamma y)\,,\ x,y \in D\,(A^\gamma)$, with $H_0 = H$, $(x,y)_0 = (x,y)\,, x,y \in H$.

Denote by $L_2\,(R_+;H)$ the Hilbert space of all vector-valued functions defined on $R_+ = (0,+\infty)$ with the values from H and the finite norm

$$\|f\|_{L_2(R_+;H)} = \left(\int_0^{+\infty} \|f(t)\|_H^2\,dt\right)^{1/2}.$$

Following [1, Ch. 1], we introduce the Hilbert space

$$W_2^4\,(R_+;H) =$$
$$= \left\{u\,(t) : u^{(4)}\,(t) \in L_2\,(R_+;H)\,, A^4 u\,(t) \in L_2\,(R_+;H)\right\}$$

equipped with the norm

$$\|u\|_{W_2^4(R_+;H)} = \left(\left\|u^{(4)}\right\|^2_{L_2(R_+;H)} + \left\|A^4 u\right\|^2_{L_2(R_+;H)}\right)^{1/2}.$$

By $L\,(X,Y)$ we mean the set of linear bounded operators acting from the Hilbert space X to another Hilbert space Y.

Consider the following operator pencil in the space H:

$$P\,(\lambda) = \lambda^4 E + A^4 + \lambda^3 A_1 + \lambda^2 A_2 + \lambda A_3, \qquad (1)$$

where λ is the spectral parameter, $A = A^* \geq cE$, $c > 0$, $A_j,\ j = 1,2,3$, are linear and, in general, unbounded operators.

We associate with the operator pencil (1) the boundary value problem

$$P(d/dt)u\,(t) = 0,\;\; t \in R_+, \tag{2}$$

$$u\,(0) = \varphi,\;\; u''\,(0) - Ku'\,(0) = \psi, \tag{3}$$

where $K \in L(H_{5/2}; H_{3/2})$, $\varphi \in H_{7/2}$, $\psi \in H_{3/2}$, $u\,(t) \in W_2^4\,(R_+; H)$.

In this paper, we prove a theorem on double completeness in the space of traces of a system of derivative chain constructed by eigen- and adjoint vectors corresponding to eigenvalues from the left half-plane and to the boundary value problem (2) and (3).

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Solvability of a boundary value problem for second order elliptic differential-operator equations with a complex parameter in the equation and in the boundary condition

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In separable Hilbert space H we consider the following boundary value problem for a second order elliptic differential equation:

$$L(\lambda)u := \lambda^2 u(x) - u''(x) + Au(x) = f(x),\ x \in (0,1), \tag{1}$$

$$\begin{gathered} L_1(\lambda)u := u'(0) + \alpha\lambda u(1) = f_1, \\ L_2 u := u(0) = f_2, \end{gathered} \tag{2}$$

where λ is a complex parameter; A is a linear, closed, densely defined operator in H.

Theorem. *Let the following conditions be fulfilled:*

1) *A is a linear, closed, densely defined operator in H and $\|R(\lambda, A)\|_{B(H)} \leq c(1+|\lambda|)^{-1}$ for $|\arg\lambda| \geq \pi - \varphi$, where $\varphi \in (0,\pi)$ is some number, $c > 0$ is some constant independent on λ;*

2) *$\alpha \neq 0$ is a complex number.*

Then for $f \in L_p((0,1); H(A^{1/2}))$, $f_1 \in (H(A), H)_{\frac{1}{2}+\frac{1}{2p},p}$, $f_2 \in (H(A^2), H)_{\frac{1}{4}+\frac{1}{4p},p}$, $p \in (1, +\infty)$, *and for sufficiently large* $|\lambda|$ *from the angle* $|\arg \lambda| \le \varphi < \frac{\pi}{2}$, *problem (1) and (2) has a unique solution from* $W_p^2((0,1); H(A), H)$ *and for the solution we have the following noncoercive estimation*

$$|\lambda|^2 \|u\|_{L_p((0,1);H)} + \|u''\|_{L_p((0,1);H)} + \|Au\|_{L_p((0,1);H)} \le$$

$$\le c \left(|\lambda| \|f\|_{L_p((0,1);H(A^{1/2}))} + \|f_1\|_{(H(A),H)_{\frac{1}{2}+\frac{1}{2p},p}} + \right.$$

$$\left. + \|f_2\|_{(H(A^2),H)_{\frac{1}{4}+\frac{1}{4p},p},} + \|f_1\|_H + |\lambda|^{\frac{3}{2}-\frac{1}{2p}} \|f_2\|_H \right),$$

where $c > 0$ is a constant independent of λ.

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Isomorphism and coerciveness with a defect of the irregular boundary value problems of ordinary differential equations

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Consider a nonlocal boundary value problem:

$$L(\lambda)u \ := 2\lambda^2 u(x) - \lambda u'(x) - u''(x) +$$

$$+b(x)u(x) = f(x), x \in (0,1) \qquad (1)$$

$$L_1 u \ := u'(0) + u'(1) = f_1, \qquad L_2 u \ := u(0) - 2u(1) = f_2, \ (2)$$

where $f(x)$ and $b(x)$ are given functions, f_1 and f_2 are given complex numbers. The root of the equations $-\omega^2 - \omega + 2 = 0$ are $\omega_1 = -2, \ \ \omega_2 = 1$. Problem (1) and (2) is Birkhoff-irregular [1]. Let us show that problem (1) and (2) is 1-regular with respect to a system of numbers $\omega_1 = -2, \ \ \omega_2 = 1$ (see [2]) and is not 1-regular with respect to a system of numbers $\omega_2 = 1, \ \omega_1 = -2$. Then, problem (1) and (2) is 1-regular with respect to a system of numbers $\omega_1 = -2, \ \ \omega_2 = 1$, since

$$\theta \ := \begin{vmatrix} \omega_1 & \omega_2 \\ 1 & -2 \end{vmatrix} = \begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} = 3 \neq 0,$$

and is not 1-regular with respect to a system of numbers $\omega_2 = 1,\ \omega_1 = -2$, since

$$\theta\ := \begin{vmatrix} \omega_2 & \omega_1 \\ 1 & -2 \end{vmatrix} = \begin{vmatrix} 1 & -2 \\ 1 & -2 \end{vmatrix} = 0.$$

It is proved in the paper that, inside the angle

$$-\frac{\pi}{2} + \varepsilon < \arg\lambda < \frac{\pi}{2} - \varepsilon, \tag{3}$$

the resolvent of problem (1) and (2) decreases like to the regular case and, inside the angle

$$\frac{\pi}{2} + \varepsilon < \arg\lambda < \frac{3\pi}{2} - \varepsilon, \tag{4}$$

the resolvent of problem (1) and (2) increases with respect to the spectral parameter λ.

Theorem.

a) Let $b \in W_p^{k+1}(0,1)$, *where* $1 < p \leq \infty$, $b^{(j)}(0) = b^{(j)}(1) = 0$, $j = 0, \ldots, k-1$, $b^{(k)}(1) + 2(-1)^{k+1} b^{(k)}(0) \neq 0$ *for some* $k \in \mathbb{N}$.

Then, for any $\varepsilon > 0$ *there exists* $R_\varepsilon > 0$ *such that for all complex numbers* λ *satisfying* $|\lambda| > R_\varepsilon$ *and which belong to angle (4), the operator* $\mathbb{L}(\lambda) : u \to \mathbb{L}(\lambda)u = (\mathbb{L}(\lambda)u,\ \mathbb{L}_1 u, \mathbb{L}_2 u\ )$ *from* $W_{q,\gamma}^2(0,\ 1)$ *onto* $L_{q,\gamma}(0,\ 1) + \mathbb{C}^2$, *where* $q \in (1,\ \infty)$, $-\frac{1}{q} < \gamma < \min\left\{\frac{1}{q},\ 1 - \frac{1}{q}\right\}$, *is an isomorphism and for these* λ *the following noncoercive estimate holds for a solution of problem (1) and (2)*

$$|\lambda|^2 \|u\|_{L_{q,\gamma}(0,\ 1)} + \|u\|_{W_{q,\gamma}^2(0,\ 1)} \leq$$

$$\leq C(\varepsilon)\,|\lambda|^{k+4-\gamma-\frac{1}{q}}\left(\|f\|_{L_{q,\gamma}(0,\ 1)}+|f_1|+|f_2|\right)$$

b) Let $b \in L_{q,\gamma}(0,\ 1)$. *Then, the same isomorphism as in item a, is fulfilled in angle (3),* $\lambda > R_{\varepsilon}$, *and for these* λ *the following coercive estimate holds for a solution of problem (1) and (2)*

$$\sum_{k=0}^{2}|\lambda|^{2-k}\|u\|_{W^{k}_{q,\gamma}(0,\ 1)} \leq$$

$$\leq C(\varepsilon)\left(\|f\|_{L_{q,\gamma}(0,\ 1)}+\sum_{\upsilon=1}^{2}|\lambda|^{\upsilon-\gamma-\frac{1}{q}}\,|f_{\upsilon}|\right).$$

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Spectral properties of a fourth order differential operator with a spectral parameter in three of the boundary conditions

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We consider the following eigenvalue problem

$$y^{(4)}(x) - (q(x)y'(x))' = \lambda y(x),\ 0 < x < 1, \tag{1}$$

$$y''(0) = 0,\ Ty(0) - a\lambda y(0) = 0, \tag{2}$$

$$y''(1) - b\lambda y'(1) = 0,\ Ty(1) - c\lambda y(1) = 0, \tag{3}$$

where $\lambda \in \mathbb{C}$ is a spectral parameter, $Ty \equiv y''' - qy'$, $q(x)$ is a positive absolutely continuous function on the interval $[0,1]$, a, b and c are real constants such that $a > 0$, $b > 0$ and $c < 0$.

Problem (1)-(3) describes bending vibrations of a homogeneous rod, on the right end of which a tracing force acts and on the right end an inertial mass is concentrated (see [1, 2]). We study oscillation properties of eigenfunctions and basis properties of subsystems of eigenfunctions in the space $L_p(0,1)$, $1 < p < \infty$ of this problem.

Theorem 1. *There exists an infinitely nondecreasing sequence* $\{\lambda_k\}_{k=1}^{\infty}$ *of eigenvalues of problem (1)-(3) such that* $\lambda_1 = \lambda_2 = 0$ *and* $\lambda_k > 0$, $k \geq 3$. *Moreover, the corresponding eigenfunctions and their derivatives have the following oscillation properties: (i) the eigenfunction* $y_k(x)$, *corresponding to the eigenvalue* λ_k *for* $k \geq 3$ *has either* $k-2$ *or* $k-1$ *simple zeros in* $(0,1)$; *(ii) the function* $y_k'(x)$ *for* $k \geq 3$ *has exactly* $k-2$ *simple zeros in the interval* $(0,1)$.

Theorem 2. *If* $r = 1$, $l = 2$ *and* s *is a sufficiently large number of odd multiplicity, then the system of eigenfunctions* $\{y_k(x)\}_{k=1,\, k\neq r,\, l,\, s}^{\infty}$ *of problem (1)-(3) forms a basis in the space* $L_p(0,1)$, $1 < p < \infty$ *(which is an unconditional basis for* $p = 2$*).*

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Oscillatory integral operator on Lebesgue spaces with variable exponent

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By $L^{p(\cdot)}(\Omega)$ we denote the space of all measurable functions $f(x)$ on Ω such that

$$I_{p(\cdot)}(f) = \int_{\Omega} |f(x)|^{p(x)}\, dx < \infty.$$

Equipped with the norm

$$\|f\|_{p(\cdot)} = \inf \left\{ \eta > 0 : I_{p(\cdot)}\left(\frac{f}{\eta}\right) \le 1 \right\}.$$

A distribution kernel $K(x,y)$ is a "standard singular kernel", that is, a continuous function defined on $\{(x,y) \in \Omega \times \Omega : x \ne y\}$ and satisfying the estimates

$$|K(x,y)| \le C\, |x-y|^{-n} \quad for \;\; all \quad x \ne y,$$

$$|K(x,y) - K(x,z)| \le C \frac{|y-z|^{\sigma}}{|x-y|^{n+\sigma}},\ \sigma > 0,\ \ if \;\; |x-y| > 2\,|y-z|\,,$$

$$|K(x,y)-K(\xi,y)| \leq C\frac{|x-\xi|^{\sigma}}{|x-y|^{n+\sigma}},\ \sigma>0,\ \ if\ \ |x-y|>2\,|x-\xi|\,,$$

Calderon-Zygmund type singular operator and the oscillatory integral operator are defined by

$$Tf(x)=\int_{\Omega}K(x,y)f(y)dy,$$

$$Sf(x)=\int_{\Omega}e^{p(x,y)}K(x,y)f(y)dy,$$

where $P(x,y)$ is a real valued polynomial defined on $\Omega\times\Omega$.

Theorem. *Let $\Omega\subset R^n$ be an open unbounded set, $p\in P_{\infty}^{\log}(\Omega)$. Then the operator S is bounded in the space $L^{p(\cdot)}(\Omega)$.*

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Applications of the affine systems of Walsh type to the construction of smooth basis

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Let us consider the contraction-modulation operators $W_0; W_1$, acting in the Hilbert space $H = L_0^2 = L_0^2(0,1)$, which contains all 1-periodic functions $f(t)$, $t \in \mathbb{R}$ such that $f \in L^2(0,1)$, and $\int_0^1 f(t)\,dt = 0$. That is for $f \in L^2(0,1)$, we choose:
$W_0 f(t) = f(2t); W_1 f(t) = r(t) f(2t)$,
where, $r(t)$ is the periodic function: Haar-Rademacher-Walsh. Then the family of all possible products of these operators applied to the function f, coincides with the Walsh affine system:

$$f_n = f_\alpha = W^\alpha f = W_{\alpha_0} \ldots W_{\alpha_{k-1}} f,$$

where, W^α denote the product of the operators: the operator $W_{\alpha_{k-1}}$ acts first, W_{α_0} acts last (for $k = 0$ the empty product is equal to the identity operator I) and $n = 2^k + \sum_{\nu=0}^{k-1} \alpha_\nu 2^\nu$ is a binary expansion of a natural number $n = 1, 2, \ldots$. The operator structure $\{W_0, W_1\}$ has the following important property, which consists in the fact that this pair of operators forms a multi-shift in the Hilbert space L_0^2 in the sense of the following definition, introduced and studied in [1].

The question on affine Riesz basis of Walsh affine systems is considered. An affine Riesz basis is constructed, generated by a periodic function f continuous on the whole number axis, which has a derivative almost everywhere belonging to the space L^2; in connection with the construction of this example, we note that the functions of the classical Walsh system suffer a discontinuity at separate binary rational points and their derivatives almost vanish everywhere. A method of regularization (improvement of differential properties) of the generating function of an affine Walsh system is proposed, and a criterion for an affine Riesz basis for a regularized generating function that can be represented as a sum of a series in the Rademacher system is obtained.

Theorem 1. *For each $h = \frac{1}{2^{n+1}}$, $n = 1, 2, \dots$, the affine Walsh system generated by the function $f = r_h$, (Denote r_h, $h > 0$, are the Steklov average of the function r) is a Riesz basis.*

Theorem 2. *Suppose that the function $f \in L_0^2$ is representable as a sum of a series in the Rademacher system $f = \sum_{k=0}^{\infty} c_k r_k$. Then for an affine Walsh system generated by the function $J_0 f$ ($J_0 f(t)$ is an antiperiodization of $Jf(t) = \int_0^t f(s)\, ds$ - integration operator), to be a Riesz basis it is necessary and sufficient that analytic function $\widehat{f}(z) = \sum_{k=0}^{\infty} c_k z^k$ satisfied the condition $\frac{1}{2\pi} \int_0^{2\pi} |\widehat{f}(\frac{1}{2} e^{it})|^2\, dt \leq 3 |\widehat{f}(\frac{1}{2})|^2$.*

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Inverse problems for the diffusion operators with impulse

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We consider the differential equation

$$l(y) = -y'' + [q(x) + 2\lambda p(x)]y = \lambda^2 y, \quad x \in [0, a) \cup (a, \pi] \quad (1)$$

with the boundary conditions

$$U(y) := y'(0) = 0, \quad V(y) := y(\pi) = 0 \quad (2)$$

and discontinuous conditions

$$y(a + 0) = \alpha y(a - 0), \quad y'(a + 0) = \alpha^{-1} y'(a - 0), \quad (3)$$

where λ is spectral parameter, $y = y(x, \lambda)$ is an unknown function, $q(x) \in L_2(0, \pi), p(x) \in W_2^1 1(0, \pi)$ are real-valued functions and $\alpha > 0, \alpha \neq 1, a \in (\pi/2, \pi)$.

In this work, a distinct method, unlike other studies of the inverse spectral problems for the diffusion operator, is presented for the reconstruction of the diffusion operator by the spectral datas. Namely, the integral equations of Gelfand-Levitan-Marchenko type are obtained for the problem (1)-(3) and is proved sufficient condition for the inverse problem by these integral equations and the spectral datas of the problem (1)-(3).

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Completeness of the system of eigen and associated vectors of operators generated by partial operator-differential expressions in Hilbert space

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Let $H_0, H_1, ..., H_{2m}$ be Hilbert spaces, where $H_{i+1} \subset H_i,\ i = = 0, 1, 2, ..., 2m-1$, and all the imbeddings are compact. Denote by $\widetilde{H}_i$ a Hilbert space $\widetilde{H}_i = L_2\left(H_i, R^n\right)$.

We consider $2m$-th order operator-differential equation

$$A(x,D)u = \sum_{|\alpha|\le 2m} A_\alpha(x) D^\alpha u.$$

The function $u(x) \in H_{2m}$ is such that $D^\alpha u \in H_{2m-|\alpha|}$. It is assumed that for every $x \in R^n$ the operator $A_\alpha(x),\ \alpha \neq 0$ is a bounded operator: $H_{2m-|\alpha|} \to H_0,\ A_\alpha(x) = A_0 + \gamma(x)E$, where A_0 is a positive-definite self-adjoint operator such that A_0^{-1} is completely continuous.

It is assumed that the complex-value function $\gamma(x)$ is measurable and locally bounded.

Let us formulate conditions that in the sequel should be satisfied by the coefficients $A(x,D)$.

1. The operator-function $R_0\,(x,\xi) = \left[\sum\limits_{|\alpha|=2m} A_\alpha(x)\ (i\xi)^\alpha\right]^{-1}$, $\xi = (\xi_1, \xi_2 ... \xi_n), \xi^\alpha = \xi_1^{\alpha_1}, \xi_2^{\alpha_2} ... \xi_n^{\alpha_n}$ for all $\xi \in R^4\,|0,\quad x \in R^n$, is a bounded operator and $H_0 \to H_{2m}$, and

$$\sum_{j\le 2m} |\xi|^j \, \|R_0(x,\xi)\|_{H_0 \to H_{2m-j}} \le \delta_1,$$

where δ_1 is independent of x and ξ.

2. There exists the ray $l = \{\lambda : \arg\lambda = \beta\}$ of a complex plane λ such that the operator

$$R\,(x,\xi,\lambda) = \left[\sum_{|\alpha|\le 2m} (i\xi)^\alpha\, A_\alpha(x)\ + \lambda E\right]^{-1}$$

is a bounded operator $H_0 \to H_{2m}$ for $\lambda \in l,\ \xi \in R^n,\ \ |x| > c$ and

$$(|\gamma(x)| + |\lambda|) \left\| R(x, \xi, \lambda) \right\|_{H_0 \to H_{2m}} +$$

$$+ \sum_{j<2m} |\xi|^{2m-j} \left\| R(x, \xi, \lambda) \right\|_{H_0 \to H_{2m-j}} \le \delta_2.$$

3. The quantities $\sup\limits_{|x-x_0| \le h} \left\| A_\alpha(x) - A(x_0) \right\|,\ \ \sup \left| \frac{\gamma(x)-\gamma(x_0)}{\gamma(x_0)} \right|$ tend to zero as $h \to 0$ uniformly with respect to $x_0,\ \ |\alpha| = 2m$.

4. $\sum\limits_{0<|\alpha| \le 2m} \left\| A_\alpha(x) \right\| \le \delta_3$.

5. A_0 is a self-adjoint positively-definite operator $H_0 \to H_0$ bounded in $H_{2m} \to H_0$ and such that A_0^{-1} is completely continuous and $\lambda_k \left(A_0^{-1} \right) \ge C \cdot K^{-\frac{n}{2m}},\ \lambda_k$ are eigenvalues of the operator A_0^{-1}.

We have a theorem.

Theorem. *Let conditions 1-5 be fulfilled, and condition 2 be fulfilled for all β such that $\beta_2 < \beta < \beta_2$. We accept $\beta_0 = \max \left\{ (\beta_1 - \beta_2),\ 2\pi - (\beta_1 - \beta_2) \right\} < \frac{2m\pi}{n}$.*

Let $|\gamma(x)| \ge C\, |x|^\gamma$, where $\gamma > \dfrac{2m\beta_0}{\frac{2m\pi}{n} - \beta_0},\ \ |\alpha| > a$.

Then the system of eigen and associated functions of the operator A is complete in H_0.

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On an inverse coefficient problem for a longitudinal wave propagation equation with non-self-adjoint boundary conditions

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In this paper, we study the unique solvability of the inverse problem of determining the triplete of functions $\{u(x,t), a(t), b(t)\}$ satisfying the equation [1]

$$u_{tt}(x,t) - \alpha u_{ttxx}(x,t) - u_{xx}(x,t) =$$

$$= a(t)u(x,t) + b(t)u_t(x,t) + f(x,t) \quad (x,t) \in D_T, \qquad (1)$$

with the initial conditions

$$u(x,0) = \varphi(x), \quad u_t(x,0) = \psi(x), \quad 0 \le x \le 1, \tag{2}$$

the not-self-adjoint boundary condition

$$u(0,t) = 0, \quad u_x(0,t) = u_x(1,t), \quad 0 \le t \le T, \tag{3}$$

and the additional conditions

$$u\left(\frac{1}{2}, t\right) = h_1(t), \quad u(1,t) = h_2(t), \quad 0 \le t \le T. \tag{4}$$

Here $D_T := \{(x,t) : 0 \le x \le 1, 0 \le t \le T\}$ is a rectangular domain, $T, \alpha > 0$ are fixed numbers, $f(x,t)$, $\varphi(x)$, $\psi(x)$, $h(t)$ are given functions, while the functions $u(x,t)$, $a(t)$ and $b(t)$ are unknowns.

We introduce the following set of functions:

$$\tilde{C}^{(2,2)}(D_T) := \{u(x,t) : u(x,t) \in C^2(D_T), u_{ttx}(x,t),$$

$$u_{txx}(x,t), u_{ttxx}(x,t) \in C(D_T)\}.$$

Definition. The triplete $\{u(x,t), a(t), b(t)\}$ is said to be a classical solution of problem (1)–(4), if the functions $u(x,t) \in \tilde{C}^{(2,2)}(D_T)$, and $a(t), b(t) \in C[0,T]$ satisfying the equation (1) in D_T, the condition (2) on $[0,1]$, and the statements (3), (4) on the interval $[0,T]$.

First, the original problem is reduced to an equivalent problem in a certain sense. Further, using the Fourier method, the equivalent problem is substituted with the system of integral equations. Applying the contraction mapping principle, we prove the existence and uniqueness of the solution of the system of integral equations, which is also the unique solution to the equivalent problem. Then, using equivalence, the existence and uniqueness of the classical solution of the original problem are proved.

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Absence of a global solution of a semilinear parabolic equation with a biharmonic operator in the principal part

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We introduce the following notation: $x = (x_1, ..., x_n) \in R^n$, $n > 4$, $r = |x| = \sqrt{x_1^2 + ... + x_n^2}$, $B_R = \{x; |x| < R\}$, $B_R^{'} = \{x; |x| > R\}$, $B_{R_1,R_2} = \{x; R_1 < |x| < R_2\}$, $Q_R = B_R \times (0; +\infty)$, $Q_R^{'} = B_R^{'} \times (0; +\infty)$, $\partial B_R = \{x; |x| = R\}$, $\nabla u = \left(\frac{\partial u}{\partial x_1}, ..., \frac{\partial u}{\partial x_n}\right)$.

In the domain $Q_R^{'}$ we consider the following problem

$$\begin{cases} |x|^{\lambda} \frac{\partial u}{\partial t} = -\Delta^2 u^p + \frac{C_0}{|x|^4} u^p + |x|^{\sigma} |u|^q, \\ u|_{t=0} = u_0(x) \geq 0, \\ \int\limits_0^{\infty} \int\limits_{\partial B_R} u dx dt \geq 0, \qquad \int\limits_0^{\infty} \int\limits_{\partial B_R} \Delta u^p dx dt \leq 0, \end{cases} \tag{1}$$

where $q > 1$, $0 \leq C_0 \leq \left(\frac{n(n-4)}{4}\right)^2$, $\sigma > -4$, $u_0(x) \in C\left(B_R^{'}\right)$, $\Delta^2 u = \Delta(\Delta u)$, $\Delta u = \sum\limits_{i=1}^{n} \frac{\partial^2 u}{\partial x_i^2}$.

We will study the question of the existence of global solutions of problem (1). Such problems are studied quite intensively and a review of such results can be found in the monograph [1] and in the book [2]. We will understand the solution of the problem in the classical sense.

The main result of this paper is the following theorem.

Theorem. *Let $n > 4$, $\sigma > -4$, $1 \leq p < q$, $0 \leq C_0 \leq (\frac{n(n-4)}{4})^2$ and $q \leq p + \frac{\sigma+4}{\frac{n+4}{2} + \lambda + \alpha_-}$. If $u(x,t)$ is a solution of problem (1), then $u(x,t) \equiv 0$.*

Consider the function

$$\xi\left(|x|\right)=\frac{1}{2}\left(1+\frac{\sqrt{D}-\alpha_{+}}{\alpha_{-}}\right)|x|^{-\frac{n-4}{2}+\alpha_{-}}+$$

$$+\frac{1}{2}\left(1-\frac{\sqrt{D}-\alpha_{+}}{\alpha_{-}}\right)|x|^{-\frac{n-4}{2}-\alpha_{-}}-|x|^{-\frac{n-4}{2}-\alpha_{+}}.$$

It can be shown that $\xi\left(x\right)$ satisfies the following conditions:

$$\xi\big|_{|x|=1}=0,\quad \left.\frac{\partial\xi}{\partial r}\right|_{|x|=1}\geq 0,\quad \Delta\xi\big|_{|x|=1}=0,\quad \left.\frac{\partial\left(\Delta\xi\right)}{\partial r}\right|_{|x|=1}\leq 0.$$

Consider the following functions:

$$\varphi\left(x\right)=\begin{cases}1 & \text{for } 1\leq|x|\leq\rho,\\ \left(\frac{1}{2}\cos(\pi(\frac{|x|}{\rho}-1))+\frac{1}{2}\right)^{\tau} & \text{for } \rho\leq|x|\leq 2\rho,\\ 0 & \text{for } |x|\geq 2\rho;\end{cases}$$

$$T_{\rho}\left(t\right)=\begin{cases}1 & \text{for } 0\leq t\leq\rho^{\kappa},\\ \left(\frac{1}{2}\cos(\pi(\rho^{-\kappa}t-1))+\frac{1}{2}\right)^{\mu} & \text{for } \rho^{\kappa}\leq t\leq 2\rho^{\kappa},\\ 0 & \text{for } t\geq 2\rho^{\kappa},\end{cases}$$

where β,μ are large positive numbers, and β is such that for $|x| = 2\rho$

$$\psi = \frac{\partial \psi}{\partial r} = \frac{\partial^2 \psi}{\partial r^2} = \frac{\partial^3 \psi}{\partial r^3} = 0,$$

and $\kappa = \sigma \frac{p-1}{q-p} + 4\frac{q-1}{q-p} + \lambda$.

Considering

$$\psi (x, t) = T_\rho (t) \, \xi (x) \, \varphi (x)$$

as a test function and using the method of test functions Pohozhayev, we prove this theorem.

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The asymptotic behavior of the spectrum of the perturbed Airy operator with Neumann boundary condition

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Consider the operator $L(q)$, defined on the space $L_2(0,+\infty)$ by the differential expression

$$l(y) = -y'' + xy + q(x)\,y$$

with the domain

$$D(L(q)) =$$
$$= \{y \in L_2(0,\infty) : y \in W^2_{2,loc},\ l(y) \in L_2(0,\infty),\ y'(0) = 0\},$$

where the coefficient $q(x)$ is real and satisfy the conditions

$$q(x) \in C[0,+\infty),\ \int_0^\infty x^4 |q(x)|\, dx < \infty,\ q(x) = o(x),\ x \to \infty.$$

In the present work we investigated the asymptotic behavior of the spectrum of the operator $L(q)$. Similar question for the

Airy operator with Dirichlet boundary condition has been studied in the works [1]-[2].

It is well known [3], the spectrum of the operator $L(0)$ is discrete and consists of a sequence of simple eigenvalues $\lambda_n(0), n = 1, 2, ...,$ where

$$\lambda_n(0) = f\left(\frac{3\pi(4n-3)}{8}\right),$$

$$f(z) \sim z^{\frac{2}{3}}\left(1 - \frac{7}{48}z^{-2} + \frac{35}{288}z^{-4} - \frac{181223}{207360}z^{-6} + \frac{18683371}{1244160}z^{-8} - ...\right), z \to \infty.$$

In this paper using the transformation operator (see [4],[5]), we find the asymptotic behavior of the spectrum of the operator $L(q)$.

Theorem. *The spectrum of the operator $L(q)$ is discrete and consists of a sequence of simple eigenvalues $\lambda_n(q), n = 1, 2, ...$ The spectrum $\lambda_n(q)$ has the asymptotic expansion*

$$\lambda_n = \left(\frac{3\pi(4n-3)}{8}\right)^{\frac{2}{3}} + O\left(n^{-\frac{2}{3}}\right), n \to \infty.$$

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Formula for calculating the normal derivative double layer logarithmic potential

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Consider the double layer logarithmic potential

$$W(x) = \int_L \frac{\partial \Phi(x, y)}{\partial \vec{n}(y)} \rho(y) dL_y, \quad x \in L,$$

where $L \subset R^2$ is a simple closed Lyapunov curve, $\vec{n}(y)$ an outer unit normal at the point $y \in L$, $\rho(y)$ a continuous function on the curve L, and $\Phi(x, y)$ a fundamental solution of the Laplace equation, i.e.,

$$\Phi(x,\, y) = \frac{1}{2\pi} \ln \frac{1}{|x-y|}, x, y \in R^2, x \neq y.$$

We denote by $C\,(L)$ the space of all continuous functions on L with the norm $\|\rho\|_\infty = \max\limits_{x \in L} |\rho\,(x)\,|$, and we introduce a modulus of continuity of the form

$$\omega(\varphi,\, \delta) = \delta \sup_{\tau \geq \delta} \frac{\bar{\omega}(\varphi,\, \tau)}{\tau}, \quad \delta > 0,$$

for the function $\varphi(x) \in C\,(L)$, where $\bar{\omega}(\varphi,\, \tau) = \max\limits_{\substack{|x-y| \leq \tau \\ x,\, y \in L}} |\varphi(x) - \varphi(y)|\,.$

Theorem. *Let L be a simple closed Lyapunov curve, $\rho(x)$ is a continuously differentiable function on L, and*

$$\int_0^{diam\, L} \frac{\omega(\overrightarrow{grad}\,\rho,\, t)}{t} dt < +\infty.$$

Then the double layer logarithmic potential $W(x)$ has continuous normal derivatives on L and

$$\frac{\partial W(x)}{\partial \vec{n}\,(x)} = -\frac{1}{\pi} \int_L \frac{(\overrightarrow{yx},\, \vec{n}(y))\; (\overrightarrow{yx},\, \vec{n}(x))}{|x-y|^4} \,(\rho(y) - \rho(x))\, dL_y +$$

$$+\frac{1}{2\pi} \int_L \frac{(\vec{n}(y),\, \vec{n}(x))}{|x-y|^2} \;(\rho(y) - \rho(x))\, dL_y, \;\; x \in L,$$

where the last integral is understood as the Cauchy principal value. Moreover,

$$\left\| \frac{\partial W}{\partial \vec{n}} \right\|_\infty \leq M \left(\|\rho\|_\infty + \left\| \overrightarrow{grad}\rho \right\|_\infty + \int_0^d \frac{\omega\left(\overrightarrow{grad}\rho, t\right)}{t} dt \right),$$

where by M we denote a positive constant depending on L.

On the Noetherness of the Riemann problem in weighted Smirnov classes

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Weighted Smirnov classes with power weight in bounded and unbounded domains are defined in this work. Homogeneous and nonhomogeneous Riemann problems with a measurable coefficient whose argument is a piecewise continuous function are considered in these classes. In case of homogeneous problem, a sufficient condition on general weight function is found which is satisfied by Muckenhoupt class weights, and the general solution of this problem is constructed. In case of nonhomogeneous problem, a Muckenhoupt type condition is imposed on the power type weight function and the orthogonality condition is found for the solvability of nonhomogeneous problem in weighted Smirnov classes, and the formula for the index of the problem is derived. Some special cases with power type weight function are also considered, and conditions on degeneration order are found.

Let $G(\xi) = |G(\xi)| e^{i\theta(\xi)}$ be complex-valued functions on the curve Γ. We make the following basic assumptions on the coefficient $G(\cdot)$ of the considered boundary value problem and Γ:

(i) $|G(\cdot)|^{\pm 1} \in L_\infty(\Gamma)$;

(ii) $\theta(\cdot)$ *is piecewise continuous on* Γ*, and* $\left\{ \xi_k\,,\; k = \overline{1,r} \right\} \subset \Gamma$ *are discontinuity points of the function* $\theta(\cdot)$.

We impose the following condition on the curve Γ.

(iii) Γ *is either Lyapunov or Radon curve (i.e. it is a limited rotation curve) with no cusps.*

Consider the nonhomogeneous Riemann problem

$$F^+(z(s)) - G(z(s))\, F^-(z(s)) = g(z(s))\ ,\ s \in (0, S)\,, \qquad (1)$$

where $g \in L_{p,\rho}(\Gamma)$ is a given function. By the solution of the problem (1) we mean a pair of functions $(F^+(z); F^-(z)) \in E_{p,\rho}(D^+) \times_m E_{p,\rho}(D^-)$, whose boundary values $F^\pm$ on Γ a.e. satisfy (1). Before the formulation of next results introduce the following weight function

$$\nu(s) =: \sigma^p(s)\, \rho(z(s))\ ,\ s \in (0, S)\,, \qquad (2)$$

where the weight function $\rho(\cdot)$ is defined by the expression

$$\rho(s) = \rho(z(s)) = \prod_{k=0}^{m_0} |z(s) - z(t_k)|^{\alpha_k}\,. \qquad (3)$$

We will assume that the weight $\rho(\cdot)$ satisfies the condition

$$\alpha_i < \frac{q}{p}, \quad i = \overline{0, m_0}. \tag{4}$$

Let's find a particular solution $F_1(z)$ of nonhomogeneous problem that corresponds to the argument $\theta(\cdot)$. Let $Z_\theta(z)$ be a canonical solution of homogeneous problem corresponding to the argument $\theta(\cdot)$. Consider the following piecewise analytic function

$$F_1(z) \equiv \frac{Z_\theta(z)}{2\pi i} \int_\Gamma \frac{g(\xi)\, d\xi}{Z_\theta^+(\xi)(\xi - z)}, \; z \notin \Gamma. \tag{5}$$

We are ready to prove the following main

Theorem. *Let the coefficient* $G(z(s))$, $0 \le s \le S$, *of nonhomogeneous problem (1) and the curve* $\Gamma = z([0, S])$ *satisfy the conditions (i)-(iii). Assume that the weight function* $\nu(\cdot)$ *defined by (2), where the weight* $\rho(\cdot)$ *has a form (3) and the condition (4) holds. Then the following assertions are true with regard to the solvability of the problem (1) in the classes* $E_{p;\rho}(D^+) \times_m E_{p;\rho}(D^-), 1 < p < +\infty$:

1) for $m \ge -1$, *the problem (1) has a general solution of the form*

$$F(z) = Z_\theta(z) P_m(z) + F_1(z),$$

where $Z_\theta(\cdot)$ *is a canonical solution of corresponding homogeneous problem,* $P_m(z)$ *is an arbitrary polynomial of degree* $k \le m$ *(for* $m = -1$ *we assume* $P_m(z) \equiv 0$*), and* $F_1(\cdot)$ *is a particular solution of nonhomogeneous problem (1) of the form (5);*

2) for $m < -1$, the nonhomogeneous problem (1) is solvable only when the right-hand side $g(\cdot)$ satisfies the following orthogonality conditions

$$\int_{\Gamma} \frac{g(\xi)}{Z_{\theta}^{+}(\xi)} \xi^{k-1} d\xi = 0 \ , \ \ k = \overline{1, -m-1},$$

and the unique solution $F(z) = F_1(z)$ is defined by (5).

On unique solvability of a boundary value problem for a fourth order operator-differential equation in Hilbert space

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The paper studies a unique solvability of a boundary value problem for a fourth order operator-differential equation on a finite segment in Hilbert space.

To this end, at first we study the solution of a boundary value problem for an appropriate homogeneous equation. It is shown that under the given boundary condition, a boundary value problem for a homogeneous equation has only a trivial solution.

Let H be a separable Hilbert space with a scalar product $(x, y)_H$, where $x, y \in H$. Denote by $L_2([0,1];H)$ Hilbert space of all vector-functions determined on $[0,1]$ with values in H, that has the norm $\|f\|_{L_2([0,1];H)} = \left(\int\limits_0^1 \|f(t)\|_H^2 \, dt\right)^{1/2}$.

Let A be a self-adjoint positive-definite operator in H with domain of definition $D(A)$.

The domain of definition of the operator $A^p (p \geq 0)$ is a Hilbert space H_p, with respect to the scalar product $(x, y)_{H_p} = (A^p x, A^p y)$, $x, y \in D(A^p)$.

For $p = 0$ we assume $H_0 = H$, $(x, y)_{H_0} = (x, y)_H$, $x, y \in H$.

We consider the boundary value problem

$$Lu = u^{(4)}(t) + A^4 u = f(t), \ t \in [0,1], \tag{1}$$

$$u(0) = u(1) = 0, \quad u'(0) = u'(1) = 0. \tag{2}$$

Definition. *If for all $f(t) \in L_2([0,1];H)$ boundary value problem (1) and (2) has a regular solution and the estimation*

$$\|u\|_{W_2^4([0,1];H)} \leq C \|f\|_{L_2([0,1];H)}$$

is fulfilled, then boundary value problem (1) and (2) is called uniquily solvable or regularly solvable.

In order to study a unique solvability of problem (1) and (2), it is necessary to show that the boundary value problem

$$\begin{cases} u^{(4)}(t) + A^4 u(t) = 0, \quad t \in [0,1], \\ u(0) = u(1) = 0, \quad u'(0) = u'(1) = 0 \end{cases}$$

has only a trivial solution.

The following theorem is proved.

Theorem. *Let A be a self-adjoint positive-definite operator in space H. Then boundary value problem (3) has only a trivial solution.*

On the semiclassical Laplacian with magnetic field having a quadratic zero set

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Let $\Omega \subset R^2$ be a bounded and simply connected domain with a smooth boundary $\partial\Omega$ and $\Delta_A^N(h)$ be a self-adjoint operator in space $L_2(\Omega)$, generated by the magnetic Laplace expression

$$\Delta_A(h) = -\sum_{k=1}^{2}\left(-ih\frac{\partial}{\partial x_k} + a_k(x)\right)^2 \equiv \left(-ih \bigtriangledown + A(x)\right)^2$$

with the domain of definition

$$D\left(\Delta_A^N\right) =$$

$$= \left\{u(x) \in W_2^2(R_n) : \left(-ih \bigtriangledown + A(x)\right) u(x) \cdot \nu = 0 \ on \ \partial\Omega\right\},$$

where $i=\sqrt{-1}$, $h>0$, is the semiclassical parameter, $x=(x_1,x_2)\in R^2, A(x)=(a_1(x),a_2(x))$, is a real magnetic potential from the class $C^\infty\left(\overline{\Omega}\right)$, ν is an outward normal to $\partial\Omega$.

Obviously, the positive operator $\Delta_A^N(h)$ has a purely discrete spectrum. Denote by $\lambda_n(h)$ (in order of increase, taking into account their multiplicity) its eigenvalues.

We introduce the following two sets:

$$\Gamma=\left\{x\in\overline{\Omega}:B(x)=0\right\} \text{ and } \sum=\{x\in\Gamma:\nabla B(x)=0\},$$

where $B(x)=\nabla\times A$ is a magnetic field.

In the theory of II type superconductivity, where model is described by the Ginzburg-Landau functional, the main task is to study the behavior of the first eigenvalue $\lambda_1(h)$ and the first eigenfunction $\psi_{1,h}(x)$ of the operator $\Delta_A^N(h)$ when $h\to 0$. When $\Gamma=\emptyset$, there are numerous works devoted to this problem . When $\sum\neq\emptyset$, but $\sum\cap\partial\Omega=\emptyset$ this problem was investigated in [1]. In this paper, this problem is investigated with $\sum\cap\partial\Omega\neq\emptyset$.

Using curvilinear coordinates in the neighborhoods of the curves $\Gamma,\sum$ and $\partial\Omega$ and applying Ismagilov, Morgan, and Simon's formula the problem reduces to investigation in the entire space R^2 and half-plane $R_+^2=\left\{(x_1,x_2)\in R^2:x_2>0\right\}$ with the Neumann boundary condition a model magnetic Schrödinger operator $\widetilde{\Delta}_A=\left(-i\nabla+\widetilde{A}(x)\right)^2$ with a magnetic field of the form

$$\widetilde{B}(x)=a_{11}x_1^2+2a_{12}x_1x_2+a_{22}x_2^2.$$

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Regularity of solutions of classes nonlinear elliptic-parabolic problems

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Let $\Omega \subset R^n$be a smooth bounded domain and $T > 0$. Let us denote $\Omega = \Omega \times (0, T]$. We consider the following problem:

$$\partial_t(u) - \operatorname{div}(\Phi(b(u))Du) = 0 \text{ in } \ Q, \tag{1}$$

$$u = g = f \ \text{ on } \ \partial\Omega \times [0, T],$$

$$u(0) = u_0 \ \text{ on } \Omega.$$

For the function $b : R \to R$ we assume that: b is increasing and Lipschitz; $b(s) = 0$ for $b \leq 0$, $b \in C(R) \cap C^1([0, \infty])$; there exists a constant $c > 0$ such that $b'(s) > 0$ for $s \in (0, \infty)$.

The function $\Phi \in C^1([0,\infty])$ is a positive function. The class of operators given in (1) represent the well-known Richards equation, which serves as a basic model for the filtration of water in unsaturated soils (see [1],[2]).

For weak solution of problem we proved boundedness. Also we proved comparison principle in following form.

Theorem 1. *Let u be a weak solution and ν be a super solution of (1) on $Q = \Omega \times (0,T]$ for some $T > 0$. If $u < \nu$ on the parabolic boundary $\partial_p Q = (\partial\Omega \times [0,T]) \cup \{\Omega \times \{t = 0\}\}$, then $u < \nu$ on Q.*

Equipped with the comparison principle, we can use Perron's method and obtained existence and stability theorem.

Theorem 2. *For initial data $u_0 \in C^{1,1}$ the following holds:*

1) There exists a minimal $\underline{u}$ and $\overline{u}$ a maximal weak solution and of (1) with initial data u_0;

2) $\underline{u}$ and $\overline{u}$ are stable under perturbations of initial data with appropriate ordering;

3) $\underline{u}$ and $\overline{u}$ can be obtained as a limit of solutions solving the regularized parabolic equation;

4) if $(\Phi(b(u))Du)$ is linear, then there exists a unique weak solution u with initial data u_0.

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Coefficient-inverse problems on hydraulic networks

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The hydraulic model of steady-state gas flow can be determined by the following implicit dependence:

$$p_j = p_j(Q, P_1, \lambda), j = 1, ..., m \tag{1}$$

(in the right part, the pressure value at one of the vertices, for example, in the first of I_h is used to provide hydraulic calculation), $\lambda = (\lambda_1, \lambda_2, ..., \lambda_m)$ hydraulic resistance coefficients of network sections, P_1 is given pressure in one of the vertices, $Q = (Q_1, Q_2, ..., Q_r)$ external sources consumptions. To determine $p_j, j = 1, ..., m$, by the mathematical model, as mentioned above (1), it is necessary to solve the system of m nonlinear and n linear equations.

Let the coefficients of hydraulic resistance of some of the sections are not specified, for example, the at the first t sections:

$\overline{\lambda} = (\overline{\lambda_1}, \overline{\lambda_2}, ..., \overline{\lambda_t})$ and it is required to determine them on the basis of existing resistances.

The task of determining (identification) of unknown values of hydraulic resistances by the least squares method leads to minimization of the function

$$\min_{\overline{\lambda}} \left\{ F(\overline{\lambda}) = \sum_{i=1}^{N} \sum_{j \in I_h} \left[p_j(Q^i, P_1^i, \overline{\lambda}) - p_j^i \right]^2 \right\}.$$

Taking into account that the first t components of the vector λ are unknown, the other values are known and specified, then in, instead of the full vector λ we will use the vector $\overline{\lambda}$, that is, only those components of vector λ, whose values need to be determined.

The mathematical model of gas movement in a pipeline network in the stationary mode consists of m nonlinear and n linear equations. Using graph theory, the mathematical model is reduced to a system of $l = m + n - 1$ nonlinear equations for the circulation coefficients in closed contours.

To determine the hydraulic resistance coefficients of the sections , observations are made at the tops of the network for flow and pressure values. From these data, the square deviation function between the calculated by the model and the observed values of gas flow is constructed. Next, first-order iterative minimization methods are applied to this function.

For this purpose, analytical formulas for the gradient of the objective function are obtained by identifiable parameters.

Application of finite integral transformation method to solving mixed problems for hyperbolic equations with irregular boundary conditions

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Problem statement. Find the classic solution of the equation

$$\begin{gathered}\frac{\partial^2 u}{\partial t^2} = a(x)\frac{\partial^2 u}{\partial x^2} + b(x)\frac{\partial u}{\partial x} + c(x)u + f(x,t),\\ \beta_0 < x < \beta_1, \quad 0 < t < T,\end{gathered} \tag{1}$$

under the boundary conditions

$$\begin{gathered}\sum_{j=0}^{2}\sum_{m=0}^{1}\sum_{k=0}^{1}\alpha_{j,m}^{(i,k)} \left.\frac{\partial^{j+m}u(x,t)}{\partial t^j \partial x^m}\right|_{x=\beta_k} = \varphi_i(t),\\ 0 < t < T, \quad i = 1,2,\end{gathered} \tag{2}$$

and initial conditions

$$\left.\frac{\partial^j u(x,t)}{\partial t^j}\right|_{t=0} = f_j(x), \quad x \in (\beta_0, \beta_1), \quad j = 0,1, \tag{3}$$

where $u \equiv u(x,t)$ is the desired classic solution, $\alpha_{j,m}^{(i,k)}, \beta_m$ are the known number, T is some positive number, the remaining ones are the known functions.

1°. Let $a(x) > 0$ for $x \in [0,1]$; $a(x) \in C^{2+n}([0,1])$, $b(x) \in C^{1+n}([0,1])$, $c(x) \in C^{n}([0,1])$, where n is some natural number ; $f_j(x) \in C^1([0,1]), j = 0,1$; the functions $f(x,t)$,$\varphi_i(t), i = 1,2...$ are conditions for $x \in [0,1], t \in [0,1]$.

2°. Let boundary conditions (2) be well-posed; for $x \in \sigma_\varepsilon, 0 \leq t \leq 1$ there exist conditions derivatives $\frac{\partial^k u(x,t)}{\partial t^k}, k = \overline{2, m+1}$.

Theorem. *Let constrains* 1°-2° *be fulfilled. Then of problem (1)-(3) has a classic solution, then i) is unique ii) this solution for* $0 < \varepsilon \leq x \leq 1-\varepsilon < 1$, $0 < t \leq 1$ *is represented by the formula*

$$u(x,t) = f_0(x) + \frac{t}{1!} f_1(x) + ... + \frac{t^m}{m!} f_m(x) +$$

$$+\frac{1}{2\pi\sqrt{-1}} \int_L \Phi(x,t,\lambda) d\lambda,\ 0 < \varepsilon \leq x \leq 1-\varepsilon < 1,\ 0 < t \leq T.$$

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Solvability of a class of boundary value problems for third-order differential operator equations

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Consider the following boundary value problem in the Hilbert space H:

$$P\left(d/dt\right)u\left(t\right)=\left(\frac{d}{dt}+A\right)\left(\frac{d}{dt}-A\right)^{2}u\left(t\right)+$$

$$+A_{1}u\left(t\right)+A_{2}u\left(t\right)=f\left(t\right),t\in R_{+}\left(0,\infty\right),$$

$$u(0)=0,$$

where $f(t)$, $u(t)$ are functions is defined almost everywhere $R_{+}=(0,\infty)$ with values in H , and the coefficients of operators satisfies the following conditions:

1) A is positive definite self-adjoint operator with completely continuous inverse A^{-1} ;

2) operators $B_1 = A_1A^{-1}$, $B_2 = A_2A^{-2}$ are completely continuous in H.

Let $L_2(R_+;H)$ be a Hilbert space of all functions $f(t)$ which is defined almost everywhere in R_+, with values in H, square integrable, since,

$$\|f\|_{L_2(R_+;H)} = \left(\int\limits_0^\infty \|f\,(t)\|^2\,dt\right)^{1/2}$$

and $W_2^3\,(R_+;H) = \left\{u : A^3u \in L_2\,(R_+;H)\,, u''' \in L_2\,(R_+;H)\right\}$ a Hilbert spaces with norm

$$\|u\|_{W_2^3(R_+;H)} = \left(\left\|A^3u\right\|^2_{L_2(R_+;H)} + \|u'''\|^2_{L_2(R_+;H)}\right)^{1/2}.$$

Let $\overset{\circ}{W}{}_2^3\,(R_+;H) = \left\{u : u \in W_2^3\,(R_+;H)\,, u(0) = 0\right\}$. Define the following operator $\overset{\circ}{W}{}_2^3\,(R_+;H) \to L_2\,(R_+;H):$

$$Pu = P\,(d/dt)\,u, u \in \overset{\circ}{W}{}_2^3\,(R_+;H)\,.$$

Definition. *Operator P is said to be the $\Phi-$ operator, if the range JmP is closed in $L_2(R_+;H)$ and $dimKerP < \infty$, $codimJmP < \infty$.*

The following theorem is proved.

Theorem 1. *Let the following conditions are true 1), 2) and with respect to an imaginary axis $P^{-1}(\lambda)$*

$$\sup_\lambda \left(\left\|\lambda P^{-1}\,(\lambda)\right\| + \left\|A^3P^{-1}\,(\lambda)\right\|\right) < \infty,$$

where

$$P\,(\lambda) = (\lambda + A)\,(\lambda - A)^2 + \lambda A_1 + \lambda A_2, \qquad (1)$$

then the operator P is Φ-operator.

From this theorem it follows.

Theorem 2. *Let the conditions of the theorem 1 are true, $A^{-1} \in \sigma_p (0 < p < \infty)$. Then root vectors of pencil (1) corresponding to eigen values from the left half-plane are complete in $H_{5/2}$ with finite defect.*

Here $H_{5/2} = D(A^{5/2})$ is a Hilbert space with the scalar product $(x,y)_{5/2} = \left(A^{5/2}x, A^{5/2}y\right), (x,y) \in D\left(A^{5/2}\right)$.

Bounds for the fundamental solution of the heat equation in special domains

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Let

$$P_B(t,x) = \int_B F\left(t-\tau, x-\xi\right) d\mu\left(\tau,\xi\right)$$

be the heat potential, where $F(t,x)$ is the fundamental solution of the heat equation, $B \subset R^{n+1}$ is a Borel set and μ is a Borel measure.

The measure μ on B called admissable, if

$$P_B(t,x) \leq 1, \quad (t,x) \in R^{n+1}.$$

Denote for $\lambda > 1$ and $m, k \in N$ the paraboloids

$$P_m \left\{(t,x) : \left|x^2\right| < -\lambda^m t,\ t < 0\right\}$$

and clylinders $C_{m,k} \{(t,x) : -t_k < t < 0,\ \ |x| < a\rho_{m,k}\}$, where $t_{k+1} = \dfrac{t_k}{4},\ t_1 > 0,\ a > 0,\ \rho^2_{m,k} = \lambda^m t_k$.

Let

$$B_{m,k} = (P_{m+1} \backslash P_m) \cap \left[-t_k; -\frac{t_k}{4}\right],$$

and denote by $S_{m,k}$ the lateral surface of the cylinder $C_{m,k}$.

Let $T_{m,k} = C_{m,k} \backslash P_m$ and denote by $T^{(j)}_{m,k},\ j = 1, 2, ..., n_0$, the minimal finite partition $T_{m,k}$ for which the following

$$|x - \xi| \le |\xi|$$

is fulfilled, at $(t,x) \in T^{(j)}_{m,k+1}$ and $(\xi, \tau) \in T^{(j)}_{m,k}$ for every j.

Now let's formulate the main result.

Theorem. There exist the following absolute constants $C_1 > 0$ and $C_2 > 0$ depending only on fixed numbers λ, a, n such that holds

$$\sup_{S_{m,k}} P_{B_{m,k}}(t,x) \le C_1 \cdot P_{B_{m,k} \cap T^{(j)}_{m,k}}(0,0),$$

and also such finite partition that for every $j \in \{1, 2, ..., n_0\}$

$$\inf_{T^{(j)}_{m,k}} P_{B_{m,k}}(t,x) \ge C_2 \cdot P_{B_{m,k} \cap T^{(j)}_{m,k}}(0,0),$$

moreover $C_2 > C_1$.

On the existence and uniqueness of the solution of a problem with a nonlocal condition for a hyperbolic type equations

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Consider the equation

$$\begin{array}{c}\frac{\partial^2 u(x,t)}{\partial t^2} - \sum\limits_{i,j=1}^{m} \frac{\partial}{\partial x_i}\left(a_{ij}(x,t)\frac{\partial u(x,t)}{\partial x_j}\right) = \\ = f(x,t,u(x,t)), \quad (x,t) \in Q_T,\end{array} \tag{1}$$

with initial conditions

$$u(x,0) = \varphi_0(x), \quad \frac{\partial u(x,0)}{\partial t} = \varphi_1(x), \quad x \in \Omega, \tag{2}$$

and nonlocal condition

$$\begin{array}{c}\sum\limits_{i,j=1}^{m} a_{ij}(x,t)\frac{\partial u}{\partial x_j}\cos(v,x_i)|_{S_T} = \\ = \int\limits_{\Omega} K(x,y,t,u(y,t))dy \ \text{ in } \ S_T,\end{array} \tag{3}$$

where $Q_T = \{(x,t) : x \in \Omega, \quad 0 < t < T\}$ is the cylinder, $\Omega-$ bounded domain in R^m with smooth boundary $\partial\Omega$, $T > 0$ $-$

given number, $S_T = \{(x,t) : x \in \partial\Omega, \quad 0 < t < T\}$ is the lateral surface of cylinder $Q_T, v-$ external normal to S_T.

Assume that following conditions are satisfied:

1^0. $a_{ij}(x,t)$, $\frac{\partial a_{ij}(x,t)}{\partial t} \in C(\overline{Q}_T), i,j = 1,2,...,m,$ where $\forall \xi \in R^m$ and for all $(x,t) \in \overline{Q}_T$, $\sum\limits_{i,j=1}^{m} a_{ij}(x,t)\xi_i\xi_j \geq \mu \sum\limits_{i,j=1}^{m} \xi_i^2$, $\mu = const > 0$, $a_{ij}(x,t) = a_{ji}(x,t)$, $i,j = 1,...,m$;

2^0. $\varphi_0(x) \in W_2^1(\Omega)$, $\varphi_1(x) \in L_2(\Omega)$;

3^0. the function $f(x,t,u)$ is continuous on $\overline{Q}_T \times R$, it satisfies the Lipchitz condition for u uniformly with respect to $(x,t) \in \overline{Q}_T$ the function $K(x,y,t,u)$ is continuous on $\partial\Omega\times$ $\overline{Q}_T \times R$ and has continuous derivatives $\frac{\partial K}{\partial t}, \frac{\partial K}{\partial u}$, besides that $K(x,y,t,0) = 0$, $\frac{\partial K(x,y,t,0)}{\partial t} = 0$, $\left|\frac{\partial K(x,y,t,u)}{\partial u}\right| \leq c$ and $\frac{\partial K(x,y,t,u)}{\partial t}$ satisfies the Lipchitz condition on u with coefficient L, moreover $A = \alpha L \left(\frac{1}{2} + (mes\Omega)^2\right) < 1$, where $mes\Omega$ is the Lebesgue measure of the domain Ω, α is constant from the inequality $\int\limits_{\partial\Omega} |w(x)|ds \leq \alpha \int\limits_{\Omega} (|w(x)| + |\nabla w(x)|)dx$, $w \in W_2^1(\Omega)$.

The following theorem is true.

Theorem. *Under the condition 1^0, 2^0, 3^0 the mixed problem (1)-(3) has a unique solution from* $W_2^1(\Omega)$.

On the basicity of system $\{\cos(n\theta)\}_{n\in\mathbb{N}_0}$ in grand Lebesgue spaces

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In this work, the basicity of system $\{\cos(n\theta)\}_{n\in\mathbb{N}_0}$ is studied in grand Lebesgue spaces. For the basicity of this system the sufficient conditions are obtained in $L^{p)}(0,\pi)$.

Theorem. The system $\{\cos(n\theta)\}_{n\in\mathbb{N}_0}$ forms a basis for $L^{p)}(0;\pi)$.

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Hardy-Littlewood-Stein-Weiss inequality in the generalized Morrey spaces

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It coincides with the Riesz potential

$$I_\alpha f(x) = \int_{\mathbb{R}^n} \frac{f(y)dy}{|x-y|^{n-\alpha}}, \qquad 0 < \alpha < n.$$

Let $1 \le p < \infty$. The generalized weighted Morrey space $\mathcal{M}^{p,\omega,|\cdot|^\gamma}(\mathbb{R}^n)$ is defined by the norm

$$\|f\|_{\mathcal{M}^{p,\omega,|\cdot|^\gamma}} = \sup_{x\in\mathbb{R}^n, r>0} \frac{r^{-\frac{n}{p}}}{\omega(x,r)} \|f\|_{L_{p,|\cdot|^\gamma}(B(x,r))}.$$

Theorem. *Let* $0 < \alpha < n$, $1 < p < \frac{n}{\alpha}$, $\frac{1}{p} - \frac{1}{q} = \frac{\alpha}{n}$, $\alpha p - n < \gamma < n(p-1)$, $\mu = \frac{q\gamma}{p}$ *and the functions* $\omega_1(x,r)$ *and* $\omega_2(x,r)$ *fulfill the condition*

$$\int_r^\infty t^\alpha (t+|x|)^{-\frac{\gamma}{p}} \omega_1(x,t) \frac{dt}{t} \le C\,(r+|x|)^{-\frac{\gamma}{p}} \omega_2(x,r).$$

Then the operator I_α *are bounded from* $\mathcal{M}^{p,\omega_1(\cdot),|\cdot|^\gamma}(\mathbb{R}^n)$ *to* $\mathcal{M}^{q,\omega_2(\cdot),|\cdot|^\mu}(\mathbb{R}^n)$.

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On well-defined solvability of second order partial operator-differential equation in Hilbert space

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Let H be a separable Hilbert space, C be a positive-definite self-adjoint operator in H with domain of definition $D(C)$. Then the domain of definition of the operator C^p becomes a

Hilbert space H^p with respect to the scalar product $(x,y)_{H_p} = (C^p x, C^p y)_H$, $H_0 = H$.

Let $L_2(R^n; H_2)$ be a Hilbert space of vector-functions $f(x_1, x_2, ..., x_n)$, $(x_1, x_2, ..., x_n) \in R^n$ determined almost everywhere in R^n, with the values in H with the norm

$$\|f\|_{L_2(R^n;h)} = \left(\int_{R^n} \|f(x_1, x_2, ..., x_n)\|_H^2 \, dx, dx_2 ... dx_n \right)^{1/2}.$$

Denote by $D\left(R^n; \widetilde{H}\right)$ a set of infinitely differentiable vector-functions determined in R^n with compact supports. In the linear set $D(R^n; H_2)$ we define the norm

$$\|u\|_{W_2^2(R^n;H)} = \left(\sum_{k=1}^{n} \left\| \frac{\partial^2 u}{\partial x_k} \right\|_{L_2(R^n;H)}^2 + \left\|C^2 u\right\|_{L_2(R^n;H)}^2 \right)^{1/2}.$$

The linear set $D(R^n; H_2)$ will be a pre-Hilbert space with respect to the norm $\|u\|_{W_2^2(R^n;H)}$ whose completion will be denoted by $W_2^2(R^n; H)$.

In space H we consider the operator-differential equation

$$-\sum_{k=1}^{n} a_k \frac{\partial^2 u(x)}{\partial x_k^2} + C^2 u(x) = f(x), \quad x \in R^n. \tag{1}$$

Definition 1. *If for $f(x) \in L_2(R^n; H)$ there exists a vector-function $u(x) \in W_2^2(R^n; H)$ satisfying equation (1) almost everywhere in R^n, then $u(x)$ is said to be a regular solution of equation (1).*

Definition 2. *If for any $f(x) \in L_2(R^n; H)$ equation (1) has the solution $u(x) \in W_2^2(R^n; H)$, that has the estimation*

$$\|u(x)\|_{W_2^2(R^n;H)} \leq C \cdot \|f\|_{L_2(R^n;H)},$$

then equation (1) is said to well-defined solvable.

We have the following theorem.

Theorem. *Let $a_k > 0$, $k = 1, 2...n$ and C be a positive-definite self-adjoint operator in H. Then equation (1) is well-defined solvable.*

Inverse scattering problem for operator Sturm-Liouville with discontinuity conditions

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The direct and inverse problems for the equation

$$-p(x)\left[\frac{1}{p^2(x)}(p(x)y)'\right]' + q(x)y = \lambda^2 y, \ -\infty < x < +\infty,$$

are considered. It is assumed that $p(x)$ is a real-valued piecewise constant positive function with a finite number of discontinuity points and $q(x)$ is a real-valued function satisfying the condition $(1+|x|)q(x) \in L_1(-\infty, +\infty)$.

An inverse boundary-value problem for the Boussinesq-Love equation with non-classical boundary condition

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There are many cases where the needs of the practice lead to problems in determining the coefficients or the right-hand side of the differential equations according to some known data of its solutions. Such problems are called inverse value problems of mathematical physics. Inverse value problems arise in various areas of human activity such as seismology, mineral exploration, biology, medicine, quality control of industrial products, etc., that states them in a number of actual problems of modern mathematics.

Consider for the Boussinesq-Love equation [1]

$$u_{tt}(x,t) - u_{ttxx}(x,t) -$$

$$-\alpha u_{txx}(x,t) - \beta u_{xx}(x,t) = a(t)u(t,x) + f(x,t), \qquad (1)$$

in the domain $D_T = \{(x,t) : 0 \le x \le 1, 0 \le t \le T\}$ an inverse boundary problem with

$$u(x,0) = \varphi(x), \quad u_t(x,0) = \psi(x) \quad (0 \le x \le 1), \tag{2}$$

the Dirichlet boundary condition

$$u(0,t) = 0 \quad (0 \le t \le T), \tag{3}$$

the nonclassical boundary condition

$$u_x(1,t) + du_{xx}(1,t) = 0 \quad (0 \le t \le T), \tag{4}$$

and with the additional condition

$$u(x_0,t) = h(t) \quad (0 \le t \le T), \tag{5}$$

where $x_0 \in (0,1)$, $\alpha > 0$, $\beta > 0$ are the given numbers, $f(x,t), \varphi(x), \psi(x), h(t)$ are the given functions, and $u(x,t), a(t)$ are the required functions.

Definition. *A classical solution to the problem (1)–(5) is a pair $\{u(x,t), a(t)\}$ of the functions $u(x,t)$ and $a(t)$ with the following properties:*

i) the function $u(x,t)$ is continuous in D_T, together with all its derivatives contained in equation (1);

ii) the function $a(t)$ is continuous on $[0,T]$;

iii) all the conditions (1)–(5) are satisfied in the ordinary sense.

First, the initial problem is reduced to the equivalent problem, for which the theorem of existence and uniqueness of solutions is proved. Then, using these facts, the existence and uniqueness of the classical solution of initial problem is proved.

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On analogs of Riesz and Smirnov theorems in grand-Lebesgue space

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In the paper we define grand-Hardy spaces in which the analogs of the Riesz and Smirnov theorems are established.

Let $L^{p)}(-\pi;\pi)$, $1<p<+\infty$, be a grand-Lebesgue space of measurable on $[-\pi;\pi]$ function f, with the finite norm

$$\|f\|_{p)}=\sup_{0<\varepsilon<p-1}\left(\frac{\varepsilon}{2\pi}\int_{-\pi}^{\pi}|f(t)|^{p-\varepsilon}\,dt\right)^{\frac{1}{p-\varepsilon}}<+\infty.$$

Let $U=\{z\in C:|z|<1\}$ be a unit circle and $\gamma=\{\tau:|\tau|=1\}$ be a unit circumference. We define the grand-Hardy space $H^{+}_{p)}$, $p>1$, of analytic in U functions f, with the finite norm

$$\|f\|_{H^{+}_{p)}}=\sup_{0<r<1}\|f_r\|_{p)}<+\infty,$$

where $f_r(t) = f(re^{it})$. Clearly, any functions $f \in H^+_{p)}$, $p > 1$, has always everywhere on γ the boundary values $f^+(e^{it})$ along non tangential ways as $r \to 1$.

The following statements are true.

Theorem 1. *For any $f \in H^+_{p)}$, $p > 1$, we have*

$$\left\| f^+(\cdot) \right\|_{p)} = \lim_{r \to 1} \left\| f_r(\cdot) \right\|_{p)} .$$

Theorem 2. *For $\lim_{r \to 1} \left\| f_r(\cdot) - f^+(\cdot) \right\|_{p)} = 0$ for $f \in H^+_{p)}$, $p > 1$, it is necessary and sufficient that the relation*

$$\lim_{\varepsilon \to +0} \varepsilon \int_{-\pi}^{\pi} \left| f^+(t) \right|^{p-\varepsilon} dt = 0$$

to be fulfilled.

Theorem 3. *Let $f \in H^+_{p)}$, $p > 1$. Then*

1) if $\left| f^+(e^{it}) \right| \leq M$ almost everywhere on γ, then $|f(z)| \leq M$, $z \in U$;

2) if $f^+ \in L^{q)}(-\pi;\pi)$, $p < q$, then $f \in H^+_{q)}$.

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Inverse scattering problem for a first order system on the half-line

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We consider the first order system on the half-line ([3]):

$$-i\frac{dy}{dx} + Q(x)y = \lambda\sigma y,\ 0 \leq x < \infty,$$

where λ is a parameter and $\sigma = diag(\xi_1, \ldots, \xi_{2n})$ is a $(2n \times 2n)$ -order diagonal matrix with $\xi_1 < \cdots < \xi_n < 0 < \xi_{n+1} < \cdots < \xi_{2n}$, and Q is a $(2n \times 2n)$ -order matrix-valued function with exponential decaying entries and satisfies some triangular condition which the system has a transformation operator at infinity that can be expressed by the second kind Volterra operator. Let $y(x, \lambda) = (y_1(x, \lambda), y_2(x, \lambda))^T$ be a solution of this scattering problem with boundary condition given by $y_2(0, \lambda) = Hy_1(0, \lambda)$ with an invertible $n \times n$ matrix H. It is a natural question whether the scattering matrix $S_H(\lambda)$ determines the potential Q. We prove the following uniqueness theorem: Let $S_{H_1}(\lambda)$ and $S_{H_2}(\lambda)$ be two scattering matrices with $det(H_2 - H_1) \neq 0$. Then the potential Q is uniquely determined by the two scattering matrices if certain matrix Riemann-Hilbert problems are uniquely solved.

We also give an example such that two scattering matrices with $det(H_2 - H_1) \neq 0$ are needed to determine the potential uniquely.

We recall the paper [1] that the fundamental result consisting with the necessary and sufficient condition for scattering data is given for inverse scattering problem for matrix Dirac system. We also recall the paper [2], where the inverse problem for above-mentioned system on the half-line with self-adjoint potential was investigated from its spectral function by using transformation operator at $x = 0$.

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Existence and uniqueness of solutions for the system of first-order nonlinear impulsive differential equations with three-point and integral boundary conditions

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In this thesis, we study the existence and uniqueness of the system of nonlinear differential equations of the type

$$\dot{x}(t) = f(t, x(t)) \text{ for } t \in [0, T], \tag{1}$$

subject to impulsive conditions

$$x(t_i^+) - x(t_i) = I_i(x(t_i)), i = 1, 2, ..., p, t \in [0, T],$$

$$0 = t_0 < t_1 < ... < t_{p_1} < \tau < t_{p_{1+1}} < ... < t_p < t_{p+1} = T, \tag{2}$$

and three-point and integral boundary conditions

$$Ax(0) + Bx(\tau) + Cx(T) + \int_0^T n(t)x(t)dt = d, \tag{3}$$

where A, B, C are constant square matrices of order n such that $\det N \neq 0, N = A + B + C + \int_0^T n(t)dt; f : [0, T] \times R^n \to R^n$ and

$n:[0,T]\to R^{n\times n}$ are given matrix-functions; $d\in R^n$ is a given vector; and τ satisfies the condition $0<\tau<T$.

Theorem. *Let $f(t,x)\in C([0,T]\times R^n,R^n)$. Then the unique solution $x(t)\in C([0,T],R^n)$ of the boundary value problem for differential equation (1) with impulsive and boundary conditions (2)-(3) is given by*

$$x(t)=D+\int\limits_0^T G(t,s)f(s,x\left(s\right))ds+\sum_{k=1}^{p}G\left(t_i,t_k\right)I_k\left(x\left(t_k\right)\right)$$

for $t\in\left(t_i,t_{i+1}\right],i=0,1,...,p,$ where

$$G(t,s)=\begin{cases} G_1(t,s),0\leq t\leq\tau,\\ G_2(t,s),\tau<t\leq T,\end{cases}$$

$$D=N^{-1}d$$

with

$$G_1(t,s)=\begin{cases} N^{-1}\left(A+\int\limits_0^s n(\xi)d\xi\right),0\leq s\leq t,\\ -N^{-1}\left(B+C+\int\limits_s^T n(\xi)d\xi\right),t<s\leq\tau,\\ -N^{-1}\left(C+\int\limits_s^T n(\xi)d\xi\right),\tau<s\leq T,\end{cases}$$

and

$$G_2(t,s) = \begin{cases} N^{-1}\left(A + \int\limits_0^s n(\xi)d\xi\right), 0 \le s \le \tau, \\ N^{-1}\left(A + B + \int\limits_0^s n(\xi)d\xi\right), \tau < s \le t, \\ -N^{-1}\left(C + \int\limits_s^T n(\xi)d\xi\right), t < s \le T, \end{cases}$$

The theorems about existence and uniqueness of solutions of boundary value problems (1), (2) has been proved by using contraction principles. Similar problems are considered in [1]-[3].

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Global existence and blow up of solutions to initial boundary value problem for nonlinear equations of thermoelasticity type

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The talk is devoted to the study of the following initial boundary value problem:

$$\begin{cases} \partial_t^2 u - \Delta u - \kappa \nabla v = f_1(u,v), \ x \in \Omega, t > 0, \\ \partial_t v - \Delta v - \kappa \nabla \cdot \partial_t u = f_2(u,v), \ x \in \Omega, t > 0, \\ u\Big|_{\partial\Omega} = v\Big|_{\partial\Omega} = 0, \\ u\Big|_{t=0} = u_0, \ \partial_t u\Big|_{t=0} = u_1, \ v\Big|_{t=0} = v_0, \end{cases} \tag{1}$$

where $\Omega \subset R^n, n = 1,2,3$ is a bounded domain with sufficiently smooth boundary $\partial\Omega$, u is the unknown vector function, v is the unknown scalar function, $\kappa \geq 0$ is a given parameter, u_0, u_1, v_0 are given initial data and $f_1(u,v), f_2(u,v)$ are given nonlinear terms. We are going to discuss results on local and global well posedness, stabilization and blow up of solutions to the initial boundary value problem (1) The results obtained can be considered as a development of results on local and global unique solvability and stabilization of solutions of initial boundary value problems for coupled systems of nonlinear parabolic and hyperbolic equations obtained in [1], [4] and results on blow up in a finite time of solutions of initial boundary value problems for the systems of the form (1) obtained in [2,3,5].

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Absence of a finite spectrum of regular boundary value problems for differential equations

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In this report it is discussed the spectral properties of the linear operator L_Q corresponding to the boundary value problem for a linear differential equation defined by a differential expression in the domain $\Omega \subset R^n$ with homogeneous boundary conditions.

At the same time, the problem of a finite number of the spectrum of differential operators generated by regular boundary conditions remains open. So far, there are no known examples of well-posed linear boundary value problems having the finite number of eigenvalues. In the work [1] Kal'menov T.Sh. and Suragan D. proved the absence of the finite spectrum of boundary value problems for a wide class of differential equations under assumption that in some sufficiently small part of the domain, coefficients of the equation under consideration are constant. In the present report, this restriction is removed, an infinite differentiability of the coefficients is required.

Let $\Omega \subset R^n$ be a finite domain. Consider the following boundary value problem.

Problem Q**.** *Find a solution to the equation*

$$Lu = \sum_{|\alpha|\le p} a_\alpha(x) D^\alpha u = f(x), \tag{1}$$

satisfying the boundary condition

$$Qu|_{\partial\Omega} = 0. \tag{2}$$

Where Q is a linear boundary operator defined on traces of the function u and its derivatives up to the order $p-1$ on the boundary $\partial\Omega$.

By L_Q denote the closure in $L_2(\Omega)$ of the differential operator given by expression (1) on a linear manifold of the functions $u \in C^p(\overline{\Omega})$ satisfying conditions (2). The report assumes that the inverse operator L_Q^{-1} exists, is defined on the entire $L_2(\Omega)$ and is compact. In this case the spectrum of the operator L_Q can consist only of eigenvalues.

The main result of the report is the proof that under some general assumptions with respect to the differential operator L_Q, it will be either Volterra (that is, it has no eigenvalues), or the number of its eigenvalues will be infinite.

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Uniform convergence of the spectral expansions in the terms of root functions of a spectral problem for the equation of vibrating rod

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We consider the following spectral problem:

$$y^{(4)}(x) - (q(x)y'(x))' = \lambda y(x),\ 0 < x < 1, \tag{1}$$

$$y(0) = y'(0) = y''(1) = 0, Ty(1) - (a\lambda + b)y(1) = 0, \tag{2}$$

where $\lambda \in \mathbb{C}$ is a spectral parameter, $Ty \equiv y''' - qy'$, $q(x)$ is a positive absolutely continuous function on the interval $[0, 1]$, a and b are real constants such that $a > 0$, $b < 0$.

Problem (1) and (2) describes bending vibrations of a homogeneous rod, the left end of which is fixed rigidly and on the right end is concentrated an elastically fixed load [1]. The spectral properties of this problem in a more general form were investigated in the paper [2]. In this note we study the uniform convergence of the spectral expansions in terms of root functions of this problem.

Let $\{y_k(x)\}_{k=1}^{\infty}$ be the system of root functions of problem (1) and (2) and the system $\{u_k(x)\}_{k=1,k\neq r}^{\infty}$ is adjoint to the system $\{y_k(x)\}_{k=1,k\neq r}^{\infty}$, where r is an arbitrarily fixed natural number such that $s_r \neq 0$ (see [2]).

Theorem. *Let $f(x) \in C[0,1]$ and $f(x)$ has a uniformly convergent Fourier expansions in the system $\{\Phi_k(x)\}_{k=1}^{\infty}$ on the interval $[0,1]$, where $\{\Phi_k(x)\}_{k=1}^{\infty}$ is a system of eigenfunctions of problem (1) and $y(0) = y'(0) = y''(1) = y(1) = 0$. If $(f, y_r) \neq 0$, then the Fourier series $\sum\limits_{k=1,\, k\neq r}^{\infty} (f, u_k) y_k(x)$ of function $f(x)$ in the system $\{y_k(x)\}_{k=1,\, k\neq r}^{\infty}$ is uniformly convergent on every interval $[0, c]$, $0 < c < 1$. Moreover, this series is uniformly convergent on the interval $[0,1]$ if and only if $(f, y_r) = 0$.*

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Central limit theorem for Markov random walk described by the autoregreressive process of order one $(AR(1))$

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Let X_n, $n \geq 1$ be a sequence of independent identically distributed random variables determined on some probability space (Ω, F, P).

Let us consider a first order autoregression process $(AR(1))$

$$X_n = \beta X_{n-1} + \xi_n, \quad n \geq 1,$$

where $|\beta| < 1$, and the initial value of the process X_0 is independent of the innovation $\{\xi_n\}$.

The statistical estimation for the parameter β by the method of the least squares has the following form

$$\beta_n = \frac{T_n}{S_n},$$

where $T_n = \sum\limits_{k=1}^{n} X_k X_{k-1}$ and $S_n = \sum\limits_{k=1}^{n} X_{k-1}^2$.

It is noted that in the conditions $EX_0^2 < \infty$, $E\xi_1 = 0$ and $D\xi_1 = 1$ we have $\frac{T_n}{n} \overset{a.s.}{\to} \dfrac{\beta}{1-\beta^2}$ and $\frac{S_n}{n} = \frac{1}{1-\beta^2} = \alpha_1^2$ as $n \to \infty$.

In the present paper we prove the central limit theorem for T_n, $n \geq 1$.

On equiconvergence rate of spectral expansion in eigen-function of even order differential operator with trigonomteric series

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On the interval $G = (0, 1)$ we consider the following formal differential operator

$$Lu = u^{(2m)} + P_2(x)u^{(2m-2)} + ... + P_{2m}(x)u$$

with summable real coefficients $P_i(x)$, $i = \overline{2,\ 2m}$.

Let $\{u_k\,(x)\}_{k=1}^{\infty}$ be a complete orthonormed $L_2(G)$ system consisting of eigenfunctions of the operator L, while $\{\lambda_k\}_{k=1}^{\infty}$, $(-1)^{m+1}\,\lambda_k \geq 0$, an appropriate system of eigenvalues.

We introduce the partial sum of spectral expansion of the function $f(x) \in W_1^1(G)$ in the system $\{u_k(x)\}_{k=1}^{\infty}$:

$$\sigma_\nu(x,f) = \sum_{\mu_k \le \nu}^{\infty} f_k u_k(x), \quad \nu > 2,$$

where $\mu_k = \left((-1)^{m+1}\lambda_k\right)^{1/2m}$, $f_k = (f, u_k) = \int_0^1 f(x)\overline{u_k(x)}dx$.

Denote $\Delta_\nu(x,f) = \sigma_\nu(x,f) - S_\nu(x,f)$, where $S_\nu(x,f)$, $\nu > 0$ is a partial sum of trigonometric Fourier series of the function$f(x)$.

Theorem. *Let the function $f(x) \in W_p^1(G)$, $p > 1$, and the system $\{u_k(x)\}_{k=1}^{\infty}$ satisfy the condition*

$$\left| f(x)\,\overline{u_k^{(2m-1)}(x)}\,|_0^1 \right| \le C_1(f)\,\mu_k^\alpha\,\|u_k\|_\infty,$$

$$0 \le \alpha < 2m-1, \quad \mu_k \ge 1.$$

Then expansions of the function $f(x)$ in orthogonal series in the system $\{u_k(x)\}_{k=1}^{\infty}$ and in trigonometric Fourier series uniformly equiconverge on any compact $K \subset G$, and the following estimation is valid

$$\max_{x\in K} |\Delta_\nu(x,f)| = O(\nu^{\beta-1}), \ \nu \to +\infty,$$

where $\beta = 0$, if the system $\{u_k(x)\}_{k=1}^{\infty}$ is uniformly bounded; $\beta = \frac{1}{2}$, if the system $\{u_k(x)\}_{k=1}^{\infty}$ is not uniformly bounded.

On behavior of solution to fourth order nonlinear pseudohyperbolic equation with nonlinear boundary conditions

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We consider the following problem:

$$u_{tt} + \alpha\Delta^2 u - \beta\Delta u_t + \gamma u_t + f(u) = 0, (x,t) \in \Omega \times [0,T]\,, \quad (1)$$

$$u(x,0) = u_0(x), u_t(x,0) = u_1(x), x \in \Omega \quad (2)$$

$$\frac{\partial u}{\partial n} = 0, (x,t) \in \partial\Omega \times [0,T]\,, \quad (3)$$

$$\frac{\partial \Delta u}{\partial n} = g(u), (x,t) \in \partial\Omega \times [0,T]\,, \quad (4)$$

where $\Omega \subset R^n$ is a bounded domain with smooth boundary $\partial\Omega, f(u)$, and $g(u)$ are some nonlinear functions, α, β, γ are some positive numbers, $\Delta u = \sum\limits_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$ is Laplace operator, $\frac{\partial}{\partial n}$ is a derivative on the external normal at $\partial\Omega$.

To the questions of behavior of solutions for the equations of type (1) with different boundary conditions were devoted a lot of papers. Generally, these works deal with nonlinearity in the boundary conditions.

In this work, we study a question of stabilization of solution for a problem (1)-(4) when boundary function has some smoothing properties. For the problem (1)-(4) it is proved the following theorem.

Theorem. *Let the functions $f(t)$ and $g(s)$ satisfy the following conditions:*

$$F(u) = \int_0^u f(\tau)d\tau \geq 0, f(0) = 0,$$

$$G(u) = \int_0^u g(s)ds \geq 0, f(0) = 0$$

however

$$uf(u) - F(u) \geq 0,$$

$$ug(u) - G(u) \geq 0 \text{ for every } u \in R^1$$

and besides it $g(\theta)$ is such a function that $G(u) \geq Mu^2$, $\forall u \in R^1$ where M is some positive number. Then for every solution $u(x,t) \in W_2^1(0,T; W_2^2(\Omega)) \cap W_2^2(0,T; L_2(\Omega))$:

1) there exists $0 < \eta \leq \frac{3}{\delta}$ such that $\frac{\eta(10-\eta)}{2(4-\eta)} \leq k \leq \frac{2M - C\eta_1 + 2\eta}{2}$, $M \geq \frac{\eta(10-\eta)}{2(4-\eta)} + \frac{\delta}{2}\eta - \eta$ the solution stabilizing in the sense $\|u_t\|_{L_2} + \|u(x,t)\|_{W_2^1(\Omega)} \to 0$ as $t \to \infty$;

2) there exists $0 < \eta \leq \frac{3}{2\delta}$ such that, $\frac{\eta(8-\eta)}{2(4-\eta)} \leq k \leq \frac{2M - \eta(2\delta-1)}{2}$, $M \geq \frac{\eta(8-\eta)}{4-\eta} + \delta\eta + \frac{1}{2}\eta$ this solution is stabilized in the sense $\|u_t\|_{L_2} + \|u(x,t)\|_{L_2(\Omega)} \to 0$ as $t \to \infty$.

Geometrical middle problem in the non-classical treatment for one generalized Aller equation with non-smooth coefficients

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In this work substantiated for a generalized Aller equation with non-smooth coefficients in the geometrical middle problem with non-classical boundary conditions is considered, which requires no matching conditions. Equivalence of these conditions boundary condition is substantiated classical, in the case if the solution of the problem in the anisotropic S. L. Sobolev's space is found.

On a scattering problem for a discontinuous Sturm-Liouville problem

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We consider the boundary value problem for the equation

$$-u'' + q(x)u = \lambda^2 \rho(x)u, \qquad 0 < x < +\infty, \tag{1}$$

with the boundary condition

$$-(\alpha_1 u(0) - \alpha_2 u'(0)) = \lambda \left(\beta_1 u(0) - \beta_2 u'(0)\right), \tag{2}$$

where λ is a complex parameter, $q(x)$ is a real valued function satisfying the condition

$$\int_0^{+\infty} (1+x)\,|q(x)|\,dx < +\infty, \tag{3}$$

$\rho(x)$ is a positive piecewise-constant function with a finite number of points of discontinuity, $\alpha_i,\ \beta_i,\ (i=1,2)$ are real numbers and $\alpha_1\alpha_2 > 0$. The aim of present paper is to investigate the direct and inverse scattering problem on the half line $[0,+\infty)$ for the boundary value problem (1)-(3). The eigenvalues of this problem are distributed in the complex plane, are not simple and are not only imaginary. The main equation is obtained and the uniqueness of solution of this main equation is proved. The construction of potential function is given according to scattering data. Different boundary problems are investigated in [1-5].

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Finite approximation of solution of one Riccati-type boundary problem

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In the process of solving of optimal control problem for a parabolic system core of feedback operator in the area $Q = (0,1) \times (0,1) \times (0,T]$ is found as the solution of one Riccati - type boundary problem:

$$K_t(t,\, x,\, s) + a^2(K_{xx}(t,\, x,\, s) + K_{ss}(t,\, x,\, s)) =$$

$$= \beta \int_0^1 \int_0^1 K(t,\, s,\, 1)K(t,\, \xi,\, 1)dsd\xi; \qquad (1)$$

$$\begin{aligned} K_x(t,\, 0,\, s) = K_x(t,\, 1,\, x) + \alpha K(t,\, 1,\, s) = 0, \\ K_s(t,\, x,\, 0) = K_s(t,\, x,\, 1) + \alpha K(t,\, x,\, 1) = 0; \end{aligned} \qquad (2)$$

$$K(T,\, x,\, s) = \delta(x - s). \qquad (3)$$

Here $\alpha,\ \beta = const > 0$, $\delta(x - s)$- Dirac function.
The solution of (1) –(3) problem is found as Fourier series:

$$K(t,\, x,\, s) = \sum_{i,\, j=0}^{\infty} k_{ij}(t)X_i(x)X_j(s), \qquad (4)$$

here $X_i(x)$ $X''(x)+\lambda^2X(x)=0,\ X'(0)=X'(1)+\alpha X(1)=0$ are own functions of boundary problem and form a complete system of functions in $L_2(0,1)$.

Substituting (4) in equation (1) we find that Fourier coefficients $k_{ij}(t)$ are solution of the following infinite system of differential equations

$$\begin{cases} \frac{dk_{ij}(t)}{dt}=a^2(\lambda_i^2+\lambda_j^2)k_{ij}(t)+ \\ +\beta\sum_{m,\,n=1}^{\infty}X_m(1)X_n(1)k_{im}(t)k_{nj}(t),\ \ i,\,j=0,\,1,\,2,... \end{cases} \tag{5}$$

Based on (3), we obtain that the last system is solved under the additional condition

$$k_{ij}(T)=\delta_{ij}. \tag{6}$$

If we denote

$K(t)=\{k_{ij}(t)\}_{i=\overline{1,\,\infty}}^{j=\overline{1,\,\infty}},\ \ B=\{X_i(1)\}_{i=1}^{\infty}$ and $A=\left\{-a^2\lambda_i^2\right\}_{i=1}^{\infty}$, then we write the problem (5) and (6) in the form of a matrix:

$$\frac{dK}{dt}=-A^*K=KA+KBB^*K,\quad K(T)=E. \tag{7}$$

For approximate solution of this problem the successive approximations are constructed as follows:

$$\begin{cases} \frac{dK^{(n+1)}}{dt}=(-A^*+K^{(n)}BB^*)K^{(n+1)}+ \\ +K^{(n+1)}(-A+BB^*K^{(n)})+Q^{(n)}(t),\ K^{(n+1)}(T)=E, \\ n=0,1,2,... \end{cases}$$

and $K^{(0)}(t)\equiv 0,\quad Q^{(n)}(t)=K^{(n)}(t)BB^*K^{(n)}(t)$.

As can be seen from the above the linear differential equation is solved at each step. It is shown that successive approximations converge and the sequence limit is a solution of the problem (7).

One mixed problem for the heat equation with time advance in the boundary conditions

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The mixed problem is studied in present paper :

$$u_t = a^2 u_{xx} + f(x,t), 0 < x < 1, t > 0, \qquad (1)$$

$$u(x,0) = \varphi(x), 0 < x < 1, \qquad (2)$$

$$\left.\begin{array}{l} l_1 u \equiv u(0, t+\omega) + \alpha\, u(1,t) = \psi_1(t) \\ l_2 u \equiv u(0,t) + \beta\, u(1, t+\omega) = \psi_2(t) \end{array}\right\} t > 0, \qquad (3)$$

$$\left.\begin{array}{l} l_3 u \equiv \alpha_1 u_x(0,t) + \beta_1\, u_x(1,t) = \psi_3(t) \\ l_4 u \equiv \alpha_2 u\,(0,t) + \beta_2\, u\,(1,t) = \psi_4(t) \end{array}\right\} 0 \le t \le \omega, \qquad (4)$$

where $f(x,t)$, $\varphi(x)$, $\psi_k(t)$ ($k = 1,\ 2,\ 3,\ 4$) are known, and $u\,(x,t)$ desired are functions.

Here, by combining the residue method and the method of the contour integral of academician M.L. Rasulov [1], prove the existence and uniqueness of the solution of the mixed problem (1) - (4) and obtain an explicit analytic representation for it.

Theorem. *Suppose, that* $\alpha_1\beta_2 + \alpha_2\beta_1 \neq 0, \varphi(x) \in C^2\,[0,\ 1]$, $l_j\varphi = 0$ *(*$j = 1,\ 2$*),* $f(x,t) \in C^{2,1}\,([0,\ 1]\times[0,\ \infty))$, $l_j f = 0$ *(*$j = 1,\ 2$*),* $\psi_k(t) \in C^1\,[0,\ \infty)$, *(*$k = 1,\ 2$*) and* $(\alpha_1+\beta_1)\ (\alpha_2+\beta_2) \neq 0$*. Then the problem (1)-(4) has a unique solution and it is represented by the formula*

$$u\,(x,t) = \varphi(x) - \int_0^t f(x,\ \tau)d\tau + \frac{a^2}{\pi\, i}\int_S \lambda^{-1}e^{\lambda^2 t}\left[\int_0^1 G(x,\xi,\lambda)\cdot\right.$$

$$\left.\cdot\left(\varphi''(\xi) + \int_0^t e^{-\lambda^2\tau} f''_{\xi\xi}(\xi,\tau)d\tau\right)d\xi - Q(x,\lambda,\varphi(0),\varphi\ (1))\right]d\lambda +$$

$$+\frac{1}{\pi\, i}\int_l \lambda\, e^{\lambda^2 t}Q(x,\lambda,\, p(\lambda), q(\lambda)), \qquad (5)$$

where $G(x,\xi,\lambda)$ *is the Green's function of the spectral problem, corresponding to mixed problem (1)-(3),*

$$S = \left\{\lambda:\quad \lambda = \sigma\ \exp\left(-\frac{3\pi}{8}i\right),\quad \sigma\in\left[2c\sqrt{1+\sqrt{2}},\ \infty\right)\right\}\bigcup$$

$$\bigcup\left\{\lambda:\quad \lambda = c(1+i\eta),\ \eta\in\left[-1-\sqrt{2},\ 1+\sqrt{2}\right]\right\}\bigcup$$

$$\bigcup\left\{\lambda:\ \ \lambda = \sigma\exp\left(\frac{3\pi}{8}i\right),\right.$$

$$\left.\sigma\in\left[2c\sqrt{1+\sqrt{2}},\ \infty\right)\right\},$$

$$l=\left\{\lambda:\ Re\,\lambda^2=c,\ \ Re\lambda>0\right\},c>\max\left(0,\ln\left|\frac{\alpha}{\beta}\right|\right),$$

$$Q(x,\lambda,p(\lambda),q(\lambda))=\left(e^{-\frac{\lambda}{a}}-e^{\frac{\lambda}{a}}\right)^{-1}\left[e^{\frac{\lambda}{a}x}\left(p(\lambda)\,e^{-\frac{\lambda}{a}}-q(\lambda)\right)+\right.$$
$$\left.+e^{-\frac{\lambda}{a}x}\left(q(\lambda)-p(\lambda)\,e^{\frac{\lambda}{a}}\right)\right],$$

$$p(\lambda)=\left(\beta\ e^{2\lambda^2\omega}-\alpha\right)^{-1}\left[\beta\,e^{\lambda^2\omega}\left(\tilde{\psi}_1(\lambda)+e^{\lambda^2\omega}\int_0^\omega e^{-\lambda^2t}u(0,\,t)dt\right)-\right.$$
$$\left.-\alpha\left(\tilde{\psi}_2(\lambda)+\beta e^{\lambda^2\omega}\int_0^\omega e^{-\lambda^2t}u(1,\,t)dt\right)\right],$$

$$q(\lambda)=\left(\beta\ e^{2\lambda^2\omega}-\alpha\right)^{-1}\left[e^{\lambda^2\omega}\left(\tilde{\psi}_2(\lambda)+\beta e^{\lambda^2\omega}\int_0^\omega e^{-\lambda^2t}u(1,\,t)dt\right)-\right.$$
$$\left.-\left(\tilde{\psi}_1(\lambda)+e^{\lambda^2\omega}\int_0^\omega e^{\lambda^2t}u(0,\,t)dt\right)\right],$$

$$\tilde{\psi}_k(\lambda)=\int_0^\infty e^{-\lambda^2t}\psi_k\,(t)dt,k=1,\ 2,$$

$u(s,t)$ is the boundary value of the solution of mixed problem (1), (2), (4) on the parts $\{(s,t):\ 0\le t\le\omega,\ \ s=0,\ 1\}$ of the lateral boundary of the domain $\{(x,t):\ 0<x<1,\ \ t>0\}$.

Remark. Mixed problem for the heat equation with deviating arguments in boundary conditions also was considered in paper [2].

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Application of the generalized dispatch method to the solution of the boundary-value problem for parabolic equation

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In the work, we consider an approach that reduces the solution of the boundary-value problem for the parabolic equation to the solution of the sequence of ordinary differential equations.

The statement of the problem is as follows. We consider the solution to the thermal conductivity equation

$$\frac{\partial u}{\partial t} = \mu^2 (x,t) \frac{\partial^2 u}{\partial x^2} + f(x,t), 0 < x < l, 0 < t \leq T. \qquad (1)$$

The initial and boundary conditions are given in the form:

$$u(x,0) = \varphi(x), \qquad (2)$$

$$\begin{aligned} \frac{\partial u(0,t)}{\partial x} &= \alpha_0(t)\,u(0,t) + \beta_0(t)\,, \\ \frac{\partial u(l,t)}{\partial x} &= \varphi_0(t)\,u(l,t) + \omega_0(t)\,. \end{aligned} \qquad (3)$$

If we choose $t_j = j\tau$, $j = 0, 1, 2, ...$, the mesh points for the time variable, we approximate the equation (1) on the horizontal lines $t = t_j$ by the ordinary differential equation:

$$\tau\mu^2(x, t_j)\,\frac{d^2 u^{j+1}(x)}{dx^2} = u^{j+1}(x) - u^j(x) - \tau f(x, t_j)\,, \qquad (4)$$

where $u^j(x) = u(x, t_j)\,,\ j = 0, 1, 2, ...$

Similarly, we approximate the boundary conditions (3) on the horizontal lines $t = t_j$ by

$$\begin{cases} \frac{du^{j+1}(0)}{dx} = \alpha_0^{j+1} u^{j+1}(0) + \beta_0^{j+1}, \\ \frac{du^{j+1}(l)}{dx} = \varphi_0^{j+1} u^{j+1}(l) + \omega_0^{j+1}. \end{cases} \qquad (5)$$

Let's assume that (5) holds true not only at the endpoints of the segment $[0, l]$, but also at each point $x \in (0, l)$, that is, the following relation is true:

$$\frac{du^{j+1}(x)}{dx} = \alpha(x)\,u^{j+1}(x) + \beta(x)\,.$$

If we differentiate each side of (6), we obtain

$$\frac{d^2 u^{j+1}(x)}{dx^2} = \left(\alpha'(x) + \alpha^2(x)\right) u^{j+1}(x)\,\beta'(x) + \alpha(x)\,\beta(x)\,. \quad (6)$$

If we compare the last expression with equation (4), we will see that unknown coefficients $\alpha(x)$ and $\beta(x)$ can be found from the following differential equations:

$$\left\{\begin{array}{l}\alpha'(x)+\alpha^2(x)=\frac{1}{\tau\mu^2(x,t_j)},\\ \beta'(x)+\alpha(x)\beta(x)=-\frac{u^j(x)+\tau f(x,t_j)}{\tau\mu^2(x,t_j)}.\end{array}\right. \quad (7)$$

Comparing (5) and (6), we see that

$$\alpha(0)=\alpha_0^{j+1},\beta(0)=\beta_0^{j+1},u^{j+1}(l)=\frac{\omega_0^{j+1}-\beta(l)}{\alpha(l)-\varphi_0}. \quad (8)$$

Thus, the solution of the original boundary-value problem is reduced to solution of the series of first-order ordinary differential equations (7) and (6) with initial conditions (8) at each time layer.

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Inverse spectral and inverse nodal problems for Sturm-Liouville equations with point δ and δ'–interactions

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We study inverse spectral and inverse nodal problems for Sturm-Liouville equations with point δ and δ'−interactions on finite interval. Inverse spectral problems consist in recovering operators from their spectral characteristics. Such problems play an important role in mathematics and have many applications in natural sciences (see, for example, monographs [1-3]). Inverse nodal problems consist in constructing operators from the given nodes (zeros) of eigenfunctions (see [2,4,6]). In this study, uniqueness results are proved, and using the nodal set of eigenfunctions the given problem reconstructed.

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Necessary conditions for extremum in calculus of variation for problems with higher derivatives

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Denote by $KC^m(I, R^n)$ the class of functions $x(\cdot): I \to K^n$, having continuous derivatives of up to $(m-1)$-th order and a piecewise continuous derivatives of order m (at the end points of t_0 and t_1 segments of $I := [t_0, t_1]$, supposed that $x^{(m)}(t_0) = x^{(m)}(t_0+0),\ x^{(m)}(t_1) = x^{(m)}(t_1-0)$).

In the class of functions $x(\cdot) \in KC^m(I, R^n)$, consider the following variational problem with higher derivatives:

$$J(x(\cdot)) = \int_{t_0}^{t_1} L(t, x(t),\ \dot{x}(t), ..., x^{(m)}(t))dt \to \min_{x(\cdot)}, \qquad (1)$$

$$x(t_0) = x_{00}, \quad \dot{x}(t_0) = x_{01}, ..., x^{(m-1)}(t_0) = x_{0m-1}, \qquad (2)$$

$$x(t_1) = x_{10}, \quad \dot{x}(t_1) = x_{11}, ..., x^{(m-1)}(t_1) = x_{1m-1},$$

where $x_{ij} \in R^n, \quad i = 0, 1, \; j = \overline{0, m-1}, t_0$ Đ¸ t_1 are given points.

It is assumed that given function (integrand) $L(t, x, \dot{x}, ..., x^{(m)}) : I \times R^n \times \times R^n \times ... \times R^n \to R$ is continuous with respect to all variables.

Functions $x(\cdot) \in KC^m \left(I, R^n\right)$, satisfying marginal condition (2) are called admissible. An admissible function $\overline{x}(\cdot)$ is called a strong (weak) local minimum in problem (1), (2) if there exists $\overline{\delta} > 0 \quad (\widehat{\delta} > 0)$ such that inequalities $J\left(x\left(\cdot\right)\right) \geq J\left(\overline{x}\left(\cdot\right)\right)$ are satisfied for all admissible functions $x(\cdot)$ for which,

$$\|x(\cdot) - \overline{x}(\cdot)\|_{C^{m-1}(I,R^n)} \leq \delta$$

$$(\max\{\|x(\cdot) - \overline{x}(\cdot)\|_{C^{m-1}(I,R^n)}, \; \|x^{(m)}(\cdot) - \overline{x}^{(m)}(\cdot)\|_{L_\infty(I,R^n)}\} \leq \widehat{\delta}) \, .$$

In this case, we say that the admissible function $\overline{x}(\cdot)$ delivers a strong (weak) local minimum in problem (1), (2) with $\overline{\delta}(\widehat{\delta})$-neighborhood.

The problem (1), (2) is investigated for the case when $m = 2$, i.e. $L(\cdot) = L\left(t, x, \dot{x}, \ddot{x}\right)$.

Let $\overline{x}(\cdot)$ be some admissible function in the problem (1), (2) for $m = 2$ and $T \subseteq [t_0, t_1]$ be the set of continuity points of a function $\ddot{x}(\cdot)$. Let us define the following function that is corresponding to integrant $L\left(t, x, \dot{x}, \ddot{x}\right)$ and function $\overline{x}(\cdot)$:

$$Q(t,\lambda,\xi;\overline{x}(\cdot)) = \lambda[L(t,\overline{x}(t),\dot{\overline{x}}(t),\ddot{\overline{x}}(t)+\xi) - \overline{L}(t)]+$$
$$+ (1-\lambda)[L(t,\overline{x}(t),\dot{\overline{x}}(t),\ddot{\overline{x}}(t)+\frac{\lambda}{\lambda-1}\xi) - \overline{L}(t)],$$
$$(t,\lambda,\xi) \in T_2 \times [0,1) \times R^n,$$

where $\overline{L}(t) = L(t,\overline{x}(t),\dot{\overline{x}}(t),\ddot{\overline{x}}(t))$.

The following theorem is proved.

Theorem. *Let an integrand $L(t,x,\dot{x},\ddot{x}) : I\times R^n\times R^n\times R^n \to R$ be continuous with respect to all variables. Then:*

(a) if an admissible function $\overline{x}(\cdot)$ is a strong local minimum in the problem (1), (2) for $m = 2$, then we have:

$$Q\left(t,\lambda,\xi;\overline{x}\left(\cdot\right)\right) \geq 0, \quad \forall\left(t,\lambda,\xi\right) \in T_2 \times [0,1) \times R^n;$$

(b) if an admissible function $\overline{x}\left(\cdot\right)$ is a weak local minimum in the problem (1), (2) for $m = 2$, then there exists a number $\delta > 0$ such that the following inequality is satisfied:

$$Q(t,\lambda,\xi;\overline{x}(\cdot)) \geq 0, \quad \forall(t,\lambda,\xi) \in T_2 \times [0,\frac{1}{2}] \times B_\delta(0),$$

where $B_\delta(0)$- is a closed ball with the radius δ and the center at $0 \in R^n$.

As the consequences of this theorem, analogues of Weiesstrass and Legendre conditions are obtained.

In addition, in the paper, equality and inequality type of necessary conditions for weak local extremum are obtained for the case when Legendre condition degenerates i.e. $det\bar{L}_{x^{(m)}x^{(m)}}(t^*) = 0$, where $t^* \in T_m$ are some points.

Note that the obtained necessary conditions for extremum generalize the corresponding known necessary conditions (see, for example [1,2]) in the classical calculus of variations.

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Asymptotics of solutions to differential equations with singular coefficients

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Let $a_1, ..., a_n$ and λ be complex numbers, $p_1, ..., p_n$ be complex-valued functions on $R_+(:= [0, +\infty))$. Suppose also that

$$|p_1| + (1 + |p_2 - p_1|)(|p_2| + ... + |p_n|) \in L^1_{loc}(R_+).$$

The purpose of the report is to present the construction that allows us under this condition to determine in what sense the equation of the form

$$l(y) := y^{(n)}+(a_1+p_1)y^{(n-1)}+(a_2+p_2')y^{(n-2)}+\ldots+(a_n+p_n')y = \lambda y$$

should be understood, where the derivatives are understood in the sense of the theory of distributions.

Using this construction, it is established that if functions $p_1, p_2, \ldots, p_n$ satisfy some integral decay conditions at infinity then the main term of the asymptotics as $x \to +\infty$ of the fundamental system of solutions of this equation and their derivatives determined, as in the classical case, by the roots of a polynomial

$$Q(z) = z^n + a_1 z^{n-1} + a_2 z^{n-2} + \ldots + a_n - \lambda.$$

Let us state one of the theorems that we are going to discuss in the report.

Theorem. *Consider the equation* $l(y) = \lambda y$. Let $\lambda \neq a_n$ and functions p_1, $p_2, \ldots, p_n$ be such that

$$x^{m-1}(|p_1| + (1 + |p_2 - p_1|)(|p_2| + ... + |p_n|)) \in L^1(R_+),$$

where m *is the largest of the numbers equal to the multiplicities of the roots of* $Q(z)$*. Let* z_1 *be a root of the polynomial* $Q(z)$ *with multiplicity* $l_1 \leqslant m$*. Then this equation has a subsystem* $\{y_j\}$, $j = 1, 2, \ldots l_1$, *of fundamental solutions of the form*

$$y_j(x) = e^{z_1 x} x^{j-1}(1 + o(1)).$$

The same asymptotic behavior is exhibited by the subsystem $\{y_j\}$, $j = l_1 + 1, \dots l_1 + l_2$, *of fundamental solutions corresponding to a root* z_2 *of the polynomial* $Q(z)$ *with multiplicity* l_2, *and so on.*

The report is based on [1].

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On completeness of the root vectors of the fourth-order operator pencil corresponding to eigenvalues of the quarter-plane

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Let us consider a fourth-order polynomial operator pencil

$$P(\lambda) = \lambda^4 E - A^4 + \lambda^3 A_1 + \lambda^2 A_2 + \lambda A_3, \tag{1}$$

in a separable Hilbert space H, where E is an identity operator, λ is a spectral parameter, and the remaining coefficients satisfy the following conditions:

1) A is a positive definite self-adjoint operator with completely continuous inverse operator A^{-1};

2) the operators $B_j = A_j A^{-j}, j = \overline{1,3}$, are bounded in H.

In this paper, we find the conditions on the coefficients of the operator pencil (1) that ensure the completeness of its root vectors corresponding to the eigenvalues from the sector

$$\tilde{S}_{\frac{\pi}{4}}\{\lambda : |arg\lambda - \pi| < \frac{\pi}{4}\}.$$

Jordan blocks in spectral decompositions of Hill-Schroedinger operator

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In 1980 M.Gasymov brought attention to Hill operators with the potentials analytic in the unit disc; in particular, in this case the spectral gaps vanish. More generally, complex valued potentials could lead to any finite system of Jordan blocks (V. Tkachenko; Y. Hryniv, Y. Mykytyuk).

We'll focus on weak smoothness restrictions imposed on potentials which still permit to find and analyze the structure of a finite dimensional root space with Jordan blocks (P. Djakov and the speaker).

On classes of saturation of linear operators in spaces $L_p(-\infty,\infty)\,(1\le p\le 2)$

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Let $K(t)$ be some function summable on $(-\infty,\infty)$, and

$$\int_{-\infty}^{\infty} K(t)dt = 1.$$

We consider approximation of functions $f(x)\in L_p(-\infty,\infty)$ by $[m]-$ singular integrals

$$M_{\lambda}^{[m]}(f,x)=\lambda\int_{-\infty}^{\infty}\sum_{j=1}^{m}(-1)^{j-1}\binom{m}{j}f(x-jt)K(\lambda t)dt \qquad (1)$$

R.G. Mamedov has studied the classes and orders of saturation of approximation of functions $f(x)\in L_p(-\infty,\infty)$ by $[m]$ -singular integrals (1). It was proved that if the kernel $K(t)$ satisfies the condition

$$\lim_{\lambda\to\infty}\frac{1-\sqrt{2\pi}\sum\limits_{j=1}^{m}(-1)^{j-1}\binom{m}{j}K^{\wedge}\left(\frac{u}{\lambda}j\right)}{\left(\frac{u}{\lambda}\right)^{n}}=D\neq 0,$$

where n is some numberer and $K^{\wedge}(u)$ is Fourier transformation of the function $K(t)$, then $[m]-$singular integral (1) is saturated in the space $L_p\left(-\infty,\infty\right)$ with order $O(\lambda^{-n})$. In the present paper, based on the given singular integral (1) we construct a linear operator giving higher order approximation to the functions from the space $L_p\left(-\infty,\infty\right)$.

Theorem. *Let the kernel $K(t)$ of $[m]$ -singular integral (1) satisfy the condition*

$$\lim_{\lambda\to\infty}\frac{A^{[m]}\left(\frac{u}{\lambda}\right)-\sum\limits_{\nu=0}^{N-1}b_\nu^{[m]}\left(\frac{|u|}{\lambda}\right)^{\alpha_\nu}}{\left(\frac{|u|}{\lambda}\right)^{\alpha_N}}=D\neq 0,$$

where

$$A^{[m]}\left(\frac{u}{\lambda}\right)=\sum_{j=1}^{m}(-1)^{j-1}\left(\begin{array}{c} m \\ j \end{array}\right)j^{\alpha_\nu}a_\nu \quad (\nu=0,1,2,...)$$

and $\{a_\nu\}$ be the given sequence of real numbers $(a_0=1)$. Then, if for the function $l_N(x)\in L_p\left(-\infty,\infty\right)$ the condition

$$\left\|\lambda^{\alpha_N}\left[Q_{\lambda,N}^{[m]}(f,x)-f(x)-l_N(x)\right]\right\|_{L_p}=o(1)$$

is fulfilled as $\lambda\to\infty$, then the function $f(x)$ has a Riesz derivative of order a_N, and $l_N(x)=Df^{\{\alpha_N\}}(x)$ everywhere for $p=1$, almost everywhere for $1<p\leq 2$, where

$$Q_{\lambda,N}^{[m]}(f,x)=M_\lambda^{[m]}(f,x)-\sum_{\nu=1}^{N-1}\frac{b_\nu^{[m]}}{\lambda^{\alpha_\nu}}f^{\{a_\nu\}}(x).$$

Convergence of iterates of operator representations

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Let G be a locally compact abelian group with the dual group Γ. Let $M(G)$ be the measure algebra of G. A measure $\mu \in M(G)$ is said to be power bounded if $\sup_{n\geq 1} \|\mu^n\| < \infty$. For a power bounded measure $\mu \in M(G)$, we put $\mathcal{F}_\mu = \{\gamma \in \Gamma : \widehat{\mu}(\gamma) = 1\}$ and $\mathcal{E}_\mu = \{\gamma \in \Gamma : |\widehat{\mu}(\gamma)| = 1\}$, where $\widehat{\mu}$ is the Fourier-Stieltjes transform of μ.

An *action* $\mathbf{\Theta}$ of G in a $\sigma-$finite positive measure space (Ω, Σ, m) is a family $\mathbf{\Theta} = \{\theta_g : g \in G\}$ of invertible measure preserving transformations of (Ω, Σ, m) satisfying: 1) $\theta_0 = id$; 2) $\theta_{g+s} = \theta_g \theta_s$, for all $g, s \in G$; 3) The mapping $G \times \Omega \to \Omega$ defined by $(g, \omega) \to \theta_g \omega$ is mesurable with respect to the product $\sigma-$algebra $\Sigma_G \times \Sigma$ in $G \times \Omega$.

Any action $\mathbf{\Theta}$ induces a representation $\mathbf{T} = \{T_g : g \in G\}$ of G on $L^p(\Omega) := L^p(\Omega, \Sigma, m)$ $(1 \leq p < \infty)$ by invertible isometries, where $(T_g f)(\omega) = f(\theta_g \omega)$. If $\mathbf{\Theta}$ is continuous, then for any $\mu \in M(G)$, we can define a bounded linear operator $\mathbf{T}_\mu$ on $L^p(\Omega)$ $(1 \leq p < \infty)$ by

$$(\mathbf{T}_\mu f)(\omega) = \int_G f\left(\theta_g^{-1} \circ \omega\right) d\mu(g).$$

Theorem. Let $\mathbf{\Theta}$ be a continuous action of G in (Ω, Σ, m) and let $\mu \in M(G)$ be power bounded. If $\mathcal{E}_\mu = \mathcal{F}_\mu$, then the sequence $\left\{ \mathbf{T}_{\mu}^{n} f \right\}$ converges in L^p-norm for every $f \in L^p (\Omega)$ $(1 < p < \infty)$.

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On sufficient conditions for the solvability of the inverse problem for a differential operator

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In this work, we present sufficient conditions for the solvability of the inverse problem of recovery of the Sturm-Liouville operator with non-separated boundary conditions, one of which contains a spectral parameter. In contrast to the problems considered earlier, here only one spectrum and a sequence of signs are used as spectral data.

Consider the boundary value problem generated on the interval $[0, \pi]$ by the Sturm-Liouville equation

$$-y'' + q\,(x)\,y = \lambda^2 y \tag{1}$$

and the non-separated boundary conditions of the form

$$\begin{aligned} y'\,(0) + (\alpha\lambda + \beta)\,y\,(0) + \omega y\,(\pi) = 0\,, \\ y'\,(\pi) + \gamma\,y\,(\pi) - \omega y\,(0) = 0\,, \end{aligned} \tag{2}$$

where $q(x)$ is a real function belonging to the space $L_2[0, \pi]$, λ is a spectral parameter, and α, β, γ, ω are real numbers with $\alpha\,\omega \neq 0$. We denote this problem by P.

Let $c(x,\lambda)$, $s(x,\lambda)$ be a fundamental system of solutions of equation (1), defined by the initial conditions $c(0,\lambda)=s'(0,\lambda)=1$, $c'(0,\lambda)=s(0,\lambda)=0$. It is easy to see that the characteristic function of the boundary value problem P is

$$\delta(\lambda)=2\omega-\eta(\pi,\lambda)+\omega^2 s(\pi,\,\lambda)+(\alpha\lambda+\beta)\,\sigma(\pi,\,\lambda),$$

where $\eta(x,\,\lambda)=c'(x,\,\lambda)+\gamma\,c(x,\,\lambda)$, $\sigma(x,\,\lambda)=s'(x,\,\lambda)+\gamma\,s(x,\,\lambda)$.

The zeros μ_k, $k=\pm0,\,\pm1,\,\pm2,\,...$ of the function $\delta(\lambda)$ are the eigenvalues of the problem P. Denote $\sigma_n=\operatorname{sign}\,[1-|\omega\,s(\pi,\,\theta_n)|]$ $(n=\pm1,\ \pm2,...)$, where θ_n's are the zeros of the function $\sigma(\pi,\lambda)$, whose squares are the eigenvalues of the boundary value problem generated by the equation (1) and boundary conditions $y(0)=y'(\pi)+\gamma\,y(\pi)=0$.

Inverse problem. Given the spectral data $\{\mu_k\}$, $\{\sigma_n\}$ of boundary value problems, construct the coefficient $q(x)$ of the equation (1) and the parameters $\alpha,\,\beta,\,\gamma,\omega$ of the boundary conditions (2).

We now give sufficient conditions for the solvability of the inverse problem.

Theorem 1. *In order for the sequences of real numbers* $\{\mu_k\}$ $(k=\pm0,\,\pm1,\,\pm2,\,...)$ *and* $\{\sigma_n\}$ $(\sigma_n=-1,\,0,\,1\,;\ n=\pm1,\,\pm2,\,...)$ *to be spectral data of the boundary value problem of the form P, it is sufficient that the following conditions hold:*

1) *the asymptotic formula* $\mu_k=k+a+\frac{(-1)^{k+1}A-B}{k\pi}+\frac{\tau_k}{k}$ *is true, where* $A=2\omega\cos\pi a$, ω, a, B *are real numbers,* $0<|a|<\frac{1}{2}$, $\omega\neq0$, $\{\tau_k\}\in l_2$;

2) $...\leq\theta_{-3}\leq\mu_{-2}\leq\theta_{-2}\leq\mu_{-1}\leq\theta_{-1}\leq\mu_{-0}<\ 0<\mu_{+0}\leq$

$\theta_1 \leq \mu_1 \leq \theta_2 \leq \mu_2 \leq \theta_3 \leq \dots$, *where* θ_n*'s are the zeros of the function* $\delta(\lambda) - \delta(-\lambda)$,

$$\delta(\lambda) = \frac{\pi(\mu_{-0} - \lambda)(\mu_{+0} - \lambda)}{\cos \pi a} \prod_{k=-\infty, k \neq 0}^{\infty} \frac{\mu_k - \lambda}{k},$$

with $\theta_n \neq \theta_m$ *for* $n \neq m$;

3) *the inequality* $b_n \overset{def}{=} |\delta(\theta_n) - 2\omega| - 2|\omega| \geq 0$ *holds;*

4) σ_n *is equal to zero if* $b_n = 0$, *and to 1 or -1 if* $b_n > 0$; *besides, there exists* $N > 0$ *such that* $\sigma_n = 1$ *for all* $|n| \geq N$.

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On the basicity of a perturbed system of exponents with a unit in Morrey-type spaces

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In this paper a perturbed system of exponents with a unit, whose phase is a linear function depending on a real parameter is considered. The necessary and sufficient conditions for the parameter for the basis properties (completeness, minimality and basicity) of this system are found in the subspace of Morrey space in which continuous functions are dense.

Consider the following exponential system with a unit

$$1\bigcup\left\{e^{i(n-\beta\, sign\, n)t}\right\}_{n\neq 0}\equiv 1\bigcup E^0_\beta, \tag{1}$$

where

$$E_\beta\equiv\left\{e^{i(n-\beta\, sign\, n)t}\right\}_{n\in Z}, E_{\beta 0}=E_\beta\backslash\{1\}, \tag{2}$$

$\beta\in C-$is a complex parameter.

With respect to the basis properties of the system E_β the following theorem is true.

Theorem 1. *Let* $2Re\beta+\frac{\alpha}{p}\notin Z$. *Then the system* E_β *forms a basis for* $M^{p,\alpha}$, $0<\alpha<1, 1<p<+\infty$, *if and only if* $\left[2Re\beta+\frac{\alpha}{p}\right]=0$ *(*$[\cdot]-$ *is the integer part). Its defect is equal to* $d(E_\beta)=\left[2Re\beta+\frac{\alpha}{p}\right]$. *For* $d(E_\beta)<0$, *it is not complete, but is minimal in* $M^{p,\alpha}$; *for* $d(E_\beta)>0$, *it is complete, but it is not minimal in* $M^{p,\alpha}$.

We also proved the following main theorem.

Theorem 2. *Let* $2Re\beta+\frac{\alpha}{p}\notin Z$. *Then the system of exponents*

$$1\bigcup\left\{e^{i(n-\beta\, sign\, n)t}\right\}_{n\neq 0},$$

forms a basis for $M^{p,\alpha}, 0<\alpha<1, 1<p<+\infty$, *if and only if* $d(\beta)=\left[2Re\beta+\frac{\alpha}{p}\right]=0$. *For* $d(\beta)<0$ *it is not complete,* *but it is minimal in $M^{p,\alpha}$; for $d(\beta) > 0$ it is complete, but not minimal in $M^{p,\alpha}$.*

Spectral analysis of a fourth-order differential bundle with a triple characteristic root

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Consider a sheaf of differential operators L_λ^α in space $L_2(0,\infty)$, generated by a differential expression

$$l\left(x,\frac{d}{dx},\lambda\right)Y \equiv \left(\frac{d}{dx}-i\lambda\right)^3\left(\frac{d}{dx}+i\lambda\right)Y+$$

$$+r(x)\frac{dY}{dx}+(\lambda p(x)+q(x))Y=0\,, \qquad (1)$$

and boundary conditions

$$U_\nu(Y) \equiv \alpha_{\nu 0}Y(0,\lambda)+\alpha_{\nu 1}Y'(0,\lambda)+$$

$$+\alpha_{\nu 2}Y''(0,\lambda)+\alpha_{\nu 3}Y'''(0,\lambda)=0\ ,\ \nu=\overline{1,3}, \qquad (2)$$

where λ is a spectral parameter, the functions $r(x)$, $p(x)$, $q(x)$ complex functions defined and continuous on $[0,\infty)$, which have order continuous derivatives to order 3,4,5 respectively and integrals

$$\int_0^\infty x^4 \left| r^{(s)}(x) \right| dx < \infty,\ s = \overline{0,3};$$

$$\int_0^\infty x^4 \left| p^{(s)}(x) \right| dx < \infty,\ s = \overline{0,5};$$

$$\int_0^\infty x^4 \left| q^{(s)}(x) \right| dx < \infty,\ s = \overline{0,4},$$

converge. $\alpha_{\nu k}$, $\nu = \overline{1,3}$, $k = \overline{0,3}$, are such fixed complex numbers that the forms $U_\nu (Y)$ are linearly independent, the number of boundary conditions varies depending on the location of the parameter λ in the complex plane.

In the works [1,2], equation (1) was studied and transformation operators were constructed that transferred the solution of the equation$\left(\frac{d}{dx} - i\lambda\right)^3 \left(\frac{d}{dx} + i\lambda\right) Y = 0$ to the solution of equation (1). In particular, in [2] it was obtained that equation (1) has a fundamental system of solutions $Y_j (x, \lambda)$, $j = \overline{1,4}$, which satisfy the conditions:

$$\lim_{x\to\infty} \left[Y_j (x, \lambda) - x^{j-1} e^{i\lambda x} \right] = 0 \,,\ j = \overline{1,3},\ Im\lambda \geq 0;$$

$$\lim_{x\to\infty} \left[Y_j (x, \lambda) - e^{-i\lambda x} \right] = 0 \,,\ \ Im\lambda \leq 0;$$

there are kernels $K_j^{\pm} (x, t)$ such that

$$\begin{aligned} Y_j (x, \lambda) &= x^{j-1} e^{i\lambda x} + \textstyle\int_x^\infty K_j^+ (x, t)\, e^{i\lambda t} dt \,,\ Im\lambda \geq 0, \\ Y_4 (x, \lambda) &= e^{-i\lambda x} + \textstyle\int_x^\infty K_4^- (x, t)\, e^{-i\lambda t} dt \,,\ Im\lambda \leq 0, \end{aligned}$$

where in $K_j^{\pm}(x,t)$, $j=\overline{1,4}$, satisfy the equations

$$l\left(x,\frac{\partial}{\partial x},\ \pm i\frac{\partial}{\partial t}\right)K_j^{\pm}(x,t)=0\,,$$

and takes place

$$\lim_{x+t\to\infty}\frac{\partial^{\alpha+\beta}K_j^{\pm}(x,t)}{\partial x^{\alpha}\partial t^{\beta}}=0\,,\ \ \alpha+\beta\le 4\,,$$

$$\int_x^{\infty}\left|K_j^{\pm}(x,t)\right|^2 dt<\infty\,,$$

in addition, the functions $K_j^{\pm}(x,t)$ and their derivatives satisfy certain integral conditions on the characteristic $t=x$.

We first study the structure of the discrete spectrum of the sheaf L_λ^α. Denote by D the set of all functions $Y(x,\lambda)\in L_2(0,\infty)$ such that: 1) for each $\lambda:\pm Im\lambda\ge 0$ the derivatives $Y^{(\nu)}(x,\lambda)$, $\nu=\overline{0,3}$, exist and are absolutely continuous in each finite interval $[0,b]$, $b>0$; 2) $l\left(x,\frac{d}{dx},\ \lambda\right)Y\in L_2(0,\infty)$. Further, by D_α we denote the set of those functions from D, which satisfy the conditions (2). We define L_λ^α as follows: its domain is D_α and $L_\lambda^\alpha=l\left(x,\frac{d}{dx},\lambda\right)Y$ at $Y\in D$. Denote $A(\lambda)=\det\|U_i(Y_k)\|_{i,k=1}^3$, and consider the upper half-plane $\lambda:Im\lambda\ge 0$. In its open part, the solutions $Y_k(x,\lambda)$, $k=\overline{1,3}$ belong to the $L_2(0,\infty)$, and $Y_4(x,\lambda)\,\overline{\in}\,L_2(0,\infty)$. If λ is in the open lower half-plane, none of the solutions of $Y_k(x,\lambda)$, $k=\overline{1,3}$, belongs to this space, but $Y_4(x,\lambda)\in L_2(0,\infty)$. Then the eigenvalues of the sheaf L_λ^α in the open upper half-plane are determined from the equation $A(\lambda)=0$.

The eigenvalues of this sheaf in the open lower half-plane can be determined by one boundary condition $B(\lambda) = U_\nu(Y_4) = 0$, where ν can be one of the numbers 1,2,3.

From [3] methods have taken following theorems.

Theorem 1. *The operator L_λ^α in the open upper and in the open lower half-planes has eigenvalues, which are respectively the roots of the equation $A(\lambda) = 0$ and $B(\lambda) = 0$. This operator has no eigenvalues on the real axis. If the numbers λ_0 and λ_1 with $Im\lambda_0 = 0$ and $Im\lambda_1 = 0$, are the roots of the equation $A(\lambda) = 0$ and $B(\lambda) = 0$, respectively, then these numbers are spectral characteristics of the sheaf L_λ^α.*

Theorem 2. *A sheaf of differential operators L_λ^α can have only a finite or countable number of eigenvalues forming a bounded set in a complex λ-plane with a cut along the real axis. The limit points of this set can be located only on the real axis.*

Theorem 3. F*or all values of the spectral parameter λ from the open upper and open lower half-planes, that are not the roots of the equation $A(\lambda) = 0$ and $B(\lambda) = 0$, the resolvent of the operator L_λ^α, defined on the whole space$L_2(0,\infty)$ in it, is a bounded integral operator with Carleman type kernels. When λ approaches to the real axis, the norm of the resolvent increases indefinitely and the entire real axis belongs to the continuous spectrum of the sheaf L_λ^α.*

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Inverse problem for the quadratic pencil of differential operators with periodic coefficients

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In this study, inverse problem for the differential operator

$$L_\lambda = \frac{d^2}{dx^2} + p(x)\frac{d}{dx} + \left[\lambda^2 + i\lambda p(x) + q(x)\right]$$

has been investigated in the space $L_2(\mathbb{R})$. Here, λ is a complex parameter, $p(x) = \sum_{n=1}^{\infty} p_n e^{inx}$, $q(x) = \sum_{n=1}^{\infty} q_n e^{inx}$ belong to the class Q^+ of periodic functions $q(x) = \sum_{n=1}^{\infty} q_n e^{inx}$ with $\|q(x)\| = \sum_{n=1}^{\infty} |q_n| < +\infty$.

In the case $p(x) \equiv 0$ for the operator L_λ, the sequence $\{S_n\}$ of spectral data has been defined and with respect to this sequence, the inverse problem has been investigated in [1], [2]. Two sequences $\{S_n^+\}$, $\{S_n^-\}$ of spectral data for the operator

$$L_\lambda^0 = -\frac{d^2}{dx^2} - \lambda^2 + 2\lambda p(x) + q(x)$$

have been defined and the inverse problem with respect to these sequences has been investigated in [3] when $p'(x), q(x) \in Q^+$.

In this study, sequences $\{S_n^+\}$, $\{S_n^-\}$ of spectral data have been defined for pencil L_λ of differential operators as similar to in [3]. It has been shown that coefficients $p(x)$ and $q(x)$ are determined as uniquely with respect to these sequences. Moreover, necessary and sufficient conditions have been obtained for given two complex sequences $\{S_n^+\}$, $\{S_n^-\}$ which are the set of spectral data for the operator

$$L_\lambda = \frac{d^2}{dx^2} + p(x)\frac{d}{dx} + \left[\lambda^2 + i\lambda p(x) + q(x)\right]$$

with $p'(x), q(x) \in Q^+$.

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Uniform estimates of solutions of one class of equations in finite dimensional space

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Let H be a real finite dimensional space with the scalar product $\langle \cdot, \cdot \rangle$ and the norm $\|\cdot\|$. We will be interested in an equation of the following kind

$$f(u) = u + L(u) = g \in H, \tag{1}$$

where $L(\cdot)$ is a nonlinear continuous transformation.

If $u(\xi)$ is a vector-function continuously differentiable with respect the parameter ξ, then we assume that $L(u(\xi))$ is continuously differentiated (and also vector-functions arising in the future).

By L_u denote the Gateaux derivative of the transformation $L(u)$:

$$(L(u(\xi)))_\xi = L_u u_\xi, \;\; (f(u_\xi))_\xi = H_\xi + L_u u_\xi.$$

Let Q be an invertible linear transformation and $G = Q^*Q$. Enter notations:

$$D_u^* = E + (L_u)^*, \;\; (D_u^* f(u))_\xi = M_u(u_\xi).$$

$$\mu(u) = \|Qu\|^2 \|u\|^{-2}, \;\; \gamma(u) = \langle D_u^* f(u), u\rangle \|u\|^{-2},$$

$$R(u) = (G - \mu(u)) u, \;\; K(u) = \langle D_u^* f(u), R\rangle \|R(u)\|^{-2}.$$

We assume the following conditions are met:

U1. For any $0 \neq u$ the estimates

$$\gamma(u) \geq -\|f(u)\| \|u\|^{-1}, \;\; \|Q(u)\| \leq \delta_0 \|f(u)\|$$

are met, where $0 < \delta_0$ is a constant independent of u.

U2. If $0 \neq u \in H$ is an eigenvector of the operator $G = Q^*Q$, then

$$\|u\| \leq \delta_1 \|f(u)\|,$$

where δ_1 is a constant independent of u.

U3. If $K(u) > \delta_2$ and $\mu(u) < \frac{\delta_1}{4}$, then

$$\inf_{\|a\|_H = 1} \left[\langle M_u a, a\rangle - \gamma(u) \|a\|^2 - K(u) \langle (G - \mu) a, a\rangle\right] < 0,$$

where $\delta_2 > -\infty$ a constant independent of u.

Theorem. *Let Conditions U1, U2 and U3 be met. Then, if* $d = \delta_2 [\delta_1 + \delta_0] < \frac{1}{8}$, *then the a-priori estimate*

$$\|u\|_H \leq C \|f(u)\|_H$$

is valid, where C *depends of the constant number* d*, but does not depend of* $u \in H$.

It is well-known that many nonlinear differential equations can be written in a limited form. Equation (1) under our consideration can be a finite dimensional approximation of such equations. Since in Theorem the constant C does not depend of u (depends only of d!), then tending the dimension of the space H to infinity, we get strong estimates for solutions of nonlinear differential equations.

About inverse problem for the periodic Sturm-Liouville operator

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In this speech we consider the two: periodic and antiperiodic Sturm-Liouville problems

$$\begin{array}{l} -y'' + q_i(x)y = \lambda y,\ i = 1, 2, \\ y(0) = y(\pi), y'(0) = y'(\pi); \end{array} \tag{I}$$

$$\begin{array}{l} -y'' + q_i(x)y = \lambda y, i = 1, 2, \\ y(0) = -y(\pi),\ y'(0) = -y'(\pi). \end{array} \tag{II}$$

Along with the problems (I) and (II), we consider get another problem for the equation Sturm-Liouville

$$\begin{array}{l} -y^{''} + q_i(x)y = \lambda y, i = 1, 2, \\ y(0) = y(\pi) = 0. \end{array} \tag{III}$$

We assume that the potentials $q_i(x)$ is a smooth periodic function of period π. We denote the spectrum of the problems (I),(II) and (III) respectively

$$\lambda_0 < \lambda_3 \leq \lambda_4 < \lambda_7 \leq \lambda_8 < \ldots,$$

$$\lambda_1 \leq \lambda_2 < \lambda_5 \leq \lambda_6 < \ldots,$$

$$\nu_1 < \nu_2 < \nu_3 \ldots.$$

In the case of an N-zoned potential we have, for all $n > N$, $\lambda_{2n+1} = \nu_n = \lambda_{2n+2}$.

If the potentials $q_1(x)$ and $q_2(x)$ have one and the same N-zoned Lyapunov function and $\tilde{\nu}_n = \nu_n$ for $n > N$, then the integral equation inverse problem is degenerate in the extended sense.

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Solvability of one Riemann boundary value problem in generalized weighted Lebesgue spaces

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In this work, the homogeneous Riemann boundary problem with the piece-wise Holder coefficients is considered in the generalized weighted Lebesgue spaces. The sufficient conditions for the noetherness of the coefficient and the weight, and the general solution of the solution of the homogeneous problem is constructed.

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Central limit theorem for perturbed Markov random walk described by the autoregreressive process of order one $(AR(1))$

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Let $X_n,\ n \geq 1$ be a sequence of independent identically distributed random variables determined on some probability space (Ω, F, P).

Let us consider a first order autoregression process $(AR(1))$

$$X_n = \beta X_{n-1} + \xi_n, \quad n \geq 1,$$

where $|\beta| < 1$, and the initial value of the process X_0 is independent of the innovation $\{\xi_n\}$.

The statistical estimation for the parameter β by the method of the least squares has the following form

$$\beta_n = \frac{T_n}{S_n},$$

where $T_n = \sum\limits_{k=1}^{n} X_k X_{k-1}$ and $S_n = \sum\limits_{k=1}^{n} X_{k-1}^2$.

It is noted that in the conditions $EX_0^2 < \infty,\ E\xi_1 = 0$ and $D\xi_1 = 1$ we have $\frac{T_n}{n} \overset{a.s.}{\to} \dfrac{\beta}{1-\beta^2}$ and $\frac{S_n}{n} = \frac{1}{1-\beta^2} = \alpha_1^2$ as $n \to \infty$.

Let us consider the process of the form

$$Z_n = \frac{T_n^2}{S_n}, \quad n \geq 1.$$

In the present paper we prove the central limit theorem for $Z_n,\ n \geq 1$.

Asymptotics of the solution of a boundary value problem for a quasilinear parabolic equation with angular boundary layer

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In the rectangular $D = \{(t,x)\,|0 \le t \le T,\ 0 \le x \le 1\}$ we consider the following boundary value problem:

$$\frac{\partial u}{\partial t} - \varepsilon^p \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)^p - \varepsilon \frac{\partial^2 u}{\partial x^2} + a\frac{\partial u}{\partial x} + F(t,x,u) = 0, \quad (1)$$

$$u\,|_{t=0} = 0 \ \ (0 \le x \le 1),$$

$$u\,|_{x=0} = u\,|_{x=1} = 0 \ \ (0 \le t \le T), \quad (2)$$

where $\varepsilon > 0$ is a small parameter, $p = 2k+1$, k is an arbitrary natural number, $F(t,x,u)$ is a given function in D, satisfying the condition

$$\frac{\partial F(t,x,u)}{\partial u} \ge \alpha^2 > 0 \quad \text{for} \ \ (t,x,u) \in D \times (-\infty, +\infty). \quad (3)$$

Assuming that the function $F(t,x,u)$ vanishes with its derivatives for $x = at$, condition (3) and condition

$$\frac{\partial^i F(t,1,0)}{\partial t^i}\,|_{t=0} \ = 0, \quad i = 0,1,...,2n+1, \quad (4)$$

are fulfilled, the complete asymptotics of the solution of boundary value problem (1) and (2) was constructed in our previous works.

In this note, rejecting from condition (4) on the function $F(t,x,u)$, we construct complete asymptotics of the solution of boundary value problem (1) and (2) with angular boundary layer in the form

$$u=\sum_{i=0}^{n}\varepsilon^i w_i+\sum_{j=0}^{n+1}\varepsilon^j v_j+\sum_{i=1}^{n}\varepsilon^{1+i}\eta_i+z.$$

Here the functions w_i, v_j, η_i are determined by the corresponding iterative processes, the functions w_i are the solutions of hyperbolic equations, v_j are boundary layer type functions near the angular point $x=1$, η_i is a remainder term, and the following estimation is valid for it:

$$\int\limits_0^1 z^2dx+\varepsilon^p\int\limits_0^t\int\limits_0^1\left(\frac{\partial z}{\partial x}\right)^p dxd\tau+\varepsilon\int\limits_0^t\int\limits_0^1\left(\frac{\partial z}{\partial x}\right)^2 dxd\tau+$$

$$+c_1\int\limits_0^t\int\limits_0^1 z^2dxd\tau\le c_2\varepsilon^{2n+2},$$

where $c_1>0,\ c_2>0$ are the constants independent of $\varepsilon,\ t\in[0,1]$.

On a boundary value problem for a bisingular perturbed quasilinear elliptic equation

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Let Ω be a unique circle centered at the origin of coordinates, Γ be its boundary.

In Ω we consider the boundary value problem

$$-\varepsilon^p \left[\frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} \right)^p + \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} \right)^p \right] - \varepsilon \Delta u -$$

$$-\frac{\partial u}{\partial x} + h(x,y)u = f(x,y), \tag{1}$$

$$u \left|_{\Gamma} \right. = 0, \tag{2}$$

where $\varepsilon > 0$ is a small parameter, $p = 2k+1$, k is an arbitrary natural number, $h(x,y)$, $f(x,y)$ are the given smooth functions.

The degenerated equation corresponding to equation (1) (for $\varepsilon = 0$) has the form

$$-\frac{\partial w_0}{\partial x} + h(x,y)w_0 = f(x,y). \tag{3}$$

Its characteristics are the straighlines $y = c = const$. Only two characteristics $y = -1$ and $y = 1$ touch the boundary Γ, i.e. the circle $x^2 + y^2 = 1$.

$A(0; -1)$ and $B(0; 1)$ are the tangential points of the characteristics with Γ. The points A and B divide Γ into two parts, Γ_1 and Γ_2. The semicircle Γ_2 has the equation: $x = \sqrt{1 - y^2}$, $-1 \le y \le 1$.

The degenerated problem is in solving the Cauchy problem for equation (3) under the initial condition

$$w_0 \,|_{\Gamma_2} = 0. \tag{4}$$

It is easy to see that the derivatives of the function $w_0(x, y)$ with respect to y have singularities at the points $A(0; -1)$ and $B(0; 1)$.

We prove the following theorem.

Theorem. *Let $h(x, y)$, $f(x, y)$ be sufficiently smooth functions, the function $h(x, y)$ satisfy the condition (3), the function $f(x, y)$ satisfy the condition*

$$f\left(\sqrt{1 - y^2}, y\right) = O\left[(1 - y^2)^{2n + \frac{3}{2}}\right] \quad \text{form} \quad y \to \pm 1.$$

Then the generalized solution of problem (1), (2) is representable in the form

$$u = \sum_{i=0}^{n} \varepsilon^i w_i + \sum_{j=0}^{n+1} \varepsilon^j v_j + z,$$

where the functions w_i are determined by the first iterative process, v_j are boundary layer type functions near the boundary Γ_1, and z is a uniformly bounded remainder term.

The general solution of non-homogeneous Riemann-Hilbert problem in the weighted Smirnov classes

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In this work the Riemann-Hilbert problem of the theory of analytic functions in weighted Smirnov classes is considered. Under certain conditions on the coefficients the Noetherness of this problem is proved.

Let $A(\xi) =: |A(\xi)| e^{i\alpha(\xi)}$, $B(\xi) =: |B(\xi)| e^{i\beta(\xi)}$ be complex-valued functions given on the curve Γ. We will assume that they satisfy the following basic conditions:

i) $|A|^{\pm 1}, |B|^{\pm 1} \in L_\infty(\Gamma)$;

ii) $\alpha(\xi), \beta(\xi)$ *are piecewise-continuous on* Γ *and let* $\{\xi_k, k=\overline{1,r}\} \subset \Gamma$ *be discontinuity points of the function* $\theta(\xi) =: \beta(\xi) - \alpha(\xi)$.

For the curve Γ we require the following conditions be fulfilled:

iii) Γ *is any Lyapunov or Radon curve (i.e. it is a curve of bounded rotation without cusps).*

Consider the following non homogeneous Riemann-Hilbert problem

$$F^+(z(s)) - D(s) F^-(z(s)) = g(z(s)), \; s \in (0,S), \quad (1)$$

where $g \in L_{p,\rho}(\Gamma)$ is a given function. By the solution of problem (1) we mean the pair $(F^+(z); F^-(z)) \in E_{p,\rho}(D^+) \times {}_m E_{p,\rho}(D^-)$, whose non-tangential boundary values $F^{\pm}$ satisfy relation (1) a.e. on Γ. It is clear that the general solution of the problem (1) has the representation

$$F(z) = F_0(z) + F_1(z),$$

where $F_0(z)$ is a general solution of the corresponding homogeneous problem, and $F_1(z)$ is some particular solution of the non-homogeneous problem (1).

The following theorem is true.

Theorem 1. *Let the complex-valued functions $A(\xi)$, $B(\xi)$ and the curve Γ satisfy conditions i)-iii). Take the weight ρ of the form*

$$\rho(z(s)) = \prod_{k=1}^{m} |s - t_k|^{\alpha_k},$$

and define the numbers $\{\beta_k\}_{k=1}^{l}$ by the expressions

$$\beta_k = -\frac{p}{2\pi} \sum_{i=1}^{r} h_{-i} \chi_{T_k}(s_i) + \sum_{i=1}^{m} \alpha_{-i} \chi_{T_k}(t_i), \ k = \overline{1,l}.$$

Let the inequalities $-1 < \beta_k < \frac{p}{q}, k = \overline{1,l}$ and $\alpha_k < \frac{q}{p}$, $k = \overline{1,m}$ be fulfilled. Then the general solution of the problem (1) in classes $E_{p,\rho}(D^+) \times {}_m E_{p,\rho}(D^-)$ has the representation

$$F(z) = F_0(z) + F_1(z),$$

where $F_0(z)$ is a general solution of appropriate homogeneous problem, $F_1(z)$ defined by the expressions

$$F_1(z) := \frac{Z(z)}{2\pi i} \int_\Gamma \frac{g(\xi)\, d\xi}{Z^+(\xi)(\xi - z)}, \tag{2}$$

in which $Z(z)$ is a canonical solution, and $m \geq 0$ is an integer.

From this theorem we directly get

Corollary. *Let all the conditions of Theorem 1 be fulfilled. Then the problem (1) under the condition $F(\infty) = 0$ has a unique solution of the form (2) in the class $E_{p,\rho}(D^+) \times {}_mE_{p,\rho}(D^-)$.*

Laplace operator with nonlocal Samarskii-Ionkin type boundary conditions in a disk

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In the report we consider a Samarskii-Ionkin type boundary value problem for the Laplace operator in the unit disk $\Omega = \{z = (x, y) = x + iy \in C : |z| < 1\}$. Let $r = |z|$ and $\varphi = \arctan(y/x)$.

Problem SI_α. *Find a solution of the Poisson equation*

$$-\Delta u(r, \varphi) = f(r, \varphi), \;\; r < 1 \tag{1}$$

satisfying the following boundary conditions

$$u(1,\varphi) - \alpha u(1, 2\pi - \varphi) = \tau(\varphi),\ \ 0 \le \varphi \le \pi, \tag{2}$$

$$\frac{\partial u}{\partial r}(1,\varphi) - \frac{\partial u}{\partial r}(1, 2\pi - \varphi) = \nu(\varphi),\ \ 0 \le \varphi \le \pi \tag{3}$$

or

$$\frac{\partial u}{\partial r}(1,\varphi) + \frac{\partial u}{\partial r}(1, 2\pi - \varphi) = \nu(\varphi),\ \ 0 \le \varphi \le \pi, \tag{4}$$

where $\alpha \in \mathbb{R}$, $f \in C^{\gamma}(\overline{\Omega})$, $\tau(\varphi) \in C^{1+\gamma}[0,\pi]$ and $\nu(\varphi) \in C^{\gamma}[0,\pi]$, $0 < \gamma < 1$.

We consider well-posedness and spectral properties of the formulated problem. The possibility of separation of variables is justified. We obtain an explicit form of the Green function for this problem and an integral representation of the solution.

We consider the spectral problem corresponding to the Laplace operator

$$-\Delta u(r,\varphi) = \lambda u(r,\varphi), r < 1$$

with the nonlocal boundary conditions

$$u(1,\varphi) - \alpha u(1, 2\pi - \varphi) = 0,\ 0 \le \varphi \le \pi,\ \alpha \in \mathbb{R},$$

and

$$\frac{\partial u}{\partial r}(1,\varphi) - \frac{\partial u}{\partial r}(1, 2\pi - \varphi) = 0,\ \ 0 \le \varphi \le \pi$$

or

$$\frac{\partial u}{\partial r}(1,\varphi) + \frac{\partial u}{\partial r}(1, 2\pi - \varphi) = 0,\ \ 0 \le \varphi \le \pi.$$

The eigenfunctions and eigenvalues of these problems are constructed in the explicit form. Moreover, we prove the completeness of these eigenfunctions.

In addition, we note that unlike the one-dimensional case the system of root functions of the problems consists only of eigenfunctions.

The boundedness generalized fractional integrals in the Morrey spaces, associated with the Bessel differential expansions

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Let R^n is an n-dimensional Euclidean space, $x = (x_1, ..., x_n)$, are vectors in R^n, $|x|^2 = \sum_{i=1}^{n} x_i^2$, $R_+^n = \{x = (x_1, \ldots x_n) : x_n > 0\}$, $E_+(x,r) = \{y \in R_+^n : |x-y| < r\}$, $|E_+(0,r)|_\gamma = \int_{E_+(0,r)} x_n^\gamma dx = Cr^{n+\gamma}$.

The Bessel differential expansion B_n is defined by $B_n = \frac{\partial^2}{\partial x_n^2} + \frac{\gamma}{x_n}\frac{\partial}{\partial x_n}, \gamma > 0$. The operator of generalized shift (B_n-shift operator) is defined by the following way:

$$T^y f(x) = C_\gamma \int_0^\pi f\left(x' - y', \sqrt{x_n^2 - 2x_n y_n \cos\alpha_n + y_n^2}\right) \sin^{\gamma-1} \alpha d\alpha.$$

For a function $K:(0,+\infty)\to(0,+\infty)$, let

$$T_{K,\gamma}f(x)=\int_{\mathbb{R}^n_+}T^yK(|x|)f(y)y_n^\gamma dy.$$

We consider the following conditions on K :

$(K_{1,\gamma})$ $0\le K(t),$ is decreasing on, $(0,\infty)$, $\lim\limits_{t\to 0}K(t)=\infty$;

$(K_{2,\gamma})$ $\exists C_1>0, \exists\sigma>0, \forall R>0, \int_0^R K(t)t^{n+\gamma-1}dt\le B_1R^\sigma$;

$(K_{3,\gamma})$ $\exists C_2>0, \exists\gamma(p)>0, \forall R>0, \left(\int_R^\infty K^{p'}(t)t^{n+\gamma-1}dt\right)^{1/p'} \le B_2R^{-\gamma(p)}$.

Let $1\le p<\infty$, $0\le\lambda<\gamma(p)p$. We denote by Morrey spaces $L_{p,\lambda,\gamma}(\mathbb{R}^n_+)$, associated by Bessel differential operator B_n ($\equiv$ B_n–Morrey spaces) the sets of locally integrable functions $f(x)$, $x\in\mathbb{R}^n_+$, with finite norms

$$\|f\|_{L_{p,\lambda,\gamma}(\mathbb{R}^n_+)}=\sup_{t>0,\,x\in\mathbb{R}^n_+}\left(t^{-\lambda}\int_{E_+(0,t)}T^y\,|f(x)|^p\,y_n^\gamma dy\right)^{1/p},$$

and weak B_n–Morrey spaces $WL_{p,\lambda,\gamma,a}(\mathbb{R}^n_+)$

$$\|f\|_{WL_{p,\lambda,\gamma}(\mathbb{R}^n_+)}=\sup_{r>0}r\sup_{t>0,\,x\in\mathbb{R}^n_+}\left(t^{-\lambda}\int_{\{y\in E_+(0,t):\,T^y|f(x)|>r\}}y_n^\gamma dy\right)^{1/p}.$$

Theorem. *Let $0\le\lambda<\gamma(p)p$, $1\le p<\infty$: a) if $1<p<\infty$, $\frac{1}{p}-\frac{1}{q}=\frac{\sigma p}{(\gamma(p)p-\lambda)q}$, $f\in L_{p,\lambda,\gamma}(R^n_+)$, and the kernel K satisfies $(K_{1,0})-(K_{3,0})$ conditions, then it follows that*

$$\|T_{K,\gamma}f\|_{L_{q,\lambda,\gamma}(R^n_+)}\le C_{p,\lambda}\|f\|_{L_{p,\lambda,\gamma}(R^n_+)}.$$

b) For $p = 1$ *if* $1 - \frac{1}{q} = \frac{\sigma}{(\gamma(1)-\lambda)q}$, *the kernel* K *satisfies* $(K_{1,\gamma})$, $(K_{2,\gamma})$. *Then*

$$\left|\left\{x \in R_+^n : \left|T_{K,\gamma}\right| f(x) > \beta\right\}\right|_\gamma \leq \left(\frac{C}{\beta} \cdot \|f\|_{L_{1,\lambda,\gamma}(R_+^n)}\right)^q,$$

where C *is independent of* f.

Nonlocal boundary value problem for Davis linear equation

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Consider the following nonlocal boundary value problem in the rectangle $Q_T = \{(x,t) : 0 \leq x \leq 1, 0 \leq t \leq T\}$: find a function $u(x,t) \in C^{(4,1)}(Q_T)$ which satisfies in Q_T the equation [1]

$$u_t(x,t) - u_{txx}(x,t) - \alpha u_{xx}(x,t)$$
$$+\beta u_{xxxx}(x,t) + a(x,t)u(x,t) = f(x,t), \qquad (1)$$

the nonlocal initial conditions

$$u(x,0) + \delta u(x,T) = \varphi(x), 0 \leq x \leq 1, \qquad (2)$$

and the boundary conditions

$$u_x(0,t) = u_x(1,t) = u_{xxx}(0,t) = 0,$$

$$\int_0^1 u(x,t)dx = 0, 0 \le t \le T, \tag{3}$$

where $\alpha > 0, \beta > 0, \delta \ge 0$ is a given numbers, and $a(x,t)$, $f(x,t), \varphi(x)$ are the given functions.

The aim of this work is to prove the existence and uniqueness of solutions of the boundary value problem (1)-(3).

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The basicity of the system of sines in the grand-Sobolev spaces

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Let $1 < p < +\infty$. Space of measurable functions f on $(a,b) \subset R$ such that

$$\|f\|_{p)} = \sup_{0<\varepsilon<p-1} \left(\frac{\varepsilon}{|b-a|} \int_a^b |f(t)|^{p-\varepsilon} dt \right)^{\frac{1}{p-\varepsilon}} < +\infty,$$

is called $L_{p)}(a,b)$ Grang-Lebesgue space [1] . Similarly, the Grand-Sobolev space [2] $W^1_{p)}(a,b) = \{f : f, f' \in L_{p)}(a,b)\}$ is introduced with a norm

$$\|f\|_{W_{p)}} = \|f\|_{p)} + \|f'\|_{p)}. \tag{1}$$

It is known that this is not separable Banach space. Denote by $\tilde{G}W^1_{p)}(a,b)$ the set of all functions from $W^1_{p)}(a,b)$ for which $\|\hat{f}'(\cdot+\delta) - \hat{f}'(\cdot)\| \to 0$, for $\delta \to 0$, where

$$\hat{f}(t) = \begin{cases} f(t), t \in (a,b), \\ 0, t \notin (a,b). \end{cases}$$

It is clear that $\tilde{G}W^1_{p)}(a,b)$ is a manifold in $W^1_{p)}(a,b)$. Let $GW^1_{p)}(a,b)$ be a closure of $\tilde{G}W^1_{p)}(a,b)$ with respect to the norm (1).

Theorem. *The system* $1 \cup \{sin\, nt\}_{n\geq 1}$ *forms a basis for the space* $GW^1_{p)}(0,\pi)$.

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On the existence of solution of the Dirichlet problem for semilinear elliptic equations

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Let E_n n -dimensional Euclidean space of the points $x = (x_1, x_2, ..., x_n)$, Ω is a bounded domain in E_n with the boundary $\partial\Omega \in C^2$.

Consider Ω the following Dirichlet problem

$$\sum_{i,j=1}^{n} a_{ij}(x)u_{x_i x_j} + g(x, u_x) = f(x), x \in \Omega, \qquad (1)$$

$$u|_{\partial\Omega} = 0. \tag{2}$$

Assume, that the coefficients $a_{ij}(x)$ are measurable bounded functions satisfying the following conditions:

$$\gamma|\xi|^2 \le \sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \le \gamma^{-1}|\xi|^2, \tag{3}$$

$\forall x \in \Omega$, $\forall \xi \in E_n$, $\gamma \in (0,1)$- const,

$$ess \sup_{x\in\Omega} \frac{\sum\limits_{i,j=1}^{n} a_{ij}^2(x)}{\left[\sum\limits_{i=1}^{n} a_{ij}(x)\right]^2} \le \frac{1}{n-1} - \delta, \tag{4}$$

where $\delta \in \left(0; \frac{1}{n}\right)$ is some number and $g(x,u_x)$ is a Caratheodory function and in addition to this, satisfying the following growth condition:

$$|g(x,u_x)| \le b_0 \, |u_x|^q \,, b_0 > 0. \tag{5}$$

We denote by $\dot{W}_2^2(\Omega)$ the closure of the class of functions $u \in C^\infty(\overline{\Omega}) \cap C(\overline{\Omega})$, $u|_{\partial\Omega} = 0$ with respect to the norm

$$\|u\|_{W_2^2(\Omega)} = \left[\int\limits_{\Omega} \left(|u|^2 + \sum_{i=1}^{n} |u_{x_i}|^2 + \sum_{i,j=1}^{n} \left|u_{x_i x_j}\right|^2\right) dx\right]^{1/2}.$$

The following theorem is proved:

Theorem 1. *Let $n > 2$, $1 \le q < \frac{n}{n-2}$ and conditions (3)-(5) be satisfied, $\partial\Omega \in C^2$. Then there exists a sufficiently* *small constant* $C > 0, C = C = (n, \gamma, \delta, q, b_0)$ *such that problem (1)and(2) has at least one solution from* $\dot{W}_2^2(\Omega)$ *for any* $f(x) \in L_2(\Omega)$ *satisfying the condition*

$$\|f\|_{L_2(\Omega)} \leq C(mes_n\Omega)^{\frac{-n+(n-2)q}{2n(q-1)}}.$$

Separability of the generalized Cauchy–Riemann system in space $L_2\,(E)$

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In this paper, for a wide class of coefficients, we establish a unique solvability in a Hilbert space for the operator of a generalized system of Cauchy-Riemann type, the separation-in-space theorem $L_2\,(E)$ is proved and the coercive conditions of the corresponding operator.

One application of theory of correct restrictions to problem of minimization of rod deviation

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In seismically active areas vertical steel beams attached to foundation are used for reinforcement of high-rising buildings. Usually six beams are used. We assume that in case of earthquake far located detectors inform about seismic wave which is moving at the speed comparable to the speed of sound in terrestrial compartment. The message arrives (with the speed of light) to the computer which has a program that is able to "prepare" the building to face the seismic wave. In the model it will be considered that arriving seismic wave sets non-homogeneous boundary condition in the lower rod end. And the bias of the rod satisfies the hyperbolic equation of the rod. As the equation of the rod the hyperbolic second order equation is considered (for simplicity), although for the rod there exists the fourth order equation. The program must define what forces to apply on rods to provide the safety of the building. These forces and connections will be expressed (we assume so) as local and nonlocal boundary conditions. The actions of the structure on the beams that are served by these beams will be on the right-hand side of the equation. For simplicity let us consider case of one rod (beam). The objective will be to find the required deviation of the rod. For description of the beam deviation let us take the hyperbolic equation

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = f(t, x), \tag{1}$$

where $f(t, x)$ is some function responsible (in our model) for actions of the structure on the rod. Equation (1) is considered in the area $0 < t < T$, $0 \leq x \leq l$, where $T, l > 0$. Seismic wave will be expressed in boundary condition when $x = 0$:

$$u(t, x)|_{x=0} = \varphi(t).$$

Now we set local and nonlocal boundary conditions which must be well-posed in order for the problem to have one unique solution. Thus we must use only correct restriction of operator (1) in terms of theory from the works [1], [2]. As actual correct restriction consider Cauchy problem by time and Dirichlet problem by X-axis. Denote it by $L_\phi^{-1} f$. Thus by Theorem 1 from [1] all correct restrictions of maximal operator will be as follows:

$$u(t, x) = L_K^{-1} f = L_\phi^{-1} f \; + K\, f.$$

Here K is an operator acting in the core of operator (1). In the following denote:

$$K\, f(t, x) = \Psi(t, x).$$

Now the goal is to choose the operator K, minimally deviating from the initial value (in some metric)

$$\inf\left\{\int_0^T\int_0^l (|u(t,x)-\phi(t,x)|\,dxdt)\right\}.$$

So the best K will be found. And for the approximate calculation $\Psi(t,x)$ let us choose it as follows:

$$\Psi(t,x)=\sum_{m=1}^{N} C_m e^{imx},$$

$$C_m(t)=a_m\sin 2\alpha\pi t+b_m\cos 2\alpha\pi t.$$

(Generally speaking the choice of $\Psi(t,x)$ must be coordinated with the constructors. It is the task of the future). We receive the task for minimization defining $a_m, b_m, m\le N$.

Remark. In this work we have constructed a model of the rod that is used for reinforcement of the rod. The seismic wave has been displayed in the boundary condition in the form of the function $\varphi(t)$. To apply the obtained results, collaboration with engineers is required. It should be noted that this application of theory of correct restrictions has never been presented anywhere. This problem can be extended by taking into consideration other rods (as planned in the future). Also similar minimization of functional using the theory of correct restrictions can be applied to other applied problems, for example, to fluctuation of helicopter blades.

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Multipliers in Sobolev spaces and their applications in the theory of differential operators

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Denote by $H_p^s(\mathbb{R}^n),\ p > 1, s \in \mathbb{R}$, the Bessel potential spaces (for integer s they coincide with the Sobolev spaces $W_p^s(\mathbb{R}^n)$). We shall present the last results on the description of the spaces of multipliers acting from the space $H_p^s(\mathbb{R}^n)$ to another space $H_q^{-t}(\mathbb{R}^n)$. The main attention we will pay to the case when the smooth indices are of different signs, i.e. $s, t \geqslant 0$. Such a space of multipliers (we denote it by $M[H_p^s(\mathbb{R}^n) \to H_q^{-t}(\mathbb{R}^n)]$) consists of distributions $u \in \mathcal{D}'$ which obey the estimate

$$\|u\varphi\|_{H_p^{-t}} \leqslant C\|\varphi\|_{H_p^s} \quad \forall\, \varphi \in \mathcal{D},$$

where $\mathcal{D}$ is the space of the test functions and a constant C is independent of φ.

It turns out that always the following embedding with the norm estimate holds

$$M[H_p^s(\mathbb{R}^n) \to H_q^{-t}(\mathbb{R}^n)] \subset H_{q,\,unif}^{-t}(\mathbb{R}^n) \cap H_{p',\,unif}^{-s}(\mathbb{R}^n),$$

where $H_{r,\,unif}^{\gamma}(\mathbb{R}^n)$ is the so-called uniformly localized Bessel potential space.

In the case when $p \leqslant q$ and one of the following conditions

$$s \geq t \geqslant 0,\ s > n/p \quad \text{or} \quad t \geq s \geq 0,\ t > n/q' (\,1/q + 1/q' = 1),$$

holds one has an explicit representation

$$M[H_p^s(\mathbb{R}^n) \to H_q^{-t}(\mathbb{R}^n)] = H_{q,\,unif}^{-t}(\mathbb{R}^n) \cap H_{p',\,unif}^{-s}(\mathbb{R}^n),$$

where p and p' are Holder conjugate numbers. Representations of this kind are impossible, provided that $s > n/p$ or $t > n/q'$. Then the spaces of multipliers should be described in terms of capacities. In the important case $s = t < n/\max(p, q')$ we can establish the two-sided embeddings with the norm estimates

$$H_{r_1,\,unif}^{-s}(\mathbb{R}^n) \subset M[H_p^s(\mathbb{R}^n) \to H_{q'}^{-s}(\mathbb{R}^n)] \subset H_{r_2,\,unif}^{-s}(\mathbb{R}^n),$$

where the numbers $r_1 > r_2 > 1$ can be written down explicitly.

The obtained results have important applications in the theory of differential operators with distribution coefficients. Some simple applications will be presented in the talk.

The talk is based on the joint papers with Alexey A. Belyaev. The work is supported by the Russian Scientific Fund, grant number No 17-11-01215.

On some applications of spectral asymptotics in Wiman-Valiron theory

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More than a century spectral asymptotics is an object of intensive study of many outstanding mathematicians and physicists. Calculation of asymptotics of the function of distribution of $N(\lambda)$ eigenvalues of a self-adjoint differential operator is one of the classic problems of mathematical physics. H.Weyl first ([4, 1913) established the following asymptotic formula

$$N(\lambda) = \frac{V}{6\pi^2}\lambda^3 + \frac{S}{16\pi}\lambda^2 + o(\lambda^2), \quad \lambda \to +\infty,$$

in a spectral problem for a Laplace operator in a bounded domain $\Omega \subset R^n$:

$$-\Delta v = \lambda^2 v \;\text{ in }\; \Omega, \quad v\,|_{\partial\Omega} = 0,$$

where $V = vol\ \Omega$, S is the square of the surface $\partial\Omega$. Afterwards, very famous mathematicians established similar results for different classes and different domains in R^n.

The strongest results belong to Ivriy ([1], 1980) who has obtained the exact value of the remainder term in the asymptotic formula for the function $N(\lambda)$ for an elliptic boundary value problem for a second order operator, and also found the second term of the asymptotics in Weyl's formula.

L. Hórmander (1968) by the PDO method for m-th order self-adjoint elliptic operator A in a compact manifold $M \subset R^n$ established A the following (unimpoved estimation of the remainder term) formula

$$N(\lambda) = \lambda^{n/m} \left(1 + O\left(\lambda^{\frac{n-1}{m}}\right)\right) .$$

The principal, sometimes final results, were established only in the eighties, ninetieth years in the papers of Ivriy, Vasil'ev, Safarov, Mikhailets, Shubin, Tulovskii and others.

In the joint monograph of Yu. Safarov and D. Vasilev ([2], 1997 USA) in the spectral problem

$$Av = \lambda^{2m} v \ \text{ in } \ M \subset R^n; \quad B^{(i)} v\,|_{\partial M} = 0, \quad j = 1, 2, ..., m,$$

where A is a positive self-adjoint elliptic differential-operator of $2m$ order on a compact manifold $M \subset R^n$ asymptotic formulas (classic one-term and also nonclassic two-tern) of the form

$$N(\lambda) = c_0 \lambda^{n/2m} + c_1 \lambda^{\frac{n-1}{2m}} + o\left(\lambda^{\frac{n-1}{2m}}\right), \quad \lambda \to +\infty$$

were established for the function $N(\lambda)$.

One of the exapmles of fruitful applications of the spectral asymptotics is the theory of Wiman-Valiron type estimations constructed in the papers of N.M. Suleymanov in his monograph ([3], Moscow University publishers, 2012). In the present paper, using relatively late results on spectral asymptotics (Irviy, Vasilev, Safarov, Mikhaileva, Shubin and others) Wiman-Valiron type estimations are established for evolution equations of the form

$$u'(t) - A(t)u(t) = 0$$

in Hilbert space, where $A(t)$ is a lower-bounded self-adjoint differential operator with a discrete spectrum. We establish the estimations of type

$$\|u(t)\| \le \mu(t)\psi\left(\mu(t)\right), \quad t \to \infty,$$

where $\mu(t) = \sup\limits_k \left|(u(t),\ \varphi_k(t))\right|; \quad \{\varphi_k(t)\}$ is an orthonormed system of eigen-functions of the operator $A(t)$, while ψ is some slowly increasing function on $(0, \infty)$, for example $\psi(t) = (\log t)^{1/2+\varepsilon}$, $\varepsilon > 0$.

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The spectral properties of singular differential operators with oscillatihg coefficients

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It is well known, that positive Sturm-Liouville operator with growing potential function has discrete spectrum and if this function satisfies to Titchmarsh-Levitan condition,then the asymptotic formula for the function of eigenvalues may be calculated.

If operator is nonsemibounded the problem is more complicated.The growing of potential does not garantee the discreetness of spectrum.If potential function is oscillating growing function, then it is impossible to use Karleman method of investigating of spectral asymptotic.We are going to discuss in our talk the last results in the spectral theory of singular differential operators with oscillating coefficients.

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Five-electron systems in the Hubbard model. Third quartet state

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We consider the energy operator of five-electron systems in the Hubbard model and describe the structure of the essential spectra and discrete spectrum of the system for third quartet states.

The Hamiltonian of the chosen model has the form

$$H = A\sum_{m,\gamma} a^+_{m,\gamma}a_{m,\gamma} + B\sum_{m,\tau,\gamma} a^+_{m+\tau,\gamma}a_{m,\gamma} +$$

$$+U\sum_{m} a^+_{m,\uparrow}a_{m,\uparrow}a^+_{m,\downarrow}a_{m,\downarrow}.$$

Here A is the electron energy at a lattice site, B is the transfer integral between neighboring sites (we assume that $B>0$ for convenience), $\tau=\pm e_j, j=1,2,...,\nu$, where e_j are unit mutually orthogonal vectors, which means that summation is taken over the nearest neighbors, U is the parameter of the on-site Coulomb interaction of two electrons, γ is the spin index, and $a^+_{m,\gamma}$ and $a_{m,\gamma}$ are the respective electron creation and annihilation operators at a site $m\in Z^\nu$. In the five-electron systems exists sextet state, four type quartet states, and five type doublet states. The Hamiltonian H acts in the antisymmetric Fo'ck space $\mathcal{H}_{as}$. Third quartet state is consists of states ${}^3q^{3/2}_{m,n,r,t,l\in Z^\nu} = a^+_{m,\uparrow}a^+_{n,\uparrow}a^+_{r,\downarrow}a^+_{t,\uparrow}a^+_{l,\uparrow}\varphi_0$. The subspace ${}^3\mathcal{H}^q_{3/2}$, corresponding to the third five-electron quartet state is the set of all vectors ${}^3\psi^q_{3/2} = \sum_{m,n,r,t,l\in Z^\nu} f(m,n,r,t,l){}^3q^{3/2}_{m,n,r,t,l\in Z^\nu}, f\in l_2^{as}$, where l_2^{as} is the subspace of antisymmetric functions in the space $l_2((Z^\nu)^5)$. The restriction ${}^3H^q_{3/2}$ of operator H to the subspace ${}^3\mathcal{H}^q_{3/2}$, is called the five-electron third quartet state operator.

Theorem 1. *If* $\nu = 1$ *and* $U < 0$, *or* $U > 0$, *then the essential spectrum of the third five-electron quartet state operator* ${}^3\widetilde{H}^q_{3/2}$ *is consists of the union of seven segments, and discrete spectrum of operator* ${}^3\widetilde{H}^q_{3/2}$ *is consists of no more one point.*

Theorem 2. *If* $\nu = 3$ *and* $U < 0$, *or* $U > 0$, *then the essential spectrum of the third five-electron quartet state operator* ${}^3\widetilde{H}^q_{3/2}$ *is consists of the union of seven, or four, or two, or single segments, and discrete spectrum of operator* ${}^3\widetilde{H}^q_{3/2}$ *is consists of no more one point.*

Existence of a saddle-point in differential game

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It is known that many problems of the economy are brought to the solution of differential games. In this thesis, we consider a differential game involving two players. Let some process be described by the system of differential equations

$$\dot{x}(t) = f(t, x(t), u(t), v(t)), t \in [t_0, T], \qquad (1)$$

where $x \in, u \in R^r, v \in R^q; f : [0,T] \times R^n \times R^r \times R^q \to R^n$ is given function.

Assume, that the studied process starts at the time moment t_0 and finishes after the fixed time interval at the moment T . Let at the moment t_0 the initial

$$x\left(t_0\right) = x_0 \tag{2}$$

be given. It is assumed that U and V are the given sets in the Hilbert spaces $L_2^r[t_0,T]$ and $L_2^r[t_0,T]$, respectively. On the totality $(u,v) \in U \times V$ let's consider the functional

$$J((u,v)) = \varphi(x(T)), \tag{3}$$

where the vector-function $x(t) = x(t;u,v), t \in [t_0,T]$ is determined as a solution of system (1) under condition (2), corresponding to the control$(u,v) \in U \times V$.

Let the controls $(u,v) \in U \times V$ be selected at each moment of time from the domain of determination depending on the state of the process x to optimize quality criterion (3). If u and v belong to two sides whose interests in the sense of selected criterion are opposite, then such a controlled process is called a differential game, optimized criterion charge.

Definition. *[1] The admissible controls and we call optimal strategies if for them*

$$J\left(u_\bullet, v\right) \leq J\left(u_\bullet, v^\bullet\right) \leq J\left(u, v^\bullet\right)$$

are fulfilled.

The trajectory $x_\bullet(t)$ generated by the optimal strategies $u_\bullet\left(t\right), v^\bullet\left(t\right)$ is called an optimal trajectory.

Under some conditions on the initial data of the problem, sufficient conditions for the existence of a saddle point in a differential game (1)-(3) are obtained.

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