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ABSTRACTS

Optimization problem for semilinear elliptic equations with nonlinear control

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Optimization control problems for nonlinear infinite dimensional systems with linear controls are well known for small enough values of the set dimension and the velocity of the increment of the nonlinearity. If these parameters are large enough, then the control-state mapping become non-differentiable, and the adjoint system has weak enough properties. The known methods are non-applicable for this case. We could obtain necessary conditions of optimality by means of extended derivatives. However this technique was used for systems with linear controls only. We will use this idea for the equation with nonlinear control.

Consider homogeneous Dirichlet problem for the equation

$$-\Delta y = f(x, v, y),$$

where x is independent variable on the open bounded n-dimensional set Ω, y is the state function, v is the control. It is determined on a convex closed bounded set U of the space $L_r(\Omega)$. Suppose f is Caratheodory function, which satisfies the standard conditions of coercivity, monotony, and boundedness of the increment degree. Then for all function $v \in L_r(\Omega)$ this equation has a unique solution $y = y[v]$ from the space $Y = H_0^1(\Omega) \cap L_s(\Omega)$, besides the mapping $v \to y[v]$ is continuous, where s is the velocity of the increment of the function f with respect to the third argument.

We have the minimization problem for the functional

$$I(v) = \int_\Omega F\big(x, v, y[v]\big)dx$$

on the set U, where F is Caratheodory function, which is convex with respect to its second argument and satisfies a condition of the boundedness of the increment degree with respect to its second and third arguments. Using standard optimization methods, we prove the solvability of this problem.

Suppose the function f has the derivatives with respect to the second and third arguments, which satisfy a condition of the boundedness of the increment degree. Using Implicit Function Theorem, we prove the following result.

Lemma 1. *The mapping $v \to y[v]$ is Frechet differentiable for small enough values of n and s.*

Using known optimization methods, we obtain necessary conditions of the optimality.

Theorem 1. *Under the conditions of Lemma* 1 *the optimal control satisfies the variational inequality*

$$\int\limits_{\Omega} \frac{\partial H}{\partial u}(v-u)dx \geq 0 \; \forall v \in U,$$

where $H(x,v,y,p) = f(x,v,y)p - F(x,v,y)$, *and* p *is the solution of the homogeneous Dirichlet problem for the adjoint equation*

$$-\Delta p = -\frac{\partial H}{\partial y}.$$

Lemma 2. *The mapping* $v \to y[v]$ *is not Gateaux differentiable for large enough values of* n *and* s.

This result and weak enough properties of the adjoint equation put obstacles in the way of using of the standard optimization methods. The extended differentiability of the control-state mapping was proved before for systems that are linear with respect to the control. However we prove the following result.

Lemma 3. *The mapping* $v \to y[v]$ *is extended differentiable.*

Using this property we obtain necessary optimality conditions for the general case.

Theorem 3. *The assertions of Theorem* 1 *are true for all value of* n *and* s.

Thus the extended differentiation theory is applicable for systems with nonlinear control. We can use it for obtaining necessary conditions of optimality if the standard optimization methods are not applicable. We could analyze other optimization control problems, for example, optimization problems for systems described by nonlinear parabolic equations with nonlinear control.