6th International IFS and Contemporary Mathematics Conference

June, 07-10, 2019 Mersin Turkey

CONFERENCE PROCEEDING BOOK

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ISBN 978-605-68670-2-6

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PREFACE

We are very pleased to introduce the abstracts of the 6th International IFS and Contemporary Mathematics Conference (IFSCOM2019).
As previous conferences, the theme was the link between the Mathematics by many valued logics and its applications.

In this context, there is a need to discuss the relationships and interactions between many valued logics and contemporary mathematics.

Finally,in the previous conference, it made suceessful activities to communicate with scientists working in similar fields and relations between the different disciplines.

This conference has papers in different areas; multi-valued logic, geometry, algebra, applied mathematics, theory of fuzzy sets, intuitionistic fuzzy set theory, mathematical physics, mathematics applications, etc.

Thank you to all paticipants scientists offering the most significant contribution to this conference.

Thank you to Scientific Committee Members, Referee Committee Members, Local Committee Members, University Administrators, Mersin University Mathematic Department.

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JUNE, 07-10, 2019, MERSIN, TURKEY
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ISBN:978-605-68670-2-6

NOETHERITY OF A SINGULAR INTEGRAL OPERATOR WITH CAUCHY KERNEL AND WITH CARLEMAN'S SHIFT IN FRACTIONAL SPACES

N.K. BLIEV AND ZH.KH. ZHUNUSSOVA

ABSTRACT. Singular integrals and equations containing singular integrals are closely related to boundary value problems for piecewise analytic functions of a complex variable and have important applications, for example, in the theory of boundary value problems for partial differential equations.

1. INTRODUCTION

Let Γ be Lyapunov closed contour of a class C^1_ν, $\frac{2}{p} - 1 < \nu \leq 1$, $1 < p < 2$. We consider in Besov's space $B(\Gamma) \equiv B^r_{p,1}(\Gamma), 1 < p < 2, r = \frac{1}{p'}$ the singular integral equation (SIE):

$$M\varphi \equiv a(t)\varphi(t) + b(t)\varphi[\alpha(t)] + \frac{c(t)}{\pi i}\int_\Gamma \frac{\varphi(\tau)}{\tau - t}d\tau +$$

$$\frac{d(t)}{\pi i}\int_\Gamma \frac{\varphi(\tau)}{\tau - \alpha(t)}d\tau + \int_\Gamma K(t,\tau)\varphi(\tau)d\tau = g(t), \tag{1}$$

where $a(t), b(t), c(t), d(t)$ and $g(t)$ belong to $B(\Gamma), \alpha(t)$ is a Carleman's shift: homeomorphically translates Γ onto itself while maintaining or changing its orientation on Γ and satisfies the condition

$$\alpha[\alpha(t)] = t. \tag{2}$$

We assume, that there exists a derivative $\alpha'(t)$ belongings to space $H_\mu(\Gamma), 0 < \mu \leq 1$, of the functions which are continuous by Helder with indicator μ. The kernel $K(t,\tau)$ has such a weak singularity that the corresponding integral operator is completely continuous in $B(\Gamma)$. Note, that $B(\Gamma)$ embedded in space $C(\Gamma)$ of continuous functions but not embedded in $H_\mu(\Gamma), 0 < \mu \leq 1$, [1] and is a commutative Banach algebra with the usual operations of addition and multiplication [2, 3]. Operator M is limited in $B(\Gamma)$ [2] (see pp. 45 - 47). Equation (1) is considered in [4] in the spaces $H_\mu(\Gamma), 0 < \mu \leq 1$, and $L_p(\Gamma), 1 < p < \infty$, where there is a necessary bibliography.

Date: xxxx a, 2019, accepted yyyy b, 2019.
2000 *Mathematics Subject Classification.* Primary 45E05, 30E25; Secondary 32A38, 32A26.
Key words and phrases. singular integrals, analytic functions, Besov's space, integral equation, Carleman's shift.

Characteristic equation ($K(t,\tau)=0$) (1) equivalent to a boundary value problem of finding a piecewise analytic function vanishing at infinity $\{\Phi^+(z), \Phi^-(z)\}$,

$$a_1\Phi^+(t) + b_1\Phi^+[\alpha(t)] + c_1\Phi^-(t) + d_1(t)\Phi^-[\alpha(t)] = g(t), \tag{3}$$

where

$$a_1(t) = a(t) + c(t), c_1(t) = c(t) - a(t),$$
$$b_1(t) = b(t) + d(t), d_1(t) = d(t) - b(t), \tag{4}$$

Following [4], for SIE with a shift (1), we construct a system of two equations without a shift, called the corresponding one. To do this, we add to (1) another equation derived from (1) by replacing t with $\alpha(t)$ And we introduce new unknown functions

$$\rho_1(t) = \varphi(t), \rho_2(t) = \varphi[\alpha(t)].$$

As a result, we obtain the corresponding system of two SIE with the Cauchy kernel with respect to the vector function $\rho(t) = \{\rho_1(t), \rho_2(t)\}$:

$$a(t)\rho_1(t) + b(t)\rho_2(t) + \frac{c(t)}{\pi i}\int_\Gamma \frac{\rho_1(\tau)}{\tau - t}d\tau + \frac{\gamma d(t)}{\pi i}\int_\Gamma \frac{\alpha'(\tau)\rho_2(\tau)}{\alpha(\tau)-\alpha(t)}d\tau +$$
$$+\int_\Gamma K(t,\tau)\rho_1(\tau)d\tau = g(t),$$
$$b[\alpha(t)]\rho_1(t) + a[\alpha(t)]\rho_2(t) + \frac{d[\alpha(t)]}{\pi i}\int_\Gamma \frac{\rho_1(\tau)}{\tau - t}d\tau + \frac{\gamma c[\alpha(t)]}{\pi i}\int_\Gamma \frac{\alpha'(\tau)\rho_2(\tau)}{\alpha(\tau)-\alpha(t)}d\tau +$$
$$\gamma\int_\Gamma K[\alpha(t),\alpha(\tau)]\alpha'(\tau)\rho_2(\tau)d\tau = g[\alpha(t)], \tag{5}$$

Here the coefficient γ takes the value +1 or -1 if, respectively, $\alpha(t)$ maps Γ onto itself with preservation or change of orientation Γ. The operator which application to the vector function (ρ_1, ρ_2) gives the left-hand side of (5), can be written as

$$L \equiv p(t)J + q(t)S + D_1, \tag{5'}$$

where the matrixes $p(t)$ and $q(t)$ have the form

$$p(t) = \begin{pmatrix} a(t) & b(t) \\ b[\alpha(t)] & a[\alpha(t)] \end{pmatrix}, \quad q(t) = \begin{pmatrix} c(t) & \gamma d(t) \\ d[\alpha(t)] & \gamma c[\alpha(t)] \end{pmatrix},$$

D_1 is a completely continuous operator. Here S is a scalar singular operator

$$S\varphi = \frac{1}{\pi i}\int_\Gamma \frac{\varphi(\tau)}{\tau - t}d\tau$$

The operator L acts on $B(\Gamma)$. It is introduced more SIE with Carleman's shift (accompanying (1))

$$K_\chi \equiv a(t)\chi(t) - b(t)\chi[\alpha(t)] + \frac{c(t)}{\pi i}\int_\Gamma \frac{\chi(\tau)}{\tau - t}d\tau - \frac{d(t)}{\pi i}\int_\Gamma \frac{\chi(\tau)}{\tau - \alpha(t)}d\tau +$$
$$+\int_\Gamma K(t,\tau)\chi(\tau)d\tau = 0. \tag{6}$$

It is easy to see that from (6) using the procedure described above, we can go to the corresponding system (5), if we set $\rho_1(t) = \chi(t), \rho_2(t) = -\chi[\alpha(t)]$.

2. Preliminaries

We also consider two SIEs with Carleman's shift, one of which is union with (1), and the other is union with accompanying equation (6). The union operators are understood as meaning identities

$$\int_\Gamma (M\varphi)(t)\psi(t)dt = \int_\Gamma \varphi(t)(M'\psi)(t)dt.$$

$$M'\psi \equiv a(t)\psi(t) + \gamma\alpha'(t)b[\alpha(t)]\psi[\alpha(t)] - \frac{1}{\pi i}\int_\Gamma \frac{c(\tau)\psi(\tau)}{\tau - t}d\tau$$

$$-\frac{\gamma}{\pi i}\int_\Gamma \frac{d[\alpha(t)]\alpha'(\tau)\psi[\alpha(t)]}{\tau - t}d\tau + \int_\Gamma m(\tau,t)\psi(\tau)d\tau = 0. \quad (7)$$

$$K'\omega \equiv a(t)\omega(t) + \gamma\alpha'(t)b[\alpha(t)]\omega[\alpha(t)] - \frac{1}{\pi i}\int_\Gamma \frac{c(\tau)\omega(\tau)}{\tau - t}d\tau$$

$$+\frac{\gamma}{\pi i}\int_\Gamma \frac{d[\alpha(\tau)]\alpha'(\tau)\omega[\alpha(t)]}{\tau - t}d\tau + \int_\Gamma m(\tau,t)\omega(\tau)d\tau = 0. \quad (8)$$

It is easy to see that equation (8) is corresponding to the union equation (7). Corresponding to equations (8) and (7), the system of SIE without a shift is

$$a(t)\omega_1(t) + b[\alpha(t)]\omega_2(t) - \frac{1}{\pi i}\int_\Gamma \frac{c(\tau)\omega_1(\tau)}{\tau - t}d\tau - \frac{1}{\pi i}\int_\Gamma \frac{d[\alpha(t)]\omega_2(\tau)}{\tau - t}d\tau +$$

$$+\int_\Gamma m(\tau,t)\omega_1(\tau)d\tau = 0. \quad (9)$$

$$b(t)\omega_1(t) + a[\alpha(t)]\omega_2(t) - \frac{\gamma\alpha'(t)}{\pi i}\int_\Gamma \frac{d(\tau)\omega_1(\tau)}{\alpha(\tau) - \alpha(t)}d\tau - \frac{\gamma\alpha'(t)}{\pi i}\int_\Gamma \frac{c[\alpha(t)]\omega_2(\tau)}{\alpha(\tau) - \alpha(t)}d\tau +$$

$$+\gamma\alpha'(t)\int_\Gamma m[\alpha(\tau),\alpha(t)]\omega_2(\tau)d\tau = 0.$$

It is easy to verify that system (9) is union with the corresponding system of equations (6). The following lemmas are proved similarly to Lemmas 6.1 - 6.3, and 6.6 of [4], taking into account the above information.

Lemma 2.1. *The number of linearly independent solutions of the homogeneous ($\gamma(t) \equiv 0$) corresponding system of equations (5) is equal to the sum of the numbers of the linearly independent solutions of this homogeneous equation (1) and the accompanying equation (6).*

Let l^* be the number of linearly independent solutions of system (9), union with the corresponding system of equations (5), l_1^* and l_2^* are the numbers of linearly independent solutions respectively union with (1) equation (7) and equation (8) which is union with accompanying equation (6). The following lemmas are held.

Lemma 2.2. *The numbers l^*, l_1^* and l_2^* satisfy to the equality $l^* = l_1^* + l_2^*$.*

Lemma 2.3. *The inhomogeneous SIE with the Carleman shift (1) is solvable if and only if the inhomogeneous corresponding system of equations (5) is solvable.*

Lemma 2.4. *If one of the two operators M or L is noetherian, then the second one is noetherian, and*

$$IndM = IndL.$$

3. Noetherity of a SIE with Carleman's shift

Let $B_2(\Gamma)$ be a set of all 2-dimensional vectors with components from $B(\Gamma)$ and $B_{2\times2}$ is a set of all square matrices of order 2 with elements from $B(\Gamma)$.

The set $B_2(\Gamma)$ can be supplied with the norm, taking, as the norm of the vector $X = (x_1, x_2)$ the sum of the norms of the individual components:

$$\|X\| = \|x_1\|_{B(\Gamma)} + \|x_2\|_{B(\Gamma)}.$$

In this case, the norm of the matrix $A = \{a_{kj}\}_1^2 \in B_{2\times2}(\Gamma)$ can be determined, for example, by relation

$$\|A\| = 2\max_{j,k} \|a_{j,k}\|_{B(\Gamma)}$$

Then the space $B_{2\times2}(\Gamma)$ with this norm, there will also be a Banach's algebra.

Let $L(B)$ be a set of all continuous linear operators in $B(\Gamma)$, then every operator $A \in L(B_2)$ can be interpreted as a two-dimensional matrix $A = \{a_{kj}\}$, where $a_{jk} \in L(B)$. Moreover, the matrix operator A is completely continuous if and only if all completely continuous operators are multiplied by a_{jk} in $B(\Gamma)$. The operator L in $(5')$belongs to $L(B_2)$. Matrix operator $L(5')$ is written in the form

$$L = CP_1 + DQ_1 + T, \tag{10}$$

where C and D are multiplication operators in $B_2(\Gamma)$ correspondingly on the matrix functions

$$C(t) = p(t) + q(t) \quad D(t) = p(t) - q(t),$$

$$S_1 = \{S\delta_{jk}\}_1^2, P_1 = \frac{1}{2}(I + S_1), Q_1 = \frac{1}{2}(I - S_1),$$

here S is a scalar singular operator

$$S\varphi = \frac{1}{\pi\varepsilon}\int_\Gamma \frac{\varphi(\tau)}{\tau - t}d\tau$$

is limited in $B(\Gamma)$ [2], l is an unitary operator in $B(\Gamma)$, δ_{jk} is Chronecker's symbol.

Operators in the form L form an algebra [3]. We introduce the notation

$$\alpha(L) = dimkerL, \quad \beta(L) = dim\ co\ kerL,$$

It is said, that the operator L has a finite d- characteristic or a finite index if both numbers $\alpha(L)$ and $\beta(L)$ are finite. A closed normally solvable operator L is called a noetherian or F -operator if its d -characteristic is finite, and a semi- noetherian operator if at least one of the numbers $\alpha(L)$ and $\beta(L)$ is finite. In the case of semi-noetherity, we distinguish F_+ - operators $(\alpha(L) < \infty)$ and F_- operators $\beta(L) < \infty$.

Let G be a domain of the complex plane E bounded by a closed Lyapunov contour $\Gamma, G^+ = G, G^- = E - \overline{G^+}$ We consider, that $0 \in G^+, z = \infty \in G^-$. Right factorization for everywhere on Γ the non-singular matrix $A(t) \in B_{2\times2}(\Gamma)$ is called its representation in the form

$$A(t) = A_-(t)D(t)A_+(t), t \in \Gamma, \tag{11}$$

with a diagonal matrix $D(t)$ in the form $D(t) = \{t^{jk}\delta_{jk}\}_1^2 \quad t \in \Gamma$. Here $k_1 \geq k_2$ are some numbers uniquely determined by the matrix $A(t)$, $A_\pm$ are second-order square matrices that can be continued analytic in the domains $G_\pm$ and are continuous in $\overline{(G_\pm)}$, moreover

$$detA_+(z) \neq 0 \quad z \in \overline{G_\pm}, \quad detA_-(z) \neq 0 \quad z \in \overline{G}_-.$$

The factorization of the matrix $A(t)$ which is obtained by changing the factors $A_{\pm}(t)$ in (11), is called the left factorization. Obviously, each right (left) factorization of the transposed matrix $A'(t)$ as well as the inverse matrix $A^{-1}(t)$.

$B(\Gamma)$ is a decaying R-algebra [5] (Section 4), therefore, every non-singular matrix $A(t) \in B_{2\times 2}(\Gamma)$ admits a right (and left) factorization [6] (p. 309).

Now we can formulate the conditions for the noetherity of the operator (the noetherian solvability of system (5)) from $L(B_2)$ written in the form (5′).

From theorems 2 and 3 of [5] under $n = 2$ follows the next theorem.

Theorem 3.1. *The operator $L = CP_1 + DQ_1 + T$ is F_+ or (F_-) operator if and only if, when*

$$detC(t) \neq 0, \quad detD(t) \neq 0 (t \in \Gamma). \tag{12}$$

If the conditions are hold, then $R = C^{-1}P_1 + D^{-1}Q_1$ is a two-sided regularizer of the operator L. If conditions (12) are hold, then the matrix $D^{-1}(t)C(t)$ allows the right factorization

$$D^{-1}(t)C(t) = C(t)U(t)C_T(t), U(t) = \{t^{K_j}\delta_{Kj}\}.$$

Theorem 3.2. *Let condition (12) is held. Then if the characteristic equation*

$$(CP_1 + DQ_1)\rho = f \quad (f \in B_{2(n)})$$

is solvable, then the solution is given by formula

$$\rho = (CP_1 + DQ_1)^{-1}(f),$$

in which

$$(CP_1 + DQ_1)^{-1} = (C_+^{-1}P_1 + C_-Q_1)(U^{-1}P_1 + Q_1)C_-^{-1}D^{-1}$$

is a left and right inverse operator to $CP_1 + DQ_1$ in dependence on $k = ind(D_1^{-1}C_1) \geq 0$ or $k \leq 0$.

The index of the noetherian operator is calculated by the formula

$$I = \frac{1}{2\pi}\{arg\frac{detD(t)}{detC(t)}\}_\Gamma \tag{13}$$

By [4], condition (12) and formula (13) can be written to the cases: $\alpha(t)$ is a direct shift ($\gamma = 1$) or $\alpha(t)$ is an inverse shift ($\gamma = -1$).

Let $\alpha(t)$ keeps the orientation on $\Gamma(\gamma = 1)$. Then

$$detD(t) = det(p(t) - q(t)) = c_1(t)c_1[\alpha(t)] - d_1(t)d_1[\alpha(t)] = \Delta_1(t),$$

$$detC(t) = det(p(t) + q(t)) = a_1(t)a_1[\alpha(t)] - b_1(t)b_1[\alpha(t)] = \Delta_2(t),$$

Let $\alpha(t)$ changes orientation on $\Gamma(\gamma = -1)$. Then

$$detD(t) = -b_1(t)d_1[\alpha(t)] + c_1(t)a_1[\alpha(t)] = \Delta(t),$$

$$detC(t) = -b_1(t)[\alpha(t)]d_1(t) + c_1[\alpha(t)]a_1(t) = \Delta(t).$$

Theorem 3.3. *In order to SIE with Carleman's shift (1) be noetherian sufficiently the conditions to be held*

A. $\Delta_1(t) \neq 0, \Delta_2(t) \neq 0$ under preserving orientation on $\alpha(t)$.

B. $\Delta(t) \neq 0$ under changing orientation $\alpha(t)$.

Theorem 3.4. *Under the noetherian conditions the SIE index with Carleman's shift (1) is calculated by the formulas:*

A. If $\alpha(t)$ keeps orientation, then

$$IndM = \frac{1}{4\pi}\{arg\frac{\Delta_1(t)}{\Delta_2(t)}\}_\Gamma$$

B. If $\alpha(t)$ changes orientation, then

$$IndM = \frac{1}{\pi}\{arg\Delta(t)\}_\Gamma.$$

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