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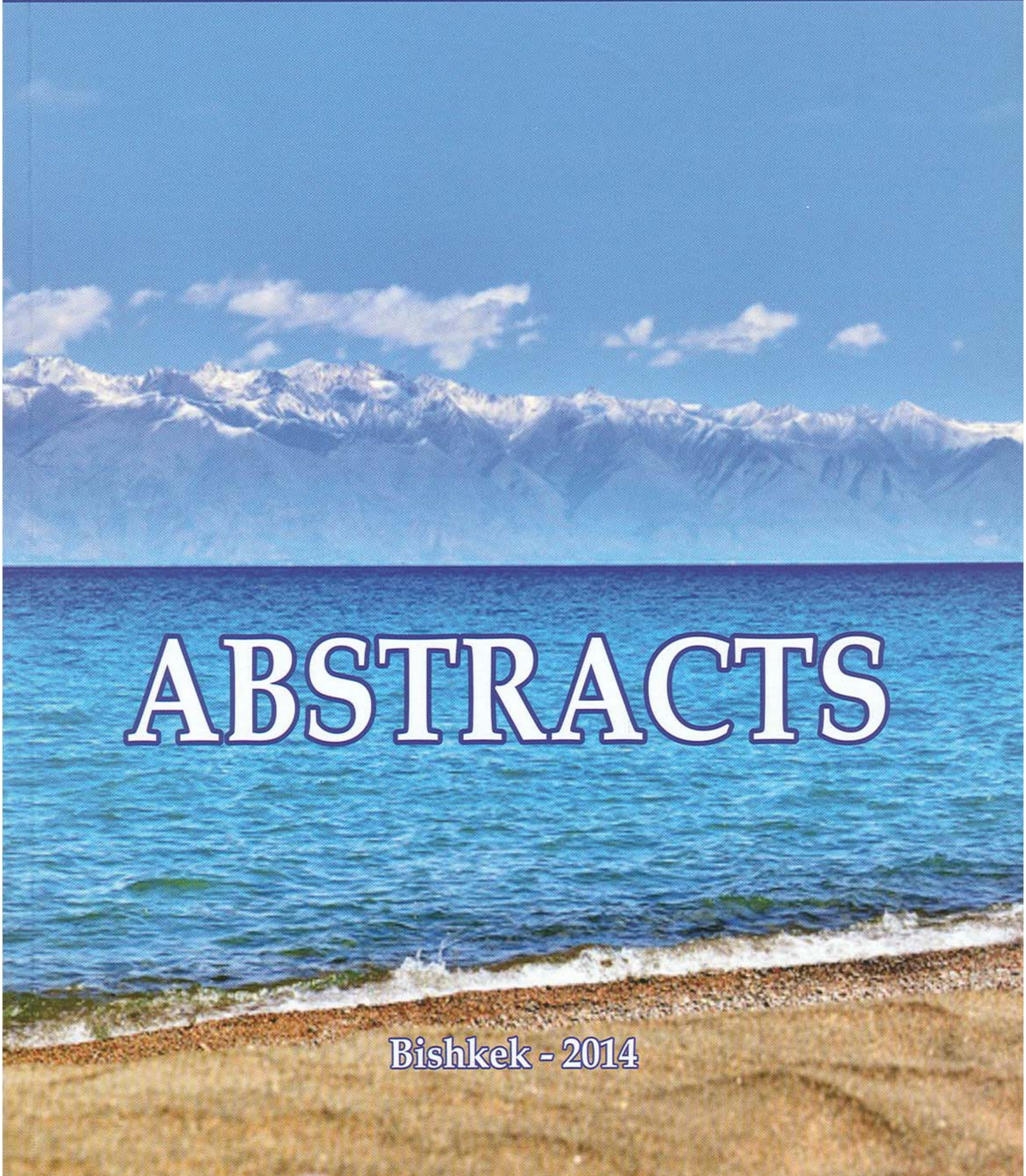

ABSTRACTS

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JACOBI OPTIMALITY CONDITIONS FOR EXTREMUM PROBLEMS WITH EQUALITIES CONSTRAINTS

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This work continues investigations [1].

Lagrange multipliers method is used for constraints extremum problems. The necessary optimality conditions consist of equalities that are relations between unknown values and Lagrange multipliers. However there exists another form of optimality conditions. It is not consist of Lagrange multipliers. This relation is characterized the matrix of partial derivatives of the state functional and constraints operators.

Let V, Y, Z be Banach spaces, $A : V \times Y \to Z$ be a smooth operator, and $I : V \times Y \to \mathbb{R}$ be a smooth functional. We have the problem of the minimization of the functional I on the zeros set of the operator A. Determine $F = (I, A)$, $Q = (E, L)$, where E is the unit operator of the control space V, and $L : V \to Y$ is implicit operator determined by the state equation $A(v, y) = 0$ (control-state mapping).

Theorem 1. *If the pair (v, y) is a solution of the given problem, and the partial derivative $A_y(v, y)$ is invertible, then $F'(v, y)Q'(v) = 0$.*

This optimality condition includes the matrix of partial derivatives (Jacobean) of F. Consider finite dimensional case. We have the problem of the minimization of the function $f_0 = f_0(x_0, x_1, ..., x_n)$ under the conditions $f_i(x_0, ..., x_n) = 0$, $i = 1, ..., n$. All functions are smooth.

Theorem 2. *If $x = (x_0, x_1, ..., x_n)$ is the local extremum of the function f_0 under the given conditions, then Jacobean of functions $f_0, f_1, ..., f_n$ at the point x is equal to zero.*

Consider the problem of the minimization of the function $f_0 = f_0(x)$, where $x = (x_1, ..., x_{n+r})$ under the conditions $f_i(x_1, ..., x_{n+r}) = 0$, $i = 1, ..., n$. All functions are smooth. Fixed the multi index $\alpha = (\alpha_1, ..., \alpha_n)$, where $\alpha_i \in \{1, ..., n + r\}$, $\alpha_i \neq \alpha_j \; \forall i \neq j$. Determine

$$F^{\alpha}_{\beta_i}(x) = \begin{pmatrix} \partial_{\alpha_1} f_0(x) & \dots & \partial_{\alpha_n} f_0(x) & \partial_{\beta_i} f_0(x) \\ \partial_{\alpha_1} f_1(x) & \dots & \partial_{\alpha_n} f_1(x) & \partial_{\beta_i} f_1(x) \\ \dots & \dots & \dots & \dots \\ \partial_{\alpha_1} f_n(x) & \dots & \partial_{\alpha_n} f_n(x) & \partial_{\beta_i} f_n(x) \end{pmatrix}, \; i = 1, 2, ..., r,$$

where $\beta_i \in 1, ..., n + r$, $\beta_i \neq \beta_j$ for all $i \neq j$ and $\beta_i \neq \alpha_j$ for all i, j.

Theorem 3. *If $x = (x_1, ..., x_{n+r})$ is the local extremum of the function f_0 under the given conditions, then $\left|F^{\alpha}_{\beta_i}(x)\right| = 0$, $i = 1, 2, ..., r$.*

References

[1] Serovajsky S.Ya. Optimization and differentiating (in Russian). Vol. 1. Minimization of functionals. Stationary systems (2006); Vol. 2. Nonstationary systems (2009), Almaty.

SECTION 8. HISTORY AND TEACHING OF MATHEMATICS