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DYFERENCJALNE I INTEGRALNE ZRYWNANIE

TEORIA PRAWDOPODOBIECSTW I MATEMATYCZNA STATYSTYKIEM

STOSOWANA MATEMATYKA

MATEMATYCZNE MODELOWANIE

STOSOWANA MATEMATYKA

К.ф.-м.н. Джомартова Ш. А.
Казахский национальный университет имени аль-Фараби, Казахстан

INTERVAL MATHEMATICS FOR PRACTICAL CALCULATIONS

In scientific researches, the engineering and mass manufacture frequently should carry spend measurements of any sizes (length, weight, force of a current etc.). At recurrence of measurements of the same object which is carried out with the help of the same measuring device with identical carefulness because of influence of the various factors, identical data never turn out. The casual vibrations of separate parts of the device, physiological changes of sense organs of the executor, various not taken into account changes in environment (temperature, optical, electrical and magnetic properties etc.) concern to number of such factors. Though the result of each separate measurement at presence of casual dispersion cannot beforehand be predicted, it corresponds «to a normal curve of distribution» (figure).

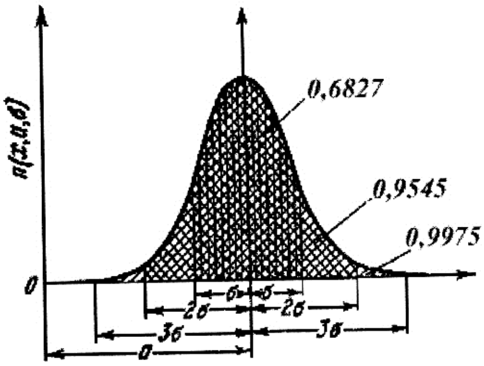

Figure Normal curve of distribution

From figure it is visible, that basic weight of received results will be grouped about some central or average meaning *a*, which the «true size» measurely of object answers unknown. The deviations in this or that of the party will occur by that less often, than more absolute size of such deviations, and are characterized by size σ – average quadratic deviation. On a site from $a-\sigma$ up to $a+\sigma$ There is on the average share equal 0,6287 (of 68,27 %) of all weight of made repeated measurements. In borders $(a-2\sigma, a+2\sigma)$ is placed on the average 0,9545 (95,45%) all measurements, and on a site $(a-3\sigma, a+3\sigma)$ – already 0,9973 (99,73%), so for «three sigma» limits leaves only 0,0027 (0,27%) all number of measurements, i.e. their insignificant share [1].

«Classical» interval arithmetics assumes, that all meanings of an interval are equiprobable [2,3]. Therefore all results received with its help, cover every possible meanings and are «supersufficient».

In the given work new interval arithmetics which is taking into account non-uniformity of distribution of meanings inside an interval is offered.

Let's enter formal concept of an interval a in the following kind:

$$a = \left[\bar{a} - e_a, \bar{a} + e_a\right] = \left(\bar{a}, e_a\right); \tag{1}$$

Where $\bar{a}$ – middle of an interval (or mathematical expectation), ε_a – width of an interval (or дисперсия). Let's designate set of all such intervals as $I_{вер}(R)$.

Let a, b, c – intervals from $I_{вер}(R)$. Let's enter the following interval arithmetic operations (in the assumption, that the intervals are the independent normally distributed sizes):

1. Addition of two intervals $a,\ b \in I_{вер}(R)$: $a,\ b \in I_{вер}(R)$,

$$\bar{c} = \bar{a} + \bar{b};\ \varepsilon_c = \sqrt{\varepsilon_a^2 + \varepsilon_b^2}; \tag{2}$$

2. Subtraction of two intervals $a,\ b \in I_{вер}(R)$: $c = a - b$,

$$\bar{c} = \bar{a} - \bar{b};\ \varepsilon_c = \sqrt{\varepsilon_a^2 + \varepsilon_b^2}; \tag{3}$$

3. Multiplication of two intervals $a,\ b \in I_{вер}(R)$: $c = a * b$,

$$\bar{c} = \bar{a} \cdot \bar{b};\quad \varepsilon_c = \sqrt{\bar{a}^2 \cdot \varepsilon_b^2 + \bar{b}^2 \cdot \varepsilon_a^2}; \tag{4}$$

4. Return interval $a \in I_{вер}(R)$: $c = \frac{1}{a}$;

$$\bar{c} = \frac{1}{\bar{a}};\ \varepsilon_c = \frac{\varepsilon_a}{\bar{a}^2}; \tag{5}$$

5. Division of two intervals $a,\ b \in I_{вер}(R)$: $c = \frac{1}{a}$;

$$\bar{c} = \frac{\bar{a}}{\bar{b}};\ \varepsilon_c = \sqrt{\frac{\bar{a}^2 \cdot \varepsilon_b^2}{\bar{b}^4} + \frac{\varepsilon_a^2}{\bar{b}^2}}. \tag{6}$$

Let x – independently distributed (independently allocated) interval characterized by mathematical expectation (by middle of an interval) $Mx = v$ and дисперсией (in width of an interval) $Dx = \sigma^2$.

Function $U = f(x)$ let is given interval meaning, which argument is in turn interval. Meaning of this function will be an interval, which we shall designate $U = \left(\bar{u}, \varepsilon_u\right)$, determined on the formula:

$$\bar{u} = f(v),\ \varepsilon_u = \left|\left(\frac{\partial f}{\partial x}\right)\right|_v \sigma. \qquad (7)$$

Let $x_1, x_2, ..., x_n$ – independently distributed intervals characterized by mathematical expectation (by middle of an interval) $Mx_1 = v_1, Mx_2 = v_2, ..., Mx_n = v_n$ and дисперсией (in width of an interval) $Dx_1 = \sigma_1^2, Dx_2 = \sigma_2^2, ..., Dx_n = \sigma_n^2$. Let's designate $V = (v_1, v_2, ..., v_n)$.

Function $U = f(x_1, x_2, ..., x_n)$ let is given interval meaning, which arguments are in turn intervals. Meaning of this function will be an interval, which we shall designate $U = \left(\bar{u}, \varepsilon_u\right)$, determined on the formula:

$$\bar{u} = f(v_1, v_2, ..., v_n), \varepsilon_u = \sqrt{\left(\frac{\partial f}{\partial x_1}\right)_1^2 \cdot \sigma_1^2 + \left(\frac{\partial f}{\partial x_2}\right)_2^2 \cdot \sigma_2^2 + ... + \left(\frac{\partial f}{\partial x_n}\right)_n^2 \cdot \sigma_n^2}. \qquad (8)$$

Let's compare entered interval arithmetics with «classical» on examples.

1. Operation of addition (subtraction). Intervals $a = [1.750, 2.250] = (2.0, 0.250)$ and $b = [3.250, 4.750] = (4.0, 0.750)$ let are given.

Then for new interval mathematics we shall receive the following results:

$c = a + b = [5.209, 6.791] = (6.0, 0.791)$,

$c = a - b = [-2.791, -1.209] = (-2.0, 0.791)$,

Similarly for classical interval mathematics:

$c = a + b = [5.0, 7.0] = (6.0, 1.0)$,

$c = a - b = [-3.0, -1.0] = (-2.0, 1.0)$.

Thus, the centres of both intervals coincide, however width of the entered interval (1.582) is less than width of a «classical» interval (2.0).

2. Operation of multiplication. Intervals $a = [1.750, 2.250] = (2.0, 0.250)$ and $b = [3.250, 4.750] = (4.0, 0.750)$ let are given.

Then for new interval mathematics we shall receive the following results:

$c = a * b = [6.197, 9.803] = (8.0, 1.803)$

Similarly for classical interval mathematics:

$c = a * b = [5.688, 10.688] = (8.188, 2.50)$,

Thus, width of the entered interval (1.803) is less than width of a «classical» interval (2.50) and the centre of a «classical» interval is displaced on size 0.188.

3. Operation of calculation of a return interval.

In «classical» calculation of a return interval it is supposed, that $0 \notin a_k$. In the entered definition of a return interval it is supposed, that $0 \in a$, if only $\bar{a} \neq 0$.

Example. Intervals $a = [1.750, 2.250] = (2.0, 0.250)$ and

$b = [-0.250, 1.250] = (0.5, 0.750)$ let are given.

Then for new interval mathematics we shall receive the following results:

$$c = \frac{1}{a} = [0.438, 0.563] = (0.50, 0.063),\ c = \frac{1}{b} = [-1.00, 5.00] = (2.0, 3.0).$$

Similarly for classical interval mathematics:

$$c = \frac{1}{a} = [0.444, 0.571] = (0.508, 0.063),\ c = \frac{1}{b}, \text{ does not exist.}$$

Thus, the centre of a «classical» interval $\frac{1}{a}$ is displaced on size 0.08.

4. Operation division of two intervals.

For example, for intervals $c = \frac{a}{a}$ and $c_k = \frac{a_k}{a_k}$ we shall receive

$$\bar{c} = 1,\ \varepsilon_c = \frac{\sqrt{2} \cdot \varepsilon_a}{|\bar{a}|},\ \bar{c}_k = \frac{\bar{a}_k^2 + \delta_a^2}{\bar{a}_k^2 - \delta_a^2},\ \delta_c = \frac{2 \cdot \delta_a \cdot |\bar{a}_k|}{\bar{a}_k^2 - \delta_a^2}.$$

The centre of the entered interval is equal 1. The centre of a «classical» interval is displaced rather 1, though contains 1.

At $\varepsilon_a = \frac{\bar{a}}{2}$ and $\delta_a = \frac{\bar{a}_k}{2}$ we shall receive

$$c = \left(1, \frac{1}{\sqrt{2}}\right) = \left[1 - \frac{1}{\sqrt{2}}, 1 + \frac{1}{\sqrt{2}}\right] = [0.293, 1.707],$$

$$c_k = [1.666 - 1.33, 1.666 + 1.33] = [0.33, 2.999].$$

Thus, width of the entered interval is equal 1.414 and less than width of a «classical» interval, which is equal 2.666. Besides the centre of a «classical» interval is displaced from 1 on size 0.666.

Example. Intervals $a=[1.750,2.250]=(2.0,0.250)$ and $b=[3.250,4.750]=(4.0,0.750)$ let are given.

Then for new interval mathematics we shall receive the following results:

$$c=\frac{a}{b}=[0.387,0.613]=(0.50,0.113)$$

Similarly for classical interval mathematics:

$$c=\frac{a}{b}=[0.368,0.692]=(0.530,0.162).$$

Thus, width of the entered interval (0.113) is less than width of a «classical» interval (0.162) and the centre of a «classical» interval is displaced on size 0.030. Example. Intervals $a=[1.750,2.250]=(2.0,0.250)$ and

$b=[-0.250,1.250]=(0.5,0.750)$ let are given.

Then for new interval mathematics we shall receive the following results: $c=\frac{a}{b}=[-2.021,10.021]=(4.00,6.021)$, is similar for classical interval mathematics: $c=\frac{a}{b}$, does not exist.

5. Operation of calculation of interval functions

Let $f(x)$ – interval meaning function of interval argument.

Then for $c=f(a)$ we shall receive: for new interval mathematics

$$\bar{c}=f(\bar{a}),\ \varepsilon_c=\left|\left(\frac{\partial f}{\partial x}\right)\right|_{\bar{a}}\cdot\varepsilon_a.\ (9)$$

For classical interval mathematics

$$\overline{c_k}=\frac{(f_{\max}+f_{\min})}{2},\qquad \delta_c=\frac{(f_{\max}-f_{\min})}{2},\ \text{where}$$

$$f_{\min}=\min_{y\in a}f(y),\quad f_{\max}=\max_{y\in a}f(y) \qquad (10)$$

As it is visible from the above mentioned formulas for differentially of functions of calculation of meanings of function on new interval mathematics (9) more structurally in view of ending of carried out arithmetic operations. At the same time calculations under the formulas (10) require the decision two optimization of tasks, for the decision generally is necessary for each of which realization of iterative calculations. Thus there are problems of convergence of iterative process and choice of an index point.

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Лавриненко Н.М.

Доктор физико-математических наук, профессор

Томашевский П.В.

Донецкий Национальный Университет Экономики и Торговли имени Михаила Туган-Барановского

DEMAND FUNCTION. THE COEFFICIENT OF ELASTICITY

As a result of solving problems of optimal choice, it is possible to trace the relationship between changes in systems of price and income groups of consumers, on the one hand, and the demand of this group of goods and services on the other, as well as construct a function of optimal demand.

In a sufficiently general form, the optimal demand is expressed by functions of the form – $D_j = x_j(I; p_1,...,p_j,...,p_n)$, where $(j = 1,...,n)$.

In some cases, the optimal demand functions are particularly simple form. Thus, if the utility function has a logarithmic form, then the optimal demand expressed by the formula $D_j = x_j^0 + \frac{c_j(I - I_0)}{p_j \sum_{j=1}^{n} c_j}$, $(j = 1,...,n)$ where $I_0 = \sum_{j=1}^{n} p_j x_j^0$.

The exact form of the demand function is determined by statistical processing of results of special observations for revenues and expenditures for various social groups. A study of the demand functions are typically installed some classification features of goods.

If for some goods, the condition $\frac{\partial D_j}{\partial p_j} < 0$, they are called normal goods, as the demand for it decreases with the increase of its price. However, there are goods for which demand increases, despite the price increase. This paradoxical situation arises when an increase in price an ineffective product (e.g., potatoes), a group of consumers with low incomes simply cannot acquire more high-calorie products (meat) and is forced to compensate for the lack of calories enhanced buying potatoes. Goods for which the inequality $\frac{\partial D_j}{\partial p_j} > 0$, they are called anomalous or goods Giffin. With a fixed income, and for practical purposes for normal products are used as a rule, the demand function of two types: