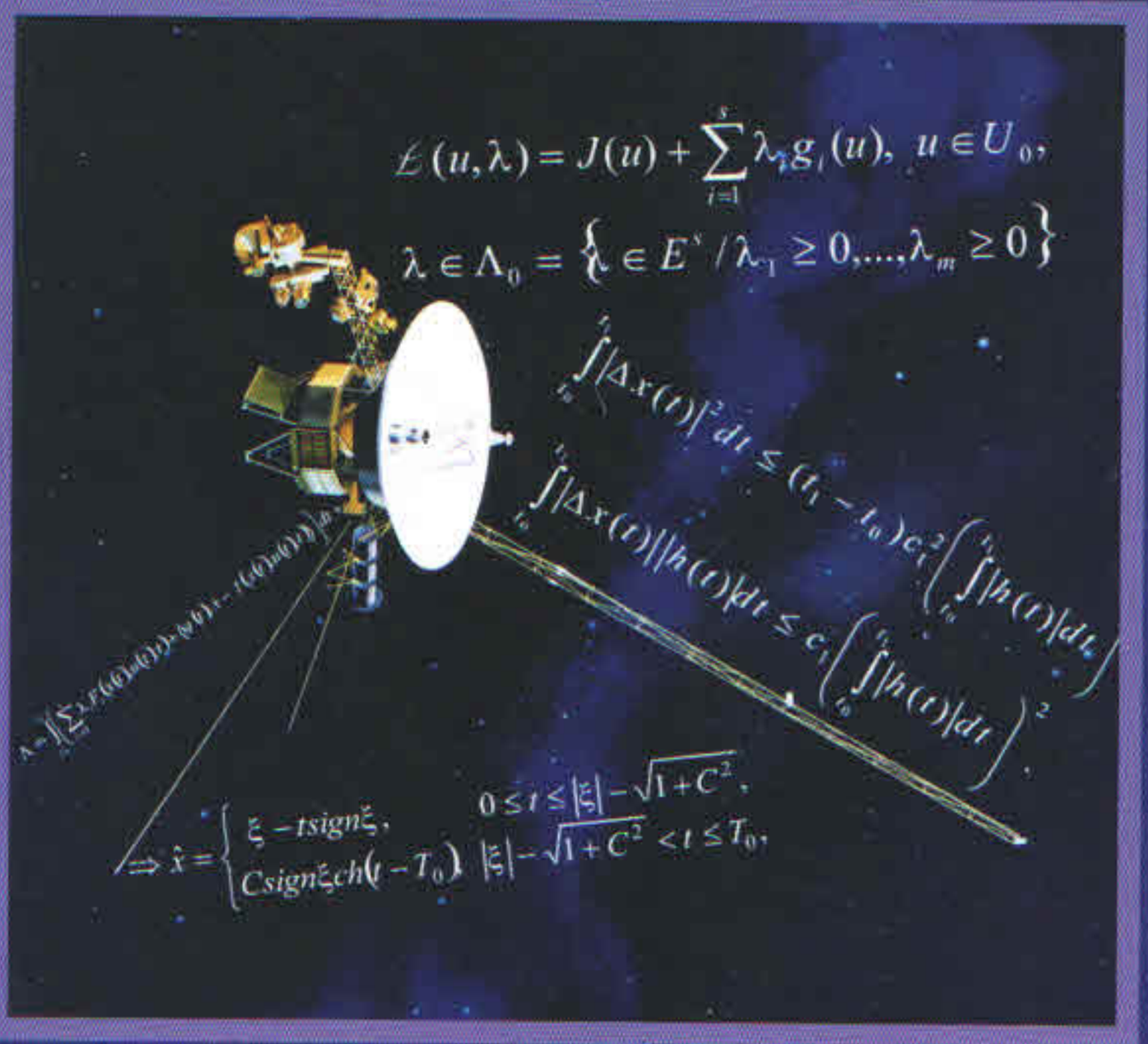
S. A. Aisagaliev,
Zh. Kh. Zhunussova

# OPTIMAL CONTROL

Textbook

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Some theoretical foundations of optimal control problem are expounded in the textbook: methods of variation calculus, maximum principle, dynamical programming for solving of topical problems of economic macromodels.

The tasks for independent work with solving of concrete examples, brief theory and solution algorithms of the problems, term tasks on sections of optimal control and variation calculus are presented in the appendix.

It is intended as a textbook for students of the high schools training on specialties "mathematics", "mechanics", "economic cybernetics" and "informatics". It will be useful for the post-graduate students and scientific workers of the economic, mathematical and naturally-technical specialties.

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Textbook

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