Offsetting, relations and blending with perturbation functions

Sergey I. Vyatkin*[a], Alexander N. Romanyuk[b], Zenon Y. Gotra[c], Oksana V. Romanyuk[b], Waldemar Wójcik[d], Ryszard Romaniuk[e], Yedilkhan Amirgaliyev[f], Azat Assembay[g]

[a]Institute of Automation And Electrometry, Siberian Branch of The Russian Academy of Sciences, Novosibirsk, Russia; [b]Vinnytsia National Technical University, Vinnytsia, Ukraine; [c]Lviv Polytechnic National University, Ukraine; [d]Lublin University of Technology, Lublin, Poland; [e]Warsaw University of Technology, Warsaw, Poland; [f]Institute of Information And Computational Technologies, Almaty, Kazakhstan; [g]Kazakh National Research Technical University After K. I. Satpaev, Almaty, Kazakhstan

ABSTRACT

Transformations of geometric objects are described for offsetting. Free forms based on the perturbation functions have an advantage of spline representation of surfaces, that is, a high degree of smoothness, and an advantage of arbitrary form for a small number of perturbation functions.

Key words: Perturbation Functions, Offsetting, Relations, Collision Detection, Blending

1. INTRODUCTION

Several representations of geometric objects are currently used in computer graphics. Each of the objects, according to its properties, is used in different fields, beginning from 3-D simulation and CAD systems up to real-time visualization systems.

The functional representation describes most accurately the object geometry and has the smallest size of the required data. Procedures of functional representation demonstrate compact and flexible representation of surfaces and objects that are the results of logical operations on volumes.

The paper describes free forms based on the analytical perturbation functions with offsetting and blending operations. It is shown that an adequate surface smoothness and a compact object description can be achieved using a limited number of base and perturbation functions. We suggest expanding the notion of primitives and making it possible to process them by easy and effective method without approximation by polygons. A method to display curved surfaces allows obtaining picture quality which cannot be achieved by the traditional means (even with great number of polygons) and is described below. New techniques for specifying free forms without their approximation by polygons or spline-patches are considered.

2. GEOMETRIC OPERATIONS

Two major types of elements of the set of geometric objects are simple geometric objects and complex geometric objects. A complex geometric object is a result of operations on simple geometric objects [1].

The set of geometric operations Φ is expressed mathematically in the following form:

$$\Phi_i: M^1 + M^2 + \ldots + M^n \rightarrow M, \tag{1}$$

where n is the number of operation operand.

* E-mail: ran12345@mail.ru

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Let the object G_1 be defined as $f_1(X) \geq 0$. The unary operation (n =1) of the object

G_1 means operation G_2=$\Phi_j(G_1)$ with the definition

$$f_2=\psi(f_1(\mathbf{X})) \geq 0, \quad (2)$$

where ψ is a continuous real function of one variable.

3. OFFSETTING OPERATIONS

Offsetting operations in geometric modeling generate expanded or contracted versions of an original object. These operations have been applied: to plan collision-free trajectories of a computer-controlled manipulator; to describe cutter path generation for numerically controlled machining; in the algorithms for computing volume, moments of inertia and other integral properties of solid; in design of mold profiles; to generate fillets and rounds in modeled parts, such as constant-radius offsetting [2, 3].

There are several mathematical definitions for offsetting. Normal offsetting defines an offset curve or surface as a set of points of an initial curve displaced along the normal direction by a given distance. Constant-radius offset is as a set of points with the fixed minimal distance to an initial point set. This operation is a Minkowski or vector sum of an initial object and a disk or ball. It can be thought also as sweeping by a disk with its center moving throughout an initial object. For an initial parametric curve a distance-like implicit function can be defined with an offset curve as a level set of this function.

Iso-valued offsetting is

$$F = f(\mathbf{X}) + C, \quad (3)$$

Negative constant C defines negative (internal) offset, positive C defines positive (external) offset.

Offsetting along the normal are

$F = f(\mathbf{X}+D\mathbf{N})$, for positive (external) and

$F = f(\mathbf{X}-D\mathbf{N})$, for negative (internal) offsetting,

where D is the given distance value, **N** is the gradient vector of function f in the point with **X** coordinates.

This definition differs from standard one. The gradient is calculated in a given point.

Positive constant-radius offsetting with distance D: define an initial object by $f(x, y, z) \geq 0$; for any given point (x, y, z) defined the sphere of radius D with the center in the point; find maximal value f_0 of $f(x', y', z')$ for points (x', y', z') belonging to the sphere's surface; assign $f(x, y, z) = f_0$.

The offsetting operation with perturbation functions[1] was implemented by transformation of perturbation function coefficients. Thus, one can create an enlarged or diminished copy of the initial object, i.e., makes positive or negative offsetting, respectively. For example, solid beats can be simulated. Let the initial object be defined by the function $f(X)>0$, then in the case of this operation, the obtained solid will be described by the function $F=f(X)+C$, where $C<0$ determines the negative offsetting (compression) and $C>0$ determines the positive offsetting (extension). Otherwise, adding together the positive or negative constant and the free term of the perturbation function yields extension or compression of the whole object

4. BLENDING OPERATION

The operation joining several surfaces in a complex object with a smooth surface is called blending [4]. The main difficulties and requirements to blending are: tangency of a blend surface with the base surfaces; easy intuitive control of the blending surface shape; necessity to perform for blended objects all the computations possible for unblended objects including set-theoretic operations; blend interference or ability to blend on blends and as the particular case complex vertices or corners blending; at least C^1 continuous blending function in the entire domain of definition; blending definition of basic set-theoretic operations, such as intersection, union and subtraction; single edge blending or localizing the blend to a region about intersection curve of two faces; added and subtracted materials blends; the ability to produce constant-radius blending; no restriction of circular cross sections or the requirement of variable-radius blends; exact representation for blends instead of many approximation; automatic clipping of unwanted parts of the blending surface; blending of two non-intersecting surfaces; functional constraints; aesthetic blends constrained by appearance.

An example of 2D method is

Let A=0 represents a line of the form:

$$x_A + y_A + c_A = 0, \tag{4}$$

Starting from four line equations A=0, B=0, P=0, Q=0, we can combine them into a curve as follows:

$$(1-u)AB + uPQ = 0, \tag{5}$$

If P=0 and Q=0 come together, the equation is

$$(1-u)AB + uP^2 = 0, \tag{6}$$

$$u = \frac{P}{P+Q}. \tag{7}$$

Examples of 3D methods are bellow.

A solid is defined as $f(P) \le 1$[5].

Intersection is

$$I(f_1, f_2 ..., f_n) = (f_1^p + f_2^p + ... + f_n^p)^{1/p}, \tag{8}$$

Union is

$$U(f_1, f_2 ..., f_n) = (f_1^{-p} + f_2^{-p} + ... + f_n^{-p})^{-1/p} \tag{9}$$

p is a positive real number.

$$\lim_{p \to \infty} I(f_1, f_2 ..., f_n) = \min(f_1, f_2, ... f_n) \tag{10}$$

$$\lim_{p \to \infty} U(f_1, f_2 ..., f_n) = \max(f_1, f_2, ... f_n) \tag{11}$$

The blend surface is given as [6]:

$$1 - (1 - \frac{A}{r_A})^w - (1 - \frac{B}{r_B})^w = 0, \tag{12}$$

where r_A and r_B determine the range of the blend: when $r_A = A$, the blend surface becomes B=0.

The exponent w is called "thumb weight" and controls the nearness of the blend to the surfaces. Larger values of w give a super elliptic cross-section which approximates sharp intersection more and more.

Another example is

Substitute (A+M-r) for P in the Liming formula:

$$(1-u)A^{э}B^{э}+u(A^{'}+B^{'}-r)^{2}=0, \tag{13}$$

where $A^{э}=c_{A}A, \quad B^{э}=c_{B}B$.

Constant-radius blends can be implemented with the composition of constant-radius offsets.

We propose describing complex geometric objects by specifying the function of deviation (an implicit second-order function) from the base surfaces. The freeform is a composition of the base surface and the perturbation functions

$$F'(x,y,z)=F(x,y,z)+\sum_{i=1}^{N}R_{i}(x,y,z) \tag{14}$$

where the perturbation function $R(x, y, z)$ is found as follows

$$R_{i}(x,y,z)=\begin{cases}Q_{i}^{3}(x,y,z), if\ Q_{i}(x,y,z)\geq 0\\ 0, if\ Q_{i}(x,y,z)<0\end{cases} \tag{15}$$

Herein, $Q(x, y, z)$ is the perturbing quadric.

Since $\max[Q + R] \leq \max[Q] + \max[R]$, for estimating the maximum Q on some interval we have to calculate the maximum perturbation function on the same interval. The obtained surfaces are smooth, and creation of complex surface forms requires few perturbation functions.

Thus, the problem of object construction reduces to the problem of quadric surface deformation in a desired manner rather than to approximation by primitives (polygons or patches represented by B-spline surfaces).

5. RELATIONS FOR PERTURBATION FUNCTIONS

We have investigated geometric operations on functionally defined objects on the basis of the perturbation functions.

5.1 Inclusion relation

This relation is described as $G_{2} \subset G_{1}$ and means that the object G_{2} is a subset of G_{1}. If G_{2} is a point P, the relation can be described by the following bivalued predicate:

$$S_{2}(P,G_{1})=\begin{cases}0, if\ f_{1}(x,y,z)<0\ for\ P\notin G_{1}\\ 1, if\ f_{1_{i}}(x,y,z)\geq 0\ for\ P\in G_{1}\end{cases} \tag{16}$$

5.2 Point membership relation

Let iG_{1} be the interior of G_{1} and bG_{1} be the boundary of G_{1}. The point membership relation is described by the 3-valued predicate:

$$S_{3}(P,G_{1})=\begin{cases}0, if\ f_{1}(x,y,z)<0\ for\ P\notin G_{1}\\ 1, if\ f_{1_{i}}(x,y,z)=0\ for\ P\in bG_{1}\\ 2, if\ f_{1}(x,y,z)>0\ for\ P\in iG_{1}\end{cases} \tag{17}$$

This predicate can be correctly evaluated for G_1 without internal zeroes, internal points with $f_1(x,y,z) = 0$.

5.3 Set theoretic operations

Let the objects G_1 and G_2 be defined as $f_1(X) \geq 0$ and $f_2(X) \geq 0$. The binary operation (n=2) of the objects G_1 and G_2 means operation $G_3=\Phi_i(G_1,G_2)$ with the definition

$f_3=\psi(f_1(X),f_2(X)) \geq 0$,

where ψ is the continuous real function of two variables. Let us dwell on the binary operations: set-theoretic operations.

For function-based objects on the bases of perturbation functions we propose the following. To create a complex scene, one should describe in it a certain number of primitives necessary for a concrete task. The rendered object with which the rendering algorithm interacts by means of query represents the whole 3D scene. Hence, the geometric model should allow designing of objects and their compositions of infinite complexity. This is primarily achieved by means of Boolean operations of uniting and intersection.

Let the objects $G_1 : f_1(x,y,z) \geq 0$ and $G_2 : f_2(x,y,z) \geq 0$, then $G_3 = G_1 \cup G_2$ is the union operation, $G_3 = G_1 \cap G_2$ is the intersection operation, $G_3 = G_1 \setminus G_2$ is the subtraction operation and

$$f_3 = \psi\ (f_1(x,y,z),\ f_2(x,y,z)) . \quad (18)$$

5.4 Intersection relation

The intersection or interference and collision relation is defined by the bivalued predicate:

$$S_C(G1,G2) = \begin{cases} 0, if\ G1 \cap G2 = \varnothing \\ 1, if\ G1 \cap G2 \neq \varnothing \end{cases} \quad (19)$$

$$G_1 : f_1(x,y,z) \geq 0\ ,\ G_2 : f_2(x,y,z) \geq 0 \quad (20)$$

A function $f_3(x,y,z) = f_1(x,y,z)\ \&\ f_2(x,y,z)$ defining the result of the intersection can be used to evaluate S_c. It can be stated that $S_c = 0$ if $f_3(x,y,z) < 0$ for any point of E^3.

6. COLLISION DETECTION

An example of the relations is collision detection for objects. The binary relation is a set of the set $M^2=M\times M$. It may be defined as $S_i: M\times M \rightarrow I$.

Collision detection is a complicated problem solved in various computer programs. This means that for each animation frame, one should test whether any two or more objects collided.

The ideal case is collision detection of any complexity between two arbitrary objects in the minimal time. Since the control of collisions between all pairs of objects is a resource-consuming process, such tests are usually done only for part of objects. The detection algorithm can be simplified prior to testing the presence of the given point (belonging to one of the objects), e.g., inside the cube confining the second object. The problem of simulating the behavior of interacting bodies having irregular shape arises in some applications such as dynamics of body collisions and celestial mechanics, molecular dynamics, graphics simulations for the problem of nano-assembly automation and its application in medicine using collective robotics, computer games and haptic interactions.

Particularly in calculating motions of many objects that move under changing constraints and frequently make collisions, one of the key issues of dynamic simulation methods is calculation of collision impulse between rigid bodies. A fast algorithm for calculating contact force with friction by formulating the relation between force and relative acceleration as a linear complementary problem was equally demonstrated and this model was based on solving the linear complementary problem.

We propose the collision detection algorithm by means of recursive object space subdivision.

After calculation of the intersection,

$$S_c(G_1,G_2)=\begin{cases}0, if\ G_1 \cap G_2 = \varnothing \\ 1, if\ G_1 \cap G_2 \neq \varnothing\end{cases} \tag{21}$$

i.e., application of the Boolean operation of intersection, the search for the contact point of collided objects is done by means of recursive subdivision of the object (model) space. Hence, it is sufficient to find at least one point (or more) belonging to intersection. Let we deal with the object-intersection that has the property of answering the request on intersection with a rectangular parallelepiped or a bar. The negative answer guarantees that the object is not intersected and has no common points belonging to the intersection is done by recursive subdivision of the space inside the cube defined by boundaries of ±1 along each coordinate. The center of the cube matches the origin of the model coordinate system M whereas the plane Z= −1 coincides with the screen plane. At the first step of recursion, the initial cube is subdivided into four smaller subcubes in the screen plane. At the stage of subdivision of space along the quaternary tree, 2-times compression and transfer by ±1 along two coordinates are performed. Assume, that domain of point search is a cube in which embed our object-intersection.

Then recursive subdivision of the domain applied: domain cuts by 2 planes, that perpendicular to the screen plane XY, into 4 bars. For each bar intersection test are executed. If the object intersects with given bar, then bar subdivides further. Otherwise, we exclude bar from subdivision. This corresponds with exclusion of the square areas in the screen, on which given bar (and therefore, object- intersection) are projected.

If in the equation of quadric Q(x, y, z) = 0 (4) the values of the variables x, y, z vary within the length [-1, 1], then

max[|Q(x, y, z) – A44|] maxF = |A11|+|A22|+|A33|+|A12|+|A13|+|A23|+|A14| +|A24|+|A34|

We should note that if |A44 < max[|Q(x, y, z) – A44|] < max F, then, probably, a point M0 = (x0, y0, z0) (-1 < x0, y0, z0 < 1) exists such that Q(x0, y0, z0) = 0. If maxF < |A44|, then such points do not knowingly exist, and the sign of the coefficient A44 distinguishes location of the bar inside or outside with respect to the quadric surface Q=0 (if A44 ≥ 0, then the subbar is inside the quadric). Using results of this test, we perform subdivision of subbars that fall within the quadric completely or, probably, partially, and the knowingly external subbars are eliminated from processing. A test for intersection of subbars with freeform is somewhat different. For the basic quadric the test for intersection looks as follows:

if ((A44+R) < 0) && (|A11|+|A22|+|A33|+|A12|+ |A13|+|A23|+|A14|+|A24|+|A34| < -(A44+R)) then the subbar is outside.

Here R is the maximum perturbation function on the current interval; Aij are the coefficients of quadratic function. The following test is performed for the perturbation function:

if (|A11|+|A22|+|A33|+|A12|+|A13|+|A23|+ |A14|+|A24|+|A34| < |A44|)

then the subbar is outside of the range of definition of perturbation,

where Aij are the coefficients of the quadratic perturbation function and a value of R is additionally calculated and added to the basic function.

7. CONCLUSION

In this paper, we have considered geometric operations on function-based objects[7-15]. New methods for program implementation of complex geometric offsetting and blending operations on the basis of the perturbation functions have been proposed. Thus, the proposed method for describing objects of 3D scenes by the base surfaces and the perturbation functions has a more compact description than the known methods for defining the function-based objects.

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