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Theorem 1 (Tarski-Vaught test). *[3] Let $\mathbb{M}$ be a substructure of $\mathbb{N}$. Then* $\mathbb{M} \prec \mathbb{N} \iff \forall \psi(x, \bar{a}),\ \bar{a} \in \mathbb{M}\ \ [\mathbb{N} \models \exists x\ \psi(x, \bar{a}) \Rightarrow \exists b \in \mathbb{M},\ \ \mathbb{N} \models \psi(b, \bar{a})]$.

Theorem 2 (B.Baizhanov). *[4-5] Any expansion of a model of weakly o-minimal theory by unary convex predicate has a weakly o-minimal theory.*

Theorem 3. *Let $\mathbb{M} :=< M; \Sigma >$ be a model of a weakly o-minimal, model complete theory and $\mathbb{M}^+ :=< M; \Sigma^+ >$ be an expansion of $\mathbb{M}$ by unary convex predicate P^1 from $\Sigma^+ := \Sigma \cup \{P^1\}$. Then elementary theory $T^+ = Th(\mathbb{M}^+)$ of $\mathbb{M}^+$ is model complete.*

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On question on definability of non-orthogonal types

Definition 1. [1] Let A be a set in theory T, $p(\bar{x}), q(\bar{y}) \in S(A)$. We say that p is weakly orthogonal to q ($p \perp^w q$) if $p(\bar{x}) \cup q(\bar{y})$ is complete type over A.

Definition 2. Let A be a set in theory T, $p, q \in S(A)$. We say that p is not almost orthogonal to q ($p \not\perp^a q$) if there exists A-formula $\varphi(\bar{x}, \bar{y})$ for some $\bar{a} \in p(M)$ such that $\emptyset \neq \varphi(M, \bar{a}) \subset q(M)$.

Note that for finite A in small theory the notion of non-almost orthogonality of two types coincides with the notion of Rudin-Keisler preorder.

We have an example for two types $p, q \in S_1(A)$ such that $p \not\perp^w q$ when p is isolated and q is non-isolated [2].

Definition 3. Type p is called *definable* , if there exists a formula $\phi(\bar{x}, \bar{y})$ for every formula $\theta_\phi(\bar{x}, \bar{a}_\phi) \in p$ this condition $\forall \bar{x}(\theta_\phi(\bar{x}, \bar{a}_\phi) \rightarrow \phi(\bar{x}, \bar{y}))$ is true.

Note that T is stable if, and only if for any set $\mathbb{A}$ every type over an $\mathbb{A}$ is definable [1].

Definition 4. Let Γ be a non-isolated consistent set of A-definable formulas. We say that Γ is a *quasimodel set* if, for every formula $\theta \in \Gamma$, there exists $\bar{a} \in A$ such that $N \models \theta(\bar{a})$. If we assume that Γ is closed under formation of finite conjunction, the Γ can be extended to a *quasimodel types* over an A.

Fact 1. If we have non-definable type q then there exists a quasimodel type p such that $p \not\perp^w q$.

Fact 2: If there are isolated type p and non-isolated type q such that $p \not\perp^w q$, then $p \perp^a q$.

Theorem: If $p \not\perp^a q$ and p is definable then q is definable.

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Small theories with a definable linear order

We say that a complete countable theory T is *small* if $|\bigcup_{n<\omega} S_n(T)| = \omega$, where $S_n(T)$ is the set of all n-types of T over $\emptyset$. A complete countable theory T has *a few number of countable models* if the number of countable non-isomorphic models $I(T, \omega)$ is less than 2^ω. It is clear that a theory with few countable models is small.

Fact 1 Let T be a small ordered theory, A be a finite subset of a model of T. Then the linear order on the equivalence classes of A-definable 1-formulas with respect to formulas' right (left) borders is a discrete order.

Theorem 1 For any theory T of a finite signature there exists a complete theory T' with a definable linear order such that $I(T, \omega) = I(T', \omega)$.

Let A be a finite subset of a saturated model N and $H(x), \Theta(x)$ be A-definable 1-formulas such that $H(N) \subset \Theta(N)$. Denote

$E_{H,\Theta}(x, y) := H(x) \wedge H(y) \wedge (x < y \rightarrow \forall z((x < z < y \wedge \Theta(z)) \rightarrow H(z))) \wedge (y < x \rightarrow \forall z((y < z < x \wedge \Theta(z)) \rightarrow H(z)))$.

$E_{H,\Theta}(x, y)$ is an A-definable equivalence relation on $H(N)$ such that any $E_{H,\Theta}$-class is a convex subset of $\Theta(N)$. This equivalence is said to be a *convex equivalence of $H(x)$ in* $\Theta(x)$. On the set of all $E_{H,\Theta}$-classes $\{E_{H,\Theta}(N, \alpha) | \alpha \in H(N)\}$ there is A-definable linear order $<^c$ such that for any $\alpha, \beta \in H(N)$, $E(N, \alpha) \neq E(N, \beta)$ the following holds: $E(N, \alpha) <^c E_{H,\Theta}(N, \beta)$ if and only if $\forall \gamma \in E(N, \alpha), \forall \delta \in E(N, \beta)$ we have $N \models \gamma < \delta$.

Recall, that an ordered structure M is *weakly o-minimal* if for any M-definable 1-formula $H(x)$ the structure $\langle \{E_{H,x=x}(M, a) | a \in H(M); <^c\} \rangle$ is finite and M is *o-minimal* if this structure is finite and every such convex $E_{H,x=x}$-class is an interval (opened, closed or semi-closed) or a singleton.

Further, for any $A \subset_{\text{finite}} N$, for any A-definable 1-formulas $H(x)$ and $\Theta(x)$ ($H(N) \subset \Theta(N)$) we consider the properties of a linear order of

$$\langle \{E_{H,\Theta}(N,\alpha)/\alpha \in H(N)\}; =, <^c \rangle.$$

We say that an ordered theory T has the *property of finiteness of discrete chains of convex equivalences* (*FDCCE*) if for every two 1-formulas $H(x)$ and $\Theta(x)$ such that $H(N) \subset \Theta(N)$, for any k $(1 < k < \omega)$ every discrete chain of convex $E_{H,\Theta}$-classes is finite.

Let $\phi_1(x)$ and $\phi_2(x)$ be two A-definable 1-formulas. Then for every k $(1 < k < \omega)$, for every i $(0 < i < k+1)$ define an A-definable 1-formula $\phi_1(x) \lhd_k^i \phi_2(x)$ by induction:

$$\phi_1(x) \lhd_k^1 \phi_2(x) := D_0^k[\phi_1(x) \wedge \phi(x), \phi_2(x)](x);$$

$0 < i < k+1$, $\phi_1(x) \lhd_k^i \phi_2(x) := \exists x_1, \ldots, x_k \, (\wedge_{j<k}((\phi_1 \wedge \phi_2)(x_j) \wedge \neg E_{\phi_1 \wedge \phi_2, \phi_2}(x_j, x_{j+1}) \wedge x_j < x_{j+1}) \wedge D_0^k[\phi_\wedge \phi_2, \phi_2](x_1) \wedge D_1^k[\phi_\wedge \phi_2, \phi_2](x_k) \wedge E_{\phi_1 \wedge \phi_2, \phi_2}(x, x_i) \wedge \forall y((\phi_1(y) \wedge \phi_2(y)) \to \vee_{0<j<k+1} y = x_j);$

$$\phi_1(x) \lhd_k^k \phi_2(x) := D_1^k[\phi_1(x) \wedge \phi(x), \phi_2(x)](x).$$

The set of A-definable 1-formulas $C \subset F_1(A)$ is called a $BH - algebra$ if it is closed under the following logical operations: $\wedge, \neg, \vee, \lhd_k^i$ $(0 < i < k, 1 < k < \omega)$.

Theorem 2 Let T be a small ordered theory with FDCCE, A be a finite subset of a countable saturated model N of the theory T. Then for every finite set of A-definable 1-formulas $\{\phi_1(x), \ldots, \phi_n(x)\}$, $n < \omega$ the BH-algebra generated by this set is finite.

Theorem 3 Let T be a small theory of a pure order. Then T is ω-categorical if and only if T has FDCCE.

Corollary 1 Let T be an ω-categorical theory of a pure order. Then T is finitely axiomatizable.

Corollary 2 Let T be a non-ω-categorical small theory of a pure order. Then there is $\emptyset$-definable 1-formula $\phi(x)$ such that for some elements $\alpha, \beta \in \phi(N)$ $(\alpha < \beta)$, $(\alpha, \beta) \cap \phi(N)$ is an infinite discrete chain.

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Countable sets in small theory

For any model $\mathfrak{M}$ of theory T, $\mathfrak{D}(\mathfrak{M})$ is called a *finite diagram* of $\mathfrak{M}$, which is the set of all types realizing in this model.

Theorem 1. For any countable subset $\mathbb{A}$ of saturated model of small theory there is a model $\mathfrak{M}$ containing $\mathbb{A}$ minimal with finite diagram. In other words, for any $\mathfrak{N}$ such that $\mathbb{A} \subset \mathfrak{N}$, we have $\mathfrak{D}(\mathfrak{M}) \subseteq \mathfrak{D}(\mathfrak{N})$.

Earlier in the paper [1] we proved Theorem 2, which is the corollary of the Theorem 1.

Theorem 2. Let $\mathfrak{M}$ be a countable, non-homogeneous model of a small theory T, $\mathfrak{N}$ be a ω_1-saturated model ($\mathfrak{M} \prec \mathfrak{N}$) and $p(\bar{x}) \in S(T)$ be a non-isolated type such that $p(\bar{x})$ is

weakly (almost) orthogonal to any non-isolated type from the finite diagram of $\mathfrak{M}(\mathfrak{D}(\mathfrak{M}))$ and for any $q(x, \bar{y}) \in \mathfrak{D}(\mathfrak{M})$, such that there exists $\bar{\alpha} \in M$, $q(N, \bar{\alpha}) \cap M = \emptyset$, for any $\bar{\beta} \in M$ for $p' \in S(\bar{\alpha}\bar{\beta})$, for $q'(x, \bar{\alpha}, \bar{\beta})$ such that $p \subset p'$, $q \subset q'$ we have $p' \perp^w q'(x, \bar{\alpha}, \bar{\beta})$. Then the following conditions hold:
1) There exists a countable elementary extension $\mathfrak{M} \prec \mathfrak{N}(M, p(\bar{c})) \prec \mathfrak{N}$, such that $\mathfrak{N}(M, p(\bar{c}))$ is also non-homogeneous.
2) For any non-homogeneous countable $\mathfrak{M}' \not\cong \mathfrak{M}$, with equal finite diagrams $\mathfrak{D}(\mathfrak{M}) = \mathfrak{D}(\mathfrak{M}')$ we have $\mathfrak{D}(\mathfrak{N}(M, p(\bar{c}))) = \mathfrak{D}(\mathfrak{N}(M', p(\bar{c})))$ and $\mathfrak{N}(M, p(\bar{c})) \not\cong \mathfrak{N}(M', p(\bar{c}))$.

Proof of this theorem is based on S.V.Sudoplatov's construction in his work [2], where the theorem states that every countable model of a small theory can be represented as an increasing chain of prime models over some finite tuples.

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Invariant theory of relatively free right-symmetric and Novikov algebras

We fix a field K of characteristic 0 and consider nonassociative K-algebras. An algebra A is called *right-symmetric* if it satisfies the polynomial identity

$$(x_1, x_2, x_3) = (x_1, x_3, x_2), \text{ where } (x_1, x_2, x_3) = x_1(x_2x_3) - (x_1x_2)x_3$$

is the associator, i.e., $(a_1, a_2, a_3) = (a_1, a_3, a_2)$ for all $a_1, a_2, a_3 \in A$. A right-symmetric algebra is *Novikov* if it satisfies additionally the polynomial identity of left-commutativity

$$x_1(x_2x_3) = x_2(x_1x_3).$$

We denote by $\mathfrak{R}$ and $\mathfrak{N}$ the varieties of all right-symmetric algebras and all Novikov algebras. For details on the history of right-symmetric and Novikov algebras we refer to the introduction of the paper by Dzhumadil'daev and Löfwall [7]. The origins of the right-symmetric algebras can be traced back till the paper by Cayley in 1857. Later they were studied under different names: Vinberg, Koszul, Gerstenhaber, and pre-Lie algebras. Novikov algebras appeared first in 1985 in the study of equations of hydrodynamics. In a series of papers, see, e.g., [7, 5, 6] Dzhumadil'daev, with coauthors or alone, studied free