$\leq \frac{\varepsilon^n}{\alpha} \|J^n f\|$, $Jf(x) = \frac{d}{dx}\frac{f(x)}{q(x)}$, $J^0 = I$ *- unit operator.*

Keywords: spectral expansion, self - adjoint operators, singularly perturbed differential equations.

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Duality theorems for the noncommutative spaces

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Abstract: In this work, we introduce the noncommutative $H_p(A,l_\infty)$ and $H_p(A,l_1)$ spaces, and give results on duality.

Let M be a finite von Neumann algebra equipped with a normal faithful tracial state τ. Let D be a von Neumann subalgebra of M, and let $\phi: M \to D$ be the unique normal faithful conditional expectation such that finite subdiagonal algebra of M with respect to ϕ is a w* -closed subalgebra A of M satisfying the following conditions:

(i) $A + A^*$ is w *-dense in M;
(ii) ϕ is multiplicative on A;
(iii) $A \cap A^* = D$,

where A^* is the family of all adjoint elements of the element of A. The algebra D is called the diagonal of A (see [1], [3]). We define $H_p(A,l_\infty)$ and $H_p(A,l_1)$ spaces by a similar way as in [3].

Theorem. *Let* $1 \leq p < \infty$ *such that* $1/p + 1/p' = 1$. *Then*

(i) $H_p(A;l_1)^* = L_{p'}(M;l_\infty)/H_{p'}^0(A;l_\infty)^*$ *and* $(L_p(M;l_1)/H_p^0(A;l_1))^* = H_{p'}(A;l_\infty)$

isometrically via the following duality bracket

$$((x_n),(y_n)) = \sum_{n=1}^{\infty} \tau(y_n^* x_n)$$

for $x \in H_p(A;l_1)$ *and* $y \in H_{p'}(A;l_\infty)$.

Keywords: von Neumann algebra, subdiagonal algebras, vector valued noncommutative Hardy spaces, duality theorems.

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