About Problems of Nonlinear Dynamics of the Elastic Rod Elements in Practice of Chisel Works

L. Khajiyeva

Al-Farabi Kazakh National University, KAZAKHSTAN, e-mail: khadle@mail.ru

Abstract The purpose of the paper is modelling of nonlinear vibrations and stability of movement of boring columns at various complicating factors taking into account finite deformations in particular. Movement of boring columns for shallow drilling (up to 500 m) applied in oil-gas extractive industry is considered.

Various types of loading and deformations of rod elements are developed in the paper. Nonlinear models of movement of a compressed-torsioned drill rod within the nonlinear theory of finite deformations of V.V. Novozhilov are constructed. A method for its analysis and criterion of dynamic stability are offered.

The case of a flat curve of a rotating drill rod under the action of variable longitudinal force is investigated at an assumption of finiteness of elastic deformations. The numerical analysis of its elastic dislocations and instability zones of the basic resonance is carried out, which confirm the efficiency of the offered nonlinear dynamic model of rod elements and techniques for their calculation.

Key words: Dynamics, nonlinear vibrations, finite deformations, boring column, stability, instability zone.

1 Introduction

The article is devoted to applied problems of dynamics of nonlinear deformable mediums. Practical application of models of nonlinear deformable mediums to the analysis of movement of drill rods is considered in the view of finite deformations.

From boring practice it is known that up to 30 % of drill holes are rejected. Major factors for a borehole rejection are its curving and drill rods breakage [1-2]. They can be caused by reasons of various nature - geological, technological and technical.

Many researchers consider that geological conditions for position of rock, distinction in their hardness and drilling are a principal cause of a borehole curvature.

Under another concept the principal cause of a borehole curvature is instability of the rectilinear form of the rod. It can arise under the action of such factors as dynamic cross-section influences; the big inertial forces arising at drilling; initial curvature of the rod; stress concentrators, etc.

In the literature linear models of movement of drill rods are known, which basic restriction is the assumption of deformations by the small. The finite deformations of drill rods arising under the action of big axial loadings and twisting moments can complicate their dynamics essentially. They are poorly studied and represent scientific and practical interest.

The purpose of the work is modeling of movement of drill rods at various complicating factors in view of finite deformations in particular.

The solution of the given problem assumes: 1) development of nonlinear model of movement of a drill rod in view of finite deformations; 2) development of a technique of the dynamic analysis of a drill rod with definition of finite dislocations and stability of movement; 3) the numerical analysis of the dynamic model of a drill rod with application of the developed techniques.

2 Nonlinear model of movement of a drill rod for shallow drilling.

Movement of a drill rod for shallow drilling (up to 500 м) is considered, which is used in oil and gas extraction industry (fig. 1).

The admitted in work finiteness of deformations of a drill rod can be caused by changeability of axial forces N (t) and twisting moments M (t):

$$N(x,t) = N_0(x) + N_t(x)\Phi_N(t) \quad , \tag{1}$$

$$M(x,t) = M_0(x) + M_t(x)\Phi_M(t) \tag{2}$$

where $N_0(x)$ – is the longitudinal force, caused by the construction body weight mgx and by constant in time compression force N_1:

$$N_0(x) = N_1 + mgx, \tag{3}$$

g – gravity acceleration,

x – distance from the top end of the rod.

Φ(t) – the periodic function of time defining a weighting mode.

The elementary variant of function Φ(t) corresponds to harmonious influence:

$$\Phi_N(t) = \cos\omega t \tag{4}$$

Similarly for twisting moment M(t): M_0 - is the nominal moment, constant in time; M_t – defines contribution of a variable component; $\Phi_M(t)$ – is the periodic function.

Within the framework of the nonlinear theory of deformations of V.V.Novozhilov [3], where components of tensor of deformations for the general three-dimensional case of deformation are defined as:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right), \qquad i,\ j = 1,2,3, \tag{5}$$

nonlinear model of rotation of a drill rod is constructed in view of finite deformations. For this purpose, accepting the second system of simplifications by V.V.Novozhilov, when in comparison with a unit not only extensions e_{ii} and shears e_{ij}, but also corners of turn ω_i are small, the elastic potential of volumetric deformation [4-5] is received:

$$\begin{aligned}\Phi = G\Bigg[&\left(1+\frac{\nu}{1-2\nu}\right)(\alpha + U_X\alpha_X + \alpha_X^2 + V_y\alpha_y + \alpha_y^2 + W_Z\alpha_Z + \alpha_Z^2) + \\ &+\frac{2\nu}{1-2\nu}((W_Z + 0{,}5\alpha_Z)(U_X + V_y + 0{,}5\alpha_X + 0{,}5\alpha_y) + (U_X + 0{,}5\alpha_X)\times(V_y + 0{,}5\alpha_y)) + \\ &+\frac{1}{2}\left((U_y\beta_X + V_X\beta_y + W_XW_y\right)^2 + (V_Z\beta_y + W_y\beta_Z + U_yU_Z)^2 + (W_X\beta_Z + U_Z\beta_X + V_XV_Z)^2)\Bigg],\end{aligned} \tag{6}$$

where indexes at components of elastic dislocation $U(x,y,z,t)$, $V(x,y,z,t)$ and also $W(x,y,z,t)$ mean differentiation of these functions on the specified variables, and the following designations are entered:

$$\alpha = U_x^2 + V_y^2 + W_z^2; \quad \alpha_x = U_x^2 + V_x^2 + W_x^2; \quad \alpha_y = U_y^2 + V_y^2 + W_y^2; \tag{7}$$
$$\alpha_z = U_z^2 + V_z^2 + W_z^2; \quad \beta_x = 1 + U_x; \quad \beta_y = 1 + V_y; \quad \beta_z = 1 + W_z.$$

The potential (6) - (7) has the general character and allows to pass to individual cases of deformation of elastic systems. So, for the considered case of a drill rod as rod element [6], with the circuit of deformation fig. 2 and rotating at angular speed ω, the equations of its movement are received:

$$EJ_V \frac{\partial^2}{\partial x^2}\left[\frac{\partial^2 V}{\partial x^2}\left(1 - \frac{3}{2}\left(\frac{\partial V}{\partial x}\right)^2\right)\right] + \frac{\partial^2}{\partial x^2}\left[M(x,t)\frac{\partial U}{\partial x}\right] + \frac{\partial}{\partial x}\left[\mathrm{N}(x,t)\frac{\partial V}{\partial x}\right] + K_1 V = -\frac{\gamma F}{g}\frac{\partial^2 V}{\partial t^2},$$
$$EJ_U \frac{\partial^2}{\partial x^2}\left[\frac{\partial^2 U}{\partial x^2}\left(1 - \frac{3}{2}\left(\frac{\partial U}{\partial x}\right)^2\right)\right] + \frac{\partial^2}{\partial x^2}\left[M(x,t)\frac{\partial V}{\partial x}\right] + \frac{\partial}{\partial x}\left[\mathrm{N}(x,t)\frac{\partial U}{\partial x}\right] + K_1 U = -\frac{\gamma F}{g}\frac{\partial^2 U}{\partial t^2}, \tag{8}$$

where $K_1 = \gamma F \omega^2 / g$.

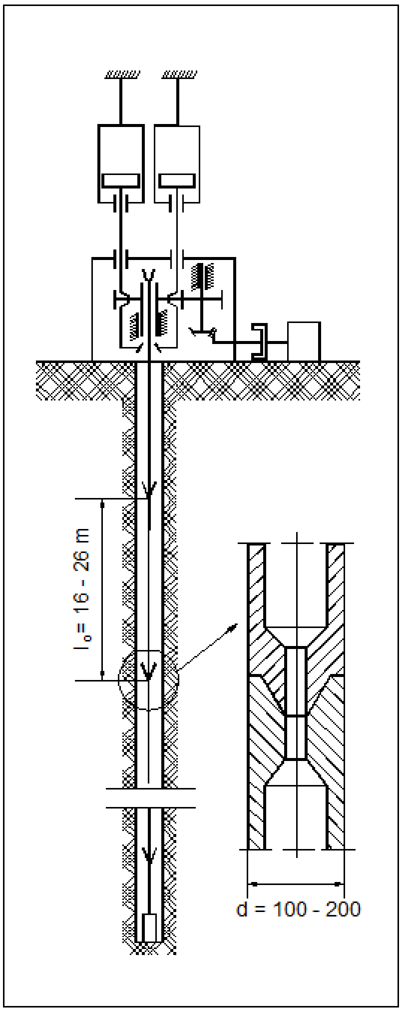


Fig. 1 The kinematic circuit of the boring machine

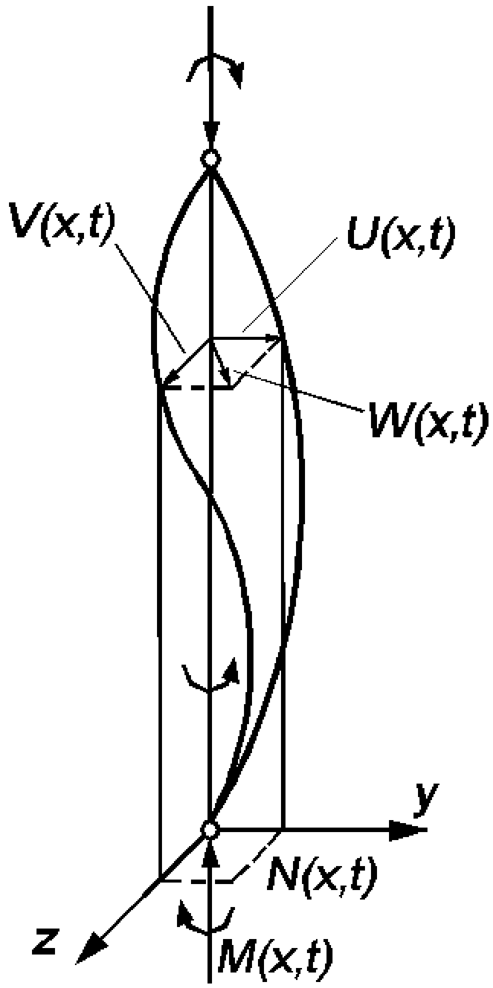


Fig. 2 The form of the curved axis of a rod

2 Analysis of finite dislocations of a drill rod and stability of its movement.

A special case of a model (8) is considered - a case of a flat curving of the rod rotating at speed ω under the action of longitudinal force $N(t) = N_0 + N_t \Phi(t)$. Presuming the drill rod is hingedly supported on the ends, the boundary conditions are set:

$$V = EJ_v \frac{\partial^2 V}{\partial x^2} = 0, \quad U = EJ_u \frac{\partial^2 U}{\partial x^2} = 0, \qquad (x = 0,\ x = l) \tag{9}$$

The following solution satisfies these conditions

$$U(x,t) = \sum_{k=1}^{\infty} f_k(t) \sin \frac{k\pi x}{l}. \tag{10}$$

Substituting the solution (10) in the equation of the curved axis of a rod and applying direct variational method of Bubnov-Galyorkin, the latter leads to to the nonlinear parametrical equations with one degree of freedom:

$$\ddot{f} + C_k^2 (1 - 2\nu_k \cos \Omega t) f + \alpha f^3 = 0, \tag{11}$$

where $C_k = \frac{k^2 \pi^2}{l^2} \sqrt{\frac{EI}{m}\left(1 - \frac{N_0}{N_k}\right)} = \omega_0$, $N_k = \frac{k^2 \pi^2 EI}{l^2}$, $\nu_k = \frac{N_t}{2(N_k - N_0)}$, $\alpha = \frac{3Ek^4 \pi^4}{8\rho l^4}$ (12)

Stability of the set modes of movement of a drill rod has basic value for maintenance of its trouble-free operation. Here steady movement of a drill rod will be understood as its movement in absence of dangerous resonant modes of fluctuations. For this purpose stability of the basic resonance is investigated:

$$f_0 = r_1 \cos(\Omega t - \varphi_1).$$

The technique of definition of instability zones of resonant vibrations of a drill rod with application of Flock theory is offered. According to it the solution of the equation of the perturbed condition of model (11):

$$\frac{d^2 \delta f}{dt^2} + \delta f \left[C_k^2 + 1{,}5\alpha r_1^2 - 2C_k^2 \nu \cos \Omega t + 1{,}5\alpha r_1^2 \cos 2\varphi_1 \cos 2\Omega t + 1{,}5\alpha r_1^2 \sin 2\varphi_1 \sin 2\Omega t \right] = 0 \tag{13}$$

set as follows:

$$\delta f = e^{\mu t} P(t), \tag{14}$$

The type of expression $P(t)$ defines the instability zone of fluctuations.

Having set $P(t) = b_1 \cos(\Omega t - \psi_1)$, the border of the first zone of instability of the basic resonance [7-8] is received:

$$\Delta(\mu=0)=\Omega^4+\frac{27}{16}\alpha^2 r_4+(3c_k^2\alpha-3\Omega^2\alpha)-2\Omega^2 c_k^2+c_k^4=0 \qquad (15)$$

The numerical analysis of dislocations of nonlinear fluctuations (11) and definition of an instability zone (15) represent no hard work. The given approach can be distributed for research of fluctuations on the maximum modes and with definition of their instability zones.

3 Numerical example

The numerical analysis of nonlinear fluctuations and instability zones of the basic resonance of the rotating drill rod is carried out, undergone to a flat curving under the action of variable longitudinal force $N(t)=N_0+N_t\Phi(t)$.

The numerical analysis of the equations (11) - (12) is carried out at entry conditions [9]:

$$f_k(t=0)=0, \qquad \frac{\partial f_k}{\partial t}(t=0)=0{,}001 \qquad (16)$$

Steel and dural drill rods are considered. From the comparative analysis of the research results one can see that the amplitude of fluctuations of nonlinear model (11) in both cases is less than in linear position (fig. 3).

The influence of forms of a rod curving on amplitude of its fluctuations is investigated. It is established that at the basic form of a curving the amplitude of dislocations of a drill rod exceeds the amplitudes of the big forms. Thus, duralumin drill rod is subject to smaller deflections from the rectilinear form than the steel one at the same modes of drilling (fig. 4).

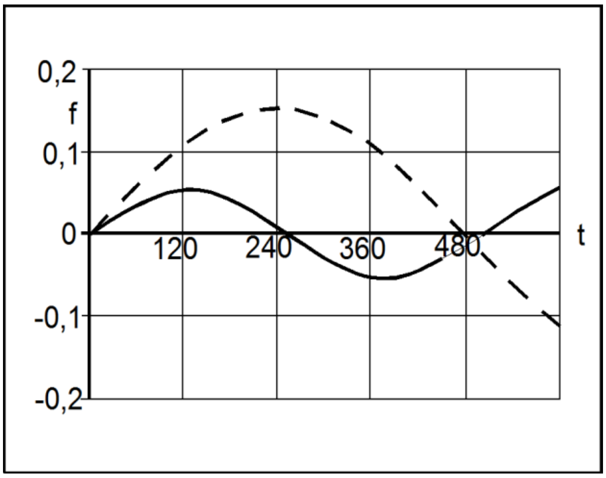


Fig. 3 Amplitude of fluctuations of linear and nonlinear models (_ _ _ _ linear, ______ nonlinear) at $E_д = 0{,}7\cdot10^5$ МПа; $\rho_д = 2698{,}9$ кг/м^3

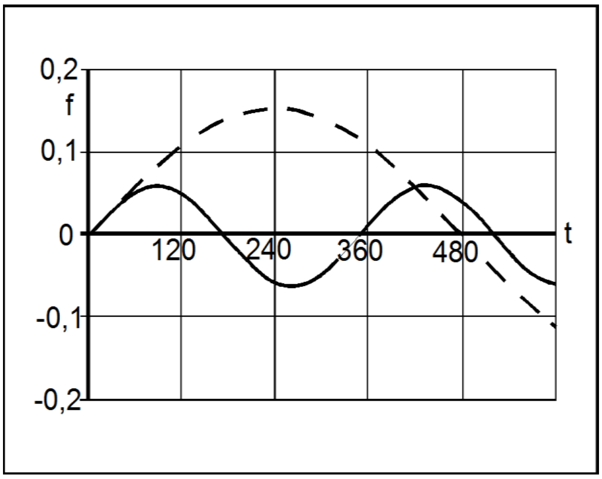
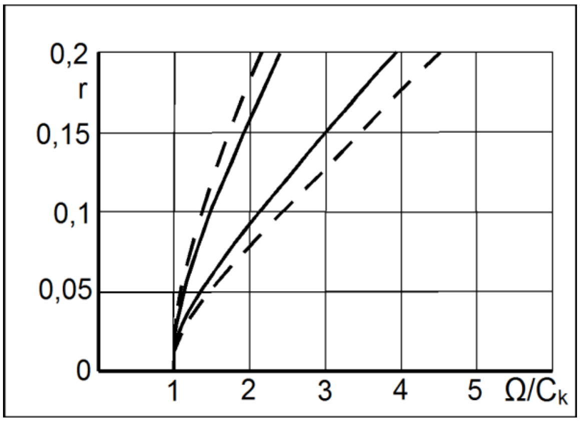


Fig. 4 Influence of properties of the material of a rod (_ _ _ _ steel, ______ duralumin) at $E_c = 2{,}1\cdot10^5$ МПа; $\rho_s = 7{,}8\cdot10^3$ кг/м^3; $E_д = 0{,}7\cdot10^5$; $\rho_d = 2698{,}9$ кг/м^3

l=500м; $D_1 = 0{,}12$м; $D_2 = 0{,}2$м

In figures 5-7 the results of the research of influence of forms of fluctuations of a rod, its length and properties of the material on the width of instability zones of the basic resonance are submitted. It is established that the first two forms of fluctuations influence the width of the instability zone (fig. 5). The increase in length of a rod leads to the expansion of instability zone (fig. 6). In all the considered cases instability zones for a duralumin rod are larger than for a steel one.



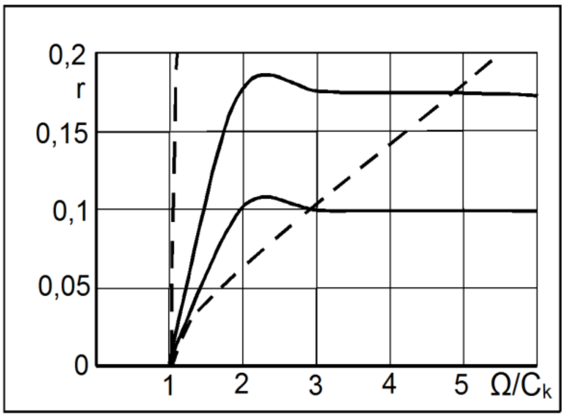


Fig. 5 Influence of the forms of fluctuations on the instability zone of a rod (_ _ _ k=2, ____ k=1) at L=300м,

Fig.6 Influence of the length of a rod on the instability zone of a rod (_ _ _ L=500 m, ____ L=1000 m) at

$$\boldsymbol{N}_0 = 500\boldsymbol{н},\ \boldsymbol{N}_t = 2195,5\ \boldsymbol{Кн},\ \boldsymbol{d} = 120\boldsymbol{мм}, \boldsymbol{D} = 200\,\boldsymbol{мм},\ \ \boldsymbol{E}_{\boldsymbol{cm}} = 2,1\cdot 10^5\,\boldsymbol{МПа}$$

4 Conclusions

In the work practical application of the nonlinear model of elastic deformation of mediums developed by the author for the general spatial case is considered. The results of the numerical analysis of movement of compressed - torsioned drill rod without restrictions of the deformation sizes testify to the efficiency of a nonlinear model. It is established that the account of nonlinear factors leads to essential specification of the dynamic model of drill rods - to the downturn of fluctuation amplitudes and displacement of basic resonance zones into the area of big frequencies. In spite of the fact that the suggested techniques were applied to the research of stability of the basic resonance of elastic dynamic systems, they can also be successfully applied to the analysis of resonances on the maximum frequencies.

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