

Proceedings of the Twelfth Asia-Pacific International Conference on

# Gravitation, Astrophysics, and Cosmology

Dedicated to the Centenary of Einstein's General Relativity



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**Preface**

The conference is dedicated to the centenary of Einstein's General Relativity theory



## Wormhole solutions in GR with two phantom scalar fields

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Wormhole solutions for two scalar fields in general relativity are considered. The physical characteristics of such objects are discussed.

**Keywords:** Phantom scalar fields; wormholes.

### 1. Wormhole Solutions

At the present time it is widely believed that observed acceleration of our Universe is the consequence of some special form of matter – dark energy (DE). One of the key features of the latter is its ability to violate various energy conditions. In the most extreme case, DE is modeled by matter violating the null energy condition. Such matter is called exotic. One of DE models is phantom fields. Here we use two phantom scalar fields to construct wormholes.

We consider Einstein + scalar field equations in the form

$$G_{\mu}^{\nu} = 8\pi GT_{\mu}^{\nu}, \quad (1)$$

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^{\mu}} \left[ \sqrt{-g} g^{\mu\nu} \frac{\partial(\varphi, \chi)}{\partial x^{\nu}} \right] = -\frac{\partial V}{\partial(\varphi, \chi)}. \quad (2)$$

The potential for these two phantom scalar fields is

$$V(\varphi, \chi) = \frac{\lambda_1}{4}(\phi^2 - m_1^2)^2 + \frac{\lambda_2}{4}(\chi^2 - m_2^2)^2 + \phi^2 \chi^2 - V_0. \quad (3)$$

The metric for our wormhole solution filled with two phantom scalar fields is

$$ds^2 = B(r)dt^2 - dr^2 - A(r)(d\theta^2 + \sin^2 \theta d\phi^2), \quad (4)$$

Using this metric, one can obtain Einstein equations:

$$A''A - \frac{1}{2} \left( \frac{A'}{A} \right)^2 - \frac{1}{2} \frac{A'B'}{AB} = \varphi'^2 + \chi'^2,$$

$$\frac{A''}{A} + \frac{1}{2} \frac{A'B'}{AB} - \frac{1}{2} \left( \frac{A'}{A} \right)^2 - \frac{1}{2} \left( \frac{B'}{B} \right)^2 + \frac{B''}{B} = 2 \left[ \frac{1}{2}(\varphi'^2 + \chi'^2) + V \right]$$

The corresponding field equation for  $\varphi$  is (the equation for  $\chi$  is analogous)

$$\varphi'' + \left( \frac{A'}{A} + \frac{1}{2} \frac{B'}{B} \right) \varphi' = \varphi [2\chi^2 + \lambda_1(\varphi^2 - m_1^2)].$$

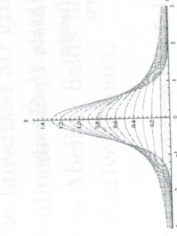


Fig. 1.  $\chi(r)$

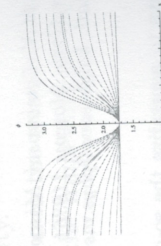


Fig. 2.  $\phi(r)$

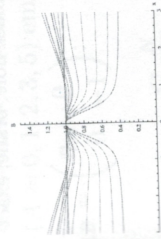


Fig. 3.  $A(r)$

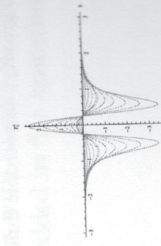


Fig. 4.  $B(r)$

Fig. 5. The profile of energy density

Fig. 6. The profile of mass of wormhole vs

### 2. Conclusions

(a) Wormhole solutions in GR created with two phantom scalar fields investigated; (b) solutions with different values of scalar field  $\chi(0)$  origin are obtained; (c) the wormhole mass by different values of calculated.

### References

1. V. Dzhumshaliyev and V. Folomeev, "4D static solutions with interaction phantom fields," *Int. J. Mod. Phys. D* **17**, 2125 (2008); arXiv:0710.1071 [gr-qc].