Compact objects with two phantom scalar fields in GR

V. Dzhunushaliev, V. Folomeev, A.Urazalina, A.Makhmudov

IETP, al-Farabi KazNU, Almaty, Kazakhstan
Dept. Theor. Nucl. Phys. al-Farabi KazNU, Almaty, Kazakhstan

April 15, 2016

At the present time it is widely believed that the accelerated expansion of our Universe is due to the presence in the Universe of some special form of matter – dark energy (DE).

In modeling the accelerated expansion of the Universe, it is usually assumed that DE is distributed homogeneously and isotropically on the largest scales. This, however, does not exclude a possibility that DE might cluster on relatively small scales comparable to sizes of galaxies or even separate stars.

One of DE models are phantom fields. In this talk we use two phantom scalar fields ϕ, χ to construct (phantom) wormholes, boson stars and domain walls.

Equations

We consider Einstein + scalar field equations in the form

$$
\begin{aligned}
G_\mu^\nu &= 8\pi G T_\mu^\nu, \\
\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left[\sqrt{-g}\; g^{\mu\nu}\frac{\partial(\varphi,\chi)}{\partial x^\nu}\right] &= -\frac{\partial V}{\partial(\varphi,\chi)}.
\end{aligned}
$$

Potential

The potential for these two phantom scalar fielfs is

$$V(\phi,\chi)=\frac{\lambda_1}{4}(\phi^2-m_1^2)^2+\frac{\lambda_2}{4}(\chi^2-m_2^2)^2+\phi^2\chi^2-V_0.$$

The metric for our wormhole solution filled with two phantom scalar fiels is

$$ds^2 = B(r)dt^2 - dr^2 - A(r)(d\theta^2 + \sin^2\theta d\phi^2),$$

Using this metric, one can obtain Einstein equations :

$$A''A - \frac{1}{2}\left(\frac{A'}{A}\right)^2 - \frac{1}{2}\frac{A'}{A}\frac{B'}{B} = \varphi'^2 + \chi'^2 \ ,$$

$$\frac{A''}{A} + \frac{1}{2}\frac{A'}{A}\frac{B'}{B} - \frac{1}{2}\left(\frac{A'}{A}\right)^2 - \frac{1}{2}\left(\frac{B'}{B}\right)^2 + \frac{B''}{B} =$$

$$2\left[\frac{1}{2}(\varphi'^2 + \chi'^2) + V\right] \ ,$$

$$\frac{1}{4}\left(\frac{A'}{A}\right)^2 - \frac{1}{A} + \frac{1}{2}\frac{A'}{A}\frac{B'}{B} = -\frac{1}{2}(\varphi'^2 + \chi'^2) + V \ ,$$

Part1. Phantom wormholes. Scalar field equations

The corresponding field equations from are

$$\varphi'' + \left(\frac{A'}{A} + \frac{1}{2}\frac{B'}{B}\right)\varphi' = \varphi\left[2\chi^2 + \lambda_1(\varphi^2 - m_1^2)\right] \ ,$$

$$\chi'' + \left(\frac{A'}{A} + \frac{1}{2}\frac{B'}{B}\right)\chi' = \chi\left[2\varphi^2 + \lambda_2(\chi^2 - m_2^2)\right] \ .$$

Eigenvalues for these equations: $m_{1,2}$. Solutions are found numerically.

Main feature of these equations that they have regular solutions with special choice of $m_{1,2}$ parameters. That means that this problem should be considered as a non-linear eigenvalue problem for eigenvalues $m_{1,2}$ and eigebfunctions ϕ, χ.

Part1. Phantom wormholes. Profiles of ϕ and χ

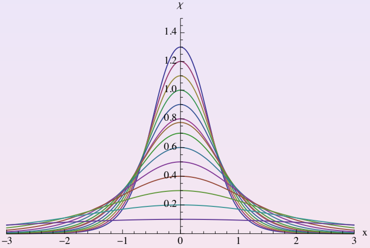

Figure: $\chi(r)$

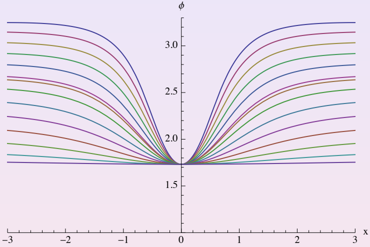

Figure: $\phi(r)$

The solutions are obtained numerically.

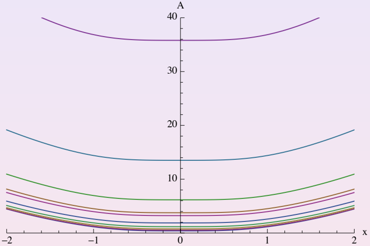

Figure: $A(r)$

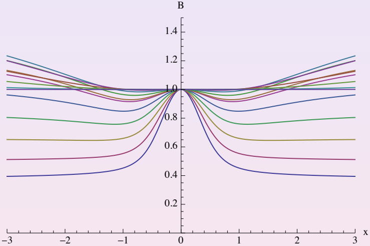

Figure: $B(r)$

Part1. Phantom wormholes. Asymptotical behaviour

$$
\begin{aligned}
A &\approx r^2 + r_0^2 \to r^2, \\
B &\approx B_\infty \left(1 - \frac{r_0^2}{r^2}\right) \to B_\infty \\
\varphi &\approx m_1 - C_\varphi \frac{\exp\left(-r\sqrt{2\lambda_1 m_1^2}\right)}{r} \to m_1, \\
\delta\chi &\approx C_\chi \frac{\exp\left(-r\sqrt{2m_1^2 - \lambda_2 m_2^2}\right)}{r} \to 0,
\end{aligned}
$$

Part1. Phantom wormholes. Profiles of energy density $\epsilon(r)$ and wormhole mass

The dimensionless energy density:

$$\epsilon = -\left[\frac{1}{2}\left({\phi'}^2 + {\chi'}^2\right) + V(\phi, \chi)\right]$$

The dimensionless mass:

$$m = \frac{A_0}{2} + \int_0^\infty x^2 \epsilon dx$$

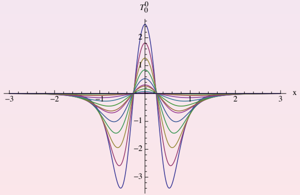

Figure: The profile of energy density

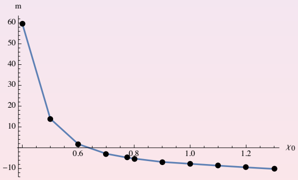

Figure: The profile of the mass of wormhole vs χ_0

For consideration of the phantom boson stars we take the metric in Schwarzschild coordinates:

$$ds^2 = B(r)dt^2 - A(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

Part2. Phantom boson stars. Asymptotic behavior

The asymptotic behavior of the metric functions $A(r), B(r)$ and scalar fields ϕ, χ are:

$$
\begin{aligned}
A &\approx \frac{1}{1+\frac{r_0}{r}}, \\
B &\approx B_\infty \left(1+\frac{r_0}{r}\right) \\
\varphi &\approx m_1 - C_\varphi \frac{\exp\left(-r\sqrt{2\lambda_1 m_1^2}\right)}{r}, \\
\chi &\approx C_\chi \frac{\exp\left(-r\sqrt{2m_1^2-\lambda_2 m_2^2}\right)}{r},
\end{aligned}
$$

There are configurations with some ϕ_0, χ_0 with ZERO mass !

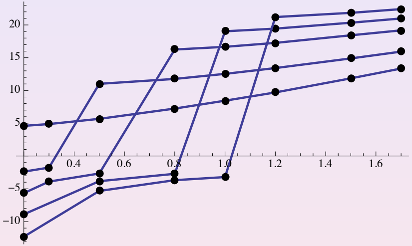

Figure: The profile of mass of phantom boson star

In this section we consider a brane-like solution in 4D. Let us chose the metric in the form:

$$ds^2 = a^2(x)(dt^2 - dy^2 - dz^2) - dx^2,$$

Einstein equations are

$$3\left(\frac{a'}{a}\right)^2 = -\frac{1}{2}\left(\varphi'^2+\chi'^2\right)+V,$$
$$\frac{a''}{a}-\left(\frac{a'}{a}\right)^2 = \frac{1}{2}\left(\varphi'^2+\chi'^2\right),$$

and the scalar field equations:

$$\varphi''+3\frac{a'}{a}\varphi' = \varphi\left[2\chi^2+\lambda_1(\varphi^2-m_1^2)\right],$$
$$\chi''+3\frac{a'}{a}\chi' = \chi\left[2\varphi^2+\lambda_2(\chi^2-m_2^2)\right],$$

Part3. Phantom thick brane-like solutions. Energy density.

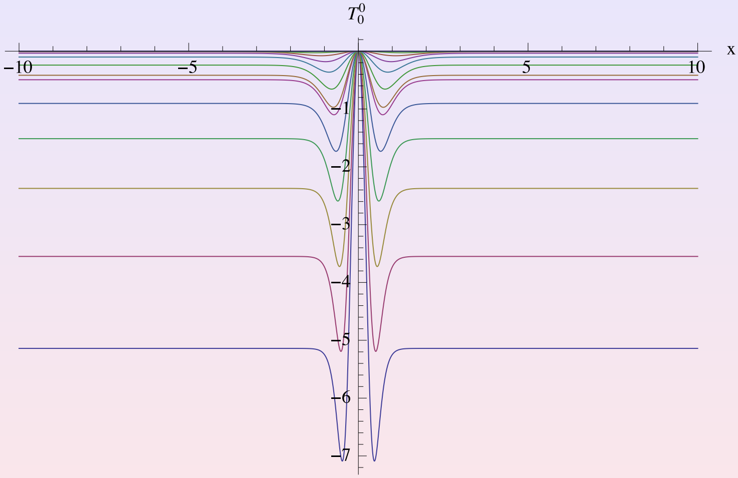

Figure: The profile of mass of phantom boson star

Conclusions

- Phantom wormholes, boson stars, and thick brane-like solutions in GR created with two phantom scalar fields are investigated.
- The profiles of solutions with different values of phantom fields at the origin are investigated.
- It is shown that phantom wormholes and boson stars may have positive, negative, and zero masses.

THANKS FOR YOUR ATTENTION !