Quantum torsion with non-zero standard deviation: non-perturbative approach for cosmology

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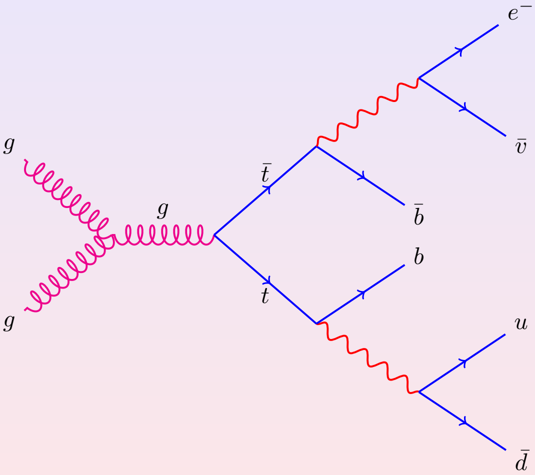
The problems

- Gravity quantization
- Torsion

Feynman diagrams



Operator equations

$$\hat{\Gamma}^{\rho}{}_{\mu\nu} = \hat{G}^{\rho}{}_{\mu\nu} + \hat{K}^{\rho}{}_{\mu\nu},$$

$$\hat{R}_{\mu\nu} - \frac{1}{2}\hat{g}_{\mu\nu}\hat{R} = \varkappa \hat{T}_{\mu\nu},$$

$$\hat{G}^{\rho}{}_{\mu\nu} = \frac{1}{2}\hat{g}^{\rho\sigma}\left(\frac{\partial \hat{g}_{\mu\sigma}}{\partial x^{\nu}} + \frac{\partial \hat{g}_{\nu\sigma}}{\partial x^{\mu}} - \frac{\partial \hat{g}_{\mu\nu}}{\partial x^{\sigma}}\right)$$

Let us note that all quantities in Eqs are operators. This leads to a difficult problem because now one has to solve the operator differential equations. Actually this is the main problem of nonperturbative quantization (that is well known.)

The non-perturbative quantization for Einstein gravity means that the quantum operators of metrics $\hat{g}_{\mu\nu}$, Christoffel symbols $\hat{G}^{\rho}{}_{\mu\nu}$ and torsion $\hat{Q}_{\mu\nu}{}^{\rho}$ obey the operator Einstein - Cartan equations.

Vector field approximation for non-perturbative quantization of torsion

In our approach we consider the torsion with zero expectation value

$$\left\langle \hat{Q}^{\rho}{}_{\mu\nu} \right\rangle = 0$$

but with non-zero dispersion

$$\left\langle \left(\hat{Q}^{\rho}{}_{\mu\nu} \right)^2 \right\rangle \neq 0.$$

Vector field approximation for non-perturbative quantization of torsion

$$\left\langle \hat{Q}_{\rho_1\mu_1\nu_1}(x_1)\hat{Q}_{\rho_2\mu_2\nu_2}(x_2)\right\rangle \approx \epsilon_{\rho_1\mu_1\nu_1\alpha}\epsilon_{\rho_2\mu_2\nu_2\beta}A^\alpha(x_1)A^\beta(x_2).$$

The expectation values of the Ricci and scalar curvature operators

$$\left\langle \hat{R}_{\mu\nu} \right\rangle = \tilde{R}_{\mu\nu} + 2\left(g_{\mu\nu} A_\alpha A^\alpha - A_\mu A_\nu\right),$$

$$\left\langle \hat{R} \right\rangle = \tilde{R} + 6 A^\mu A_\mu,$$

$$\tilde{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \tilde{R} - \left(g_{\mu\nu} A_\alpha A^\alpha + 2 A_\mu A_\nu \right) = \varkappa T_{\mu\nu},$$

In order that the Einstein equations are not overdetermined we demand that

$$\left\langle \left(\hat{R}^\mu_\nu - \frac{1}{2}\delta^\mu_\nu \hat{R} \right) \right\rangle_{;\mu} = 0$$

The desired equation for the vector field A_μ

$$\left(\delta^\mu_\nu A^\alpha A_\alpha + 2A^\mu A_\nu\right)_{;\mu} = 0$$

Now we would like to consider cosmology with quantum corrections coming from the torsion.

$$ds^2 = a^2(\eta)\left\{d\chi^2 - \left[d\chi^2 + \left(\frac{\sin\sqrt{k}\chi}{\sqrt{k}}\right)^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right]\right\}$$

We take the vector field A_μ as follows

$$A_\mu = (\phi(\eta), 0, 0, 0) .$$

The averaged Einstein equations in the presence of matter in the form of dust are

$$\frac{a'^2}{a^2} + \left(k - \phi_0^2\right) = \varkappa \frac{\varepsilon_0}{3a},$$

$$2\frac{a''}{a} - \frac{a'^2}{a^2} + \left(k - \phi_0^2\right) = 0,$$

$$k = \pm 1, 0$$

Quantum torsion may lead to a qualitative change of the evolution of the Universe. For example, a closed Universe with fluctuating quantum torsion may have an evolution similar to a closed, open or flat (non-torsion) Universe, depending on the value of the quantum fluctuation dispersion of the torsion.