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Phase Shifts and Scattering Cross Sections of the Particles of Non-ideal Semiclassical Plasmas Based on the Dynamic Interaction Potential

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The elastic scattering processes of the charged particles in the dense nonideal plasma on the basis of the dynamic interaction potential were investigated. It is shown that dynamic charge screening increases the phase shifts and the scattering cross sections in comparison with static screening. The problem was solved on the basis of the Calogero equation for finding the phase shifts.

1 Introduction

Collisional processes determine almost all properties of the plasma: its structure, thermodynamics, transport properties, electrodynamic properties, etc. So, it is especially important to be able correctly and reliably to carry out research on the level of the elementary processes. For an adequate theoretical description of the scattering processes in the plasma and the reliable quantitative estimation of the collision characteristics it is necessary to use the methods of the quantum mechanics. In general, the Schrödinger equation has to be solved with the corresponding interaction potential. In order to find the phase shifts it is sufficient to solve the equation of Calogero.

To describe the interaction of particles in the semiclassical nonideal fully and partionally ionized plasmas the potentials that take into account the effects of diffraction and the Debye screening were proposed [1-6]. Work [1] presented a potential for charge-charge interaction. This potential does not depend on the velocity of the colliding particles, and the magnitude of the charge screening is determined only by the temperature and density of charged particles:

$$\Phi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta e^2}{\sqrt{1-4\lambda_{\alpha\beta}^2/r_D^2}}\left(\frac{e^{-B_{\alpha\beta}r}}{r} - \frac{e^{-A_{\alpha\beta}r}}{r}\right), \tag{1}$$

here $A_{\alpha\beta}^2 = \frac{1}{2\lambda_{\alpha\beta}{}^2}(1+\sqrt{1-4\lambda_{\alpha\beta}^2/r_D^2})$; $B_{\alpha\beta}^2 = \frac{1}{2\lambda_{\alpha\beta}{}^2}(1-\sqrt{1-4\lambda_{\alpha\beta}^2/r_D^2})$;

where $\lambda_{\alpha\beta} = \hbar/\sqrt{2\pi m_{\alpha\beta}k_B T}$ is the de Broglie thermal wavelength; $m_{\alpha\beta} = m_\alpha m_\beta/(m_\alpha + m_\beta)$ is the reduced mass of α and β interacted particles; $r_D = (k_B T/(4\pi e^2 \sum_\alpha n_\alpha Z_\alpha^2))^{1/2}$ is the Debye length, $Z_\alpha e$, $Z_\beta e$ are the electrical charges of the types of particles, labeled with α and β.

Collision cross sections directly depend on the relative velocity of the colliding particles. The energy of the static interaction usually does not depend on this velocity. This consideration is not entirely correct, and accounting of the influence of the different dynamic effects, in particular the dynamic screening of the incident charge's field, on the interaction energy of the particles is more consistent [7,8]. In work [8] the elastic differential cross sections of charges in a dense semiclassical plasma on the basis of the interaction potential, taking into

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account the effects of diffraction and the effect of dynamic screening, were investigated. This potential has been obtained by replacing the usual Debye length in the interaction potential (1) by the screening radius, depending on the relative velocity of the colliding particles [9]:

$$r_0 = r_D(1 + \upsilon^2/\upsilon_{Th}^2)^{1/2} \tag{2}$$

Here υ is the relative velocity of the colliding particles, υ_{Th} is the thermal velocity. Then the energy of the electron-electron interaction, which takes into account dynamic screening, in a dimensionless form is:

$$\Phi_{ee}(R)/k_BT = \frac{\Gamma_{ee}}{\sqrt{1-24\Gamma_{ee}^2/(\pi r_s(1+\delta^2))}}\left(\frac{e^{-B_{ee}R}}{R} - \frac{e^{-A_{ee}R}}{R}\right) \tag{3}$$

where $A_{ee}^2 = \frac{\pi r_s}{4\Gamma_{ee}}\left(1+\sqrt{1-24\Gamma_{ee}^2/(\pi r_s(1+\delta^2))}\right)$;
$B_{ee}^2 = \frac{\pi r_s}{4\Gamma_{ee}}\left(1-\sqrt{1-24\Gamma_{ee}^2/(\pi r_s(1+\delta^2))}\right)$;

where $\delta = \upsilon/\upsilon_{Th}$ is the parameter of the relative velocity. $R = r/a$ is the distance between particles in terms of the average distance. $\Gamma_{ee} = \frac{e^2}{ak_BT}$ is the coupling parameter, $a = \left(\frac{3}{4\pi n}\right)^{1/3}$ is the average distance between particles, $r_s = \frac{a}{a_B}$ is the density parameter, $a_B = \frac{\hbar^2}{m_e e^2}$ is the Bohr radius. Potential (3) for small velocities of the colliding particles tends to the potential (1), and at high velocities tends to the Deutsch potential, which takes into account the effect of diffraction and does not take into account the screening [10,11].

In the case when the influence of the scattering center is not large, the calculation of the scattering cross sections can be carried out in the Born approximation [12]. In work [8] based on the results obtained by the Born method, it was shown that dynamic screening increases the differential scattering cross sections in comparison with the Debye static screening, especially at small scattering angles. At large scattering angles differential cross sections converge. More accurate quantum mechanical solution of the problem of the scattering cross section's estimation gives the method of phase functions [12-13], in which the scattering cross sections are calculated on the basis of the scattering phases of the waves at different values of the orbital quantum number ℓ. For large values ℓ the scattering phase can be found from the semiclassical representation of a particle moving in the field of a fixed force center (WKB approximation). Another method for the scattering phase calculation is called as the phase-function method (PFM), it reduces to solving of the first order differential equation for the phase function [12].

2 The scheme of calculations and results

To determine the phase shifts δ_ℓ in this work the phase-function method was used, where the Calogero equation was solved [12,13]:

$$\frac{d}{dr}\delta_\ell(r) = -\frac{1}{k}\frac{2m}{\hbar^2}\Phi_{\alpha\beta}(r)[\cos\delta_\ell(r)j_\ell(kr) - \sin\delta_\ell(r)n_\ell(kr)]^2, \qquad \delta_\ell(0) = 0. \tag{4}$$

where $\delta_\ell(r)$ is the phase function; $\Phi_{\alpha\beta}(r)$ is the interaction potential; k is the wave number of the particle; $j_\ell(kr)$ and $n_\ell(kr)$ are regular and irregular solutions of the Schrödinger equation. The phase shift is the asymptotical value of the phase function at the large distances:

$$\delta_\ell = \lim_{r\to\infty}\delta_\ell(r). \tag{5}$$

Partial cross sections for scattering in quantum-mechanical approximation is calculated on the basis of the phase shifts:

$$Q_\ell^P(k) = \frac{4\pi}{k^2}(2\ell+1)\sin^2\delta_\ell. \tag{6}$$

Total scattering cross section is defined as the sum of the partial ones:

$$Q^F(k) = \sum_{\ell=0}^{n} Q_\ell^P(k). \tag{7}$$

Transport scattering cross sections of different orders, used for calculation of the transport coefficients, are also determined by the phase shifts. Thus, the transport cross section of the first order was calculated by the following formula:

$$Q_{\alpha\beta}^T(k) = \frac{4\pi}{k^2} \sum_{\ell=0}^{\infty} (\ell+1) \sin^2(\delta_{\ell+1}^{\alpha\beta} - \delta_\ell^{\alpha\beta}), \tag{8}$$

The results of studies of the collisional processes within the static model (1) has been previously obtained and presented in [14-16]. The comparison with results obtained on the basis of the Debye-Huckel and Deutsch models, which take into account one of two effects, screening or diffraction, respectively, was also presented. In this paper, we present the data obtained in the framework of the potential (1) for comparison with the results obtained on the basis of a new interaction potential (3) taking into account the dynamic screening and the effect of diffraction. The goal of this work is to identify the differences in the characteristics of the collisional processes associated with the use of the dynamic screening in the charged particles interaction. All comparisons with experiments are planned to be done at the level of the transport coefficients.

Equation (4) was solved numerically. Figure 1 shows the dependences of the phase functions of the electron-electron scattering on the distance obtained in the framework of the models of static and dynamic screening. As can be seen in Figure 1, the obtained phase functions demonstrate proper asymptotic behavior, at large distances they tend to some steady-state values, which are actually the phase shifts. Phase shifts calculated within the dynamic potential (3) are larger then phase shifts derived from the static model (1), since the dynamic screening of the field is weaker than the static one. It is also seen that with increase in r_s (decrease in the density) phase shifts grow, since screening as a phenomenon is more pronounced in the more dense and cold plasma.

The same can be seen in Figure 2, which shows the phase shifts as a function of the orbital quantum number. With increase in ℓ they tend to zero, that allows further to limit the calculation of the total scattering cross-section by some ℓ_{max}.

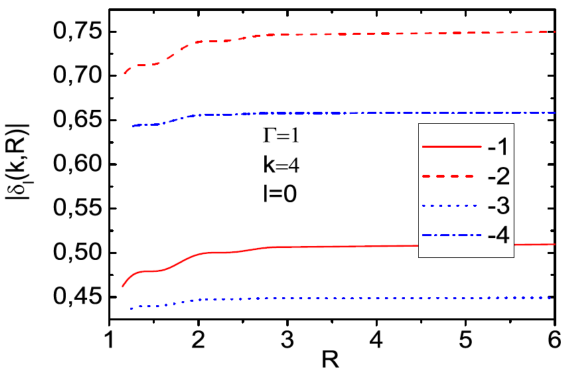

Fig. 1 Phase functions of electron-electron scattering. On the basis of the dynamic potential: 1) $r_s = 10$; 2) $r_s = 15$; on the basis of the static potential: 3) $r_s = 10$; 4) $r_s = 15$.

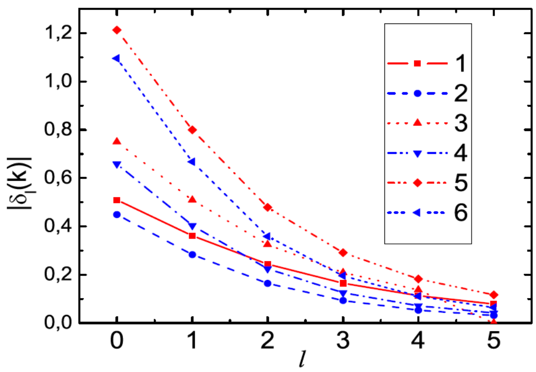

Fig. 2 Phase shifts of electron-electron scattering, $\Gamma = 1$, $k = 4$.On the basis of the dynamic potential: 1) $r_s = 10$; 3) $r_s = 15$; 5) $r_s = 25$; on the basis of the static potential: 2) $r_s = 10$; 4) $r_s = 15$; 6) $r_s = 25$.

Figure 3 shows the phase shifts as the functions of the reduced wave vector ka . The absolute values of the phase shifts obtained on the basis of the dynamic potential exceed the data obtained on the basis of the static model, and the two curves converge at low values of ka , since the dynamic screening becomes an ordinary static one.

Recall that the total scattering cross section is expressed in terms of phase shifts by the equation (7), where each term is the contribution of the partial scattering cross section. Figure 4 shows the partial cross sections for electron-electron scattering as a function of ka . This figure shows that the cross sections obtained on the basis of potential (3) lie above the cross sections obtained in the framework of the static interaction potential (1) as a result of the increase of the phase shifts. Also, Figure 4 shows that the main contribution to the cross section is from the partial wave with $\ell = 0$.

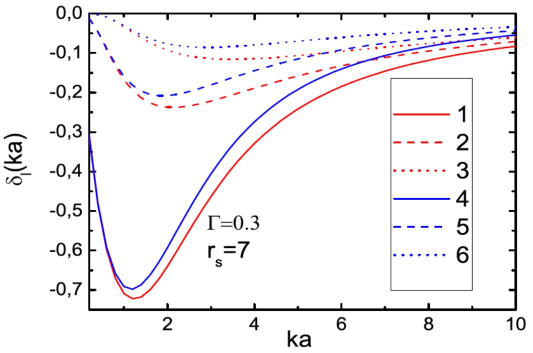

Fig. 3 Phase shifts of electron-electron scattering,$\Gamma = 0.3$, $r_s = 7$. On the basis of the dynamic potential: 1) $\ell = 0$; 2) $\ell = 1$; 3) $\ell = 2$; on the basis of the static potential: 4) $\ell = 0$; 5) $\ell = 1$; 6) $\ell = 2$.

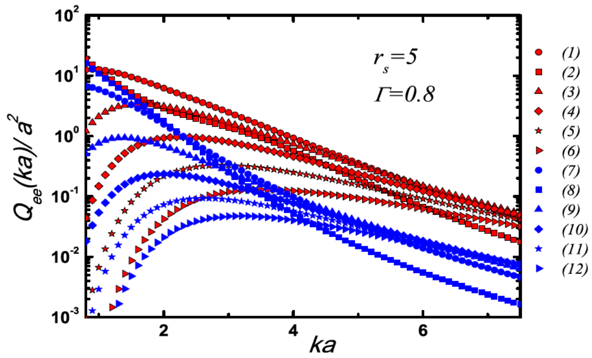

Fig. 4 Partial cross sections for electron-electron scattering on the basis of the static and dynamic potentials.On the basis of the dynamic potential: 1) $\ell = 0$; 2) $\ell = 1$; 3) $\ell = 2$; 4) $\ell = 3$; 5) $\ell = 4$; 6) $\ell = 5$; on the basis of the static potential: 7) $\ell = 0$; 8) $\ell = 1$; 9) $\ell = 2$; 10) $\ell = 3$; 11) $\ell = 4$; 12) $\ell = 5$.

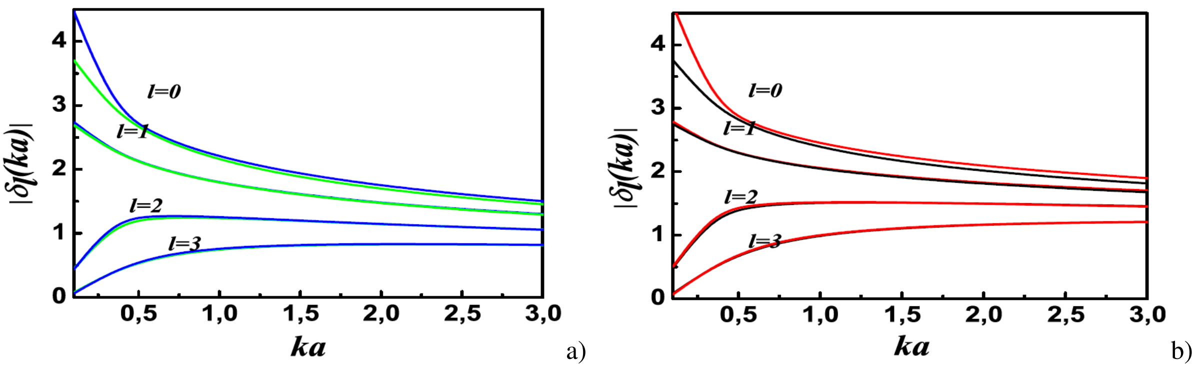

Fig. 5 Phase shifts of electron-electron scattering, $\Gamma = 1$, $r_s = 10$. a) On the basis of the static potential: blue(PFM) and green (WKB). b) On the basis of the dynamic potential: red (PFM) and black (WKB).

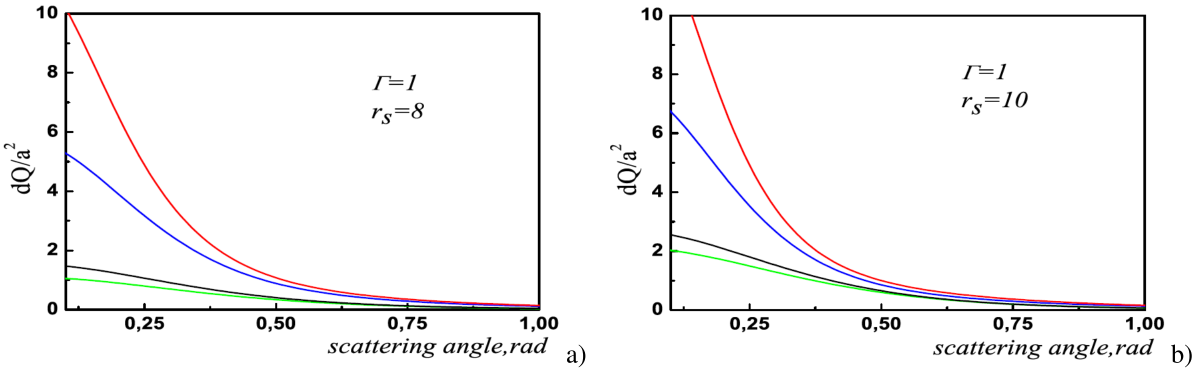

Fig. 6 Differential cross sections for electron-electron scattering. On the basis of the static potential: blue(PFM) and green (the Born method). On the basis of the dynamic potential: red (PFM) and black (the Born method). a) $\Gamma = 1$, $r_s = 8$; b) $\Gamma = 1$, $r_s = 10$.

Figures 5 a,b show the phase shifts obtained on the basis of the phase function method (PFM) and in the semiclassical approximation (WKB) for the potential (1) and (3). As one can see results of the both methods tend to each other with increase of orbital quantum number.

Figures 6 a,b present differential cross sections of electron-electron scattering in the Born and PFM approximations. As one can see there are significant differences between results of these two methods. It once more indicates the importance of using of quantum mechanical description for scattering processes.

Figures 7 and 8 show the plots of the total scattering cross sections and transport cross sections on the basis of dynamic and static interaction models.

It has been found that the inclusion of the dynamic screening leads to an increase in total scattering cross section (Figure 7), due to the increase of the partial cross sections.

Figure 8 shows the transport cross sections of electron-electron scattering for different values of the density parameter r_s. It can be seen that with increasing r_s (decrease in density) transport cross sections obtained in the framework of both models converge. It is caused by the weakening of the screening in a rarefied plasma.

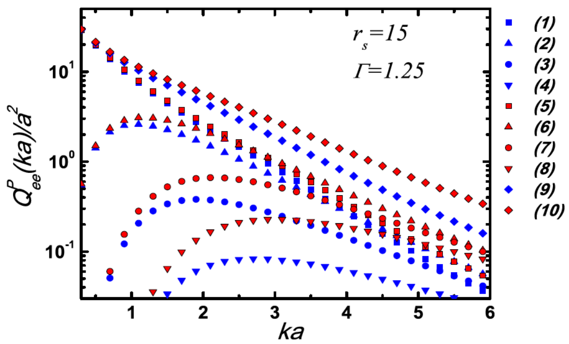

Fig. 7 Total and partial cross sections for electron-electron scattering on the basis of the static and dynamic potentials. On the basis of the static potential: 1) $\ell = 0$; 2) $\ell = 1$; 3) $\ell = 2$; 4) $\ell = 3$; 9) the total cross section; on the basis of the dynamic potential: 5) $\ell = 0$; 5) $\ell = 1$; 6) $\ell = 2$; 8) $\ell = 3$; 10) the total cross section.

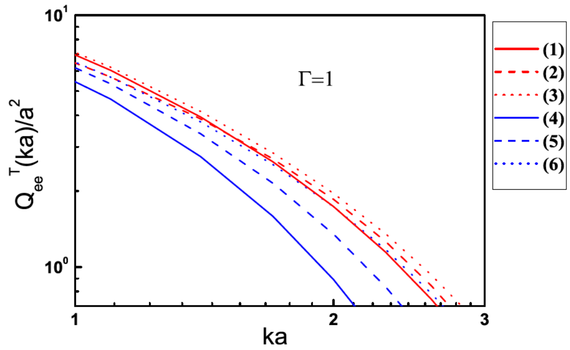

Fig. 8 Transport cross sections for electron-electron scattering on the basis of the static and dynamic potentials. On the basis of the dynamic potential: 1) $r_s = 6$; 3) $r_s = 9$; 5) $r_s = 12$; on the basis of the static potential: 2) $r_s = 6$; 4) $r_s = 9$; 6) $r_s = 12$.

3 Conclusion

Based on the dynamic model of the charged particles interaction in nonideal semiclassical plasma phase shifts, partial, total and transport scattering cross sections of the plasma particles were investigated. Quantum mechanical method has been used for their calculation. Analysis of the results showed that the phase shifts, as well as the cross section of electron-electron scattering obtained with taking into account the dynamic screening of the field are larger than the data obtained with consideration of static charge screening.

The results can be used to calculate the various transport coefficients of the semiclassical dense plasma. Knowledge of these characteristics plays a major role in the design of technical installations associated with the use of the dense nonideal plasma, for example, inertial confinement fusion facilities.

References

[1] T.S. Ramazanov and K.N. Dzhumagulova, Phys.Plasmas **9**, 3758 (2002).
[2] T.S. Ramazanov, K.N. Dzhumagulova, and M.T. Gabdullin, Phys. Plasmas **17**, 042703 (2010).
[3] T.S. Ramazanov, K.N. Dzhumagulova, and Y.A. Omarbakiyeva, Phys.Plasmas. **12**, 092702 (2005).
[4] Zh.A. Moldabekov, T.S. Ramazanov, and K.N. Dzhumagulova, Contrib. Plasma Phys. **52**, 207 (2012).
[5] T.S. Ramazanov, K.N. Dzhumagulova, and Y.A. Omarbakiyeva, Contrib. Plasma Phys. **47**, 267 (2007).
[6] T.S. Ramazanov, K.N. Dzhumagulova, and Y.A. Omarbakiyeva, J. Phys. A: Math. Gen. **39**, 4369 (2006).
[7] Ch.-G. Kim and Y.-D. Jung, Phys. Plasmas **19**, 014502 (2012).
[8] K.N. Dzhumagulova, G.L. Gabdullina, and E.O. Shalenov, Physics of Plasmas **20**, 042702 (2013).

[9] D. Kremp, M. Schlanges, and W.-D. Kraeft, "Quantum Statistics of Nonideal Plasmas", Springer, Berlin (2005).
[10] C. Deutsch, Phys. Letters A **60**, 317 (1977).
[11] C. Deutsch, Y. Furutani, and M.M. Gombert, Phys. Rep. **69**, 85 (1981).
[12] L.D. Landau and E.M. Lipschitz, "Quantum mechanics (nonrelativistic theory)", Nauka, Moscow (1989).
[13] G.F. Drukarev, "The Theory of Electron-Atom Collisions", Translated from the Russian edition (1962) by S. Chomet. J. B. Hasted, Translation Ed. Academic Press, New York, 1965.
[14] T.S. Ramazanov, K.N. Dzhumagulova, and A.Zh. Akbarov, J. Phys. A: Math. Gen. **39**, 1 (2006).
[15] T.S. Ramazanov, K.N. Dzhumagulova, M.T. Gabdullin, and A.Zh. Akbarov, J. Phys. A: Math. Theor. **42**, 214049 (2009).
[16] T.S. Ramazanov, K.N. Dzhumagulova, M.T. Gabdullin, A.Zh. Akbarov, and G.H. Nigmetova, Contrib. Plasma Phys. **47**, 262 (2007).

