ABSTRACTS

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Optimization method of solving of the Gel'fand-Levitan equation

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We consider the inverse problem of determining the coefficient of $q(x)$ from equations

$$u_{tt} = u_{xx} - q(x)u, \quad x \in R, \quad t > 0 \tag{1}$$

$$u|_{t=0} = 0, \quad u_t|_{t=0} = \delta(x) \tag{2}$$

$$u(0,t) = r(t), \qquad t \geq 0 \tag{3}$$

This problem can be reduced to the equation of I.M. Gel'fand and B.M. Levitan [2].

$$\frac{1}{2}\left[r(t-x) + r(t+x)\right] + \int_{-x}^{x} r(t-\tau)w(x,\tau)d\tau = 0 \tag{4}$$

We use the minimization method to solve equation (4). We rewrite equation (4) in operator form

$$Ag = f \tag{5}$$

Here $f(t) = -\frac{1}{2}\left(r(t-x) + r(t+x)\right), \; g(s) = w(x,\tau).$

We will find the pseudo-solution g of the integral equation (5) minimizing the cost function

$$J(g) = \left\| Ag - f \right\|^2 \rightarrow \min, \tag{6}$$

We compare two methods of solving (4) namely, the method of conjugate gradient and Cholesky decomposition, which is a decomposition of a Hermitian, positive-definite matrix into the product of a lower triangular matrix and its conjugate transpose. When it is applicable, the Cholesky decomposition is roughly twice as efficient as the LU decomposition for solving systems of linear equations. In a loose, metaphorical sense, this can be thought of as the matrix analogue of taking the square root of a number.

The conjugate gradient method is an iterative method, so it can be applied to sparse system that are too large to be handled by direct methods such as the Cholesky decomposition.

To solve (5). We apply the conjugate gradient method

$$g_{n+1}(s) = g_n(s) - \alpha_n p_n, \;\; g_n \in Q, \;\; \alpha_n > 0 \tag{7}$$

Here

$$p_n = J'g_n + \left\| J'g_n \right\|^2 \left\| J'g_{n-1} \right\|^{-2} g_{n-1},$$

Gradient of functional $J(g)$

$$J'g_n = 2A^*\left(Ag_n - f\right).$$

Can be found using the representation of the adjoin operator

$$J'g_n = 2\int_{-x}^{x} r(\xi - s)\int_{-x}^{x} r(\xi - \beta) g_n(\beta) d\beta d\xi - 2\int_{-x}^{x} r(\xi - s) g(\xi) d\xi. \quad (8)$$

We denote

$$I_\beta(\xi) = \int_{-x}^{x} r(\xi - \beta) g_n(\beta) d\beta. \quad (9)$$

We repalce the integral quadrature sum of the trapezoid rule

$$I_\beta(\xi) \approx 0.5 \sum_{j=-n+1}^{n} \left(r(\xi - \beta_j) g_n(\beta_j) + r(\xi - \beta_{j-1}) g_n(\beta_{j-1}) \right) h$$

We denote

$$I(s) = \int_{-x}^{x} r(\xi - s) I_\beta(\xi) d\xi. \quad (10)$$

We repalce the integral (10) the sum of the square

$$I(s) \approx 0.5 \sum_{i=-n+1}^{n} \left(r(\xi_i - s) I_\beta(\xi_i) + r(\xi_{i-1} - s) I_\beta(\xi_{i-1}) \right) h$$

We get similary, using a quadrature formula

$$I_h(s) = \int_{-x}^{x} r(\xi - s) g(\xi) d\xi \approx 0.5 \sum_{i=-n+1}^{n} h\left(r(\xi_i - s) g(\xi_i) + r(\xi_{i-1} - s) g(\xi_{i-1}) \right).$$

We obtain

$$J'g_n = 2I(s^n) - 2I_h(s^n). \quad (11)$$

The report presents the results of the calculations.

References

1. S. I . Kabanikhin, Inverse and Ill-Posed Problems. Theory and Applications. De Gruyter, Germany, 2011, 459 pp.
2. I. M. Gelfand and B. M. Levitan, On the determination of a differential equation from its spectral function, Izv. Akad. Nauk. SSSR Ser. Math. 15 (1961), 309-360. (in Russian)
3. S. I. Kabanikhin, A. D. Satybaev, M. A. Shishlenin Direct methods of solving inverse hyperbolic problems. 2004. VSP/BRILL, the Netherlands. 179 pp.

Short-Bio

Temirbekova Laura is a PhD student of second year specialty Mathematics from the Al-Farabi Kazakh National University, Almaty, Kazakhstan. Since 2010, she deals with the Gel'fand - Levitan equation of solving the inverse problem for hyperbolic equations. She graduated the Mechanics and Mathematics department of the Al-Farabi Kazakh National University (specialty Mechanics) in 2007. Masters degree has been obtained in the East Kazakhstan State Technical University named after D.Serikbaev (specialty Mathematics).