

Bənəvəb E.

MINISTRY OF EDUCATION OF THE REPUBLIC OF AZERBAIJAN

ABSTRACTS

of the IV Congress of the

TURKIC WORLD MATHEMATICAL SOCIETY

1-3 July, 2011



The Ministry of Education
of the Azerbaijan Republic



Azerbaijan National
Academy of Science



Baku State University



Institute of Applied
Mathematics BSU

BAKU-2011



WEIGHTED ESSENTIALLY NON OSCILLATORY SCHEMES FOR SUPERSONIC AIR FLOW

A. Abdalla, A.Zh. Naimanova, A. Kaltayev, Ye. Belyayev
Institute of Mathematics MES RK, Almaty, Kazakhstan
Kazakh National University al- Farabi, Almaty, Kazakhstan
e-mail: Hassan.Amr@kaznu.kz

In this work a planar supersonic turbulent multi-species flow is numerically simulated in the channel with perpendicular injection of hydrogen from slots located symmetrically on the lower and upper walls of the channel. Height of the channel is taken as 3cm, length 10cm. The slots of the width 0.1cm is located on distance 5cm from entrance. For the mathematical modelling of such flows the Favre-averaged Navier-Stokes equations for multi-species gas are used. For convenience of calculation the jet injection is considered from the bottom wall.

The WENO (Weighted Essentially Non oscillatory) scheme of 4th order has been used to approximate the inviscid fluxes of the system of the Favre-averaged Navier- Stokes equations. This scheme is based on the idea of ENO (Essentially Non oscillatory) - scheme developed in [1], which was constructed according to a principle of Godunov's scheme. Instead of choosing the smoothest stencil for interpolating polynomial for the ENO here was used a convex combination of all candidates to achieve the essentially non-oscillatory property. The algebraic Baldwin-Lomax's [2] turbulence model has been used to calculate the eddy viscosity coefficient. The boundary conditions are taken as: adiabatic no-slip walls at the bottom, supersonic inlet at the left boundary and sonic inlet on the slot; on the top condition of symmetry; on an outlet non- reflection condition [3]. In regions of large gradients, that is in the boundary layer, near the wall and at the slot level, the grid clustering is introduced. Then the system of the Navier- Stokes equations was formed in transformed generalized coordinates system. The computations were done on a staggered spatial grid with parameters: $2 \leq M_\infty \leq 4$, $M_0 = 1$, $Pr = 0.7$, $2 \leq n \leq 15$, here index " ∞ " concerns to values of parameters of a flow, index "0" - to values of parameters of a jet, $n = P_0/P_\infty$, where P_0 -pressure in jet and P_∞ -pressure in a flow.

For testing the method a following numerical experiment was performed: the sound hydrogen jet was injected perpendicularly to the main air flow through a slot of width 0.1 cm with parameters: $M_\infty = 3.75$, $M_0 = 1$, $Re = 62.73 \times 10^6$, $n = 10.29$, $T_0 = 800K$, and $T_\infty = 629.34K$, the results are compared with solution obtained by using ENO scheme [1] and experimental work [4], it gave good agreements. The numerical experiment showed, that because of deceleration of the incoming flow, the pressure ahead of the jet increases, and a bow shock wave is formed. An oblique shock wave emanates upstream from the bow shock wave. In addition to the separation region, there is also a supersonic flow region behind the oblique shock wave; subsequent flow deceleration is accompanied by emergence of a second shock wave: barrel shock parallel to the jet axis. The detached shock wave, oblique shock wave, and barrel shock wave intersect at one point to form a complicated λ -shape structure of shock waves.

REFERENCES

- [1] A.O. Beketaeva, A.Zh. Naimanova. Application of ENO (Essentially nonoscillatory) scheme for modelling the flow of a multicomponent gas mixture // Journal of Computational Technologies, Vol.12, No.4, 2007, pp. 17-25.
- [2] C.A. Fletcher. J. Computational Technique for Fluid Dynamics // M.: World, Vol.2, 1991, pp. 474.
- [3] Kevin W. Thompson. Time dependent boundary conditions for hyperbolic systems // Journal of computational physics, Vol.68, 1987, pp. 1-24.
- [4] C.F. Chenault, P.S. Beran. $K - \epsilon$ and Reynolds Stress Turbulence Model Comparisons for Two-Dimensional Injection Flows // AIAA Journal, Vol.36, 1999, pp. 1401-1412.



TURBULENCE MODELING IN SIMULATION OF SUPERSONIC FLOW WITH PERPENDICULAR INJECTION OF THE GAS

A. Naimanova, A. Kaltayev, Ye. Belyayev, A. Abdalla
Institute of Mathematics, Ministry of Education and Science, Almaty, Kazakhstan
Al-Farabi Kazakh National University, Almaty, Kazakhstan
e-mail: Yerzhan.Belyaev@kaznu.kz

Numerical study of perpendicular injection jets in supersonic flow were conducted by many researchers. In practice, the problem of interaction of the gas jet with a supersonic flow is a major problem in the simulation of supersonic combustion chambers of scramjet engines. The flow field in such devices is very complex: the turbulent fuel-air mixing, chemical reactions, shock waves, separation region ahead of the jet and behind of it. One of the important problems in the correct physical description and in numerical realization of such tasks is the turbulence modeling, namely, definition of influence of turbulence on shock wave, supersonic and subsonic regions of the flow. The given characteristics are well described with the Jones-Launder k-e turbulent model with and without compressibility correction, where Navier-Stokes equation for k and e consist of low-Reynolds number terms for near-wall turbulent boundary layer modeling in subsonic region and compressibility correction in turbulent kinetic energy equation for supersonic region.

In this work the numerical simulation of supersonic air flow with perpendicular injection of sonic hydrogen jet in 2D channel is investigated. A mathematical model of this process is described by two-dimensional Favre-averaged Navier-Stokes equations for multi-component gas flow. Turbulence effects are modeled with algebraic Baldwin-Lomax's model, Jones-Launder k-e model with and without compressibility correction. The boundary conditions are taken as: supersonic inlet at the left boundary, adiabatic no-slip walls at the bottom, sonic inlet on the slot; on the top condition of symmetry; on an outlet non-reflection condition. The following parameters were taken at the left boundary and on the slot: $M_0 = 1$, $T_0 = 642$, $M_\infty = 3,75$, $T_\infty = 800$, $n = 10,26$. The space grid was taken as 241×181 , the height of the channel - 7,62 cm, and length - 15 cm, width of slot 0,0559 cm. The slot was located on distance of 10 cm from entrance.

The comparative analyses of influence of turbulence models (algebraic Baldwin-Lomax's model, Jones-Launder k-e model with and without compressibility effect) on the surface pressure profiles, supersonic and subsonic zones, recirculation zones was made. The qualitative agreement of results is obtained, namely, boundary layer separation regions ahead of the jet and behind of it, and barrel structure of the expanding jet. The quantitative difference in results is seen in near-jet region.

Numerical experiments show, the constructed numerical model and computer code for studied the turbulent supersonic multi-species flow with different turbulent models allows to study influence of parameters to the shock wave structure and character of jet-supersonic flow interaction.

REFERENCES

- [1] D.P. Rizzetta. Numerical Investigation of Supersonic Wing-Tip Vortices // 26th AIAA Fluid Dynamics Conference, 1995.
- [2] F. Clarence. Chenault and Philip S. Beran, k-e and Reynolds Stress Turbulence Model Comparisons for Two-Dimensional Injection Flows // AIAA Journal, Vol.36, No.8, August 1998
- [3] A.O. Beketaeva, A.Zh. Naimanova. Numerical simulation of a supersonic flow with transverse injection of jets // Journal of Applied Mechanics and Technical physics, Vol.45, 2004, pp. 367-374.