



**The 28th International Conference  
on Applications of Computer Algebra ACA'2023**

# **PROGRAM & ABSTRACTS**

**Warsaw University of Life Sciences – SGGW  
Institute of Information Technology  
July 17 – 21, 2023**

WWW: <https://aca2023.iit.sggw.pl>

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## ACA2023 – General Schedule

Times	Monday	Tuesday	Wednesday	Thursday	Friday									
09:00 – 09:30														
09:30 – 10:00	Registration, 3rd floor Aula IV Build. 34	S6	S9	S10	S3	S8	S4	S6	S8	S3	S10	S2		
10:00 – 10:30		Coffee break (room 3/82)												
10:30 – 11:00		Plenary Lecture Jon McLoone			Plenary Lecture Werner M. Sieler			Plenary Lecture Adam Strzeboński			Closing			
11:00 – 11:30		OPENNING Aula IV												
11:30 – 12:00	Lunch (Limba)													
12:00 – 12:30	Lunch (Limba)													
12:30 – 14:00	Lunch (Limba)													
14:00 – 14:30	S1	S7	S9	S6	S2	S10	S3	S8	S2	Excursion: Warsaw City Center, Royal Castle;  Conference dinner at the Green GardenHotel Restaurant 19:00-22:30				
14:30 – 15:00	Coffee break (room 3/82)													
15:00 – 15:30	Coffee break (room 3/82)													
15:30 – 16:00	Coffee break (room 3/82)													
16:00 – 16:30	S1	S7	S9	S1	S9	S10	S10	S8	S2	Excursion: Warsaw City Center, Royal Castle;  Conference dinner at the Green GardenHotel Restaurant 19:00-22:30				
16:30 – 17:00	Coffee break (room 3/82)													
17:00 – 17:30	Coffee break (room 3/82)													
17:30 – 18:00	Coffee break (room 3/82)													
18:00 – 19:00	Welcome reception (Limba)			ACA-WG meeting Aula IV			ACA-WG meeting Aula IV			Excursion: Warsaw City Center, Royal Castle;  Conference dinner at the Green GardenHotel Restaurant 19:00-22:30				

S1 – Computer Algebra in Education

S2 – Computer Algebra Modeling in Science and Engineering

S3 – D-Finite Functions and Beyond: Algorithms, Combinatorics  
and Arithmetic

S4 – Computer Algebra Systems and Interval Methods

S6 – Computer Algebra Applications in the Life Sciences

S7 – Computational Differential and Difference Algebra and its Applications

S8 – Algebraic Geometry from an Algorithmic Point of View

S9 – Effective Ideal Theory and Combinatorial Techniques in Commutative  
and Non-Commutative Rings and Their Applications

S10 – Algebraic and Algorithmic Aspects of Differential and Integral Operators

## Schedule for Invited Talks

### Tuesday, July 18, 2023

Build. 34, 3d floor, Lecture Hall "Aula IV"

11:30 – 12:30 **Jon McLoone**

Wolfram's Vision for Unified Computation

### Wednesday, July 19, 2023

Build. 34, 3d floor, Lecture Hall "Aula IV"

11:30 – 12:30 **Werner M. Seiler**

Theoretical and Numerical Analysis of Singular Initial  
and Boundary Value Problems

### Thursday, July 20, 2023

Build. 34, 3d floor, Lecture Hall "Aula IV"

11:30 – 12:30 **Adam Strzebonski**

Recent Symbolic Computation Developments in Mathematica

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## The problem of many bodies with isotropically varying masses

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The number of confirmed exoplanetary systems is more than 4000 to date [1] and it is growing up every day. The parent star and exoplanets are non-stationary [2]. It means that the investigation of a multi-planetary system with variable masses is actual in celestial mechanics and astronomy. Due to the non-stationarity of celestial bodies, the mathematical model of their motion becomes more complicated.

In the present talk, we investigate the dynamic evolution of the system of many bodies with isotropically varying masses. We apply the method of canonical perturbation theory developed for solutions of such non-stationary problems in [3]. Doing quite cumbersome symbolic calculations with the computer algebra system Wolfram Mathematica [4], we calculated the perturbing function in the form of power series in small parameters (analogues of eccentricities and inclinations). Averaging the perturbing function over the mean longitudes and computing its derivatives with respect to the canonical variables, we derived the evolution equations describing the secular perturbations of the orbital elements in analytical form [5]. As an example, we have considered the K2-3 exoplanetary system (see [6]) and obtained numerical solutions of the evolution equations.

**Keywords:** four body problem, variable mass, dynamic evolution, secular perturbations

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### References

- [1] EXOPLANET EXPLORATION, *url:https://exoplanets.nasa.gov/*. Last update: May 11, 2023.
- [2] T.B. OMAROV, Non-Stationary Dynamical Problems in Astronomy. *New-York: Nova Science Publ.Inc. P.260. ISBN:1-59033-331-4, (2002).*

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