The Problem of Tree-Body-Points with Masses, Changing Isotropically in Different Specific Rates

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The problem of three-body-points under Newton interaction is considered. Masses of bodies are assumed to be comparable, but the laws of mass changing are arbitrary. These masses are change isotropically in different specific rates:

$$m_0 = m_0(t), \quad m_1 = m_1(t), \quad m_2 = m_2(t), \quad \frac{\dot m_i}{m_i} \neq \frac{\dot m_j}{m_j}, \quad i \neq j. \tag{1}$$

Equations of motion in the Jacobi coordinates are given as:

$$\mu_1 \ddot{\vec r}_1 = \mathrm{grad}_{\vec r_1} U, \quad \mu_2 \ddot{\vec r}_2 = \mathrm{grad}_{\vec r_2} U - \mu_2 (2\dot\nu_1 \dot{\vec r}_1 + \ddot\nu_1 \vec r_1), \tag{2}$$

$$\mu_1 = \mu_1(t) = \frac{m_1 m_0}{m_0 + m_1} \neq const, \quad \mu_2 = \mu_2(t) = \frac{m_2(m_1 + m_0)}{m_0 + m_1 + m_2} \neq const, \tag{3}$$

where μ_i are the reduced masses, $\nu_1 = \nu_1(t) = m_1/(m_0 + m_1)$, U is the power function [1]. In general, non-autonomous differential equations (1) do not have any integral in contrast to the classical problem of three bodies with constant masses.

Aperiodically motion on quasiconic section is used as an initial unperturbed intermediate motion. Eccentricities and inclinations of the orbits of bodies are the small values. Equations of secular perturbations in the analogues of the second system of the Poincare elements [1], [2] have the form:

$$\dot\xi_i = \frac{\partial R_{sec}}{\partial \eta_i}, \quad \dot p_i = \frac{\partial R_{sec}}{\partial q_i}, \quad \dot\eta_i = -\frac{\partial R_{sec}}{\partial \xi_i}, \quad \dot q_i = -\frac{\partial R_{sec}}{\partial p_i}, \quad i = 1, 2. \tag{4}$$

here R_{1sec}, R_{2sec} are the corresponding expressions of the secular disturbing functions:

$$R_{1sec} = -\frac{1}{\gamma_1^2(t)} \cdot \frac{\beta_1^4}{2\mu_{10}\Lambda_1^2} + \frac{1}{\nu_1}\left[-\frac{b_1}{2}\frac{\gamma_1^2 \mu_0^2}{\Lambda_1^2}\left(1 + \frac{3}{2}\frac{\xi_1^2 + \eta_1^2}{\Lambda_1}\right) + F_{sec}\right], \tag{5}$$

$$R_{2sec} = -\frac{1}{\gamma_2^2(t)} \cdot \frac{\beta_2^4}{2\mu_{20}\Lambda_2^2} + \frac{1}{\nu_2}\left[-\frac{b_2}{2}\frac{\gamma_2^2 \mu_0^2}{\Lambda_2^2}\left(1 + \frac{3}{2}\frac{\xi_2^2 + \eta_2^2}{\Lambda_2}\right) + F_{sec}\right] -$$
$$- \frac{\mu_2}{\nu_2}\left[(2\dot\nu_1 \dot x_1 + \ddot\nu_1 x_1)\, x_2 + (2\dot\nu_1 \dot y_1 + \ddot\nu_1 y_1)\, y_2 + (2\dot\nu_1 \dot z_1 + \ddot\nu_1 z_1)\, z_2\right]_{sec}. \tag{6}$$

where $\tilde{\beta}_i$ are constant depending on the initial values of masses, $\gamma_i(t)$, $b_i(t)$, $\psi_i(t)$ are function of time depending on the laws of mass changing.

Main complication is consist of finding expression:

$$F_{sec} = [m_1 m_2 / r_{12}]_{sec}. \tag{7}$$

Secular part of perturbed function which is obtained by the computer algebra system *Mathematica* [3] are very cumbersome and difficultly foreseeable. Therefore, expression (7) can be written as:

$$F_{sec} = \sum_{j=1}^{NM} \Pi_j^*(t) P_j(\ni_k) + \sum_{j=1}^{3} \Pi_j(\Lambda_1, \Lambda_2, t). \tag{8}$$

In expression of the perturbing function (8) the members are preserved including the second degree inclusively concerning small values. And even in this case analytical calculations are very cumbersome and difficultly foreseeable.

Under these assumptions the expression of secular part of the perturbing function in analogues of the second system of the Poincare elements are obtained. Equations of secular perturbations are obtained in explicit form. Solutions of these equations by the Picard method in the first approximation have the structure:

$$\ni_k(t) = \ni_k(t_0) + \sum_j P_j(\ni_j(t_0)) \int_{t_0}^{t} \Pi_j^*(t)dt, \quad \ni_k(t_0) = \ni_k(t)|_{t=t_0} = const, \tag{9}$$

where $\ni_k$ are elements ξ_i, η_i, p_i, q_i. Solutions (9) give possibility to analyze [1], [4] the changes of eccentricities e_i, inclinations of orbits i_i, argument of pericentres ω_i and motions of ascending nodes longitude Ω_i, longitude of pericentres π_i. In particular, we obtain:

$$e_i^2 = \frac{\xi_i^2 + \eta_i^2}{\Lambda_i}, \quad \sin^2 i_i = \frac{p_i^2 + q_i^2}{\Lambda_i}, \quad i = 1, 2. \tag{10}$$

Analogously we obtain simple formulas for the rest of osculating elements.

References

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