THE SYNTHESIS OF THE PROPORTIONAL-DIFFERENTIAL REGULATORS FOR LINEAR SYSTEMS WITH CONSTRAINTS

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Introduction. Optimal control problem for nonstationary linear systems under the external influences is considered. The maximum principle provides the basis for program control construction when solving optimal control problem, and it reduces the solving to the appropriate two point boundary value problem. Solution of the same problem in the form of optimal feedback control synthesis is based on the method of dynamic programming. Development of the various ways for construction of the proportional-integral-differential regulators and the appropriate algorithms for control possessing the properties required for application is topical problem of the modern information technology.

Statement of the problem. Consider the following linear controllable system:

$$\dot{x} = A(t)x + B(t)u + f(t), \quad t \in (t_0, T), \quad x(t_0) = x_0, \tag{1}$$

$$u(t) \in U(t) = \Big\{ u \;\big|\; \alpha(t) \le u(t) \le \beta(t), \quad t \in (t_0, T) \Big\}, \tag{2}$$

where $x(t)$ is the state vector function of order n , $u(t)$ is the $m \times 1$ control vector function.

Let the functional of control function, state of the object and its derivative

$$J(\dot{x}, x, u) = \frac{1}{2} \int_{t_0}^{T} \Big[\dot{x}^* D(t) \dot{x} + x^* Q(t) x + u^* R(t) u \Big] dt + \frac{1}{2} x^*(T) F x(T) \tag{3}$$

is given.

Problem: Find the feedback control such that the corresponding pair $\{\tilde{x}(t), \tilde{u}(t)\}$ minimizes functional (3) and satisfies differential equation (1) under constraint on control values (2).

The method based on Lagrange multipliers of a special form is used to solve optimal control problem (1)–(3) [1].

Theorem 1. *For the pair $(\tilde{x}(t), \tilde{u}(t))$ to be optimal for problem (1)–(3) it is necessary and sufficient that $\tilde{x}(t)$ be chosen such that*

$$\dot{x} = A_1(t)x(t) - B_1(t)q(t) + B(t)\phi(x,t) + (E - B_1(t)D)f(t), \quad x(t_0) = x_0. \tag{4}$$

The control $(\tilde{u}(x(t), t)$ is determined by

$$\tilde{u}(x(t), t) = -R^{-1}(t)[B^*(t)(D(t)\dot{x} + K(t)x(t) + q(t)) - \lambda_1(t) + \lambda_2(t)], \tag{5}$$

where the function $\lambda_0 = K(t)x + q(t)$ ensures that constraints in the form of system of differential equations (1) hold, and the summands $\{\lambda_1(t), \lambda_2(t)\}$ provide that the control function satisfies condition (2).

References

[1] Murzabekov Z.N., Murzabekov A.Z. (2012) Optimizatsiya odnogo klassa upravlyaemykh nelineinykh system na konechnom otrezke vremeni. *Problemy informatiki.* Sibirskoe otdelenie RAN, s. 5-9.