MINISTRY OF EDUCATION OF THE PERUBLIC OF AZERBAIJAN

ABSTRACTS

of the IV Congress of the

TURKIC WORLD MATHEMATICAL SOCIETY

1-3 July, 2011

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The book contains abstract of the participants of the IV Congress of the Turkic World Mathematical Society.

The book will be useful for the specialists in the field of Mathematics and its applications, as well as for the students in Mathematics and Information Technologies.

Preface

The Turkic World Mathematical Society (TWMS) held its 4th Congress in July 1-3, 2011 in Baku, Azerbaijan. The aim of the Congress was to provide a forum where scientists and mathematicians from academia and industry can meet to share ideas of latest research work in wide branches of mathematics. The Turkic World Mathematical Society (TWMS) was founded in 1999 and unites mathematicians from Turkey, Kazakhstan, Azerbaijan, Kyrgyzstan, Uzbekistan, Turkmenistan and also Russia, Iran, China, USA and European countries. It had been held three congresses of this society since 1999: the First Congress of the TWMS in Firat University (Elazig, Turkey) in 1999, the Second Congress of the TWMS in Sakarya University (Adapazari, Turkey) in 2007, the Third Congress of the TWMS in al-Faraby Kazakh National University (Almaty, Kazakhstan) in 2009.

This issue of abstracts was presented at the IV Congress of the TWMS. The Congress is organized by the Ministry of Education of Azerbaijan Republic, Institute of Applied Mathematics of Baku State University with the collaboration and support of the Azerbaijan National Academy of Sciences. The 562 abstracts presented during three Congress days. More than 650 participants from more than 20 countries including France, Iran, Turkey, Uzbekistan, USA, Russia, Ukraine, Kazakhstan, Turkmenistan, Azerbaijan, Kyrgyzstan, Germany, Latvia, Italy, Czech Republic, India, Pakistan, Spain participated in the Congress. The programm contained 9 invited talks, selected by the International Program Committee, 470 contributions were selected for oral presentation. Congress included 10 topics.

Organizing and Program Committees of the IV Congress of the Turkic World Mathematics Society established competition in three nominations.

1. One award for the best young (under 30) participant's talk.
2. One award for the best talk on theoretical mathematics.
3. One award for the best talk on applied mathematics.

All submitted papers were reviewed by two independent reviewers. The selected papers will be published in: "Applied and Computational Mathematics" (ISSN 1683-3511, indexed in Scopus and in Science Citation Index Expanded, www.science.az/acm), "TWMS Journal of Pure and Applied Mathematics (ISSN 2076-2585)" and "TWMS Journal of Applied and Engineering Mathematics (ISSN 2146-1147). We wish to thank all the authors for their co-operation. We also wish to mention with appreciation the significant role of the reviewers from the international community whose diligent contribution led to the successful completion and publishing of this special edition. Many people contributed time and effort to make the Congress a success: The authors and speakers have prepared a great collection of high-quality contributions, the program committee spent time and effort reviewing the submissions, the members of the organizing committee all took on additional responsibilities, and many student volunteers helped with practical aspects.

We thank the Ministry of Education of Azerbaijan Republic for major funding for the Congress; without their support the meeting can not have taken place. We thank also the Baku State University and the Azerbaijan National Academy of Sciences for his support. I would like to thank the members of the Organizing Committee, the International Advisory Committee, the International Program Committee and the Local Committee for organization and successful passing of the Congress.

Aliev Fikret
Vice-president of TWMS
Editor in Chief

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SOME CHARACTERIZATIONS OF RIESZ SETS

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Let G be a compact abelian group equipped with its normalized Haar measure, L^1G its group algebra and $M(G)$ its measure algebra. For a subset E of the dual group $\hat{G}$, let $L^1_E(G) = \{a \in L^1(G) : \hat{a}(\gamma) = 0 \ \ for \ \gamma \in \hat{G} \backslash E\}$ and $M_E(G) = \{a \in M(G) : \hat{a}(\gamma) = 0 \ \ for \ \ \gamma \in \hat{G} \backslash E\}$. These are ideals of the algebras L^1G and $M(G)$, respectively. If the equality $L^1_E(G) = M_E(G)$ holds (via Radon-Nikodym Theorem) the set E is said to be a Riesz set. In this talk we present a series of characterizations of Riesz sets.

POINCARE SERIES OF MULTI-INDEX FILTRATIONS, INTEGRATION WITH RESPECT TO THE EULER CHARACTERISTIC AND MONODROMY ZETA FUNCTIONS

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There are a number of observations when Poincare series of natural filtrations on rings of germs of functions are related (sometimes coincide) with certain monodromy zeta functions. These relations are not really understood. They are obtained by direct computations of their sides in one and the same terms and comparison of the obtained results. The definition of the Poincare series of a multi-index filtration is related with the notion of integration with respect to the Euler characteristic over the projectivization of the ring of germs of functions (which is an infinite dimensional space). Integration with respect to the Euler characteristic is also a useful tool for computing Poincare series and monodromy zeta functions. I'll discuss the basic notions and statements and also some recent results. The talk is based on joint works with A.Campillo, F.Delgado and W.Ebeling.

The 4th Congress of the Turkic World Mathematical Society (TWMS) Baku, Azerbaijan, 1-3 July, 2011

DEFINED MATHEMATICAL CONCEPT WITH VALUES IN THE EXTENDED SET OF REAL NUMBERS

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This article discusses the problem concerning the definition of some basic mathematical concepts on which there is no common view among mathematicians. Such problematic issues include: the concept of existence of the derivative, limit, continuity of functions at a point, as well as their associated definitions of certain mathematical objects, such as a tangent to the graph of the function, the concept of smoothness,etc. For example, consider the well-known definition of a smooth curve. Definition (see [1], §5.15). The function f is smooth on the interval$[a, b]$, if it has a continuous derivative on this interval. By this definition, for example, the function$y = \sqrt[3]{x}$,$x \in [a, b]$, $a < 0$, $b > 0$, is not smooth at a specified interval - in this case mathematics "unanimous." But because of its lack of smoothness of the mathematicians there is no single view: Some attribute this to the non-existence of the derivative and the other with a discontinuity of the derivative of the function at point $x = 0$.is that the cases $f'(x) = +\infty$ or $f'(x) = -\infty$, some mathematicians are seen for the "existence of the derivative function at a point x", but some believe that derivative is not here (there is also math, counting indefinitely without a specific character $f'(x) = \infty$, as "the existence of the derivative). In our approach, the function $y = \sqrt[3]{x}$ is smooth on the interval $[a, b]$, $a < 0$, $b > 0$! This is proved in the article, but here only the following conclusion: in the sense of this definition, function $y = x^3$, in an obvious way smooth, so that the inverse function $y = \sqrt[3]{x}$ must also refer to a class of "smooth". After all, if the metal wire shapes $y = x^3$ to rotate relative to the line $y = x$ on 180^0, her smoothness remain unchanged. In an article for a smooth curve we are three equivalent definitions of the three mathematical languages, some of the above mathematical concepts are defined in a different aspect than they exist in well-known textbook definition, and on the basis of these definitions are removed differences mathematicians on the above areas of concern.

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DIFFERENTIAL ALGEBRA OF BIQUATERNIONS BIWAVE EQUATIONS AND THEIR GENERALIZED SOLUTIONS

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The differential algebra of biquaternions is designed , which is very suitable for solving the broad class of the mathematical physics problems. This algebra is associative but not commutative.

With use of differential operator - mutual complex gradients (*bigradients*), which generalize a notion of the gradient on the functional space of biquaternions $\mathsf{B}(\mathsf{M})$ on the Minkovskiy space M we consider *biwave equation* on $\mathsf{B}(\mathsf{M}) = \{\mathbf{F} = f(\tau, x) + F(\tau, x)\}$, $(\tau, x) \in \mathsf{M}$, f is complex function and F is complex three dimensional vector-function:

$$\nabla^{\pm}\mathbf{B} = (\partial_\tau \pm i\nabla) \circ (b(\tau, x) + B(\tau, x)) =$$
$$= (\partial_\tau b \mp i \operatorname{div} B) + \partial_\tau B \pm i \operatorname{grad} B \pm i \operatorname{rot} B = \mathbf{F}(\tau, x)$$

Invariance of this equation for the group of the Lorentz transformations are proved.

Using the property of mutual bigradients:

$$\nabla^{-}\left(\nabla^{+}\mathbf{B}\right) = \nabla^{+}\left(\nabla^{-}\mathbf{B}\right) = \left(\nabla^{-} \circ \nabla^{+}\right)\mathbf{B} = \Box\mathbf{B},$$

where $\Box = (\frac{\partial^2}{\partial\tau^2} - \Delta)$ is the wave Dalamber's operator, the generalized decisions of biwave equations are built, including steady-state case and the case of stationary vibrations [1,2,3].

With use of bigradients it is possible the many systems of the equations of utter mediums to bring about solving the biwave equation. In particular, it is built the biwave equation, which is equivalent to the system of Maxwell equations and their generalized biquaternionic decisions have been built [3,4].

The equation of quantum mechanics - a matrix equation of Dirac is considered and its biquaternionic form is built:

$$\nabla^{\pm}\mathbf{B} + i\rho\mathbf{B} = \mathbf{F}(\tau, x), \quad \text{here } \rho \text{ is real constant.}$$

The generalized decisions of Dirac equation in biquaternionic form also are constructed. The biquaternionic form of spinors and spinors fields are presented.

On the base of this algebra in [3,5] biquaternionic model of electro-gravimagnetic fields and their interactions has been built.

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The 4th Congress of the Turkic World Mathematical Society (TWMS) Baku, Azerbaijan, 1-3 July, 2011

ALMOST TAME AUTOMORPHISMS OF TWO GENERATED FREE LEIBNIZ ALGEBRAS

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Recall that a Leibniz algebra $\mathfrak{g}$ over an arbitrary field K is a non-associative algebra with a bilinear bracket $[-,-]$ satisfying the Leibniz identity

$$[x,[y,z]] = [[x,y],z] - [[x,z],y].$$

The Leibniz algebras were first introduced in 1965 by A. Blokh [3] as a non-antisymmetric version of Lie algebras and were rediscovered in 1993 by J.-L. Loday [5]. These algebras are related in a natural way to several topics as differential geometry, homological algebra, classical algebraic topology, and noncommutative geometry [5].

Two generated free Leibniz algebras have wild automorphisms. It is proved in [1] that the automorphism

$$\sigma = (x + [[y,y],x], y)$$

of $LB\langle x,y\rangle$ the free Leibniz algebra in two variables x, y over K is not tame.

An automorphism φ of $LB\langle x,y\rangle$ is called *almost triangular* if $\varphi(y) = \underline{y}$ and an automorphism of $LB\langle x,y\rangle$ is called *almost tame* if it is a product of almost triangular and linear automorphisms. The automorphism σ gives an example of an almost tame but not tame automorphism of $LB\langle x,y\rangle$. Hence it seems natural to ask whether every automorphism of $LB\langle x,y\rangle$ is almost tame. Denote by $GL_2(K)$, $AT(LB\langle x,y\rangle)$, and $ATame(LB\langle x,y\rangle)$ the groups of all linear, almost triangular, and almost tame automorphisms of $LB\langle x,y\rangle$, respectively.

Theorem 1. *$ATame(LB\langle x,y\rangle) = GL_2(K) *_H AT(LB\langle x,y\rangle)$ where $H = GL_2(K) \cap AT(LB\langle x,y\rangle)$.*

This is an analogue of the well-known decomposition theorem for groups of automorphisms of polynomial algebras and free associative algebras in two variables [4]. *An analogue of this result for tame automorphisms of $LB\langle x,y\rangle$ is proved in* [2].

Theorem 2. *The automorphism $\delta = (x + f, y + g)$, where*

$$f = [[[x,x],[[y,x],x]],x] - [[[[y,x],x],[[y,x],x]],x],$$
$$g = [[x,x],[[y,x],x]] - [[[y,x],x],[[y,x],x]],$$

of the free two generated Leibniz algebra $LB\langle x,y\rangle$ is not almost tame.

The question about a "convenient" set of generators of $Aut(LB\langle x,y\rangle)$ is still open.

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The 4^{th} Congress of the Turkic World Mathematical Society (TWMS) Baku, Azerbaijan, 1-3 July, 2011

ON EXTENSION OF $\mathcal{P}$-SEQUENTIALLY CONTINUOUS MAPPING

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It was studied in [1] a sequentially continuous mapping and an extension of sequentially continuous mapping into sequentially complete uniform space. Here we consider a generalization of a sequentially continuous mapping and its extension into a uniform space with certain properties of completeness due to some ideas of Bernstein [2].

Suppose $\mathfrak{M}$ - is an infinite cardinal, and $\mathfrak{M} = \beta\mathfrak{M}\backslash\mathfrak{M}$ is Stone-Cech remainder of the discrete space of power $\mathfrak{M}$- or the space of all free ultrafilters on the set of power $\mathfrak{M}$. Below by $\mathfrak{M}$ we understand the first ordinal number of cardinality $\mathfrak{M}$. Let $p \in \mathfrak{M}^*$ and $\mathcal{P} \subset \mathfrak{M}^*$, where $\mathcal{P} \neq \emptyset$. Arbitrary set of points $\{x_\xi, \xi \in \mathfrak{M}\}$ in a topological space is called a $\mathfrak{M}$ -sequence. Following[2], we say that $x = p - \lim x_\xi$, or x is a $p-$ limit point of $\{x_\xi, \xi \in \mathfrak{M}\}$ in (X, τ) if for any neighborhood O_x of x we have$\{\xi, x_\xi \in O_x\} \in p$. If for any $p \in \mathcal{P}$, there is $x = p - \lim x_\xi$ then we say that the sequence $\{x_\xi, \xi \in \mathfrak{M}\}$ $\mathcal{P} - s$- converges to x, or $\{x_\xi, \xi \in \mathfrak{M}\}$ is $\mathcal{P} - s$ convergent and x is its $\mathcal{P} - s$ limit.

Theorem 1. *Suppose that* $X = \cup\{H_\alpha : \alpha \in \mathfrak{M}^+\}$, $H_\alpha \subset H_\beta$ *for any* $\alpha < \beta$ *and* $F : X \to Y$ *is a map with the following property:* $F|_{H_\alpha}$ *is a* $\mathcal{P} - s-$ *sequentially continuous mapping of* H_α *into* Y *for any* α. *Then* F*is a* $\mathcal{P} - s-$ *sequentially continuous mapping of* X *into* Y.

Suppose that $(X, \mathfrak{U})$ *is a uniform space. The sequence* $\{x_\xi, \xi \in \mathfrak{M}\}$ *in* $(X, \mathfrak{U})$ *is called a Cauchy* $\mathcal{P} - s-$ *sequence if for any cover* $\alpha \in U$ *there is a set* $V \in \alpha$ *such that* $\{\xi, x_\xi \in V\} \in p$ *for any* $p \in \mathcal{P}$.

Theorem 2. *Let* H *be a* $\mathcal{P}$*-s-sequentially dense subset in* $(X, \mathfrak{U})$ *and* (Y, V)*is a* $\mathcal{P}$*-s-sequentially complete uniform space. In order a* $\mathcal{P}$*-s-sequentially continuous by mapping* $f : H \to (Y, \tau_V)$ *could be* $\mathcal{P} - s-$ *sequentially continuous extended on* X *it is necessary and sufficient that for any* $\beta \in V$ *there exists* $\mathcal{P} - s$ *sequentially open cover* α *of* (X, τ) *such that* $\alpha \wedge \{H\}$ *is inscribed in cover* $f^{-1}(\beta) = \left\{f^{-1}(B);\ B \in \beta\right\}$.

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POWER SUM FOR BINOMIAL COEFFICIENTS

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For positive integers k, m and N set

$$f_{k,m}(N) = \sum_{i=0}^{N-1} \binom{i+k-1}{k}^m.$$

Well-known that $f_{1,m}(N) = \sum_{i=1}^{N-1} i^m$ is a polynomial in N of degree $m+1$. By Faulhaber theorem $f_{1,m}(x)$ is divisible by polynomial $f_{1,1}(x) = x(x-1)/2$. Moreover, for odd m it is divisible by $f_{1,1}(x)^2$ and remained divisor is a polynomial in $f_{1,1}(x)$; If m is even, then $f_{1,m}(x)$ can be presented as a polynomial of $f_{1,1}(x)$ multiplied by $(2x-1)$ (see [1]). We establish the following analog of Faulhaber theorem for a sum of powers of binomial coefficients.

Theorem 1. *The power sum $f_{k,m}(x)$ is a polynomial of degree $km+1$ with rational coefficients and there exist polynomials $Q_{k,m}(x) \in \mathbf{Q}[x]$, such that*

$$f_{k,m}(x) = \begin{cases} \binom{x+k-1}{k+1}^2 Q_{k,m}((2x+k-2)^2), & \text{if } m>1, k \text{ are odd numbers,} \\ \binom{x+k-1}{k+1}(2x+k-2)Q_{k,m}((2x+k-2)^2), & \text{if } k \text{ is odd and } m \text{ is even,} \\ \binom{x+k-1}{k+1} Q_{k,m}((2x+k-2)^2), & \text{otherwise.} \end{cases}$$

Let us consider negative power sum

$$\zeta(k,m) = \sum_{i=1}^{\infty} \binom{i+k-1}{k}^{-m}.$$

Theorem 2. *For any positive integers k and m there exist rational numbers $\lambda_0, \lambda_1, \ldots, \lambda_{\lceil m/2 \rceil}$ such that*

$$\zeta(k,m) = \begin{cases} \lambda_0 + \sum_{i=1}^{\lceil m/2 \rceil} \lambda_i \zeta(2i), & \text{if } km \text{ is even,} \\ \lambda_0 + \sum_{i=1}^{\lceil m/2 \rceil} \lambda_i \zeta(2i-1), & \text{if } km \text{ is odd.} \end{cases}$$

For example, for $k=2$ takes place the following formula

$$\frac{\zeta(2,m)}{2^m} = (-1)^{m-1}\binom{2m-1}{m} + (-1)^m \sum_{i=1}^{\lfloor m/2 \rfloor} \binom{2m+2i-1}{2i}(-1)^{i+1}\frac{(2\pi)^{2i}}{(2i)!}B_{2i}.$$

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CONSTRUCTION OF ANALOGS OF KRAMER DETERMINANTS FOR ALGEBRAIC SYSTEM

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Let the algebraic system be

$$\{a_{0,i}+\sum_{k=1}^{k_{1,i}}a_{1,k,i}x_1^k+\sum_{k=1}^{k_{2,i}}a_{2,k,i}x_2^k+...+\sum_{k=1}^{k_{n,i}}a_{n,k,i}x_n^k\}=0(i=1,2,...,n). \tag{1}$$

1-st index of factor $a_{r,k,i}$ specifies an index of a variable x_r;k there is a degree from which x_r entering in $i-$th the equation of system (1).

In (1) we shall introduce following labels: $k_r=\max k_{r,i}, i=1,2,...,k;$ $x_s^r=\lambda_{k_1+k_2+...+k_{s-1}+r}$, $a_{r,s,i}=d_{k_1+k_2+...+k_{s-1}+s,i}, r=1,2,...,n; s=1,2,...,k_r; i=1,2,...,n$ $a_{r,s,i}=d_{k_1+k_2+...+k_{s-1}+s,i}, r=1,2,...,n; s=1,2,...,k_r; i=1,2,...,n$

In new labels in tensor product $R\otimes R\otimes...\otimes R(n$ of times) the system (1) can be noted as

$$\left\{\sum_{r=0}^{n}\sum_{k=1}^{k_r}(d_{k_1+k_2+...+k_{s-1}+k,i}\ \lambda_{k_1+k_2+...+k_{s-1}+k})x_i=0; k_0=0; k_{-i}=0; i=1,2,...,n\right.$$

where $x_i\in R$ eigenvectors of $i-$th equation of last system.

Let's add system (1) by means of the auxiliary equations according to requirements of system (1).

$$\begin{array}{l}(t_2+\lambda_{k_1+k_2+...+k_{i-1}+1}t_0+\lambda_{k_1+k_2+...+k_{i-1}+k_i}t_1)x_{k_1+...+k_{i-1}+1}=0\\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\ (\lambda_{k_1+k_2+...+k_i-2}t_2+\lambda_{k_1+k_2+...+k_i-1}t_0+\lambda_{k_1+...+k_i}t_1)x_{k_1+k_2+...+k_i}=0; i=1,...,n;\end{array}, \tag{2}$$

where operators t_0,t_1,t_2 are set by means of matrixes

$$\overline{H}t_1=\begin{pmatrix}1&0\\0&0\end{pmatrix}, t_2=\begin{pmatrix}0&0\\0&1\end{pmatrix} \tag{3}$$

((1), (2)), considered together, form the multiparameter system consisting of $k_1+k_2+...+k_n$ the equations and containing $k_1+k_2+...+k_n$ parameters. Analogs of determinants of Cramer for the system (1) are obtained with help of expansions of corresponding determinants of the system ((1),(2)) considering, that tensor product of k any matrixes from(3)there is a matrix with number of lines and columns are equal to 2^k.

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NEW ALGEBRAIC STRUCTURES ON DIFFERENTIAL OPERATORS

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Let $A = (A, \circ)$ be an algebra with vector space A and multiplication $\circ$. Denote by $A^{(q)} = (A, \circ_q)$ an algebra with vector space A under q-commutator $a \circ_q b = a \circ b + q\, b \circ a$. For a category of algebras $\mathcal{C}$ denote by $\mathcal{C}^{(q)}$ a category of algebras of a form $A^{(q)} = (A, \circ_q)$, where $A = (A, \circ) \in \mathcal{C}$. For example, if $A = \mathcal{A}ss$ is a category of associative algebras, then $\mathcal{A}ss^{(-1)}$ is a category of Lie algebras.

A vector field is a differential operator of first order. A composition of operators is a natural operation on a space of differential operators. Space of vector fields is not close under composition. It is close under Lie commutator, $[D_1, D_2] = D_1 D_2 - D_2 D_1$. Let us define N-commutator of diferential operators as a skew-symmetric sum of $N!$ compositions

$$s_N(D_1, \ldots, D_N) = \sum_{\sigma \in S_N} sign\, \sigma\, D_{\sigma(1)} \cdots D_{\sigma(N)}$$

Let $\mathcal{D}_{n,p}$ be a space of differential operators with n variables of differential degree p. For example, $\mathcal{D}_{n,1}$ coincides with a space of vector fields on n-dimensional manifold $Vect(n)$. Another example, laplacian $\Delta = \sum_{i=1}^{n} u_i \partial_i^2 \in \mathcal{D}_{n,2}$.

Theorem 1. *Let $q^2 \neq 1$. Then the categories of algebras $\mathcal{C}$ and $\mathcal{C}^{(q)}$ are isomorphic. Moreover, if $\mathcal{C}$ is a variety, i.e. is generated by polynomial identities, then $\mathcal{C}^{(q)}$ is also variety.*

Theorem 2. *For any positive integers n and p there exists $N = N(n,p)$ such that N-commutator is well-defined on a space of differential operators $\mathcal{D}_{n,p}$.*

If $N = n^2 + 2n - 2$, then N-commutator is well-defined on $Vect(n)$. The space $\mathcal{D}_{n,2}$ has nontrivial N-commutator for $N = (n^2 + 5n + 2)/2$ and $\mathcal{D}_{n,3}$ has N-commutator for $N = (n^3 + 9n^2 + 26n + 6)/6$. For example, 4-commutator of four differential operators of second order on the line can be calculated by wronskians

$$s_4(u_1\partial^2, u_2\partial^2, u_3\partial^2, u_4\partial^2) = -2 \begin{vmatrix} u_1 & u_2 & u_3 & u_4 \\ \partial(u_1) & \partial(u_2) & \partial(u_3) & \partial(u_4) \\ \partial^2(u_1) & \partial^2(u_2) & \partial^2(u_3) & \partial^2(u_4) \\ \partial^3(u_1) & \partial^3(u_2) & \partial^3(u_3) & \partial^3(u_4) \end{vmatrix} \partial^2.$$

We study identities of these kind N-algebras on differential operators. For example, we establish that an algebra $\mathbf{C}[x]$ under Jordan commutator $a \star b = \partial(ab)$ satisfies the following identity

$$(a \star b) \star (c \star d) - (a \star d) \star (c \star b) = (a, b, c) \star d - (a, d, c) \star b,$$

where $(a, b, c) = a \star (b \star c) - (a \star b) \star c$. Partially these results was published in the following papers of author.

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LIFTS OF TENSOR FIELDS TO THE COFRAME BUNDLE

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K.P.Mok, L.A.Cordero and M.Leon studied ([1], [2]) complete and horizontal lifts of tensor fields from a differentiable manifold M to its frame bundle FM. The present work is devoted to the construction of complete and horizontal lifts of tensor fields to coframe bundle.

The differentiable manifold M_n of class C^∞ with a affine connection ∇ is considered. Let L^*M_n be the coframe bundle over manifold M_n. We consider $\left(x^i, X_i^\alpha\right) = \left(x^i, x^{i_\alpha}\right)$, $\alpha = 1, ..., n, i_\alpha = n + 1, ..., n + n^2$, as local coordinates in a neighborhood $\pi^{-1}(U)$ (π is the natural projection L^*M_n onto M_n). The skew-symmetric tensor field S of type $(1, 2)$, having coordinates S_{kj}^i in natural frame $\{\partial_i\}$ is considered. The complete lift of tensor field S to coframe bundle L^*M_n is defined as a tensor field ${}^cS$ of type $(1, 2)$ with components:

$$
{}^cS_{kj}^i = S_{kj}^i,\ {}^cS_{k_\gamma j}^i = {}^cS_{kj_\beta}^i = {}^cS_{k_\gamma j_\beta}^i = {}^cS_{k_\gamma j_\beta}^{i_\alpha} = 0,
$$
$$
{}^cS_{kj}^{i_\alpha} = X_m^\alpha(\partial_k S_{ij}^m + \partial_j S_{ki}^m - \partial_i S_{kj}^m),
$$
$$
{}^cS_{k_\gamma j}^{i_\alpha} = \delta_\gamma^\alpha S_{ij}^k,\ {}^cS_{kj_\beta}^{i_\alpha} = \delta_\beta^\alpha S_{ki}^j,
$$

where δ_β^α is the Kronecker delta.

The horizontal lift ${}^HS$ of tensor field S is defined by analogy of the complete lift. Relation between the complete and horizontal lifts of tensor field S to coframe bundle L^*M_n is investigated.

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LORENTZIAN HOLONOMY ALGEBRAS AND THEIR APPLICATIONS [1]

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The classical result of Berger states that the holonomy algebra of an indecomposable not locally symmetric n-dimensional Riemannian manifold is one of the following subalgebras of $\mathfrak{so}(n)$: $\mathfrak{so}(n)$, $\mathfrak{u}(m)$, $\mathfrak{su}(m)$ $(n=2m)$, $\mathfrak{sp}(k)$, $\mathfrak{sp}(k)\oplus\mathfrak{sp}(1)$ $(n=4k)$, G_2 $(n=7)$, $\mathfrak{spin}(7)$ $(n=8)$. In general, a Riemannian manifold is locally a product of a flat Riemannian manifold and of Riemannian manifolds with irreducible holonomy algebras. This result has a lot of consequences and applications both in geometry and physics, see [9] and the references there. Riemannian manifolds with the most of possible holonomy algebras are automatically Einstein or Ricci flat, and they admit parallel spinors.

We study the holonomy algebras of Lorentzian manifolds. Any Lorentzian manifold is locally a product of a Riemannian manifold and of a Lorentzian manifold that is further indecomposable. Hence, it is enough to consider an indecomposable Lorentzian manifold (M,g) (let $\dim M = n+2 \geq 3$). In this case the holonomy algebra $\mathfrak{g}\subset\mathfrak{so}(1,n+1)$ does not preserve any non-degenerate subspace of the tangent space. If $\mathfrak{g}\neq\mathfrak{so}(1,n+1)$, then $\mathfrak{g}$ must preserve an isotropic line of the tangent space, hence it is contained in the similitude Lie algebra, $\mathfrak{g}\subset\mathfrak{sim}(n)=\mathbb{R}\oplus\mathfrak{so}(n)\oplus\mathbb{R}^n$. There are the following 4 types of the Lorentzian holonomy algebras $\mathfrak{g}\subset\mathfrak{sim}(n)$:

$$\mathfrak{g}=\mathbb{R}\oplus\mathfrak{h}\oplus\mathbb{R}^n,\quad \mathfrak{g}=\mathfrak{h}\oplus\mathbb{R}^n,\quad \mathfrak{g}=\{\varphi(A)+A|A\in\mathfrak{h}\}\oplus\mathbb{R}^n,\quad \mathfrak{g}=\{A+\psi(A)|A\in\mathfrak{h}\}\oplus\mathbb{R}^m,$$

where $\mathfrak{h}\subset\mathfrak{so}(n)$ is a Riemannian holonomy algebra, $\varphi:\mathfrak{h}\to\mathbb{R}$ is a non-zero homomorphism; for the last algebra $\mathbb{R}^n=\mathbb{R}^m\oplus\mathbb{R}^{n-m}$ is an orthogonal decomposition, $\mathfrak{h}\subset\mathfrak{so}(m)$ and $\psi:\mathfrak{h}\to\mathbb{R}^{n-m}$ is a surjective homomorphism. This result is obtained by Berard-Bergery, Ikemakhen, Leistner and the author, see the review [4]. For example, the manifolds with the holonomy algebras $\mathbb{R}^n\subset\mathfrak{sim}(n)$ are pp-waves, thay are used in theoretical physics.

For each of the above algebras we construct a Lorentzian metric, realizing it as the holonomy algebra. We describe the curvature tensors of Lorentzian manifolds with each possible holonomy algebra. Motivated by [8], we find all possible holonomy algebras of Einstein and Ricci-flat Lorentzian manifolds, and we use this in [6] for simplifying the Einstein equation on Lorentzian manifolds with special holonomy. In [7], conformally flat Lorentzian manifolds with special holonomy are described. In [1], two-symmetric Lorentzian manifolds, i.e. satisfying $\nabla^2 R=0$, are classified. Possible physical applications are discussed, e.g. in [2, 3, 8].

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KAZHDAN PROPERTY (T) AND C*-DYNAMICAL SYSTEM[1]

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In this paper we studied the property (T) for the discrete C^*-dynamical system (A, G, α) and obtained some important results. Our results can be considered as a generalization of some results from the C^*-algebras to the C^*-dynamical systems obtained by B. Bekka [2].

Let A be anunitalC^*-algebra with a tracial state. A is said to have property (T) if there exists a finite subset F of A and $\varepsilon > 0$, such that for every Hilbert bimodule having a (F, ε)-central unite vector, there is a non-zero central vector [2]. This definition is similar to Kazhdan property (T) for locally compact groups [6, 1]. Many authors have studied the property (T) for various structures. For example, study of this property for Von Neumann algebras has been done by A. Connes. In their works E. Bedos, G.J. Murphy and others defined the property (T) for algebraic quantum groups. Also a type of the property (T) was studied for Hopf algebras by the authors of the present paper [4]. In this work we defined the property (T) for the C^*-dynamical systems. A C^*-dynamical system is a triple (A, G, α), where A is an unital C^*-algebra, G is a discrete countable group and α is an action of G on A. Let us say that (A, G, α) has the property (T) if there exists (F, ε, Q, r), such that for every Hilbert system module, having a (F, ε, Q, r)-central unit vector, there is a non-zero (A, G)-central vector, where F is a finite subset of A, Q is a finite subset of G and both ε and r are greater than zero.

Our results are the following:

(1) If the C^*-dynamical system (A, G, α) has the property (T), then its C^*-crossed product has property (T) as an unital C^*-algebra as well.

(2) If (A, G, α) is a C^*-dynamical system with countable discrete group G, then the property (T) of the locally compact group G together with the strong property (T) of A imply the property (T) of (A, G, α), as well as the property (T) of C^* - crossed product and its reduced C^*-crossed product.

(3) Let A be a commutative C^*-algebra , and let α be a trivial action such that there exists a faithful representation on A to the Hilbert space $l^2(G)$, then their (T) properties are equivalent.

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THE CUBICAL HOMOLOGY OF SETS OVER TRACE MONOIDS

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For an arbitrary set with an action of a trace monoid, we prove that its simplicial and cubical homology groups are isomorphic. It allows us to get the finite complexes to calculate the homology groups for some mathematical models of concurrency.

Let E be a set with an irreflexive symmetric relation $I \subseteq E \times E$. Denote by E^* the free monoid of words with the empty word $1 \in E^*$. For the set $R = \{(ab, ba) | (a, b) \in I\}$ there is a smallest congruence relation $\equiv_I$ containing R. A *trace monoid* $M(E, I)$ is a quotient monoid $E^*/\equiv_I$.

Consider a pair $(M(E, I), X)$ where X is a set with a right action of a trace monoid $M(E, I)$, written as $x \mapsto x \cdot \mu$ for $x \in X$ and $\mu \in M(E, I)$. Denote by $\mathscr{C}(M(E, I), X)$ a category with objects $Ob(\mathscr{C}(M(E, I), X)) = X$ and morphisms defined as triples $x \xrightarrow{\mu} x'$ for which $x \cdot \mu = x'$.

We define a *homology groups* $H_n(M(E, I), X)$, $n \geq 0$, as the homology groups of the nerve of the category $\mathscr{C}(M(E, I), X)$.

A *precubical set* $Q_\diamond = (Q_n, \partial_i^{n,\varepsilon})$ is a sequence of sets Q_n, $n \geq 0$, and a family of maps $\partial_i^{n,\varepsilon}$, where $n \geq 0$, $1 \leq i \leq n$, $\varepsilon \in \{0, 1\}$, which satisfy the equations $\partial_i^{n-1,\alpha} \circ \partial_j^{n,\beta} = \partial_{j-1}^{n-1,\beta}\partial_j^{n,\alpha}$ for all $n \geq 2$, $1 \leq i < j \leq n$, and $\alpha, \beta \in \{0, 1\}$.

For a set S, denote by $L(S)$ a free abelian group generated by S. Let $H_n(Q_\diamond)$ be homology groups of the complex

$$0 \leftarrow L(Q_0) \overset{d_1}{\leftarrow} L(Q_1) \overset{d_2}{\leftarrow} L(Q_2) \leftarrow \cdots \leftarrow L(Q_{n-1}) \overset{d_n}{\leftarrow} L(Q_n) \leftarrow \cdots$$

where $d_n(q) = \sum\limits_{i=1}^{n} (-1)^i (\partial_i^{n,1}(q) - \partial_i^{n,0}(q))$ for all $q \in Q_n$.

Let $(M(E, I), X)$ be a set over a trace monoid with a totally order relation on E. For any $n \geq 1$, denote by E_n the set of all n-tuples $e_1 < \cdots < e_n$ in E satisfying the condition $(e_i, e_j) \in I$ for all $1 \leq i < j \leq n$. Define $E_0 = 1$. We get a precubical set $Q_n(M(E, I), X) = \{(x, e_1, \cdots, e_n) | x \in X, (e_1, \cdots, e_n) \in E_n\}$, $\partial_i^{n,\varepsilon}(x, e_1, \cdots, e_n) = (x \cdot e_i^\varepsilon, e_1, \cdots, e_{i-1}, e_{i+1}, \cdots, e_n)$, where $e_i^0 = 1$, $e_i^1 = e_i$.

Theorem 1. *If E does not contain infinite subsets of pairwise commuting elements, then independently of the order relation there are the isomorphisms $H_n(M(E, I), X) \cong H_n(Q_\diamond(M(E, I), X))$ for $n \geq 0$. A set S with a partially action of $M(E, I)$ can be considered as a pointed set $S_* = S \sqcup \{*\}$ with the extension of the action by $x \cdot \mu = *$ when $x \cdot \mu$ is not defined, and $* \cdot \mu = *$. An* asynchronous system *$(M(E, I), S_*, s_0)$ can be defined as a pointed set over a trace monoid with distinguished element $s_0 \in S$. An element $s \in S$ is* reachable *if there is $\mu \in M(E, I)$ such that $s_0 \cdot \mu = s$. Let $S(s_0)$ the set of reachable elements and $S_*(s_0) = S(s_0) \sqcup \{*\}$. Following* [1] *we define the* homology groups *$H_n(M(E, I), S, s_0)$ as $H_n(M(E, I), S_*(s_0))$.*

It follows from Theorem 1 that $H_n(M(E, I), S, s_0) \cong H_n(Q_\diamond(M(E, I), S_*(s_0)))$. Each Petri net can be considered as an asynchronous system. A prefix-closed trace languages can be defined as an asynchronous system. Theorem 1 allows us to build the asynchronous systems, the Petri nets, and the prefix-closed trace languages the homology groups of which can have arbitrary amounts of torsion in dimensions $n \geq 2$.

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THE PROOF OF LEGENDRE HYPOTHESIS

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There are a lot of unsolved problems related with prime numbers. The most famous of which were listed by Edmund Landau at the Fifth International Congress of Mathematicians (Cambridge Great Britain, 1912). One of these problems-is Legendre hypothesis (Landau's third problem): In the interval (n2; (n+1)2)there is at least one prime number.

Namely, this article is devoted to solving of this problem.

KELLER ENDOMORPHISMS OF POLYNOMIALS' ALGEBRA IN TWO VARIABLES

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We consider and investigate two variables' polynomials' algebra endomorphisms' ring over zero characteristics field. Here we give Keller endomorphism definition and its properties are investigated. In two variables' polynomials' algebra two polynomials' Jacobean requests Lee structure. Then we can get the Lee algebra's homology group. In this work we prove the following theorem.

Theorem. *Keller endomorphism of two variables' polynomials' algebra over zero characteristics field with identical endomorphism is compared precisely to the boundaries.*

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FOUR COLOR THEOREM

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It is generally known all difficulties, are connected with to find of *chromatic number* $h(G)$ given simple (n, m)-Graph G, where $n(G) = n$ - number of nodes, $m(G) = m$ - number of edges and the well-known *Four color Problem.*

Subject of this Report is to message the important results more than a quarter prescription's century [1] which have remained in "shadow". Concept *k-partite Graph*, are entered into the Graph's Theory most likely, in connection with studying of *chromatic number h* of Graphs G. And though concepts: *k-partite Graph* and *k-colorable Graph* are designate the same mathematical object, however, actually, from a point of view of studying the problem Graph's *colorability*, there was convenient and perspective the concept of *k-partite Graphs.*

In the above mentioned work it was noted, that *k-partite Graph* will have independent interest for Graph's Theory and outside of a problem of chromatic number as they appeared closely connected with numerous combinatory problems. There the interesting appendix some property of *k-partite Graph's* for the decision of one known problem having old history. It is a problem, so-called, *Ramsey numbers* in *Ramsey's Theorem.*

But a basic result of this work was: the *constructive proof of a sufficient condition of k-colorability Graph* G. From a *case at k=4* follows of the above mentioned well-known *Four color Theorem.* Here is this interesting

Theorem. [Jamshid G'ofur, 1981]. Every (n, m, φ)-Graph G with number of nodes n, with number of edges m and with number of density $\varphi(G) \leq k$, is k-colorable Graph, if:

$$m(G) \leq \mu^*(n, k)$$

where $\mu^*(n, k)$ - a least chromatic invariant k-partite Graph's, and $k \in \{1, 2, ..., n\}$ [1].

In the report it has been considered also other important theorem which was till now the next problem of the Graph's Theory. It is known as *Hadwiger's Hypothesis.* In the middle of last century Hugo Hadwiger, in a context of the Four color problem has put forward such *Hypothesis*:

Hadwigwer's hypothesis. [2] Every k-colourable Graph with the chromatic number $h(G) = k$, by means of shrinkage some edges, it is being reduced in a full Graph F_k with a k nodes.

It is known, that at $k = 5$ from this Hypothesis follows the *Four color Theorem.* The constructive proof of this interesting Hypothesis, also follows from a properties *k-partite Graph* [1].

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TRIPLE LIE DIFFERENTIATION OF ALGEBRAS OF MEASURABLE OPERATORS

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We prove that every Triple Lie derivation on algebras of measurable operators for von Neumann algebras is in standard form, that is, it can be uniquely decomposed into the sum of a derivation and a center-valued trace.

Let A be an algebra. A linear operator $L : A \to A$ is said to be a triple Lie differentiation if $L[[x,y],z] = [[L(x),y],z] + [[x,L(y)],z] + [[x,y],L(z)]]$, $\forall x,y,z \in A$. A linear operator $D : A \to A$ is called *differentiation* if $D(xy) = D(x)y + xD(y)$ for all $x,y \in A$. Any element $a \in A$ defines the differentiation D_a by the rule $D_a(x) = ax - xa$, $x \in A$. Differentiations in the form of D_a are said inner.

Denote by $Z(A)$ the center of A.

A linear operator $\tau : A \to Z(A)$ is called *a center-valued trace* if $\tau(xy) = \tau(yx)$, $\forall x,y \in A$.

Theorem 1. *Let $L : S(M) \to S(M)$ be a triple Lie differentiation. Then $L = D + \tau$, where D is an associated differentiation and τ is a center-valued trace from $S(M)$ into $Z(S(M))$.*

Let $L^0(\Omega) = L^0(\Omega,\Sigma,\mu)$ be an algebra of equivalence classes of all complex measurable functions on (Ω,Σ,μ). Consider arbitrary differentiation $\delta : L^0(\Omega) \to L^0(\Omega)$, and D_δ — the "coordinate-wise" differentiation on $M_n\left(L^0(\Omega)\right)$ of the $n\times n$ matrices over $L^0(\Omega)$, defined by the rule

$$D_\delta\left((\lambda_{ij})_{i,j=1}^n\right) = \left(\delta\left(\lambda_{ij}\right)_{i,j=1}^n\right)$$

where $(\lambda_{ij})_{i,j=1}^n \in M_n\left(L^0(\Omega)\right)$. The operator D_δ is a differentiation on $M_n\left(L^0(\Omega)\right)$. Taking this into account, we can define the differentiation D_δ on $S(M)$ [1] where M is an I-type von Neumann algebra I, setting

$$D_\delta(x) = \left(D_{\delta_\alpha}(x_\alpha)\right), \quad x = (x_\alpha) \in S(M). \tag{1}$$

We obtain from Theorem 3.6 [1] the following

Corollary. *Let M be an I-type von Neumann algebra. Then any Triple Lie differentiation can be uniquely represented in the form of*

$$L = D_\alpha + D_\delta + \tau$$

where D_α is an inner differentiation, D_δ is the differentiation in the form of (1).

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DESCRIPTION OF ENDOMORPHISMS OF FINITE-DIMENSIONAL KAPLANSKY-HILBERT MODULES

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Let L^0 be the algebra of equivalence classes of all measurable complex valued functions defined on measure space (Ω, Σ, μ) with finite measure, $B(\Omega)$ be the Boolean algebra of all idempotents in L^0. Let X be the left module over L^0, and $\langle x|y\rangle$ be the scalar product on X with values in L^0. If X is complete under the L^0-value norm $\|x\| = \sqrt{\langle x|x\rangle}$, then X is called the Kaplansky-Hilbert module over L^0. For each $x \in X$ we set $s(x) = \mathbf{1} - \sup\{e \in B(\Omega) : ex = 0\}$, where $\mathbf{1}$ is the identity in $B(\Omega)$. If X has the orthonormal set $\{x_i\}_{i=1}^n = E$ with $E^\perp = \{0\}$, then the Kaplansky-Hilbert module X is called n-homogeneous. We say that X is a finite-dimensional Kaplansky-Hilbert module over L^0, if there exists the finite partition $\{e_i\}_{i=1}^m$ of the identity $\mathbf{1}$, such that e_iX is n_i-homogeneous Kaplansky-Hilbert module over e_iL^0, $n_i \in \mathbf{N}$, $i = \overline{1,m}$, $n_1 < n_2 < \ldots < n_m$.

Theorem 1. *The following conditions are equivalent:*

(i) *X is finite-dimensional;*

(ii) *X is finitely generated, i. e. there exists finite set $\{x_i\}_{i=1}^k \subset X$, such that $X = \left\{\sum\limits_{i=1}^k \alpha_i x_i : \alpha_i \in L^0, i = \overline{1,k}\right\}$;*

(iii) *there exists $n \in \mathbf{N}$, such that for all nonzero $e \in B(\Omega)$ any eL^0-linearly independent system from eX contains no more than n elements.*

Let $B(X)$ be the set of all endomorphisims in X.

Theorem 2. *Let X be a finite-dimensional Kaplansky-Hilbert module, $T \in B(X)$. Then there exist orthogonal sets $\{x_i\}_{i=1}^k$, $\{y_i\}_{i=1}^k$ in X and the set $\{\lambda_i\}_{i=1}^k \subset L^0$, such that $0 \neq s(\lambda_i) \leq s(x_i)$, $\langle x_i|x_i\rangle \in B(\Omega)$, $0 \neq \langle y_i|y_i\rangle \in B(\Omega)$, $i = \overline{1,k}$ and*

$$Tx = \sum_{i=1}^k \lambda_i \langle x|x_i\rangle y_i$$

for all $x \in X$.

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THE CONSTRUCTIVIZABILITY CRITERION OF THE ONE CLASS TORSION-FREE NILPOTENT GROUPS

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The study of constructive groups began with Mal'cev posing in [1] the general question of "finding given abstractly defined groups, which constructive enumerations they admit". This question is considered for torsion-free nilpotent groups, whose commutant has finite dimension.

Theorem 1. S*uppose that* (G, ν) *is a constructive group and* B *is a computable enumerable subgroups of* G *such that the quotient* G/B *is a torsion-free abelian group and for all finite set* $F \subseteq G$ *the dimension of the quotient of the centralizer* $C(F)$ *of the set* F *by* B *is infinite. Then there exists an enumeration* μ *of* G *possessing the following properties:*

1) the group (G, μ) is constructive;
2) the subgroup B is computable in (G, μ);
3) there exists a computable enumerable system $\{c_i \mid i \in I\}$ of elements in (G, μ) such that the cosets $\{c_iB\}$ form a basic of the quotient G/B.

Theorem 2. *Suppose that* G *is a torsion-free nilpotent group and the quotient of* G *by the isolator* $I(G')$ *of the commutant* G' *has infinite dimension. If the dimention of* G' *is finite then any finite set* $F \subseteq G$ *the dimension of the quotient* $C(F)/I(G')$ *is infinite.*

On the basis of this theorem we received the following criteria for the constructivizability of a group G.

Theorem 3. *Suppose that* G *is a torsion-free nilpotent group such that the dimension of* G' *is finite. Then* G *is constructivizable if and only if there exists the following central series*

$$G_0 = G \geq G_1 \geq \ldots \geq G_n = e$$

and a contructivizations $\bar{\nu}_i$ *of central factors* $\bar{G}_i = G_i/G_{i+1}$ *such that the group* (G_i, ν_i) *is the extension of constructive group* (G_{i+1}, ν_{i+1}) *by the constructive group* $(\bar{G}_i, \bar{\nu}_i)$ *and some recursive system of factor from* $(\bar{G}_i, \bar{\nu}_i)$ *in* (G_{i+1}, ν_{i+1}), $i < n$.

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ON BINARY TYPES IN WEAKLY O-MINIMAL THEORIES

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Let L be a countable first-order language. Everywhere in this paper we consider L-structures and assume that L contains a binary relation symbol $<$ that is interpreted as a linear ordering in these structures.

This talk concerns the notion of *weak o-minimality* originally deeply studied by D. Macpherson, D. Marker and C. Steinhorn in [1]. *A weakly o-minimal structure* is a linearly ordered structure $M = \langle M, =, <, \ldots \rangle$ such that any definable (with parameters) subset of M is a finite union of convex sets in M. Real closed fields with a proper convex valuation ring provide an important example of weakly o-minimal structures.

Let M be a weakly o-minimal structure, $A \subseteq M$, $p, q \in S_1(A)$ be non-algebraic. We say that p is *not weakly orthogonal* to q ($p \not\perp^w q$) if there are an A-definable formula $H(x, y)$, $\alpha \in p(M)$ and $\beta_1, \beta_2 \in q(M)$ such that $\beta_1 \in H(M, \alpha)$ and $\beta_2 \notin H(M, \alpha)$.

We say a tuple $\bar{a} = \langle a_1, a_2, \ldots, a_n \rangle \in M^n$ is *increasing* if $a_1 < a_2 < \ldots < a_n$. Let $A \subseteq M$, A be finite, $p_1, p_2, \ldots, p_s \in S_1(A)$ be non-algebraic. We say that the family of 1-types $\{p_1, \ldots, p_s\}$ is *weakly orthogonal over* A if every s–tuple $\langle a_1, \ldots, a_s \rangle \in p_1(M) \times \ldots \times p_s(M)$ satisfies the same type over A. We say that the family of 1-types $\{p_1, \ldots, p_s\}$ is *orthogonal over* A if for every sequence $(n_1, \ldots, n_s) \in \omega^s$ and for every increasing tuples $\bar{a}_1, \bar{a}'_1 \in [p_1(M)]^{n_1}$, …, $\bar{a}_s, \bar{a}'_s \in [p_s(M)]^{n_s}$ such that $\mathrm{tp}(\bar{a}_1/A) = \mathrm{tp}(\bar{a}'_1/A)$, …, $\mathrm{tp}(\bar{a}_s/A) = \mathrm{tp}(\bar{a}'_s/A)$ we have $\mathrm{tp}(\langle \bar{a}_1, \ldots, \bar{a}_s \rangle/A) = \mathrm{tp}(\langle \bar{a}'_1, \ldots, \bar{a}'_s \rangle/A)$. If $A \subseteq M$, $p_1, p_2 \in S_1(A)$ and $p_1 \perp^w p_2$ then obviously $\{p_1, p_2\}$ is weakly orthogonal over A.

Let $p \in S_1(A)$ be non-algebraic. We say p is *binary over* A if for every $n < \omega$ and for every $b_1 < b_2 < \ldots < b_n$, $b'_1 < b'_2 < \ldots < b'_n \in p(M)$ such that $\mathrm{tp}(\langle b_i, b_j \rangle/A) = \mathrm{tp}(\langle b'_i, b'_j \rangle/A)$ for all $1 \leq i < j \leq n$ we have $\mathrm{tp}(\bar{b}/A) = \mathrm{tp}(\bar{b}'/A)$. If $p \in S_1(\emptyset)$ is non-algebraic and p is binary over $\emptyset$, we say p is *binary.*

Example 1 *Let $M = \langle Q \cup W, <, E^3, P^1 \rangle$ be a linearly ordered structure, where Q is the set of rational numbers; W is the set of all Q-sequences from $\{0, 1\}$ with finitely many non-zero coordinates excepting the only Q-sequence consisting only from $\{0\}$, ordered lexicographically; $P(M) = Q$, $\neg P(M) = W$ and $P(M) < \neg P(M)$. For every $a \in P(M)$ $E(a, y_1, y_2)$ is an equivalence relation on $\neg P(M)$ defined as follows: for every $a \in P(M)$, $b_1, b_2 \in \neg P(M)$ $E(a, b_1, b_2) \Leftrightarrow b_1(q) = b_2(q)$ for all $q \leq a$, i.e. q–th coordinates of b_1 and b_2 coincide for all $q \leq a$.* It can be proved M is an $\aleph_0$–categorical weakly o-minimal structure and $p_2 := \{\neg P(x)\}$ is not binary.

In [2] orthogonality of all families of pairwise weakly orthogonal 1-types for $\aleph_0$–categorical weakly o-minimal theories of finite convexity rank has been proved. Here we discuss on orthogonality of all families of non-algebraic pairwise weakly orthogonal binary 1-types for an arbitrary $\aleph_0$–categorical weakly o-minimal theory.

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AXIOMATIZABILITY OF THE CLASS OF POSITIVELY EXISTENTIALLY CLOSED MODELS

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We take definitions of notions of the positive logic from [1] and repeat some of them.

We consider a signature Σ with an equality and relations, and we form as usually the first order formulas using $\neg$, $\wedge$, $\vee$ and $\exists$. *Positive* formulas are formed without negation, i.e. only using $\wedge$, $\vee$ and $\exists$; they can be written as $(\exists\overline{y})f(\overline{x},\overline{y})$, where $f(\overline{x},\overline{y})$ is a quantifier free positive formula.

A homomorphism from the Σ-structure $\mathcal{M}$ into the Σ-structure $\mathcal{N}$ is an application h from the underlying set M of $\mathcal{M}$ into the underlying set N of $\mathcal{N}$ such that, for each $\overline{a}$ from M, if $\overline{a}$ satisfies the atomic formula $A(\overline{x})$, so does $h(\overline{a})$; we do not assume the reciprocal, so that $h(\overline{a})$ may satisfy furthermore atomic formulas than $\overline{a}$, and in particular h may not be injective. If there exists an homomorphism from $\mathcal{M}$ to $\mathcal{N}$, we say that $\mathcal{N}$ is a *continuation* of $\mathcal{M}$, and that $\mathcal{M}$ is a *beginning* of $\mathcal{N}$. (We use the words extension/restriction only when h is an embedding, i.e. when $\overline{a}$ and $h(\overline{a})$ satisfy the same atomic formulas).

If h is an homomorphism, then every positive formula $(\exists\overline{y})f(\overline{x},\overline{y})$ satisfied by $\overline{a}$ is also satisfied by $h(\overline{a})$. We say that h is an *immersion* if we have the converse, that is if $\overline{a}$ and $h(\overline{a})$ satisfy the same positive formulas; we say then that $\mathcal{M}$ is *immersed*, or *positively existentially closed*, in $\mathcal{N}$. An immersion is an embedding, but positively existentially closed is weaker than the robinsonian notion, since we consider only positive existential formulas.

An *h-universal sentence* is by definition the negation of a positive sentence; it can be written $\neg(\exists\overline{y})f(\overline{y})$, or equivalently $(\forall\overline{y})\neg f(\overline{y})$, where $f(\overline{y})$ is free and positive.

If C is a class of Σ-structures, we say that an element $\mathcal{M}$ of C is *positively existentially closed* in C if every homomorphism from $\mathcal{M}$ into any member of C is an immersion.

Author find the example of class which axiomatized with infinite number of h-universal sentences where the subclass of positively existentially closed models is not elementary.

At the same time it had been proved that if class axiomatized with finite number of h-universal sentences then its subclass of positively existentially closed models is axiomatized. And found the example where this axiomatization of the subclass of positively existentially closed models can not be finite.

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FINDING THE SOLUTIONS OF UNILATERAL QUADRATIC MATRIX EQUATION

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Problems of the theory of oscillations continue to take the important place in various engineering problems. Here it is necessary to note the theory of strongly damped systems [1]. In this theory the central place occupy the problems of determination of matrix (or operator [1]) roots of the equations

$$A_2X^2 + A_1X + A_0 = 0.$$

In [2] matrix equation (1) is called as the unilateral quadratic matrix equation. Here the wide range of problems of control in which it is necessary to find a solution of (1) is noted. As it is noted in [1], one of motivations of determination of roots of (1) can be the following reasons. In a scalar case the equation (1) can be considered as a characteristic equation of the differential equation with constant coefficients:

$$A_2\ddot{q} + A_1\dot{q} + A_0q = 0.$$

The general solution of this equation, as is known, can be write down in the form:

$$q = c_1e^{x_1t} + c_2e^{x_2t},$$

where x_1, x_2 are roots of the characteristic equation. Thereupon there is a problem on a possibility of construction of similar relations in a matrix case. That, in turn, will demand determination the roots of (1).

It will be shown, that in some cases, for construction of a solution (1) it is possible to use the method of doubling transformation [3]. This method, unlike algorithm [2], does not demand invertibility of the matrix in (1).

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ON POINT-LINE DISPLACEMENT IN 3-SPACE $E^3_{\alpha\beta}$

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In kinematics, the motions of a various objects are of major concern. An object may be expressed in the form of a rigid body, a line, a plane, a point, or combinations. Point-line is the combination of an oriented line and an endpoint on the line. Many tools used in manufacturing, such as milling, welding, drilling, screwing, laser jet, etc., can be represented by point-line kinematics. Zhang and Ting[5] showed a framework and the relevant algebraic treatment concerning point-line positions and displacements by using dual quaternion algebra. In Euclidean *3*-space, a screw or a dual vector is used to represent a point-line and the pitch is used to measure the endpoint location along the point-line. Aydogmus and et al.[*1*] studied point-line displacement of a line in Minkowski *3*-space. In [*1*], Dual split quaternion algebra is used to express point-line displacement operation. In previous work, we studied real and dual generalized quaternions and presented some of their algebraic properties[*2,3*]. Here, we generalize the concept of point-line operator in $E^3_{\alpha\beta}$. The operators offers a simple and unique geometrical interpretation for point-line displacement.

Difinition: In *3-space* $E^3_{\alpha\beta}$, *a point-line* $\widehat{A}$ *can be expressed by multiplying a dual number* $\exp(\varepsilon h)$, *to* $\widehat{a}$, *namely* $\widehat{A} = \widehat{a}\exp(\varepsilon h) = \overrightarrow{a} + \varepsilon(\overrightarrow{a}^* + h\overrightarrow{a})$, *where* $\widehat{a} = \overrightarrow{a} + \varepsilon\overrightarrow{a}^*$ *is a unit dual vector in* $D^3_{\alpha\beta}$.*So,* $\widehat{A}$ *is a dual vector with dual length* $\exp(\varepsilon h)$.

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ISOMORPHISMS AND ALGORITHMIC COMPLEXITY RELATIONS OVER STRUCTURES WITH BIPARTITE BINARY PREDICATE[1]

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We will consider the problems on algorithmic complexity of isomorphic and definable properties on models and connections with Scott families.

Let $\mathcal{A}$ be a computable structure.

We say that $\mathcal{A}$ is Δ^0_α *categorical* if for all computable $\mathcal{B} \cong \mathcal{A}$, there is a Δ^0_α isomorphism from $\mathcal{A}$ to $\mathcal{B}$.

We say that $\mathcal{A}$ is *relatively* Δ^0_α categorical if for all computable $\mathcal{B} \cong \mathcal{A}$, there is a $\Delta^0_\alpha(\mathcal{B})$ isomorphism from $\mathcal{A}$ to $\mathcal{B}$.

The Δ^0_α dimension of the structure $\mathcal{A}$ is the number of computable presentations of $\mathcal{A}$ up to Δ^0_α isomorphisms.

A *Scott family* for $\mathcal{A}$ is the set Φ of formulas, with a fixed tuple of $\bar{c}$ in $\mathcal{A}$, such that 1) each tuple of parameters in $\mathcal{A}$ satisfies some formula $\varphi \in \Phi$, and 2) if both $\bar{a}$, $\bar{b}$ satisfy the same formula $\varphi \in \Phi$, then there is an automorphism of $\mathcal{A}$ mapping $\bar{a}$ to $\bar{b}$.

A *formally* Σ^0_α *Scott family* is a Σ^0_α Scott family that is made up of "computable Σ_α" formulas.

Let $\mathcal{A}$ be a computable structure and R be a relation on $\mathcal{A}$. We say that R is *intrinsically* Σ^0_α if in all computable $\mathcal{B} \cong \mathcal{A}$ the image of R in $\mathcal{B}$ is Σ^0_α.

We say that R is *relatively intrinsically* Σ^0_α if in all computable $\mathcal{B} \cong \mathcal{A}$, the image of R is $\Sigma^0_\alpha(\mathcal{B})$.

We say that R is *intrinsically* if for each automorphism f of the structure $\mathcal{A}$ the image $f(R) \subseteq R$.

The structure $\mathcal{A}$ with binary predicate $P(x,y)$ is called the structure with bipartition binary predicates $P(x,y)$ if for the sets $K_1 = \{x : \mathcal{A} \models \exists y P(x,y)\}$ and $K_2 = \{x : \mathcal{A} \models \exists y P(y,x)\}$ satisfy the conditions: $K_1 \cap K_2 = \emptyset$ and $K_1 \cup K_2 \neq \emptyset$. Let $\mathcal{A}$ structure with bipartition binary predicates $P(x,y)$.

Theorem 1. *For each computable successor ordinal α there is computable structure $\mathcal{A}$ that is Δ^0_α categorical but not relatively Δ^0_α (and without formally Σ^0_α Scott family).*

Theorem 2. *For each computable successor ordinal α there is computable structure $\mathcal{A}$ that is Δ^0_α with a relation that is intrinsically Σ^0_α but not relatively intrinsically Σ^0_α.*

Theorem 3. *For each computable successor ordinal α for each finite n there is computable structure $\mathcal{A}$ with Δ^0_α dimension n.*

Theorem 4. *For each computable successor ordinal α there is structure $\mathcal{A}$ with presentations in just the degrees of sets X such that $\Delta^0_\alpha(X) \neq \Delta^0_\alpha$. In particular, for each finite n there is structure $\mathcal{A}$ with presentations in just the $non-low_n$ degrees.*

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EQUIVALENCE OF PATHS BY ACTION OF THE SPECIAL PSEUDOORTHOGONAL GROUP

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In this work, the criterion of equivalence of regular paths with respect to the identical action of the special pseudoorthogonal group is determined.

Let $E = C^n_{p,q}$ (where $p + q = n$) be n-dimensional pseudo-euclidean space, indefinite metrics of which is identified by the equation: $[x, y] = x_1y_1 + \ldots + x_py_p - x_{p+1}y_{p+1} - \ldots - x_ny_n$, where $x = (x_i)^n_{i=1}$, $y = (y_i)^n_{i=1} \in C^n_{p,q}$, C is the field of complex numbers.

We denote by $SO(p, q)$ the special pseudoorthogonal subgroup of $GL(n, C)$ consisting of all transformations $g \in GL(n, C)$, for which $\det g = 1$ and $g^T Ig = I$, where $I = Diag(\underbrace{1, ..., 1}_{p}, \underbrace{-1, ..., -1}_{q})$, g^T is the transpose of g. Infinitely differentiable mapping $x : (0, 1) \to C^n_{p,q}$ is called a path. Two paths $x(t)$ and $y(t)$ are called $SO(p, q)$-equivalent if there exists element $g \in SO(p, q)$ such that $y(t) = gx(t)$ for any $t \in (0, 1)$.

Let $x(t)$ be a path in $C^n_{p,q}$ and $M(x)(t)$ be a $n \times n$-matrix $\left(x(t)x'(t) \ldots \overset{(n-1)}{x}(t)\right)$, where k-th column has coordinates $\overset{(k-1)}{x_i}(t)$, $\overset{(k-1)}{x_i}(t)$ – $(k-1)$-derivative of $x_i(t)$, $i, k = 1, \ldots, n$. The path $x(t)$ is called regular path if $\det M(x)(t) \neq 0$ for any $t \in (0, 1)$.

The following theorem provides necessary and sufficient conditions for $SO(p, q)$-equivalence of two regular paths.

Theorem. *Two regular paths $x, y \in C^n_{p,q}$ are $SO(p, q)$-equivalent if and only if the following equalities are satisfied* $\left[\overset{(i-1)}{x}(t), \overset{(i-1)}{x}(t)\right] = \left[\overset{(i-1)}{y}(t), \overset{(i-1)}{y}(t)\right]$ *and* $\det M(x)(t) = \det M(y)(t)$ *for all* $t \in (0, 1)$ *and* $i = 1, \ldots, n - 1$.

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ON EXISTENTIALLY AND POSITIVELY EXISTENTIALLY CLOSED STRUCTURES

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The notion of existentially closed structure of a given theory is one of the most classical and nontrivial objects of the Algebraical Model Theory [1]. The two first Nurtazin's theorems are devoted to such structures. On the other hand their analogues for a Positive Logic were introduced by Bruno Poizat a few years ago and are new. The third Kugozgin's theorem says that the class of positively existentially closed models of a finitely axiomatized h-universally axiomatized theory is axiomatizable.

We take definitions of notions of the positive logic from [2] and repeat some of them. We consider a signature Σ with an equality and functional and relational symbols and form as usually the first order formulas. *Positive* formulas are formed without negation.

A very simple and useful enough criterion of an existentially closedness gives us the next theorems

Theorem 1. *A model $\mathcal{A}$ is existentially closed if and only if it's every countable elementary submodes is existentially closed.*

Corollary. *(1) A first order theory T is modelly complete iff there is an existentially closed ω-saturated model in T. (2) (A known fact.) A complete, categorical in some infinite power, inductive theory T is modelly complete.*

Theorem 2. *A complete inductive theory T of a finite signature is modelly complete.*

Corollary. *In a complete first order theory T, axiomatizable by axioms with n exchanges of quantifiers, every formula is equivalent to the formula with quantifiers of the same form. A homomorphism is an application h the underlying set of $\mathcal{M}$ into the underlying set N of $\mathcal{N}$ such that, for each $\overline{a}$ from M, if $\overline{a}$ satisfies the atomic formula $A(\overline{x})$, so does $h(\overline{a})$; we do not assume the reciprocal, so that $h(\overline{a})$ may satisfy furthermore atomic formulas than $\overline{a}$, and in particular h may not be injective. If there exists an homomorphism from $\mathcal{M}$ to $\mathcal{N}$, we say that $\mathcal{N}$ is a continuation of $\mathcal{M}$, and that $\mathcal{M}$ is a beginning of $\mathcal{N}$. (We use the words extension/restriction only when h is an embedding, i.e. when $\overline{a}$ and $h(\overline{a})$ satisfy the same atomic formulas). If h is a homomorphism from the Σ-structure $\mathcal{M}$ into the Σ-structure $\mathcal{N}$, then every positive formula $(\exists\overline{y})f(\overline{x},\overline{y})$ satisfied by $\overline{a}$ from M is also satisfied by $h(\overline{a})$. We say that h is an immersion if we have the converse, that is if $\overline{a}$ and $h(\overline{a})$ satisfy the same positive formulas; we say then that $\mathcal{M}$ is immersed, or positively existentially closed, in $\mathcal{N}$. An h-universal sentence is by definition the negation of a positive sentence; it can be written $\neg(\exists\overline{y})f(\overline{y})$, or equivalently $(\forall\overline{y})\neg f(\overline{y})$, where $f(\overline{y})$ is free and positive. If C is a class of Σ-structures, we say that an element $\mathcal{M}$ of C is positively existentially closed in C if every homomorphism from $\mathcal{M}$ into any member of C is an immersion.*

The second author had found the example of class which axiomatized with infinite number of h-universal sentences where the subclass of positively existentially closed models is not elementary.

Theorem 3. *A class of positively existentially closed models of a finitely axiomatizable h-universal class is axiomatizable.*

At the same time it had been proved that if and found the example where this axiomatization of the subclass of positively existentially closed models cannot be finite.

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PARALLEL COMPUTATION OF SOLVING LINEAR SYSTEM WITH ILL-CONDITIONED MATRIX

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In the paper [1] we consider

$$Ax = f,$$

where$A$ is $n \times n$ real matrix and f is a vector of dimension n. There was proposed one method of solving and parallel computation system (1) with singular or ill-conditioned matrix A.

Consider functional

$$J_\varepsilon(x) = |Ax - f|^2 + \varepsilon|x|^2.$$

We will find $\mathring{x}$ which is the solution of problem to minimize functional:

$$\inf J_\varepsilon(x) = J_\varepsilon(\mathring{x}). \tag{1}$$

Here $infimum$ is given by all vectors x in R^n. Unit ball is compact in R^n, it follows that the solution of the minimization problem exists.

Remark 1. *If matrix A is non-singular, then $\varepsilon = 0$ and minimization problem has unique solution $\mathring{x} = A^{-1}f$, but if A is singular, then $A\mathring{x}$ implies the best approximation of f by vectors Ax. When $\varepsilon \neq 0$ and A is non-singular then $\mathring{x}$ is the approximate solution of equation* $Ax = f$.

Define a sequence of vectors $x_j (j = 1, 2, ...)$ by the following relations

$$x_j = \delta \sum_{k=0}^{j-1} [E - \delta(A^*A + \varepsilon E)]^k A^* f, \tag{2}$$

where $\mathring{x}$ is the solution of problem to minimize functional $J_\varepsilon(\cdot)$ and $\delta \in \big(0, 2(\|A^*A\| + \varepsilon)^{-1}\big)$.

Theorem 1. *Let $\varepsilon \geq 0$, $\mathring{x}$ be the solution of (1) and x_j from (2). Then*

$$x_j - \mathring{x} = -[E - \delta(A^*A + \varepsilon E)]^j \mathring{x} \tag{3}$$

and $x_j \to \mathring{x}$ as $j \to +\infty$ with geometric rate. Thus, there exists $\rho > 0$ for all $j = 1, 2, \ldots$ holds

$$|x_j - \mathring{x}| \leq C \cdot \rho^j, \tag{4}$$

where C is the constant which depends on numbers δ, ε.

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SOME PROPERTIES OF A DISCRETE DYNAMICAL SYSTEMS FAMILY DEPENDING ON A MATRIX OF PARAMETERS

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We consider a class of dynamical systems F^m generated by a map F,

$$F : \mathbb{R}^n \to \mathbb{R}^n, \quad Fy = \Phi(y)Ay,$$

in a compact set $\mathrm{X} \subset \mathbb{R}^n$ chosen as a phase space,

$$\mathrm{X} = \{y \in \mathbb{R}^n \mid y \geq 0, \ \|y\| \leq a\}, \ \ a > 0.$$

Here $\mathbb{R}^n$ is the n-dimensional real vector space, A is a $(n \times n)$- matrix and $\Phi(y)$ is a scalar function ($y = (y_1, \ldots, y_n) \geq 0$ means $y_i \geq 0$, $i = \overline{1, n}$, and $\| \cdot \|$ is a norm in $\mathbb{R}^n$).

Let

$$\mathrm{cone}(y) = \{\alpha y \mid y \in \mathbb{R}^n, \ \alpha \in \mathbb{R}^+\}$$

be a ray directed to y where $\mathbb{R}^+$ be a set of real nonnegative numbers. The system of noncoincident rays $(l_1, ..., l_p)$, $p < \infty$, is called *the cycle of rays of period* p if

$$Al_k = l_{k+1}, \ \ k = 1, \ldots, p-1, \quad Al_p = l_1.$$

The description of the dynamics of the system F^m in X leads to a study of the behavior of its trajectories in so-called *cyclic invariant sets* $\mathrm{M}_p \subseteq \mathrm{X}$ *of the map* F *of finite period* $p \geq 1$ containing all ω- limit sets of the system F^m [1]. The set M_p is the intersection of X with invariant subspace of the matrix A consisting of cycles of rays of periods $\leq p$. The location of the sets M_p in X and their periods are determined by the matrix A, and these periods are independent of $\Phi(y)$. The period p of the set M_p coincides with the maximum number of parameters $\mu_1, \ldots, \mu_p$, by which the dynamics of the system F^m is described in M_p. There may be a few (and even continuum-many) sets M_p of the map F of different periods in different parts of X. Therefore, the dynamics of the system F^m in X is described, generally, by the sets of parameters $(\mu_1, \ldots, \mu_p)$, $p \in N$. So, a question arises as to how to get the greatest period $p^* > 1$ (for a given $n > 1$) of cyclic invariant sets M_p of a family of n- dimensional dynamical systems F^m. The answer to this enables to determine a maximum number of parameters $(\mu_1, \ldots, \mu_{p^*})$ by which the dynamics of the family of systems F^m depending on n^2 parameters, i.e. coefficients of the matrix A, is described in X. The research conducted into the system F^m made it clear that p^* is a function of n and while increasing n $p^*(n)$ becomes not only bigger than n but (essentially) bigger than n^2. Let formulate this result as follows.

Property.

$$p^*(n) \begin{cases} \leq n, \text{ if } n \leq 4, \\ > n, \text{ if } \ n \geq 5 \ \ (\textit{except } n = 6), \\ \leq n^2, \text{ if } n \leq 18, \\ > n^2, \text{ if } \ n \geq 19 \ (\textit{except } n = 21, 22). \end{cases}$$

The last one means that the number of parameters by which the dynamics of the family of n-dimensional systems F^m is described may be more than the number of parameters given by the matrix A.

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ON SCORES AND LOSING SCORES IN HYPERTOURNAMENTS

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A k-hypertournament is a complete k-hypergraph with each k-edge endowed with an orientation, that is, a linear arrangenment of the vertices contained in the hyperedge. In a k-hypertournament, the score $s(v_i)$ (losing score $r(v_i)$) of a vertex v_i is the number of arcs containing v_i and in which v_i is not the last element (v_i is the last element). The score (losing score) sequence is formed by listing the scores (losing scores) in non-decreasing order. Zhou et al obtained a characterization of score and losing score sequences in k-hypertournaments, a result analogous to Landau's theorem on tournament scores. A bipartite hypergraph is a generalization of a bipartite graph.

We discuss various results on the sequences of non-negative integers to be losing scores or scores of various classes of hypertournaments.

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CATEGORY OF CHAİN COMPLEXES OF SOFT MODULES

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Molodtsov initiated the concept of soft sets in [1]. Maji et al. defined some operations on soft sets in [2]. Aktaş et al. generalized soft sets by defining the concept of soft groups in [3]. After then, Qiu-Mei Sun et al. gave soft modules in [4].

In this study, methods of homology algebra are applied to the category of soft modules. For this reason, chain complexes of soft modules and their soft homology modules are defined. The exact sequence of soft homology modules is obtained. Homotopy relation is given in the category of chain complexes of soft modules, and according to this relation is proved the invariance of soft homology modules.

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SOME NOTES ON PURE HYPERCOMPLEX CONNECTIONS

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A hypercomplex algebra is a real associative algebra with unit 1. A Π-structure on a manifold $M$is a family of affinor fields (i.e. tensor fields of type (1,1)) on M. If Π is an algebra (under the natural operations) isomorphic to a hypercomplex algebra, the Π-structure is called hypercomplex. In this paper we define some operators which are applied to pure tensor fields. In this context Tachibana, Vishnevskii operators and the operator applied to pure connection can be found [1]. Using these operators we study some properties of integrable commutative hypercomplex structures endowed with a holomorphic torsion-free pure connection whose curvature tensor satisfy the purity condition with respect to the covariantly constant structure affinors. These connections are naturally appeared in the context of pseudo-Riemannian manifolds with Kahler-Norden(i.e. anti-Kahler) metrics. Recently, manifolds with Norden metrics have been an object of investigation in differential geometry and theoretical physics(see [2], [3], [4]).

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ON SOME FIBONACCI -TYPE POLYNOMIALS

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In this study, the zeros of some generalized polynomials defined recursively are investigated. Some properties related with these polynomials are given.

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THE PELL-PADOVAN NUMBERS AND POLYNOMIALS

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In this paper, we define new a recurrence relation and polynomials called Pell-Padovan numbers and Pell-Padovan polynomials, respectively. We also derive formulae related with theseThe Pell-Padovan sequence is the sequence of integers $T(n)$ defined by the initial values

$$T(0) = 1, T(1) = 1, T(2) = 1$$

and the recurrence relation

$$T(n) = 2T(n-2) + T(n-3).$$

The first few values of $T(n)$ are

$$1, 1, 1, 3, 3, 7, 9, 17, 25, 43, 67, 111, 177, 289, 465, \ldots.$$

In this study we denote the ratio of two consecutive Pell-Padovan numbers converges to the Golden Proportion, $\alpha = \frac{1+\sqrt{5}}{2}$. We also present a relation between the Pell-Padovan numbers with Fibonacci numbers and Lucas numbers. Moreover we introduce Pell-Padovan polynomials and give some formulae related with these polynomials.

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EUCLIDEAN CRYPTOSYSTEM WITHOUT REPETITION

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The goal of this paper is to construct and analyze the cryptosystem with the following property: (A) The encryption algorithm encrypts a plaintext of any length so that after encryption with one fixed key all letters of cipher text are different, i.e. the cipher text has no repetitions.

Let Z be the ring of integers and the cryptosystem has the following algorithms of encryption and decryption:

(*) **Encryption:** Let k be an integer. If $p \in Z,\ p \in Z\ |p| < |k|$; $s \in Z$ and s satisfies conditions:

$$(s,p) = 1,\ (s,k) = 1 \tag{1}$$

We obtain the cipher text c as follows:

Step 1: we find u using the equality $us + \nu p = 1$. This is well known that it may be evaluated from generalized Euclidean algorithm [1]. Step 2: we find a_p as the remainder of division pu by k. Step 3: $c = a_p s$.

Decryption: $p = c(mod\ k)$.

Definition. *An integer s is called the partial key of a pair (k,p) if it satisfies the conditions (1).*

The cryptosystem with the following algorithms of encryption and decryption satisfies the property (A).

(C) Algorithm of encryption: let k be an integer, $p_1, p_2, ..., p_m$ be some sequence of integers such that $|p_i| < |k|,\ i = 1, ..., m,\ s_1, s_2, ..., s_m$ be the sequence of different partial keys. We encrypt an element p_i by using the partial key s_i and the algorithm (*). As the result all cipher texts $c_1, c_2, ..., c_m$ are different. So we have: k is the secret key, $p_1, p_2, ..., p_m$ is the sequence of plaintexts, $s_1, s_2, ..., s_m$ is the sequence of the corresponding partial keys, $c_1, c_2, ..., c_m$ are corresponding cipher texts.

(D) Algorithm of decryption: $p_i = c_i(mod\ k), i = 1, 2, ..., m$.

Attack and modification.

Now we consider the following kind of attack against our cryptosystem.

(*) Let we have some set of plaintexts $p_1, p_2, ..., p_n$ and the set of corresponding cipher texts $c_1, c_2, ..., c_n, k$ is the secret key. From the decryption algorithm it follows that there exist elements $d_1, ..., d_n \in Z$ such that $a_i = c_i - p_i = d_i k,\ i = 1, 2 = ..., n$ Let us have some $(x_1, ..., x_n)$ be the greatest common divisor of elements $x_1, x_2, ..., x_n \in Z$. Then we have $(a_1, ..., a_n) = dk,\ d = (d_1, ..., d_n)$. And this attack may be effective for finding of a secret key. So we modify our cryptosystem as follows:

Algorithm of encryption: let $k, a, b \in Z, |b| < |ak|$ and $\overline{c} = (q;r) \in Z \times Z$, where $ac = bq + r$ and $ac \neq r$ because $|c| > |k|$.

$$E_k(p) = c,\ \overline{E_{a,b}}(c) = (q,r).$$

Then $\overline{c} = \overline{E_{a,b}}(E_k(p))$ and we have: p is the plaintext, $\overline{c}$ is the cipher text.

Algorithm of decryption: $[\frac{bq+r}{a}](mod\ k) = p$.

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EUCLIDEAN RINGS AND THEIR APPLICATION IN CRYPTOGRAPHY

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The goal of this paper is to construct cryptosystems with the following properties:
(A) The encryption algorithm encrypts a plaintext of any length so that after encryption with one fixed key all letters of cipher text are different, i.e. the cipher text has no repetitions. The length of the alphabet is limited to the key length.
(B) The encryption algorithm encrypts a plaintext of any length so that after encryption with one fixed key all letters of cipher text are different. The length of the alphabet is not limited.

Further we suppose that all Euclidean rings satisfy the following condition:
(E). Let $c \in K$ and $C = cK$ be the ideal of the ring K generated by the element c. If elements $a, b \in K$ satisfy the following conditions $\delta(a) < \delta(c),\ \delta(b) < \delta(c),\ a - b \equiv 0(mod\, C)$ then $a = b$.

Remark 1. *Examples of rings, which satisfy condition (E) are the ring of integers and rings of polynomials over fields. The example of a ring without condition (E) is the ring of Gauss integers [1].*

Theorem 1. *Let K be a Euclidean ring and k be an arbitrary element of K. Then for any element $p \in K$ such that p is not invertible and $\delta(p) < \delta(k)$, and for any element $s \in K$ such that*

$$(s, k) = e,\ (s, p) = e \tag{1}$$

there exist elements $a_p, c \in K$ which satisfy the following conditions:

1) $\delta(a_p) < \delta(k)$.

2)$c = a_p s,\ c(mod\, \overline{k})$, *where $\overline{k}$ is the ideal of the ring K generated by the element k*

3) $\delta(k) \leq \delta(c)$.

4) *If $p_1 \neq p_2$ and $\delta(p_1) < \delta(k),\ \delta(p_2) < \delta(k)$, then $c_1 \neq c_2$, and vice versa if $c_1 \neq c_2$, then $p_1 \neq p_2$.*

(*) Algorithm of encryption: let k be an element of the Euclidean ring K, p be any element of K such that $\delta(k) > \delta(p)$, and s be any element of K, which satisfies conditions (1). Then, using theorem 1 we obtain the element c. So we have: k is the secret key, p is the plaintext, c is the cipher text.

Definition. *An element $s \in K$ is called the partial key of the pair (k, p) if it satisfies conditions (1). If S is a finite set of distinct partial keys, then $|S|$ is the period of S. If a set S is infinite, then its period is infinity.*

Now let K be a Euclidean ring, k be the element of the ring K, $p_1, p_2, ..., p_m$ be some sequence of elements of K such that $\delta(p_i) < \delta(k)$. Suppose that the ring K satisfies the following condition:
(In) The set $S = \{s \in K, s \text{ prime}, \delta(s) > \delta(k)\}$is infinite.
(C) Algorithm of encryption: let k be an element of the ring K, $p_1, p_2, ..., p_m$ be some sequence of elements of K such that $\delta(p_i) < \delta(k),\ i = 1, ..., m,\ s_1, s_2, ..., s_m$, be the sequence of distinct elements from S. We encrypt element p_i by using the partial key s_i and algorithm (*). As the result all cipher texts $c_1, c_2, ..., c_m$ are different.
(D) Algorithm of decryption: $p_i = c_i(mod\, k), i = 1, 2, ..., m$.

Theorem 2. *Let $K = Z$ be the ring of integers. Then the cryptosystem with algorithm of encryption (C) and the algorithm of decryption (D) satisfies property (A).*

Theorem 3. *Let $K = Q[x]$ be the polynomial ring over the field of rational numbers Q. Then the cryptosystem with algorithms of encryption and decryption (C), (D) satisfy the property (B).*

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UNIVERSAL ENVELOPING ALGEBRAS AND UNIVERSAL DERIVATIONS OF POISSON ALGEBRAS[1]

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We study the universal enveloping algebras of Poisson symplectic algebras and free Poisson algebras. For any Poisson algebra P denote by P^e its universal enveloping algebra. Note that the universal enveloping algebra A^e of an associative algebra A in the variety of associative algebras is $A \otimes_k A^{op}$ where A^{op} is an anti-isomorphic copy of A. It is proved that the universal enveloping algebras of the Poisson symplectic algebra P_n and the Weyl algebra A_n are isomorphic. The canonical isomorphism between P_n^e and A_n^e naturally leads to the Moyal product.

Let $P = k\{x_1, \ldots, x_n\}$ be the free Poisson algebra in the variables $x_1, \ldots, x_n$. We construct a linear basis of the universal enveloping algebra P^e. Unfortunately left ideals of P^e are not free left P^e-modules, i.e., an analogue of Cohn's theorem for free associative algebras is not true in this case. We prove a weaker result which says that the left dependency of a finite set of elements P^e over P^e is algorithmically recognizable. We prove that any two elements of a free Poisson algebra over a field of characteristic 0 either generate a free Poisson algebra in two variables or commute. We prove that an analogue of the Jacobian Conjecture for two generated free Poisson algebras is equivalent to the two dimensional classical Jacobian Conjecture. A new proof of the tameness of automorphisms of two generated free Poisson algebras is also given.

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THE STRUCTURAL PROPERTIES OF LINEAR MATRIX INEQUALITIES

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Our previous research **"On the Feasibility Problem of Linear Matrix Inequalities"** has focused on the characterization of the feasibility region of second order linear matrix inequalities (LMIs) and defining this region as a finitely generated convex cone. More precisely, firstly we have proposed a simple method which gives the full characterization of the feasibility region if there exists at least one strictly definite element in a given LMI. Then, we have introduced an outer cone for an LMI with two nondefinite matrices. Finally, we have proposed a procedure for refining this cone so that it will intersect the feasibility region of the given LMI. We have also defined an inner cone for higher order LMIs with at least one strictly definite element and shown that the distance between the boundaries of the inner cone and the feasibility region can be directly calculated using the original problem data.

In this research, we focus on the characterization of the feasibility region of higher order LMIs with nondefinite elements. In this context, let $A_1, A_2, ..., A_k \in \mathbb{R}^{n\times n}$ $(2 < k \leq n)$ be nondefinite symmetric matrices whose diagonal elements are linearly independent. Then, $\mathcal{F}_\alpha := \left\{ \alpha \,\middle|\, \sum_{i=1}^{k} \alpha_i A_i \geq 0, A_i^T = A_i \in R^{n\times n} \right\}$ is a finitely generated cone in $\mathbb{R}^k$. It can be easily shown that $\mathcal{F}_\alpha$ and $\mathcal{F}_\beta := \left\{ \beta \,\middle|\, \sum_{i=1}^{k} \beta_i B_i \geq 0, B_i^T = B_i \in R^{n\times n} \right\}$ are equivalent LMIs as defined in [1] and the i^{th} column of $D := \begin{bmatrix} I_k & K_o^T \end{bmatrix}^T$ consists of the diagonal elements of B_i. We assume without loss of generality that $B_1, B_2, ..., B_k$ are nondefinite. Then, the outer cone $\mathcal{C}_{out}$ for $\mathcal{F}_\beta$ can be defined as

$$\mathcal{C}_{out} := \left\{ \beta := [\,\beta_1 \ldots \beta_k]^T \mid K_o\beta \succeq 0,\ \beta \succeq 0 \right\}$$

which is also finitely generated. Based on the theories in [2] and [8], $\beta \in \mathcal{C}_{out}$ can be expressed as the conic combinations of its extreme directions. In this work, we first propose a procedure for row by row calculation of the extreme directions of the $\mathcal{C}_{out}$. Then, we give a method which reduces the size of the outer cone until it intersects the set $\mathcal{F}_\beta$.

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ON A SIMILARITY OF THE POSITIVE JOHNSON'S THEORIES IN ENRICHED SIGNATURE

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One of the classic problems of mathematical science is the question of the classification of objects in order to study for some common features. In mathematics a role of such objects performing sets which predetermined by its relations. Using mathematical logic, these objects were related to the formulae of the language of calculus of predicates. This relationship between syntax and semantics of language is the main essence of the model theory. Therefore, it is clear that finding syntax and semantics features of the similarity can be useful for classifying objects in the model theory.

In this abstract we present the results associated with the Δ-PM-theories in the enriched signature.

Let T - an arbitrary Δ - PM-theory in the language of signatures Σ. Consider all the completions of a center T^* theory T in new signature σ_Γ, where $\Gamma = \{c\}$. By Δ - PM-surface theory T^*, its center there and we denoted as T^c. Under the restriction T^c to the signature Σ, the theory of T^c becomes a complete type. This type, like in Δ-PM-case, we call the central theory of the type T. Let C-semantic model of T, $A \subseteq C$. Let $\sigma_\Gamma(A) = \sigma \cup \{c_a \mid a \in A\} \cup \Gamma$, where $\Gamma = \{P\} \cup \{c\}$.

Consider the following theory $T_\Gamma^{PM}(A) = Th_{\Pi^+_{\alpha+2}}(C, a)_{a\in A} \cup \{P(c)\} \cup \{P(c)\} \cup \{"P \subseteq "\}$, where $\{"P \subseteq "\}$ there is an infinite set of sentences which says that the interpretation of the symbol P is a positive existentially closed submodel in the signature σ. The requirement of existential closure is essential to the sense that the interpretation of the symbol P must be infinite. It is clear that the considered set of proposals is a theory and this theory in general is not complete. Let T - an arbitrary Δ - PM-theory, then $E^+(T) = \bigcup n, m < \omega E^+_{n,m}(T)$, where $E^+_{n,m}(T)$ - a lattice of positive existential formulas with n free variables and m-changes of quantifiers.

Let T_1 and T_2 Δ - PM-theory. We say that T_1 and T_2 Δ-PM syntactically similar if there exists a bijection $j : E^+(T_1) \longrightarrow E^+(T_2)$ such that:

1) the restriction f to $E^+_{n,m}(T_1)$ is a lattice isomorphism $E^+_{n,m}(T_1)$ and $E^+_{n,m}(T_2)$, $n < \omega$,
2) $f(\exists \nu_{n+1}\varphi) = \exists \varphi_{n+1} f(\varphi)$, $\varphi \in E^+_n(T)$, $n < \omega$,
3) $f(\nu_1 = \nu_2) = (\nu_1 = \nu_2)$.

One of the results obtained under the above definitions as follows:

Theorem *Let T_1 and T_2 - Σ^+_{m+1}-complete, perfect, Jonsson's Δ - PM-theory. Then the following conditions are equivalent:*

1) T_1^* *and* T_2^* Δ - *PM-syntactically similar,*
2) T_1^c and T_2^c - *syntactically similar (in the sense of [1]).*

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SECTION II
Geometry and Topology

DOUBLE KNOT SURGERIES TO S^4 AND $S^2 \times S^2$

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It is known that S^4 is a union of two fishtails, and $S^2 \times S^2$ is a union of two cusps (glued along their boundaries). Here we prove that for any choice of knots $K, L \subset S^3$ performing knot surgery operations $S^4 \rightsquigarrow S^4_{K,L}$ and $S^2 \times S^2 \rightsquigarrow (S^2 \times S^2)_{K,L}$ along both of these fishtails and cusps, respectively, do not change the diffeomorphism type of these manifolds.

ON THE GEOMETRY OF THE TANGENT BUNDLE

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The tangent bundle of a smooth manifold has itself a natural structure as a smooth manifold and constitutes the natural domain of discourse of analytical dynamics in the manner of Lagrange. Thus the tangent bundle hosts phenomena with geometric as well as dynamical portent. The purpose of this talk will be to introduce the basic concepts in a historical perspective and present a short report of my recent work on conformal vector fields on tangent bundles.

INTEGRAL GEOMETRY PROBLEM FOR GENERAL FAMILY OF CURVES ON FINSLER SURFACE

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We consider a general family of curves Γ on a compact oriented Finsler surface (M,F) with boundary ∂M. Let $\varphi \in C^\infty(M)$ and ω a smooth 1-form on M. We show that

$$\int_{\gamma(t)} \{\varphi(\gamma(t)) + \omega_{\gamma(t)}(\dot{\gamma}(t))\}\, dt = 0$$

holds for every $\gamma \in \Gamma$ whose endpoints belong to ∂M, $\gamma(a) \in \partial M$, $\gamma(b) \in \partial M$ if and only if $\varphi = 0$ and ω is exact. This generalizes Mukhometov‘s well-known theorem in several directions.

CHARACTERIZATIONS OF SPHERICAL HELICES IN EUCLIDEAN 3-SPACE

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In this study, we calculate Frenet frames of the *tangent indicatrix* (t), *principal normal indicatrix* (n) and *binormal indicatrix* (b) of the curve α in $\mathbb{R}^3$. Also, we give some differential equations which they are characterizations for (t), (n) and (b) to be general helix. Morever we give a characterization for *tangent indicatrix* (t) to be a circle.

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SPECIAL CONTACT HYPERSURFACES OF SASAKIAN SPACE FORM

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A submanifold M of a contact manifold $\widetilde{M}^{2m+1}$ tangent to ξ is called an invariant (resp. anti-invariant) submanifold if $\phi(T_pM) \subset T_pM, \forall p \in M$ (resp. $\phi(T_pM) \subset T_p^{\perp}M, \forall p \in M$).

A submanifold M tangent to ξ of a contact manifold $\widetilde{M}^{2m+1}$ is called a contact CR-submanifold if there exists a pair of orthogonal differentiable distributions D and $D^{\perp}$ on M, such that:

(1) $TM = D \oplus D^{\perp} \oplus \mathbb{R}\xi$, where $\mathbb{R}\xi$ is the 1−dimensional distribution spanned by ξ;
(2) D is invariant by ϕ, i. e., $\phi(D_p) \subset D_p, \forall p \in M$;
(3) $D^{\perp}$ is anti-invariant by ϕ, i. e., $\phi(D_p^{\perp}) \subset T_p^{\perp}M, \forall p \in M$.

Let (M, g) be a real connected hypersurface of $\widetilde{M}$ and N be a unit normal vector field on M. Then we have

$$TM = D \oplus D \perp \oplus \mathbb{R}\xi$$

where D is ϕ-invariant subspace and $D^{\perp}$ is one dimensional subspace of spanned by $V = \phi(N)$ such that it is orthogonal component of D. Let A be the shape operator of M and the plan spanned by $\{\xi, V\}$ bean invariant subspace of A.

With above structure we proved:

Let M^{2n} be a contact hypersurfaces of Sasakian space form $(\widetilde{M}^{2n+1}, \phi, \xi, \eta)$ with weakly $\phi-$sectional constant curvature. Then M^{2n} have constant principal curvature.

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A CONTINUOUS MOTION DERIVED FROM A REGULAR CURVE

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Let

$$\alpha : I \subseteq \mathbb{R} \to \mathbb{R}^3$$

be a regular curve not passing origin. $\beta(t) = \frac{\alpha(t)}{\|\alpha(t)\|}$, normed projection of α on unit sphere, is a spherical curve. Every point of β corresponds on equivalence class. If $\overrightarrow{b(t)} \in \beta(t)$, then $\left\|\overrightarrow{b(t)}\right\| = 1$ and for $\forall t$, $b(t) \in S^2$. If we choose an arbitrary rotating angle, θ, we can define a rotating matrix which their axis is d_b and rotating angle is θ. So we have an equivalence class for $0 \leq \theta \leq 2\pi + k\pi\theta$. One parameter curve of this equivalence class is one parameter action set which derivated from α.

$$C(t) : \mathbb{R}^3 \to \mathbb{R}^3,$$

$$p \to C(t)(p) = P(t)$$

is a curve in $\mathbb{R}^3$ and it is called as orbit of P under $\alpha(t)$. The action set is invariant on cone surface which has α as striction curve. In other words, we assume that a cone surface is given as

$$K(t,v) = (1+v)\overrightarrow{O\alpha(t)}.$$

For every curve on $K(t,v)$ which generates the same surface, we have the same $C(t)$ action set. This means that all of cone surface forms the same action set.

The relations between the orbit of $p \in \mathbb{R}^3$ under this action set and α are very important. Finally Matlab application is given.

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MANNHEIM CURVES IN 4D GALILEAN SPACE

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One of the principal problems that has been studied by researchers are special curves and their characterizations. A space curve is called "Mannheim curve" if and only if for some constants, it satisfies the following relation

$$k_1 = \beta(k_1^2 + k_2^2)$$

where k_1 and k_2 are curvature and torsion, respectively.

On the other hand Galilean and Pseudo-Gallilean spaces have been recently investigated by [6],[7], [9],[10],[11]. In this paper, we consider the idea of Mannheim curves in 3D Galilean space. Then we give the definition of Mannheim curves for 4D Galilean space and some characterizations are also obtained.

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STATISTICAL CONVERGENCE OF DOUBLE SEQUENCES OF ORDER (α, β)

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In the present study we introduce and examine the concepts of statistical convergence of order (α, β) and strong $p-$Cesàro summability of order (α, β) for double sequences of complex (or real) numbers, where α, β are any real numbers such that $0 < \alpha \leq 1$ and $0 < \beta \leq 1$. We give some results and establish some inclusion relations between the set $S^2_{(\alpha,\beta)}$ of all double statistical convergent sequences of order (α, β) and the set $w^2_{p(\alpha,\beta)}$ of all double strongly $p-$Cesàro summable sequences of order (α, β).

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FREE BICOMMUTATIVE ALGEBRAS

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Algebra with identities $a(bc) = b(ac)$, $(ab)c = (ac)b$, is called bicommutative. Let $F(X)$ be free bicommutative algebra over a field K of characteristic $p \geq 0$. For $\mathbf{m} = (m_1, \ldots, m_q)$ denote by $F^{\mathbf{m}}(X)$ a space generated by products of elements $x_1, \ldots, x_q$, where x_1 appears m_1 times, x_2 m_2 times, etc x_q appears m_q times. Let F_n^{multi} be multilinear part of free bicommutative algebra generated by n elements, i.e., $F_n^{multi} = F_n^{11\cdots 1}(X)$, with $|X| = n$.

Let $\mathbf{Z}_+$ be set of non-negative integers, $\mathbf{Z}_+^q = \mathbf{Z}_+ \oplus \cdots \oplus \mathbf{Z}_+$ is a direct sum of q copies of $\mathbf{Z}_+$ and $\epsilon_i = (0, \ldots, 0, 1, 0, \ldots, 0) \in \mathbf{Z}_+^q$, all components except i-th are 0. Let $F(q)$ be a linear space with base

$$V = \{x_i | i = 1, \ldots, q\} \cup \{v_{\alpha,\beta} | \alpha, \beta \in \mathbf{Z}_+^q, \ \sum_{i=1}^q \alpha_i \neq 0, \ \sum_{i=1}^q \beta_i \neq 0\}.$$

Endow $F(q)$ by a multiplication $\circ$ given by the following rules

$$x_i \circ x_j = v_{\epsilon_i,\epsilon_j}, \quad x_i \circ v_{\alpha,\beta} = v_{\alpha+\epsilon_i,\beta}, \quad v_{\alpha,\beta} \circ x_j = v_{\alpha,\beta+\epsilon_j}, \quad v_{\alpha,\beta} \circ v_{\gamma,\delta} = v_{\alpha+\gamma,\beta+\delta}.$$

Let $S^{[n]}$ be trivial S_n-module, and $S^{[n-i,i]}$ be S_n-module corresponding to Young diagramm with two rows with $n-i$ and i boxes

$$\begin{matrix} \bullet & \cdots & \cdots & \bullet \\ \bullet & \cdots & \bullet & \end{matrix}$$

Theorem. *Let $X = \{x_1, \ldots, x_q\}$ be set of generators. Then*

a *$F(q)$ is isomorphic to a free bicommutative algebra $F(X)$.*

b *($p = 0$) As a module of symmetric group S_n*

$$F_n^{multi} \cong (n-1)S^{[n]} \oplus \bigoplus_{i=1}^{\lfloor \frac{n-1}{2} \rfloor} (n-2i+1)S^{[n-i,i]}, \quad n > 1,$$

where $S^{[n]} \cong K$ is a trivial S_n-module and $S^{[n-i,i]}$ is an irreducible S_n-module corresponding to partitions $\{n-i, i\} \vdash n$ and $\lfloor \alpha \rfloor$ is a integer part of α.

c *Bicommutative operad is not Koszul.*

Corollary. *For a Hilbert generating function $H(bicom, t_1, \ldots, t_q) = \sum_{\mathbf{m}} \dim F^{\mathbf{m}}(X) t^{\mathbf{m}}$ takes place the following formula*

$$H(bicom, t_1, \ldots, t_q) = 1 + \sum_{i=1}^q t_i + \prod_{i=1}^q \frac{1}{(1-t_i)^2} - 2\prod_{i=1}^q \frac{1}{1-t_i}.$$

In particular,

$$\dim F^{\mathbf{m}}(X) = (m_1+1)\cdots(m_q+1) - 2 + \delta_{|\mathbf{m}|,\mathbf{1}} + \delta_{|\mathbf{m}|,0}$$

and

$$\dim F_n(X) = \binom{n+2q-1}{n} - (2 - \delta_{n,1})\binom{n+q-1}{n}.$$

Codimensions sequence is given by the formula

$$c_q = \dim F^{\mathbf{1}}(X) = 2^q - 2 + \delta_{q,1}.$$

Exponent of the variety of bicommutative algebras is 2.

Here $\delta_{x,y}$ denotes Kronecker symbol, it is 1 if $x = y$ and 0, if $x \neq y$.

THE NORMAL CURVATURES AND THE FUNDAMENTAL FORMS OF INVERSE SURFACES

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In this study, we studied inverse surfaces in 3-dimensional Euclidean space E^3. We investigated third fundamental forms of inverse surfaces and obtained some relations about these. Also, we gave a formula concerning the normal curvatures of inverse surfaces.

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THE NONDEGENERATE FOCAL SURFACE OF A TUBE

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A tube surface is the envelope of a one-parameter family spheres with constant radius and variable center. In this study we consider the nondegenerate focal surface of a tube. We obtain that it is developable and never is contained in a plane.

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A CONTRIBUTION TO THE LINE GEOMETRY

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As known the geometry of a trajectory surfaces tracing by an oriented line (spear) is important in line geometry and spatial kinematics. Until early 1980s, although two real integral invariants, the pitch of angle λ_x and the pitch ℓ_x of an x- trajectory surface were known, any dual invariant of the surface were not. Because of the deficiency, line geometry wasn't being sufficiently studied by using dual quantities.

A global dual invariant, Λ_x, of an x- closed trajectory surface is introduced and shown that there is a magic relation between the real invariants, $\Lambda_x = \lambda_x - \varepsilon\ell_x, [1]$. It gives suitable relations, such as $\Lambda_x = 2\pi - A_x = \oint G_x$ or $\lambda_x = 2\pi - a_x = \oint g_x ds$ $\ell_x = a_x^* = \oint(\partial_u + \partial_v) dudv$ and also the dual distribution parameter which have the new geometric interpretations of x- trajectory surface where a_x is the measure of the spherical area on the unit sphere described by the generator of x- closed trajectory surface and ∂_u and ∂_v are the distribution parameters of the principal surfaces of the $X(u; v)$ - closed congruence. Therefore all the relations between the global invariants, λ_x, ℓ_x, a_x, a_x^*, g_x, g_x^*, K, T, σ and s_1 of x- c.t.s. are worth reconsidering in view of the new geometric explanations. Thus, some new results and new explanations are gained $[1, 2, 3, 4,]$. Furthermore, as a limit position of the surface, some new theorems and comments related to space curves are obtained.

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ON THE QUATERNIONIC $\mathbf{B_2}$ SLANT HELICES IN THE SEMI EUCLIDEAN SPACE $\mathbf{E_2^4}$

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In this paper, we studied quaternionic slant helices in four dimensional semi- Euclidean space E_2^4 which we called semi-real quaternionic B_2 slant helix. Also we define the harmonic curvature functions for semi-real quaternionic curves and we give a vector field D which we call Darboux quaternion for semi-real quaternionic B_2 slant helix. And then we obtained some characterizations for semi-real quaternionic B_2 slant helix in the terms of the harmonic curvatures and by using D Darboux vector field.

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ON THE ISOMETRY GROUP OF CHINESE CHECKER SPACE

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In this work, we, firstly, find that the spheres in the Chinese Checkers space are deltoidal icositetrahedrons. Then we show that the group of isometries of the 3-dimensional space with respect to Chinese Checkers metric is the semi-direct product of deltoidal icositetrahedron group $G(D)$ and $T(3)$, where $G(D)$ is the (Euclidean) symmetry group of the deltoidal icositetrahedron and $T(3)$ is the group of all translations of the 3-dimensional space.

The talk will be based on the [1].

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DIRECTED HOMOTOPY THEORY OF DIGITAL IMAGES

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In this paper we recall some concepts like digital image, digital path, directed space, directed path and directed homotopy. We give the definition of digital directed path and digital directed map and present some fundamental examples of directed spaces in digital images. Directed homotopy in digital images is introduced and some categorical structures are constructed via directed digital homotopy.

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A NEW CHARACTERIZATION FOR THE TIMELIKE SPACE CURVE IN THE LORENTZ SPACE

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Let

$$\alpha : I \subset IR \to L^3,$$
$$s \to \alpha(s) = (\alpha_1(s), \alpha_2(s), \alpha_3(s))$$

be a smooth regular curve in L^3, (i.e $\alpha^{'}(t) \geq 0$ for any $t \in I$) where I is an open interval. For any $t \in I$ the curve α is called spacelike curve, lightlike curve or a timelike curve if $\langle \alpha, \alpha \rangle > 0, \langle \alpha, \alpha \rangle = 0, \langle \alpha, \alpha \rangle < 0$, respectively. Let $\{t, n, b\}$ be the Frenet trihedron of the differentiable timelike space curve in Lorentz space. Then the Frenet equations are

$$t^{'} = \kappa n,$$
$$n^{'} = \kappa t + \tau b,$$
$$b^{'} = -\tau n,$$

where κ is curvature and τ is torsion. In addition, Darboux vector is defined as follows[3]

$$w = -\tau t - \kappa b.$$

In this paper, we give arc lengths of spherical representations of tangent vector field t, principal vector field n, binormal vector field n and the vector field $c = \frac{w}{\|\overrightarrow{w}\|}$, where w is the Darboux vector field of a timelike space curve α in Lorentz space.

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CONDITION OF BULGE AND CONCAVITY OF A CURVE IN THE POLAR SYSTEM OF COORDINATES IN THE EXTREME POINTS OF RADIUS OF FUNCTIONS

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The method of research of functions in the polar system of coordinates is given in the work [1]. In this work is obtained the condition of concavity and bulge of the graph of functions in extreme points. We consider a curve, given in the polar system of coordinates by the equation $\rho = \rho(\varphi)$.

Definition. *The graph of functions in the polar system of coordinates is protuberant in the extreme points of radius of functions if the normal in this point, is directed to the pole; concave, if in these points the normal is directed to an opposite side.*

It ensues from definition of a circle of curvature, that the center of curvature always lies on the normals to the curve in the examined point from the side of concavity (I.e. outside, reverse to that, where the bulge of curve is directed).

On the tangent to the curve the positive is consider on the direction towards the growth of arc the curve. On the normal for positive direction we will choose such kind of direction, that must be relative (positively directed) to the tangent as it also oriented, as an y-axis to the of x-axis. For example, at ordinary direction of these axes normal must make corner $+\pi/2$ with the tangent anticlockwise. Now, considering the radius of curvature as a directed segment, lying on the normal, we add him a sign plus, if he is put aside on the normal in the positive directions and the sign minus, otherwise. So in the case (I) the radius of the curvature will have a sign of "+", and in a case (II) p sign of "–" (Fig.1).

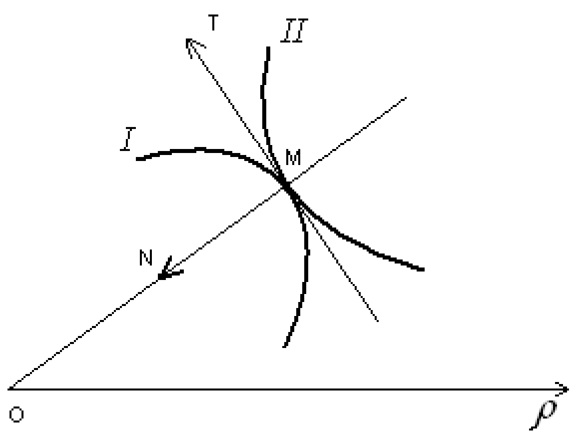

Figure 1

Considering into account that in extreme points the first derivate $\rho'(\varphi_0) = 0$, get terms, at $\rho - \rho'' > 0$, a curve is protuberant, and at $\rho - \rho'' < 0$, a curve is concave.

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MIRROR PRINCIPLE AND HORI-VAFA CONJECTURE ON HOMOGENEOUS SPACES

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In a series of papers Lian, Liu, and Yau ([1], [4]) proposed a machinery called "Mirror Principle" to solve some enumerative problems including the conjectural formula of Candelas et.al.([2]) for counting the number n_d of rational curves in every degree d on a general quintic in $\mathbf{P}^4$. Basically, mirror principle proposes that certain sequences of multiplicative equivariant characteristic classes on stable map moduli spaces can be computed in terms of certain hypergeometric type classes.

Given a projective, non-singular variety X, $d \in H_2(X)$ and a vector bundle V with a multiplicative class b, we can construct a cohomology valued "Generalized Hyperbolic Series" which is shown to carry important information about the intersection numbers

$$K_d = \int_{[M_{0,1}(d,X)]} b(V_d)$$

where $\overline{M}_{0,k}(d,X)$ is the moduli space of $k-$pointed genus 0, degree d stable maps with target X, $[M_{0,1}(d,X)]$ is the virtual fundamental class ([3]) and V_d is a vector bundle on $\overline{M}_{0,k}(d,X)$ induced by V whose fiber at $(C, x_1, \ldots, x_k, f)$ is given as $H^0(C, f^*V) \oplus H^1(C, f^*V)$. This cohomology valued series is called A series associated to V and b and denoted by $A^{V,b}(t)$.

In this presentation, we explain the computation of A series of a flag manifold using $\mathbf{T}$-equivariant tangent bundle and Chern polynomial where $\mathbf{T}$ is an algebraic torus. We also explain the connections of similar computation to a generalization of Hori-Vafa conjecture which originally describes a generating function for 1-point descendant Gromov-Witten invariants of a Grassmannian in terms of the $J-$ function of a product of projective spaces.

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ON THE CURVATURE AND NATURAL LIFTS OF THE SPHERICAL INDICATORS OF MANNHEIM CURVE PAIRS

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Let $\alpha : I \to E^z$ be a Mannheim curve and $\alpha : I \to E^z$ be Mannheim partner curve of α. Let $\{T^*, N^*, B^*\}$ be the Frenet frame and C^* be the unit vector of this Frenet frame.

The following feets have been proved.

Corollary 1. *There is not such a α Mannheim curve that (T^*) tangent indicator is a big circle on unit sphere. Thus $\overline{(T^*)}$ natural lift of (T^*), can not be an integral curve of geodesic spray on $T\left(S^2\right)$ tangent bunch.*

Corollary 2. *If the α Mannheim curve is a helix then (B^*) binormal indicator of Mannheim partner curve is a big circle on unit sphere. In this case, $\overline{(B^*)}$ natural lift of $(B^*$ binormal indicator is an integral curve of geodesic spray on $T\left(S^2\right)$ tangent bunch.*

Corollary 3. *There is not a α^* curve such that (C^*) constant pol curve is a big circle on unit sphere. Thus, $\overline{(C^*)}$ natural lift of (C^*) curve can not be an integral curve of geodesic spray on $T\left(S^2\right)$ tangent bunch.*

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GEOMETRY OF DECODING OF A GENETIC CODE

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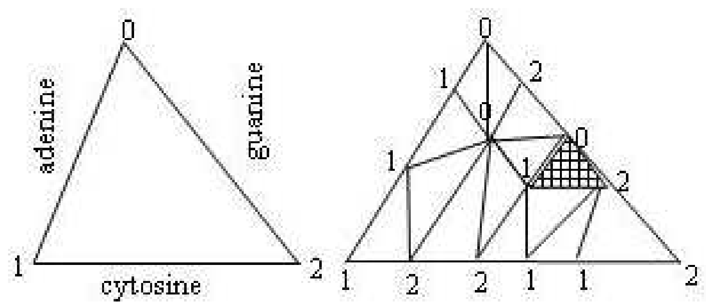
Modern genetics represents any live cell as formation, consisting of two basic parts, i.e. the *kernel* containing *chromosomes* - that carriers the hereditary information, and *cytoplasm*- which is the main mass of the cell and contains the *enzymes, playing a role* of catalysts ensuring various biochemical reflection, necessary for development and organism survival.

Enzymes and the majority of hormones are long protein molecules. Proteins represent the linear chains consisting of rather simple molecules of various *amino acids,* connected among themselves so-called *peptide connections.* Note that, under the influence of heat and weak acids of the proteins decay on amino acids. Amino acids in proteins organize chains with probability properties. It is interesting that in organic chemistry by association to *any* molecule of atom of hydrogen, amino group (-NH2) and carboxylic groups (-SOON) it is possible to obtain thousand various amino acids. However, expansion of molecules of fiber leads *only to twenty* various amino acids.

Protein chains in various enzymes and hormones are defined genetic material, containing in chromosomes. This material is known as a nucleonic acid molecule by which are separated from cell kernels, represent the chains consisting of rather simple atomic groups - *nucleotides.* However in contrast to fibers nucleotide exist only in four aspects: *Adenine* (A), *Thymine (T), Guanine (G), Cytosine (C).*



The principal not solved completely genetics problem is an establishment of the one to one correspondence between triples of organized of twenty proteins and from triples from four nucleotides.

This problem belongs to the problems of combinatorial mathematics. The considered problem is directly connected with this problem. Using the Shperner's theorem and geometrical representation a triangle with sides the basic of three nucleotides from four. We prove that any triple from four nucleotides is definitely in the same disposition will be met repeatedly.

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GENERALIZED EULER-LAGRANGE AND HAMILTON EQUATIONS ON HORIZONTAL AND VERTICAL DISTRIBUTIONS

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In the work given in [8], mathematical models of mechanical systems are introduced on the horizontal and vertical distributions of tangent and cotangent bundles. In this study, we generalize Euler-Lagrange and Hamilton equations deduced in [8]. Also we give some propositions and examples about generalized Euler-Lagrange and Hamilton equations.

Finally, some geometrical and physical results related to the obtained mechanical systems and equations are discussed.

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SIMPLICITY OF NONASSOCIATIVE ALGEBRAS WITH SWITCHES

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This article is devoted to proving the fact that simple algebras with switches are exhausted by the trivial one-dimensional abelian algebra. Algebra with the switches are of great use in the theory of dynamical systems. Let's consider an algebra A, satisfying the identity

$$(a \circ b) \circ c = (b \circ a - a \circ b) \circ c$$

. We call such algebra A as an algebra with switches. The first problem on nonassociative algebras is classification of simple objects. The object under investigation is the problem of finding simple algebras in variety of algebras with switches. Let us consider Jacobian element $I(a, b, c) = a \circ (b \circ c) + b \circ (c \circ a) + c \circ (a \circ b)$ for some $a, b, c \in A$. We will prove the following important lemma.

Lemma. *$\forall d \in A$ the following statement is true: $I(a, b, c) \circ d = 0$.*

Using this lemma we can obtain the proof of main theorem.

Theorem. *Algebra with the switches A is simple only when it is trivial abelian algebra of dimension one. Also we give the classification of finite-dimensional algebras with the switches of small dimension.*

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ON INVOLUTE-EVOLUTE CURVE COUPLES IN RIEMANN-OTSUKI SPACE

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Riemann-Otsuki spaces are the special type of Weyl-Otsuki spaces of which metric tensor satisfies Otsuki and $\nabla_k g_{ij} = 0$ relations. In [1] Nadj obtained Frenet formula of this space with respect to covariant and contravariant part of the connection.

On the other hand, consider the curves α and β with their coordinate neighborhoods (I, α) and (I, β) in E^n. Let Frenet r- frames of α and β be $\{V_1, ..., V_r\}$ and $\{V_1^*, ..., V_r^*\}$, respectively. β is called involute of α (α is called evolute of β) if

$$g_{ij}(V_1, V_1^*) = 0$$

holds.

In this paper after a short brief of Riemann-Otsuki spaces, we define involute -evolute curve couples of 3-dimensional Riemann-Otsuki space $R - O_3$ and obtained characterizations in this space for this type curves.

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ELASTIC CURVES ON THE CYLINDER[1]

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In this study, we examine elastic curves on cylinder surface in 3−dimensional Euclidean space. First, we define elastic curve on cylinder surface as a curve minimizing the functional defined by the integral of the sum of the squared geodesic curvature and squared normal curvature (bending energy functional) among regular curves of a fixed length satisfying given boundary conditions (the same initial point, the same initial direction, the same end point and the same end direction). Then, we derive differential equation determining elastic curves on cylinder using elements of differential geometry and Euler-Lagrange equations. Finally, by means of this differential equation we obtain that geodesics (straight line, circle and helix) on cylinder are elastic curves.

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ON SOLITON SURFACE[1]

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One of the new stage in interaction of the differential geometry and nonlinear partial derivative differential equations begins with opening of the inverse problem method. Method of the inverse problem allows to solve Cauchy problems for such nonlinear partial derivative differential equations as Korteweg de Vries equation, modified Korteweg de Vries equation, nonlinear Schrodinger equation, Landau-Lifschitz equation [1].

Let's consider well-known from differential geometry Gauss-Codacci-Mainardi equation in space (1+1)-dimensionality [2]

$$A_t - B_x + [A, B] = 0, \tag{1}$$

where A is 3×3 matrix, B is an arbitrary 3×3 matrix, $[A, B] = AB - BA$.

We imply the following (1+1)-measured spin system by isotropic Landau-Lifschitz equation [1]

$$\mathbf{S}_t = \mathbf{S} \times \mathbf{S}_{xx}, \tag{2}$$

where $\times$ is vector product, $\mathbf{S} = (S_1, S_2, S_3), \mathbf{S}^2 = S_1^2 + S_2^2 + S_3^2 = 1$.

It is known Gauss-Weingarten equation [2]

$$\mathbf{r}_{xx} = \Gamma_{11}^1 \mathbf{r}_x + \Gamma_{11}^2 \mathbf{r}_t + L\mathbf{n}, \ \ \mathbf{r}_{xt} = \Gamma_{12}^1 \mathbf{r}_x + \Gamma_{12}^2 \mathbf{r}_t + M\mathbf{n}, \tag{3a}$$

$$\mathbf{r}_{tt} = \Gamma_{22}^1 \mathbf{r}_x + \Gamma_{22}^2 \mathbf{r}_t + N\mathbf{n}, \tag{3b}$$

$$\mathbf{n}_x = p_{11}\mathbf{r}_x + p_{12}\mathbf{r}_t, \ \ \mathbf{n}_t = p_{21}\mathbf{r}_x + p_{22}\mathbf{r}_t, \tag{3c}$$

where Γ_{ij}^k are Christoffel symbols, $p_{ij}, i = 1, 2$ are coefficients.

Working in orthogonal base is convenient in soliton geometry. We enter an orthogonal base

$$\mathbf{e}_1 = \frac{\mathbf{r}_x}{\sqrt{E}}, \quad \mathbf{e}_2 = \mathbf{n}, \quad \mathbf{e}_3 = \mathbf{e}_1 \times \mathbf{e}_2. \tag{4}$$

In this case Gauss-Weingarten equation (3) is reduced to certain form.

We use an united spin approach to soliton equations [1]. With help of this approach a surface corresponding to certain (1+1)-measured spin system is built. According to the to this approach following identity

$$\mathbf{S} \equiv \mathbf{e}_1. \tag{5}$$

holds. We present the following statement.

Statement. *At performing of the condition (5) from equations (1), (2), (3) we get following expressions for* $\omega_j, j = 1, 2, 3:$

$$\omega_1 = \frac{1}{k}(\sigma_t - \omega_{2x} + \tau\omega_3), \ \omega_2 = -k_x - \tau\sigma, \ \omega_3 = \sigma_x - \tau k, \tag{6}$$

where ω, k, σ, τ *are functions defined the surface and expressed in Christoffel symbols.*

It is possible to express (6) in coefficient term of the first and second quadratic forms.

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SECTION III

Function Theory and Functional Analysis

SYMBOLS OF MULTIDIMENSIONAL SINGULAR INTEGRALS AND OSCILLATING MULTIPLIERS IN THE WEIGHTED SPACES OF BESSEL POTENTIALS ON THE SPHERE

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Let S be the Beltrami operator on the sphere S^{n-1} of the space R^n (spherical part of the Laplace operator Δ) and let $H_p^l\left(S^{n-1}\right)$ be the space of Bessel potentials on this sphere - the completeness of the space $C^\infty\left(S^{n-1}\right)$ by the norm $\left\|(E+\delta)^{l/2}g\right\|_{L_p(S^{n-1})}$, where E is the identity operator, l positive number and $p \geq 1$, so that

$$\|g\|_{p,l} = \left\|(E+\delta)^{l/2}g\right\|_p .$$

For the points $x', y' \in S^{n-1}$ we denote $x' = (\theta, \theta')$ and $y' = (\omega, \omega')$, where $\theta', \omega' \in S^{n-2}$ and $0 \leq \omega, \theta < n$ and introduce the weighted spaces of Bessel potentials on the sphere. Given real number β we denote by $C_\beta H_p^l\left(S^{n-1}\right)$ the space of functions with finite norm

$$\|g\|_{p,l,C_\beta} = \|g\|_p + \left\|(\cos\theta)^\beta \delta^{l/2}g\right\|_p$$

and by $C_\beta H_p^l\left(S^{n-1}\right)$ the space of functions with finite norm

$$\|g\|_{p,l,S_\beta} = \|g\|_p + \left\|(\sin\theta)^\beta \delta^{1/2}g\right\|_p .$$

We will give the theorems on connections between the differential properties of Calderon-Zygmud multidimensional singular integral in these weighted spaces. By using these theorems we formulate the corresponding theorems on oscillating multipliers $i^m a(m)$ of spherical harmonic expansions of functions in the weighted spaces $C_\beta H_p^l$ and $S_\beta H_p^l$.

ON EXISTENCE OF U - ULTRAFILTERS AND THEIR PROPERTIES

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In this thesis the existence of u - ultrafilters is proved, which are analogous to the regular results of Aleksandrov ([3]).

Let $C^*(uX)$ be the ring of all uniformly continuous bounded functions on uniform space uX.

Definition 1. ([1]). *A subset $F \subset X$ in uniform space uX is called uniform zero set if $F = f^{-1}(0)$ for some $f \in C^*(uX)$.*

Complement of *uniform zero set* is called uniform *cozero set*. In other words a subset $U \subset X$ in uniform space uX is uniform *cozero set*, if $U = f^{-1}(\mathbb{R}\setminus\{0\})$ for some $f \in C^*(uX)$.

Definition 2. ([1]). *A function $f : uX \to I = [0;1]$ is called C_u^*-function,* if *$f^{-1}(U)$ is uniform cozero set for any open $U \subset I$, or equivalently, $f^{-1}(F)$ is uniform zero set for any closed $F \subset I$, i.e. $f^{-1}(F) \in \mathfrak{Z}(uX)$ and $f^{-1}(U) \in \mathfrak{L}(uX)$.*

By $\mathfrak{Z}(uX)\,(\mathfrak{L}(uX))$ we denote the family of all uniform zero (cozero) sets.

Definition 3. ([2]).*Two subsets $A, B \subseteq X$ in uniform space uX are called u*-separated, *if there exists some C_u^*- function $f : uX \to I = [0,1]$, such that $f(A) = 0$ and $f(B) = 1$. If A is u- separated from $X\setminus B$, then B is called u-neighborhood of A.*

Definition 4. ([2]). *Let $A, B \subseteq X$, then A is called u- embedded in B, if A u–separated from $X\setminus B$, i.e. there exists C_u^* - function $f : uX \to I$, with $f(A) = 0$ and $f(X\setminus B) = 1$.*

Definition 5. ([2]). *A family ξ of subsets in uniform space uX is called u- system, if for any $K \in \xi$ one can find $B \in \xi$ such that B u- embedded in K, or equivalently, each $K \in \xi$ is u- neighborhood of some $B \in \xi$.*

Definition 6. *u - centered system is a centered u - system consisting of open subsets.*

Definition 7. *Prefilter (filter) which is u – system is called u - prefilter (filter).*

Definition 8. *Each u - centered system which cannot be proper subsystem of any other u - centered system is called maximal u -centered system.*

Theorem 9. *Each maximal u - centered system is u -ultrafilter.*

Definition 10. *Maximal u–centered systems is called of u–ultrafilters.*

Theorem 11. *Every u – centered system contains at least one u – ultrafilter.*

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PROBLEM OF INTEGRAL GEOMETRY WITH THREE - DIMENSIONAL SPACE CURVE

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This work analyzes the problem of the restoration of unknown functions according to the given value of their integrals with non-invariant (depending not only of the difference of an independent variable and one of integration) weighted functions along collections of curves in critical cases (denominators in the arising integral equations tending toward zero).

Suppose in a band $\Omega = (\xi,\eta) : \xi = (\xi_1,\xi_2) \in R^2, \eta \in [0,h] \in R^3$ we are given a set of curves

$$\gamma(x,y) = \begin{cases} \xi_1 = x_1 - \phi_1(t,y), \\ \xi_2 = x_2 - \phi_2(t,y), \\ \eta = y - t,\ 0 \le t \le y \end{cases}$$

with variables $(x,y) \in \Omega$ and the ends of elements lying in the axis $\eta = 0$. The function $\xi_1 = x_1 - \phi_1(t,y)$ increases with respect to η and the function $\xi_2 = x_2 - \phi_2(t,y)$ increases with respect to η in the interval $[0,y]$, $\phi_1(0,y) = \phi_2(0,y) = 0, \quad y \in [0,h]$.

For the weight function $a(x,y,\xi,\eta)$ and the function $u(\xi,\eta)$ we set

$$Au \equiv \int\limits_{\gamma(x,y)} a(x,y,\xi,\eta)u(\xi,\eta)ds = g(x,y).$$

We need to recover the function $u(x,y)$ by the function $g(x,y)$ given in Ω. We shall assume that the function $a(x,y,\xi,\eta)$ as follows

$$a(x,y,\xi,\eta) = a_0(x-\xi,y,\eta) + a_1(x,y,\xi,\eta)$$

we obtain:

$$Au \equiv \int\limits_0^y [a_0(\phi_1(t,y),\phi_2(t,y),y,y-t) + a_1(x,y,x_1-\phi_1(t,y),x_2-\phi_2(t,y),y-t)] \times$$

$$\times u(\phi_1(t,y),\phi_2(t,y),y-t)\sqrt{1+(\phi_1'(t,y))^2+\phi_2'(t,y))^2}dt = g(x,y),\ (x,y) \in \Omega.$$

Further, by means of the Fourier transformation, these problems are reduced to integro-differentials equations and the method of the scale of Banach spaces is applied.

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THE GROWTH OF THE NORM ALGEBRAIC POLYNOMIALS OF THE WHOLE COMPLEX PLANE

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Let G be a finite region bounded by a Jordan curve $L := \partial G$ and let X and Y be norm spaces of functions defined in G. Let $\wp_n$ denote the set of arbitrary algebraic polynomials $P_n(z)$, $\deg P_n \leq n$, $n = 0, 1, 2, ...$ The comparison of norms of polynomials type

$$\|P_n^{(k)}\|_X \leq A(k, n, G)\|P_n\|_Y$$

for all $P_n \in \wp_n$ and all $k = 0, 1, 2, ...$ have been studied in many works, where $A(k, n, G)$ is a constant depending on k, n and G.

Let $A_p(G)$, $p > 0$, denote the class of functions f analytic in G and satisfying the condition

$$\|f\|_{A_p(G)} := \left(\iint_G |f(z)|^p \, d\sigma_z\right)^{1/p} < \infty,$$

where σ be the two-dimensional Lebesque measure.

Let $w = \Phi(z)$ be the conformal mapping of $\Omega := C\overline{G}$ onto the $|w| > 1$ normalized by $\Phi(\infty) = \infty$, $\Phi'(\infty) > 0$; $\Omega_R := \{z : |\Phi(z)| > R > 1\}$.

In this paper we discuss the estimation of the following type

$$\left|P_n^{(k)}(z)\right| \leq cons.\|P_n\|_{A_p(G)} \begin{cases} n^{\alpha}, & z \in \overline{G} \\ \frac{n^{\beta}}{d(z,L)}, & z \in \Omega \end{cases}, \; p > 1,$$

for different regions of the complex plane, where the numbers $\alpha := \alpha(k, p, G)$ and $\beta := \beta(k, p, G)$ explicitly defined depending of the properties of the given regions.

We give one of these results.

Theorem. *Assume that L is $K-$quasiconformal. Then, for any $P_n \in \wp_n$ we have*

$$|P_n(z)| \leq cons.\|P_n\|_{A_2(G)} \begin{cases} n^{\nu}, & z \in \overline{G}, \\ \frac{n^{\nu-\nu^{-1}}}{d(z,L)} |\Phi(z)|^{n+1}, & z \in \overline{\Omega}_{1+\frac{1}{n}}, \end{cases}$$

where $\nu := \min\left\{2, K^2\right\}$.

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SOLVING NONLINEAR FREDHOLM-HAMMERSTEIN INTEGRAL EQUATIONS BY USING CARDINAL LEGENDRE FUNCTIONS

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Legendre's functions are developed to approximate the solutions of nonlinear Fredholm-Hammerstein integral equations. Properties of these functions are first presented; these properties are then used to reduce the computation of integral equations to some algebraic equations. The method is computationally attractive, and applications are demonstrated through some illustrative example.

In the present paper we apply Legendre's functions on $[0, 1]$, to solve the nonlinear Fredholm-Hammerstein integral equations of the form

$$y(x) = f(x) + \int_0^1 K(x,t)g(t,y(t))dt, \;\; 0 \le x \le 1, \tag{1}$$

where f, g and K are given continuous functions, $g(t, y)$ is nonlinear for y and y is unknown function and should be found.

Our method consists of reducing (1) to a set of algebraic equations by expanding all functions as Legendre's functions.

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THE APPROXIMATION OF FUNCTIONS OF TWO VARIABLES OF BOUNDED P-VARIATION BY POLYNOMIALS WITH RESPECT TO HAAR SYSTEM

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Let $f(x,y)$ be defined on the closed square $[0,1]^2$ and $\tau=\xi\times\eta$, where $\xi=\{x_0<x_1<...<x_m=1\}$, $\eta=\{y_0<y_1<...<y_n=1\}$- be an arbitrary partition of $[0,1]^2$. Let $1\le\ p<\infty$. The quantity

$$\aleph^p_{\xi,\eta}(f)=\left(\sum_{k=1}^{n}\sum_{l=1}^{m}|f(x_k,y_l)-f(x_{k-1},y_l)-f(x_k,y_{l-1})+f(x_{k-1},y_{l-1})|^p\right)^{1/p}.$$

is called variational sum of order p of the function $f(x,y)$ with respect to the partitions τ.

The quantity

$$\omega_{1-1/p}(f,\delta_1,\delta_2)=\sup_{\substack{|\xi|\le\delta_1\\|\eta|\le\delta_2}}\aleph^p_{\xi,\eta}(f),$$

is called variational modulus of continity of order $1-\frac{1}{p}$ of the function $f(x,y)$. Here $|\xi|=\max\limits_{1\le k\le n}(x_k-x_{k-1})$, $|\eta|=\max\limits_{1\le l\le m}(x_l-x_{l-1})$.

We say that $f\in BV_p[0,1]^2$, $1\le\ p<\infty$, if $V_p\left(f,[0,1]^2\right)\equiv\omega_{1-1/p}(f,1,1)<\infty$, and if $f\in C_p[0,1]^2$, $1<\ p<\infty$, if $\lim\limits_{\substack{\delta_1\to0\\\delta_2\to0}}\omega_{1-1/p}(f,\delta_1,\delta_2)=0$.

Endowed with the norm $\|f\|_{BV_p}=max\left\{\sup\limits_{(x,y)\in[0,1]^2}|f(x,y)|\,,V_p\left(f,[0,1]^2\right)\right\}$. This space is a Banach space. Let $h_n(x)$ be functions of the Haar system. For the case of $[0,1)^2$ we put $h_{m,n}(x,y)=h_m(x)h_n(y)$.

Denote by $E^h_{m,n}(f)_X$ the best approximation of $f\in X[0,1]^2$ by Haar polynomials of degree $\le m\times n$ $(m,n\in N)$ in the norm of $X[0,1]^2$, where $X[0,1]^2=C_p[0,1]^2$, $1<p<\infty$ or $X[0,1]^2=BV_p[0,1]^2$, $1\le p<\infty$

$$E_{m,n}(f)_X=\sup_{c_{ij}}\left\|f(x,y)-\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}\chi_i(x)\chi_j(y)\right\|_X.$$

Denote by $S^h_{m,n}(f)$ rectangular partial sum of Fourier series of f by Haar system. By $K_{\alpha,\beta,\gamma}$ we denote a constant whose value may be different at each occurrence.

Theorem 1. *That* $1<p<\infty$ $f\in C_p[0,1]^2$, $m,n\in P$. *Then*

$$E^h_{m,n}(f)_{C_p}\le K_p\omega_{1-\frac{1}{p}}\left(f,\frac{1}{m},\frac{1}{n}\right)$$

Theorem 2. *That* $1<p<\infty$ $f\in C_p[0,1]^2$, $m,n\in P$. *Then*

$$\omega_{1-\frac{1}{p}}\left(f,\frac{1}{m},\frac{1}{n}\right)\le K_pE^h_{m,n}(f)_{C_p}.$$

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FIBONACCI TYPE POLYNOMIALS

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In this study the generalized bivariate Fibonacci-type polynomials are investigated. Furthermore, some identities concerning with these polynomials are derived.

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A GENERALIZATION OF EXHAUSTIVENESS AND α CONVERGENCE FOR FUNCTION SEQUENCES

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The concept of continuous convergence which is stronger than the pointwise convergence for sequences of functions was introduced in the first half of the twentieth century ([3],[4]) and called as the $\alpha-$convergence later ([1],[2]). The concepts of continuous convergence and $\alpha-$convergence are equivalent for sequences of functions, but they are not equivalent for nets of functions ([2]). Gregoriades and Papanastassiou [2] introduced a new concept which is called the exhaustiveness for sequences and nets of functions.

In this work, we consider sequences of functions defined between two metric spaces. As a generalization of the concepts mentioned above, we introduce the concepts of $\mathcal{F}-\alpha-$*convergence* and $\mathcal{F}-$*exhaustiveness,* where $\mathcal{F}$ is a filter on the set $\mathbb{N}$ of all natural numbers. We also generalize the concepts of Cauchy sequence of functions, pointwise convergence and uniform convergence, via the notion of filter. Next, we investigate some properties of these new concepts. We also examine the relations between the classical concepts and their analogues that we introduced.

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ON A CLASS OF OPERATOR-DIFFERENTIAL EQUATIONS OF THIRD ORDER WITH MULTIPLE CHARACTERISTICS ON THE WHOLE AXIS IN THE WEIGHTED SPACE

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For the function $u(t)$ defined in $R = (-\infty, +\infty)$, with values in a separable Hilbert space H, we introduce the following spaces with a weight $e^{-\frac{\kappa}{2}t}$, $\kappa \in R$:

$$L_{2,\kappa}(R;H) = \left\{ u(t) : \|u\|^2_{L_{2,\kappa}(R;H)} = \int_{-\infty}^{+\infty} \|u(t)\|^2_H e^{-\kappa t} dt < +\infty \right\},$$

$$W^3_{2,\kappa}(R;H) = \left\{ u(t) : \|u\|^2_{W^3_{2,\kappa}(R;H)} = \int_{-\infty}^{+\infty} \left(\left\| \frac{d^3 u(t)}{dt^3} \right\|^2_H + \left\| A^3 u(t) \right\|^2_H \right) e^{-\kappa t} dt < +\infty \right\},$$

where A is a self-adjoint positive definite operator in H, and the derivatives are understood in the sense of the theory of distributions.

Consider the following third order operator-differential equation whose principal part has a multiple characteristic:

$$\left(-\frac{d}{dt} + A\right)\left(\frac{d}{dt} + A\right)^2 u(t) + A_1 \frac{d^2 u(t)}{dt^2} + A_2 \frac{du(t)}{dt} = f(t), t \in R, \tag{1}$$

where $f(t) \in L_{2,\kappa}(R;H)$, $u(t) \in W^3_{2,\kappa}(R;H)$, A is the same self-adjoint positive definite operator with the lower bound of the spectrum λ_0 ($A = A^* \geq \lambda_0 E$ $(\lambda_0 > 0)$, E is the identity operator), and A_1, A_2 are linear operators, moreover the operators $A_1 A^{-1}$, $A_2 A^{-2}$ are bounded in H.

Applying the calculations from [1], we prove the following theorem.

Theorem. *Let* $|\kappa| < 2\lambda_0$ *and holds the inequality*

$$b(\kappa)\left(\left[1 + \frac{4\lambda_0 |\lambda_0 + \kappa|}{(2\lambda_0 + \kappa)^2} \right] \left\| A_1 A^{-1} \right\|_{H \to H} + \frac{2\lambda_0}{2\lambda_0 + \kappa} \left\| A_2 A^{-2} \right\|_{H \to H} \right) < 1,$$

where

$$b(\kappa) = \begin{cases} \frac{\lambda_0}{2^{1/2}\left(2\lambda_0^2 - \kappa^2\right)^{1/2}}, & if \ 0 \leq \frac{\kappa^2}{4\lambda_0^2} < \frac{1}{3}, \\ \frac{2\lambda_0 |\kappa|}{4\lambda_0^2 - \kappa^2}, & if \ \frac{1}{3} \leq \frac{\kappa^2}{4\lambda_0^2} < 1. \end{cases}$$

Then, for any $f(t) \in L_{2,\kappa}(R;H)$, *equation* (1) *has a unique solution* $u(t)$ *from the space* $W^3_{2,\kappa}(R;H)$.

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ON THE PROBLEM ABOUT FLUCTUATIONS OF THE SYSTEM OF A VISCOUS STRATIFIED LIQUID IN THE ELASTIC VESSEL

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We consider a bounded domain $S = \mathbf{R}^3$ and let $\overline{\Omega_0} \subset S_3$ be a subdomain and $S = S_3 \backslash \overline{\Omega_0}$, in which $\gamma = \partial S_3, \sum = \partial \Omega_0$ from C^2.

Let in the elastic vessel from S be the system from two viscous stratified impressing liquid, which be fulfilled the domain Ω_0. Let Γ be the boundary, which divide this liquids and let Ω_1 and Ω_2 from Ω_0 such that Ω_1 lie above from Γ and Ω_2 lie below from Γ, $\sum_1 = \partial\Omega_1 \backslash \Gamma$, $\sum_2 = \partial\Omega_2 \backslash \Gamma$. Corresponding parts of the elastic vessel we denote by $S_k, (k = 1, 2)\,, \gamma_k = \partial S_k, (k = 1, 2)$.

Considering the normal fluctuations we have the following problem

$$\begin{cases} L\mathbf{u}_k + \lambda^2 \rho \mathbf{u}_k = 0 \,(S_k, k = 1, 2)\; \sigma(\mathbf{u}_k)\, \mathbf{n}_k|_{\gamma_k \cup \sum_k} = 0 \,(k = 1, 2) \\ -\nu_k \Delta \mathbf{V}_k + \nabla P_k - \lambda \mathbf{V}_k + \lambda^{-1} g \rho_{0k}^{-1} \nabla \rho_0 \mathbf{V}_k = 0, di\vartheta \mathbf{V}_k = 0 \,(\Omega_k, k = 1, 2) \\ -\lambda \mathbf{u}_k = \mathbf{V}_k, (\sum_k, k = 1, 2)\,, \sigma(\mathbf{u}_k)\, \mathbf{n}_k = -T(\mathbf{V}_k)\, \mathbf{n}_k, (\sum_k, k = 1, 2) \\ \mathbf{V}_1 = \mathbf{V}_2, T_1(\mathbf{V}_1)\, \mathbf{n}_1 = T_2(\mathbf{V}_2)\, \mathbf{n}_2\, (\Gamma) \end{cases} \tag{1}$$

where $\overline{u_k}\,(\overline{x}, t) -$ is the removal vector of elastic body, $\mathbf{V}_k\,(\mathbf{x}, t)$ is the speed of fluid, $P_k\,(\mathbf{x}, t)\,, (k = 1, 2)$ is the deviation from equilibrium pressure,

$$\sigma(\mathbf{u_k}) = (\sigma_{ij}(\mathbf{u_k}))_{i,j=1}^3\,,\; \sigma_{ij}(\mathbf{u}_k) = \lambda_0 \delta_{ij} di\vartheta \mathbf{u}_k + \mu_0 \Big(\partial (u_k)_i / \partial x_j + \partial (u_k)_j / \partial x_i\Big)$$

is the tensor of pressure, $\lambda_0, \mu_0 > 0-$ is a Lyame constants, ρ is the density of elastic body, $g$is the accelerate, $(-L\mathbf{u}_k)_i = \sum\limits_{j=1}^{3} \partial \sigma_{ij}(\mathbf{u}_k) / \partial x_j, (i = 1, 2, 3, k = 1, 2)\,.$

$$T(\mathbf{V}_k) = (T_{ij}(\mathbf{V}_k))_{i,j=1}^3\,, T_{ij}(\mathbf{V}_k) = \delta_{ij} P_k + \nu_k \Big(\partial (V_k)_i / \partial x_j + \partial (V_k)_j / \partial x_j\Big)\,,\; (k = 1, 2)$$

is the tensor of pressure.

Theorem. *The spectrum of the problem (1) consists from eigenvalues λ_n having limiting points in 0 and on ∞, and lie in the right-hand side of halfspase, and is symmetric concerning of real axis. For large $|\lambda|$ all eigenvalues λ_n, get in sufficiently small angles between imaging axis and positive halfaxis.*

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COMPUTABLE NUMBERINGS IN THE ERSHOV HIERARCHY

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For undefined notions we refer to [1] and [2].

Σ_a^{-1}-computable numbering of a family $\mathcal{A}$ is a sequence of sets from $\mathcal{A}$ which is uniform in Σ_a^{-1}. Here a is Kleene's notation for a computable ordinal. More precisely, surjective mapping $\alpha : \omega \mapsto \mathcal{A}$ is Σ_a^{-1}-computable numbering of $\mathcal{A}$ if there exist a computable function $f(n, x, s)$ and a partial computable function $g(n, x, s)$ such that for all n, x, t,

(1) $x \in \alpha(n) \Leftrightarrow \lim_t f(n, x, t) = 1$, with $f(n, x, 0) = 0$ and range$(f) \subseteq \{0, 1\}$;
(2) $g(n, x, t) \downarrow \Rightarrow g(n, x, t+1) \downarrow$; $g(n, x, t+1) \leq_O g(n, x, t) <_O a$; and
(3) $f(n, x, t+1) \neq f(n, x, t) \Rightarrow g(n, x, t+1) \downarrow \neq g(n, x, t)$.

If α, β are numberings of a same family, let $\alpha \leq \beta$ if there is a computable function h such that $\alpha = \beta \circ h$. The relation $\leq$ is a reducibility and gives rise to a degree structure (called a *Rogers semilattice*), where degree of a numbering is the equivalence class of the numbering under the equivalence relation $\equiv$ generated by $\leq$.

Rogers semilattice of a family is upper semilattice, it is considered as algebraic structure which describes a scale of algorithmic complexity in whole under relation $\leq$ of all uniform computations of the family. We will represent recent results of the author and his students on extremal elements of the Rogers semilattices for the families of sets of any given level (finite or infinite) of the Ershov hierarchy.

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CONDITION OF NON-SYMMETRY OF THE RELATION OF SEMI-ISOLATEDNESS

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The property of non-symmetry of the relation of semi-isolatedness is a key property in studying the class of Eurenfeucht theories. We find necessary and saficient conditions for non-symmetry of the relation of semi-isolatedness in terms of coloring of a neighbourhood of a type and in terms of quasineighbourhoods. By using new terms we clarify and symplify well-known theorems of non-symmetry of the relation of semi-isolatedness, taking from [1], as well as Tsuboi's theorem [2] on non-representability of Eurenfeucht theory, which does not have the strict order property in a union of countably categorical theories.

Definition (A. Pillay [3]). *Let $\mathcal{M}$ be a model of T, $\bar{a}$ and $\bar{b}$ tuples from M, A a subset of M. We say that $\bar{a}$ semi-isolate the tuple $\bar{b}$ over A, if there exists a formula $\varphi(\bar{a},\bar{y}) \in \mathrm{tp}(\bar{b}/(\bar{a}\cup A))$ such that $\varphi(\bar{a},\bar{y}) \vdash \mathrm{tp}(\bar{b}/A)$.*

The following definition is due to B.S. Baizhanov [4].

Definition. *Let $p(\bar{x})$ and $q(\bar{y})$ be some (possibly partial) types over $A \subseteq M$ in some model $\mathcal{M}$ of T. A formula $\varphi(\bar{x},\bar{y})$ with parameters in A is called (p,q)-stable $(p \to q)$-formula or $(q \leftarrow p)$-formula if for any realization $\bar{a}$ of p it holds that $\varphi(\bar{a},\bar{y}) \vdash q(\bar{y})$. A formula $\varphi(\bar{x},\bar{y})$ is called $(p \leftrightarrow q)$-formula* if *$\varphi(\bar{x},\bar{y})$* is *$(p \to q)$-formula as well as $(p \leftarrow q)$-formula. If $p = q$, then (p,q)-stable formula is called p-stable or $(p \to p)$-formula.*

Definition. *Let $p(\bar{x})$ be some n-type over $A \subseteq M$ in a model $\mathcal{M}$ of T, B a subset in $\mathcal{M}$. The quasineighbourhood of B in p is the following set $QV_{p,\mathcal{M}}(B)$ which consists of all tuples $\bar{c} \in M$, such that there exists a tuple $\bar{b} \in B$ and $(\mathrm{tp}(\bar{b}/A),p)$-stable formula $\varphi(\bar{x},\bar{y})$, such that $\mathcal{M} \models \varphi(\bar{b},\bar{c})$.*

The quasineighbourhood of B in $S_n(A)$ is the set

$$QV^n_{A,\mathcal{M}}(B) \rightleftharpoons \bigcup_{p\in S_n(A)} QV_{p,\mathcal{M}}(B).$$

The quasineighbourhood of B in $S(A)$ is *$QV_{A,\mathcal{M}}(B) \rightleftharpoons \bigcup_{n\in\omega} QV^n_{A,\mathcal{M}}(B)$.*

Theorem *The relation $\bar{a} \in QV_A(\bar{b})$ forms a preorder. In a powerfull type this relation is not symmetric.*

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THE ASYMPTOTIC BEHAVIOR OF EIGENVALUES AND TRACE FORMULA OF SECOND ORDER DIFFERENTIAL OPERATOR EQUATION

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Consider in $L_2((0,1),H)$ (H is a separable Hilbert space) the following spectral problem

$$-y''(x)+Ay(x)+q(x)y(x)=\lambda y(x), \quad (1)$$

$$y'(0)=0, \quad (2)$$

$$y(1)-hy'(1)=\lambda y(1), \quad (3)$$

where A=A $^*>E$ (E *is an identity operator in* H), $A^{-1}\in\sigma_\infty$. Denote the eigenfunctions and eigenvalues of A by $\varphi_1,\varphi_2,...$ and $\gamma_1\le\gamma_2\le...$ respectively. Assume that exists second order weak derivative of $q(x)$ and the following conditions are satisfied:

1. $\left[q^{(l)}(x)\right]^*=q^{(l)}(x)$, $l=0,1,2$; $\sum_{k=1}^{\infty}\left|\left(q^{(l)}(x)\varphi_k,\varphi_k\right)\right|<\infty$ for each x$\in[0,1]$;
2. $q'(0)=q'(\pi)=0$.
3. $\int_0^1\left(q(x)\varphi_k,\varphi_k\right)dx=0$.

Theorem 1. *Eigenvalues of problem (1)-(3) form two sequences:* $\lambda_k=-h\sqrt{\gamma_k}+1-\frac{h^2}{2}+O\left(\frac{1}{\sqrt{\gamma_k}}\right)$, $\lambda_{k,n}=\gamma_k+(\pi n)^2+\pi^2 n+O(1)$.

Define in $L_2=L_2((0,1),H)\oplus H$ a selfadjoint operator L_0 for $q(x)=0$ as:

$D(L_0)=\{(y(x),y(1))\in L_2/\ -y"(x)+Ay(x)\in L_2((0,1),H),\ y'(0)=0,\ y_1=y(1)\}$,

$L_0\{y(x),y(1)\}=\{-y"(x)+Ay(x),\ y(1)-hy'(1)\}$.

Operator corresponding to the case $q(x)\ne0$ denote by L. Let $\lambda_1,\lambda_2,...$ and $\mu_1,\mu_2,...$ be eigenvalues of operators L and L_0 respectively. The statement of the following theorem is true.

Theorem 2.

$$\sum_{n=1}^{\infty}{}'(\lambda_n-\mu_n)=\sum_{k=1}^{\infty}\frac{(q(0)\varphi_k,\varphi_k)-(q(\pi)\varphi_k,\varphi_k)}{4}(\lambda_n-\mu_n)=\sum_{k=1}^{\infty}\frac{(q(0)\varphi_k,\varphi_k)-(q(\pi)\varphi_k,\varphi_k)}{4}.$$

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ON BASICITY OF SYSTEMS FROM FABER GENERALIZED POLYNOMIALS[1]

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In the paper, a double system from Faber generalized polynomials

$$\left\{A(\xi)\ F^{+}_{p;n}(\xi)\ ;B(\xi)\ F^{-}_{p;k}(\xi)\ \right\}_{n\geq 0;k\geq 1}, \tag{1}$$

with complex coefficients $A(\xi)\equiv|A(\xi)|e^{i\alpha(\xi)}$ and $B(\xi)\equiv|B(\xi)|e^{i\beta(\xi)}$ on a closed, rectifiable curve Γ on a complex plane C is considered . It is assumed that the following conditions are fulfilled.

1) $|A|^{\pm 1};|B|^{\pm 1}\in L_{\infty}$.

2) $\alpha(\xi);\beta(\xi)$ are piecewise-continuous on Γ and $\{\xi_k, k=\overline{1,r}\}\subset\Gamma$ are discontinuity points of the function $\theta(\xi)\equiv\beta(\xi)-\alpha(\xi)$.

3) $\{h_k-\dfrac{2\pi}{p}:k=\overline{0,r}\}\cap Z=\emptyset$, where $h_k=\theta(\xi_k+0)-\theta(\xi_k-0)$, $k=\overline{1,r}$; $h_0=\theta(a+0)-$ $-\theta(a-0)$, $1<p<+\infty$. Assume that the following inequalities are fulfilled

$$-\frac{1}{q}<\frac{h_k}{2\pi}<\frac{1}{p},\ \ k=\overline{0,r},\ \ \frac{1}{p}+\frac{1}{q}=1. \tag{2}$$

Under definite conditions on the curve Γ and if the inequalities (2) are fulfilled, the following facts are proved:

a) basicity of the system $\left\{F^{+}_{p;n}(\xi)\right\}_{n\geq 0}$ $\left(\left\{F^{-}_{p;k}(\xi)\right\}_{k\geq 1}\right)$ in Smirnov spaces $E^{+}_{p}(int\Gamma)$ $\left({}_{-1}E^{-}_{p}(ext\Gamma)\right)$;

b) basicity of the system (1) in Lebesgue spaces $L_p(\Gamma)$, $1<p<+\infty$.

$F^{\pm}_{p;n}(z)$ are generalizations of the classical Faber polynomials (see. for example [1]). These generalizations are probably introduced by the author of [2]. Note that, under studying the basicity, we used methods of boundary value problems of the theory of analytic functions in Smirnov classes (see for example [3]).

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THE APPROXIMATION OF FUNCTIONS OF SEVERAL VARIABLES OF BOUNDED P-FLUCTUATION BY POLYNOMIALS WITH RESPECT TO WALSH SYSTEM

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Let $\bar{x} = (x_1, ..., x_n)$, $\bar{k} = (k_1, ..., k_n)$, $k_j = 1, 2, ...$, $j = 1, 2, ..., n$, $n \in N$, $f(\bar{x})$ be defined on the $[0, 1)^n$. Let $J_j^k = \left[\frac{j-1}{2^k}, \frac{j}{2^k}\right), (j = 1, 2, ..., 2^n, k \in N)$ be the dyadic intervals and let $J_{\bar{j}}^{\bar{k}} = J_{j_1}^{k_1} \times J_{j_2}^{k_2} \times ... \times J_{j_n}^{k_n}$; $osc(f, J_{\bar{j}}^{\bar{k}}) = \sup\limits_{\bar{x},\bar{y} \in J_{\bar{j}}^{\bar{n}}} |f(\bar{x}) - f(\bar{y})|$. Denote by $H_p(f, \bar{k}) = \left(\sum\limits_{j_1=1}^{2^{k_1}} ... \sum\limits_{j_n=1}^{2^{k_n}} \left(osc\left(f, J_{j_1,...,j_n}^{k_1,...,k_n}\right)\right)^p\right)^{\frac{1}{p}}$, $H_p(f, 0) = osc(f, [0, 1)^n)$. If $V_p(f) = \sup\limits_{\bar{k} \in N} H_p\left(f, \bar{k}\right) < \infty$, then f is called function of bounded p-fluctuation [1]. The quantity $V_p(f)_{\bar{m}} = \sup\limits_{\bar{k} \geq \bar{m}} H_p\left(f, \bar{k}\right)$ is called discrete modulus of continuity. Here $\bar{k} \geq \bar{m}$ means $k_1 \geq m_1, ..., k_n \geq m_n$. We say that $f \in FV_p[0, 1]^n$, $(1 \leq p < \infty)$, if $V_p\left(f\right)_{\bar{m}} < \infty$, and $f \in FC_p[0, 1]^n$, $(1 < p < \infty)$, if $\lim\limits_{m_j \to \infty, (j=1,...n)} V_p(f)_{\bar{m}} = 0$. Endowed with the norm $\|f\|_{p,F} = max\left(V_p\left(f\right), \sup\limits_{\bar{x}} \in [0, 1)^n \left|f(\bar{x})\right|\right)$.

Let $w_k(x)$ be the Walsh system by enumeration Paley. $w_{\bar{k}}(\bar{x}) = w_{k_1}(x_1)...w_{k_n}(x_n)$ is the multiples Walsh systems. Denote by $\hat{a}_{\bar{k}}\left(f\right)$ coefficients of Walsh-Fourier of $f \in FC_p[0, 1]^n$. Denote by $E_{\bar{k}}\left(f\right) = E_{k_1,...,k_n}\left(f\right)$ the best approximation of functions by Walsh polynomials of degree $\leq k_1 \times ... \times_n$ $(k_j \in N)$ in the norm of $FC_p[0, 1]^n$.

Theorem 1. *Let* $f \in FC_p[0, 1]^n$ $(1 < \ p < \infty)$. *Then the following inequalities*

$$V(f)_{\bar{m}} \leq E_{2^{m_1},...,2^{m_n}}(f)_{FC_p} \leq 2V(f)_{\bar{m}}$$

holds true.

Theorem 2. *Let* $f \in FC_p[0, 1]^n$, $0 < \beta < 2$.

1) $1 < p < 2$. *Then for the convergence of series* $\sum\limits_{\bar{n}}^{\infty} |a_{\bar{n}}(f)|^{\beta}$ a *sufficient condition is the following* $\sum\limits_{\bar{n}}^{\infty} E_{\bar{n}}^{\beta}(f)_{FV_p} \bar{n}^{-\beta} < \infty$.

2) $2 \leq p < \infty$. *Then for the convergence of series* $\sum\limits_{\bar{n}}^{\infty} |a_{\bar{n}}(f)|^{\beta}$ *a sufficient condition is the following* $\sum\limits_{\bar{n}}^{\infty} E_{\bar{n}}^{\beta}(f)_{FV_p} (\bar{n})^{\frac{-\beta}{p} - \frac{\beta}{2}} < \infty$.

In the case of one variables analogies results was proved in [2].

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QUASI-PLUMED UNIFORM SPACES

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Definition 1([1]). *A space* X *is called quasi-plumed, if there is a normal sequence* $\{\alpha_n\}_{n\in N}$ *of an open coverings, its provided*

($*$) *if* $\{\mathrm{K}_n\}$ *is a centered system of a closed sets such that the some point* $x_0 \in \mathrm{X}$ *exists* $\mathrm{K}_n \subset \alpha_n(x_0)$ *for any* $n \in N$, *then* $\cap [K_n] \neq \emptyset$.

Definition 2. *A uniform space* $(X, \mathcal{U})$ *is called (strongly) uniformly quasi-plumed, if there is a normal sequence* $\{\alpha_n : n \in N\} \subset \mathcal{U}$, *its provided*

($*$) if $\{\mathrm{K}_n\}$ *is a centered system of a closed sets such that the some point* $x_0 \in \mathrm{X}$ *exists and* $\mathrm{K}_n \subset \alpha_n(x_0)$ *for any* $n \in N$, *then* $\cap\{\mathrm{K}_n : n \in N\} \neq \emptyset$ ($\mathcal{U} = \sup\{\mathcal{V}, \mathcal{U}_p\}$, *where* $\mathcal{V}$ *is a pseudouniformity, induced by a normal sequence* $\{\alpha_n : n \in N\}$ *and* $\mathcal{U}_p \subset \mathcal{U}$ *be a maximal precompact uniformity of* $\mathcal{U}$[2]).

Theorem 1. *A uniform space* $(X, \mathcal{U})$ *is a (strongly) uniformly quasi-plumed, if and only if there is a pseudouniformity* $\mathcal{V} \subset \mathcal{U}$ *such that* ($\mathcal{U} = \sup\{\mathcal{V}, \mathcal{U}_p\}$)

(1) $w(\mathcal{V}) \leq \aleph_0$;

(2) $\bigcap\{\alpha(x) : \alpha \in \mathcal{V}\} = K_x$ *is countably compact for any* $x \in X$;

(3) $\{\alpha(K_x) : \alpha \in \mathcal{V}\}$ *is a basis of* K_x *for any* $x \in X$.

Definition 3. A *mapping* $f : (X, \mathcal{U}) \to (Y, \mathcal{W})$ *of a uniform space* $(X, \mathcal{U})$ *into a uniform space* $(Y, \mathcal{W})$ *is called (uniformly) quasi-perfect, if it is (precompact) uniformly continuous, closed and* $f^{-1}(y)$ *is countable compact for any* $y \in Y$.

Theorem 2. *A uniform space* $(X, \mathcal{U})$ *is a (strongly) uniformly quasi-plumed, if and only if* $(X, \mathcal{U})$ *is (uniformly) quasi-perfect mapping onto some metric uniform space.*

Theorem 3. *If uniform space* $(X, \mathcal{U})$ *is uniformly quasi-plumed, then its* $\aleph_0$*-completion* $\left(X^{\aleph_0}, \mathcal{U}^{\aleph_0}\right)$ *is uniformly plumed.*

Corollary. *If* $(X, \mathcal{U})$ *is uniformly quasi-plumed, then its completion* $\left(\tilde{X}, \tilde{\mathcal{U}}\right)$ *is uniformly plumed.*

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LATTICE OF UNIFORMITIES

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Throughout this work, all spaces under consideration are supposed to be Tychonoff spaces.

By $U(X)$ ($U_p(X)$) we denote a set of all uniformities (precompact uniformities) of space X. $U(X)$ ($U_p(X)$) naturally represents a partially ordered set. By $K(X)$ we denote a set of all bicompact extensions of space X. $K(X)$ is partially ordered. $b_2\% \leq b_1\%$ if and only if there is such continuous mapping $f : b_1X \to b_2X$ that $fx = x$ for every $x \in X$.

Partially ordered set $U(X)$ of all uniformities of space X is called a lattice if each of its two-element subsets $\{U_1, U_2\}$, $U_1, U_2 \in U(X)$ has both the least upper bound and the greatest lower bound i.e. $\sup\{U_1, U_2\}$ and $\inf\{U_1, U_2\}$. Also, set $K(X)$ of all bicompact extensions of space X is called a lattice if each of its two-element subsets $\{b_1X, b_2X\}$, $b_1X, b_2X \in K(X)$ has $\sup\{b_1X, b_2X\}$ and $\inf\{b_1X, b_2X\}$. A set is called a complete lattice if each of its subsets has the least upper bound and the greatest lower bound. It is obvious that a complete lattice is a lattice. In his work [3], A.A. Borubaev sets up the problem: to find the necessary and sufficient conditions applied to space X in a way that $U(X)$ is a lattice.

This work demonstrates a partial solution of this problem. It is well known that there has to be natural isomorphism between sets $K(X)$ and $U_p(X)$. Japanese mathematician T. Shirota [5] proved that set $U_p(X)$ is a full lattice if and only if X is locally bicompact. Later, German mathematicians J. Visliseni and J. Flachsmeyer [4] proved that for every Tychonoff space X with the first axiom of countability, set $K(X)$ is a lattice if and only if X is locally bicompact. J.Vislileni and J.Flachsmeyer theorem amplifies the non-trivial half of T.Shirota theorem for the case of a space with the first axiom of countability. They showed that such a property as local bicompactness should not be a necessity. Below is a proof of two theorems on lattices.

Theorem 1. *In order for $K(X)$ to be a lattice, it is necessary that X must be sequentially open in βX and sufficient for X to be sequentially open in βX and $\beta X \backslash X$ must be a Freshe - Urysohn space.*

There is natural isomorphism between sets $U_p(X)$ and $K(X)$, so $K(X)$ can be replaced with $U_p(X)$ in this theorem.

Now let us indicate one sufficient constraint at which $U(X)$ is a lattice.

Let U and V be any uniformities from $U(X)$. Assume that for any $N \in U$ and $M \in V$ there exist $N_1 \in U$ and $M_1 \in V$ such that $N_1 \circ M_1 \subset N \bigcup M$ and $M_1 \circ N_1 \subset N \bigcup M$ (1)

Theorem 2. *If condition (1) is true for any $U, V \in U(X)$ then $U(X)$ is a lattice.*

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ON SATURATION CLASS FOR HOLDERS FOURIER SERIES SUMMATION METHOD

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Let $f(x)$ be a continuous 2π periodic function, and $\frac{a_0}{2}+\sum\limits_{k=1}^{\infty}(a_k\cos kx+b_k\sin kx)$ be its Fourier series. Denote by γ the summation process of this series determine by a sequence of constants $\gamma_k^n(k=1,2,...,n;n=1,2,...)$, i.e. γ is a process of approximation of the function $f(x)$ by means of the trigonometric polynomials of the sequence $P_n^{\gamma}(x)=\frac{a_0}{2}+\sum\limits_{k=1}^{n}\gamma_k^n(a_k\cos kx+b_k\sin kx)$.

If there exists an increasing function $\varphi_\gamma(n)$ such that for any continuous 2π-periodic function $f(x)$ differ from a constant and for any natural n we have $\max\limits_x\varphi_\gamma(n)\left|P_n^{\gamma}(x)-f(x)\right|>a$, where a is a positive constant dependent on f, and there exist the function $f(x)$ for which $\max\limits_x\varphi_\gamma(n)\left|P_n^{\gamma}(x)-f(x)\right|<b$, where $b>0$ is another constant dependent on f, well say that the summation process γ is saturated. We call the saturation class belonging to the process γ the set of continuous 2π-periodic functions differ from constant and such that $\left|P_n^{\gamma}(x)-f(x)\right|=O\left(\frac{1}{\varphi_\gamma(n)}\right)$.

It would be interesting to determine the saturation class for other summation methods, in particular, for Holders summation method H^r where $r>0$.

Let for the series

$$\sum_{k=0}^{\infty}A_k, \tag{1}$$

and let $s_k=\sum\limits_{\nu=0}^{k}A_\nu(k=0,1,2,...)$ be its partial sums. Then the Holder sums of order $r>0$ of series (1) are called the sums $H_n^r=\sum\limits_{k=0}^{n}c_n^k s_k\Delta^{n-k}\frac{1}{(k+1)^r}$, where $\Delta^{n-k}\frac{1}{(k+1)^r}=\sum\limits_{p=0}^{n-k}\frac{(-1)^p c_{n-k}^p}{(k+p+1)^r}$. Find the expression of H_n^r by the members of the series (1)

$$H_n^r=\sum_{k=0}^{n}c_n^k\Delta^{n-k}\frac{1}{(k+1)^r}\sum_{\nu=0}^{n}A_\nu=\sum_{k=0}^{n}A_k\sum_{\nu=k}^{n}c_n^\nu\Delta^{n-\nu}\frac{1}{(\nu+1)^r}=\sum_{k=0}^{n}\gamma_k(n)A_k,$$

where $\gamma_k(n)=\sum\limits_{\nu=k}^{n}c_n^\nu\Delta^{n-\nu}\frac{1}{(\nu+1)^r}$.

Taking into account that $c_n^l c_{l-1}^{k-1}=\frac{n}{l}c_{n-1}^{k-1}c_{n-k}^{n-1}$, we get

$$\gamma_k(n)=nc_{n-1}^{k-1}\sum_{l=k}^{n}\frac{(-1)^{l+k}c_{n-k}^{n-1}}{l(l+1)^r}=nc_{n-1}^{k-1}\sum_{p=0}^{n-k}\frac{(-1)^p c_{n-k}^p}{(p+1)(p+k+1)^r}(k=1,2,...,n).$$

Theorem. $1-\gamma_k(n)=\frac{k\lg^{r-1}n}{n\Gamma(r)}(1+\varepsilon_{n,k}^r)$, *where* $\lim\limits_{n\to\infty}\varepsilon_{n,k}^r=0$, *for the fixed* k *and* r. *Hence we deduce that the cernel of the Holder method for* $r>1$ *is non-negative. Thus, the Holders summation method* H_r *for any real* $r\geq 1$ *is saturated with approximation of saturation of order* $O\left(\frac{\lg^{r-1}n}{n}\right)$.

FUCTIONALLY PARACOMPACT UNIFORM SPACES

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Let $(X, \mathcal{U})$ be a uniform space. A closure α of the uniform space $(X, \mathcal{U})$ is called *uniformly local finite*, if there exists uniform covering $\beta \in \mathcal{U}$, which every element of β is intersecting only finite quantity of covering elements α ([1]). A uniform space $(X, \mathcal{U})$ is called *uniformly R - paracompact*, if in any open covering may inscribe uniformly local finite open covering. A Tychnoff space X is *strongly collectivise normal* ([2]), if all neighborhoods of diagonal $\Delta = \{(x, x) : x \in X\}$ form the universal uniformity $\mathcal{U}_X$ ([3]) of space X. A subset $O \subset X$ of the uniform space $(X, \mathcal{U})$ is called *uniformly open*, if there exists such uniformly continuous mapping $f : (\mathrm{X}, \mathcal{U}) \to (M, \rho)$ of the uniform space $(X, \mathcal{U})$ into metric space (M, ρ), that $O = f^{-1}(V)$ for a some open set $V \subset M$ ([4]). A covering, is consisting of uniformly open sets, is called *uniformly open covering.*

Definition. *A uniform space $(X, \mathcal{U})$ is called functionally uniformly R – paracompact, if in any uniformly open covering may inscribe uniformly local finite uniformly open covering.*

Theorem 1. *A Tychonoff space X is strongly collectivise normal if and only if the uniform space $(X, \mathcal{U}_X)$ is functionally uniformly R- paracompact.*

Theorem 2. *A uniform space $(X, \mathcal{U})$ is functionally uniformly R - paracompact if and only if for any uniformly open covering α the covering $\alpha^{\angle} = \{\cup\alpha' : \alpha' \subset \alpha$ and $\alpha'-$finite$\}$ is a uniform covering, i.e. $\alpha^{\angle} \in \mathcal{U}$.*

Theorem 3. *A uniform space $(X, \mathcal{U})$ is functionally uniformly R - paracompact if and only if for any bicompact $B \subseteq s_{\mathcal{U}}X \backslash X$ there exists uniformly open covering $\alpha \in U$ such that $B \cap [A]_{s_{\mathcal{U}}X} = \emptyset$ for any $A \in \alpha$.*

In the Theorem 3 the Samuel bicompact extension of the uniform space $(X, \mathcal{U})$ denote by $s_{\mathcal{U}}X$ ([6]).

Theorem 4. *A Tychonoff space X is strongly collectivise normal if and only if for any bicompact $B \subseteq \beta X \backslash X$ there exists functionally open covering α of the space X such that $B \cap [A]_{\beta X} = \emptyset$ for any $A \in \alpha$.*

In the Theorem 4 the Stone-Čech bicompact extension of the Tychonoff space X denote by βX ([3]) and the functional open covering is called a covering is consisting of a functional open sets ([5]).

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ON VARIOUS CHARACTERIZATIONS OF SUPERCOMPLETE MAPPINGS

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Let $f:(X,\mathcal{U})\to(Y,\mathcal{V})$ be a uniformly continuous mappings of a uniform space $(\mathrm{X},\mathcal{U})$ into a uniform space $(Y,\mathcal{V})$.

Definition 1([1]). *A filter* $\mathcal{F}$ *in* $(\mathrm{X},\mathcal{U})$ *is called a stable, if for any covering* $\alpha\in\mathcal{U}$ *there exists such* $\mathrm{F'}\in\mathcal{F}$ that $\mathrm{F}\subset\alpha(\mathrm{F'})$ *for any* $\mathrm{F}\in\mathcal{F}$.

Proposition 1. *A continuous image* $f\mathcal{F}$ *of any stable filter* $\mathcal{F}$ *in* $(X,\mathcal{U})$ *is a basis of some stable filter in* $(Y,\mathcal{V})$.

Definition 2. *A mapping* $f:(\mathrm{X},\mathcal{U})\to(Y,\mathcal{V})$ *is called a supercomplete, if for any stable filter* $\mathcal{F}$ *in* $(X,\mathcal{U})$, *the next assertion* $\bigcap\left\{\overline{f(\mathrm{F})}:\mathrm{F}\in\mathcal{F}\right\}\neq\emptyset$ *implies* $\bigcap\left\{\overline{\mathrm{F}}:\mathrm{F}\in\mathcal{F}\right\}\neq\emptyset$.

Proposition 2.*Let* $f:(\mathrm{X},\mathcal{U})\to(Y,\mathcal{V})$ *be a uniformly continuous supercomplete mapping of a uniform space* $(\mathrm{X},\mathcal{U})$ *onto a uniform space* $(\mathrm{Y},\mathcal{V})$. *Then* $\left(f^{-1}(y),\mathcal{U}|_{f^{-1}(y)}\right)$ *is supercomplete for any point* $y\in Y$.

Lemma 1) *Any Cauchy net in a uniform space* $(\exp X,\exp\mathcal{U})$ *generate a stable Cauchy filter* F *in a uniform space* $(X,\mathcal{U})$.

2) *Any stable Cauchy* $\mathcal{F}$ *in a uniform space* $(X,\mathcal{U})$ *generate a Cauchy net in a uniform space* $(\exp X,\exp\mathcal{U})$.

Definition 3. *A uniformly continuous mapping* $f:(\mathrm{X},\mathcal{U})\to(Y,\mathcal{V})$ *is called complete, if for any Cauchy net* $\{x_i\colon x_i\in X,i\in I\}$ *in* $(X,\mathcal{U})$ *a net* $\{f(x_i):x_i\in X,i\in I\}$ *is converging in* $(Y,\mathcal{V})$, *then* $\{x_i\colon x_i\in X,i\in I\}$ *is converging in* $(X,\mathcal{U})$.

Theorem 1. *A uniformly continuous mapping* $f:(\mathrm{X},\mathcal{U})\to(Y,\mathcal{V})$ *is supercomplete, if and only if* $\exp f:(\exp\mathrm{X},\exp\mathcal{U})\to(\exp Y,\exp\mathcal{V})$ *is complete.*

Theorem 2. *Let* $f:(\mathrm{X},\mathcal{U})\to(Y,\mathcal{V})$ *be a supercomplete uniformly continuous mapping of a uniform space* $(X,\mathcal{U})$ *onto a supercomplete uniform space* $(Y,\mathcal{V})$. *Then a uniform space* $(X,\mathcal{U})$ *is supercomplete.*

Corollary 1. *Let* $f:(\mathrm{X},\mathcal{U})\to(Y,\mathcal{V})$ *be a supercomplete uniformly continuous mapping of a uniform space* $(X,\mathcal{U})$ *onto a paracompact uniform space* $(Y,\mathcal{V})$ *and* $\lambda\mathcal{V}=\mathcal{V}_Y$ *be a maximal uniformity of* Y. *Then a uniform space* $(X,\mathcal{U})$ *is paracompact and* $\lambda\mathcal{U}=\mathcal{U}_X$.

Corollary 2. *If* $f:(\mathrm{X},\mathcal{U})\to(Y,\mathcal{V})$ *is a supercomplete uniformly continuous mapping of the uniform space* $(X,\mathcal{U})$ *into a metric space* $(Y,\mathcal{V})$, *then a uniform space* $(X,\mathcal{U})$ *is supercomplete.*

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INVERSE BOUNDARY PROBLEMS FOR MAGNETIC FLOWS

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We develop a method for reconstructing the conformal factor of a Riemannian metric and the magnetic field on a surface from the scattering relation associated to the corresponding magnetic flow. The scattering relation maps a starting point and direction of a magnetic geodesic into its end point and direction. The key point in the reconstruction is the interplay between the magnetic ray transform, the fiberwise Hilbert transform on the circle bundle of the surface, and the LaplaceBeltrami operator of the underlying Riemannian metric. This is a joint work with G. Uhlmann.

NONLOCAL BOUNDARY VALUE PROBLEMS FOR SYSTEMS OF INTEGRO-DIFFERENTIAL EQUATIONS WITH PARTIAL DERIVATIVES

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On $\bar{\Omega} = [0,\omega] \times [0,T]$ we consider the system of integro-differential equations

$$\frac{\partial v}{\partial t} = A(x,t)v + \int_0^x K_0(x,\xi,t)v(\xi,t)d\xi + \int_0^T K_1(x,t,s)v(x,s)ds + f(x,t), \quad v \in R^n \tag{1}$$

with nonlocal boundary value conditions

$$B(x)v(x,0) + C(x)v(x,T) = d(x), \qquad x \in [0,\omega], \tag{2}$$

where $A(x,t)$, $K_0(x,\xi,t)$, $K_1(x,t,s)$, $B(x)$, $C(x)$ are continuous matrices and $f(x,t)$, $d(x)$ are continuous vectors on their domains.

In [1] a method for solving the linear boundary value problem for an integro-differential equation is proposed.

In communication by this method necessary and sufficient conditions of correct solvability to problem (1), (2) are obtained.

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THE ANALOGUE HARTOGS-BOCHNER THEOREM IN DOMAIN WITH THE SINGULAR WEDGES

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This work considers limited domains D from $\mathbb{C}^n$, the boundary which contains finite number of singular wedges. In such domains have been got generalized Hartogs-Bochner theorem. Bochner [1] and Martinelli [2] in 1943 independently found strict proof of Hartogs theorem about the elimination of compact peculiarities of holomorphic functions. Bochner, as a matter of fact, proved the theorem in smooth case ∂D $f \in \mathcal{C}^1(\partial D)$ (however he, didn't know about function CR-, operator $\bar{\partial}_\tau$ and etc.). Therefore the given theorem is usually called Hartogs-Bochner theorem. In 1969 Weinstock [3] (see also. [4]) proved it for continued f $(\partial D \in \mathcal{C}^\infty)$.

Let's give generalization of Hartogs-Bochner theorem for bounded domains D in $\mathbb{C}^n$ with singular wedges. Let D — limited domain in $\mathbb{C}^n$, $n > 1$. We suppose that the boundary D has $\partial D = Y \cup S_1 \cup S_2 \cup \ldots \cup S_m$, where Y smooth class $\mathcal{C}^1$ hypersurface, but each of S_ν diffeamorphic of singular hypersurface S, which was considered above (with different p_ν and different variety X'_ν, X''_ν).

Each singular hypersurface S_ν suppose to be $S_\nu = M_\nu \cup F_\nu$, where M_ν — smooth part, defined by variety X'_ν, X''_ν and by function φ_ν (considered above), and F_ν singular wedge.

Theorem. *Let multitude $\mathbb{C}^n \setminus \overline{D}$ be connected. I the function $f \in \mathcal{L}^1(\partial D)$, continued in $\partial D \setminus (F_1 \cup \ldots \cup F_m)$ and satisfies the terms*

$$\int\limits_{\partial D} f\bar{\partial}\omega = \int\limits_{\partial D \setminus (F_1 \cup \ldots \cup F_m)} f\bar{\partial}\omega = 0 \tag{1}$$

for all differential forms ω like $(n, n-2)$ with coefficient $\mathcal{C}^\infty$ $\mathbb{C}^n$, and not only in the cross of each singular wedge satisfies the terms $s(f) = O(\varphi_\nu^N(\varepsilon))$ when $\varepsilon \to 0$ for $N > 2n-2-p_\nu$ for all $\nu = 1, \ldots, m$, then exists the function F like $\mathcal{H}^\infty(D)$, when F is continued even up to each $z \in \partial D \setminus (F_1 \cup \ldots \cup F_m)$ and coincides with function f in this multitude.

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BERNOULLI NUMBERS AND EULERIAN NUMBERS

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Let $B_m = 1, -1/2, 1/6, 0, -1/30, \cdots$ be Bernoulli numebrs and B'_m be modified Bernoulli numbers, $B'_1 = 1/2$ and $B'_n = B_n$ for $n \neq 1$. Let $\mathbf{n} = 1^{k_1} \cdots n^{k_n}$ be multiset of type $k_1 \dots k_n$ and $S_{\mathbf{n}}$ be set of permutations on $\mathbf{n}$. This means that $S_{\mathbf{n}}$ is a set of sequences with components in $[n] = \{1, 2, \dots, n\}$, whose number of components equal to $i \in [n]$ is k_i. Let $K = k_1 + \cdots k_n$. It is clear that any permutation $\sigma \in S_{\mathbf{n}}$ has length K. For $\sigma \in S_{\mathbf{n}}$ say that i is *descent* index if $\sigma(i) > \sigma(i+1)$, $i < K$ or $i = K$. Let

$$a_{n,l} = |\{\sigma \in S_n | des(\sigma) = i\}|$$

be the number of permutations with l descents. If $k_i = 1$, for all $i \in [n]$, we will use Knuth notation, $\left\langle {n \atop i} \right\rangle$ instead of $a_{n,i}$. Similarly, set

$$\left\langle\!\left\langle {n \atop i} \right\rangle\!\right\rangle = a_{1^2 \cdots n^2, i}, \quad \left\langle\!\left\langle\!\left\langle {n \atop i} \right\rangle\!\right\rangle\!\right\rangle = a_{1^3 \cdots n^3, i}.$$

Let

$$H_a^{(0)} = 1, \quad H_a^{(m)} = \sum_{0 < i_1 < \cdots < i_m \leq a} \frac{1}{i_1 \cdots i_k}, \quad m > 0,$$

$$Q_{a,b}^{(c)} = \sum_{i=0}^{c} (-1)^i H_a^{(i)} H_b^{(c-i)}.$$

Theorem 1. *Let $K = k_1 + \cdots + k_n$ and $k^* = Max\{k_1, \dots, k_n\}$. Then*

$$\sum_{i=1}^{K-k^*+1} (-1)^i a_{1^{k_1} \cdots n^{k_n}, i} \binom{K-1}{i-1}^{-1} = 0, \tag{1}$$

$$\sum_{i=1}^{2n-1} (-1)^i \left\langle\!\left\langle {n \atop i} \right\rangle\!\right\rangle \binom{2n}{i}^{-1} = \frac{2n+1}{2^n} \sum_{j=0}^{n} \binom{n}{j} B_{n+j}, \tag{2}$$

$$\sum_{i=1}^{3n-2} (-1)^i \left\langle\!\left\langle\!\left\langle {n \atop i} \right\rangle\!\right\rangle\!\right\rangle \binom{3n}{i}^{-1} = -\frac{3n+1}{6^n} \sum_{j=0}^{n} (-1)^{n-j} \binom{n}{j} B'_{n+2j}. \tag{3}$$

If $n > 2$ then for any $r \geq 1$,

$$\sum_{p=1}^{\lfloor rn/2 \rfloor} (-1)^p \Big(\sum_{j=p}^{rn-p} \frac{1}{j}\Big)(a_{1^r \cdots n^r, p} + (-1)^{rn} a_{1^r \cdots n^r, p-r+1}) \binom{rn-1}{p-1}^{-1} = 0. \tag{4}$$

Theorem 2. *Let $p > 0$. Then*

$$\sum_{i=1}^{n} (-1)^i Q_{i,n-i}^{(p-1)} \left\langle {n \atop i} \right\rangle \binom{n}{i}^{-1} = (-1)^{n+p+1} \binom{n+1}{p} B'_{n+1-p},$$

$$\sum_{i=1}^{2n-1} (-1)^i Q_{i,2n-i}^{(p-1)} \left\langle\!\left\langle {n \atop i} \right\rangle\!\right\rangle \binom{2n}{i}^{-1} = \frac{(2n+1)}{2^n p} \sum_{i=0}^{n} \binom{n}{i} \binom{n+i}{p-1} B_{n+i+1-p},$$

$$\sum_{i=1}^{3n-2} (-1)^i Q_{i,3n-i}^{(p-1)} \left\langle\!\left\langle\!\left\langle {n \atop i} \right\rangle\!\right\rangle\!\right\rangle \binom{3n}{i}^{-1} = \frac{(3n+1)}{6^n p} \sum_{i=0}^{n} (-1)^{n+i} \binom{n}{i} \binom{n+2i}{p-1} B'_{n+2i+1-p}.$$

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EXTREMAL PROBLEMS IN THE CLASS $W^2_{\sigma,\nu}$

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Extremal problems in the various classes of the polynomials and the entire functions have been studied by mathematicians from many different countries and more significant results [1-5] have obtained reached. In the existing works, norm was determined by the weight function that provides condition of $\varphi(x) \geq 1$. In this talk, the weight function was taken as $\varphi(x) = x^\nu (0 \leq x < +\infty, \nu \geq 0$ - arbitrary real numbers).

We consider the functional as

$$L(f) = \frac{1}{2\pi i} \int\limits_{\Gamma} f(\xi) K(\xi) d\xi, \tag{1}$$

where $K(\xi)$ is a given function and the problem is to find the norm $\|L\|_v = \sup |L(f)|\,,\ f \in W^{(2)}_{\sigma,\nu}$. The $W^{(2)}_{\sigma,\nu}$ consists of even entire functions finite degree $\leq \sigma$ and belong to $L^{\varphi}_2(0,+\infty)$.

$$\|f\|_{2,\nu} = \left(\int\limits_0^\infty |x^\nu f(x)|^2 \, dx \right)^{\frac{1}{2}},\ W^{(2)}_{\sigma,\nu}[1] = \left\{ f \in W^{(2)}_{\sigma,\nu} : \|f\|_{2,\nu} \leq 1 \right\}. \tag{2}$$

We answer following questions which are related this problem, mentioned above:

Does extremal function exist for functional (1)? If it exists, is it odd? What are the characteristic features of this extremal function? Can this function be found correctly? What is the correct quantity for $\|L\|_\nu$? Hence, following theorem is proved.

Theorem. *The following statements are true.*

1) *$f_0(x)$ extremal function exists for given every function $K(\xi)$.*

2) *Extremal function $f_0(x)$ is unique. (Unique up to a factor $e^{i\gamma}$, $0 \leq \gamma < 2\pi$).*

3) *For the function $f_0(x) \in W^{(2)}_{\sigma,\nu}[1]$ to be extremal for $L(f)$, it is sufficiantly and necessary condition is that function $f_0(x)$ admit a representation of the following form:*

$$f_0(x) = \frac{e^{i\gamma}}{\|\tau \phi_\nu(\tau)\|^0_2} \int\limits_0^\tau x^{\frac{1}{2}-\nu} J_{\nu-\frac{1}{2}}(xt) t \phi_\nu(t) dt \tag{3}$$

where J_ν is a Bessel function and

$$\phi_\nu(t) = \frac{1}{2\pi i} \int\limits_{\Gamma} K(\xi) \xi^{\frac{1}{2}-\nu} \int\limits_{\nu-\frac{1}{2}} (\xi t) d\xi,\ \ \|.\|^0_2 = \left(\int\limits_0^\sigma \|.\|^2 \, dt \right)^{\frac{1}{2}}$$

4) $\|L\|_\nu = \left\| \sqrt{t} \phi(t) \right\|^0_2$ is true.

From this theorem several special results of the type

$$L(f) = A(f)^{(k)}(x_0) + Bf^{(m)}(x_1)...,$$

may be obtained.

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CATEGORIES AND QUASIPERIODIC MANIFOLD

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The structure on the quasi periodic manifold W_∞ is different of the structure periodic manifold only with, that $\varphi : W_k \to V$ homeomorphisms chanced to homeomorphisms

$$\varphi_k : W_k \to V_\alpha,\ \alpha \in \alpha(k),$$

where indices $\alpha \in A$ have some finite set value, i.e. manifold W_k model rued by several manifolds V_α, such that function $\alpha \in \alpha(k)$ is not constant in any interval $(-\infty, k]$ or $[k, \infty)$.

The structure of the quasi periodic manifold maybe defined two a priory operation: granulation and enlargement.

Granulation - meaning that, the manifold V_α can be represented as finitely granulation of the type which give as more smallest granulation of manifold W_∞.

Enlargement - meaning that, by given strongly monotone sequences n_k, with condition $n_{k+1} - n_k <$ Const as instead granulation we take manifolds $W'_k = W_{[n_k \ldots n_{k+1}-1]}$.

Let two quasiperiodic manifold W_∞ and W'_∞. Then a mapping $f : W_\infty \to W'_\infty$ is called quasi periodic if there exists a sequins n_k and constant λ, such that:

1. $1 \le n_{k+1} - n_k \le \lambda =$ const,
2. $f(W_k) \subset W'_{[n_k \ldots n_{k+1}+\lambda]}$,
3. A mapping

$$u_k = (\varphi_k \cup \varphi_{k+1}) \circ f \circ \varphi_k^{-1} : V_{\alpha(k)} \to \left(\cup V'_{\alpha'(f)}\right)$$

have finitely many valued.

Theorem. *The identity mapping quasi periodic structure to its enlargement is a quasi periodic mapping. Composition of quasi periodic mapping is a quasi periodic. If quasi periodic mapping is a diffeomorphism, then its inverse is quasi periodic mapping also.*

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ABOUT LOCAL SUPERPARACOMPACT SPACES

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In the present work it is studied some properties and cardinal invariants of local suerparacompact spaces which generalize respectively suerparacompact and local bicompact topological spaces. Below as spaces we mean topological spaces, as mappings continuous mappings.

Definition 1 (Musaev D.K.). *The space is called: (1) local (weakly) Pi-complete, (a2) local superparacompact if every point $x \in X$ has neighborhood $O_x \subseteq X$ closure of which respectively $\bar{O}_x$ (A1) (weakly) Pi-complete [1], (A2) superparacompact [2].*

It is clear that any superparacompact and any local bicompact space is superparacompact, and any (weakly) superparacompact [1] Hausdorff space is local (weakly) Π-complete. Inverse, generally speaking, is not valid.

Theorem 1. *Any local superparacompact, in particular, local bicompact Hausdorff is Tychonoff space.*

Theorem 1 is generalization of theorem 3.3.1 [3].

Proposition 1. *a) Closed suspace as well as open subspace of local superparacompact Hausdorff space is local superparacompact; b) Closed subspace of local (weakly) Π-complete space is local (weakly) Π complete.*

Proposition 2. *The discrete sum [3]: a) of local superparacompact spaces is the same; b) local (weakly) Π-complete space is the same.*

Lemma 1. *For local connected space X the following assertions are equivalent: a) X is local bicompact; b) X is local superparacompact.*

Proposition 3. *For any local connected of local superparacompact Hausdorff spaces X the following equalities hold: 1) $nw(X) = w(X)$ [3]; 2) $wd(X) = d(X)$ [3]; 3) $\psi(A, X) = \chi(A, X)$ for any bicompact subspaces $A \subset X$ [3].*

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EVALUATION OF K-th MOMENT S_k USING THE RESIDUE THEORY

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The k-th moment of the neutrino nucleus scattering nuclear matrix elements are studied microscopically within quasiparticle Random Phase Approximation (QRPA). Using the analytical expressions of the nuclear matrix elements $\langle n|\sum\limits_i \sigma(i)t_3(i)|0\rangle$ and the dispersion equation $D(\omega_n) = 0$ for the energy of the $I^\pi = 1^+$ states the nuclear part of the neutrino nucleus scattering cross section $S_k = \sum\limits_n \omega_n^k |\langle n|\sum\limits_i \sigma(i)t_3(i)|0\rangle|^2$ moments are calculated by help of the contour integrals and the residue theorem of the analytical functions. We have derived analytical expressions for S_k moments and show that exact calculation of S_k is possible. The great practical value of our expressions is that, in contrast to the traditional QRPA, our calculations of S_k moments in any single particle basis with an arbitrarily large number of levels present no particular technical difficulty. The machine time necessary for these computations is reduced by several orders of magnitude.

FUNCTORS AND MORPHISMS OF CATEGORY OF UML CLASS DIAGRAM

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The centerpiece of the object-oriented programming is the development of a logical model of the system in the form of class diagrams. Notation of classes in the UML is simple and intuitive to everyone who has ever had experience with CASE tools. Class diagram is used to represent the static structure of a model system in terms of classes of object-oriented programming. The class diagram may reflect, in particular, various relationships between individual entities subject area, such as objects and subsystems, and describes their internal structure and relationship types. This diagram does not specify details about the time aspects of the system. The study of a specific UML class diagrams, models using the algebraic theory of categories allows the initial stages of creating software to avoid conflicts and inadequate operation of the software, which leads to malfunction expensive program.

In this article we construct DC category, which contains morphisms reflecting various communication class diagrams. Sibling morphisms are used for modeling the relationship of objects in DC. For hierarchical morphisms of fundamental importance direction from objects located at the upper levels of the hierarchy, the objects on the bottom. Hierarchical morphisms from top to bottom objects form cones.

We prove the following theorems.

Theorem 1. *DC category is small category.*

Theorem 2. *Cones of hierarchical morphisms of DC category are products.*

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ASYMPTOTICS OF EIGENVALUES OF DISCRETE SCHRÖDINGER OPERATORS

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Let $\mathbb{T}^d$ be the d - dimensional torus and $L_2^e(\mathbb{T}^d)-$ the Hilbert space of square-integrable even functions on $\mathbb{T}^d$. The discrete Schrödinger operator $H_\mu(K)$, $K\in\mathbb{T}^d$, $\mu>0$ acts in $L^{2e}(\mathbb{T}^d)$ by $H_\mu(K)=H_0(K)-\mu V, (H_0(K)f)(q)=\mathcal{E}_K(q)f(q),\quad \mathcal{E}_K(q)=2\sum\limits_{j=1}^{d}\left[1-\cos\frac{K_j}{2}\cos q_j\right], (Vf)(q)=\int\limits_{\mathbb{T}^d} f(q)dq$. Notice that the essential spectrum of $H_\mu(K)$ fills the segment $[\min\limits_{q\in\mathbb{T}^d}\mathcal{E}_K(q), \max\limits_{q\in\mathbb{T}^d}\mathcal{E}_K(q)]$.

Theorem. *Let $d\ge 3$, $K\in\mathbb{T}^d$ and none of coordinates of K equals to π. Then there exists $\mu_0(K)>0$ that for any $\mu>\mu_0(K)$ the operator $H_\mu(K)$ has a unique eigenvalue $z(\mu,K)$ lying below $\mathcal{E}_{\min}(K)=\min\limits_{q\in\mathbb{T}^d}\mathcal{E}_K(q)$. Moreover, as $\mu\to\mu_0(K)$, the following asymptotics hold true:*

(i) *if $d=3$, then*

$$\mathcal{E}_{\min}(K)-z(\mu,K)=c_3(K)\left[\mu-\mu_0(K)\right]^2+O\left(\left[\mu-\mu_0(K)\right]^3\right),$$

where $c_1(K)=\frac{2}{c\,\pi}\left(\mu_0(K)\right)^{-2}\sqrt{\cos\frac{K_1}{2}\ldots\cos\frac{K_d}{2}},\quad c>0;$

(ii) *if $d=4$, then*

$$\mathcal{E}_{\min}(K)-z(\mu,K)=c_4(K)\,\frac{\mu-\mu_0(K)}{-\ln(\mu-\mu_0(K))}\;\left[1+o(1)\right],$$

with $c_4(K)=\frac{2}{c}\left(\mu_0(K)\right)^{-2}\sqrt{\cos\frac{K_1}{2}\ldots\cos\frac{K_d}{2}},\quad c>0;$

(iii) *if $d\ge 5$ and odd, then*

$$\mathcal{E}_{\min}(K)-z(\mu,K)=c_5(K)\left[\mu-\mu_0(K)\right]+O\left(\left[\mu-\mu_0(K)\right]^{3/2}\right),$$

where

$$c_5(K)=\left(\left(\mu_0(K)\right)^2\int\limits_{\mathbb{T}^d}\left(\mathcal{E}_K(q)-\mathcal{E}_{\min}(K)\right)^{-2}dq\right)^{-1/2};$$

(iv) *if $d\ge 6$ and even, then*

$$\mathcal{E}_{\min}(K)-z(\mu,K)=c_6(K)\;\left(\mu-\mu_0(K)\right)\;\left[1+o(1)\right],$$

with

$$c_6(K)=\left(\left(\mu_0(K)\right)^2\int\limits_{\mathbb{T}^d}\left(\mathcal{E}_K(q)-\mathcal{E}_{\min}(K)\right)^{-2}dq\right)^{-1/2}.$$

Note that, in [1,2], analogously results for the continuous and discrete Schrödinger operators have been obtained.

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IV Congress of the Turkic World Mathematical Society BAKU, AZERBAIJAN- 2011

ABOUT ONE CONVOLUTION OPERATOR

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Let Z be a set of integer numbers and $\alpha \in Z$. Let's indicate a set of all numerical sequences on an integral point $\alpha \in Z$ as A, i.e. $A = \{a : a = \{a_\alpha\}_{\alpha \in Z}\}$. Let $\rho > 0$ and $\sigma > 0$. Let's introduce a space $A_{\rho,\sigma}$ of sequences $a = \{a_\alpha\}_{\alpha \in Z}$ as satisfying an inequality $|a_\alpha| \le C(a)e^{\sigma|\alpha|^\rho}$, where $C(\alpha)$is a constant dependent on the sequence $a = \{a_\alpha\}_{\alpha \in Z}$ and $\sigma > 0$. Let's consider a space $A_{\rho,\sigma}$ in the case of $\rho = 1$, i.e. $A_{1,\sigma} = \{a : a = \{a_\alpha\}_{\alpha \in Z}, |a_\alpha| \le C(a)e^{\sigma|\alpha|}\}$. In the space $A_{1,\sigma}$ let's define a norm as

$$\|a\|_{1,\sigma} = \sup_\alpha \frac{|a_\alpha|}{e^{\sigma|\alpha|}} < \infty. \tag{1}$$

With the norm (1), the space $A_{1,\sigma}$ is a normalized space. The spaces $A_{1,\sigma}$ are Banach spaces (proving is conducted according to the definition of Banach spaces. Please see [1], p. 139).

Let's consider the space

$$A_{1,\infty} = \bigcup_{\sigma>0} A_{1,\sigma} = \bigcup_{\sigma>0} \{a : a = \{a_\alpha\}_{\alpha \in Z}, \in A, |a_\alpha| \le C(a)e^{\sigma|\alpha|}, \sigma = \sigma(a)\}.$$

$A_{1,\infty}$ is an inductive limit of the spaces $A_{1,\sigma}$, i.e. $A_{1,\infty} = \lim\limits_{\sigma>0} ind A_{1,\sigma}$ (see [2]). Let's consider an space $A^*_{1,\infty} = \bigcap\limits_{\sigma>0} A^*_{1,\sigma} = \bigcap\limits_{\sigma>0} \{b : b = \{b_\alpha\}_{\alpha \in Z}, \in A, |b_\alpha| \le B(b)e^{-\widetilde{\sigma}|\alpha|}, \widetilde{\sigma} = \widetilde{\sigma}(b)\}$, which is adjoint to the space $A_{1,\infty}$. The space $A^*_{1,\infty}$ is a projective limit of the spaces$A^*_{1,\sigma}$, i.e. $A^*_{1,\infty} = \lim\limits_{\sigma>0} pr A^*_{1,\sigma}$ (see. [2]).

Definition 3. *The operator M_F is called a convolution operator generated by a functional$F \in A^*_{1,\infty}$, and functioning according to the following rules:*
$M_F[a = \{a_\alpha\}_{\alpha \in Z}] = (F, \{a_{\alpha+m}\})_{m \in Z} = c\{c_m\}_{m \in Z}$, where $c = \{c_m\}_{m \in Z} = (F, (a_{\alpha+m}))_{\alpha \in Z}$ and $a = \{a_\alpha\}_{\alpha \in Z} \in A_{1,\infty}$. The functional $F = b = \{b_\alpha\}_{\alpha \in Z} \in A^*_{1,\infty}$ defines the convolution operator as

$$M_F[a] = M_b[a] \sum_{\alpha=-\infty}^{+\infty} b_\alpha a_{\alpha+m}, m = 0, \pm 1, \pm 2, \pm \cdots. \tag{2}$$

Lemma 1. $M_b[a] \in A_{1,\infty}$.

Let's consider homogeneous equation of convolution $M_b[a] = 0$ and write its solution. Let $\widetilde{z} = (z^\alpha)_{\alpha \in Z} \in A_{1,\infty}$.

Theorem 1. *For every j, $\widetilde{z}_j = \{z_j^\alpha\}$ is a solution of a homogeneous equation of convolution $M_b[a] = 0$.*

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ON FOURIER ANALYSIS IN INFINITE DIMENSIONS AND IT'S APPLICATIONS

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Bohr H. (see [1-4]) showed that the almost periodic function can be represented as a "diagonal" function of a limit-periodic function, defined in the multidimensional or infinite (enumerable) dimensional space as a uniform limit of periodic continuous functions.

Let trigonometric series be given

$$\sum_{n=-\infty}^{\infty} a_n e^{it\lambda_n}; \lambda_{-n} = -\lambda_n.$$

Let the sequence (λ_n) to have a whole bases(ν_m)(see [4, p.31]), so that, every finite subsequence of the sequence (ν_m) is linearly independent over the field of rational numbers, and for every of λ_n we have a unique representation

$$\lambda_n = \sum_{k=1}^{\infty} l_k(n)\nu_k;\ l_k(n) \in Z, \tag{1}$$

where the numbers $l_k(n)$ are zero with exception of finite number of them. Let $\Lambda(k)$ to denote the set of such numbers $\lambda_{\pm n}$ for which the representation (1) contains only numbers $\nu_1, ..., \nu_k$. In addition to talked above we suppose that $\nu_k \to \infty$ as $k \to \infty$ and $\sum_{n=-\infty}^{\infty} |a_n|^2 < +\infty$.

Theorem. *Let for every k the series $B_k(t) = \sum_{\lambda_n \in \Lambda(k)} a_n e^{it\lambda_n}$ be absolutely convergent. Then there exist an almost everywhere finite function $B(t)$ such that*

$$B(t) = \lim_{k\to\infty} B_k(t)$$

for almost all real t.

It should be noted that the functions $B_k(t)$ are almost periodic functions in the Bohr sense. But the convergence in the formulation of the theorem is not uniform. For the proof of the theorem we use a new measure introduced in the works [5-7].

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PROPERTIES OF BIORTHOGONAL POLYNOMIALS SUGGESTED BY JACOBI POLYNOMIALS

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In the work [1] (part 1-3) the systems of polynomials $\left\{U_n^{(\alpha,\beta)}(y;k)\right\}$ and $\left\{V_n^{(\alpha,\beta)}(z^k;k)\right\}$ biorthogonals in intervals (-1, 1) with weight function $h(x)=(1-x)^\alpha(1+x)^\beta$, where $\alpha>-1,\ \beta>-1,\ y=\frac{1-x}{2},\ z=\frac{1+x}{2}$, k- odd natural number have been studied.

In [1] following polynomials are explicit representation to generating function and some recurrente formulas.

Theorem 1. *All zeros of the biorthogonal polynomials* $U_n^{(\alpha,\beta)}(y;k)$ *and* $V_n^{(\alpha,\beta)}(w;k)$ *are real, different and located in intervals (0, 1), where* $w=z^k$.

Theorem 2. *The biorthogonal polynomials* $U_n^{(\alpha,\beta)}(y;k)$ *are shown as*

$$U_n^{(\alpha,\beta)}(y;k)=\frac{1}{\sqrt{G_{n-1}G_n}}\begin{vmatrix} M_{00} & M_{01} & ... & M_{0,n-1} & 1\\ M_{10} & M_{11} & ... & M_{1,n-1} & y\\ ... & ... & ... & ... & ...\\ M_{n0} & M_{n1} & & M_{n,n-1} & y^n\end{vmatrix},$$

$$V_n^{(\alpha,\beta)}(z^k;k)=\frac{1}{\sqrt{G_{n-1}G_n}}\begin{vmatrix} M_{00} & M_{01} & ... & M_{0n}\\ ... & ... & ... & ...\\ M_{n-1,0} & M_{n-1,1} & ... & M_{n-1,n}\\ 1 & z^k & ... & z^{kn}\end{vmatrix},$$

where G_n*- determinant of Gramian,* $G_{-1}=1,\ G_n=\begin{vmatrix} M_{00} & M_{01} & ... & M_{0n}\\ M_{10} & M_{11} & ... & M_{1n}\\ ... & ... & ... & ...\\ M_{n0} & M_{n1} & ... & M_{nn}\end{vmatrix}$,

$$M_{ms}=\int_{-1}^{1}(1-x)^\alpha(1+x)^\beta\left(\frac{1-x}{2}\right)^m\left(\frac{1+x}{2}\right)^{ks}dx,\ \ m\geq 0,\ \ s\geq 0.$$

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ROOT FUNCTIONS' BASIS PROPERTY OF SCHRÖDINGER OPERATOR WITH NONLOCAL PERTURBATION OF BOUNDARY CONDITION

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We consider the operator L given by the differential expression

$$L(y) = -y^{''}(x) + q(x)y(x), 0 < x < 1 \tag{1}$$

and by the boundary conditions

$$U_1(y) = 0,\ U_2(y) = \int_0^1 \overline{p(x)} y(x)dx,\ p(x) \in L_2(0,1), \tag{2}$$

where

$$U_j(y) = a_{j1}y^{'}(0) + a_{j2}y^{'}(1) + a_{j3}y(0) + a_{j4}y(1),\ \ j = 1, 2.$$

The question of the eigen and adjoint functions (EAF) basis of L with more generalized integral boundary conditions is positively resolved in [1], where Riesz basis with brackets is proved as long as the boundary conditions $U_1(y) = 0, U_2(y) = 0$ are regular by Birkhoff. And as long as an additional assumption of strongly regular Riesz basis EAF is proved. A double differentiation operator with the integral perturbation of the periodic boundary conditions is investigated in [2].

We consider the case when the boundary conditions are regular, but not strongly regular:

$$A_{12} \ = \ 0,\ A_{14} + A_{23} \neq 0,\ A_{14} + A_{23} = \pm(A_{13} + A_{24}).$$

Herein as usual $A_{ij} = a_{1i}a_{2j} - a_{1j}a_{2i}$ is denoted.

Theorem. *Let the unperturbed operator L_0 (in case $p(x) \equiv 0$) to have an EAF system forming Riesz basis in $L_2(0,1)$. Then:*

1) *set P of $p(x) \in L_2(0,1)$ functions such that the EAF system of L forms Riesz basis in $L_2(0,1)$ is dense in $L_2(0,1)$;*

2) *if $A_{14} = A_{23}, A_{34} = 0$ then $L_2(0,1)\backslash P$ is also dense in $L_2(0,1)$;*

3) *if $A_{14} = A_{23}, A_{34} \neq 0$ then $P = L_2(0,1)$;*

4) *if $A_{14} \neq A_{23}$ then the P set and $L_2(0,1)\backslash P$ are dense in $L_2(0,1)$. To ensure that the EAF system operator L is Riesz basis in $L_2(0,1)$ it is necessary and sufficient that all eigenvalues λ_k^1 of L except may be a finite number are multiple (in other words asymptotically multiples).*

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APPROXIMATION PROBLEMS IN WEIGHTED SMIRNOV CLASSES

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In this talk we discuss the direct and inverse problems of approximation theory in the weighted Smirnov classes, defined on the simple connected domain G with a regular boundary L. By ω we denote the weight function satisfying the well known Muckenhoupt condition on L and by $L^p(L,\omega)$- the weighted Lebesgue spaces of functions defined on L.

Under some restrictive conditions on the weight function ω we prove the direct theorem of approximation theory in the Smirnov classes $E^p(G,\omega)$, $1 < p < \infty$, i.e. we estimate the degree of approximation $L^p(L,\omega)$ norm by a r^{th} integral modulus of continuity $\omega_k(f,h)_{L^p(L,\omega)}$ defined as following:

$$\omega_k(f,h)_{L^p(L,\omega)} := \sup_{\delta \le h} \|\sigma_\delta^r f\ \|_{L^p(L,\omega)} := \left(\int_0^{2\pi} |\sigma_\delta^r f_0\,(x)|^p \,\omega\,(x)\,dx \right)^{1/p},$$

where

$$\sigma_\delta^r f\,(x) = \frac{1}{\delta} \int_0^{\delta} |\Delta_t^r f\,(x)|^p \,dt$$

and

$$\Delta_t^r f\,(x) = \sum_{s=0}^{r} (-1)^{r+s+1} \binom{r}{s} f\,(x+st)\,.$$

Later in the weighted Smirnov spaces $E^p(G,\omega)$ we prove, using the same modulus of smoothness, the appropriate inverse theorem. In particular, combining the direct and inverse theorems we obtain the constructive description of the generalized Lipschitz classes of functions.

Note that in term of some other modulus of smoothness the similar results were obtained by us in [1].

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THE RIEMANN HYPOTHESIS

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Appearance of the zeta function and analytical methods in Number Theory is connected with L.Euler's name. In 1748 Euler introduced the zeta function

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s}, s > 1,$$

considering it as a function of real variable s. B. Riemann in 1859 considered, for the first time, the zeta function as a function of complex variable and connected the problem of distribution of prime numbers with an arrangement of complex zeroes of the zeta function. He had continued, analytically, the zeta function to the all complex plane and formulated some hypotheses about the zeta function. One of them later becomes a central problem for all of Mathematics. This hypothesis asserts that all of complex zeroes of the zeta function, located in the critical strip $0 < Res < 1$, lay on the critical line $Res = 0.5$. In this work we formulate our result (see [1]).

Theorem 1. *Let $0 < r < 1/4$ be a real number. Then there exist sequence (θ_n) in Ω ($\theta_n \in \Omega$, $n = 1, 2, ...$) and a sequence (m_n) of integers that for every real t*

$$\lim_{n\to\infty} F_n(s + it, \theta_n) = \zeta(s + it)$$

uniformly in the circle $|s - 3/4| \le r$; here

$$F_n(s + it, \theta_n) = \prod_{p \le m_n} \left(1 - \frac{e^{-2\pi i \vartheta_p^n}}{p^{s+it}}\right)^{-1}; \ \theta_n = (\theta_p^n),$$

and the product is taken over all prime numbers and the components of θ_n are indexed by prime numbers.

It should be noted that the length of a partial product, approximating $\zeta(s)$, depends on t.

Corollary. *The Riemann Hypothesis is true, i. e.*

$$\zeta(s) \ne 0$$

when $\sigma > 0.5$.

The result is based on a new measure introduced in the infinite dimensional unite cube distinct from the Haar or product Lebesgue measures and closely connected with the Tichonov metric.

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ON A CONSTRUCTIVE CHARACTERISTIC OF CLASSES OF HARMONIC FUNCTIONS[1]

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Let $w(\delta)$, $\delta > 0$, be a function of the type of modulus of continuity, i.e. a positive nondecreasing (with $w(+0) = 0$) function satisfying for some $c = const > 0$ the condition $w(t\delta) \le ctw(\delta)$, $\delta > 0$, $t > 1$. Let us consider a Jordan curve Γ and two arbitrary points z_1 and z_2 on it. By $\Gamma(z_1, z_2)$ we denote one of the two curves (with less diameter) on which the points z_1 and z_2 divide the curve Γ. The feasibility of the relation diam $\Gamma(z_1, z_2) \preccurlyeq |z_1 - z_2|$ is the necessary and sufficient condition for the quasiconformal of the curve Γ (see [1],p.100). Let us denote by $C^w_\Delta(\Gamma)$ the class of real-valued, continuous in $\mathbb{C}$, harmonic in $\bar{\mathbb{C}}\backslash\Gamma$ functions u satisfying, for any z and $\zeta \in \mathbb{C}$, the condition $|u(z) - u(\zeta)| \le cw(|z - \zeta|)$, $c = c(u) = const > 0$. By $B^w_\Delta(\Gamma)$ being the class of real-valued, continuous in $\bar{\mathbb{C}}$, harmonic in $\bar{\mathbb{C}}\backslash\Gamma$ functions such that, for any $n \in \mathbb{N}$, there is a harmonic rational function

$$R_n(z) = Re\sum_{j=-n}^{n} a_j z^j,\ n = 1, 2, ...,\ a_j \in \mathbb{C} \tag{1}$$

satisfying the relation $|u(z) - R_n(z)| \le c_1 w\left[\rho_{1/n}, (z)\right], z \in G_n$. In this study the constructive characterization of classes of harmonic functions with singularities on a quasiconformal curves has been studied. To prove the inverse theorem in this study, we use the standard scheme for the proofs of inverse theorem [2] , [3] and [4]. The main results of this work are as follows:

Theorem 1. *Let Γ be a quasiconformal curve and $f \in B^w_\Delta(\Gamma)$. Then $f \in C^\mu_\Delta(\Gamma)$, where*

$$\mu(\delta) = \delta\int_\delta^1 \frac{w(t)}{t^2}dt,\ 0 < \delta < 1/2\ .$$

Corollary 1. *If*

$$\delta\int_\delta^1 \frac{w(t)}{t^2}dt \le c_2 w(\delta), \quad 0 < \delta < 1/2$$

then

$$C^w_\Delta(\Gamma) = B^w_\Delta(\Gamma).$$

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COMPACT LAW OF THE ITERATED LOGARITHM FOR HILBERT SPACE-VALUED RANDOM VARIABLES AND AUTOREGRESSIVE PROCESSES

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Let $\{Y_n,\ n \geq 1\}$ be a sequence of random variables (r.v.'s) with values in separable Hilbert space H (with a norm $\|\cdot\|$) satisfying φ_m– mixing condition. Coefficients of φ_m– mixing are defined as following

$$\varphi_m(n) = \sup_{H_m} \sup_n \left\{ |\ (B/A) - \ (B)| : B \in \sigma^m(F_{k+n}^{\infty}),\ \ A \in \sigma^m(F_1^k),\ P(A) > 0,\ k \in N \right\}$$

where $\sigma^m(F_a^b)$ – σ-algebra generated by r.v.'s ${}_mY_a, ...,{}_mY_b$, ${}_m$- operator of projection from H to m-dimensional space $H_m \subset H$. Denote

$$U_n = Y_1 + Y_2 + ... + Y_n, a_n = \sqrt{2n \ln \ln n},\ \ n \geq 3\,,\quad Lx = \max(1, \ln x),$$

H^*-dual space, $V(f,g) = \lim_{n\to\infty} \frac{1}{n} Ef(U_n)\, g(U_n)\ f, g \in H^*$.

Theorem *Let $\{Y_n,\ n \geq 1\}$ be a sequence of identically distributed r.v.'s with values in H satisfying the following conditions: $E\|Y_1\| = 0$, $E\|Y_1\|^2 < \infty$, $\lim_{n\to\infty} \frac{1}{n} Ef^2(U_n) = \sigma_f < \infty$ for all$f \in H^*$, $\sum_{i=1}^{\infty} \varphi_m^{\frac{1}{\theta}}(2^k) < \infty$, for some $\theta > 3$ and $m = 1, 2, ...$, $V(f,g)$ $g, f \in H^*$ is continuous in weak $*$-topology.*

Then there exists a compact set $K \subset H$ such that almost surely $\lim_{n\to\infty} \inf_{x\in K} \left\| \frac{U_n}{a_n} - x \right\| = 0$ and $C\left(\left\{\frac{U_n}{a_n}\right\}\right) = K$, where $C\left(\left\{\frac{U_n}{a_n}\right\}\right)$ -all limit points of $\left\{\frac{U_n}{a_n}\right\}$ in H.

The compact law of the iterated logarithm for autoregressive processes in separable Hilbert space H, which are defined by the following equation

$$X_n - m = T(X_{n-1} - m) + \varepsilon_n,\ \ n \in Z,$$

where $T : H \to H$ is bounded linear operator, $m \in H$, $\{\varepsilon_n, n \in Z\}$ is a stochastic process of innovations will be discussed as well.

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QUADRATIC STOCHASTIC OPERATORS CORRESPONDING TO GRAPHS[1]

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The notion of quadratic stochastic operator (QSO) was first formulated by Bernshtein [1]. We denote by S the following set:

$$S = \{x = (x_i) : x_i \geq 0, \ \sum_{i=1}^{\infty} x_i = 1\}. \tag{1}$$

We define an operator $V : S \to S$ as follows

$$(V(x))_k = \sum_{i,j=1}^{\infty} p_{ij,k} x_i x_j, \quad k \in N, x = (x_i) \in S, \tag{2}$$

where

$$p_{ij,k} \geq 0, \quad p_{ij,k} = p_{ji,k}, \quad \sum_{k=1}^{\infty} p_{ij,k} = 1, \quad i, j \in N. \tag{3}$$

Definition 1. *An operator defined as above is called an infinity dimensional quadratic stochastic operator.*

Let $G = (\Lambda, L)$ be a graph without multiple edges, where Λ is the set of vertices which is at most a countable set, $L-$ is the set of edges of the graph G. Enumerate the vertices of the graph G by elements of $E = \{0, 1, 2, ...\}$.

We denote $\langle i, j\rangle$ if vertices i and j are the neighboring points of graph, i.e. vertices i and j connect with edges of a graph. By $\rangle i, j\langle$ we denote non neighboring points. The coefficients of heredity we define as the following:

$$p_{ij,k} = \begin{cases} 1, & \text{if } \ \rangle i, j\langle, \quad k = 0, \quad i, j \in E; \\ 0, & \text{if } \ \rangle i, j\langle, \quad k \neq 0, \quad i, j \in E; \\ \geq 0, & \text{if } \ \langle i, j\rangle, \quad i, j \in E \setminus \{0\}; \\ 0, & \text{if } \ \langle i, j\rangle, \quad k \neq 0, \quad i = 0 \ \text{ or } \ j = 0. \end{cases} \tag{4}$$

Definition 2. *For any fixed graph G, QSO satisfying conditions* (2),(3) *and* (4) *is called the quadratic stochastic operator corresponding to the graph (QSOCG).*

Arbitrary QSOCG has the form

$$V : \begin{cases} x'_0 = x_0^2 + 2x_0 \sum\limits_{i \in E\setminus\{0\}} x_i + 2 \sum\limits_{\substack{i,j \in E: \\ \rangle i,j\langle}} x_i x_j + \sum\limits_{\substack{i,j \in E: \\ \langle i,j\rangle}} p_{ij,0} x_i x_j \\ x'_k = \sum\limits_{\substack{i,j \in E: \\ \langle i,j\rangle}} p_{ij,k} x_i x_j, \quad k \in E \setminus \{0\}. \end{cases} \tag{5}$$

Theorem 1. *Any QSOCG* (5) *has a unique fixed point (1,0,0,...). Besides, for any $x^{(0)} \in S$, the trajectory of operator* (5) *tends to this fixed point exponentially rapidly.*

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ON THE SUMMABILITY OF FOURIER COEFFICIENTS OF FUNCTIONS FROM LORENTZ SPACE

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Let $1 \le p < \infty, 0 < q \le \infty$. A set of measureable functions on $[0, 2\pi]$ is called the Lorentz space $L_{pq}[0, 2\pi]$, if

$$\|f\|_{L_{pq}[0,1]} = \left(\int_0^1 \left(t^{\frac{1}{p}} f^*(t) \right)^q \frac{dt}{t} \right)^{\frac{1}{q}},$$

for $0 < q < \infty$

$$\|f\|_{L_{p\infty}[0,1]} = \sup_{0 \le t \le 1} t^{\frac{1}{p}} f^*(t)$$

for $q = \infty$ are finite. Here $f^*(t)$ - non-increasing permutation of function $f(t)$.

Let $f \sim \sum\limits_{k=1}^{\infty} a_k \cos kx$. The following result is provedin [1]:

If $1 < p < \infty$, $p' = \frac{p}{p-1}$, $1 \le q \le \infty$, $f \in L_{pq}[0, 2\pi]$, then

$$\left(\sum_{k=1}^{\infty} k^{\frac{q}{p'}} (\bar{a}_k)^q \frac{1}{k} \right)^{\frac{1}{q}} < \infty, \tag{1}$$

where $\bar{a}_k = \frac{1}{k} \left| \sum\limits_{m=1}^{k} a_m \right|$.

In this paper we consider inequality (1) type for distinct averagings Fourier coefficients.

Theorem 1. *Let* $1 < p < \infty$, $p' = \frac{p}{p-1}$, $1 \le q < \infty, f \sim \sum\limits_{k=1}^{\infty} a_k \cos kx$. If $f \in L_{pq}[0, 2\pi]$, *then the following inequality holds*: $\left(\sum\limits_{k=1}^{\infty} k^{\frac{q}{p'}} (\tilde{a}_k)^q \frac{1}{k} \right)^{\frac{1}{q}} < \infty$, where $\tilde{a}_k = \left| \sum\limits_{m=k}^{\infty} \frac{a_m}{m} \right|$.

Theorem 2. *Let* $1 < p < \infty$, $p' = \frac{p}{p-1}$, $\alpha > \frac{1}{p'}$, $f \sim \sum\limits_{k=1}^{\infty} a_k \cos kx$, *a sequence* $\lambda = \{\lambda_k\}$ *satisfy to the next conditions:*

1) $\sup\limits_{r \le k} \frac{1}{r^{\alpha}} \left| \sum\limits_{m=1}^{r} \lambda_m \right| \le D \frac{1}{k^{\alpha}} \left| \sum\limits_{m=1}^{k} \lambda_m \right|$,

2) $|\lambda_k - \lambda_{k+1}| \le D \frac{1}{k^2} \left| \sum\limits_{m=1}^{k} \lambda_m \right|$, *D- some constant, that undepends from index k.* If $f \in L_{pq}$. *Then* $\left(\sum\limits_{k=1}^{\infty} \left(k^{\frac{1}{p'}} \bar{a}_k(\lambda) \right)^q \frac{1}{k} \right)^{\frac{1}{q}} < \infty$ *holds, where* $\bar{a}_k(\lambda) = \frac{1}{\left| \sum\limits_{m=1}^{k} \lambda_m \right|} \left| \sum\limits_{m=1}^{k} \lambda_m a_m \right|$, $k \in N$.

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ABOUT SOME CARDINAL INVARIANTS AND METRIZATION OF SUPERPARACOMPACT AND COABSOLUTE SPACES

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In the present work it is studied some cardinal invariants of coabsolute [1] spaces. Below as space we mean topological space, as mapping of spaces continuous mappings. It is known [1] that if two regular spaces are coabsolute then their density [1], and also Suslin's number [1] are coincide.

It turns out for coabsolute spaces the following holds

Theorem 1. *If two regular spaces X and Y are coabsolute then their wekly density [2] and also weakly Lindelof's number [2] are concide.*

Proposition 1. *Any (O-C)-finite [3] superparacompact [3]* T_2 *space with fisrt axiom of countablity is metrizable.*

Proposition 2. *For any local connected superparacompact* X *the following equalities hold*

$$1)\ n\omega(\mathrm{X}) = \omega(\mathrm{X})\ \ [2]\ ;\ \ 2)\ \ \omega d(\mathrm{X}) = d(\mathrm{X})\ \ [2]\ ;\ \ 3)\, \psi(A,\mathrm{X}) = \chi(A,X)\ \ [2]$$

for any bicompact subspace $A \subset \mathrm{X}$.

Theorem 2. *Local bicompact superparacompact* X *is metrizable if and only if when any its open bicompact subspace has countable base.*

Theorem 3. *Local connected superparacompact groupp [3] G metrizable if and only if whet components unity of the group G has countable closeness and countable chatacter in the group G.*

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ON APPROXIMATION PROPERTIES OF THE ROOT FUNCTIONS GENERATED BY THE CORRECTLY SOLVABLE BOUNDARY VALUE PROBLEMS FOR THE HIGH ORDER ORDINARY DIFFERENTIAL EQUATIONS

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In this work we study properties of systems of root functions generated with differential operators. Let L be a differential operator with the following ordinary differential expression in the $L_2(0,1)$ space

$$l(y) \equiv y^{(n)}(x) + p_{n-2}(x)y^{(n-2)}(x) + p_{n-3}(x)y^{(n-3)}(x) + ... + p_0(x)y(x), \tag{1}$$

and with inner boundary conditions

$$y^{(\nu)}(0) = 0, \nu \neq k, \nu = \overline{0, n-1},$$
$$y^{(k)}(0) - \int_0^1 (l(y(x)))\overline{\sigma(x)}dx = 0, \tag{2}$$

where $\sigma(x) \in L_2(0,1)$ and $\overline{\sigma(x)}$ denotes complex conjugate of the $\sigma(x)$. Obviously, the inverse operator L^{-1} is completely continuous operator. Then it follows from [1, pp.10] that spectrum of L is finite or it is a countable set of isolated eigenvalues with finite algebraic multiplicity without points of accumulation. For every eigenvalue λ_s with geometric multiplicity m_s is set in correspondence the following chain of the eigenfunction and associated function of the operator L

$$E_s = \{y_{s,0}(x,), y_{s,1}(x), ..., y_{s,m_s-1}(x)\}.$$

Union of all different chains of the root functions

$$E \equiv \{E_s : \lambda_s - \text{eigenvalue of the operator L}\}$$

is called the system of root functions of the operator L. Thus, the differential operator L is a derivation of some system of root functions. The main problem is to study properties of root functions generated with differential operator L.

The following theorem is valid.

Theorem. *Let $\sigma(x) \in C^{n-k-2}[0,1]$ for $k < n-1$ and $\sigma(x) \in L_2(0,1)$ for $k = n-1$. If there exist non vanishing limits*

$$\lim_{\varepsilon \to +0} \frac{1}{\varepsilon} \int_0^\varepsilon (1 + (-1)^{n-k}\sigma^{(n-k-1)}(x))dx = \alpha_1, \lim_{\varepsilon \to 1-0} \int_\varepsilon^1 \sigma(x)dx = \alpha_2,$$

then the system of root functions of the operator L is complete and minimal in $L_2(0,1)$.

From [2] followed that for all $\sigma(x) \in L_2(0,1)$ conditions (2) are described correctly solvable problems corresponding to the expression (1).

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CERTAIN SASAKIAN MANIFOLDS OF CONSTANT SECTIONAL CURVATURE

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In this paper we study ϕ-recurrent and generalized ϕ-recurrent Sasakian manifolds and prove that their sectional curvature is constant. We show that any generalized concircular ϕ-recurrent Sasakian manifold is a concircular ϕ-recurrent and both of them are of constant curvature.

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INTEGRAL OPERATORS IN WEIGHTED SOBOLEV SPACES AND APPLICATIONS

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For the integral operators of the form $Kf(x) = \int\limits_a^x K(x,t)f(t)dt$ and $Kg(x) = \int\limits_x^b K(t,x)g(t)dt$, $x \in (a,b) \subseteq \mathbb{R}$, under some assumptions on the kernel $K(x,t) \geq 0$ we find necessary and sufficient conditions for the validity of the following weighted inequalities

$$\|wKf\|_q \leq C\left(\|\rho f'\|_p + \|vf\|_p\right), \quad \forall f \in A\overset{o}{C}\ (a,b), \tag{1}$$

$$\|u(Kf)'\|_r \leq C\left(\|\rho f'\|_p + \|vf\|_p\right), \quad \forall f \in A\overset{o}{C}\ (a,b), \tag{2}$$

where $A\overset{o}{C}\ (a,b)$ is a set of absolutely continuous functions with compact supports.

Studying the inequalities (1) and (2) leads to for instance the problem on discreteness of spectrum of the Friedrichs extension of operator

$$A_n u \equiv (-1)^{n+1}\left(\rho^2(x)\left(\frac{u}{w}\right)^{(n+1)}\right)^{(n+1)} + (-1)^n\left(v^2(x)\left(\frac{u}{w}\right)^{(n)}\right)^{(n)}$$

with domain $D(A_n) = C_0^\infty(I)$, $n \geq 1$.

INVESTIGATION OF A RENEWAL-REWARD PROCESS WITH A GENERALIZED REFLECTING BARRIER

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In this study, two sequences $\{\xi_n\}$ and $\{\eta_n\}$ of independent and identically distributed positive - valued random variables are considered. By means of these random variables a stochastic process $X_\lambda(t)$ which is called "Renewal - reward process with a generalized reflecting barrier" is constructed. In the study, some ergodic characteristics of the process $X_\lambda(t)$ are investigated and the following main results are obtained.

Theorem 1. *Assume that the sequences* $\{\xi_n\}$ and $\{\eta_n\}$ a*re satisfied the following supplementary conditions:*

i) $0 < E(\xi_1) < \infty$,

ii) $E(\eta_1) > 0$,

iii) $E(\eta_1^{n+2}) < \infty$,

iv) η_1 *s non - arithmetic random variable.*

Then, for n^{th} ergodic moment ($E(X^n)$) of the process $X_\lambda(t)$ the following asymptotic expansion with three terms can be written, when $\lambda \to \infty$:

$$E(X^n) = \frac{2m_{n+2}}{(n+1)(n+2)m_2}\lambda^n + B_n\lambda^{n-1} + C_n\lambda^{n-2} + o(\lambda^{n-2}),$$

where $m_n = E(\eta_1^n), n \geq 1$.

The explicit expressions for the coefficients B_n and C_n are exist in the study.

Theorem 2. *Under the conditions of the Theorem 1, the process $Y_\lambda(t) \equiv X_\lambda(t)/\lambda$ is ergodic and the ergodic distribution ($Q_Y(X)$) of the process $Y_\lambda(t)$ weakly converges to the limit distribution $G(X)$, when $\lambda \to \infty$, i.e., for each $x > 0$,*

$$Q_Y(X) \to G(X) \equiv (2/m_2)\int_0^x \int_v^\infty (1 - F(u))dudv.$$

Here, $F(u)$ is the distribution function of the random variable η_1 .

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THE NORM OF THE ERROR FUNCTIONAL OF WEIGHT OPTIMAL CUBATURE FORMULAS IN $\tilde{C}^{(m)}(T_n)$ SPACE

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The present paper is devoted for functions $f(x_1, x_2, ...x_n)$ of n - variables, which belong to the space $\tilde{C}^{(m)}(T_n)$, i.e. $f(x_1, x_2, ...x_n) \in \tilde{C}^{(m)}(T_n)$, where T_n is n dimensional torus.

Definition 1. *The set* $T_n = \{x = (x_1, x_2, ...x_n); \quad x_k = \{t_k\}, \quad t_k \in R\}$ *is called* n *dimensional torus, where* $\{t_k\} = t_k - [t_k]$, *i.e. fractional part of* t_k [1].

Definition 2. *The space* $\tilde{C}^{(m)}(T_n)$ *is defined as closure of the set of finite Fourier series*

$$\sum_{\gamma} \widehat{f}[\gamma] e^{-2\pi i(\gamma,x)} = f(x)$$

in semi norm

$$\left\| f(x)|\tilde{C}^{(m)}(T_n) \right\| = \max_{x\in T_n} \left| \sum_{\gamma\neq 0} |\gamma|^m \, \hat{f}[\gamma] \, e^{-2\pi i(\gamma,x)} \right|,$$

where $(\gamma, x) = \sum\limits_{k=1}^{n} \gamma_k x_k$ and $\widehat{f}[\gamma] =< f(x), e^{2\pi i(\gamma,x)} >= \int\limits_{T_n} f(x)\, e^{2\pi i(\gamma,x)} dx$, *i.e. are Fourier coefficients.*

We consider the following cubature formula

$$\int\limits_{T_n} P(x) f(x) dx \approx \sum_{\lambda=1}^{N} C_\lambda f(x^{(\lambda)}), \tag{1}$$

where $P(x)$ is the weight function, C_λ is the coefficients and $x^{(\lambda)}$ is the nodes of cubature formula (1).

To the cubature formula (1) we correspond the following generalized function

$$\ell(x) = P(x)\varepsilon_{T_n}(x) - \sum_{\lambda=1}^{N} C_\lambda \delta(x - x^{(\lambda)}). \tag{2}$$

And we call it by the error functional. Here $\delta(x)$ is Dirac's delta function, $\varepsilon_{T_n}(x)$ is the characteristic function of the torus T_n.

The following holds

Theorem. *The norm of the error functional* (2) of *cubature formula* (1) *in the space* $\tilde{C}^{(m)}(T_n)$ *has the form*

$$\left\| \ell(x)|\tilde{C}^{(m)*}(T_n) \right\| = \inf_{\chi} \int\limits_{T_n} \left| \sum_{\gamma\neq 0} \frac{\widehat{P}[\gamma] - \sum\limits_{\lambda=1}^{N} C_\lambda \, e^{-2\pi i(\gamma, x^{(\lambda)})}}{|\gamma|^m} \cdot e^{2\pi i(\gamma,x)} + \chi \right| dx, \tag{3}$$

where χ *is a constant.*

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THE INEQUALITIES OF HARDY-LITTLEWOOD TYPE FOR THE FOURIER COEFFICIENTS

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In this work some inequalities of Hardy-Littlewood type with respect to a regular system for the generalized Lorentz spaces $\Lambda_q(\omega)$ are obtained.

Let $0 < q \leq \infty$ and let ω be a nonnegative function on $[0,1]$. The generalized Lorentz spaces $\Lambda_q(\omega)$ consists of the mesurable functions f on $[0,1]$ such that: $\|f\|_{\Lambda_q}(\omega) < \infty$, where

$$\|f\|_{\Lambda_q(\omega)} := \begin{cases} \left(\int_0^1 \left(f^*(t)t\omega\left(\frac{1}{t}\right)\right)^q \frac{dt}{t}\right)^{\frac{1}{q}} & \text{for} \quad 0 < q < \infty, \\ \sup_{0\leq t\leq 1} f^*(t)t\omega(\frac{1}{t}) & \text{for} \quad q = \infty, \end{cases}$$

where $f^*(t)$ is the nondecreasing rearrangement of the function $|f(t)|$.

Let the function f be periodic with period 1 and integrable on [0,1] and let $\Phi = \{\varphi_k\}_{k=1}^{\infty}$ be an orthonormal system. The numbers

$$a_k = a_k(f) = \int_0^1 f(x)\overline{\varphi_k(x)}dx, \; k \in \mathbb{N},$$

are called the Fourier coefficients of the function f with respect to the system $\Phi = \{\varphi_k\}_{k=1}^{\infty}$.

Let $\delta > 0$ and $\omega(t)$ be a nonnegative function on [0,∞). We define the following class

$$A = \bigcup_{\delta>0} A_\delta = \bigcup_{\delta>0} \{\omega(t): \; \omega(t)t^{-\delta} \nearrow \text{ and } \omega(t)t^{-1+\delta} \searrow\}.$$

Theorem 1. *Let $\Phi = \{\varphi_k\}_{k=1}^{\infty}$ be a regular system and let $1 \leq q \leq \infty$. If $\omega(t)$ belongs to the class A, then*

$$\left(\sum_{k=1}^{\infty}\left(\overline{a}_k k\omega(\frac{1}{k})\right)^q \frac{1}{k}\right)^{\frac{1}{q}} \leq c_1\|f\|_{\Lambda_q(\omega)},$$

where $\overline{a}_k = \frac{1}{k}\left|\sum\limits_{m=1}^{k} a_m(f)\right|$, $a_k(f)$ are the Fourier coefficients with respect to the system Φ.

Theorem 2. *Let $\Phi = \{\varphi_k\}_{k=1}^{\infty}$ be a regular system, $f \overset{a.e.}{=} \sum\limits_{k=1}^{\infty} a_k\varphi_k$ and $1 \leq q \leq \infty$. If $\omega(t)$ belongs to the class A, then*

$$\left(\int_0^1 \left(\overline{f(t)}\omega(t)\right)^q \frac{dt}{t}\right)^{\frac{1}{q}} \leq c_2\left(\sum_{k=1}^{\infty}\left(a_k^* k\omega(\frac{1}{k})\right)^q \frac{1}{k}\right)^{\frac{1}{q}},$$

where $\overline{f(t)} = \frac{1}{t}\left|\int\limits_0^t f(s)ds\right|$. The definition of the regular system Φ was presented in [1].

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NUMBER OF EIGENVALUES OF THE FAMILY OF FRIEDRICHS MODEL UNDER ONE PERTURBATION

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Let $\mathbb{T}^d$ be the $d-$dimensional torus and $L_2(\mathbb{T}^d)$ be the Hilbert space of square-integrable functions on $\mathbb{T}^d$. The Friedrichs model operator $h_\lambda(p)$, $p \in \mathbb{T}^d$ is of the form [1]:

$$h_\lambda(p) = h_0(p) - \lambda v.$$

The non perturbed operator $h_0(k)$ on $L_2(\mathbb{T}^d)$ is a multiplication operator by the function $\omega(p,q)$

$$(h_0(p)f)(q) = \omega_p(q)\, f(p),$$

the perturbation v is an integral operator of rank one

$$(vf)(p) = \varphi(p) \int\limits_{\mathbb{T}^d} \varphi(t)\, f(t)\, dt,$$

where $\omega_p(q) := \omega(p,q)$ and $\varphi(q)$ are analytic functions on $(\mathbb{T}^d)^2$ and $\mathbb{T}^d$ respectively. The essential spectrum $\sigma_{ess}(h_\lambda(p))$ of $h_\lambda(p)$ fills the segment $[\min\limits_{q\in\mathbb{T}^d} \omega_p(q), \max\limits_{q\in\mathbb{T}^d} \omega_p(q)]$. Let $d = 1, 2$. We assume that there exists such open connected set $\mathcal{G} \subset \mathbb{T}^d$ that for any $p \in \mathcal{G}$ there exists a unique non degenerated minimum $q_0(p)$ of the function $\omega_p(q)$. We remark that if $\varphi(q_0(p)) = 0$ then there exists

$$\lambda(p) = \left(\int\limits_{\mathbb{T}^d} \varphi^2(s)\ (w_p(s) - m(p))^{-1}\, ds \right)^{-1} > 0,$$

where $m(p) = \min\limits_{q\in\mathbb{T}^d} \omega_p(q)$, and if $\varphi(q_0(p)) \neq 0$, then we set $\lambda(p) = 0$. Then the following results are hold true:

(i) if $\lambda > \lambda(p)$ then the operator $h_\lambda(p)$, $p \in \mathcal{G}$ has a unique eigenvalue $E(\lambda, p) \in (\infty, m(p))$.

(ii) If $\varphi(q_0(p)) = 0$, $\nabla\varphi(q_0(p)) \neq 0$ then $\lambda(p) > 0$ and for any $\lambda \in (0, \lambda(p))$, the operator $h_\lambda(p)$, $p \in \mathcal{G}$ has none eigenvalue in $(-\infty, m(p)]$;

(iii) If $\varphi(q_0(p)) = 0$, $\nabla\varphi(q_0(p)) \neq 0$ and $\lambda = \lambda(p)$, then the equation

$$h_\lambda(p)f = m(p)f, \qquad p \in \mathcal{G}$$

has non-zero solution

$$f = \frac{\lambda(p)\varphi(q)}{\omega_p(q) - m(p)} \notin L_1(\mathbb{T}^d) \setminus L_2(\mathbb{T}^d)$$

(iv) If $\varphi(q_0(p)) = 0$, $\nabla\varphi(q_0(p)) = 0$, then the number $z = m(p) = w_p(q_0(p))$ is eigenvalue of $h_\lambda(p)$, $p \in \mathcal{G}$ and the corresponding eigenfunction has a form

$$f(q) = C\, \frac{\lambda(p)\varphi(q)}{w_p(q) - m(p)},$$

where $C \neq 0$ – the normalization coefficient.

Notice that in [1] the existence of eigenvalues of the operator h_λ have been studied for $d \geq 3$.

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NECESSARY AND SUFFICIENT CONDITIONS OF GEODESIC OF THE IMAGE OF THE GIVEN NET IN THE MAPPING OF P- DIMENSIONAL SURFACES OF N-DIMENSIONAL EUCLIDEAN SPACE

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Let F, F' be p-dimensional surfaces of Euclidean space E_n

$$\Re = (X, \overrightarrow{e}_i, \overrightarrow{e}_\alpha) \qquad (X \in F, i, j, k = 1, ..., p; \qquad \alpha = p+1, ..., n)$$

is moving frame on F, where $\overrightarrow{e}_i \in T_p(X)$ - tangent vector space of $F, \overrightarrow{e}_\alpha \in N_{n-p}(X)$ - orthogonally complementary vector space to $T_p(X)$. Derivation formulas of the frame $\Re$ are following:

$$d\overrightarrow{x} = \omega^i \overrightarrow{e}_i, \qquad d\overrightarrow{e}_i = \omega_i^j \overrightarrow{e}_j + \omega_i^\alpha \overrightarrow{e}_\alpha, \qquad d\overrightarrow{e}_\alpha = \omega_\alpha^i \overrightarrow{e}_i + \omega_\alpha^\beta \overrightarrow{e}_\beta$$

differential forms ω^i, ω_i^j, ω_i^α are satisfied structure equations of Euclidean space.

It is defined the differential mapping $f : F \to F'$ so that

$$f(X) = Y \in F',\ Y = F' \cap (X, \overrightarrow{e}_{p+1}).\Re' = (Y, \overrightarrow{a}_i,\ \overrightarrow{a}_\alpha)$$

- moving frame on F', where $\overrightarrow{a}_i = p_i^j \overrightarrow{e}_j + p_i^\beta \overrightarrow{e}_\beta,\ \overrightarrow{a}_\alpha = \overrightarrow{e}_\alpha$.

Derivation formulas of the frame $\Re'$ are following:

$$d\overrightarrow{y} = \omega^j \overrightarrow{a}_j,\ d\overrightarrow{a}_i = \overline{\omega}_i^j \overrightarrow{a}_j + \overline{\omega}_i^\alpha \overrightarrow{a}_\alpha,\ d\overrightarrow{a}_\alpha = d\overrightarrow{e}_\alpha = \overline{\omega}_\alpha^i \overrightarrow{a}_i + \overline{\omega}_\alpha^\beta \overrightarrow{a}_\beta.$$

It is find the connection between differential form ω_i^j, ω_i^α and $\overline{\omega}_i^j$, $\overline{\omega}_i^\alpha$:

$$\overline{\omega}_i^j = \widetilde{p}_k^j (dp_i^k + p_i^l \omega_l^k + p_i^\alpha \omega_\alpha^k),$$

$$\overline{\omega}_i^j = dp_i^\alpha + p_i^\ell \omega_\ell^\alpha + p_i^\beta \omega_\beta^\alpha - p_\ell^\alpha \widetilde{p}_k^\ell (dp_i^k + p_i^\ell \omega_\ell^k + p_i^\beta \omega_\beta^k),$$

$$p_i^j \widetilde{p}_k^i = \delta_k^j.$$

The necessary and sufficient conditions of geodesic [1] of the image of given net $\sum_p \subset F$ are proved.

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ON THE FOURIER-WALSH COEFFITIENTS OF THE CONTINUOUS FUNCTIONS OF TWO VERIABLES

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In the work the conditions are given for Fourier-Walsh coefficients of the continous functions. Let $w_k(x)$ be a Walsh system by enumeration Paley, $w_{m,n} = w_m(x)w_n(y)$. Denote by $\hat{f}(k_1, k_2)$ the Fourier-Walsh coefficients of f.

Theorem 1. *Let f be continuous function on $[0,1]^2$ and*

$$\lim_{\substack{p\to\infty \\ r\to\infty}} 2^{p+r} \sum_{\nu=p}^{\infty} \sum_{l=r}^{\infty} R_{\nu l} = 0,$$

where

$$R_{\nu l} = \sum_{k_1=2^\nu}^{2^{\nu+1}-2} \sum_{k_2=2^l}^{2^{l+1}-2} |f(k_1, k_2) - (k_1+1, k_2+1)| = 0.$$

Then $f(u,\nu) = const$ on $[0,1]^2$.

Corollary 1.*Let f be continuous function on $[0,1]^2$ and*

$$\lim_{\substack{\nu\to\infty \\ r\to\infty}} 2^{\nu+r} \sum_{k_1=2^\nu}^{\infty} \sum_{k_2=2^r}^{\infty} |f(k_1, k_2) - f(k_1+1, k_2+1)| = 0.$$

Then $f(u,\nu) = const$ on $[0,1]^2$.

Theorem 2.*Let f be continuous function on $[0,1]^2$ and for Fourier-Walsh coefficients the conditions*

$$f(2^{k_1}; 2^{k_2}) \geq f(2^{k_1}+1; 2^{k_2}+1) \geq ... \geq f(2^{k_1}-1; 2^{k_2}-1)$$

are satisfied $k_1, k_2 \in N$ and

$$\lim_{\substack{\nu\to\infty \\ r\to\infty}} 2^{\nu+r} \sum_{k=2^\nu}^{\infty} \sum_{l=2^r}^{\infty} |f(k_1, k_2) - f(k_1+1, k_2+1)| = 0.$$

Then $f(u,\nu) = const$ on $[0,1]^2$.

Corollary 2.*Let f be continuous function on $[0,1]^2$ and $\hat{f}(n_1; n_2) \geq \hat{f}(k_1; k_2)$, $\forall n_1 \geq k_1$, $\forall n_2 \geq k_2$ and*

$$\lim_{\substack{p\to\infty \\ r\to\infty}} 2^{k+r} \hat{f}\left(2^k, 2^r\right) = 0.$$

Then $f(u,\nu) = const$ on $[0,1]^2$

In the case of one variable the similar results were proved in [1].

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ON THE INTEGRABILITY WITH WEIGHT OF DOUBLE TRIGONOMETRIC SERIES WITH COEFFICIENTS FROM THE $R_0^+BVS^2$

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In the present work we solve the following problem: to find necessary and sufficient conditions for the pth-power integrabality of the sums of double sine and cosine series with weight γ. We consider the following series:

$$g(x,y)=\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\lambda_{jk}\sin jx\sin ky,\ \ f(x,y)=\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\lambda_{jk}\cos jx\cos ky,$$

Definition. *The zero sequence of positive numbers* $\{a_{jk}\}$ *belongs* $R_0^+BVS^2$, if $a_{jk}\to 0$ by $j+k\to\infty$ *and*

$$\sum_{j=m}^{\infty}\sum_{k=n}^{\infty}|\Delta_{11}a_{jk}|\le Ca_{mn},\ \ m,n\in N;\sum_{j=m}^{\infty}|\Delta_{10}a_{jk}|\le Ca_{mk}\ \forall k\in N;\sum_{k=n}^{\infty}|\Delta_{01}a_{jk}|\le Ca_{jn}\ \forall j\in N;$$

where $\Delta_{11}a_{jk}=a_{jk}-a_{j+1,k}-a_{j,k+1}+a_{j+1,k+1},\ \Delta_{10}a_{jk}=a_{jk}-a_{j+1,k},\Delta_{01}a_{jk}=a_{jk}-a_{j,k+1}$.

Let $\gamma:=\{\gamma_{mn}\}$ a sequence of positive numbers. We define the function $\gamma(x,y)$ by the sequence $\{\gamma_{mn}\}$ as follows: $\gamma\left(\frac{\pi}{m},\frac{\pi}{n}\right)=\gamma_{mn}\ \forall m,n\in N$, and there exist positive constants A and B such that

$$A\gamma_{mn}\le\gamma(x,y)\le B\gamma_{m+1,n+1}\ \forall x\in\left(\frac{\pi}{m+1},\frac{\pi}{m}\right)y\in\left(\frac{\pi}{n+1},\frac{\pi}{n}\right).$$

Theorem. *Suppose that* $\{\lambda_{jk}\}R_0^+BVS^2$ *and* $1\le p<\infty$.

A) *If the sequence* $\{\gamma_{jk}\}$ *satisfies the conditions: there exist* $\varepsilon_1,\varepsilon_2>0$ *such that the sequence* $\{\gamma_{jk}\cdot j^{-1+\varepsilon_1}\}$ *is almost decreasing for any* k, *and the sequence* $\{\gamma_{jk}\cdot k^{-1+\varepsilon_2}\}$ *is almost decreasing for any* j, *then the condition*

$$\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\gamma_{jk}\cdot(jk)^{p-2}\lambda_{jk}^{p}<\infty \tag{1}$$

s sufficient for the validity of the condition

$$\gamma(x,y)|g(x,y)|^p\in L(0,\pi)^2. \tag{2}$$

B) *If the sequence* $\{\gamma_{jk}\}$ *satisfies the conditions: there exist* $\varepsilon_3,\varepsilon_4>0$ *such that the sequence* $\{\gamma_{jk}\cdot j^{p-1-\varepsilon_3}\}$ *is almost increasing for any* k *, and the sequence* $\{\gamma_{jk}\cdot k^{p-1-\varepsilon_4}\}$ *is almost increasing for any* m *, then condition (1) is necessary for the validity of condition (2).*

The similar theorem is true and for double cosine series, but with an additional condition

$$\sum_{j=m}^{\infty}\sum_{k=n}^{\infty}\left|\frac{\lambda_{jk}}{jk}-\frac{\lambda_{j+1,k}}{j+1,k}\right|\le\frac{C\lambda_{mn}}{mn}.$$

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NONSMOOTH IMPLICIT FUNCTIONS AND ITS PROPERTIES[1]

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Our aim is to study the problem on the existence of nonsmooth implicit functions and their properties for some types of nonsmooth functions. A new tool of nonsmooth analysis - upper and lower exhausters - is employed. The notion of exhausters was introduced by V. Demyanov (1999) and closely related to exhaustive families of upper convex and lower concave approximations proposed by V. Demyanov and A. Rubinov (1982).

Let $f_i(x, y)$ $(i \in 1 : n)$ be continuous on $S = S_1 \times S_2 \subset \mathbb{R}^m \times \mathbb{R}^n$, where $S_1 \subset \mathbb{R}^m$ and $S_2 \subset \mathbb{R}^n$ are open sets. Put $f = (f_1, \ldots, f_n)$.

Consider the system

$$f_i(x, y) = 0 \quad \forall\, i \in 1 : n. \tag{1}$$

In the nonsmooth case it makes sense to introduce a directional implicit function. Fix a direction $g \in \mathbb{R}^m$, $g \neq 0$, and consider the system

$$f_i(x_0 + g, y) = 0 \quad \forall\, i \in 1 : n. \tag{2}$$

We say that there exists an implicit function in the direction g if $\alpha_0 > 0$ and a vector function $y(\alpha)$ given on $[0, \alpha_0]$ exists such that

$$y(\alpha) y_0, \quad f(x_0 + \alpha g, y(\alpha)) = 0_n \quad \forall\, \alpha \in [0, \alpha_0]. \tag{3}$$

Assume that all functions $f_i(z)$ are directionally differentiatable at a point $z_0 = [x_0, y_0]$ and directional derivative $\tilde{h}_i(\eta) = f_i'(z_0, \eta)$, where $\eta = [g, q] \in \mathbb{R}^{m+n}$ continuous as function of η and bounded from above. Then from (3) the following expansions hold

$$f_i(z_0 + \alpha\eta) = f_i(z_0) + \alpha \tilde{h}_i(\eta) + o_{\eta i}(\alpha), \tag{4}$$

where

$$\tilde{h}_i(\eta) = \min_{\tilde{C}_i \in \tilde{E}_i^*} \max_{v \in \tilde{C}_i} (v, \eta), \quad \frac{o_{\eta i}(\alpha)}{\alpha} \underset{\alpha \downarrow 0}{\longrightarrow} 0 \quad \forall\, \eta \in \mathbb{R}^{m+n}, \forall\, i \in 1 : n, \tag{5}$$

$$\tilde{E}_i^* \text{ is an upper exhauster of function } \tilde{h}_i. \tag{6}$$

Note that exhausters exist for any directionally differentiable function whose directional derivatives are continuous as functions of direction. For non-Lipschitz quasidifferentiable functions differentiable in directions it is sometimes possible to find an implicit function with the help of exhausters. However, even in the cases when functions are Lipschitz or quasidifferentiable it is easier to conduct a research with the help of exhausters since an exhauster can turn out to be "'smaller"' than Clark's subdifferential and quasidifferential.

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ON A GENERALIZATION OF THE FREUDENTHAL'S THEOREM

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In this paper for superparacompact complete metrizable spaces the Freudenthal's theorem for compact irreducible standard polyhedric representation is generalized. Furthermore, for superparacompact metric spaces are reinforced: 1) the Morita's theorem about universality of the product $Q^\infty \times B(\tau)$ of Hilbert cube Q^∞ to generalized Baire space $B(\tau)$ of the weight τ in the space of all strongly metrizable spaces of weight $\leq \tau$; 2) the Nagata's theorem about universality of the product $\Phi^n \times B(\tau)$ of universal n- dimensional compact Φ^n to $B(\tau)$ in the space of all strongly metrizable spaces $\leq \tau$ and dimension $dimX \leq n$.

Theorem 1. *Any n-dimensional complete metric superparacompact [1] space X is limit of inverse sequence $S = \left\{ \widetilde{K}_i, \pi_i^{i+1} \right\}$, $i = 1, 2, \ldots$, from n-dimensional polyhedrons $\widetilde{K}_i$, being bodies of standard triangulation [2] K_i decomposing to discrete sum of compact polyhedrons; in addition projections π_i^{i+1} are simplicial [2] with respect to K_{i+1} and some triangulation $K_{\underset{\sim}{i}}^*$ of the polyhedron $\widetilde{K}_i$, being subdivision [2] of the triangulation K_i. Every projection $\pi_i : X \to \widetilde{K}_i$ is irreducible [2] with respect to triangulation K_i, $i = 1, 2, \ldots$.*

This theorem is generalization of the Freudenthal's theorem [3].

Theorem 2. *For metrizable space X following statements are equivalent: a) X is superparacompact complete metrizable space of weight $\leq \tau$; b) X is perfectly mapping into Baire space $B(\tau)$ of the weight τ; c) X is closed included into product $B(\tau) \times Q^\infty$ of Baire space $B(\tau)$ of the weight τ on Hilbert cub Q^∞.*

We note, that theorem 2 is extension of the theorem Morita [4] about universality of the product $B(\tau) \times Q^\infty$ in the class of all strongly metrizable space of the weight $\leq \tau$.

Theorem 3. *For Hausdorff space X following statement are equivalent: a) X is superparacompact (complete) metrizable space of the weight $\leq \tau$ and $dimX \leq n$; b) X is closed imbedded into product (Baire space $B(\tau)$ of the weight τ) of 0-dimensional in the sense dim of metrizable space of the weight τ onto universal n-dimensional compact Φ^n.*

Theorem 3 is expansion of the Nagata's theorem [5] about embedding n-dimensional strongly metrizable space in $B(\tau) \times \Phi^n$ to the case superparacomact.

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THE THEOREM ON COMPACTNESS OF THE EMBEDDING OPERATORS IN WEIGHT CLASSES OF THE ABSTRACT FUNCTIONS DEFINED IN DOMAIN

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The weight abstract anisotropic space $W^l_{p,g}(\Omega;\mathrm{E}_0,\mathrm{E})$ arises occurs by the investigation of the boundary problems for the partial derivative differential-operational equations. From this viewpoint there is a necessity of study of smoothness of the mixed derivative functions from space $W^l_{p,g}(\Omega;\mathrm{E}_0,\mathrm{E})$ depending as on differential, weight properties and also on the degree of summation p, and on the smoothness of interpolation space $[\mathrm{E}_0,\mathrm{E}]_\theta, 0\le\theta\le 1$.

In this work the theorem of compactness of embedding operators in the sence of theory of interpolation is proved in the case =H $\mathrm{E}_0=\mathrm{H}_0$, where H_0 and H are Hilbert spaces.

Let $g_i(x) i=\overline{0,n}$ be positive measurable functions in any domain $\Omega\subset R^n$. Let then H_0 and H be Hilbert spaces, H_0 be maps into H continuously and densely. Through $[\mathrm{H}_0,\mathrm{H}]_\theta$,$0<\theta\le 1$ we will designate the interpolation spaces between H_0 and H. $\Omega_N=\Omega\backslash\Omega^N$, where $\Omega^N=\{x;x\in\Omega,|x|\le N,N>0\}$.

Let $Q_y=\left\{x;x\in\Omega^N,|x_i-y_i|\le g_i(y),i=\overline{1,n}\right\}$ and also there is a constant $A>0$ such, that $A^{-1}g_i(y)\le g_i(x)\le c\cdot g_i(y)$,$\forall x\in Q_y$,$i=\overline{0,n}$.

From the corresponding reasonings, which was leaded at the proving the theorem on the continuity of the embedding and from the definition of interpolation spaces the following theorem is proved.

Theorem. *Let* $g_i(x)$ *satisfies* $\int\limits_\Omega g_i^{\frac{-1}{p-1}}(x)\,dx<\infty$, $\int\limits_\Omega g_o(x)\,g_j^{-1}(x)\,]^{1/p}\,dx<\infty$, $i=\overline{0,n}$, $k=\overline{1,n}$, $1<p<\infty$. *Then by* $|m:l|+\mu<1$ *the embedding* $D^m W^l_{p,g,g_0}(\Omega;\mathrm{H}_0,\mathrm{H})\subset L_{p,g_m}\left(\Omega;[\mathrm{H}_0,\mathrm{H}]_{|m:l|+\mu}\right)$ *is compact.*

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ON TWO NEW TYPES OF CONVERGENCE OF DENSITIES

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Let g be a summable function on R and consider the family $\{\lambda g(\lambda t) : \lambda > 0\}$. It is well known that, as $\lambda \to \infty$, this family converges to a point mass at zero: if ϕ is continuous and bounded on R, then

$$\int_R \phi(t)\lambda g(\lambda t)dt \to \phi(0) \int_R g(t)dt, \ \lambda \to \infty. \tag{1}$$

(A more general case $\int_R \phi(t+x)\lambda g(\lambda t)dt$ reduces to this one by way of a linear transformation.) We consider the question of what happens in the other limit case, when $\lambda \to 0$. This question arises in testing hypotheses of autocorrelation in linear regression models. We show that the answer involves the generalized mean of ϕ over R defined by

$$M\phi = \lim_{r\to\infty} \frac{1}{2r} \int_{-r}^{r} \phi(t)dt. \tag{2}$$

The counterpart of (1) happens to be

$$\int_R \phi(t)\lambda g(\lambda t)dt \to M\phi \int_R g(t)dt, \ \lambda \to 0. \tag{3}$$

In fact, applications require a more complex result, when the function g has many arguments and the above transformation with $\lambda \to 0$ is applied with respect to some of them. For simplicity, suppose that $n = 2$. Then the family to consider is $\{\lambda g(\lambda y_1, y_2) : \lambda > 0\}$ and (3) can be used to prove

$$\int_R [\lambda \int_R \phi(y_1, y_2) g(\lambda y_1, y_2) dy_1] dy_2 \to \int_R [(M_1\phi)(t_2) \int_R g(t_1, t_2)dt_1]dt_2, \tag{4}$$

where ϕ is bounded and continuous on R^2 and $M_1\phi$ denotes the result of application of (2) to ϕ with respect to the first argument. The right sides of (3) and (4) determine distributions supported at infinity because, as one can show, they vanish on all continuous functions with compact support. We prove general (multidimensional) versions of (3) and (4) for g summable and spherically symmetric. Such results allow us to show that the format of the main statements from [1] and [2] is wrong if probability content is used for measuring the rejection region.

We also introduce a different type of convergence, in which a set of points where the values of the density are close to the density maximum is used to measure the rejection region. Under this other type of convergence the format of Martellosio's statement becomes correct.

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THE BASICITY PROPERTY OF SINES AND COSINES SYSTEMS IN WEIGHT SPACES

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Let $L^0_{p,\omega} \equiv L_{p,\omega}(0,\pi)$, $\omega(t)$ be a weight function of the form

$$\omega(t) \equiv \prod_{k=0}^{r} |t-\tau_k|^{p\alpha_k}, \tag{1}$$

where $0 = \tau_0 < \tau_1 < ... < \tau_r = \pi$, $\{\alpha_k\}_0^r \subset R$.

Let the inegualities

$$-1-\frac{1}{p} < \alpha_0 < 2-\frac{1}{p},\ -1-\frac{1}{p} < \alpha_r < 2-\frac{1}{p}, -\frac{1}{p} < \ \alpha_k < \frac{1}{q}\ ,\ k=\overline{0,r-1}, \tag{2}$$

be fulfilled. Then the following statements are valid.

Statement 1. *Let the weight function $\omega(t)$ be defined by expression (1). The system of sines $\{\sin nt\}_{n\in N}$ is minimal in $L^0_{p,\omega}$, if the inequalities (2) are fulfilled. It is complete in $L^0_{p,\omega}$, if the inequalities*

$$\alpha_k > -\frac{1}{p}\ ,\ k=\overline{0,r}\ , \tag{3}$$

are fulfilled. Moreover, it forms a basis in $L^0_{p,\omega}$, if the inequalities

$$-\frac{1}{p} < \alpha_k < \frac{1}{q}\ ,\ k=\overline{0,m}. \tag{4}$$

hold.

Statement 2. *Let the weight function $\omega(t)$ be defined by expression (1). The system of cosines $1 \cup \{\cos nt\}_{n\in N}$ is minimal (forms a basis) in $L^0_{p,\omega}$, if the inequalities (4) are fulfilled. It is complete in $L^0_{p,\omega}$, if the inequalities (3) holds.*

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ON EXISTENCE OF z_u ULTRAFILTERS

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The set $C^*(uX)$ of all uniformly continuous bounded functions of uniform space uX ([1], [2], [3]) with respect of point wise addition and multiplication form a commutative ring with unit, where for unit stands a function which maps all elements in $1 \in \mathbb{R}$ and ring operations are defined as following: $(f+g)(x) = f(x) + g(x)$ and $(fg)(x) = f(x)g(x)$ for any $f, g \in C^*(uX)$, $f(x) \equiv 1$ and $x \in X$.

The set $f^{-1}(0) \subseteq X$ for $f \in C^*(uX)$ is called uniformly zero set and is denoted as $\boldsymbol{Z}(f) = f^{-1}(0) = \{x \in X : f(x) = 0\}$. By $\mathfrak{Z}\,(uX)$ we denote the set of all uniformly zero sets. We can consider $\boldsymbol{Z}$ as mapping of the ring $C^*(uX)$ onto the of all uniform zero sets of uniform spaceuX. Thus, $\boldsymbol{Z} : C^*(uX) \rightarrow \mathfrak{Z}(uX)$ and $\boldsymbol{Z}(f) \in \mathfrak{Z}(uX)$ for any $f \in C^*(uX)$.

Ideal of the ring $C^*(uX)$ is called a proper subring I of the ring $C^*(uX)$ with the following property: if $f \in I$ and $g \in C^*(uX)$, then $gf \in I$. Any ideal is contained by inclusion in some maximal ideal.

Definition 1. *Nonempty collection $\mathcal{F} \subseteq \mathfrak{Z}\,(uX)$ is called as z_u - filter on uX provided the following are fulfilled:*

1^0. $\emptyset \notin \mathcal{F}$;
2^0. *if* $Z_1, Z_2 \in \mathcal{F}$ *then* $Z_1 \cap Z_2 \in \mathcal{F}$;
3^0. *if* $Z \in \mathcal{F}$, $Z' \in \mathfrak{Z}\,(uX)$ *and* $Z \subset Z'$, *then* $Z' \in \mathcal{F}$.

Maximal z_u- filter is called *z_u-ultrafilter.* Thus, z_u- ultrafilter is a maximal subfamily of $\mathfrak{Z}\,(uX)$ possessed centered property.

Theorem 1. *If I is an ideal in $C^*\,(uX)$, then the family $\mathbf{Z}\,(I) = \{\mathbf{Z}\,(f) : f \in I\}$ is z_u-filter on uX.Conversely, of $\mathcal{F}$ is z_u-filter on uX, then $\mathbf{Z}^{-1}\,(\mathcal{F}) = \{f : \mathbf{Z}\,(f) \in \mathcal{F}\}$ is ideal in $C^*(uX)$.*

Theorem 2. *Each z_u-centered family in $\mathfrak{Z}\,(uX)$ is contained is some z_u-ultrafilter.*

Theorem 3. *(a) if M is a maximal ideal in $C^*(uX)$, then $\mathbf{Z}\,(M)$ is z_u-ultrafilter in uX.*
(b) If $\mathcal{F}$ is z_u-ultrafilter on uX, then $Z^{-1}\,(\mathcal{F})$ is a maximal ideal in $C^*(uX)$.

Corollary 1. *A mapping $Z : C^*(uX) \rightarrow \mathfrak{Z}\,(uX)$ is a bijection between the set of all maximal ideals of the ring $C^*(uX)$ and the set of all z_u-urltafilters in uX.*

If follows from Kuratovskii- Zorn' principle ([1]).

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TO THE PROBLEM OF DETERMINATION OF "INNER PROPERTIES" OF GRIDS FOR EFFECTIVE RECONSTRUCTION

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Let a function f describes *some process* in a domain Ω. Let also in nodes ξ_1, ..., ξ_N from Ω be devices. The question arises whether it is possible to describe global properties of f by inner properties of this finite sequence (grid) ξ_1, ..., ξ_N and values of f on this grid $f(\xi_1), ..., f(\xi_N)$.

Let us give some global properties necessary to be described: this is the problem of approximate calculation of an integral, then the problem of approximate representation of a function on entire set of, and finally discretization of solutions of the partial differential equations with initial and boundary conditions. A closer examination of the problem immediately shows the need for many a priori conditions.

Of course, to start learning about the general problem posed in the most natural is the special theory - the theory of uniform distribution of points on the multidimensional unit cube dedicated to quantifying *inner properties* of the complex from a specified number of points.

Following results are obtained:

- In terms of the behavior of nodes and weights of quadrature formulas written out a criterion for its effectiveness in the class of Korobov E_s^r.
- Two-side estimate to within constant, the rate of decrease of the discrepancy of the Smolyak's grid is obtained.
- It was established that the rate of uniform distribution grid on the unit cube by itself does not characterize the effectiveness of the corresponding quadrature formula with the same grid and arbitrary weights on the the class of Korobov E_s^r. As it turned out, also plays a significant role "structure " of the grid.
- We write the conditions on the grid with rational coordinates, which do not exclude an effective remedy for the class of of integrals Korobov E_s^r.

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RECONSTRUCTION PROBLEM IN CLASSES OF INFINITELY SMOOTH FUNCTIONS

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The report focuses on the problem of sample solutions of the heat equation for from the values of the distribution function of the initial temperature of $U_2((0,0),(\theta_1,\theta_2),(1,1))$ - classes Ul'yanov [1] (in [2] this problem was solved for the case of trigonometric Fourier coefficients).

The following fact is valid ($\overline{m_j} = \max\{1,|m_j|\}$)

Theorem. *Let a* $0<\theta_1,\,\theta_2<1,\ 2\le p\le\infty,$ *– a positive integer and* $V_T \equiv \left\{(n_1,n_2)\in Z^2:\ \bar{n}_1 + c\bar{n}_2 < T\right\}$. *Then the relations* ($N=\frac{2T^2}{c}$, *where* $c=\log_{\theta_1}\theta_2$)

a)

$$\sup_{f\in U_2((0,0),\,(\theta_1,\theta_2),\,(1,1))}\left\|u(t,(x_1,x_2);f)-\sum_{(n_1,n_2)\in V_T}\Lambda_{(T,(n_1,n_2))}(f)\,e^{2\pi i(n_1x_1+n_2x_2)}e^{-4\pi^2(n_1^2+n_2^2)t}\right\|_{L^\infty[0;\infty)\times[0;1]^2}\succ\prec$$

$$\succ\prec N^{\frac12}\theta_1^{\sqrt{\frac12 N}\log_{\theta_1}\theta_2},$$

b)

$$\sup_{f\in U_2((0,0),\,(\theta_1,\theta_2),\,(1,1))}\left\|u(t,(x_1,x_2);f)-\sum_{(n_1,n_2)\in V_T}\Lambda_{(T,(n_1,n_2))}(f)\,e^{2\pi i(n_1x_1+n_2x_2)}e^{-4\pi^2(n_1^2+n_2^2)t}\right\|_{L^2[0;\infty)\times[0;1]^2}\succ\prec$$

$$\succ\prec N^{\frac14}\theta_1^{\sqrt{\frac12 N}\log_{\theta_1}\theta_2},$$

$$\Lambda_{(T,(n_1,n_2))}(f)=\frac{c}{2T^2}\sum_{|k_1|+c|k_2|\le 2T-1}f\left(\frac{k_1-k_2}{2T},c\frac{k_1+k_2}{2T}\right)e^{-2\pi i\left(n_1\frac{k_1-k_2}{2T}+n_2c\frac{k_1+k_2}{2T}\right)}.$$

Note that if [3] involved in the grid, constructed for classes of functions whose Fourier coefficients decrease at a rate no higher power, there is an explicit statement issued restoring the "algebraic" grids with equal weights that are optimal for classes of functions with exponential speed decrease of Fourier coefficients.

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NORM INEQUALITIES FOR THE CONVOLUTION OPERATOR[1]

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Let (Ω, μ) be a measurable space and $L_p(\Omega, \mu)$ be the collection of all those measurable functions f satisfying $\|f\|_{L_p(\Omega,\mu)} = \left(\int_\Omega |f(x)|^p \, d\mu\right)^{\frac{1}{p}} < \infty$. The distribution of a measurable function f on Ω is defined by

$$m(\sigma, f) = \mu\{x \in \Omega : |f(x)| > \sigma\}.$$

Then $f^*(t) = \inf\{\sigma : m(\sigma, f) \le t\}$ is the decreasing rearrangement of f.

Let $0 < p < \infty$ and $0 < q \le \infty$. The Lorentz space $L_{p,q}(\Omega, \mu)$ is defined by those measurable functions f such that

$$\|f\|_{L_{p,q}} = \left(\int_0^\infty (t^{1/p} f^*(t))^q \frac{dt}{t}\right)^{1/q} < \infty,$$

when $0 < q < \infty$ and

$$\|f\|_{L_{p,\infty}} = \sup_t \; t^{1/p} f^*(t) < \infty,$$

then $q = \infty$. We also define

$$f^{**}(x) = \frac{1}{x} \int_0^x f^*(t) dt.$$

In this paper we study norm estimates for the convolution operator

$$(Af)(x) = (K * f)(x) = \int_\Omega K(x - y) f(y) dy \tag{1}$$

in the Lebesgue and Lorentz spaces.

We study norm convolution inequalities in Lebesgue and Lorentz spaces. First, we improve the well-known O'Neil's inequality for the convolution operator and prove corresponding estimate from below. Second, we obtain Young-O'Neil-type estimate in the Lorentz spaces for the limit value parameters, i.e., $\|K * f\|_{L(p,h_1) \to L(p,h_2)}$. Finally, similar estimates in the weighted Lorentz spaces are presented.

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ON EQUIVALENT CONE METRIC SPACES[1]

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Given a nonempty set X and a real Banach space E, if the mapping $d : X \times X \to E$ satisfies the metric axioms with respect to the ordering on a cone P of E, then the pair (X, d) is called a *cone metric space*. In this talk, we introduce the concept of equivalent cone metrics on the same cone, and explore the necessary conditions for the two cone metrics to be equivalent. We also present an alternative definition for the equivalence of cone metrics, which is called the Lipschitz equivalence. Finally, we compare these two definitions.

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ON ESTIMATION OF THE CORRELATION FUNCTION OF A HARMONIC RANDOM FIELD

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Let $\xi(P)$ be real -valued Gaussian harmonic random field on the unit sphere in Euclidean space which has second order moment and is continuous in quadratic mean, $E\xi = 0$. The random field $\xi(P)$ is said to isotropic if its first and second moment are invariant under group of rotation in Euclidean space. Suppose that we have T - independent realizations of the random fields. In this paper we estimate of the correlation function first and second-order moments.

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EXTENSION OF INDEX THEORY TO ARBITRARY REAL FACTORS

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Let $R \subset B(H)$ be a finite complex or real factor with the finite commutant R'. The *coupling constant* $\dim_R(H)$ of R is defined as $\mathrm{tr}_R(E_\xi^{R'})/\mathrm{tr}_{R'}(E_\xi^R)$, where ξ is a non-zero vector in H, tr_A denotes the normalized trace and E_ξ^A is the projection of H onto the closure of the subspace $A\xi$. The number $\dim_R(H)$ does not depend on ξ.

It is known, that if $M \subset B(H) = B(H_r) + iB(H_r)$ is a finite factor and $(M, \alpha) \subset B(H_r)$, where H_r is a real Hilbert space with $H_r + iH_r = H$, α is the involutive *-antiautomorphism of M and $(M, \alpha) = \{x \in M : \alpha(x) = x^*\}$, then $\dim_M(H) = \dim_{(M,\alpha)}(H_r) = \frac{1}{2}\dim_{(M,\alpha)}(H)$. Consider a subfactor $N \subset M$ such that $\alpha(N) \subset N$. The *index* of N in M, denoted by $[M : N]$ is defined as $\dim_N(L^2(M))$, where $L^2(M)$ the completion of M with respect to the norm $\|x\|_2 = \tau(x^*x)^{1/2}$. Similarly, the *index* of (N, α) in (M, α), denoted by $[(M, \alpha) : (N, \alpha)]$, or by $[R : Q]$, is defined as $\dim_{(N,\alpha)}(L^2(M, \alpha))$. Between real and complex indices there is the relation: $[(M, \alpha) : (N, \alpha)] = [M : N]$. Considering a complex factor M as a real W*-algebra in view of above we may put $[M : (M, \alpha)] = 2[(M, \alpha) : (M, \alpha)] = 2$.

For example, if M is a factor of type I_4, then up to isomorphisms it has seven real W*-subalgebras different from M, which are real or complex subfactors of M: $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, $M_2(\mathbb{R})$, $M_2(\mathbb{C})$, $M_2(\mathbb{H})$ and $M_4(\mathbb{R})$, where $\mathbb{H}$ is the quaternion algebra. The values of the indexes are respectively: $[M : M_4(\mathbb{R})] = [M : M_2(\mathbb{H})] = 2$, $[[M : M_2(\mathbb{C})] = [M_4(\mathbb{R}) : M_2(\mathbb{R})] = [M_2(\mathbb{H}) : \mathbb{H}] = 4$, $[M : M_2(\mathbb{R})] = 8$, $[M : \mathbb{C}] = [M_4(\mathbb{R}) : \mathbb{R}] = [M_2(\mathbb{H}) : \mathbb{R}] = 16$. $[M : \mathbb{R}] = 32$.

We have calculated the value of the index in the above example. It turns out that the index may be calculated also in the general case:

Theorem 1. *Let R be a finite complex or real factor, and let N be a subfactor of R with $[R : N] < \infty$. Then one has either $[R : N] = 4\cos^2\dfrac{\pi}{q}$ for some integer $q \geq 3$ or $[R : N] \geq 4$.*

Let now M be a σ-finite factor and let N be a subfactor of M with $\alpha(N) \subset N$. We fix a normal conditional expectation E from (M, α) onto (N, α). Then there exists an operator-valued weight E^{-1} from $(N, \alpha)'$ onto $(M, \alpha)'$ and it is easy to see that $E^{-1}(\mathbf{1})$ is a scalar (possibly $+\infty$). The index of $Q = (N, \alpha)$ in $R = (M, \alpha)$, denoted by $[R : Q]$ or by $[(M, \alpha) : (N, \alpha)]$, is defined as the scalar $E^{-1}(\mathbf{1})$. The value of the index also is: either $[(M, \alpha) : (N, \alpha)] = 4\cos^2\frac{\pi}{q}$ for some integer $q \geq 3$ or $[(M, \alpha) : (N, \alpha)] \geq 4$.

ON SHARP INEQUALITIES FOR TRIGONOMETRIC APPROXIMATION IN WEIGHTED ORLICZ SPACES

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In the present work exact (in the sense of order) direct and converse theorems of trigonometric approximation are proved in Orlicz spaces with weights satisfying some Muckenhoupt's A_p condition. As a consequence sharp Marchaud and its converse inequalities are obtained. Let Φ be the class of strictly increasing functions $\phi : [0,\infty) \to [0,\infty)$ satisfying $\phi(\infty) \lim_{x\to\infty} \phi(x) = \infty$. Let $-\infty < p \leq q < \infty$. By $Y[p,q]$ we denote the class of even functions $\varphi \in \Phi$ defined on $[0,\infty)$ satisfying (i) $\varphi(u)/u^p$ is non-decreasing as $|u|$ increases; (ii) $\varphi(u)/u^q$ is non-increasing as $|u|$ increases. If $p < q$ we denote by $Y\langle p,q\rangle$ the class of functions φ satisfying $\varphi \in Y[p+\varepsilon, q-\delta]$ for some small numbers $\varepsilon, \delta > 0$. By $\bar{\Phi}_p$ we will denote class of functions M such that M belongs to the class $Y\langle p,q\rangle$ for some $1 < p \leq q < \infty$. A function M of class Φ satisfying $M(0) = 0$ is said to be *quasiconvex* if there exist a convex function ϕ and a constant $c \geq 1$ such that $\phi(x) \leq M(x) \leq \phi(cx)$ for every $x \geq 0$. We set $E_n(f)_{M,\omega} := \inf\left\{\|f-T\|_{M,\omega} : T \in \mathcal{T}_n\right\}$ for $f \in L_{M,\omega}(T)$, where $\mathcal{T}_n$ is the class of trigonometric polynomials of degree not greater than n. The following unimprovable inequalities of trigonometric approximation are true:

Theorem. *Let $M \in \bar{\Phi}_p$, $1 < p < q < \infty$, ω belong to Muckenhoupt class A_p, f belong to the weighted Orlicz space $L_{M,\omega}(T)$, $\beta := \max(2, q-\delta)$ and $\gamma := \min(2, p+\varepsilon)$.*

(1) *If $n \in N$ and $r \in R^+$, then there is a positive constant c depending only on r and M such that*

$$\frac{c}{n^{2r}} \left\{ \sum_{\nu=1}^{n} \nu^{2\beta r-1} E_\nu^\beta(f)_{M,\omega} \right\}^{1/\beta} \leq \Omega_r\left(f, \frac{1}{n}\right)_{M,\omega}$$

holds.

(2) *If $M(\sqrt{x})$ is quasiconvex, then there is a positive constant C depending only on r and M such that*

$$\Omega_r\left(f, \frac{1}{n}\right)_{M,\omega} \leq \frac{C}{n^{2r}} \left\{ \sum_{\nu=1}^{n} \nu^{2\gamma r-1} E_\nu^\gamma(f)_{M,\omega} \right\}^{1/\gamma}$$

holds.

Here $\Omega_r(f,\delta)_{M,\omega}$ is the fractional order mixed moduli of smoothness which is suitable for weighted function spaces.

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FINITENESS OF THE DISCRETE SPECTRUM OF A OPERATOR MATRIX[1]

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Let $\mathbf{T}^3$ be the three-dimensional torus, $\mathbf{C}$ be the field of complex numbers, $L_2(\mathbf{T}^3)$ be the Hilbert space of square-integrable (complex) functions defined on $\mathbf{T}^3$ and $L_2^{\rm s}((\mathbf{T}^3)^2)$ be the Hilbert space of square-integrable symmetric (complex) functions defined on $(\mathbf{T}^3)^2$.

Denote by $\mathcal{H}$ the direct sum of spaces $\mathcal{H}_0=\mathbf{C}$, $\mathcal{H}_1=L_2(\mathbf{T}^3)$ and $\mathcal{H}_2=L_2^{\rm s}((\mathbf{T}^3)^2)$, i.. $\mathcal{H}=\mathcal{H}_0\oplus\mathcal{H}_1\oplus\mathcal{H}_2$.

We consider the operator matrix H acting on $\mathcal{H}$ as

$$H=\begin{pmatrix} H_{00} & H_{01} & 0\\ H_{01}^* & H_{11} & H_{12}\\ 0 & H_{12}^* & H_{22}\end{pmatrix},$$

where the matrix elements $H_{kl}:\mathcal{H}_l\to\mathcal{H}_k,\ \ k,l=0,1,2$ are defined by

$$(H_{00}f_0)_0=w_0f_0,\quad (H_{01}f_1)_0=\int\limits_{\mathbf{T}^3} v_1(t)f_1(t)dt,\quad (H_{11}f_1)_1(p)=w_1(p)f_1(p),$$

$$(H_{12}f_2)_1(p)=\int\limits_{\mathbf{T}^3} v_2(t)f_2(p,t)dt,\quad (H_{22}f_2)_2(p,q)=w_2(p,q)f_2(p,q).$$

Here $f_k\in\mathcal{H}_k$, $k=0,1,2$, w_0 is a fixed real number, $v_k(\cdot)$, $k=1,2$ are real valued analytic functions on $\mathbf{T}^3$, with $v_2(\cdot)$ being an even function on $\mathbf{T}^1$ of each variable separately, the functions $w_1(\cdot)$ and $w_2(\cdot,\cdot)$ are defined as

$$w_1(p)=\varepsilon(p)+\lambda,\quad w_2(p,q)=\varepsilon(p)+\varepsilon(p+q)+\varepsilon(q),$$

$$\varepsilon(p)=\sum_{k=1}^{3}(1-cos\,mp^{(k)}),\ p=(p^{(1)},p^{(2)},p^{(3)})\in\mathbf{T}^3,\quad m\in\mathbf{N},$$

where λ is a fixed positive number, while $\mathbf{N}$ is the set of positive integers.

To formulate the main result of the note we introduce the following bounded and self-adjoint generalized Friedrichs model $h(p)$, $p\in\mathbf{T}^3$ acting on $\mathcal{H}_0\oplus\mathcal{H}_1$ as

$$h(p)=\begin{pmatrix} h_{00}(p) & h_{01}\\ h_{01}^* & h_{11}(p)\end{pmatrix},$$

where the operators $h_{kk}(p):\mathcal{H}_k\to\mathcal{H}_k$, $k=0,1$, $p\in\mathbf{T}^3$ and $h_{01}:\mathcal{H}_1\to\mathcal{H}_0$ are defined as

$$(h_{00}(p)f_0)_0=w_1(p)f_0,\quad (h_{01}f_1)_0=\frac{1}{\sqrt{2}}\int\limits_{\mathbf{T}^3} v_2(t)f_1(t)dt,\quad (h_{11}(p)f_1)_1(q)=w_2(p,q)f_1(q).$$

Now we give the main result of the present note.

Theorem. *If the operator $h(0)$ has the zero eigenvalue, then the operator matrix H has finitely many negative eigenvalues.*

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ON THE FINE SPECTRUM OF SOME GENERALIZED DIFFERENCE OPERATORS

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The spectrum and fine spectrum of the difference operator and generalized difference operators over sequence spaces have been examined by several authors. Our paper surveys spectral properties of the difference operator Δ and the generalized difference operators $B(r,\ s)$ and $B(r,\ s,\ t)$ on some sequence spaces.

Recently, Akhmedov and El-Shabrawy [1] have introduced the generalized difference operator $\Delta_{a,b}$ over the sequence space c_0. The operator $\Delta_{a,b}$ on the space c_0 is defined by

$$\Delta_{a,b}x = \Delta_{a,b}(x_k) = (a_k x_k + b_{k-1}x_{k-1})_{k=0}^{\infty} \text{ with } x_{-1} = \ b_{-1} = 0,$$

where $x = (x_k) \in c_0$ and (a_k), (b_k) are two sequences of nonzero real numbers satisfying certain conditions. In cases, $a_k = r$, $b_k = s$ and $b_k = -a_k = -v_k$ for all $k \in \mathbb{N} = \{0,\ 1,\ 2,\ ...\}$, the operator $\Delta_{a,b}$ is reduced to the generalized difference operator $B(r,s)$ of [2] and to the generalized difference operator Δ_v of [3].

The spectrum and point spectrum of $\Delta_{a,b}$ on c_0 are given by

$$\sigma(\Delta_{a,b},\ c_0) = \{\lambda \in \mathbb{C} : |a-\lambda| \leq |b|\} \cup \{a_k : |a-a_k| > |b|\},$$

$$\sigma_p(\Delta_{a,b},\ c_0) = \begin{cases} E, & if\ there\ exists\ m \in \mathbb{N} : a_i \neq a_j\ \forall i,\ j \geq m; \\ \emptyset, & otherwise, \end{cases}$$

where $E = \{a_k : |a-a_k| > |b|\}$.

The spectrum of several special limitation matrices over the sequence space c_0 is a region enclosed by a circle. It is interesting that the spectrum of the operator $\Delta_{a,b}$ over the sequence space c_0 may includes also a finite number of points outside the region enclosed by a circle.

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SEQUENCE SPACES AS 2-CONE BANACH SPACE

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In the main part of the work the results expressing under what conditions a self-mapping T of a sequence space as 2-cone Banach space has a unique fixed point are given.

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AN APPLICATION OF EXP-FUNCTION METHOD FOR BOGOYAVLENSKY-KONOPLECHENKO EQUATION

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In this study, we present the expfunction method which was suggested by He and was implemented by He and Wu in 2006 for the analytic solutions of the Bogoyavlensky-Konoplechenko equation. By using this method, we obtain some solutions of the abovementioned equation.

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L_2 SPECIAL RISSA ELEMENTS BASIS PROPERTY FEATURES

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The issue concerning the basis property of principal vector in a certain type of function possesses a specific role in the spectral theory of linear nonself-adjoint differential operators. The majority of widely known function systems are anyway related with differential operators and their intensive study is raised somewhat owing to the requirements of the spectral theory.

Recently it was established in the paper [1] of the author that the proportional boundedness nearly everywhere principal vector modulus of the direct and second-order adjoint operators is a necessary as well as a sufficient condition for L_2 category Rissa basis property.

In case of the arbitrary system of the L_2 category, not related to a certain differential operator, facts similar to the ones stipulated above have not been observed. Therefore, type systems [2] as $\{x|^{\alpha}e^{inx}\}$, $\{x|^{-\alpha}e^{-inx}\}$, where n - whole number, $0 < \alpha < \frac{1}{2}$, are considered as biorthogonally adjoint as well as standardized. Each of them forms the special $L_2(-\pi, \pi)$ basis property, but not the Rissa basis property.

Legendre polynomials (see example, [3, p.44]) forms the Rissa special basis property $L_2(-1, 1)$ (the orthonormal basis), but they are not limited. Even so, the following is correct.

Theorem. *Let us assume that each of the biorthogonally adjoint systems* $\{u_k(x)\}$, $\{v_k(x)\}$ *of the* $L_2(G)$ *category is complete and the following uniform estimates* $\|u_k\|_{L_2(G)} \le C_1$, $\|v_k\|_{L_2(G)} \le C_2$. *Then each of these systems forms* $L_2(G)$ *Rissa basis property.*

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STRONG APPROXIMATION AND SOME EMBEDDING THEOREMS FOR TWO VARIABLE FUNCTIONS

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Let $f(x,y)$ be a 2π-periodic continuous function by the both variables. Denote by $S_{m,n}(x,y) = S_{m,n}(f;x,y)$ the rectangular partial sum of the trigonometric Fourier series of function $f \in [0,2\pi)^2$. $\omega_1(\delta), \omega_2(\delta)$ - modull of continuity.

Let

$$\omega(f;\delta_1,\delta_2) = \sup_{\substack{|h_1| \le \delta_1 \\ |h_2| \le \delta_2}} \|f(x+h_1, y+h_2) - f(x,y)\|_{C[0,2\pi)^2}$$

be the modulus of continuity of function $f \in C[0,2\pi)^2$ and let $\lambda = \{\lambda_{m,n}\},\ m,n,\in \mathbb{N}$ be a monotonic (nondecreasing or nonincreasing) sequence of positive numbers by both indexes: $\lambda_{m_2,n_2} \ge \lambda_{m_1,n_1}$ or $\lambda_{m_2,n_2} \le \lambda_{m_1,n_1}$ for $m_2 \ge m_1$ and $n_2 \ge n_1$.

Denote by $\tau_{m,n} = \tau_{m,n}(f;x,y)$ de la Vallée Poussin means: $\tau_{m,n} = \frac{1}{mn} \sum\limits_{k_1=m+1}^{2m} \sum\limits_{k_2=n+1}^{2n} S_{k_1,k_2}(f;x,y)$

And define three classes of functions:

$$H^{\omega_1,\omega_2} = \left\{ f \in C[0,2\pi)^2 : \omega(f;\delta_1,\delta_2) = O\left(\omega_1(\delta)\cdot\omega_2(\delta)\right)\right\},$$

$$S_p(\lambda) = \left\{ f : \left\| \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \lambda_{m,n}|f - S_{m,n}|^p \right\| < \infty \right\},\ V_p(\lambda) = \left\{ f : \left\| \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \lambda_{m,n}|\tau_{m,n} - f|^p \right\| < \infty \right\}.$$

We consider the following problem: under what conditions posed on a sequence $\{\lambda_{m,n}\}$ take the place embeddings: $S_p(\lambda) \subset V_p(\lambda) \subset H^{\omega_1,\omega_2}$.

The case for the one variable function was considered in [1].

Theorem 1. *If $1 \le p < \infty$ and $\{\lambda_{m,n}\}$ is a monotonic (nondecreasing or nonincreasing) sequence by the both indexes of positive numbers satisfying conditions:*

$$\frac{\lambda_{m,n}}{\lambda_{2m,n}} \le K_1, \quad \frac{\lambda_{m,n}}{\lambda_{m,2n}} \le K_2, \quad m,n,\in \mathbb{N},$$

where K_1 and K_2 - positive constants. Then $S_p(\lambda) \subset V_p(\lambda)$ holds.

Theorem 2. *Let $1 \le p < \infty$ and let $\{\lambda_{m,n}\}$ be a monotonic (nondecreasing or nonincreasing) by both indexes sequence of positive numbers, $\omega_1(\delta),\ \omega_2(\delta)$ be the moduli of continuity. Then the condition*

$$\sum_{k_1=1}^{m}\sum_{k_2=1}^{n} (k_1k_2\lambda_{k_1,k_2})^{-\frac{1}{p}} = \underset{\sim}{O}\left(mn\omega_1\left(\frac{1}{m}\right)\omega_2\left(\frac{1}{n}\right)\right) \tag{1}$$

implies the embedding

$$S_p(\lambda) \subset H^{\omega_1,\omega_2}. \tag{2}$$

And if there exists number $\theta \in [0,1)$ such that
$((m+1)n)^{\theta}\lambda_{m+1,n} \ge (mn)^{\theta}\lambda_{m,n},\ \ (m(n+1))^{\theta}\lambda_{m,n+1} \ge (m\cdot n)^{\theta}\lambda_{m,n}$ *holds, then* (2) *implies* (1).

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TO THE THEORY OF PERTURBATION OF THE OPERATOR OF THE THERMAL CONDUCTION

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It is known [1, p.322], that to any self-interfaced operator operating in a separable Hilbert space H, it is possible to add the self-interfaced operator K, not only completely continuous, but even having as much as necessary small absolute norm and such, that the system of all eigenvectors of an operator will be $A + K$ full century H.

Antipode of the self-interfaced operators are volter operators, which at all have no eigenvalues. Volter operators Hromov A.P. [2] is engaged in study of spectral properties of finite-dimensional perturbations. Principal difference of our problem from Hromova A.P.'s problem consists, that we do not assume perturbation finite-dimensional though it is completely continuous and very small in sense of norm. The theorem proved by us says, that matter not in magnitude of perturbation, and most likely in its algebraic properties this fact has been noted for the first time in work [3].

Theorem. If a - the arbitrary complex number, which is distinct from zero, satisfying to the condition

$$tg\sqrt{a^2-1} \neq \sqrt{a^2-1}, \tag{1}$$

then the system of eigenfunctions of a boundary value problem:

$$T_a u = u_t - u_{xx} + au(1-t,x) = \lambda u(t,x), \tag{2}$$

$$u|_{t=0} = 0, \tag{3}$$

$$u|_{x=0} = 0, \quad u|_{x=1} = 0 \tag{4}$$

It is full in space $L^2(\Omega)$, where $\Omega = [0,1] \times [0,1]$.

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ON A MATRIX INEQUALITY

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Let $W = \{W_i\}_{i=1}^{\infty}$, $v = \{v_i\}_{i=1}^{\infty}$, $u = \{u_i\}_{i=1}^{\infty}$ are sequences of nonnegative numbers, $u_{k.} > 0,\ k \geq 1$. Let also $f = \{f_i\}_{i=1}^{\infty}$ is arbitrary sequence of real numbers. We take $K \downarrow = \{f : f \geq 0\}$, $K \uparrow = \{f : 0 \leq f \uparrow\}$, $K \downarrow = \{f : 0 \leq f \downarrow\}$, $F_k = \sum\limits_{i=1}^{\infty} f_i,\ F_K^* = \sum\limits_{i=1}^{\infty} f_i,$ under $k \geq 1$ and $F_0 = 0$ ($\uparrow$is a sign of the non-increasing, and $\downarrow$ is a sign of the non-decreasing).

It is considered a problem of finding the following value

$$J_\infty(u, v, g, K) = \sup_{f \geq 0} \frac{\sum\limits_{i=1}^{\infty} f_i g_i}{\sup\limits_{1 \leq i \leq \infty} u_i f_i + \sup\limits_{1 \leq i \leq \infty} v_i F_i} \tag{1}$$

for $g \in K \downarrow$ and on this basis we establish an inequality which is dual to an inequality of the form

$$\sup_{1 \leq k < \infty} w_k \left(Af\right)_k \leq C \left(\sup_{1 \leq i \leq \infty} u_i f_i + \sup_{1 \leq i \leq \infty} v_i F_i \right),\ f \geq 0, \tag{2}$$

where A is a real matrix operator $\left(Af\right)_k = \sum\limits_{i=1}^{k} a_{ki} f_i,\ k \geq 1.$

For every $n \geq 1$ we derive
$\varphi_n = \left\{ \min\limits_{1 \leq k \leq n} \left[\left(\sum\limits_{i=1}^{n} u_i^{-1} \right)^{-1} + \sup\limits_{k \leq i < \infty} v_i \right] \right\}^{-1}$ and set $\varphi_0 = 0.$

Theorem 1. *Let $g \in K \downarrow$. Then*

$$J_\infty \left(u, v, g, K\right) \approx \sup_{1 \leq i < \infty} g_i \left(\varphi_i - \varphi_{i-1}\right).$$

Theorem 2. *Let elements of the matrix $\{a_{ki}\}$ of the operator A be non-negative and no increasing in the second index, i.e. $a_{ki} \geq 0,\ a_{k,i+1} \geq a_{ki},\ k \geq 1,\ i \geq 1.$*
Then the inequality (2) holds if and only if

$$\sup_{1 \leq k < \infty} w_k \sum_{i=1}^{k} a_{ki} f_i \leq C_1 \sup_{1 \leq k < \infty} f_k \left(\varphi_k - \varphi_{k-1}\right)^{-1},\ f_k \geq 0, \tag{3}$$

in this case $C \approx C_1$, where $C,\ C_1$ the smallest const in (2) and (3) respectively.

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EMBEDDING THEOREMS OF DIFFERENT METRICS IN LORENTZ SPACES WITH HERMITE WEIGHT

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Let $1 \le p < +\infty$, $0 < \theta \le +\infty$ and f be a measurable function in the meaning of Lebesgue on $\mathbb{R}_n$; $\rho_n(\bar{x}) = e^{-\frac{|\bar{x}|^2}{2}}$, $\bar{x} \in \mathbb{R}_n$; $|\bar{x}| = \left(\sum\limits_{k=1}^{n} x_k^2\right)^{\frac{1}{2}}$. By $F(|f\rho_n|;t)$ we will denote the nonincreasing rearrangement of functions $|f(\bar{x})\rho_n(\bar{x})|$ on $\mathbb{R}_n$, $t \in (0;+\infty)$. We say, that $f \in L_{p,\theta}(\mathbb{R}_n;\rho_n)$, if the next value is finite

$$\|f\|_{L_{p,\theta}(\mathbb{R}_n;\rho_n)} = \left\{ \frac{\theta}{p} \int\limits_0^{+\infty} t^{\frac{\theta}{p}-1} (F(|f\rho_n|;t))^{\theta} dt \right\}^{\frac{1}{\theta}}, \text{when} \quad 0 < \theta \le +\infty.$$

Let $\mathcal{P}_{\bar{m}}$ be a set of various algebraic polynomials, degree not higher $(m_i - 1)$ on variable quantity x_i, $i = 1,...,n$, $\bar{m} = (m_1,...,m_n)$; $E_{\bar{m}}(f)_{L_{p,\theta}(\mathbb{R}_n;\rho_n)}$ is the best approximation of $f \in L_{p,\theta}(\mathbb{R}_n;\rho_n)$ in the metric of the space $L_{p,\theta}(\mathbb{R}_n;\rho_n)$ by the algebraic polynomials of the whole $\mathcal{P}_{\bar{m}}$.

By the $C(\mathbb{R}_n;\rho_n)$ we denote the set of continuous functions on $\mathbb{R}_n$, for which the next value is finite $\|f\|_{C(\mathbb{R}_n;\rho_n)} = \max\limits_{\bar{x}\in\mathbb{R}_n} |f(\bar{x})\rho_n(\bar{x})|$.

Theorem 1. *Let $f \in L_{p,\theta}(\mathbb{R}_n;\rho_n)$, $1 \le p < +\infty$, $0 < \theta \le +\infty$. If $\left\{2^{\frac{nk}{2p}} E_{2^k,...,2^k}(f)_{L_{p,\theta}(\mathbb{R}_n;\rho_n)}\right\}_{k=0}^{+\infty} \in l_1(\mathbb{Z}^+)$, then function f might be modified on the set of 0 measure so, that it would be continuous function on $\mathbb{R}_n$ and also holds true the following inequality:*

$$\|f\|_{C(\mathbb{R}_n;\rho_n)} \le A_p \left\{ \|f\|_{L_{p,\theta}(\mathbb{R}_n;\rho_n)} + \sum_{k=0}^{+\infty} 2^{\frac{nk}{2p}} E_{2^k,...,2^k}(f)_{L_{p,\theta}(\mathbb{R}_n;\rho_n)} \right\}.$$

Here the constant $A_p > 0$ depends only on the indicated parameter.

Theorem 2. *Let $f \in L_{p,\theta}(\mathbb{R}_n;\rho_n)$, $1 \le p < +\infty$, $0 < \theta \le +\infty$. If for some numbers q and τ such, that $p < q < +\infty$, $0 < \tau \le +\infty$:*
$\left\{2^{m\left(\frac{n}{2p}-\frac{n}{2q}\right)} E_{2^m,...,2^m}(f)_{L_{p,\theta}(\mathbb{R}_n;\rho_n)}\right\}_{m=0}^{+\infty} \in L_{\tau}(\mathbb{Z}^+)$ *for $0 < \tau < +\infty$, then $f \in L_{q,\tau}(\mathbb{R}_n;\rho_n)$*. But if for $\tau = +\infty$: $\sup\limits_{m\in\mathbb{Z}^+} 2^{-\frac{nm}{2q}} \sum\limits_{k=0}^{m} 2^{\frac{nk}{2p}} E_{2^k,...,2^k}(f)_{L_{p,\theta}(\mathbb{R}_n;\rho_n)} < +\infty$, then $f \in L_{q\infty}(\mathbb{R}_n;\rho_n)$.

At the same time inequalities take place, when $0 < \tau < +\infty$:

$$\|f\|_{L_{q,\tau}(\mathbb{R}_n;\rho_n)} \le C_{pq\theta\tau n} \left\{ \|f\|_{L_{p,\theta}(\mathbb{R}_n;\rho_n)} + \left[\sum_{m=0}^{+\infty} 2^{m\left(\frac{n}{2p}-\frac{n}{2q}\right)\tau} E^{\tau}_{2^m,...,2^m}(f)_{L_{p,\theta}(\mathbb{R}_n;\rho_n)} \right]^{\frac{1}{\tau}} \right\};$$

$$\|f\|_{L_{q\infty}(\mathbb{R}_n;\rho_n)} \le C_{pq\theta n} \left\{ \|f\|_{L_{p,\theta}(\mathbb{R}_n;\rho_n)} + \sup_{m\in\mathbb{Z}^+} 2^{-\frac{nm}{2q}} \sum_{k=0}^{m} 2^{\frac{nk}{2p}} E_{2^k,...,2^k}(f)_{L_{p,\theta}(\mathbb{R}_n;\rho_n)} \right\}, \text{when} \quad \tau = +\infty.$$

ABOUT NEW FORM OF EQUATIONS IN THE PROBLEM OF A SOLID BODY ROTATIONAL MOTION $B^{r_1,\ldots,r_n}_{p_1,\ldots,p_n,\theta} \subset L^{q_1,\ldots,q_n}$

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The problem of finding necessary and sufficient conditions for embedding anisotropic Nikolskii - Besov spaces in the mixed norm is considered in this abstract.

It is valid

Theorem. *Let* $1 \le p_j < q_j \le \infty \ (j = 1, ..., n)$ $(\mathrm{L}^\infty(R^n) \equiv C(R^n))$, $0 < \theta \le \infty$. *Then*

$$B^{\omega_1,\ldots,\omega_n}_{p_1,\ldots,p_n,\theta}(R^n) \subset L^{q_1,\ldots,q_n}(R^n) \Leftrightarrow \left\{ \int_0^1 \left[\Omega(t)\, t^{-\max\limits_{1\le j\le n}\left(\frac{1}{p_j}-\frac{1}{q_j}\right)} \right]^\rho \frac{dt}{t} \right\}^{1/\rho}$$

(in necessary assumed $1/p_1 - 1/q_1 = ... = 1/p_n - 1/q_n$*).*

Here modules of smoothness $\omega_j(\delta)$ *of order of* $k_j \ge 1$*:* $\omega_j(0) = 0$, $\omega_j(1) = 1$, $\omega_j(t) \cdot t^{-\beta_j}$ *almost decreased on* $0 < \beta_j < k_j$*;* $\Omega(\delta)$ *the average modulus of smoothness of the system* $\omega_1, ..., \omega_n$*;* $q^* = \{\min q_j\}$*, if* $q_j < \infty$ *for some* $j = 1, ..., n$ *and* $q^* = 1$ *for others* $q_1 = \ldots = q_n = \infty$, $\rho = (\theta q^*)/(\theta - q^*)$ *for other* $\theta > q^*$ *and* $\rho = \infty$ *for other* $\theta > q^*$.

Corollary. *Let* $1/p_1 - 1/q_1 = ... = 1/p_n - 1/q_n$. *Then*

$$1)\ \underset{1\le q_j<\infty\ (j=1,\ldots,n)}{B^{r_1,\ldots,r_n}_{p_1,\ldots,p_n,\theta}(R^n) \subset L^{q_1,\ldots,q_n}(R^n)} \Leftrightarrow \begin{cases} 0<\theta\le+\infty,\ 1=\left(\frac{1}{r_1}+...+\frac{1}{r_n}\right)\left(\frac{1}{p_1}-\frac{1}{q_1}\right), \\ 0<\theta\le q^*,\ \ 1=\left(\frac{1}{r_1}+...+\frac{1}{r_n}\right)\left(\frac{1}{p_1}-\frac{1}{q_1}\right) \end{cases} \left(\min_{j=1,\ldots,n} q_j = q^* < +\infty\right).$$

$$2)\ \underset{(q_1=\ldots=q_n=\infty)}{B^{\mathbf{r}}_{p,\theta}(R^n) \subset C(R^n)} \Leftrightarrow \begin{cases} 0<\theta\le+\infty,\ \left(\frac{1}{r_1}+...+\frac{1}{r_n}\right)^{-1} > \frac{1}{p}, \\ 0<\theta\le 1,\ \ \left(\frac{1}{r_1}+...+\frac{1}{r_n}\right)^{-1} = \frac{1}{p}, \end{cases} \quad B^{\frac{n}{p}}_{p,\theta}(R^n) \subset C(R^n) \Leftrightarrow 0<\theta\le 1.$$

$$3)\ \underset{1<q_1\le\cdots\le q_\nu<q_{\nu+1}=\ldots=q_n=\infty}{B^{r_1,\ldots,r_n}_{p_1,\ldots,p_\nu,p_{\nu+1},\ldots,p_{\nu+1},\theta}(R^n) \subset L^{q_1,\ldots,q_\nu,+\infty,\ldots,+\infty}(R^n)} \Leftrightarrow \begin{cases} 1.\ 0<\theta\le q_1,\ \left(\frac{1}{r_1}+...+\frac{1}{r_n}\right)^{-1} = \frac{1}{p_{\nu+1}}, \\ 2.\ 0<\theta\le+\infty,\ \left(\frac{1}{r_1}+...+\frac{1}{r_n}\right)^{-1} > \frac{1}{p_{\nu+1}}, \end{cases}$$

For case $p_1 = ... = p_n$, $q_1 = ... = q_n$ *see in [1] and [2, §17].*

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LOWER AND UPPER ESTIMATES FOR THE NUMBER OF THREE PARTICLE BOUND STATES AND STRUCTURE OF ESSENTIAL SPECTRA OF THE ENERGY OPERATOR OF TWO-MAGNON SYSTEMS IN A NON-HEISENBERG FERROMAGNET WITH ARBITRARY SPIN VALUE S AND NEAREST-NEIGHBOR INTERACTIONS[1]

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We consider a two-magnon system in the isotropic ferromagnetic non-Heisenberg model with arbitrary spin value S and nearest-neighbor interactions on a $\nu-$dimensional lattice Z^ν with nearest-neighbor interactions and study the essential and discrete spectra of this system.

Hamiltonian of the system has the form

$$H = -\sum_{m,\tau}\sum_{n=1}^{2s} J_n(\vec{S}_m\vec{S}_{m+\tau})^n - \sum_{\tau}\sum_{n=1}^{2s}(J_n^0 - J_n)(\vec{S}_0\vec{S}_\tau)^n, \quad (1)$$

where $J_n > 0, n = 1, 2, ..., 2s$ are the parameters of the multipole exchange interaction between the nearest-neighbor atoms in the lattice, J_n^0 are the atom-impurity multipole exchange interaction parameters, $\vec{S}_m$ is the atomic spin operator for the spin s at the m^{th} lattice site, and the summation over τ ranges the nearest neighbors.

The Hamiltonian (1) acts in the symmetric Fock space $\mathcal{H}$. We denote by $\mathcal{H}_2$ two-magnon invariant subspace of operator H. Denoted by H_2 the restriction of H to $\mathcal{H}_2$.

Operator H_2 is a bounded self-adjoint operator. It is in the quasimomentum representation acts in the space $\widetilde{H}_2 = L_2(T^\nu \times T^\nu)$ to the formula

$$(\widetilde{H}_2 f)(x; y) = h(x; y)f(x; y) + \int_{T^\nu} h_1(x; y; t)f(t; x + y - t)dt + \int_{T^\nu} h_2(x; s)f(s; y)ds+$$

$+\int_{T^\nu} h_3(y; t)f(x; t)dt + \int_{T^\nu}\int_{T^\nu} h_4(x; y; s; t)f(s; t)dsdt,$ *where* $h(x; y), h_1(x; y; t), h_2(x; s), h_3(y; t)$ *and* $h_4(x; y; s; t)-$ *is the some concrete* 2π- *periodical continuous functions,* T^ν *is the* $\nu-$ *dimensional torus endowed with the normalized Lebesgue measure* $d\lambda : \lambda(T^\nu) = 1$.

Let $N-$ number of three-particle bound states (BS) of operator H, and

$$p(s) = -2\sum_{k=1}^{2s}(-2s)^k J_k, q(s) = -2\sum_{k=1}^{2s}(-2s)^k(J_k^0 - J_k).$$

Theorem 1. *If* $\nu = 1$ *and* $p(s) > 0, -p(s) \le q(s) < 0$ *or* $p(s) < 0, 0 < q(s) < -p(s)$, *then the essential spectrum of the operator* $\widetilde{H}_2$ *consists of a single interval:* $\sigma_{ess.}(\widetilde{H}_2) = [0; 4p(s)]$, *or* $\sigma_{ess.}(\widetilde{H}_2) = [4p(s); 0]$ *and the relation* $0 \le N \le 12$ *holds for the number of three-particle BSs.*

Theorem 2. *If* $\nu = 1$ *and* $p(s) > 0, q(s) < -p(s)$ *or* $p(s) < 0, q(s) < p(s)$ $(p(s) > 0, p(s) < q(s)$ *or* $p(s) < 0, q(s) > -p(s),)$ *then the essential spectrum of the operator* $\widetilde{H}_2$ *consists of the union of three intervals:* $\sigma_{ess.}(\widetilde{H}_2) = [0; 4p(s)]\bigcup[z_1; z_1 + 2p(s)]\bigcup[z_2; z_2 + 2p(s)]$, *or* $\sigma_{ess.}(\widetilde{H}_2) = [4p(s); 0]\bigcup[z_1; z_1 + 2p(s)]\bigcup[z_2; z_2 + 2p(s)]$, *and the relation* $3 \le N \le 15$ *holds for the number of three-particle BSs.*

Here z_1 *and* z_2 *are the eigenvalues of operator* $\widetilde{H}_1$.

It is received similar results for other dimensions to a lattice.

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LIMITING ERROR OF INEXECT INFORMATION UNDER OPTIMAL RECONSTRUCTION OF FUNCTIONS FROM THE NIKOL'SKII CLASS IN THE UNIFORM METRICS

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Let (explanation of notation, see [1])

$$\delta_N(\varepsilon_N) \equiv \delta_N(D_N;T,F,\varepsilon_N)_Y = \inf_{\left(l^{(N)},\varphi_N\right)\in D_N} \sup_{\substack{f\in F,\\ |l_j(f)-z_j|\le\varepsilon_N\\ j=1,\dots,N}} \|u(\cdot;f)-\varphi_N(z_1,\dots,z_N;\cdot)\|_Y .$$

The problem defenition of limiting error of inexect information under optimal reconstruction of functions from F consist of finding sequence $\tilde{\varepsilon}_N$ $(N=1,2,\dots)$ such as satisfy the relation $\delta_N\left(D_N,T,F;\tilde{\varepsilon}_N\right)_Y \asymp \delta_N\left(D_N,T,F;0\right)_Y$ $(N\to+\infty)$ and, simultaneously, for any tending to $+\infty$ on increasing N sequence $\{\eta_N\}$ *the equality* (a few, other formulation, see eg. [2-3])

$$\overline{\lim_{N\to\infty}}\frac{\delta_N\left(D_N,T,F;\eta_N\tilde{\varepsilon}_N\right)_Y}{\delta_N\left(D_N,T,F;0\right)_Y}=+\infty.$$

takes place.

Let us consider a particular example of the general problem being formulated (see also [1] and [4]): $F = H_p^r(0,1)$ be the Nikol'skii class, $Y = L^\infty \equiv C[0,1]$, where $1\le p<+\infty$, $r>1+\frac{1}{p}$, $D_N^{(*)}=\{(l_0(f),\dots,l_N(f)) : l_j(f)-$ are all linear functionals from the linear hull $F=H_p^r(0,1)$ such that $|l_j(f)|\le 1\}\times\{\varphi_N\}$.

Under these conditions, the following theorem is valid

Theorem. *Let numbers* $1\le p<\infty$ *and* $r>1+\frac{1}{p}$ *be given and* $\tilde{\varepsilon}_N=N^{-\left(r-\frac{1}{p}\right)}$. *Then*

$$\delta_N(D_N^{(*)},Tf=f,H_p^r(0,1),0)_{C[0,1]}\asymp$$

$$\asymp\delta_N(D_N^{(*)},Tf=f,H_p^r(0,1),\tilde{\varepsilon}_N)_{C[0,1]}\asymp N^{-\left(r-\frac{1}{p}\right)},$$

for any tending to $+\infty$ *positive sequence* $\{\eta_N\}_{N=1}^{\infty}$ *the equality*

$$\overline{\lim_{N\to\infty}}\frac{\delta_N\left(\left\{D_N^{(*)};Tf=f,H_p^r(0,1);\tilde{\varepsilon}_N\eta_N\right\}\right)_{C[0,1]}}{\delta_N\left(\left\{D_N^{(*)};Tf=f,H_p^r(0,1);0\right\}\right)_{C[0,1]}}=+\infty.$$

takes place.

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AT THE PROBLEM ON A LIMITING ERROR OF THE INEXACT INFORMATION AT OPTIMUM RECONSTRUCTION OF FUNCTIONS FROM SOBOLEV CLASS

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Let the continuous function $f(x)$ be defined on the interval $[0,1]$. We pose the problem of optimal reconstruction of functions from inaccurate numerical information of a given N, obtained from the function $f(x)$ by means of functionals $l_1(f),...,l_N(f)$ (notation explained in [1]):
1)$\delta_N(0) \asymp \delta_N(\tilde{\varepsilon}_N)$,
2)$\forall \eta_N \uparrow +\infty : \overline{\lim\limits_{N\to\infty}} \frac{\delta_N(\eta_N\tilde{\varepsilon}_N)}{\delta_N(\tilde{\varepsilon}_N)} = +\infty$,
where

$$\delta_N(\varepsilon_N) \equiv \delta_N(D_N;T,F,\varepsilon_N)_Y = \inf_{\left(l^{(N)},\varphi_N\right)\in D_N} \sup_{\substack{f\in F,\\ |l_j(f)-z_j|\le\varepsilon_N\\ j=1,...,N}} \|u(\cdot;f)-\varphi_N(z_1,...,z_N;\cdot)\|_Y .$$

Under this formulation of the problem (other formulations, see [2-3]) in circumstances - $F = W_p^r(0,1)$ the Sobolev class, $Y = L^q(0,1)$ ($L^\infty \equiv \mathrm{C}[0,1]$) - Lebesgue space, where $1 \le p < q \le +\infty$, integer $r \ge 1$, $D_N^{(*)} = \{(l_1(f),...,l_N(f)) :\ l_j(f)-$ all possible linear functionals on the linear hull such that the answer

$$\delta_N(0) \asymp \tilde{\varepsilon}_N \asymp N^{-\left(r-\left(\frac{1}{p}-\frac{1}{q}\right)\right)}$$

means the following (see also [4]): Whatever the computer unit $\varphi_N = (l_1(f),...,l_N(f);x)$ may raise, including what has been done in approximation theory and computational mathematics, including all of the Fourier series, the bases, interpolating polynomials, wavelets, Fourier widths, etc., it is better not to approximate at the prescribed speed. Figuratively speaking, "to attract any conceivable computational units" can not provide a large degree of approximation. In this case, linear functionals $l_1(f),...,l_N(f)$, which provide numerical information about the volume N by f can be calculated with precision less than or equal to $\tilde{\varepsilon}_N = N^{-\left(r-\left(\frac{1}{p}-\frac{1}{q}\right)\right)}$, but with the preservation rate of reconstruction $\delta_N(0)$, no more.

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ON SOME CHARACTERIZATION OF SUPERPARACOMPACTNESS, STRONG PARACOMPACTNESS AND COMPLETE PARACOMPACTNESS

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In this paper, we study uniform analogs of complete paracompactness [1] strong paracompactness [2], and superparacompactness [3].

Definition 1. *(i) A star finite (finite-component) U - locally finite [4] cover ω of a uniform space (X, U) is said to be uniformly star-finite (respectively, uniformly finite-component); (ii) A $\sigma-$ star-finite [3] (in particular, $\sigma-$ finite -component [3]) U -locally finite cover λ of a uniform space (X, U) is said to be uniformly $\sigma-$ star-finite (respectively, uniformly $\sigma-$ finite-component); (iii) A cover of a uniform space (X, U) wich can be represented as a countable family of uniformly star-finite (uniformly finite-component) covers is said to be $\sigma-$ uniformly star-finite (respectively, $\sigma-$ uniformly finite-component).*

Proposition 1. *If a uniformly space (X, U) is R -superparacompact (R -strong paracompact, R -completely paracompact). Moreover, if X is a superparacompact Hausdorff space (strongly paracompact Hausdorff space) and U^* is its universal uniformity, then the uniform space (X, U^*) is R -super paracompact (respectively, R -strongly paracompact).*

Proposition 2. *For a uniform space (X, U), the following conditions (a1), (b1), (c1), and (d1) are equivalent to conditions (a2), (b2), (c2), and (d2), respectively:(a1) (X, U) is R -paracompact; (a2) any finitely additive [4] open cover of (X, U) has a U -locally finite open refinement; (b1) (X, U) is R -completely paracompact; (b2) any finitely additive open cover of (X, U) has a uniform $\sigma-$ star-finite open weak refinement;(c1) R -strongly paracompact; (c2) any finitely additive open cover of (X, U) has a uniform star-finite open refinement;(d1) (X, U) is R -superparacompact; (d2) any finitely additive open cover of (X, U) has a uniform finite-component open refinement.*

Theorem 1. *For a uniform space (X, U) , the following conditions (a1) and (b1) are equivalent to conditions (a2), and (b2), respectively:(a1) the space (X, U) is uniformly R -superparacompact; (a2)the space (X, U) is uniformly R -paracompact and (X, τ_u) is superparacompact;(b1) the space (X, U) is uniformly R -strongly paracompact; (b2) the space (X, U) is uniformly R -paracompact and (X, τ_u) is strongly paracompact.*

Theorem 2. *Let(X, U) be uniform space and let bX be a compact Hausdorff extension of X.Then the following conditions are equivalent: (a) (X, U) is R -superparacompact; (b) for any compact space $K \subseteq bX \setminus X$, there exists a U-locally finite disjoint open cover λ of (X, U) which punctures the compact set K in bX.*

Proposition 3. *Any uniformly zero-dimensional R -paracompact space is R -super paracompact.*

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THE INTEGRAL EQUATION OF VOLTERRA'S TYPE OF THE SECOND ORDER

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The Volterra's type integral equation of second order is considered

$$\mathbf{K}_\lambda \mu \equiv \mu(t) - \lambda \int_0^t \mathscr{K}(t,\tau)\mu(\tau)d\tau = f(t), \;\; t \in \mathbb{R}_+, \tag{1}$$

where the kernel $\mathscr{K}(t,\tau)$ is defined by equality

$$\mathscr{K}(t,\tau) = \frac{1}{\tau^\alpha (t-\tau)^{1-\alpha}}, \quad 0 < \tau < t < \infty.$$

The right part and the equation solution (1) belong to classes of the functions summarized with corresponding spaces:

$$\{\exists\ \gamma < 1-\alpha : \;\; t^{\gamma-1} f(t) \in L_1(R_+)\}, \quad e^{-t} f(t) \in L_1(\mathbb{R}_+), \quad t^{-\alpha} e^{-t} \mu(t) \in L_1(\mathbb{R}_+). \tag{2}$$

Main result. *For any function $f(t)$ from (2) the inhomogeneous the integral equation (1) has the decision from a class (2):*

$$\begin{cases} \mu(t) = f(t) + l(-s^*) \int\limits_0^t \dfrac{\tau^{-s^*-1}}{t^{-s^*}} f(\tau)d\tau + C\, t^{-s^*}, \;\; if \;\; Re\,\lambda > 0; \\ \mu(t) = f(t) + \int\limits_0^t \sum\limits_{k=1}^{\infty} l(s_k^0) \dfrac{\tau^{s_k^0 - 1}}{t^{s_k^0}} f(\tau)d\tau, \;\; if \;\; Re\,\lambda < 0. \end{cases} \tag{3}$$

Obviously, the function (3) belong to a class (2).

The following result is proved.

Theorem. *Particular integral operator Volterra $\mathbf{K}_\lambda$, corresponding to the equation (1), is Noether's and has the index*

$$ind\{\mathbf{K}_\lambda\} = \begin{cases} 1, & \text{if} \;\; Re\lambda > 0, \\ 0\,, & \text{if} \;\; Re\lambda < 0. \end{cases}$$

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WEIGHTED APPROXIMATION BY NEW BERNSTEIN-STANCU-CHLODOWSKY POLYNOMIALS

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In the present paper, we introduce Bernstein-Stancu-Chlodowsky polynomials taking into consideration the polynomials introduced by Gadjiev and Ghorbanalizadeh [2]. The interval of convergence of the polynomials is a moved interval as polynomials given in [2] but grows as $n \to \infty$ as in the classical Bernstein-Chlodowsky polynomials. Also their knots are shifted and depend on x.

We firstly study weighted approximation properties of these polynomials and show that these polynomials are more efficient in weighted approximating to function having polynomial growth, since these polynomials contain a factor b_n tending to infinity having a certain degree of freedom. Secondly, we calculate derivative of new Bernstein-Stancu-Chlodowsky polynomials and give a weighted approximation theorem in Lipschitz space for the derivatives of these polynomials.

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CONVERGENCE OF THE $q-$MIXED SUMMATION INTEGRAL TYPE OPERATORS

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In this study, we introduce an analogue of a certain family of mixed summation integral type operators as

$$B_{n,p}^{q}(f;x) = [n+p-1]_q \sum_{k=1}^{\infty} s_{n,p,k}(x;q) \int_0^{\infty/A} q^{k(k-1)} b_{n,p,k-1}(t;q) f(t) d_q t + e_q^{-[n+p]_q x} f(0),$$

where

$$s_{n,p,k}(x;q) := e_q^{-[n+p]_q x} \frac{([n+p]_q x)^k}{[k]_q!}$$

and

$$b_{n,p,k}(t;q) := \left[\begin{array}{c} n+p+k-1 \\ k \end{array} \right]_q \frac{t^k}{(1+t)_q^{n+p+k}}.$$

We investigate their approximation properties. We establish a direct approximation theorem. Furthermore, we give a weighted approximation theorem and obtain rates of convergence of these operators for continuous functions.

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RECONSTRUCTION OF MIXED DERIVATIVES OF FUNCTIONS BELONGS TO THE CLASS W_2^r

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The problem of reconstructing the derivatives is considered by a number of papers (see, for example, [1-3]).

For a nonnegative integer r the Sobolev class W_2^r is the set of all integrable function $f(x) = f(x_1, ..., x_s)$ that are 1-periodic in each variable and, which trigonometric Fourier-Lebesgue coefficients satisfy the condition

$$\sum_{(m_1,...,m_s)\in Z^s} \left|\hat{f}\left(m_1, ..., m_s\right)\right|^2 \left(\overline{m}_1^{2r} + ... + \overline{m}_s^{2r}\right) \leq 1,$$

where Z^s is a set of all vectors $m = (m_1, ..., m_s)$ with integer components, $\overline{m}_j = \max\{1, |m_j|\}$ $(j = 1, .., s)$.

Theorem. *Let numbers $s\,(s = 2, 3, ...)$, $\alpha_j \geq 0\,(j = 1, 2, ..., s)$ and r such, that $r > \left(\sum\limits_{j=1}^{s} \alpha_j + \frac{1}{2}\right) s$. Then holds*

$$\sup_{f\in W_2^r} \left\|f^{(\alpha_1,...,\alpha_s)}\left(x\right) - \varphi_N\left(f\left(\xi_1\right), ..., f\left(\xi_N\right); x\right)\right\|_{L^2[0,1]^s} \underset{s,r,\alpha_1,...,\alpha_s}{<<} N^{-\frac{r+(\alpha_1+...+\alpha_s)}{s}} \left(p = 2, 3, ...;\ N = p^s\right),$$

where

$$\varphi_N\left(f\left(\xi_1\right), ..., f\left(\xi_N\right); x\right) = \frac{1}{N} \sum_{\xi^{(n)}\in S_N} f\left(\xi^{(n)}\right),$$

$$\sum_{\substack{|m_j| < \frac{p}{2} \\ j = 1, 2, ..., s}} {}^{*} \left(2\pi i m_1\right)^{\alpha_1} ... \left(2\pi i m_s\right)^{\alpha_s} \hat{f}\left(m\right) e^{2\pi i\left(m, x-\xi^{(n)}\right)}$$

and

$$S_N = \left\{\xi^{(n)} = \left(\frac{n_1}{p}, ..., \frac{n_s}{p}\right):\ n = (n_1, ..., n_s) \in Z^s,\ 0 \leq n_j < p,\ j = 1, 2, ..., s\right\}.$$

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SECTION IV
Partial and Ordinary Differential Equations

SOLVING OF THE MIXED CAUCHY PROBLEM FOR THE ONE-DIMENSIONAL HEAT EQUATION IN QUADRATURES BY A METHOD OF EXTERNAL POTENTIAL

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Mixed Cauchy problem: in the rectangle $\Omega = \{0 < t < T, 0 < x < 1\}$ find a regular solution of the equation

$$(\frac{\partial}{\partial t} - \frac{\partial}{\partial x^2})u = f(x,t), \tag{1}$$

satisfying to the initial condition

$$u|_{t=0} = 0, \tag{2}$$

and the boundary conditions

$$u|_{x=0} = u|_{x=1} = 0. \tag{3}$$

It is known that, as a rule, problem (1) - (3) is solved by the method of Fourier, or by the method of integral equations, but, until now it was impossible to solve it in the quadratures.

In this work the solution of the mixed Cauchy problem is obtained by the method of external thermal potential in the quadratures.

The essence of the method external potential is to find solution in the form

$$u = (\varepsilon * f)(x,t) + (\varepsilon * g)(x,t) \tag{4}$$

where $g(x,t)$ is unknown function, the support of $g(x,t)$ lies outside of Ω, $supp \subset R^2 \setminus \Omega$. Satisfying to the boundary condition (3) we obtain the Volterra integral equation of the type I and for the special choice of an unknown function it is possible to invert the integral equation for $g(x,t)$ explicitly.

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HYBRID RANS/LES METHOD FOR SOLVING NAVIER - STOKES EQUATIONS BY USING PARALLEL TECHNOLOGY

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In modern transport aircraft applied intensively Reynolds averaged Navier-Stokes(RANS) method to compute flow field. An extensive survey for RANS computations over the last 10-15 years is given by Rumsey and Ying[1]. They conclude that one of the most challenging topics for the numerical analysis of multi-element high-lift flows in the next decade is the understanding of unsteady effects. To resolve unsteady flow structures large-eddy simulation(LES) is known to be a reliable method. But, computational costs restricted LES mainly to low Reynolds number flows. Multi-element flows highly unsteady and non-equilibrium flow regions exist. These structure require LES to obtain accurate result. These unsteady flows are often limited to certain part of the flow field, usually in regions where separated flow occur. To decrease computational time, used a hybrid RANS/LES. This mean, if this is non-equilibrium regions are used LES approach, otherwise used RANS. The large eddy simulation method is based on the filtered Navier-Stokes equations for three dimensional compressible flow. Using the standard notation the filtered equations with mass-weighted variables are given by

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x_i}(\bar{\rho}\tilde{u}_i) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(\bar{\rho}\tilde{u}_i) + \frac{\partial}{\partial x_j}(\bar{\rho}\tilde{u}_i\tilde{u}_j) + \frac{\partial \bar{p}}{\partial x_i} = \frac{\partial \bar{\tau}_{ij}}{\partial x_j} + \frac{\partial \tau_{ij}^{SGS}}{\partial x_j} \tag{2}$$

$$\frac{\partial}{\partial t}(\bar{\rho}\tilde{e}_t) + \frac{\partial}{\partial x_j}(\bar{\rho}\tilde{e}_t + \bar{p})\tilde{u}_j = \frac{\partial}{\partial x_j}(\tilde{u}_i\bar{\tau}_{ij} + \tilde{u}_i\tau_{ij}^{SGS}) - \frac{\partial}{\partial x_j}(\bar{q}_j + \tau_j^{SGS}) \tag{3}$$

The applied block structures flow solver is based on a vertex centered finite-volume technique, where the equations are implicitly filtered by a top-hat filter. All algorithm paralleled by using decomposition method. [2] Due to the nonlinearity of the convection terms their discrete formulation has a strong impact on the solution and as such has to be carefully selected. It has been shown that a mixed central-upwind Advective Upstream Method (AUSM) scheme with low numerical dissipation is appropriate for the discretization of these convective fluxes [3]. The AUSM method was introduced by Liou and Steffen [4].

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ON NON-LOCAL BOUNDARY VALUE PROBLEM FOR THE SYSTEM OF PARTIAL DERIVATIVE EQUATIONS

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The non-local boundary value problem for the system of partial derivative equations is considered in $\overline{\Omega} = \{(x,t): \ t \le x \le t+\omega, \ 0 \le t \le T\}$, $T > 0$, $\omega > 0$

$$D\Big[Du\Big] = A(x,t)Du + S(x,t)u + f(x,t), \tag{1}$$

$$B(x)Du(x,0) + C(x)Du(x+T,T) = d(x), \quad x \in [0,\omega], \tag{2}$$

where $u \in R^n$; $D = \frac{\partial}{\partial t} + \frac{\partial}{\partial x}$; $(n \times n)$ - matrices $A(x,t)$, $S(x,t)$, n - vector-function, $f(x,t)$ are continuous on $\overline{\Omega}$; $(n \times n)$ - matrices $B(x)$, $C(x)$, n - vector-function $d(x)$ are continuous on $[0,\omega]$.

A method of parameterization [1], [2] was developed for the system of hyperbolic equations with mixed derivative. In given work, introduce new unknown function $v(x,t) = Du$. Then investigation problem is reduces equivalent problem for the system of hyperbolic first-order equations.

Sufficient conditions are obtained for the unique and correct solvability of the problem in the terms of invertibility of the matrix, and boundary condition.

Theorem. *Let for some* $h > 0 : Nh = T$ *and* $\nu, \ \nu = 1, 2, ...,$ $(nN \times nN)$*- matrix* $Q_\nu(\xi, h)$ *invertible for all* $\xi \in [0,\omega]$ *and are satisfied:*
a) $\|[Q_\nu(\xi,h)]^{-1}\| \le \gamma_\nu(h)$;
b) $q_\nu(\xi,h) = \gamma_\nu(h)\max\{1, h\|C(\xi)\|\}\left[e^{\alpha(\xi)h} - 1 - \alpha(\xi)h - ... - \frac{(\alpha(\xi)h)^\nu}{\nu!}\right] \le \sigma < 1$,
where $\alpha(\xi) = \max\limits_{\tau\in[0,T]} \|\widetilde{A}(\xi,\tau)\|$, $\sigma = const$.

Then following approximate $(\widetilde{v}^{(k)}(\xi,\tau), \widetilde{u}^{(k)}(\xi,\tau))$ *converges to the unique solution of the problem (1)-(2).*

In the work an algorithm is offered to find the solution of the considered problem.

Existence of the solution is established in the sense of Fridrihsu.

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ON AN UNIQUENESS OF THE SOLUTION OF THE POINCARE-TRICOMI PROBLEM FOR THE SECOND KIND ELLIPTIC-HYPERBOLIC EQUATION[1]

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We consider the equations

$$sgny\,|y|^m\,u_{xx} + u_{yy} = 0, \qquad (-1 < m < 0) \tag{1}$$

in the domain $D = D_1 \cup D_2$, where D_1 is bounded by the Jordan curve σ, resting in points $A(0;0)$, $B(1;0)$ and length AB in the direction $y = 0$, but D_2 is a bounded with the same length to axis $y = 0$ and with characteristics of the equations (1)

$$AC : x - \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 0, \quad BC : x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 1.$$

The Problem PT. *Find a function $u(x,y)$, which satisfies to the following conditions:*

1) *$u(x,y)$ is a regular solution of the equation* (1) in D_1 *and is a generalized solution of a class* R_2 in D_2 ;

2)$u_x,\ u_y \in C(\overline{D}_1 \setminus A \setminus B,\ D_2)$;

3) *on the a line of degeneration the splicing condition is satisfied*

$$-\lim_{y\to -0} u_y = \lim_{y\to +0} u_y$$

4) *the following boundary conditions are satisfied*

$$\{\alpha(s)A_s(u) + \beta(s)u\}|_\sigma = f(s), \qquad 0 \le s \le l,$$

$$u(x,y)|_{AC} = \varphi(x), \quad 0 \le x \le \frac{1}{2},$$

where l- a length of arc σ; $A_s[*]$ - canormal derivative.

Note that, this problem is considered in the work [1] for the equations with two lines of degeneration of the first type but for only elliptical domain.

Given work is devoted to the investigation of the uniqueness of the solution of the considered problem. Under some restrictions on given functions uniqueness theorem is proved.

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SOLUTION OF THE DIFFERENTIAL EQUATIONS INVOLVING NON-SEPARATED INTEGRAL AND POINTWISE CONDITIONS[1]

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We investigate numerical solution of the system of ordinary differential equations:

$$x'(t) = A(t)x(t) + B(t), \;\; t \in [t_0, T] \;\; , \tag{1}$$

involving non-separated multipoint integral conditions

$$\sum_{i=1}^{l_1} \int\limits_{\bar{t}_i}^{\bar{t}_i+\Delta_i} \bar{D}_i(\tau)x(\tau)d\tau + \sum_{j=1}^{l_2} \tilde{D}_j x(\tilde{t}_j) = C_0, \quad \bar{t}_i\,, \tilde{t}_j \, \in [t_0, T]\,, \tag{2}$$

where $x(t)$ is unknown n-dimensional vector function; $A(t) \neq const$ given n-dimensional matrix function, $B(t)$ given n-dimensional vector function; t_0,T, $\bar{t}_i$, $\tilde{t}_j$, $i = 1, 2, ..., l_1$, $j = 1, 2, ..., l_2$ given points of time; $\bar{D}_i\,(\tau)$, $\tilde{D}_j$ matrices of dimension $n \times n$; C_0 n- dimensional vector; $\bar{t}_i$, $\tilde{t}_j$ points of time from $[t_0, T]$; l_1, l_2 given values.

To be specific, we assume, without loss of generality, that

$$\min\left(\overline{t_1}\,, \; \tilde{t}_1\,\right) = t_0, \tag{3}$$

$$\max\left(\overline{t_{l_1}} \; + \Delta_1, \; \tilde{t}_{l_2}\,\right) = T, \tag{4}$$

and also, for all $i = \overline{1, l_1}$, $j = \overline{1, l_2}$ there takes place the natural condition

$$\tilde{t}_j \bar{\in} \left[\bar{t}_i, t_i + \Delta_i\right]. \tag{5}$$

In the work, we propose a numerical solution to problem (1), (2) based on the operation of convolution of integral conditions into local ones. This operation allows to reduce the solution to the initial problem to the solution to a Cauchy problem with respect to a system of ordinary differential equations and linear algebraic equations. We show the stability of the proposed calculating schemes. We also make a comparison with other possible methods and approaches to the solution to problem (1),(2).

Numerous numerical experiments have been carried out on specially constructed test problems with the application of the formulas and schemes of numerical solution proposed in the work. Results of experiments show sufficiently high efficiency of the approach described.

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MEAN MODULS OF SMOOTHNESS MODULE BAY V.I.KOLYADA IN THE PROBLEM OF DISCRETIZATION OF SOLUTIONS TO THE WAVE EQUATION

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In this paper we study the Cauchy problem for the wave equation $(s = 1, 2, \dots)$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x_1^2} + ... + \frac{\partial^2 u}{\partial x_s^2} \quad \left(u = u\left(x, t\right),\ 0 \le t \le 1,\ x \equiv (x_1, \dots, x_s) \in R^s\right), \tag{1}$$

$$u(x, 0) = f_1(x) \in W_2^{\omega_{r_1}, \dots, \omega_{r_s}}\left(0, 1\right)^s, \quad \frac{\partial u}{\partial t}\left(x, 0\right) = f_2\left(x\right) \in W_2^{\nu, \dots, \nu}\left(0, 1\right)^s. \tag{2}$$

By $W_2^{\omega_{r_1}, \dots, \omega_{r_s}}\left(0, 1\right)^s$ we understand the set of all summable 1-periodic in each of its variables functions $f(x) = f(x_1, \dots, x_s)$ with Fourier–Lesbegue coefficients $\hat{f}(m)$ satisfying the condition

$$\sum_{m=(m_1,\dots,m_s)\in Z^s} \left|\hat{f}(m)\right|^2 \cdot \left(\omega_{r_1}^{-2}\left(\frac{1}{\overline{m_1}}\right) + \dots + \omega_{r_s}^{-2}\left(\frac{1}{\overline{m_s}}\right)\right) \le 1,$$

$\overline{m_j} = \max\{1, |m_j|\}, j = 1, \dots, s.$

Class $W_2^{\omega_{r_1}, \dots, \omega_{r_s}}\left(0, 1\right)^s$ of anisotropic, i.e. every function of this class for each variable its smoothness, not necessarily power. In studying the properties of this class, we are talking about the outcome, or how else to say, the total effect, in this case, the sampling rate.

In the theorems of embedding a summary characteristics introduced V.I. Kolyada [1], though, then it turned out that not always the best possible conditions for investments are expressed through the average modulus of smoothness.

The following theorem shows that the exact order of sampling solutions to the wave equation is completely described in terms of average moduli of smoothness.

Theorem.(*informative cardinality of all admissible linear functionals). Let* $\{\omega_{r_1}, \dots, \omega_{r_s}\}$ and ω_ν *be a system and a particular function of smoothness module type of orders* $r_1, \dots, r_s$ *and* ν *respectively, satisfying conditions*

$$\sum_{m\in Z^s}\left(\left(\overline{m_1}\right)^4\omega_{r_1}^{-2}\left(\frac{1}{\overline{m_1}}\right) + \dots + \left(\overline{m_s}\right)^4\omega_{r_s}^{-2}\left(\frac{1}{\overline{m_s}}\right)\right) < \infty, \sum_{m\in Z^s}\left(\left(\overline{m_1}\right)^4\omega_{\nu}^{-2}\left(\frac{1}{\overline{m_1}}\right) + \dots + \left(\overline{m_s}\right)^4\omega_{\nu}^{-2}\left(\frac{1}{\overline{m_s}}\right)\right) < \infty$$

and the inequalities

$$\omega_\alpha\left(\eta \cdot \xi\right) \le C(\omega_\alpha) \cdot \omega_\alpha\left(\eta\right) \cdot \omega_\alpha\left(\xi\right) \ \ (\alpha = r_1, \dots, r_s, \nu)$$

for some positive $C(\omega_{\alpha_j})$ *and for all* $0 < \xi < \eta \le 1$, $(\alpha = r_1, \dots, r_s,\ \nu)$.

Then for problem (1) and (2) the relations

$$\min_{\substack{N_1+N_2=N,\\ N_1\ge 2, N_2\ge 2}} \inf_{\substack{l_1^{(1)},\dots,l_N^{(1)},\\ l_1^{(2)},\dots,l_N^{(2)}-\\ ,\varphi_N}} \sup_{\substack{f_1\in W_2^{\omega_{r_1},\dots,\omega_{r_s}}(0,1)^s\\ f_2\in W_2^{\omega_\nu,\dots,\omega_\nu}(0,1)^s}} \Big\| u\left(\cdot; f_1, f_2\right) - \varphi_N\Big(l_1^{(1)}\left(f_1\right), \dots, l_1^{(N_1)}\left(f_1\right),$$

$$l_2^{(1)}\left(f_2\right), \dots, l_2^{(N_2)}\left(f_2\right); \cdot\Big)\Big\|_{L^{2,\infty}((0,1)^s\times[0,+\infty))} \underset{s,\omega_{r_1},\dots,\omega_{r_s},\omega_\nu}{\asymp} \min_{\substack{N_1+N_2=N,\\ N_1\ge 2, N_2\ge 2}} \left(\left(\prod_{j=1}^{s}\omega_{r_j}^*\right)^*\left(\frac{1}{N_1}\right) + \frac{\left(\left(\omega_\nu^*\right)^s\right)^*\left(\frac{1}{N_2}\right)}{\sqrt[s]{N_2}}\right)$$

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ON A GLOBAL SOLVABILITY ONE OF THE INITIAL-BOUNDARY VALUE PROBLEM OF HEAT CONVECTIONS FOR THE KELVIN-VOIGHT FLUIDS

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We consider the initial-boundary value problem with free surface conditions for the modified equations (for Kelvin-Voigt fluids) of heat convection:

$$\vec{v}_t - \nu\Delta\vec{v} + v_k\vec{v}_{x_k} + \mathrm{grad}p - \chi\Delta\vec{v}_t = \vec{f}(x,t) + g\vec{\gamma}S\vec{\gamma} = (0,0,1), \tag{1}$$

$$\mathrm{div}\vec{v} = 0, \;\; S_t - \lambda\Delta S + (\vec{v}\cdot\nabla)S = q(x,t) \tag{2}$$

$$\vec{v}|_{t=0} = \vec{v}_0(x), \mathrm{S}|_{t=0} = \mathrm{S}_0(x), \quad \vec{v}_n|_{\partial \mathrm{Q_T}} = 0, (\mathrm{rot}\vec{v}\times n)|_{\partial Q_T} = 0, \mathrm{S}|_{\partial Q_T} = 0, \tag{3}$$

where, $\vec{v}_n$ is a normal component of the vector velocity of fluid $\vec{v}$ to boundary $\partial\Omega$, S-temperature, p- pressure, $\vec{f}$-external forces, ν, χ, λ-some physical positive constants.
The initial-boundary value problem (1)-(3) is studied in the cylinder $Q_T = \Omega\times[0,T]$, here $\Omega\in R^n$, $n = 2,3$ is a bounded domain with smooth boundary $\partial\Omega$. The main results of this work are the following theorems.

Theorem 1. *Suppose the following conditions are satisfied:*

$$\begin{array}{c} \vec{v}_0(x)\in \overset{\circ}{J}{}^2_n(\Omega), \;\; \Omega\subset R^m, \;\; \mathrm{m}=2,3, \;\; \partial\Omega\in \mathrm{C}^2, \;\; S_0(x)\in \overset{\circ}{W}{}^1_2(\Omega) \\ \vec{f}(x,t)\in L_2(0,T;L_2(\Omega)), \; q(x,t)\in L_2(0,T;L_2(\Omega)). \end{array} \tag{4}$$

Then the initial-boundary value problem (1)-(3) *has a unique solution* $(\vec{v},S,p)$, *such that:*

$$\begin{array}{c} \vec{v}\in L_\infty\left(0,T;J^2_n\right)\bigcap W^1_2\left(0,T;J^2_n\right), \quad \nabla p\in L_\infty(0,T;L_2(\Omega)), \\ S\in L_\infty\left(0,T;\overset{0}{W}{}^1_2(\Omega)\right)\bigcap L_2\left(0,T;W^2_2\bigcap W^1_2(\Omega)\right), \quad S_t\in L_2(0,T;L_2(\Omega)), \end{array} \tag{5}$$

and satisfies the estimates:

$$\begin{array}{c} \|\vec{v}\|^2_{L_\infty(0,T;H^2(\Omega))} + \|S\|^2_{L_\infty(0,T;\overset{\circ}{W}{}^1_2(\Omega))} + \|\vec{v}_t\|^2_{L_2(0,T;H^2(\Omega))} + \|S_t\|^2_{L_2(0,T;L_2(\Omega))} + \\ + \|\nabla p\|^2_{L_\infty(0,T;L_2(\Omega))} + \|S\|^2_{L_2(0,T;W^2_2(\Omega)\bigcap\overset{\circ}{W}{}^1_2(\Omega))} \le C_1\left(\nu^{-1},\;\chi^{-1},\;\lambda^{-1},\;\Omega,,\|\vec{v}_0\|,\|S_0\|,\;\|f,q\|^2_{2,Q_T}\right). \end{array} \tag{6}$$

Theorem 2. *Suppose the conditions* (4) *satisfied and* $\vec{f}_t(x,t)\in L_2(0,T;L_2(\Omega))$. *Then the problem* (1)-(3) *has a unique smooth solution* $(\vec{v},S,p)$, such that satisfies (5), *and* $\vec{v}\in W^1_\infty\left(0,T;J^2_n\right)$ *which satisfies the estimates* (6) *and*

$$\|\vec{v}_t\|^2_{L_\infty(0,T;H^2(\Omega))} + \|qradp\|^2_{L_\infty(0,T;L_2(\Omega))} \le C_2.$$

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ONE-VALUED SOLVABILITY OF THE INVERSE PROBLEM FOR THE MAGNETIC HYDRODYNAMIC OF INCOMPRESSIBLE VISCOUS FLUIDS

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In works [1-2] inverse problems for the system of Navier-Stokes equations with final overdetermination are investigated. Let us consider inverse problem for magnetic hydrodynamic in the cylinder $Q_T = \Omega \times [0,T],\ \Omega \subset R^2$

$$\frac{\partial \vec{v}}{\partial t} + \sum_{k=1}^{2} v_k \vec{v}_{x_k} - \frac{\mu}{\rho}\sum_{k=1}^{2} H_k \vec{H}_{x_k} - \nu\Delta\vec{v} = -\frac{1}{\rho} grad\left(p + \frac{\mu\vec{H}^2}{2}\right) + g(x,t)\vec{f}(x), \tag{1}$$

$$\frac{\partial \vec{H}}{\partial t} + \frac{1}{\sigma\mu} rotrot\vec{H} - rot\left[\vec{v}\times\vec{H}\right] = \frac{\xi(x,t)}{\sigma\mu} rot\vec{j}(x)\,, \tag{2}$$

$$div\vec{v} = 0, \quad div(\mu\vec{H}) = 0. \tag{3}$$

$$\vec{v}(x,0) = \vec{v}_0(x),\ \ \vec{H}(x,0) = \vec{H}_0(x), \tag{4}$$

$$\vec{v}|_S = 0, \quad H_n|_S = 0, \quad \left.\frac{\partial H_2}{\partial x_1} - \frac{\partial H_1}{\partial x_2}\right|_S = 0 \tag{5}$$

$$\vec{v}(x,T) = \vec{U}(x),\ \vec{H}(x,T) = \vec{\Psi}(x),\ \nabla p(x,T) = \nabla\pi(x). \tag{6}$$

Theorem 1. *Let* $\Omega \subset R^2,\ g,\ g_t \in C\left(\bar{Q}_T\right),\ \xi,\ \xi_t \in C\left(\bar{Q}_T\right),\quad |g(x,t)|g_T > 0, |\xi(x,t)|\xi_T > 0$ for $x \in \Omega,\ \vec{U}(x) \in W_2^2(\Omega) \cap \overset{0}{J}_1(\Omega),\ \vec{\Psi}(x) \in \hat{J}(\Omega), \vec{v}_0(x) \in W_2^2(\Omega) \cap \overset{0}{J}_1(\Omega),\ \vec{H}_0(x) \in \hat{J}(\Omega), \nabla\pi(x) \in G(\Omega)$. *Then operators A and B are quite continuous from* $L_2(\Omega)$ to $L_2(\Omega)$.

Theorem 2. *If the condition of theorem 1 is fulfilled, and also the following inequalities are true*

$$\frac{1}{\sigma} > \mu c_1\left(\left\|\vec{U}\right\|_{4,\Omega} + \left\|\vec{\Psi}\right\|_{4,\Omega}\right), \quad \nu > c_1\left(\left\|\vec{U}\right\|_{4,\Omega} + \frac{\mu}{\rho}\left\|\vec{\Psi}\right\|_{4,\Omega}\right).$$

Then in order for the problem (1)-(6) to have a solution it is necessary and sufficient to have a solution for equations operators in $L_2(\Omega)$.

Theorem 3. *Let the condition of theorem 2 be fulfilled, and also the inequality below is true*

$$M_1 + \left\|\vec{\aleph}\right\|_{2,\Omega} + \left\|\vec{\lambda}\right\|_{2,\Omega} < 1,$$

then a solution to the inverse problem (1)-(6) exists.

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ABOUT ONE SYSTEM OF FIRST ORDER PARTIAL DIFFERENTIAL EQUATIONS WITH SINGULAR LINES

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Let $0<\varphi_0\leq 2\pi,\ 0<\varphi_1,\varphi_2<\varphi_0$, $G=\left\{z=re^{i\varphi}:0\leq r<\infty,\ \ 0\leq\varphi\leq\varphi_0\right\}$. We consider the equation

$$2\overline{z}a_1(\varphi)\frac{\partial w}{\partial\overline{z}}+2za_2(\varphi)\frac{\partial w}{\partial z}+a_3(\varphi)w+\frac{r^\alpha b(\varphi)\overline{w}}{|y-k_1x|^\alpha}=\frac{f(\varphi)r^{\nu+\alpha}}{|y-k_2x|^\alpha} \tag{1}$$

in G, where $a_1(\varphi),\ a_2(\varphi),\ a_3(\varphi),\ b(\varphi),\ f(\varphi)\in C[0,\varphi_0]$; $k_1=\tan\varphi_1$, $k_2=\tan\varphi_2$, $0<\alpha<1$, $\nu>0$ are real numbers. Let $p>1$ if $\nu\geq 1$ and $1<p<\frac{1}{1-\nu}$ if $\nu<1$. The solutions of equation (1) we will find in the class

$$W_p^1(G)\cap C(G). \tag{2}$$

Here $W_p^1(G)$ is the Sobolev space. For $\alpha=0$ and $a_2(\varphi)\equiv 0$ such equations are studied in the papers [1] to [5]. Our goal is to study equation (1) for $\alpha\neq 0$. We proved the following theorem.

Theorem 1. *If $a_1(\varphi)\neq a_2(\varphi)$ on the interval $[0,\varphi_0]$, then the equation (1) has a variety of solutions from the class (2).*

We consider the following Dirichlet and Neuman boundary value problems.

Problem D. *Find a solution of equation (1) from the class (2) satisfying the half Dirichlet condition*

$$w(r,0)=\beta_1r^\nu, \tag{3}$$

where β_1 is a given real number.

Problem N. *Find a solution of the equation (1) from the class (2), satisfying the half Neumann condition*

$$\frac{\partial w}{\partial\varphi}(r,0)=\beta_2r^\nu, \tag{4}$$

where β_2 is a given real number.

Let $\delta=|A_\nu(0)|^2-|b_\alpha(0)|^2$, $\quad A_\nu(\varphi)=-\frac{i(\nu a_1(\varphi))+\nu a_2(\varphi)+a_3(\varphi))}{a_1(\varphi)-a_2(\varphi)}, b_\alpha(\varphi)=\frac{b(\varphi)}{|sin\varphi-k_1cos\varphi|^\alpha(\alpha(\varphi)-a_2(\varphi))}$, $a_1(\varphi)\neq a_2(\varphi)$.

We proved the following theorems.

Theorem 2. *The half Dirichlet problem D has a unique solution from class (2).*

Theorem 3. *If $\delta\neq 0$, then the half Neumann problem N has a unique solution from (2) and if $\delta=0$, then the half Neumann problem N has an infinite number of solutions from (2) having the form $w=r^\nu\psi(\varphi)$.*

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ON A SOLUTION OF VOLTERRA EQUATIONS WITH IRREGULAR SINGULARITIES

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It is known that many important issues of the asymptotic and analytic theory of Volterra integral equations still have been little studied. These are, in particular: the problem of asymptotic and analytic structure of solutions of Volterra equations with a singularity. Consider a system of Volterra equations with an irregular singularity at the point $t = 0$ of the form:

$$t^q u(t) = \int_0^t \rho(t,s)K(t,s)u(s)ds, \quad (0 < t < T), \tag{1}$$

where $u(t)$ - the unknown n - dimensional vector function, $K(t,s)$ - a square matrix of dimension $n \times n$; $\rho(t,s)$ - is a positive homogeneous $q-1$ of degree function, the function $\varphi(\tau) = \rho(1,\tau)$ is integrable on $[0,1]$, q - an integer > 1. Let

$$\varphi_k = \int_0^1 \tau^k \varphi(\tau)d\tau, \qquad (k = 0,1,2,...),$$

matrix $K(t,s)$ has continuous derivatives up to k_0 order, which k_0 is determined from

$$\varphi_{k_0} < \left\{ \max_{0<s<t<T} \|K(t,s)\| \right\}^{-1}.$$

Sufficient conditions are found under which the system of Volterra integral equations with an irregular singularity has a solution in the form

$$u(t) = p(t)\ln t + t^k u_k(t)\ln t + q(t) + t^k v_k(t),$$

where

$$p(t) = a_0 + ta_1 + t^2 a_2 + ... + t^{k-1}a_{k-1}, q(t) = b_0 + tb_1 + t^2 b_2 + ... + t^{k-1}b_{k-1},$$

with $k = k_0$, where $u_{k_0}(t)$ and $v_{k_0}(t)$ - are continuous functions on $[0,1]$, $p(t)$ and $q(t)$ - polynomials of degree $k_0 - 1$.

Example. *Volterra integral equation with irregular singular point*

$$t^2 u(t) = \int_0^t (-4t + 10s)u(s)ds$$

has a solution $u = b_0 + 4b_3 t^3$.

In the above example $K(t,s) \equiv 2$, $\rho(t,s) = -2t + 5s$ - is a homogeneous function of degree 1.

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RANDOM CHANGE OF TIME AND ASYMPTOTIC NORMALITY OF THE CAUCHY PROBLEM SOLUTION FOR ONE PARABOLIC EQUATION

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Consider the Cauchy problem for the random parabolic equation

$$\frac{\partial U_\varepsilon}{\partial t}=\frac{1}{2a(x/\varepsilon)}\frac{\partial^2 U_\varepsilon}{\partial x^2}+C(x),\quad U_\varepsilon(0,x)=f(x),\quad (t\geq 0, x\in R'),\tag{1}$$

where $a(x)>0$, $C(x)$ having finite dependency radii bounded (with probability 1) independent stationary random processes with sufficiently smooth implementations, $f(x)$ - is a given smooth non-random (or random, but independent on $a(x), C(x)$) function

The aim of the work is to prove the asymptotic normality of the solution to (1).

If we introduce a new random time with the ratio $\tau(t)=\int\limits_0^t a(W_s/\varepsilon)ds=\int\limits_{-\infty}^{+\infty} a(x/\varepsilon)\gamma(t,x)dx$ where W_s–a standard Wiener (Brownian) process, $\gamma(t,x)$ is Brownian local time ([1]), we can show that the right-hand side of (1) the operator $A_\varepsilon=\frac{1}{2a(x/\varepsilon)}\frac{\partial^2}{\partial x^2}$ is the infinitesimal operator of the process $x_\varepsilon(t)=W(\tau^{-1}(t))$, where $\tau^{-1}(t)$ is the inverse $\tau(t)$ of the function.

Then the solution of problem (1) has the representation ([2]) $U_\varepsilon(t,x)=E_x[f(x_\varepsilon(t))+\int\limits_0^t C(x_\varepsilon(s))ds]$, where the sign E_x indicates a conditional averaging over the trajectories of the process $x_\varepsilon(t)$ of the subject $x_\varepsilon(0)=x$.

Further, using properties of Brownian local time $\gamma(t,x)$ can be shown so that for $\tau_\varepsilon(t)$ there exists the relation $\tau_{\varepsilon(t)}=\sqrt{\varepsilon}y_\varepsilon(t)+\langle a\rangle t$, where $y_\varepsilon(t)=\varepsilon^{-\frac{1}{2}}\int\limits_{-\infty}^{+\infty}(a(x/\varepsilon)-\langle a\rangle)\gamma(t,x)dx$ is the asymptotic normal (with zero mathematical expectation and a finite dispersion) process. Now writing the Laplace transform solution $\nu_\varepsilon(t,x)=E_x f(x_\varepsilon(t))$ of the equation (1) when $c=0$, after the integrals obtained in the old days t replacing by new random times $\tau_\varepsilon(t)$ and reducing the resulting integrals to the convenient forms for finding the inverse transformations. Finally, passing to the inverse transformations of the solutions $\nu_\varepsilon(t,x)$ we can establish the asymptotic normality of the solution to (1) when $c=0$ (u.[3]). Similarly, but more technically difficult to prove the asymptotic normality of the solutions (1) when $f(x)=0$ (i.e. asymptotic normality $E_x\int\limits_0^t C(W_s)ds$).

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ON AVERAGING THE MAGNETIC FIELD EQUATION IN A MULTI-SCALE RANDOM FLOWS WITH UPDATE AND MUTUAL DEPENDENCES

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In this paper we consider the problem of averaging the equations of magnetic induction

$$\frac{\partial \vec{H}}{\partial t} = \nu_m \Delta \vec{H} - (\vec{V}, \nabla)\vec{H} + (\vec{H}, \nabla)\vec{V}, \quad \vec{H}(0, x) = \vec{H}_0(x) \quad (t \geq 0, x \in R^3), \tag{1}$$

where $\vec{H}(t, x) = \vec{H} = (H_1, H_2, H_3)$ is a magnetic field, ν_m is a constant coefficient of magnetic diffusion, $\vec{H}_0(x)$ –is a given initial magnetic field, $\vec{V}(t, x) = \vec{V} = (V_1, V_2 V_3)$ - is a given incompressible random velocity field of the form

$$\vec{V}(t, x) = (\sqrt{\tau})^{-1} \sum_{j=0}^{N} \alpha_j \vec{V}_{k-j}(x). \tag{2}$$

In the representation (2) N (natural) and α_j- is a given number; $\vec{V}_k(x) = \vec{V}(t, x), t \in (k\tau, (k+1)\tau), i > 0, k = 0, 1, ...,$ - independent homogeneous random velocity fields with zero mean and covariance matrices $\delta = \|\delta_{ij}\|_{i,j=1}^{3.3}$.

Following the ideas of [1], [2], we will describe the magnetic diffusion as a random walk along Brownian trajectories $\vec{W}_t$. To do this we shall construct a random process $\vec{\xi}_t$ as the solution of stochastic differential equations for the operator $A = \nu_m \Delta - (\vec{V}, \nabla)$,of the right-hand side of (1)

$$d\vec{\xi}_s = \sqrt{2\nu_m} W_s - \vec{V}(t - s, \vec{\xi}_s)ds, \vec{\xi}_0 = x. \tag{3}$$

Then, with respect of the speed, the trajectory of a fluid particle at the moment of time $k\tau, 0 < \tau << 1$ has the form

$$\vec{\xi}_{k\tau} = \vec{\xi}_{(k-1)\tau} - \int_0^\tau \vec{V}(k\tau - s, \vec{\xi}_s)ds + \sqrt{2\nu_m}(\vec{W}_{k\tau} - \vec{W}_{(k-1)\tau}), \tag{4}$$

with $\vec{\xi}_{k\tau}$ and $\vec{\xi}_{j\tau}$ independent, and if $|k - j| > N$ ($\bar{\tau} = N\tau$ -is length of "memory").

To calculate (up to τ) the coefficients of turbulent diffusion and the average helicity of an averaged equation of the magnetic field, as in we shall take into account the representation (3). Then, following the ideas of we can obtain the explicit formulas for the factors above in terms of a given velocity field (2). For example, for the turbulent diffusion coefficient β_{je} we will get the formula $\beta_{je} = (\nu_m \delta_{ji} + (\sum_{j=0}^{N} \alpha_j)^2)\delta_{ie}$.

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ON THE EXISTENCE AND UNIQUENESS OF SOLUTION OF INITIAL VALUE PROBLEM FOR DIFFERENTIAL EQUATIONS ON TIME SCALES

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Mathematical models of some natural phenomenon and physical problems are appeared as initial and boundary value problems including fractional order of ordinary and partial differential equations. We try to extend fractional order differential equations(dynamic equations) on time scales. For this, we need to define a fractional differential operator on time scales. We consider the following initial value problem:

$$ {}^{c}\Delta^{\alpha} y(t) = f(t, y(t)), \quad t \in [t_0, t_0 + \alpha] = J \subseteq \mathbb{T}, \quad 0 < \alpha < 1 \tag{1} $$

$$ y(t_0) = y_0 \tag{2} $$

where ${}^{c}\Delta^{\alpha}$ is Caputo fractional derivative operator

$$ {}^{c}\Delta^{\alpha}_{a+} h(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t (t-s)^{n-\alpha-1} h^{\Delta^n}(s)\Delta s \tag{3} $$

and the function $f : J \times \mathbb{T} \to \mathbb{R}$ is right-dense continuous function. Next we present sufficient and necessary conditions for the existence and uniqueness of problem (1)-(2).

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ON THE BOUNDARY VALUE PROBLEM FOR THE STRONG LOADED HEAT CONDUCTION OPERATOR

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We consider the boundary value problems for a strong loaded heat conduction operator in a quarter-plane which relates to the class of functional-differential operators has the form $Lu + \lambda Bu$, where L differential part and B is the loaded part.

Let $R_+ = (0, \infty)$. In the domain $Q = \{x \in R_+, t \in R_+\}$, we consider the following conjugate boundary value problems.

$$L_\lambda u = f \Longleftrightarrow \begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} + \lambda \cdot \dfrac{\partial^k u}{\partial x^k}\Big|_{x=a} = f, \\ u(x,0) = 0, \;\; u(0,t) = 0; \end{cases} \tag{1}$$

$$L_\lambda^* v = g \Longleftrightarrow \begin{cases} -\dfrac{\partial v}{\partial t} - \dfrac{\partial^2 v}{\partial x^2} + \bar{\lambda} \cdot \delta^{(k)}(x-a) \otimes \displaystyle\int_0^\infty v(\xi,t)d\xi = g, \\ v(x,\infty) = 0, \;\; v(0,t) = v(\infty,t) = v_x(\infty,t) = 0, \end{cases} \tag{2}$$

where $a = \text{const}, \; a > 0, \; k \geq 3$.

Unlike the loaded differential equations studied early [1-4], the operator in question is peculiar since the order of the derivative in the loaded summand is greater than to the order of the differential part of the operator. Moreover, the load operator B in the generalized spectral problem $Lu + \lambda Bu = 0$ is not invertible. Such operator is called *strong loaded*.

The lines, described by the equation

$$|\lambda| = \frac{\left(a/\sqrt{2}\right)^{k-2}}{\left|\arg\lambda + \left(2n + \dfrac{k-2}{4}\right)\pi\right|^{k-2}} \cdot \exp\left|\arg\lambda + \left(2n + \frac{k-2}{4}\right)\pi\right|$$

divide the complex $\lambda-$ plane into some disjoint domains D_m, $m = 0, 1, 2, \ldots$

We demonstrate that the boundary value problems (1), (2) under consideration is Noetherian, if $\lambda \in \{\mathbb{C} \setminus D_0\}$. Moreover,it is shown. if $\lambda \in D_m$, then $\dim \operatorname{Ker}(L_\lambda^*) = 2m$, and $\dim \operatorname{Ker}(L_\lambda) = -2m$.

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ABOUT CONDITIONS FOR THE SOLVABILITY OF A CLASS OF THIRD-ORDER DIFFERENTIAL EQUATIONS

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We study the qualitive properties of solutions of the following equation from $L_p \equiv L_p(R)$

$$L_\lambda y \equiv -p_1(x)\left(p_2(x)\left(p_1(x)y'\right)'\right)' + [q(x) + ir(x) + \lambda]y = f(x) \tag{1}$$

which defined on R.

The function $y(x) \in L_p(R)$ is called the solution of equation (1), if there exists a sequence $\{y_n\}_{n=1}^{\infty}$ of infinitely differentiable and compactly supported functions such that $\|y_n - y\| \to 0$, $\|L_\lambda y_n - f\|_p \to 0$ if $n \to \infty$.
$C^{(l)}(R)$ - space is continuous and bounded functions having continuous and bounded derivatives up to order l. The following theorem was proved.

Theorem. *Let $\lambda \geq 0$, the functions $q(x)$, $r(x)$ be continuous, $p_1(x) \in C^{(3)}(R)$, $p_2(x) \in C^{(2)}(R)$ and the following conditions hold*

$$q \geq 1, \;\; r \geq 1, \;\; p_j \geq 1, W(x) = |q(x) + ir(x)| \geq 1,$$

$$c^{-1} \geq \frac{q(x)}{q(\eta)}, \frac{r(x)}{r(\eta)} \leq c \;\; for \;\; |x - \eta| \leq 1,$$

$$\sup_{|x-\eta|\leq 1} \frac{|W(x) - W(\eta)|}{|W(x)|^{\nu}|x-\eta|^{\mu}} < +\infty, \;\; 0 < \nu < \frac{\mu}{3} + 1, \;\; \mu \in (0, 1].$$

Then there exists a number $\lambda_0 \geq 0$ such that the equation (1) has a unique solution $y(x)$ for all $\lambda \geq \lambda_0$ and the following estimate holds:

$$\left\| p_1(x)\left(p_2(x)\left(p_1(x)y'\right)'\right)' \right\|_p^p + \|[q(x) + ir(x)]y\|_p^p \leq c_0 \|f(x)\|_p^p. \tag{2}$$

When $p_1 = p_2 = 1$ the sufficient conditions for unique solvability of the equation (1) and estimate (2) for its solution were established when $r = 0$ and $r \geq 1$ in [1].

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THE MAXIMUM PRINCIPLE OF THE NAVIER - STOKES EQUATION

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In [1] and others of the author, some basic statements were obtained to study the maximum principle for the Navier-Stokes equation (NSE). The system of nonlinear parabolic equation for kinetic energy density, and important property of this equation - the maximum principle was derived from NSE. In this work, these results are summarized and linked to the mathematical rigor and, on this base the unique solvability of weak and the existence of strong solutions of the NSE wholly in time $t \in [0, T]$, $\forall T < \infty$ was proved. Let's consider initial-boundary value problem for NSE [2] in the domain $Q = (0, T] \times \Omega$:

$$\frac{\partial U}{\partial t} - \mu \Delta U + (U, \nabla)U + \nabla P = f(t, x), \quad \mathrm{div} U = 0, \tag{1a}$$

$$U(0, x) = \Phi(x), \quad U(t, x)\big|_{\partial\Omega} = 0, \quad x \in \partial\Omega, \quad U = (U_1, U_2, U_3), \tag{1b}$$

where $x \in \Omega \subset R_3$; **i**) $f(t, x) \in C(\bar{Q}) \cap \mathring{\mathbf{J}}(Q)$; **ii**) $\Phi(x) \in C(\bar{\Omega}) \cap W^1_{2,0}(\Omega) \cap \mathring{\mathbf{J}}(\Omega)$.

From (1a) under $f = 0$, we obtain a nonlinear parabolic equation for the density of kinetic energy $E = 1/2(U_1^2 + U_2^2 + U_3^2)$:

$$\mathbb{L}E \equiv \frac{\partial E}{\partial t} - \mu \Delta E + \mu \sum_{\alpha=1}^{3} |\nabla U_\alpha|^2 + (\nabla E, U) + (\nabla P, U) = 0. \tag{2}$$

Theorem 1. *Suppose $\bar{Q} = [0, T] \times \bar{\Omega}$ is a cylindrical domain with boundary $[0, T] \times \partial\Omega$ functions $(U, E) \in C(\bar{Q}) \cap C^2(Q) \wedge P \in C^1(Q)$ satisfy the equations (1a), (2). Then the function $E(t, x)$ takes its maximum in the cylinder $\bar{Q}$ on its lower base $\{0\} \times \bar{\Omega}$ or on lateral area $[0, T] \times \partial\Omega$, i.e.,*

$$E(t, x) \leq \max\Big\{ \sup_{t=0 \bigwedge x \in \bar{\Omega}} E(t, x), \sup_{t \in [0,T] \bigwedge x \in \partial\Omega} E(t, x) \Big\} = C - const. \tag{3}$$

and for the solution of problem (1) an estimate is valid:

$$\|U\|_{C(\bar{Q})} \leq \|\Phi\|_{C(\bar{\Omega})} + T\|f\|_{C(\bar{Q})} \equiv A_1, \ \forall T < \infty, \ \|U\|_{C(\bar{Q})} = \max_{1 \leq \alpha \leq 3} \sup_{\bar{Q}} |U_\alpha(t, x)|. \tag{4}$$

Definition[1)]**.** *We call as the weak generalized solution of the a full initial boundary value problem of the Navier-Stokes equations (1) vector-function U and function P from space*
$U \in C(\bar{Q}) \cap L_2\big(0, T; W^1_{2,0}(\Omega)\big) \cap \mathring{\mathbf{J}}(Q)$; $\qquad P \in L_2\big(0, T; W^1_2(\Omega)\big)$ *and satisfying the identities*

$$\int\limits_Q \Big(-U \frac{\partial Z}{\partial t} + \mu \sum_{k=1}^{3} \nabla U_k \nabla Z_k + (U, \nabla)UZ \Big) \, dx \, dt = \int\limits_\Omega \Phi Z(0, x) dx + \int\limits_Q f Z dx \, dt, \quad \forall Z; \tag{5}$$

$$\int\limits_Q \nabla P \, \nabla \eta \, dx \, dt = - \int\limits_Q (U, \nabla)U \, \nabla \eta \, dx \, dt, \quad \forall \eta. \tag{6}$$

Theorem 2. *If input data f and Φ satisfy requirements* **i**) *and* **ii**)*, then problem (1) has the unique weak generalized solutions U and P satisfying the identities (5), (6) at any Z and η from the definition.*

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1) Thanks to the principe of maximum, weak solution be regarded in more the restricted class of function, than in [2].

ABOUT COMPLETENESS OF ROOT VECTORS OF THE DEGENERATING ELLIPTIC OPERATOR

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In a restangle $\Omega = \{(,y) : 0 < x < 2\pi, 0 < y < 1\}$ the problem is considered:

$$Lu + \lambda u = -K(y) u_{xx} - u_{yy} + a(y) u_x + \lambda u = f(x,y) \tag{1}$$

$$\begin{cases} u(0,y) = u(2\pi,y) \\ u_x(0,y) = u_x(2\pi,y) \end{cases} \tag{2}$$

$$u(x,0) = u(x,1) = 0 \tag{3}$$

On equation factors (1) the following restrictis will be determined:

$$K(y), a(y) \in G_0, y \in [0,1].$$

In work [1] it is shown $K(y), a(y) \in G_0$,that the operator L of the problem restricted is reversible and coercive.

The basic result of the given work is:

Theorem 1. *Let $K(y),\ a(y) \in G_{\ddagger 0}$.*
Then the system of root vectors resolvent of the operator L is full in $L_2(\Omega)$.

Definition 1. *Function $u \in H$ is called as a root vector of operator A, answering to proper number, if $(A - \lambda E)^n u = 0$ at natural n.*

Definition 2. *The operator in hilbertian space is called dissipative, if*

$$Jm(Au,u) \geq 0, \quad u \in D(A).$$

Definition 3. Let A *-linear quite continuous operator and let $|A| = \sqrt{A * A}$. The proper numbers of the operator $|A|$* are $S-$numbers the operator A *and are designated*

$$S_j(A) = \lambda_j(|A|), j = 1, 2, ...$$

Definition 4. *The compact operator A is called nuclear, if $\sum_{j=1}^{\infty} S_j(A) < \infty$.*

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ON A STABILITY OF THE SOLUTION OF THE SYSTEM OF DIFFERENTIAL EQUATIONS

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Let's consider the following system of differential equations

$$\dot{x} = A(t)\,x + f(t,x)\,, \tag{1}$$

where vectorial function $f(t,x)$ continuously on t in $I = [t_0, +\infty)$ and continuously differentiable on x in the domain $G = I \times R^n$, $f(t,0) = 0$, $A(t)$ continuous matrix on I. Besides, $f(t,x)$ satisfies to the smallness condition

$$\|f(t,x)\| \leq \psi(t)\|x\|^m$$

where $m > 1$ and $\psi(t)$ is the continuous positive function on I, with the zero generalized Lyapunov's index

$$q(t) = \int\limits_{t_0}^{t} \psi(\tau)\,d\tau, \quad q(t) \uparrow +\infty \; in \; t \uparrow +\infty, \quad q(t) < t.$$

(in [1] $q(t) > t$).

Theorem 1. *Let the system* (1) *satisfies, to the following conditions:*

1. The system of the first approximation collectively correct on Lyapunov and Lyapunov's all generalized indexes toward $q(t)$ negative;

2. Function $q(t)$ satisfies to the inequality

$$\int\limits_{t_0}^{+\infty} \frac{1}{\exp(\delta q(\tau))} d\tau < +\infty, \quad (\delta > 0)$$

Trivial solution of the system (1) *its exponentially (toward $q(t)$) stable in the Lyapunov sense.*

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BOUNDARY INTEGRAL EQUATIONS METHOD AT THE BOUNDARY VALUE PROBLEMS FOR KLEIN-GORDON-FOKK EQUATIONS

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Boundary integral equations method is developed for solving the boundary value problems for Klein-Gordon-Fokk (KGF-)equations:

$$\Delta u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \pm m^2 u = f(x,t), \quad x \in S^- \subset R^N, \quad t \geq 0$$

with initial Cauchy conditions and Dirichlet's or Neumann conditions on boundary S of definition region S^-.

There is proved of unique of these problems including the shock waves, for which Adamar's conditions are satisfied under jumps of E energy and derivatives on their fronts:

$$[\dot{u}\nu_j + cu,_j]_{F_t} = 0, \quad \left[\dot{u} + c\sum_{j=1}^{N} \nu_j\, u,_j\right]_{F_t} = 0, \quad [E]_{F_t} = -c\left[\dot{u}\frac{\partial u}{\partial \nu}\right]_{F_t},$$

$\nu(x,t)$ is the normal to the wave front in R^N.

On the base of generalized functions method [1] the dynamical analog of Kirchhoff-Green formula for its solutions in generalized functions space are obtained as convolution shape:

$$\begin{aligned} u(x,t)H_S^-(x)\,H(t) = f * \hat{U} - \hat{U} * \tfrac{\partial u}{\partial n}\delta_S(x)\,H(t) - \sum_{j=1}^{N} \partial_j \hat{W} * \dot{u} n_j(x)\delta_S(x)\,H(t) - \\ - \sum_{j=1}^{N} \partial_j \hat{W} \underset{x}{*} u_0(x)\,n_j(x)\,\delta_S(x) - c^{-2}\hat{U} \underset{x}{*} H_S^-(x)\,\dot{u}_0(x) - c^{-2}\partial_t\left(\hat{U} \underset{x}{*} H_S^-(x)\,u_0(x)\right) \end{aligned} \quad (1)$$

Here $H_S^-(x)$ is the characteristic function of set S^-, which is equal to 0,5 on S, $H(t)$ is Heaviside function, $\hat{U}$ is fundamental solution of KGF-equations satisfying to radiation conditions, $\hat{W} = \hat{U} \underset{t}{*} H(t)$, $\delta_S(x)$ is the simple layer on S, $n(x)$ is the external unit normal to S.

The regular integral representation of (1) are constructed in plane case ($N = 2$). Resolute singular boundary integral equations for solving these non stationary boundary value problems are obtained.

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ON EXISTENCE AND NON-EXISTENCE OF GLOBAL SOLUTIONS OF THE CAUCHY PROBLEM FOR HIGH ORDER SEMI-LINEAR HYPERBOLIC EQUATIONS WITH ANISOTROPIC ELLIPTIC PARTS

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Lets consider the Cauchy problem for semi-linear high order hyperbolic equations with damping term

$$u_{tt} + u_t + \sum_{i=1}^{n} \sum_{l_i \le \beta_i \le L_i} (-1)^{\beta_i} D_{x_i}^{2\beta_i} u = f(u), \tag{1}$$

$$u(0,x) = \varphi(x), \qquad u_t(0,x) = \psi(x), \tag{2}$$

where $l_i, L_i \in N$, $f(u) \sim u^p$, $u \in R$.

We obtain the critical exponent question of global solvability, and show absence of global solutions for problem (1), (2). Critical exponent obtained in this article is the number

$$p_c = 1 + \frac{2}{\sum\limits_{i=1}^{n} \frac{1}{l_i}},$$

for

$$\sum_{i=1}^{n} \frac{1}{l_i} < 2,$$

and

$$p_c = 2,$$

for

$$\sum_{i=1}^{n} \frac{1}{l_i} \ge 2.$$

It is proved that if $1 < p \le p_c$ then, there exists "sufficiently small" initial data for which the corresponding problem (1), (2) has no weak solutions. If $p_c < p < \infty$ then for sufficiently small initial data there exists a global solution.

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DATA OF ONE NONLINEAR DIFFERENTIAL EQUATION WITH THE PARTIAL TO DERIVATIVES TO THE CAUCHY-RIEMANN EQUATION

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Let $D \subset R^2$-limited, convex in a direction x_2, plane domain.
Let's consider the equation:

$$\Delta u(x) + \delta\, e^{u(x)} = 0, \tag{1}$$

which after replacement

$$u(x) = \ln \vartheta(x). \tag{2}$$

Will become:

$$\vartheta(x)\Delta\vartheta(x) - \sum_{k=1}^{2}\left(\frac{\partial\vartheta(x)}{\partial x_k}\right)^2 + \delta\vartheta^3(x) = 0. \tag{3}$$

After transformation

$$\frac{\partial\vartheta(x)}{\partial x_k} = W_k(x)\vartheta(x), \qquad k = 1, 2. \tag{4}$$

We receive a following equation:

$$\frac{\partial W_1(x)}{\partial x_1} + \frac{\partial W_2(x)}{\partial x_2} + \delta\vartheta(x) = 0. \tag{5}$$

Further from (4) we will receive:

$$\frac{\partial}{\partial x_1}\left(W_2(x)\vartheta(x)\right) = \frac{\partial}{\partial x_2}\left(W_1(x)\vartheta(x)\right),$$

$$\frac{\partial W_1(x)}{\partial x_2} - \frac{\partial W_2(x)}{\partial x_1} = 0, \tag{6}$$

thus, we have received the following system:

$$\begin{cases} \frac{\partial W_1(x)}{\partial x_1} + \frac{\partial W_2(x)}{\partial x_2} = -\delta\vartheta(x), \\ \frac{\partial W_1(x)}{\partial x_2} - \frac{\partial W_2(x)}{\partial x_1} = 0. \end{cases} \tag{7}$$

Multiplying the first equation on i and the second by unit, and putting them, accepting a designation:

$$W(x) = W_1(x) + iW_2(x). \tag{8}$$

We have:

$$\frac{\partial W(x)}{\partial x_2} + i\frac{\partial W(x)}{\partial x_1} = -i\delta\vartheta(x). \tag{9}$$

Considering that

$$W(x-\xi) = \frac{1}{2\pi}\frac{1}{x_2 - \xi_2 + (x_1 - \xi_1)} \tag{10}$$

is the fundamental solution of the Cauchy-Riemann equation will consider a boundary problem for the nonlinear equation (1).

BOUNDARY PROBLEM FOR THE DIFFERENTIAL EQUATION OF FRACTIONAL ORDER

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Let's consider the following problem:

$$\sum_{k=0}^{n} a_k D^{\frac{k}{2}} y(x) = 0, \quad 0 < a < x < b, \tag{1}$$

where $n \in N$, is a fixed number, a_k- material constant factors, $D^{\frac{1}{2}}$ - differential operator of an order $\frac{1}{2}$ understood in sense of Liuvill, with boundary conditions:

$$\sum_{k=0}^{n-1} \left\{ \alpha_{jk} D^{\frac{k}{2}} y(x) \Big|_{x=a} + \beta_{jk} D^{\frac{k}{2}} y(x) \,|_{x=b} \right\} = \gamma_j, \quad j = 1, n, \tag{2}$$

where α_{jk},β_{jk} and γ_j are real-constant numbers and conditions (2) linearly independent.

Proceeding from function of Mittag-Leffera, there is equation linearly independent the decision (1). Further the common decision of the equation (1) is resulted and is any the constants entering into the common decision. These constants it is defined from boundary conditions (2).

Example.

$$Dy(x) \equiv D^{\frac{1}{2}}(D^{\frac{1}{2}} y(x)) = 0, \quad x \in (1, 2) \tag{3}$$

$$y(1) = y_1, \quad y(2) = y_2 \tag{4}$$

the common decision of the equation (3) looks like:

$$y(x) = C_0 \frac{x^0}{0!} + C_1 \frac{x^{-\frac{1}{2}}}{(-\frac{1}{2})!},$$

where $0! = 1 \quad (-\frac{1}{2})! = (\frac{1}{2})$ and - there are values scale of functions of Euler. Defining $!_0$ and $!_1$ from boundary conditions (4), we will receive function:

$$y(x) = \frac{\sqrt{2} y_2 - y_1}{\sqrt{2} - 1} + \frac{\sqrt{2}}{\sqrt{2} - 1} (y_1 - y_2) x^{-\frac{1}{2}}$$

which $y_1 = y_2$ at turns to the known classical decision the equation (3).

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INVESTIGATION OF THE MIXED PROBLEM FOR THE EQUATION OF A HIGH ORDER

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In the talk the following problem is considered

$$\sum_{k=0}^{3} a_k(x)\frac{\partial^{6-k}\vartheta(x,t)}{\partial t^k \partial x^{6-2k}} + \sum_{k=0}^{2} b_k(x)\frac{\partial^{5-k}\vartheta(x,t)}{\partial t^k \partial x^{5-2k}} = f(x,t),\ x\in(0,1)\,,\quad t>0, \tag{1}$$

with to boundary

$$\left\{\sum_{k=0}^{2}\alpha_{\nu k}\left.\frac{\partial^{5-k}\vartheta(x,t)}{\partial t^k\partial x^{5-2k}}\right|_{x=0} + \beta_{\nu k}(x)\left.\frac{\partial^{5-k}\vartheta(x,t)}{\partial t^k\partial x^{5-2k}}\right|_{x=1}\right\} = \varphi(t) \tag{2}$$

$\nu=\overline{1,6}$ and $t>0$, and initial conditions

$$\frac{\partial^s\vartheta(x,t)}{\partial t^s}\,|_{t=0} = \Phi_s(x) \qquad (s=0,1,2) \tag{3}$$

at $x\in(0,1)$. For this propose the method of planimetric integral of Rasulova M. L is applied. Under certain conditions it is possible to prove existence and uniqueness theorems problem for this in the form

$$\vartheta(x,t) = \frac{1}{\pi\sqrt{-1}}\int_S \lambda e^{\lambda^2 t}d\lambda\int_0^1 G(x,\xi,\lambda)F(\xi,\lambda)d\xi,$$

where S- infinitely opened contour entirely located in area

$$R_\delta = \left\{\lambda:\ \ |\lambda|>R\quad -\frac{\pi}{4}-\delta<\arg\lambda\le\frac{\pi}{4}\right\}.$$

Infinitely remote which parts coincide with continuations of beams, $\arg\lambda=-\frac{\pi}{4}-\delta$, and $\arg\lambda=\frac{\pi}{4}$. $G(x,\xi,\lambda)$ - Green's function of the spectral problem corresponding to the problem (1) - (3), $F(x,\lambda)$ it is defined by the formula:

$$F(x,\lambda) = f(x,\lambda)+a_3(x)\Phi_2(x)+a_3(x)\lambda^2\Phi_1(x)+b_2(x)\Phi_1'(x)+a_2(x)\Phi_1''(x)+$$

$$+a_3(x)\lambda^4\Phi_0(x)+\lambda^2(a_2(x)\Phi_0''(x)+b_2(x)\Phi_0'(x))+a_1(x)\Phi_0''''(x)+b_1(x)\Phi_0'''(x).$$

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ON THE BOUNDARY CONDITIONS IN ELLIPTIC TRANSMISSION PROBLEM THROUGH A THIN LAYER

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Let Ω be a bounded domain in R^n, $n > 1$ surrounded by a thin layer Σ_ε, where $\varepsilon > 0$ parameter which designate an order of thickness. We denote by Γ the boundary of Ω and $\Omega_\varepsilon = \bar{\Omega} \bigcup \Sigma_\varepsilon$, $\Gamma_\varepsilon = \partial\Omega_\varepsilon$. Assume that $\Sigma_\varepsilon = \{\xi + \tau\,\vec{n}(\xi) : \xi \in \Gamma,\ 0 < \tau < \varepsilon\}$, here $\vec{n}(\xi)$ the outward normal unit vector to Γ on the point ξ.

Consider transmission problem in Ω_ε:

$$-\Delta u(x) + u(x) = f(x), \qquad \forall x \in \Omega, \tag{1}$$

$$u(x) = \omega(x), \qquad \frac{\partial u(x)}{\partial n} = \delta\,\frac{\partial \omega(x)}{\partial n}, \quad \forall x \in \Gamma, \tag{2}$$

$$-\delta\,\Delta\omega(x) = \frac{1}{\varepsilon}\,g(x), \quad \forall x \in \Sigma_\varepsilon, \tag{3}$$

$$\delta\,\frac{\partial \omega(x)}{\partial n} = 0, \quad \forall x \in \Gamma_\varepsilon, \tag{4}$$

where $f(x)$, $g(x)$ given functions in Ω and Σ_ε respectively, $\partial/\partial n$ denotes the outward normal derivative, either on Γ or Γ_ε, and δ is conductivity of the layer Σ_ε. It is known that this problem is well-posed in a classical framework for elliptic equations (e.g. [1], [2]).

As $\varepsilon \to 0$ and $\delta \to \infty$ the solution of the problem (1)–(4) converges to a function $u(x)$, which is the unique solution of the elliptic equation (1) with appropriate boundary conditions. They may be of different type according to the limit values of $\alpha = \lim \varepsilon\delta$.

Theorem. *For any* $0 < \varepsilon \le \varepsilon_0$, $1 \le \delta$, $\Omega \in C^{1,1}$ *and* $f \in L_2(\Omega)$, $g \in H^1(\Sigma_\varepsilon)$ *as* $\varepsilon \to 0$ *and* $\delta \to \infty$, *the solution* $u_{\varepsilon\delta}(x)$ *of the problem* (1) – (4) *converges in* $H^1(\Omega)$ *to the solution of the elliptic equation (1) with the following boundary conditions:*

(1) *when* $\alpha = 0$: *(Neumann condition)* $\dfrac{\partial u_0(x)}{\partial n} = g_\Gamma(x)$ *on* Γ;

(2) *when* $0 < \alpha < \infty$: *(Venttsel condition)* $\dfrac{\partial u_\alpha(x)}{\partial n} + \Delta_\Gamma u_\alpha(x) = g_\Gamma(x)$ *on* Γ;

(3) *when* $\alpha = \infty$: *(nonlocal condition or flux term)* $\displaystyle\int_\Gamma \frac{\partial u_\infty(\xi)}{\partial n}\,d\xi = \int_\Gamma g_\Gamma(\xi)\,d\xi$.

Here $g_\Gamma(x)$ *is the trace of* $g(x)$ *on* Γ *and* Δ_Γ *is tangential Laplace operator on* Γ.

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A GENERATING FUNCTION AND RECURRENCE RELATIONS FOR A FAMILY OF POLYNOMIALS

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In this paper, we derive some families of polynomials. Some further results of these polynomials as generating function, rodrigues formula and recurrence relations are also dicussed.

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SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS WITH CAPUTO FRACTIONAL DERIVATIVES USING DOUBLE LAPLACE TRANSFORM

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We present the double Laplace transform of the partial fractional integrals and derivatives and use them to solve partial differential equations with Caputo fractional derivatives.

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SQUARE-LIKE FUNCTIONS GENERATED BY A WAVELET-LIKE TRANSFORM ASSOCIATED WITH THE LAPLACE-BESSEL DIFFERENTIAL OPERATOR

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In this article we introduce wavelet-like transform

$$V_t f(x) = \frac{1}{\alpha} \int_0^\infty G_{tz} f(x) w(z) dz$$

associated with Laplace-Bessel differential operator $\Delta_B = \sum_{k=1}^{n} \frac{\partial^2}{\partial x_k} + \frac{2\nu_k}{\partial x_k}\frac{\partial}{\partial x_k}$ where $G_{tz}f(x)$ is the Gene-ralized Gauss-Weierstrass semigroup and $w(z)$ is known as "function", $\int_0^\infty w(z)dz = 0$. Moreover, using the wavelet-like transform, we can define the following square-like functions

$$(Sf)(x) = \left(\int_0^\infty |V_t f(x)|^2 \frac{dt}{t} \right)^{\frac{1}{2}}.$$

So, we proved an analogue of the Calderón reproducing formula and the L_2 boundedness of the square-like functions.

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ON ASYMPTOTICAL EXPANSION OF THE SOLUTION FOR THE SINGULARLY PERTURBED DIFFERENTIAL EQUATION WITH THE TURN POINT

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It is constructed uniformly asymptotic solution of the boundary value problem for the equation

$$\varepsilon u''(x) + xu'(x) - q(x)u(x) = f(x), \tag{1}$$

$$u(0) = 0, u(1) = u^0, \tag{2}$$

where $0 < \varepsilon$ - small parameter, $x \in [0,1], u^0$ - is given constant.

Theorem. *Let is $q(x), f(x) \in C^\infty[0,1], q(0) = 2$. Then the solution of the problem (1)-(2) is presented in the form*

$$u(x) = u_0(x) + \pi_0(t) + \mu(u_1(x) + \pi_1(t)) + ... + \mu^n(u_n(x) + \pi_n(t)) + O(\mu^{n+1}),$$

$$\mu \to 0, u_k(x) \in C^\infty[0,1], \pi_k(t) \in C^\infty[0,\mu_0], \mu_0 = \mu^{-1}, u_0(1) = u^0,$$

$$u_k(1) = 0, \pi_{k-1}(\mu_0) = 0, \pi_{k-1}(0) = -u_{k-1}(0), t = \frac{x}{\mu}(k = 1, 2, ...).$$

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ON THE INCORRECT PROBLEM FOR THE POISSON EQUATION

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The problem statement. In region $\Omega = \{x, t|\ 0 < x < \pi,\ -1 < t < 1\}$ it is considered a following incorrect boundary value problem [1, 2]:

$$y_{tt}(x,t) + y_{xx}(x,t) = f(x,t),\ \{x,t\} \in \Omega, \tag{1}$$

$$y(0,t) = 0,\ y(\pi,t) = 0, \tag{2}$$

$$y(x,-1) = \varphi_1(x),\ y_t(x,-1) = \varphi_2(x). \tag{3}$$

Let's assume, that data in a problem (1)-(3) satisfy to following conditions:

$$f \in L_2(\Omega),\ \varphi_1 \in H_0^1(0,\pi),\ \varphi_2 \in L_2(0,\pi). \tag{4}$$

An optimization problem. Consider auxiliary problem: (1)-(3) with condition

$$y_t(x,1) \in \mathcal{U}_g - \text{the convex closed subset of the space } L_2(0,\pi). \tag{5}$$

Let's put in conformity (1)-(3) and (5) the following optimizing problem:

$$y_{tt}(x,t) + y_{xx}(x,t) = f(x,t), \tag{6}$$

$$y(0,t) = y(\pi,t) = 0, \tag{7}$$

$$y_t(x,-1) = \varphi_2(x),\ y_t(x,1) = \psi(x),\ \psi(x) \in \mathcal{U}_g \subset L_2(0,\pi), \tag{8}$$

and optimality functional:

$$\mathcal{J}_\alpha(\psi) = \int_0^\pi |y_x(x,-1) - \varphi_1'(x)|^2 dx + \alpha \cdot \int_0^\pi |\psi(x)|^2 dx \to \min_{\psi(x)\in\mathcal{U}_g},\ (\alpha > 0), \tag{9}$$

where ψ plays a role of control function.

As is known, in the theory of optimal control the problem (6)-(9) have unique solution. For optimization problem (6)-(9) it is received optimality conditions.

Conclusion. That the boundary problem (1)-(3) under conditions (4) has unique L_2-strong solution, necessary and safficient

$$\{\exp\{2k\} \cdot \varphi_{1k}\}_{k=1}^\infty,\ \left\{k^{-1}\exp\{2k\} \cdot \varphi_{2k}\right\}_{k=1}^\infty,\ \left\{\exp\{2k\} \cdot \|f_k(\tau)\|_{L_2(-1,1)}\right\}_{k=1}^\infty \subset l_2.$$

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DISCONTINUOUS STURM-LIOUVILLE PROBLEMS WITH EIGENPARAMETER-DEPENDENT BOUNDARY AND TRANSMISSION CONDITIONS

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In this work, we consider the following differential equation

$$ly := -p(x)[p(x)y\prime]\prime + q(x)y = \lambda y \tag{1}$$

on the set $\Omega = (0, d) \cup (d, \pi)$, where $p(x)$ and $q(x)$ are real valued functions in $L_2(o, \pi), \lambda$ is spectral parameter. Suppose that,$p(x) > 0, p(0) = 1$ and $p^{-1}(x) \in L_2(0, \pi)$.

We denote by L the boundary value problem generated by equation (1) with the boundary and transmissions conditions

$$U(y) := y(0)cos\alpha + y\prime(0)sin\alpha = 0$$

$$V(y) := a(\lambda)y(\pi) + b(\lambda)[py\prime](\pi) = 0$$

$$C(y) := \begin{cases} y(d+0) - y(d-0) = 0 \\ [p(x)y'(x)]_{x=d+0} - \beta[p(x)y'(x)]_{x=d-0} = 0 \end{cases}$$

where $a(\lambda) = a_0 + a_1\lambda + a_2\lambda^2 + ... + a_m\lambda^m$ and $b(\lambda) = b_0 + b_1\lambda + b_2\lambda^2 + ... + b_m\lambda^m$ are polynomials with real coefficients and no common zeros, $\beta \in R^+1$, $a \in [0, \pi)$ and $d \in (0, \pi)$.

In the present paper, we consider a discontinuous Sturm- Liouville operator with boundary condition that depends on the spectral parameter and transmis- sions conditions. Firstly, properties of the eigenvalues are studied. Secondly, the Prüfer's angle and the Weyl function are explanied. Finally, the unique- ness theorem for the solution of the inverse problem according to both these functions and two different eigenvalues sets in proved.

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RESOLVENT WELL POSED INNER BOUNDARY PROBLEMS FOR HELMHOLTZ EQUATION IN THE PUNCTURED REGION

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Let's consider the continuous operator K of the mapping $f(x,y) \in L_2(\Omega)$ of $h(x,y) \in \tilde{W}_2^2(\Omega_0)$. $\Omega_0 = \Omega \backslash \{M_0\}$ - punctured region, where $M_0(x_0,y_0)$ - some interior point of $\Omega = \left\{x^2+y^2<1\right\}$. Operator corresponding to the Dirichlet problem for Poisson equation the continuum region is L_0. Operator L_K corresponds to the problem

$$\Delta W(x,y) = f(x,y),\ (x,y) \in \Omega_0, \tag{1}$$

$$W(x,y)|_{\partial\Omega} - K(\Delta W)|_{\partial\Omega} = 0, \tag{2}$$

$$\alpha(W) = \alpha(K\Delta W) \tag{3}$$

$$\beta(W) = \beta(K\Delta W) \tag{4}$$

$$\gamma(W) = \gamma(K\Delta W). \tag{5}$$

Linear functional $\alpha(\cdot)$, $\beta(\cdot)$, $\gamma(\cdot)$ defined by

$$\alpha(W) = \lim_{\delta\to+0}\left\{\int\limits_{y_0-\delta}^{y_0+\delta}\left[\frac{\partial W(x_0+\delta,y)}{\partial x} - \frac{\partial W(x_0-\delta,y)}{\partial x}\right]dy + \right.$$

$$\left. + \int\limits_{x_0-\delta}^{x_0+\delta}\left[\frac{\partial W(x,y_0+\delta)}{\partial y} - \frac{\partial W(x,y_0-\delta)}{\partial y}\right]dx\right\}$$

$$\beta(W) = \lim_{\delta\to+0}\int\limits_{y_0-\delta}^{y_0+\delta}\left[W(x_0-\delta,y) - W(x_0+\delta,y)\right]dy$$

$$\gamma(W) = \lim_{\delta\to+0}\int\limits_{x_0-\delta}^{x_0+\delta}\left[W(x,y_0-\delta) - W(x,y_0+\delta)\right]dx.$$

In this case, the resolvent of L_K is given by

$$(L_K-\lambda I)^{-1}f(x,y) = (L_0-\lambda I)^{-1}f(x,y) +$$

$$+\int\limits_{\partial\Omega}\left[L_K(L_K-\lambda I)^{-1}\frac{\partial G(x,y,\xi,\eta)}{\partial \overline{n}_{\xi,\eta}}\right]KL_0(L_0-\lambda I)^{-1}f(\xi,\eta)\,ds_{\xi,\eta} -$$

$$-\alpha\left(KL_0(L_0-\lambda I)^{-1}f(x,y)\right)L_K(L_K-\lambda I)^{-1}G(x,y,x_0,y_0) -$$

$$-\beta\left(KL_0(L_0-\lambda I)^{-1}f(x,y)\right)L_K(L_K-\lambda I)^{-1}\frac{\partial G(x,y,x_0,y_0)}{\partial\xi} -$$

$$-\gamma\left(KL_0(L_0-\lambda I)^{-1}f(x,y)\right)L_K(L_K-\lambda I)^{-1}\frac{\partial G(x,y,x_0,y_0)}{\partial\eta}$$

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SOLUTION OF A BOUNDARY VALUE PROBLEM FOR THE VISCOUS TRANSONIC EQUATION

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Any viscous transonic equation (VT-equation), describing the flow of rotation bodies with the viscous and head conducting gas, has the form [1,2]

$$u_{xxx} + u_{yy} - \frac{\nu}{y} u_y = u_x u_{xx}.$$

For $\nu = 0$ the VT-equation describes the plane-parallel flow and has the form

$$u_{xxx} + u_{yy} = f(x, y).$$

In the domain $D = \{(x, y) : \; 0 < x < p, \; 0 < y < l\}$, where $p, l = const > 0$, we consider the following problem for the equation (1).

Problem $\mathbf{A_2}$. Find a regular solution of the equation (1) in the domain D, satisfying the boundary conditions

$$u_y(x, 0) = \varphi_1(x), \;\; u_y(x, l) = \varphi_2(x),$$
$$u_x(0, y) = \psi_1(y), \;\; u_{xx}(0, y) = \psi_2(y), \;\; u_{xx}(p, y) = \psi_3(y),$$

where

$$\varphi_i(x) \in C[0, p], \; i = 1, 2, \;\; \psi_j(y) \in C[0, l], j = 1, 2, 3, \;\; f(x, y) \in C^{0,2}_{x,y}\left(\overline{D}\right), \;\; f(x, 0) = f(x, l) = 0,$$

in addition, natural conditions of agreement are fulfilled.

We note that the conjugate equation to the equation (1) was studied in the works [3,4].

In the present work unique solvability of the problem A_2 is shown. In this connection, the explicit form of the solution is obtained with the help of the constructed Green function. The solution has the form:

$$2u(x, y) = \int_0^l G(x, y, p, \eta)\, \psi_3(\eta)\, d\eta + \int_0^l G(x, y, 0, \eta)\, \psi_2(\eta)\, d\eta - \int_0^l G_\xi(x, y, 0, \eta)\, \psi_1(\eta)\, d\eta -$$
$$- \int_0^p G(x, y, \xi, l)\, \varphi_2(\xi)\, d\xi + \int_0^p G(x, y, \xi, 0)\, \varphi_1(\xi)\, d\xi + \iint_D G(x, y, \xi, \eta)\, f(\xi, \eta)\, d\xi\, d\eta.$$

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TO PROPERTIES OF THE NONLINEAR DIFFUSION - REACTION SYSTEM WITH INHOMOGENEOUS DENSITY

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In the domain $Q = \left\{(t,x) : t > 0, x \in R^N\right\}$ investigated properties of the process of a nonlinear diffusion-reaction with heterogeneous density:

$$\begin{cases} \frac{\partial(\rho(x)u)}{\partial t} = div\left(|x|^n v^{m-1} |\nabla u|^{p-2} \nabla u\right) + \varepsilon\rho(x)\gamma(t)u^{\beta_1} \\ \frac{\partial(\rho(x)v)}{\partial t} = div\left(|x|^n u^{m-1} |\nabla v|^{p-2} \nabla v\right) + \varepsilon\rho(x)\gamma(t)v^{\beta_2} \end{cases} \quad (1)$$

$$u(0,x) = u_0(x) \geq 0,$$
$$v(0,x) = v_0(x) \geq 0, \ x \in R^N, \quad (2)$$

where $m, n \in R$, $\beta_1, \beta_2 \geq 1$ p ≥ 2 - given positive numbers, $\nabla(.)-\underset{x}{grad}(.)$, functions $u_0(x)$, $v_0(x) \geq 0$, $x \in R^N$, $\rho(x) = |x|^{-l}$, $l > 0$, $0 < \gamma(\mathrm{t}) \in C(0,\infty)$, $\varepsilon = \pm 1$. Particular cases ($n = 0$, $l = 0$, $p = 2$) of the system were considered in works [1-4]. There are received the conditions of existence of the solutions of the problem Cauchy on time.

The system (1) in the domain, where $u = v = 0$ is degenerated, and in the domain of degeneration it may have not the classical solution. Therefore it is studied the weak solutions of the system (1) having physical sense: $0 \leq u, v \in C(Q)$ and $|x|^n v^{m-1} |\nabla u|^{p-2} \nabla u$, $|x|^n u^{m-1} |\nabla v|^{p-2} \nabla v \in C(Q)$ satisfying to some integral identity. For the solution of the system (1) have place phenomena of the finite velocity of a propagation and localization of a disturbance, i.e. there are the functions $l_1(t)$, $l_2(t)$, that $u(t,x) \equiv 0$ and $v(t,x) \equiv 0$ when $|x| \geq l_1(t)$ and $|x| \geq l_2(t)$, accordingly. The surfaces $|x| = l_1(t)$ and $|x| = l_2(t)$ are called a free boundary or a front.

In the present work the different types estimates and the exact solutions of the system (1) based on the self - similar, approximately self-similar approaches and by the method of the standard equations are received. In the case $m + p - 3 > 0$ is established asymptotic behaviour of the weak finite solution and free boundary $l(t)$ and asymptotic of the self - similar solution when $m + p - 3 < 0$.

Using the exact solutions and the estimates of a solution and an estimate of free boundaries the numerical experiments, visualization of processes described by the system reaction diffusion with inhomogeneous density carried out.

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INVESTIGATION ONE BOUNDARY PROBLEM FOR THE EQUATION OF A HIGH ORDER

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There was an idea to investigate application method of planimetric to the of the solution problems for the equations and systems not belonging to typical classification. And as one of the basic stages of the method of planimetric integral for the equations of mathematical physics is of the boundary problems corresponding to given mixed problems. There are various methods of study of such problems one of is the method of the theory of potentials by the help of which one boundary problem in the present work will be investigated.

The problem of a finding of the solution of the equation

$$\frac{d^4y(x,\,\lambda)}{dx^4}-\lambda^4 y(x,\mu)=F(x,\,\mu),x\in(0,\ 1),\tag{1}$$

with boundary conditions

$$\left.\frac{d^sy(x,\mu)}{dx^s}\right|_{x=0}=\varphi_s(\mu),(s=0,\ 1);\ \left.\frac{d^sy(x,\mu)}{dx^s}\right|_{x=1}=\psi_s(\mu)(s=2,\ 3)\tag{2}$$

is considered. This problem corresponds to the mixed problem for the equation not belonging to typical classification, i.e. for the equation

$$\frac{\partial^2\vartheta(x,t)}{\partial t^2}=a\frac{\partial^5\vartheta}{\partial x^4\partial t}+b\frac{\partial^4\vartheta}{\partial x^4},$$

where $Re\,a>0$.

$$\mu=\frac{\lambda^2}{\sqrt[4]{a\lambda^4+b}}.\tag{3}$$

As the solution of the non-uniform equation under homogeneous boundary conditions is under construction by means of Green's function, the problem for the homogeneous equation corresponding to the equation (1), i.e. the equations should be investigated

$$\frac{d^4y(x,\,\mu)}{dx^4}-\mu^4 y(x,\mu)=0\tag{1'}$$

with boundary conditions (2). The solution of this problem is searched in the form of the sum of potentials

$$y(x,\mu)=\sum_{m=1}^{4}\mathrm{K}_m(x,\ \mu)\theta_m(\mu),\tag{4}$$

where $\mathrm{K}_m(x,\ \mu)$ $(m=\overline{1,\ 4})$are kernels which are defined by means of fundamental and private solution of the equation (1 '), and the fundamental solution is defined by the formula

$$\mathrm{P}(x-\xi,\ \mu)=\frac{1}{4\mu^2}\left\{ie^{i\mu|x-\xi|}-e^{-\mu|x-\xi|}\right\}\tag{5}$$

$\theta_m(\mu)$ $(m=\overline{1,\ 4})$ unknown density subject to definition. By direct check it is proved that at all values of parameter from sector

$$R_\delta=\left\{\mu:\ |\mu|>R\ \ \frac{\pi}{8}-\delta<\arg\mu<\frac{3\pi}{8}+\delta\right\}$$

$\mathrm{K}_m(x,\ \mu)$ kernels are regular functions in area R_δ. By means of (4) decision of a problem (1 '), (2) it is reduced to the decision of system algebraic the equation concerning unknown density $\nu_m(\mu)$ $(m=\overline{1,\ 4})$, which determinant it will be distinct from zero.

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UNIQUENESS OF SOLUTIONS OF LINEAR INTEGRAL EQUATIONS OF THE FIRST KIND WITH TWO INDEPENDENT VARIABLES

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We shall consider the integral equation

$$\int_a^b K(t,x,y)u(t,y)dy + \int_{t_0}^t H(t,x,s)u(s,x)dx + \int_{t_0}^t \int_a^x G(t,x,s,y)u(s,y)dyds = f(t,x), \qquad (1)$$

$$(t,x) \in G = (t,x) \in R^2 : \ t_0 \le t \le T, \ a \le x \le b,$$

where

$$K(t,x,y) = \begin{cases} A(t,x,y), \ t_0 \le t \le T, \ a \le y \le x \le b, \\ B(t,x,y), \ t_0 \le t \le T, \ a \le x \le y \le b. \end{cases}$$

Various issues conserning integral qeuations of the first kind and third kinds were studied in[1-3]. In this work with method of nonnegative quadratic form we investigate the uniqueness of solutions of linear integral equations of the first kind with two independent variables.

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ON INVERSE PROBLEM FOR SYSTEM OF SECOND ORDER HYPERBOLIC EQUATIONS

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We study the inverse problem for system of second order hyperbolic equations on $\Omega = [0,T] \times [0,\omega]$

$$\frac{\partial^2 u}{\partial x \partial t} = A(t,x)\frac{\partial u}{\partial x} + B(t,x)\frac{\partial u}{\partial t} + C(t,x)u + D(t,x)q(x) + f(t,x), \tag{1}$$

$$u(t,0) = \psi(t), \qquad t \in [0,T], \tag{2}$$

$$P_1(x)\frac{\partial u(0,x)}{\partial x} + P_0(x)u(0,x) = \varphi_1(x), \qquad x \in [0,\omega], \tag{3}$$

$$\frac{\partial u(T,x)}{\partial x} = \varphi_2(x), \qquad x \in [0,\omega], \tag{4}$$

where $(n \times n)$ matrices $A(t,x)$, $B(t,x)$, $C(t,x)$, $D(t,x)$, n-vector-function $f(t,x)$ are continuous on Ω, n-vector-function $\psi(t)$ is continuously differentiable on $[0,T]$, $(n \times n)$ - matrices $P_1(x)$, $P_0(x)$, n-vector-functions $\varphi_1(x)$, $\varphi_2(x)$ are continuous on $[0,\omega]$, n-vector-functions $u(t,x)$, $q(x)$ are unknown functions.

We study the problem of existence and uniqueness of the classical solution of the problem (1)–(4).

Nonlocal boundary value problems for systems of second order hyperbolic equations were considered by number of authors. Sufficient conditions for the existence and uniqueness of a solution of such problems were obtained by various methods. The nonlocal boundary value problem with data on characteristics was considered in [1-2] by the method of introduction of functional parameters. This method is a modification of the parametrization method [3] developed for the solution of two-point boundary value problems for ordinary differential equations. In [4-6] the necessary and sufficient conditions for the well-posed unique solvability of nonlocal boundary value problem are established.

In the present communication the method of introduction of functional parameters is used for investigating of questions of existence and uniqueness of classical solution of inverse problem (1)–(4). The sufficient conditions of the unique classic solvability of the inverse problem - the nonlocal boundary value problem for system of hyperbolic type with unknown parameter (1)–(4) are obtained and an algorithm of finding its solution is proposed.

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WELL-POSEDNESS OF FRACTIONAL PARABOLIC EQUATIONS

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In the present paper we consider the initial value problem for the fractional differential equation

$$\frac{du(t)}{dt} + D_t^{\frac{1}{2}} u(t) + A(t)u(t) = f(t), \;\; 0 < t < 1, \;\; u(0) = 0 \tag{1}$$

in a Banach space E with the strongly positive operator A(t): The well-posed ness of this problem in spaces of smooth functions is established. In practice, the coercive stability estimates for the solution of problems for 2m-th order multidimensional fractional parabolic equations and one dimensional fractional parabolic equations with nonlocal boundary conditions in space variable are obtained. The stable diffference scheme for the approximate solution of this problem is presented. The well-posedness of the difference scheme in difference analogues of spaces of smooth functions is established. In practice, the coercive stability estimates for the solution of difference schemes for the fractional parabolic equation with nonlocal boundary conditions in space variable and the multidimensional fractional parabolic equation with Dirichlet condition in space variables and the 2m-th order multidimensional fractional parabolic equation are obtained.

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GREEN'S FUNCTION OF THE HIGH ORDER OPERATION - DIFFERENTIAL EQUATIONS IN THE FINITE INTERVAL

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Let H be a separable Hilbert space. In the space $L_2\left[H;\ [0,\pi]\right]$ we consider the operator L, generated by expression

$$l\left(y\right)=\left(-1\right)^{n}y^{(2n)}+Q\left(x\right)y \tag{1}$$

and boundary conditions

$$\begin{aligned} B_jy\Big|_{x=0}&=y^{(l_j)}\left(0\right)+\sum_{k=1}^{l_j}\alpha_k^{(l_j)}y^{(l_j-k)}(0)=0,\\ \tilde{B}_jy\Big|_{x=\pi}&=y^{\tilde{l}_j}\left(\pi\right)+\sum_{k=1}^{l_j}\beta_k^{(\tilde{l}_j)}y^{(l_j-k)}\left(\pi\right)=0\ , \end{aligned} \tag{2}$$

where $0\le l_1<l_2<...<l_n\le 2n-1;\ 0\le\tilde{l}_1<\tilde{l}_2<...<\tilde{l}_n\le 2n-1;\ j=\overline{1,n}.$

In the work the Green's function of the equation $l\left(y\right)+\mu y=0$ with boundary conditions (2) is studied. For this purpose, first we construct the Green's function of the corresponding equation with the fixed point ξ factors, i.e. of the following problems

$$\left(-1\right)^{n}y^{(n)}+Q\left(\xi\right)y+\mu y=0, \tag{3}$$

$$B_jy\Big|_{x=0}=\tilde{B}_jy\left.\right|_{x=\pi}=0, j=\overline{1,n}. \tag{4}$$

The Green's function of the problem (3) - (4) is sought as

$$G_1\left(x,\eta;\xi,\mu\right)=G_0\left(x,\eta;\xi,\mu\right)+V\left(x;\eta;\xi,\mu\right),$$

where $G_0\left(x;\eta,\xi,\mu\right)$ is the Green's function of the equation $l\left(y\right)+\mu y=0$ in the whole axis, $V\left(x;\eta;\xi,\mu\right)$ is a solution of the corresponding homogeneous equation satisfying to the following boundary conditions

$$\begin{cases} B_j\,V\left(x,\eta,\xi;\mu\right)|_{x=0}=-B_jG_0\left(x,\eta,\xi;\mu\right)|_{x=0}\\ \tilde{B}_j\,V\left(x,\eta,\xi;\mu\right)\Big|_{x=\pi}=-\tilde{B}_j\ G_0\left(x,\eta,\xi;\mu\right)|_{x=\pi} \end{cases}$$

The following formulae is obtained for the Green's function

$$G_1\left(x;\eta;\xi,\mu\right)=\frac{K_\xi^{1-2n}}{2ni}\sum_{k=1}^{n}\omega_ke^{i\omega_kK_\xi|x-\eta|}\left(E+r\left(x,\eta;\xi,\mu\right)\right).$$

Moreover for $\mu\to\infty$ $\|r\left(x;\eta;\xi,\mu\right)\|=O\left(1\right)$ regularly with respect to (x,η).

The Green's function of the problem (1), (2) is a solution of a following integral equation

$$G\left(x,\eta;\mu\right)=G_1\left(x,\eta;\mu\right)-\int_0^\pi G\left(x,\xi;\mu\right)\left[Q\left(\xi\right)-Q\left(x\right)\right]G\left(\xi,\eta;\mu\right)d\xi.$$

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THE EXACT VALUE OF THE UNKNOWN CONSTANT IN ONE HYPERBOLIC EQUATION

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In [1-2] for the description of vibrations of a column of boring pipes the following equation was offered:

$$w_{tt} = ga^{\sigma}\left(\left(xw^{\sigma}w_x\right)_x - (\sigma/2)\,xw^{\sigma-1}w_x^2\right), \tag{1}$$

where $\sigma > 0$ - unknown constant number which characterizes a degree of influence of forces of tension on a column of the boring pipes arising at its elastic deformations $a = c\mu\,(G/E)$, μ - the factor of Poisson, G - the module of shift, E-Young's modulus, g- acceleration of free falling, c- a dimensionless constant. It is supposed, that the axis of abscissa $(x \geq 0)$ is located on the central axis of a vertical chink and directed upwards, the axis of ordinates to serve for the description of cross-section deviations of a $w = w\,(x,\ t)$ column, t - time.

It is required to find unknown numerical value of a constantσ in the equation (1).

The problem is solved on the basis of theoretical group views and the theory of resistance of materials. Dimension of maximal wide algebra of Lie as supposed by the equation (1) is equal to three:L_3, and basic infinitesimal operators of this algebra are defined by following proportions:

$$X_1 = \frac{\partial}{\partial t}, X_2 = x\frac{\partial}{\partial x} + (1/\sigma)\ w\frac{\partial}{\partial w}, X_3 = (\sigma+4)\ \ x\frac{\partial}{\partial x} + 2\,t\frac{\partial}{\partial t} + w\frac{\partial}{\partial w}.$$

It has appeared that using the group of transformations as determined by the operatorX_2:

$$x' = e^b x, t' = t, w' = e^{b/\sigma}w, \tag{2}$$

And also by defining of factor of Poisson it is possible unequivocally to calculate unknown numerical value of a constant σ from the equation (1). Here b- parameter of the group. It was found out that$\sigma = 1$.

Due to use of transformations (2) there is a arise a question: - the Algebra L_3 contains infinitely much more infinitesimal operators then why for the decision of a task we give preference exactly to the operatorX_2?

Answering to this question it is possible to note that the operator X_2, to within constant factors, appears the only thing in all algebra L_3 which defines the group of the transformations and leaving constant value of time. Therefore, it looks natural if to try to define with the use operators of algebra L_3 quantitative value of any size, indifferent to changes of time (for example, a constant σ), so for this purpose more suitable to within constant factors is the operator $X_2 \in L_3$.

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SOME PROPERTIES OF ONE STURM-LIOUVILLE TYPE PROBLEM WITH ABSTRACT LINEAR OPERATOR

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In this work, we study the differential-operator equation

$$L(u) := -u''(x) + q(x)u(x) + (Bu)(x) \quad = \quad \lambda u(x), \qquad x \in [-1,0) \cup (0,1]$$

together with boundary condition at $x = -1$

$$L_1(u) := \alpha_1 u(-1) + \alpha_2 u'(-1) = 0$$

transmission conditions at the point of discontinuity $x = 0$,

$$L_2(u) := u(+0) - \delta u(-0) \quad = \quad 0,$$

$$L_3(u) := u'(+0) - \gamma u'(-0) \quad = \quad 0$$

and eigenparameter-dependent boundary condition at $x = 1$

$$L_4(u) := \lambda(\beta_1' u(1) - \beta_2' u'(1)) + \beta_1 u(1) - \beta_2 u'(1) \quad = \quad 0,$$

where $B : L_2(-1,1) \to L_2(-1,1)$ is an abstract linear operator with domain of definition $D(B) \supset W_2^2(-1,0) \oplus W_2^2(0,1)$, $q(x)$ is a given real-valued function which is continuous in both $[-1,0]$ and $[0,1]$(that is, continuous both in $[-1,0)$ and $(0,1]$ and has finite limits $q(\pm 0) = \lim_{x\to\pm 0} q(x)$), λ is a complex eigenvalue parameter and the coefficients $\delta, \gamma, \alpha_1, \alpha_2 \beta_1', \beta_2', \beta_1, \beta_2$ are real numbers, $\delta\gamma > 0$, $\alpha_1^2 + \alpha_2^2 \neq 0$ and $\beta_1'\beta_2 - \beta_1\beta_2' > 0$. We investigated isomorphism, coerciveness and resolvent of the considered problem and derive asymptotic approximation formulas for the eigenvalues.

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SOLUTIONS BEHAVIOUR OF THE SINGULAR PERTURBED SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS IN PARTICULARLY CRITICAL CASES

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The problem under consideration is as follows:

$$\varepsilon x'(t,\varepsilon) = D(t)x(t,\varepsilon) + \varepsilon[f(t) + B(t)x(t,\varepsilon)] + g(t, x(t,\varepsilon)), \tag{1}$$

$$x(-T_0,\varepsilon) = x^0(\varepsilon), \|x^0(\varepsilon)\| = O(\varepsilon), \tag{2}$$

where $D(t) = diag(\lambda_1(t) = (t+i)^n, \lambda_2(t) = (t-i)^n), n = 4k+1, k = 0,1,2,..., \varepsilon > 0$ small parameter;

$$f(t) - colon(f_1(t), f_1(t)); \quad B(t) = (b_{kj}(t))_1^2;$$

$[-T_0, T_0]$-segment of actual axis, $0 < T_0$ open radius circle $r \geq 1+d \quad (d > 0)$ with a centre at the point of (0,0); $t \in S_r, \Delta(t,x) = (t, x_1, x_2) : t \in S_r, |x_j| < \delta (j = 1,2), 0 < \delta - const$, $\Phi(S_r)$-space of analytical functions in S_r

For the solution of $x(t,\varepsilon) = colon(x_1(t,\varepsilon), x_2(t,\varepsilon))$ we will look in the class $X_k(t,\varepsilon) \in \Phi(S_r), \ (k = 1,2)$ at t.

We will demand the solution of the following conditions:

I. Let $f_k(t) \in \Phi(S_r)$; $b_{kj}(t) \in \Phi(S_r)$; $g_k(t,x)\Phi(\Delta(t,x))(k,j = 1,2)$; in the sphere of $\Delta(t,x)$ there is inequality of $\|g(t,x) - g(t,\widetilde{x})\| \leq M\|x - \widetilde{x}\| max\{\|x\|, \|\widetilde{x}\|\}$, where $0 < M$.

Let $T_0 = tg\frac{\pi}{4(2k+1)}$. Then $Re(t \pm i)^{4k+1}, \ (k = 0,1,2,...)$ changes the digit at the segment of $[-T_0, T_0]$ $Re(t \pm i)^{4k+1} < 0 \ at \ T_0 \leq t < 0, \quad Re(t \pm i)^{4k+1} > \ at \ 0 < t \leq T.$

II. We will consider that if (t_1, t_2) -inner point of the sphere of H_0 , then the harmonic function is $Jm\lambda_1(t_1,t_2) > 0$, where $H_0 = [ABCD]$-rhombus with apexes in the points: $A(-T_0, 0), B(1,0), C(T,0)$, $D(-1,0)$. We will put a question on closeness of the solution of perturbed and unperturbed problems in the case of exchange of stabilities at the segment $[-T_0, T_0]$ at sufficiently small values ε.

In the paper [1] the solutions behavior of the tasks (1), (2) for the system of the type of (1) has been analyzed at $D(t) = diag(\lambda_1(t)) = (t+i), \lambda_2(t)) = (t-i)$ at the segment of [-1,1], when $B(t) \equiv 0; f(t) = -colon(x_1, x_2)$; $g(t,x) = \gamma x_1 x_2 colon(x_1, x_2), \ where\, \gamma = const.$

In this particular work we are analyzing the solutions behavior of the task (1), (2) at the segment of while completing the requirement of I, II.

Theorem: *Let conditions I, II be fulfilled. In that case for the problem (1), (2) at* $-T_0 \leq t < \widetilde{T_0} - \widetilde{\delta}(\varepsilon)$ *there is the only solution and the value is correct for it* $\|x(t,\varepsilon)\| \leq cw(t,\varepsilon)$, where $0 < c - const$

$$w(t,\varepsilon) = \begin{cases} \varepsilon \quad at \quad -T_0 \leq t \leq T_0 - \widetilde{\delta}(\varepsilon) \quad (0 < p < \frac{1}{3}), \\ \varepsilon^{1-p} \quad at \quad T_0 - \widetilde{\delta}(\varepsilon) \leq t \leq \widetilde{T_0} - \widetilde{\delta}(\varepsilon) \quad (\frac{1}{3} \leq p \leq \frac{1}{2}), \end{cases}$$

$\widetilde{\delta}(\varepsilon) \geq 0$ *continuous function at* $0 < \varepsilon \leq \varepsilon_0 \quad (\varepsilon_0 - const.)$, at that $lim_{\varepsilon \to 0}\widetilde{\delta}(\varepsilon) = 0$.

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SOLVABILITY OF THE NONLINEAR TWO POINT BOUNDARY VALUE PROBLEM FOR A SYSTEM OF INTEGRAL DIFFERENTIAL EQUATIONS

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Consider the nonlinear two point boundary value problem

$$\frac{dx}{dt} = f_0(t,x) + \int\limits_0^T f_1\Big(t,s,x(s)ds\Big), \quad t \in (0,T), \tag{1}$$

$$g\Big(x(0),x(T)\Big) = 0, \tag{2}$$

where $f_0 : [0,T] \times \mathbb{R}^n \to \mathbb{R}^n$, $f_1 : [0,T] \times [0,T] \times \mathbb{R}^n \to \mathbb{R}^n$ are continuous.

Problem (1),(2) is investigated by parametrization method [1]. We take step $h > 0$ and make partition of interval on N parts. Denote by $x_r(t)$ the function $x(t)$ restricted to the r-th interval $[(r-1)h, rh)$.

We introduce $\lambda_r = x_r[(r-1)h]$ and change the variable $u_r(t) = x_r(t) - \lambda_r$ on each interval $[(r-1)h, rh)$. Then, we obtain the equivalent multipoint boundary value problem with parameters

$$\frac{du_r}{dt} = f_0(t, u_r + \lambda_r) + \sum_{j=1}^{N} \int\limits_{(j-1)h}^{jh} f_1(t,s,u_j(s)+\lambda_j)ds, \quad t \in [(r-1)h, rh), \tag{3}$$

$$u_r[(r-1)h] = 0, \qquad r = \overline{1,N}, \tag{4}$$

$$g\Big(\lambda_1, \lambda_N + \lim_{t\to T-0} u_N(t)\Big) = 0, \tag{5}$$

$$\lambda_s + \lim_{t \to sh-0} u_s(t) = \lambda_{s+1}, \quad s = \overline{1, N-1}. \tag{6}$$

Cauchy problem (3),(4) at fixed values $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_N)$, is equivalent to the following integral equation

$$u_r(t) = \int_{(r-1)h}^{t} f_0(\tau, u_r(\tau) + \lambda_r) + \sum_{j=1}^{N} \int_{(j-1)h}^{jh} f_1(\tau, s, u_j(s) + \lambda_j) ds d\tau, \ t \in [(r-1)h, rh), \tag{7}$$

(6) we find $\lim\limits_{t \to rh-0} u_r(t)$, $r = \overline{1,N}$, and substituting the corresponding in (4),(5) we obtain a system of nonlinear equations

$$Q_{1h}(\lambda, u) = 0, \quad \lambda \in R^{nN}. \tag{8}$$

Propose an algorithm for finding a solution to the problem with parameters (3)-(6). On the basis of the parametrization method [1] sufficient conditions of convergence of algorithm and existence of the isolated solution of a problem (1)-(3) are received.

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ANALOGUES OF THE TRICOMI PROBLEM FOR THE LOADED THIRD ORDER DIFFERENTIAL EQUATION

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The necessity of the consideration of the parabolic-hyperbolic type equation was specified in 1956 by I.M.Gel'fand [1]. He gave an example connected to the movement of the gas in a channel surrounded by a porous environment. Inside the channel the movement of the gas was described by the were equation, outside by the diffusion equation. Recently, loaded equations have been of the great interest owing to intensive studying the problems of the optimal control of the agro economical system, of the long-term forecasting and regulating the subsoil waters layer and soil moisture. First, the most general definition of the loaded equation was given by A.M. Nakhushev. He also gave notions and detailed classification for different loaded differential, loaded integral, loaded functional equations and considered their numerical applications.

We consider the equation

$$0 = \frac{\partial}{\partial x}\begin{cases} u_{xx} - u_y - \lambda_1 u - \mu_1 u(x,0), & (x,y) \in \Omega_1, \\ u_{xx} - u_{yy} + \lambda_2 u - \mu_2 u(x,0), & (x,y) \in \Omega_2 \end{cases} \tag{1}$$

in the domain $\Omega = \Omega_1 \cup \Omega_2$, where Ω_1 is the domain bounded by segments AB, BB_0, B_0A_0, A_0A of the straight lines $y = 0$, $x = 1$, $y = h$ and $x = 0$, i.e. $\{0 < x < 1, 0 \le y \le h\}$, Ω_2 is the characteristic triangle bounded by the segment AB of the axe Ox and by characteristics $AC : x+y = 0$, $BC : x-y = 1$ of the equation (1) outgoing from the points A, B and crossing on $C\left(-\frac{1}{2}, \frac{1}{2}\right)$.

In the equation (1), λ_i, μ_i, $i = 1, 2$ are given real parameters.

To find the function $u(x, y)$ from the class of functions $W = C\left(\overline{\Omega}\right) \cap C^1(\Omega \cup AA_0 \cup AC) \cap C^{3,1}_{x,y}(\Omega_1) \cap C^{3,2}_{x,y}(\Omega_2)$ satisfying the equation (1) in the domain $\Omega_1 \cup \Omega_2$ and satisfying the following conditions

$$u(0,y) = \varphi_1(y), \quad u(1,y) = \varphi_2(y), \quad u_x(0,y) = \varphi_3(y), \quad 0 \le y \le h, \tag{2}$$

$$u(x,-x) = \psi_1(x), \qquad \left.\frac{\partial u(x,y)}{\partial n}\right|_{y=-x} = \psi_2(x), \quad 0 \le x \le 1. \tag{3}$$

Moreover, the following gluing conditions

$$u_y(x,+0) = u_y(x,-0),\ 0 < x < 1 \tag{4}$$

should be fulfilled on the lines of the type changing.

Here $\varphi_i(y)$ $(i = \overline{1,3})$, $\psi_1(x)$, $\psi_2(x)$ are given real-valued functions.

Theorem. *If* $\lambda_1 > 0, \varphi_1(0) = \psi_1(0)$,

$$\varphi_j(y) \in C[0;h] \cap C^1(0;h), \quad j = \overline{1,3}, \tag{5}$$

$$\psi_1(x) \in C^1\left[0;\frac{1}{2}\right] \cap C^3\left(0;\frac{1}{2}\right), \quad \psi_2(x) \in C\left[0;\frac{1}{2}\right] \cap C^2\left(0;\frac{1}{2}\right). \tag{6}$$

then there exists a unique solution to the problem T.

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THE SOLUTION OF THE SPEED OF THE THE WAVES AT THE POROUS PHASES

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Nonlinear waves evolution in porous medium with two-phase pore filling has been created. The mathematically posed problem for one-dimensional and isothermic case is led to the solution of the mass and momentum conservation equation

$$\frac{\partial \alpha_i \rho_i}{\partial t} + \frac{\partial \alpha_i \rho_i \upsilon_i}{\partial x} = 0, \quad \alpha_1 + \alpha_2 = 1, \tag{1}$$

$$\frac{\partial \left(\alpha_i \rho_i \upsilon_i\right)}{\partial t} + \frac{\partial \left(\alpha_i \rho_i \upsilon_i \upsilon_i\right)}{\partial x} = \delta_{1i} \frac{\partial \sigma}{\partial x} + \alpha_i \frac{\partial P}{\partial x} - (-1)^i R_{12}. \tag{2}$$

We take the connection between stress and deformation of solid phase in the form

$$\frac{\partial \left(\alpha_i \rho_i \upsilon_i\right)}{\partial t} + \frac{\partial \left(\alpha_i \rho_i \upsilon_i \upsilon_i\right)}{\partial x} = \delta_{1i} \frac{\partial \sigma}{\partial x} + \alpha_i \frac{\partial P}{\partial x} - (-1)^i R_{12}. \tag{3}$$

The system of equations (1)-(3) is completed by the thermodynamic equation of phase states

$$\rho_1 = \rho_1 \left(\sigma, P\right), \rho_2 = \rho_2 \left(P\right). \tag{4}$$

The equations (1)-(4) are completed by the following kinematic relation

$$\frac{\partial e_1}{\partial t} + \frac{\partial e_1 \upsilon_1}{\partial x} = \frac{\partial \upsilon_1}{\partial x}. \tag{5}$$

Using the length and time resealing

$$\frac{\partial e_1}{\partial t} + \frac{\partial e_1 \upsilon_1}{\partial x} = \frac{\partial \upsilon_1}{\partial x} \tag{6}$$

we rewrite the system of equations (1)-(5) in new variables.

At the first approximation the system of equations (1)-(5) is led to the system of homogeneous equations

$$\begin{array}{l} \alpha_1^{(0)} D_1 \sigma_1 + \alpha_1^{(0)} L_1 P_1 + \rho_1^{(0)} \alpha_1^{(1)} - c^{-1} \alpha_1^{(0)} \rho_1^{(0)} \upsilon_1^{(0)} = 0 \\ \rho_2^{(0)} \alpha_2^{(1)} + \alpha_2^{(1)} B_1 P_1 - c^{-1} \alpha_2^{(0)} \rho_2^{(0)} \upsilon_2^{(1)} = 0 \\ \alpha_1^{(0)} \rho_1^{(0)} \upsilon_1^{(1)} + c^{-1} \sigma_1 + c^{-1} \alpha_1^{(0)} P_1 = 0 \\ \alpha_2^{(0)} \rho_2^{(0)} \upsilon_2^{(1)} + c^{-1} \alpha_2^{(0)} P_1 = 0 \\ b_0 \left(\sigma_1 + \gamma P_1\right) = a_0 e_1, \alpha_1^{(1)} + \alpha_2^{(1)} = 0; \end{array} \tag{7}$$

The system (7) has a nontrivial solution if its determinant vanishes, that gives the following dispersion equation with respect to the velocity of linear waves c:

$$\begin{array}{l} \alpha_1^{(0)} \rho_1^{(0)} b_0 \left[\alpha_1^{(0)} \rho_2^{(0)} \left(L_1 - D_1 \gamma\right) + \rho_1^{(0)} \alpha_2^{(0)} B_1\right] c^4 + \\ + \left[\alpha_1^{(0)} \rho_2^{(0)} \left(\alpha_1^{(0)} a_0 D_1 - a_0 L_1 + \alpha_1^{(0)} \rho_1^{(0)} b_0 - \gamma \rho_1^{(0)} b_0\right) + \alpha_2^{(0)} \rho_1^{(0)} \left(\alpha_1^{(0)} \rho_1^{(0)} b_0 - a_0 B_1\right)\right] c^2 - \\ - \alpha_2^{(0)} \rho_1^{(0)} a_0 = 0 e_1 = -c^{-1} \upsilon_1^{(1)}. \end{array} \tag{8}$$

The coefficients of this equation are constants. Equation (8) has a pair of roots corresponding to propagation of longitudinal waves in solid and fluid phases.

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AN ANALOGUE OF THE TRICOMI PROBLEM WITH INTEGRAL GLUING CONDITIONS FOR PARABOLIC-HYPERBOLIC EQUATION

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Consider the equation

$$u_{xx} - D_{0y}^{\alpha H(x)+2H(-x)} u(x,t) = \lambda u \tag{1}$$

in the domain $\Omega = \Omega_1 \cup \Omega_2 \cup AA_0$. Here $H(x)$ is a Heaviside function and D_{0y}^{α} is the Riemann-Liouville fractional differential operator [1], $\Omega_1 = \{(x,y) : 0 < x < 1,\ 0 < y < 1\}$, Ω_2 is the characteristic triangle with endpoints $A\,(0,0)\,,\ D\,(-1/2,1/2)\,,\ A_0\,(0,1)$.

For $\lambda > 0$ and $0 < \alpha \leq 1$ given, we formulate the following problem

Problem T. *To find a solution of the* (1) *from the class of functions*

$$W = \left\{ u : D_{0y}^{\alpha-1} u \in C\left(\overline{\Omega_1}\right),\ u_{xx}, D_{0y}^{\alpha} u \in C\left(\Omega_1\right),\ u \in C\left(\overline{\Omega_2}\right) \cap C^2\left(\Omega_2\right) \right\},$$

satisfying the initial condition

$$\lim_{y\to 0} y^{1-\alpha} u(x,y) = \varphi(x),\ 0 \leq x \leq 1,$$

together with the boundary conditions

$$u\left(-y/2, y/2\right) = \psi_1(y),\ 0 \leq y \leq 1,$$
$$u(1,y) = \psi_2(y),\ 0 \leq y \leq 1$$

and the gluing conditions

$$D_{0y}^{\alpha-1} u\left(+0,t\right) = u\left(-0,y\right),\ 0 \leq y \leq 1,$$

$$D_{0y}^{\alpha-1} u_x\left(+0,t\right) = \int\limits_0^y u_x\left(-0,t\right) J_0\left[\sqrt{\lambda}\left(y-t\right)\right] dt,\ 0 < y < 1,$$

Here $\omega\left(x\right), \psi_j\left(y\right)\ \left(j = 1,2\right)$ *are given functions.*

We would like to note work by V.A.Nakhushev [2], where non-local problem with other gluing condition for (1) in the case when $\lambda = 0$ was studied. In this work we will equivalently reduce the formulated problem to the second kind Volterra integral equation regarding a function $u_x(0,y)$.

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ON A NON-LOCAL BOUNDARY PROBLEM FOR PARABOLIC-HYPERBOLIC EQUATION INVOLVING RIEMANN-LIOUVILLE FRACTIONAL DIFFERENTIAL OPERATOR

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We define the following sets:

$\Omega_{par} = \{(x,y) : 0 < x < 1, 0 < y < 1\}$,
Ω_{hyp0} is the characteristic triangle with endpoints $A(0,0)$, $D(-1/2,1/2)$, $A_0(0,1)$ and
Ω_{hyp1} is the characteristic triangle with endpoints $B(1,0)$, $C(3/2,1/2)$, $B_0(1,1)$.

So, for $\lambda > 0$ and $0 < \alpha_1$ given, we consider the following equation in the domain $\Omega = \Omega_{par} \cup \Omega_{hyp0} \cup \Omega_{hyp1} \cup AA_0 \cup BB_0$:

$$Lu = \lambda u, \tag{1}$$

where

$$Lu = \begin{cases} u_{xx} - D_{0y}^{\alpha} u, & \text{if} \quad (x,y) \in \Omega_{par}, \\ u_{xx} - u_{yy}, & \text{if} \quad (x,y) \in \Omega_{hyp0}, \\ u_{xx} - u_{yy}, & \text{if} \quad (x,y) \in \Omega_{hyp1}, \end{cases}$$

and D_{0y}^{α} is the Riemann-Liouville fractional differential operator [1].

Problem A. T*o find a regular solution of the* (1) satisfying the initial condition

$$\lim_{y \to 0} D_{0y}^{\alpha-1} u(x,y) = \omega(x),\ 0 \le x \le 1$$

together with the non-local boundary conditions

$$A_{0y}^{0,\sqrt{\lambda}} [u(\theta_0)] + a_0(y) u(-0,y) = b_0(y),\ 0 \le y \le 1/2,$$

$$A_{1y}^{0,\sqrt{\lambda}} \left[\frac{d}{dy} u(\theta_1) \right] + a_1(y) u_x(1+0,y) = b_1(y),\ 1/2 < y < 1,$$

and the gluing conditions

$$D_{0y}^{\alpha-1} u(+0,y) = u(-0,y),\ 0 \le y \le 1,\ D_{0y}^{\alpha-1} u(1-0,y) = u(1+0,y),\ 0 \le y \le 1,$$

$$D_{0y}^{\alpha-1} u_x(+0,y) = \int_0^y u_x(-0,t) \frac{J_0\left[\sqrt{\lambda}(y-t)\right]}{1+2a_0(y)} dt,\ 0 < y < 1,$$

$$D_{0y}^{\alpha-1} u_x(1-0,y) = \int_y^1 u_x(1+0,t)\left[1+2a_1(t)\right] J_0\left[\sqrt{\lambda}(y-t)\right] dt + u(1,1) J_0\left[\sqrt{\lambda}(y-1)\right],\ 0 < y < 1.$$

Here $\theta_0 = -\frac{y}{2} + i\frac{y}{2}$, $\theta_1 = \frac{2+y}{2} + i\frac{2-y}{2}$, $0 \le y \le 1$, $\omega(x)$, $a_j(y)$, $b_j(y)$ $(j=0,1)$ are given functions such that $a_j(y) \ne -\frac{1}{2}$, $a_0(0) \ne -1$, $A_{0y}^{n,\sqrt{\lambda}}[f(y)]$ is an integral operator defined in [2].

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THE COMPARISON OF APPROXIMATE SOLUTION OF ORDINARY DIFFERANTIAL EQUATION USING THE MODIFIED KRASNOSELSKII ITERATION METHOD

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In this paper, we used the Picard Succesive Iteration Method and Modified Krasnoselskii Iteration Method in order to solve the ordinary linear differential equation having boundary condition.Finally, it is shown that the accuracy of new iteration method (which is called Modified Krasnoselskii Iteration Method) is substantially improved by imploying variable steps which adjust themselves to the solution of the differential equation.

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ON A SOLVABILITY OF THE SINGULAR CARLEMAN-VEKUA DIFFERENTIAL EQUATION

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Existence of continuous solutions of the Carleman-Vekua differential equation with a singular point of the pole type is investigated. The necessary condition for existence of continuous solutions is given. The fact of existence of continuous solutions for a broad class of the singular Carleman-Vekua equation's coefficients is proved.

Consider the following singular at $z = 0$ Carleman-Vekua differential equation

$$\frac{\partial w}{\partial \bar{z}} + \frac{A(z)}{z} w + \frac{B(z)}{z} \overline{w} = 0, \tag{1}$$

where $z = x + iy \in G = \{|z| < 1\}$ and $\frac{\partial}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)$.

The above said is true for all differential equations that can be reduced to

$$\frac{\partial w}{\partial \bar{z}} + \frac{A_1(z)}{|z|} w + \frac{B_1(z)}{|z|} \overline{w} = 0, \tag{2}$$

where $A_1(z)$ and $B_1(z)$ belong to $L^\infty(\overline{G})$ ($A = B \equiv 0$ outside G. The equation (2) can be reduced to (1).

THE SOLUTIONS OF BURGERS' EQUATION BY THREE DIFFERENT METHODS

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In this paper we present a comparative study between three different methods for solving the Burger's equation, namely the numerical finite difference method and the semi-analytic methods; Adomian decomposition method, the Homotopy Perturbation method.

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NEW MULTIPLE SOLUTION TO THE BOUSSINESQ EQUATION AND THE BURGERS-LIKE EQUATION

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In this paper by considering an improved tanh function method, we found some exact solutions of Boussneq and Burgers-Like equation. The main idea of this method is to take full advantage of the Riccati equation which has more new solutions. We found some exact solutions of the Boussinesq equation and the Burgers-Like equation.

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TWO TRI-HARMONIC HYBRID GREEN-NEUMANN FUNCTIONS FOR THE UNIT DISC

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Convolution of the biharmonic Green and the harmonic Neumann functions leads to a hybrid tri-harmonic Green function, also harmonic Green and the biharmonic Neumann functions leads to a hybrid tri-harmonic Neumann function. Related boundary conditions are of Dirichlet-Neumann and Neumann-Dirichlet type. On the basis of the biharmonic Green function given by Almansi [1] two dual Dirichlet problems and a Dirichlet-Neumann problem arise.

Definition. *Let for* $z, \zeta \in \mathbb{D},\ z \neq \zeta,$

$$H_3(z,\zeta) = -\frac{1}{\pi}\int\limits_{\mathbb{D}} \widehat{G}_2(z,\tilde{\zeta})N_1(\tilde{\zeta},\zeta)d\tilde{\xi}d\tilde{\eta}$$

This function is called a hybrid tri-harmonic Green function. This hybrid tri-harmonic Green function is seen to be the solution to the Dirichlet problem
$\partial_z\partial_{\bar z}H_3(z,\zeta) = -\frac{1}{\pi}\int\limits_{\mathbb{D}} G_1(z,\zeta)N_1(\tilde{\zeta},\zeta)d\tilde{\xi}d\tilde{\eta} = H_2(z,\zeta),\ H_2(z,\zeta) = 0$ for $z \in \partial\mathbb{D}$.

Theorem. *The Dirichlet-Neumann problem*

$$(\partial_z\partial_{\bar z})^3 w = f \text{ in } \mathbb{D},\ w = \gamma_0,\ \partial_z\partial_{\bar z}w = \gamma_1,\ \partial_\nu\left(\partial_z\partial_{\bar z}\right)^2 w = \gamma_2 \text{ on } \partial\mathbb{D},$$

$$\frac{1}{2\pi i}\int\limits_{|\zeta|=1}\left(\partial_z\partial_{\bar z}\right)^2 w(\zeta)\frac{d\zeta}{\zeta} = c_1$$

for $f \in L_p(\mathbb{D};\mathbb{C})$, $2 < p$, $\gamma_0, \gamma_1, \gamma_2 \in C(\partial\mathbb{D};\mathbb{C})$, $c_1 \in \mathbb{C}$ *is uniquely solvable if and only if*

$$\frac{1}{2\pi i}\int\limits_{\partial\mathbb{D}}\gamma_2(\zeta)\frac{d\zeta}{\zeta} = \frac{2}{\pi}\int\limits_{\mathbb{D}} f(\zeta)d\xi d\eta.$$

The solution is given as

$$w(z) = \frac{1}{2\pi i}\int\limits_{\partial\mathbb{D}}\gamma_0(\zeta)g_1(z,\zeta)\frac{d\zeta}{\zeta} + \frac{1}{2\pi i}\int\limits_{\partial\mathbb{D}}\gamma_1(\zeta)\hat{g}_2(z,\zeta)\frac{d\zeta}{\zeta} -$$

$$-\frac{1}{4\pi i}\int\limits_{\partial\mathbb{D}}\gamma_2(\zeta)H_3(z,\zeta)\frac{d\zeta}{\zeta} - \frac{1}{\pi}\int\limits_{\mathbb{D}} f(\zeta)H_3(z,\zeta)d\xi d\eta$$

where

$$g_1(z,\zeta) = \frac{1}{1-z\bar\zeta} + \frac{1}{1-\bar z\zeta} - 1,$$

$$\hat{g}_2(z,\zeta) = (1-|z^2|)\left[\frac{1}{z\bar\zeta}\log(1-z\bar\zeta) + \frac{1}{\bar z\zeta}\log(1-\bar z\zeta) + 1\right].$$

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FRACTIONAL CALCULUS OPERATOR METHOD FOR THE CONFLUENT HYPERGEOMETRIC EQUATION

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By means of fractional calculus techniques, we find explicit solutions of confluent hypergeometric equations. We use the $N-$fractional calculus operator N^{μ} method to derive the solutions of these equations.

Theorem. *Let $y \in \{y : 0 \neq |y_{\mu}| < \infty; \mu \in \mathbb{R}\}$ and $f \in \{f : 0 \neq |f_{\mu}| < \infty; \mu \in \mathbb{R}\}$. Then the nonhomogeneous confluent hypergeometric equation*

$$L\left[y, r; \gamma, \tau\right] = y_2 r + y_1 \left(\gamma - r\right) - y\tau = f \quad \left(r \neq 0\right), \tag{1}$$

has particular solutions of the forms:

$$y = \left[\left(f_{-\tau} e^{-r} r^{\gamma-\tau-1}\right)_{-1} e^{r} r^{\tau-\gamma}\right]_{\tau-1}, \tag{2}$$

$$y = r^{1-\gamma} \left\{\left[\left(f r^{\gamma-1}\right)_{\gamma-\tau-1} e^{-r} r^{-\tau}\right]_{-1} e^{r} r^{\tau-1}\right\}_{\tau-\gamma}. \tag{3}$$

Where $y_n = \frac{d^n y}{dr^n}$ $\left(n = 0, 1, 2\right)$, $y_0 = y = y\left(r\right)$, $r \in \mathbb{C}$, γ *and* τ *are given constants.*

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ON THE SOLVABILITY OF GOURSAT PROBLEMS FOR NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS OF FOURTH ORDER

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It is considered boundary value problem

$$u(0,y) = \varphi_1(y), u_x(0,y) = \varphi_2(y), u_{xx}(0,y) = \varphi_3(y), \tag{1}$$

$$u(x,0) = \tau(x), \varphi_1(0) = \tau(0), \tau''(0) = \varphi_3(0), 0 \le x \le \ell, \tag{2}$$

for the equation

$$u_{xxxy} + au = f(x,y,u,u_x,u_y,u_{xx},u_{xy},u_{xxx},u_{xxy}), \tag{3}$$

where $0 \ne a = const, (x,y) \in D = \{(x,y) : 0 \le x \le \ell, 0 \le y \le h\}, f(x,y,u,u_x,u_y,u_{xx},u_{xy},u_{xxx},u_{xxy})$ - smooth function in arguments [1,2].

This problem will solve by the contraction mapping theorem of Banach [3-5].

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VIBRATION PROBLEMS FOR THE CUSPED PLATES ON THE BASIS OF THE REFINED THEORIES[1]

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The investigation of cusped elastic prismatic shells, in particular plates, takes its origin from the fifties of the last century, namely, in 1955 I.Vekua [1]-[3] raised the problem of investigation of elastic cusped plates, whose thickness on the whole plate or on a part of the boundary vanishes. Such bodies, considered as three-dimensional ones, occupy three-dimensional domains with non-Lipschitz boundaries, in general. In practice, such plates and beams are often encountered in spatial structures with partly fixed edges, e.g., stadium ceilings, aircraft wings, submarine wings etc., in machinetool design, as in cutting-machines, planning-machines, in astronautics, turbines, and in many other areas of engineering (e.g., dams). The problems mathematically lead to the question of posing and solving of boundary value problems for even order equations and systems of elliptic type with the order degeneration in the static case and of initial boundary value problems for even order equations and systems of hyperbolic type with the order degeneration in the dynamical case (for corresponding investigations see the surveys in [4], [5], and also I.Vekua's comments in ([3], p.86)). Some satisfactory results are achieved in this direction in the case of Lipschitz domains but in the case of non-Lipschitz domains there are a lot of open problems. To consider such problem is a main part of the objectives of the present talk. The talk is organized as follows: 1. In the first section special flexible cusped plates vibrations on the base of the classical (geometrically) non-linear bending theories (the system of equations of the classical geometrically nonlinear bending theory of isotropic plates in static case can be found, e.g., in [6-7]) is investigated; 2. In the second part concrete problems for cusped plates for Reisner-Mindlin type models are studied (case of constant thickness is considered, e.g., in [8]); 3. In the third part a fluid-solid interaction problem is considered

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THE FINITE METHOD ELEMENT FOR SOLVING THE INVERSE PROBLEM FOR THE HELMHOLTZ EQUATION

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In the report the method of the continuation of the solution the Helmholtz equation to the zone of inaccessibility is suggested. As a result of solving of continuation problem one manages to regenerate the value of the solution the Helmholtz equation in the zone of inaccessibility. The solution of the continuation problem is carried out by the way of changing this problem with some special inverse problem which is solved on the bases of combination of the finite method element [1] and the optimization method [2].

In the domain $Q = \Omega \times (0, +\infty)$, $\Omega = (-b, b) \times (-b, b) \subset R^2 = \{(x, y)\}$ consider the wave equation

$$\varepsilon v_{tt} = \Delta v - j^c,$$

where $\varepsilon > 0$. Let function v, j^c admit the separation of variables : $v(x, y, t) = u(x, y)T(t)$, $j^c(x, y, t) = f_1(x, y)T(t)$. Let $T(t) = e^{i\omega t}$ and replacing $v = ue^{i\omega t}$, we shall get the Helmholtz equation

$$\Delta u + \omega_1 u = f_1,$$

where $\omega_1 = \varepsilon\omega^2$.

The source $f_1(x, y) = \theta(a - |x|)\theta(a - |y|)$, where $|x| \leq a, |y| \leq a$ is in the center of the domain $\overline{\Omega} = [-b, b] \times [-b, b]$. Let us introduce the following notations for subdomains of the domain $\overline{\Omega}$

$$G_1 = \left\{(x, y) \in \overline{\Omega} : -b \leq x \leq -d, \quad -b \leq y \leq b\right\}, G_2 = \left\{(x, y) \in \overline{\Omega} : -d \leq x \leq d, \quad -b \leq y \leq b\right\},$$

$$G_3^+ = \{(x, y) \in \Omega : -a \leq x \leq a, \quad a \leq y \leq c\}, \; G_3^- = \{(x, y) \in \Omega : -a \leq x \leq a, \quad -c \leq y \leq -a\},$$

$G_4 = \left\{(x, y) \in \overline{\Omega} : d \leq x \leq b, \quad -b \leq y \leq b\right\}$, where G_3^+, G_3^- are antennas.

The dielectric constant is equal to

$$\varepsilon = \begin{cases} \varepsilon_1, & (x, y) \in G_3, \\ \varepsilon_2, & (x, y) \in G_2 \backslash G_3, \\ \varepsilon_3, & (x, y) \in G_1 \bigcup G_4. \end{cases}$$

In the domain Ω consider the initial-boundary value problem

$$\Delta u + \omega_1 u = f_1, \qquad (x, y) \in \Omega, \tag{1}$$

$$u(-b, y) = f(y), \qquad y \in (-b, b), \tag{2}$$

$$u_x(-b, y) = 0, \qquad y \in (-b, b), \tag{3}$$

$$u(x, -b) = u(x, b) = 0, \qquad x \in (-b, b). \tag{4}$$

It is supposed that there are the conditions of splice for the solution of (1)-(4):

$$\varepsilon_1 u_x(a - 0, y) = \varepsilon_2 u_x(a + 0, y), \; \varepsilon_2 u_x(-a - 0, y) = \varepsilon_1 u_x(-a + 0, y), \quad y \in [a, c] \cup [-c, a],$$

$$\varepsilon_1 u_x(x, c - 0) = \varepsilon_2 u_x(x, c + 0), \; \varepsilon_2 u_x(x, -c - 0) = \varepsilon_1 u_x(x, -c + 0), \quad x \in [-a, a].$$

$$\varepsilon_3 u_x(-d - 0, y) = \varepsilon_2 u_x(-d + 0, y), \; \varepsilon_2 u_x(d - 0, y) = \varepsilon_3 u_x(d + 0, y), \quad y \in [-b, b],$$

The problem (1)-(4) is ill-posed.

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THREE-POINT BOUNDARY PROBLEM FOR SINGULARLY PERTURBED INTEGRAL-DIFFERENTIAL EQUATIONS

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Three-point boundary value problem for singularly perturbed linear third order differential equations considered in [1], there analytical formulas and asymptotic in small parameter representation of initial and boundary functions are obtained using the fundamental system of solutions of singularly perturbed homogeneous linear differential equation of third order. Constructive solutions and formula, asymptotic in small parameter estimates for the solution of the three-point boundary value problem for singularly perturbed linear differential equation of third order.

In this paper we consider the linear integral-differential equation of third order with a small parameter multiplying the highest derivative in the interval [0,1]:

$$L_\varepsilon y(t,\varepsilon) \equiv \varepsilon y''' + A(t)y'' + B(t)y' + C(t)y = F(t) + \int\limits_0^1 \big(H_0(t,x)y(x,\varepsilon) + H_1(t,x)y'(x,\varepsilon)\big)\,dx \qquad (1)$$

with boundary conditions

$$H_1 y(t,\varepsilon) \equiv y\,(0,\varepsilon) = \alpha,\;\; H_2 y(t,\varepsilon) \equiv y\,(t_0,\varepsilon) = \beta,\;\; H_3 y(t,\varepsilon) \equiv y(1,\varepsilon) = \gamma, \qquad (2)$$

where α, β, γ - some known constants independent of ε, and $0 < t_0 < 1$.

In this paper an analytical formula is obtained for solving of the integral-differential boundary value problem (1), (2). Obtained asymptotic in small parameter estimates for the solution of the three-point boundary value problem (1), (2). As $\varepsilon \to 0$, these estimates allow us to set the boundary problem (1), (2) the existence of the phenomenon of an initial jump in the zero-order [2] at $t = 0$

$$y(0,\varepsilon) = O(1), \quad y'(0,\varepsilon) = O\left(\frac{1}{\varepsilon}\right), \quad y''(0,\varepsilon) = O\left(\frac{1}{\varepsilon^2}\right).$$

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ON A FREDHOLM PROPERTY OF THE SECOND ORDER CAUCHY-RIEMANN PROBLEM IN ARBITRARY DOMAIN UNDER NON-LOCAL BOUNDARY CONDITIONS

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Let‘s consider the following problem:
To find a solution of the problem

$$\frac{\partial^3 u(x)}{\partial x_2^3}+i\frac{\partial^3 u(x)}{\partial x_1 \partial x_2^2}=0, \quad x=(x_1,x_2)\in D\subset \mathrm{R}^2, \tag{1}$$

satisfying to the following boundary condition

$$\sum_{k=1}^{2}\left[\sum_{m=0}^{2}\alpha_{jmk}^{(2)}(x_1)\frac{\partial^2 u(x)}{\partial_{x_1}^{m}\partial_{x_2}^{2-m}}+\sum_{m=1}^{2}\alpha_{jmk}^{(1)}(x_1)\frac{\partial u(x)}{\partial x_m}+\alpha_{jk}^{(0)}(x_1)\,u(x)\right]\Big|_{x_2=\gamma_k(x_1)}= \\ =\alpha_j(x_1), \quad j=1,2,3;\ \ x_1\in[a_1,b_1], \tag{2}$$

where, $i=\sqrt{-1}$, the coefficients of the boundary condition (2), and the right hand sides are complex-valued continuous functions. These boundary conditions are linear independent. It should be noted that here the function

$$U(x-\xi)=\frac{\theta(x_2-\xi_2)+\theta(\xi_2-x_2)}{2\pi}\left[x_2-\xi_2+i(x_1-\xi_1)\right]\left\{\ln\left[x_2-\xi_2+i(x_1-\xi_1)-1\right]\right\} \tag{3}$$

is the fundamental solution of the equation (1). Using the given equation and fundamental solution, the main relations are obtained for u and for the derivatives x_1, x_2, x_1^2, x_2^2 and $x_{\text{-}1}x_2$ of u. The second parts of these main relations (the parts related to the boundary) are necessary conditions. From these conditions, the condition obtained for u is regular the necessary conditions obtained for the derivatives contain singular integrals. Using given boundary conditions, these singularities are regularized. The obtained regular expressions together with boundary conditions are indeed sufficient condition for the Fredholm property. The boundary value problem is also considered for the complex type equation [1], [2].

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SOLUTION OF THE LINEAR DIFFERENTIAL-ALGEBRAIC EQUATIONS BY THE LAPLACE ADOMIAN DECOMPOSITION METHOD

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In this paper, we present Laplace-Adomian decomposition method to solve constant coefficients linear differential-algebraic equations. Some examples are presented to show the ability of the method for differential-algebraic equations. The results obtained are in good agreement with the exact solution.

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INVERSE SCATTERING PROBLEM FOR THE STURM-LUIVILLE OPERATOR WITH SPECTRAL PARAMETER IN THE DISCONTINUITY CONDITION

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Consider differential equation

$$-y'' + q(x)y = \lambda^2 y, x \in (-\infty, +\infty)$$

with conditions

$$y(a+0) = y(a-0),$$
$$y'(a+0) - y'(a-0) = \lambda\beta y(a),$$

where $\beta, \alpha \in (-\infty, +\infty); \beta \neq 0; \lambda$- complex parameter $q(x)$-real valued function satisfying to the condition

$$\int_{-\infty}^{\infty} (1+|x|)\, |q(x)|\, dx$$

It is assumed that eigenvalues are absent and the inverse problem on defining of the potential $q(x)$ over the reflection coefficients. The uniqueness theorem is proved for the solution of the inverse problem and the algorithm is proposed to reconstruction of $q(x)$ over the left (right) reflection coefficients.

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ON THE LIMITING ERROR IN DIGITIZATION OF INACCURATE INFORMATION, KLEIN - GORDON EQUATION WITH INITIAL CONDITIONS FROM NIKOL'SKII CLASSES

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In this paper we consider the Cauchy problem for the Klein-Cordon equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x_1^2} + ... + \frac{\partial^2 u}{\partial x_s^2} - u \;\; (u = u(x,t),\, 0 \le t < \infty,\, x \in R^s,\, s = 1,\, 2, ...), \tag{1}$$

$$u\,(x,0) = f_1(x) \in F^{(1)}, \frac{\partial u}{\partial t}(x,0) = f_2(x) \in F^{(2)}\,(x \in R^s), \tag{2}$$

The solution describes, in particular, a free relativistic (pseudo) scalar particle of mass 1 (see e.g. [1, pp. 48-49]).

In the below-considered case, problem (1) – (2) has a closed solution in the form of the sum of an absolutely convergent multiple function series, which is completely specified by the sets $\left\{\hat{f}_1(m)\right\}_{m\in Z^s}$ and $\left\{\hat{f}_2(m)\right\}_{m\in Z^s}$ of Fourier coefficients. Therefore, we encounter the problem of the approximation of a solution (an infinite object) on the basis of finite numerical information of given size Nobtained from the functions f_1 and f_2; the mathematical statement of this problem is given in the following reconstruction problem (in [1]).

In conditions of definitions and notation from [1-2] takes place following

Theorem. *Let s $(s = 1, 2, ...)$ and $r_1 > 2 + \frac{s}{2}$ numerical sequence $\tilde{\varepsilon}_N = N^{-\frac{r_1}{s}-\frac{1}{2}}(N = 1, 2, 3, ...)$. And lets $\frac{\partial u}{\partial t}(x,0) = f_2(x) = 0$. Then the following relation is valid:*

$$\delta_N(0) \equiv \delta_N(D_N; \frac{\partial^2 u}{\partial t^2} = \Delta u - u,\, u(x,0) = f_1(x),\, \frac{\partial u}{\partial t}(x,0) = 0; H_2^{r_1}; 0)_{L^2} \asymp$$

$$\asymp \delta_N(D_N; \frac{\partial^2 u}{\partial t^2} = \Delta u - u,\, u(x,0) = f_1(x),\, \frac{\partial u}{\partial t}(x,0) = 0; H_2^{r_1}; \tilde{\varepsilon}_N = N^{-\frac{r_1}{s}-\frac{1}{2}})_{L^2} \asymp \tilde{\varepsilon}_N\sqrt{N} = N^{-\frac{r_1}{s}},$$

For any tending to $+\infty$ positive sequence $\{\eta_N\}_{N=1}^{\infty}$ the equality

$$\overline{\lim_{N\to\infty}} \frac{\delta_N\left(D_N; \frac{\partial^2 u}{\partial t^2} = \Delta u - u, u(x,0) = f_1(x), \frac{\partial u}{\partial t}(x,0) = 0; H_2^{r_1}; \tilde{\varepsilon}_N\eta_N = N^{-\frac{r_1}{s}-\frac{1}{2}}\eta_N\right)_{L^2}}{\delta_N\left(D_N; \frac{\partial^2 u}{\partial t^2} = \Delta u - u, u(x,0) = f_1(x), \frac{\partial u}{\partial t}(x,0) = 0; H_2^{r_1}; 0\right)_{L^2}} = +\infty$$

takes place.

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WAVE PROPAGATION IN ONE-DIMENSIONAL LAYERED-INHOMOGENEOUS MEDIUM WITH BARRIER

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We will investigated wave propagation in one-dimensional layered-inhomogeneous medium with barrier.

$$-y'' + \Psi(x)y + V_0[\theta(x) - \theta(x-a)]y = \lambda^2 \rho(x)y, \tag{1}$$

where $\Psi(x)$ is steplike potential of the form

$$\Psi(x) = \begin{cases} \sum\limits_{n=1}^{\infty} q_n^+ e^{inx} & x < 0, \ x > a \\ \sum\limits_{n=1}^{\infty} q_n^- e^{inx} & 0 < x < a \end{cases} \tag{2}$$

and $V_0[\theta(x) - \theta(x-a)]$ is a barrier potential with height V_0 and width a,

$$\theta(x) = \begin{cases} 0 & x < 0, \\ 1 & x \geq 0 \end{cases}$$

is a Heaviside step function and $\rho(x)$ has a form

$$\rho(x) = \begin{cases} 1 & for \quad x < 0, x > a \\ -\beta^2 & for \quad 0 < x < a. \end{cases}$$

Without changing the results any other shifted was possible.

The barrier divides the spaces in three parts $(x < 0, 0 < x < a, x > a)$ in any of these parts the potentials is complex, periodic meaning that layered-inhomogeneous medium can also absorption and emit an energy and wave propagation has different speed in each medium. The imaginary part of potential represents emission or absorbtion.

Our primary aim is to study the spectrum and solving the inverse problem for singular non-self-adjoint operator by transmission coefficient and normalizing numbers corresponding to quasi-eigenfunctions of the Sturm- Liouville operator with complex periodic potential and discontinuous coefficients on the axis.

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ON BAZISNESS OF EIGENFUNCTIONS OF WELL POSED BOUNDARY-VALUE PROBLEMS FOR THE DIFFERENTIAL EQUATION ON THE INTERVAL

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In the work [1], is presented the possibility of the expansion of function from a certain function space in eigenfunctions and associated functions of the differential operator L generated in the function space $L_2[0,b]$ where $b<\infty$ by linear differential expression with the variable coefficients

$$Ly = l(y) \equiv y^{(n)}(x) + p_{n-2}(x)y^{(n-2)}(x) + \cdots + p_0(x)y(x),$$

with sole limitation the resolvent set of the operator L – are non-empty set.

Without diminishing generality, we assume that complex number 0 belongs to the resolvent set of the operator L. Coefficients of the expression $l(.)$ satisfy to the condition

$$p_0(x) \in \mathrm{C}[0,b],\ p_1(x) \in \mathrm{C}^1[0,b],\ \ldots,\ p_{n-2}(x) \in \mathrm{C}^{(n-2)}[0,b].$$

According to wellknown Otelbaev's theorem [2] the domain of this operator is described by a set of n functions $\sigma_1(.),\ldots,\sigma_n(.)$ from this space $L_2[0,b]$

$$D(L) = \left\{ y(x) \in W_2^n[0,b] : y^{(\nu)}(0) = \langle l(y)\rangle; \sigma_{\nu+1},\ \nu = 0,\ldots,n-1 \right\},$$

where $W_2^n[0,b]$ – Sobolev space, $\langle f;g\rangle$ – is the scalar product in the space $L_2[0,b]$

Boundary functions $\sigma_1,\ldots,\sigma_n$ are selected from the space $L_2[0,b]$ such that, the boundary forms $\mathrm{U}_1(y)$ is taken the following form

$$\mathrm{U}_j(y) = \mathrm{V}_j(y) + \langle l(y); \sigma_j^1(x)\rangle, \tag{1}$$

where

$$\mathrm{V}_j(y) = \sum_{k=0}^{n-1} \left(\alpha_{jk} y^{(k)}(0) + \beta_{jk} y^{(k)}(b) \right).$$

For this it is sufficient that $\sigma_j(x)$ to had the representation $\sigma_j(x) = \sigma_j^0(x) + \sigma_j^1(x)$, where the support of σ_j^1 lies strictly inside the interval $(0,b), \sigma_j^0(\cdot)$ – the solution of the homogeneous equation $l^*(y) = 0$. Where $l^*(y)$ – corresponding formally adjoint differential expression to the differential expression $l(y)$. In this case the coefficients β_{jk} are the values of the function $\sigma_j^0(x)$ and its derivatives at points $x = b$, and the coefficients $\alpha_{jk}$are the values of the function $\sigma_j^0(x)$ and its derivatives at point $x = 0$, or they are differed from them by ± 1. Basic result

Theorem. *If the system of the boundary conditions $\{V_j(\cdot), j = 1\ldots,n\}$ are regular in Birkhoff's sense, then the system of eigenfunctions and associated functions of the operator L with boundary conditions* (1), *is formed Riesz's bazis with the brackets in the space $L_2[0,b]$. In particular, if boundary conditions are intense- regular, then the system of eigenfunctions and associated functions of the operator L forms a Riesz bazis in the space $L_2[0,b]$.*

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ON THE MULTIDIMENSIONAL INVERSE BOUNDARY VALUE PROBLEM FOR THE SYSTEM OF HYPERBOLIC EQUATIONS

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In the paper we investigate the solvability of the inverse multidimensional boundary value problem for the system of hyperbolic type equations. We propose a method by which the inverse boundary value problem is reduced to some nonlinear infinite systems of differential equations.

This method allows us to prove existence and uniqueness theorems for the solution multidimensional inverse boundary value problems in classes of functions of finite smoothness.

In this paper, $D_T = \Omega \times (0,T)$ consider the following problem:

$$\frac{\partial^2 u(x,t)}{\partial t^2} - Au(x,t) = a(x)U(x,t) + b(x)v(x,t) + c(x)f(x,t), \tag{1}$$

$$\frac{\partial^2 v(x,t)}{\partial t^2} - Av(x,t) = a_1(x)U(x,t) + b_1(x)v(x,t) + c_1(x)g(x,t), (x,t) \in D_T, \tag{2}$$

$$u(x,0) = \varphi(x), \ \left.\frac{\partial U(x,t)}{\partial t}\right|_{t=0} = 0\,, \ x \in \bar{\Omega}, \tag{3}$$

$$v(x,0) = \psi(x), \ \left.\frac{\partial V(x,t)}{\partial t}\right|_{t=0} = 0\,, \ x \in \bar{\Omega}, \tag{4}$$

$$v(x',t) = F(x',t)\,, \ V(x',t) = G(x',t)\,, \ (x',t) \in \Gamma = S \times [0,T], \tag{5}$$

$$u(x,T) = h(x)\,, \quad v(x,T) = g(x), \, x \in \bar{\Omega}, \tag{6}$$

$$\left.\frac{\partial U(x,t)}{\partial t}\right|_{t=T} = 0, \ \left.\frac{\partial V(x,t)}{\partial t}\right|_{t=T} = 0, \ x \in \bar{\Omega}, \tag{7}$$

where Ω is bounded domain in R^n, $S = \partial\Omega \in C^2$, $n \leq 3$,

$$Au(x,t) = \sum_{i,j=1}^{n} \left(a_{ij}(x)u_{x_i}(x,t)\right)_{x_j}, a_{ij}(x) = a_{ji}(x) \in C^4(\bar{\Omega}),$$

$\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \geq \mu\,|\xi|^2$, $\mu > 0\,,\varphi(x)$, $\psi(x)\,,f(x,t), g(x,t), F(x,t), G(x,t), h(x)$, $q(x)$, $a_1(x)$, $b(x)$are given functions, $a(x), b_1(x), c(x), c_1(x)$, $u(x,t)$, $v(x,t)$ are unknown functions._

Definition. *The system* $\{u(x,t),\, v(x,t),\, a(x),\, b_1(x), c(x), c_1(x)\}$ *is called the solution of the problem* (1) - (7) *if it satisfies to following conditions:*

(1) $a(x),\, b_1(x), c(x), c_1(x) \in W_2^2(\Omega)$.
(2) *Functions* $u(x,t)$ *and* $v(x,t)$*are continuous in closed domain* $\bar{D}_T$ *together with all derivatives in the equations* (1) *and* (2)*, respectively.*
(3) *Conditions* (1) - (7) *are satisfied in usual sense.*

The proof of the existence and uniqueness theorems were investigated.

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SOME GEOMETRIC PROPERTIES OF THE NEW DIFFERENCE SEQUENCE SPACE DEFINED BY DE LA VALLÉE POUSSIN MEAN

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Let w be the set of all sequences of real or complex numbers and ℓ_∞, c and c_0 be respectively the Banach spaces of bounded, convergent and null sequences $x = (x_k)$ with the usual norm $\|x\| = \sup |x_k|$, where $k \in \mathbb{N} = \{1, 2, \ldots\}$, the set of positive integers. Also by bs, cs, ℓ_1 and ℓ_p; we denote the spaces of all bounded, convergent, absolutely and $p-$absolutely convergent series, respectively.

Let $\lambda = (\lambda_n)$ be a non-decreasing sequence of positive numbers tending to ∞ such that $\lambda_{n+1} \leq \lambda_n + 1$, $\lambda_1 = 1$. The generalized de la Vallée-Poussin mean is defined by $t_n(x) = \frac{1}{\lambda_n} \sum\limits_{k \in I_n} x_k$, where $I_n = [n - \lambda_n + 1, n]$ for $n = 1, 2, \ldots$. A sequence $x = (x_k)$ is said to be $(V, \lambda)-$summable to a number ℓ if $t_n(x) \to \ell$ as $n \to \infty$. If $\lambda_n = n$, then $(V, \lambda)-$summability and strongly $(V, \lambda)-$summability are reduced to $(C, 1)-$summability and $[C, 1]-$summability, respectively.

The notion of difference sequence spaces was introduced by Kızmaz [4] and it was generalized by Et and Çolak([2], [3]). Later on difference sequence spaces have been studied by Bhardwaj and Bala [1], Malkowsky and Parashar [5], Mursaleen [6] and many others.

In this paper, we define the generalized difference sequence space $V\,[\Delta^m, \lambda, p]$ and studied topological and geometric properties this sequence space.

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CONSTRUCTION OF THE UNBIASED ESTIMATORS FOR THE SOLUTION OF THE NONLINEAR NEUMANN PROBLREM

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We consider the Monte Carlo solution of the nonlinear Neumann problem. Let D be a bounded convex domain in R^d with boundary G, $\varphi(x) \in C(G)$, $a(x) > 0$, $a(x) \in C(\overline{D})$, g - const and $n = n_x$ be an external normal in the point x to the surface of G. Consider the following problem:

$$Mu(x) \equiv \Delta u(x) + a(x)u(x) = g \, \exp\left(u(x)\right) + f(x),\ x \in D \tag{1}$$

$$\frac{\partial u}{\partial n}|_G = \varphi(x) \tag{2}$$

Suppose the functions $\varphi(x)$, $a(x)$ are such that there exists a unique continuous solution of the problem $u(x) \in C^2(D) \cap C(\overline{D})$.

Using the fundamental solution of the equation, we obtain a nonlinear integral equation with solution the same as the original partial differential equation. On the basis of this integral representation, we construct a probabilistic representation of the solution to our original Neumann problem. This representation is based on a branching stochastic process that allows one to directly sample the solution to the full nonlinear problem. Along a trajectory of these branching stochastic processes we build an unbiased estimator for the solution of original Neumann problem.We then provide results of numerical experiments to validate the numerical method and the underlying stochastic representation.

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REDUSING THE SPECTRAL PROBLEM FOR FORTH ORDER ELLIPTIC TYPE EQUATION ON A PLANE DOMAIN TO SECOND TYPE FREDHOLM INTEGRAL EQUATION

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The stated paper is devoted to the homogeneous boundary value problem with a parameter for the Cauchy-Riemann equation with non-local boundary conditions on the plane domain. The solution is sought in the form dictated by Green's second formula. Indeterminacy is eliminated by means of the obtained necessary conditions. Thus, consider the following boundary value problem:

$$\frac{\partial u(x)}{\partial x_2}+i\frac{\partial u(x)}{\partial x_1}=\lambda u(x)\ , \quad x\in D\subset R^2, \tag{1}$$

$$u(x_1,\gamma(x_1))=\alpha(x_1)\,u(x_1,0)\ , \quad x_1\in R, \tag{2}$$

where

$$D=\{x=(x_1,x_2): x_2\in(0,\gamma(x_1))\,,\ x_1\in R\}\ ,$$

$i=\sqrt{-1}$, $\lambda\in$ C is a parameter, $\alpha(x_1)$ is a complex valued continuous known function, $u(x)$ is a desired function, $\gamma(x_1)>0$, $x_1\in R$.

It is known that

$$U(x-\xi)=\frac{1}{2\pi}\frac{1}{x_2-\xi_2+i(x_1-\xi_1)}, \tag{3}$$

is a fundamental solution of the Cauchy-Riemann equations. Substituting the fundamental solution from (3), we have

$$u(\xi_1,0)=-\frac{1}{\pi i}\int_R\frac{u(x_1,0)}{x_1-\xi_1}dx_1+\frac{1}{\pi}\int_R\frac{u(x_1,\gamma(x_1))}{\gamma(x_1)+i(x_1-\xi_1)}\times$$
$$\times\left[1-i\gamma'(x_1)\right]dx_1-\frac{\lambda}{\pi}\int_D\frac{u(x)}{x_2+i(x_1-\xi_1)}dx, \tag{4}$$

$$u(\xi_1,\gamma(\xi_1))=-\frac{1}{\pi}\int_R\frac{u(x_1,0)}{-\gamma(\xi_1)+i(x_1-\xi_1)}dx_1-\frac{i}{\pi}\int_R\frac{u(x_1,\gamma(x_1))}{x_1-\xi_1}dx_1+$$
$$+\frac{i}{\pi}\int_R\frac{\gamma(x_1)-\gamma(\xi_1)-\gamma'(x_1)(x_1-\xi_1)}{\gamma(x_1)-\gamma(\xi_1)+i(x_1-\xi_1)}\times$$
$$\times u(x_1,\gamma(x_1))\,dx_1-\frac{\lambda}{\pi}\int_D\frac{u(x)}{x_2-\gamma(\xi_1)+i(x_1-\xi_1)}dx \tag{5}$$

Theorem. *Let $D\subset R^2$ be a domain of an upper half-plane with curvilinear boundaries with Lyapunov if $\alpha(x)$ belongs to some Holder class $\alpha(x)\neq 0$, then boundary value problem (1)-(2) is Fredholm.*

ON A CONTINUOUS SPECTRUM OF A PROBLEM ON SMALL FLUCTUATIONS OF THE IDEAL LIQUID IN A ROTATING ELASTIC VESSEL

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The work is devoted to studying of the structure of a spectrum of the problem on normal fluctuations of the ideal liquid which is rotating in aregular intervals around the fixed axis with small angular speed ε.

Considering the motion that is described as $u(x,t) = e^{i\lambda t}u(x), \rho_0\omega(x,t) = e^{i\lambda t}p(x), p(x,t) = e^{i\lambda t}p(x)$, we come to the following eigenvalue problem with spectral parameter λ

$$\mathrm{Lu}-\rho\lambda^2\mathrm{u} = 0(\Omega), \sigma(\mathrm{u})\mathrm{n}|_{\Sigma_1} = 0, \tag{1}$$

$$\lambda^2\omega + 2i\varepsilon\lambda\omega xk - \nabla p = 0, div\omega = 0(\Omega_0), \tag{2}$$

$$\sigma(u)n\,|_\Sigma = pn\,|_\Sigma\,, (\omega, n)\,|_\Sigma = \rho_0(u,n)\,|_\Sigma \tag{3}$$

The statement. For the spectrum of the problem (1)-(3) is valid: The interval $[-2\varepsilon, 2\varepsilon]$ is entirely filled by points of continuous spectrum. On beams $R/[-2\varepsilon, 2\varepsilon]$ the spectrum consists of isolated eigenvalues λ_n final algebraic frequency rate. If eigenvalue λ_n corresponds to the eigen function (u_n, ω_n) the number λ_n also will be an eigenvalue corresponding to the eigen function $(\bar{u}_n, \bar{\omega}_n)$. The system of all vectors $(u_n, \lambda_n u_n$, corresponding to the eigenvalues $\lambda_n \in R/[-2\varepsilon, 2\varepsilon]$, is full in space $W_2^1(\Omega) \oplus W_2^1(\Omega)$.

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SPECTRAL SINGULARITIES OF THE NON-SELFADJOINT MATRIX-VALUED DIFFERENCE EQUATIONS

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In this study we investigate spectral singularities of the non-selfadjoint matrix-valued difference equations of second order.

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THE INVESTIGATION OF ONE INITIAL VALUE PROBLEM FOR THE VIBRATION OF THE ELASTIC DISK

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We consider the axisymmetric biharmonic equation for the vibration of an elastic circular disk

$$u_{tt}(r,t) + c^2(\nabla^2 - \frac{1}{r^2})^2 u(r,t) = e^{-\alpha r} g(t), 0 < r < \infty, t > o$$

with the initial conditions

$$u(r,0) = re^{-ar^2}, u_t(r,0) = 0, 0 < r < \infty,$$

where $\bigtriangledown^2$-is the Laplase operator $\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)$, a, c and α are constants, r being the radial distance measured from its centre, $g(t)$ is a given function of t. Using the Hankel transform

$$\bar{u}(s,t) \equiv H_1 \left\{u(r,t); r \to s\right\} = \int\limits_0^\infty ru(r,t)J_1(sr)dr$$

and the formula

$$H_1 \left\{(\nabla^2 - \frac{1}{r^2})^2 u(r,t)\right\} = s^4 u(r,t),$$

there was obtained integral representation of the solution in the form

$$) = \frac{1}{4a^2}\int\limits_0^\infty s^2 \exp(-s^2/(4a)) \cos(cs^2 t) J_1(sr) ds +$$

$$+\frac{1}{c}\int\limits_0^\infty \frac{1}{(s^2+\alpha^2)^{3/2}} \left(\int\limits_0^t g(\tau)\sin(cs^2(t-\tau))d\tau\right) J_1(sr)ds$$

in the class of functions satisfying the condition

$$ru(r,t), ru_r(r,t) \to 0$$

as $r \to 0$ for $r \to \infty$.

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INVESTIGATION OF ONE BOUNDARY-VALUE PROBLEM WITH STABILIZED TEMPERATURE IN INFINITE TRUNCATED SECTOR APPLYING GENERALIZED MELLIN TRANSFORM

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The problem of stabilized distribution of temperature in infinite truncated sector ($a < r < \infty$, $|\theta| \leq \alpha < \frac{\pi}{2}$) is considered with given side temperature regimes and heat flood on the surface $r = a$:

$$r^2 u_{rr}(r,\theta) + ru_r + u_{\theta\theta} = 0,$$

$$u(r,\alpha) = \varphi(r),\ u(r,-\alpha) = \psi(r),\ a \leq r < \infty,$$

$$u_r(a,\theta) = f(\theta),\ \ |\theta| \leq \alpha;\ \varphi(a) = f(\alpha),\ \psi(a) = f(-\alpha).$$

After applying Mellin generalized transform

$$\tilde{u}_+(p,\theta) \equiv M\{u(r,\theta);\, r \to p\} = \int_a^\infty (r^{p-1} + a^{2p} r^{-p-1}) u(r,\theta) dr;\ \ 0 \leq Rep = \eta < \frac{\pi}{(2\alpha)},$$

there was obtained integral representation of the solution in the form:

$$u(r,\theta) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} r^{-p}\tilde{u}_+(p,\theta)dp;\quad 0 < c < \frac{\pi}{(2\alpha)},$$

$$\tilde{u}_+(p,\theta) = \frac{1}{\sin 2p\alpha}\{\tilde{\varphi}_+(p)\sin p(\theta+\alpha) - \tilde{\psi}_+(p)\sin p(\theta-\alpha)+$$

$$+(2a^{p+1}/p)[\sin p(\theta-\alpha)\int_{-\alpha}^{0} f(\theta_1)\sin(\alpha+\theta_1)d\theta_1 - \sin p(\theta+\alpha)\times$$

$$\times \int_0^\alpha f(\theta_1)\sin p(\alpha-\theta_1)d\theta_1 + \int_0^\theta f(\theta_1)\sin(\theta-\theta_1)d\theta_1]\},$$

in the class of functions $u(r,\theta)$ satisfying the condition

$$\lim_{r\to\infty}[(r^p + a^{2p}r^{-p})ru_r(r,\theta) - p(r^p - a^{2p}r^{-p})u(r,\theta)] = 0.$$

Particularly, on $\varphi(r) = \psi(r) = (r-a)/r^{1+\beta},\ \ \beta > 1 + \pi/(2\alpha),\ \ f(\theta) = 0,$ the was proved application of integral transform and for the solution there was obtained a more comfortable form for practical application:

$$u(r,\theta) = \frac{1}{\pi a^\beta}\int_0^\infty \frac{\beta(\beta+1)-\eta^2}{(\beta^2+\eta^2)((\beta+1)^2+\eta^2)}\cos(\eta\ln(a/r))\frac{sh(\eta\theta)}{sh(\eta\alpha)}d\eta.$$

ON A REQULARITY CRITERIA FOR THE BOUNDARY POINT FOR THE HEAT EQUATION IN THE SENCE OF DOMAIN BOUNDARY

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In the symmetrical domain

$$D_H = (x,t) : H < t < 0\, |x|^2 < 4t \log g(t) \subset R^{n+1}$$

the Dirichlet problem

$$U_t = \Delta U, (x,t) \in D_H \tag{1}$$

$$U|_{\partial D} = f(x,t) \in C(\partial D_H) \tag{2}$$

is considered for the heat equation and the problem of regularity of the boundary point $(0,0)$ is investigated. In the work the exact conditions have been obtained for the heat equation in the multidimensional case. The main result of the work is:

Theorem. *Let the following conditions hold*

1^0. $\rho(t) > 0$ *is a continuous and monotony decreasing functions to zero by* $t \to -0$;

2^0. $t \cdot \log \rho(t) \to 0$ by $t \to -0$;

3^0. $\frac{\rho(\eta)\cdot|\log \rho(\eta)|^{n/2}}{\eta} d\eta \to -\infty$ by $\varepsilon \to -0$.

Then the point $(0,0) \in \partial_\rho D$ *is regular for the problem (1), (2). If the integral in* 3^0 *converges, then the point* $(0,0)$ *is irregular for the problem (1),(2).*

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TWO-DIMENSIONAL MACROSCOPIC NON-DETERMINISTIC MODEL OF TRAFFIC FLOW MOTION "WITHOUT PREFERENCE"[1]

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Present work proceeds non-deterministic motion of the two-dimensional vehicular traffic flow, where the traffic flow is assumed as flow of particles in the investigated environment with allowed motion in both forward and opposite directions. Besides, it is assumed that at any fixed time interval in the two-dimensional flow, vehicles could change its positions on the road to any arbitrary placements at the defined probabilities, even they might be not the neighbouring ones. Such a non-deterministic motion of two-dimensional traffic flow will be named (by analogy to the work [1]) as motion "without preference". Under the pointed assumptions, first it is constructed the non-deterministic discrete mathematical model, and later by means of using the principle of continuous system there are applied limiting transitions to the constructed discrete model. As a result, the following non-deterministic continuous model relevant to the unknown vehicular traffic flow density $\rho(x_1, x_2, t)$ is obtained:

$$\begin{aligned} \frac{\partial\rho(x_1,x_2,t)}{\partial t} &= \int_{-l_1}^{+l_1} dy_1 \int_{-l_2}^{+l_2} K(t; y_1, y_2; x_1, x_2)\,\rho(y_1, y_2, t)\,dy_2 \\ &-\rho(x_1, x_2, t)\int_{-l_1}^{+l_1} dy_1 \int_{-l_2}^{+l_2} K(t; x_1, x_2; y_1, y_2)\,dy_2;\;\; x_i \in (-l_i, +l_i)\quad i = 1, 2;\; t \in (0, T], \end{aligned} \tag{1}$$

$$\rho(x_1, x_2, t)|_{t=0} = \rho_0(x_1, x_2),\;\; (x_1, x_2) \in S \overset{def}{\equiv} [-l_1, +l_1] \times [-l_2, +l_2], \tag{2}$$

where $\rho_0(x_1, x_2),\; (x_1, x_2) \in S$ is the initial traffic distribution at the considered road section; the function $K(t; z_1; z_2) \geq 0\;\; \left(t \in [0, T];\, z_1 = (z_1^1, z_2^1) :\; z_i^j \in [-l_i, +l_i]\;\, (i, j = 1, 2)\right)$ is the kernel of integral-differential equation (1), which could be treated in the following way: the probability for a vehicle within a traffic flow, which is located at some point $z_1 = (z_1^1, z_2^1) \in S$ road section at the initial time moment $t \in [0, T]$ (at this point and below it is assumed that $T = \infty$), to appear at the road section $s \overset{def}{\equiv} [z_2, z_2 + dz_2] \subset S,\;\; z_2 = (z_1^2, z_2^2)\;\; (z_1 \neq z_2)$ in the next moment of time $t + \Delta t$, is equal to the value $K(t; z_1; z_2)\,dz_1^2 dz_2^2 dt$. In other words, the kernel $K(t; z_1; z_2)$ is determined as the probability density of the traffic flow vehicles "moving"/"shift"/"carry" (divided by a time unit) from the point $z_1 = (z_1^1, z_2^1) \in S$ to the point $z_2 = (z_1^2, z_2^2) \in S$, where $z_1 \not\equiv z_2$, at the time moment $t \in [0, T]$. This means that function $K(t; z_1; z_2)$ is a relative velocity of such vehicles "moving" in traffic flow at the time moment t. It is also provided a probabilistic treatment of the both constructed discrete and continuous models in present work; it is studied the question of solution existence for the integral-differential equation (1) with the initial condition (2); there is a condition found, which allows to obtain the well-known model of "mass-flow equation" from the model (1), (2).

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ON SOLVABILITY OF DIRICHLET GENERALIZED PROBLEM FOR SECOND ORDER QUASILINEAR ELLIPTIC EQUATIONS

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I the bounded domain D of the Euclidean space R^n, $n \geq 2$ consider we equation

$$L_u = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i} \left(a_{i,j}(x) \left| \nabla u \right|^{p(x)-2} \frac{\partial u}{\partial x_j} \right) = 0, \tag{1}$$

with a measurable in D function $p(x)$, satisfying the condition

$$1 \leq p_1 \leq p(x) \leq p_2 < \infty. \tag{2}$$

If $p(x) = const$, the quasilinear equation (1) and its natural generalizations were studied on detail. The references on these equations is in the monographic [1].
Assume that the following conditions are fulfilled for the coefficients of the operator L:

$$\mu \left| \xi \right|^2 \leq \sum_{i,j=1}^{n} a_{ij}(x) \xi_i \xi_j \leq \mu^{-1} \left| \xi \right|^2, \ \ \mu \in (0,1]. \tag{3}$$

For determining the solution, the Eudroduct the class of functions

$$W_{loc}(D) = \{ u : u \in W^1_{1,loc}(D), \ \ \left| \nabla u \right|^{p(x)} \in L_{1,loc}(D) \}.$$

It is assumed that exponent satisfies the relation

$$\left| p(x) - p(y) \right| \leq \frac{C}{\ln \left| x-y \right|^{-1}}, \ x, y \in D, \ \ \left| x - y \right| < \frac{1}{2}. \tag{4}$$

In the paper the investigate the – on solvability of the generalized Dirichlet problem

$$L_{uf} = 0 in D, u_f \left| \partial D = f \right. \tag{5}$$

With a conditions on D function $f(x)$. We determine the solution of the problem as follows. We continue the boundary function $f \in C(\partial D)$ by the continuity on D, having retained the same denotation and – the sequence of infinitely differentiable in R^n functions $f_k(x)$that uniformly on D converge to $f(x)$.

Theorem. *If the conditions* (2),(3) *and* (4) *are satisfied, the generalized solution* $u_f(x)$ *of problem (5) exists, is unglue and belong to the class* $W_{\log}(D)$.

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INVESTIGATION OF THE EXISTENCE AND UNIQUENESS OF THE SOLUTION OF A CLASS OF THE SYSTEM OF THE FIRST ORDER PARTIAL EQUATIONS

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A wide class of important applied processes is described by the certain system of the first order partial differential equations [1],[2].

Let's consider in a rectangle $D = \{0 < t < T, \ 0 < s < S\}$ the following system of equations

$$\begin{array}{l} x_t = A_{11}(t,s)\,x + A_{12}(t,s)\,y + f_1(t,s)\,, \\ y_s = A_{21}(t,s)\,x + A_{22}(t,s)\,y + f_2(t,s) \end{array} \qquad (1)$$

with initial and boundary conditions

$$\begin{array}{ll} x(o,s) = \varphi_1(s)\,, & 0 \le s \le S, \\ y(t,0s) = \varphi_2(t)\,, & 0 \le t \le T, \end{array} \qquad (2)$$

where $A_{ij} - (n_i \times n_j)$, $(i,j = 1,2)$ are matrices, f_i and φ_i- n_i, $(i = 1,2)$- dimensional column vectors.

Theorem. *Let's assume that the following conditions are satisfied*:

(1) *The matrices* $A_{ij}(t,s)\,,\ (i,j = 1,2)$ and vectors $f_i(t,s)\,,\ (i,j = 1,2)$ *are defined and measurable* in$D$; *vectors* $\varphi_1(s)$ and $\varphi_2(t)$ *are defined and measurable on* $[o,S]$ and $[o,T]$, *correspondingly.*

(2) *Norms* $|A_{ij}(t,s)|\,,\ |f_i(t,s)|\,,\ |\varphi_1(s)|\,,\ |\varphi_2(t)|$ *are integrable in the definition domain where the norm* $(n \times m)$ *of the matrix* $A = (a_{ij})$ *is defined by equality* $|A| = \sum\limits_{i=1}^{n}\sum\limits_{j=1}^{m}|a_{ij}|$;

(3) *Matrices* $A_{i1}(t,s)\ \ (A_{i2}(t,s))\,, (i = 1,2)$ *for almost all* $t \in [0,T]$ *(for almost all* $s \in [0,S]$*) are continuous on* s *(are continuous on* t*).*

Then the problem (1), (2) in space $W_n^{1,0}(D) \times W_m^{1,0}(D)$ has a unique solution.

For the homogeneous adjoint system the Riemann matrix is defined and integral representation of the solution of the considered problem is obtained.

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ON A UNIQUENESS OF THE RECONSTRUCTION OF THE DIRAC OPERATOR OVER TWO SPECTRUMS

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Let $p(x)$ and $q(x)$ be real valued functions belonging to the space $L_2[0,\pi]$. Let's denote by $L(p,\,q,\,h,\,H,\,b)$ boundary problem generated on the segment $[0,\pi]$ by the canonical Dirac equation

$$By'(x)+Q(x)y(x)=\lambda\,y(x)$$

with semi-separated boundary conditions

$$y_2\,(0)-hy_1\,(0)=0,\qquad y_2\,(\pi)+Hy_1\,(\pi)+by_1\,(0)=0,$$

where $B=\begin{pmatrix}0&1\\-1&0\end{pmatrix}$, $Q(x)=\begin{pmatrix}p(x)&q(x)\\q(x)&-p(x)\end{pmatrix}$, $y(x)=\begin{pmatrix}y_1(x)\\y_2(x)\end{pmatrix}$, $h,\,H,\,b$are real numbers and $Hh-1>0$, $H+h>0$. In the present work, following results have been obtained.

Lemma. *Eigenvalues $\lambda_n\;\;(n=0,\,\pm1,\,\pm2,\,...)$ of the problem $L(p,\,q,\,h,\,H,\,b)$ satisfy the asymptotic formula*

$$\lambda_n=n+(-1)^nA+D+\xi_n,$$

where

$$A=\frac{1}{\pi}\arcsin\frac{b}{\alpha},\quad D=\frac{1}{\pi}\arccos\frac{hH-1}{\alpha},\alpha=\sqrt{(Hh-1)^2+(H+h)^2},\;\{\xi_n\}\in l_2.$$

Let's denote by $\left\{\lambda_n^{(k)}\right\}$, $\left\{\mu_n^{(k)}\right\}$ the spectrums of the initial problems $L(p,\,q,\,h,\,H_k,\,b_k)$, $L\left(\tilde{p},\,\tilde{q},\,\tilde{h},\,\tilde{H}_k,\,\tilde{b}_k\right)$ $(k=1,\,2)$, correspondingly.

Theorem. *If for all $n=0,\,\pm1,\,\pm2,...$ takes place $\lambda_n^{(k)}=\mu_n^{(k)}$ $(k=1,\,2)$, then $p\,(x)=\tilde{p}\,(x)\,,\;q\,(x)=\tilde{q}\,(x)$ almost everywhere on the segment $[0,\,\pi]$, $h=\tilde{h},\;H_k=\tilde{H}_k,\;b_k=\tilde{b}_k$.*

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ON INITIAL - BOUNDARY VALUE PROBLEM FOR LOADED PSEUDO-PARABOLIC EQUATION

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In this paper the initial-boundary value problem for loaded pseudo-parabolic equation of the third order is studied. In the domain $D = \{(x,t) : 0 < x < l,\ 0 < t < T\}$ we consider equation

$$Lu = f(x,t) + \frac{\partial}{\partial t}\int_{\alpha}^{\beta} u(x,t)dx, \qquad (1)$$

here $Lu \equiv u_{xxt} + d(x,t)u_t + a(x,t)u_{xx} + b(x,t)u_{xt} + c(x,t)u_x + e(x,t)u,$ α, β are given constants such, that $0 \le \alpha \le \beta \le l$.

Problem. Find the regular solution $u(x,t)$ of the equation (1) in D which satisfies the following initial condition

$$u(x,0) = \varphi(x), 0 \le x \le l, \qquad (2)$$

and following boundary conditions

$$u(0,t) = \psi_1(t), u(l,t) = \psi_2(t), 0 \le t \le T, \qquad (3)$$

where $\varphi(x)$, $\psi_1(t)$, $\psi_2(t)$ are given functions such that

$$\varphi(0) = \psi_1(0), \varphi(l) = \psi_2(0).$$

Theorem. *Let $d(x,t) < 0$ for any $(x,t) \in D$ and given functions satisfy following conditions*

$$a(x,t), b(x,t) \in C^1(\overline{D}) \bigcap C^2(D), c(x,t), d(x,t) \in C(\overline{D}) \bigcap C^1(D),$$

$e(x,t), f(x,t) \in C(\overline{D})$, $\varphi(x) \in C^1[0,l] \bigcap C^2(0,l)$, $\psi_1(t), \psi_2(t) \in C^1[0,T]$. *Then there exists a unique solution of the problem (1) - (3).*

The following example shows that in general the assumption that $d(x,t) < 0$, $\forall (x,t) \in D$ is necessary.

It is easy to see, that $u(x,t) = t\sin(2kx)$ is solution of

$$u_{xxt} + (2k)^2 u_t = \frac{\partial}{\partial t}\int_0^{\pi} u(x,t)dx,$$

in $D_0 = \{(x,t) : 0 < x < \pi, 0 < t < T\}$ and $u(x,t)$ satisfies the initial boundary conditions $u(x,0) = 0$, $u(0,t) = 0$, $u(\pi,t) = 0$, i.e. the solution of the initial - boundary value problem for above equation is not unique.

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THE ELLIPTIC AND PARABOLIC EQUATIONS WITH SINGULAR POTENTIALS IN CONE-LIKE DOMAINS

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We consider the following problems

$$-\Delta u = V(x)u, \quad u|_{\partial\Omega} = \phi(x) \tag{1}$$

in the domain $\Omega = G \cap B \subset R^n (n \geq 3)$, and

$$\frac{\partial u}{\partial t} - \Delta u = V(x)u + f(x,t), \quad u|_{t=0} = u_0(x), \quad u|_{\partial\Omega} = 0 \tag{2}$$

in the domain $\Omega \times (0,T)$, where $\Omega = G \cap B \subset R^n (n \geq 3), x = (x_1, ..., x_n) \in \Omega, B = B(0,\rho) = \{x \in R^n : |x| < \rho < 1\} \subset R^n$ and $\partial\Omega-$ the boundary of $\Omega, 0 < T \leq \infty, G$ be a cone with vertex at the origin. We suppose that the boundary of Ω, except the origin, is smooth enough.

Under solution to the equation in (1) (respectively to the equation in (2)) we mean the generalized function $u(x) \in D'(\Omega)$, (respectively $u(x,t) \in D'(\Omega \times (0,T))$) such that $u \geq 0$ and $Vu \in L_{1,loc}$. Assumed that $0 \leq V(x) \in L_1(\Omega), 0 \leq u_0(x) \in L_1(\Omega), f(x,t) \in L_1(\Omega \times (0,T))$ and $0 \leq \phi \in L_1(\partial\Omega), \phi(x)$ is continuous on $\partial\Omega$.

Put

$$V_0(x) = \frac{(n-2)^2}{4|x|^2} + \frac{c}{4|x|^2 \log^2 |x|} + \frac{\lambda_\rho}{|x|^2}, x \in \Omega, \tag{3}$$

where $\lambda_\rho > 0$ be a first eigenvalue of the operator $-\Delta_\omega$ on $G \cap \partial B$ with zero Dirichlet condition on $\partial G \cap \partial B$.

In this paper is studied the behavior of nonnegative solutions to the problems (1) and (2), when $V_0(x)$ is given by (3), and is proved that if $0 \leq c \leq 1$ and $V(x) \leq V_0(x)$ in Ω, then this problems has a nonnegative solutions; if $c > 1$ and $V(x) \geq V_0(x)$ in Ω, then this problems does not have nonnegative solutions.

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ON ASYMPTOTICS OF THE SOLUTION OF SOME INTEGRO-DIFFERENTIAL EQUATIONS

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The work is devoted to the construction of asymptotic solution for initial problem of some singular perturbed integro-differential equations ,analysis of its properties and shown important role of integral part for bounded ness of solution.

In the work using [1-7] the asymptotics of the following system of equations is constructed and investigated

$$\frac{dx}{dt} = L_1[x,y] + f_1(t),$$
$$\varepsilon\frac{dy}{dt} = L_2[x,y] + f_2(t), \tag{1}$$

with boundary conditions

$$x(0,\varepsilon) = x^0, y(T,\varepsilon) = y^0, \tag{2}$$

where

$$L_i[x,y] = A_{i1}(t)x + A_{i2}(t)y + \int_0^t [K_{i1}(t,s)x(s) + K_{i2}(t,s)y(s)]ds,$$

$i = 1,2; \varepsilon > 0-$small parameter, $0 \le t \le T, x, f_1 - n-$dimensional, $y, f_2 - m-$ dimensional vectors,

$A_{11}, K_{11}; A_{12}, K_{12} - (n\times n); (n\times m), A_{22}, K_{22} - (m\times m), A_{21}, K_{21} - (m\times n)$ dimensional enough smooth matrices, $K_{22}(t,s) \ne 0$.

Let the characteristic values $\lambda_i(t)$ of the matrix $A_{22}(t)$ satisfy the condition

$$Re\lambda_i(t) < 0, (i = \overline{1,m}, 0 \le t \le T) \tag{3}$$

Note that, by fulfillment of the condition (3) and when the integral part is absent problem (1), (2) generally has no bounded by $\varepsilon \to 0$solution, at the same time (1), (2) has bounded by $\varepsilon \to 0$ solution $x(t,\varepsilon), y(t,\varepsilon)$.

Therefore it is shown that appearing of the integral part leads to quantitative changes of the behavior of the solution of the boundary problem.

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INVESTIGATION OF THE ENERGY EIGENVALUE RELATIVELY TO BOUNDARY PARAMETER

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Schrödinger equation, describing the state of the particle in the spherical -symmetric (central) field is reduced to the following radial Schrödinger equation:

$$\left[-\frac{\hbar^2}{2\mu r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)+\frac{\hbar^2 l(l+1)}{2\mu r^2}+V(r)\right]R(r)=ER(r).$$

Here E is the energy eigenvalue. Let's consider the following boundary condition

$$R(s_1)=0,\ \ R(s_2)=0.$$

In this case energy eigenvalue depending from $s=(s_1,s_2)$, $\quad E=E(s)=E(s_1,s_2)$.

Theorem. *Energy eigenvalue differentiable relatively $s=(s_1,s_2)$ and*

$$\frac{\partial E}{\partial s_1}=\frac{\hbar^2}{2\mu}\left(\frac{dR(s_1)}{dx}\right)^2 s_1^2$$

$$\frac{\partial E}{\partial s_2}=-\frac{\hbar}{2\mu}\left(\frac{dR(s_2)}{dx}\right)^2 s_2^2.$$

Here we suppose that $R=R(r)$ is normalized eigenfunction, i.e.

$$\int_{s_1}^{s_2} r^2 R(r)dr=1.$$

Corollary. *Energy eigenvalue increases with s_1 and decreases with respect to s_2.*

ON AN ALGORITHM OF NUMERICAL SOLUTION TO A QUICK-ACTION PROBLEM FOR OSCILLATION PROCESS

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Within the framework of the mathematical model

$$a^2 u_{xx} + f(x,t) = u_{tt},\ 0 < x < l,\ 0 < t \le T, \tag{1}$$

$$\left.\begin{array}{l} u(x,0) = \varphi(x) \\ u_t(x,0) = \psi(x) \end{array}\right\} 0 \le x \le l, \tag{2}$$

$$\left.\begin{array}{l} \sigma_1 u(0,t) - \sigma_2 u_x(0,t) = v_1(t) \\ \sigma_3 u(l,t) + \sigma_4 u_x(l,t) = v_2(t) \end{array}\right\} 0 \le t \le T, \tag{3}$$

we put an optimal time optimal problem: to find functions $v_i(t)$ $(i = 1,2)$, $f(x,t)$ and $u(x,t)$ which satisfy conditions (1)-(3) and the constraints

$$\begin{array}{l} v_{i\min} \le v_i(t) \le v_{i\max} \quad (i = 1,2), \\ f_{\min} \le f(x,t) \le f_{\max}; \end{array} \tag{4}$$

at that the inequality

$$\int_0^l (u(x,T) - u(x))^2\, dx \le \delta^2 \tag{5}$$

takes place for a minimal time T given the function $u(x)$ and constant $\delta > 0$, where $\varphi(x)$, $\psi(x)$ are given functions; $v_{i\min},\ v_{i\max}$ $(i = 1,2)$, $f_{\min},\ f_{\max},\ \sigma_i\left(i = \overline{1,4}\right)$, $\left(\sigma_1^2 + \sigma_2^2 \ne 0,\ \sigma_3^2 + \sigma_4^2 \ne 0\right)$ given numbers.

Numerical algorithm of the solution to problem (1)-(5) is proposed. The algorithm is based on the solution to a series of optimal control problems under the fixed time. To the numerical solution to the optimal control problems we apply conditional gradient method. We obtain analytical formulas for the gradient of the functional and for conjugate boundary problem. At each iteration of the sequential method, direct and conjugate boundary problems are solved using finite difference and Fourier methods. Numerical experiments have been carried out, the results of which show the efficiency of the algorithm proposed.

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TRANSIENT VISCOELASTIC WAVES IN MATERIALS WITH VARIABLE POISSON RATIO

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Propagation of transient waves in isotropic homogeneous viscoelastic materials with variable Poisson coefficient, which is occupied the domain V with the surface S, is reduced to the solution of the integro-differential equation

$$\left(R_1^*+\frac{2}{3}R*\right)\,grad\;div\,\vec{u}-\frac{R^*}{2}rot\;rot\,\vec{u}=\rho\frac{\partial^2\vec{u}}{\partial t^2},\vec{x}\in V,\;t>0$$

with initial $\vec{u}=0\,,\;\frac{\partial\vec{u}}{\partial t}=0\,,\quad t=0\,,\;\vec{x}\in V$ and the following boundary conditions

$$\sigma_{\rho j}n_j|_{S_\sigma}=f_\rho(x,y,z)|_{S_\sigma}\,a_\rho(t)\,,\qquad t>0\quad(\rho\,,\;j=1,\;2\,,\;3),$$

$$u_\rho|_{S_u}=F_\rho(x,y,z)|_{S_u}\,b_\rho(t)\,,\qquad t>0.$$

Here $S=S_\sigma\cup S_u$, $\vec{u}$ is displacement vector, σ_{ij} and n_j are the components of stress tensor and unit vector, respectively, ρ is density, f_i,F_i,a_i,b_i are the given functions, $R\,(t)$ and $R_1\,(t)$ are the relaxation functions, and the asterisks on the letters denote the operators as

$$R^*\varphi(\vec{x}\,,\;t)=\int\limits_0^t R(t-\tau)\,d\varphi(\vec{x}\,,\;\tau).$$

Theorem. *Let the Laplace transform of the problem is represented in the form*

$$\bar{\vec{u}}(\vec{x},p)=\sum_k\left\{\bar{a}_{1k}(p)\,\bar{\vec{u}}_{1k}\left(\vec{x},\;\frac{p}{c_1}\sqrt{q_1}\right)\,\left[\chi_{10k}\,(p\sqrt{q_1})+w_1\chi_{11k}\,(p\sqrt{q_1})+\;\frac{1}{2}w_1^2\chi_{12k}\,(p\sqrt{q_1})+...\right]+\right.$$

$$\left.+\bar{a}_{2k}(p)\,\bar{\vec{u}}_{2k}\left(\vec{x},\;\frac{p}{c_2}\sqrt{q_2}\right)\left[\chi_{20k}\,(p\sqrt{q_2})+w_2\chi_{21k}\,(p\sqrt{q_2})+\frac{1}{2}w_2^2\chi_{22k}\,(p\sqrt{q_2})+...\right]\right\}\,,$$

where $w_1=\frac{q_2}{q_1}-1\,,\;\;w_2=\frac{q_1}{q_2}-1,\;q_1(p)=\frac{R_{10}+\frac{2}{3}R_0}{p\bar{R}_1+\frac{2}{3}p\bar{R}}\,,\;\;q_2(p)=\frac{R_0}{p\bar{R}}$

T*hen the solution of the problem is*

$$\vec{u}(\vec{x},t)=\sum_k[\int\limits_0^\infty\vec{U}_{10k}(\vec{x},\tau)\gamma_k(r,\tau)d\tau+\delta_{11}(t)*\int\limits_0^\infty\vec{U}_{11k}(\vec{x},\tau)\gamma_k(r,\tau)d\tau+$$

$$+\frac{1}{2}\delta_{12}(t)*\int\limits_0^\infty\vec{U}_{12k}(\vec{x},\tau)\gamma_k(r,\tau)d\tau+...+\int\limits_0^\infty\vec{U}_{20k}(\vec{x},\tau)W_k(r,\tau)d\tau+$$

$$+\delta_{21}(t)*\int\limits_0^\infty\vec{U}_{21k}(\vec{x},\tau)W_k(r,\tau)d\tau+\frac{1}{2}\delta_{22}(t)*\int\limits_0^\infty\vec{U}_{22k}(\vec{x},\tau)W_k(r,\tau)d\tau+...]\,,$$

where $\quad\bar{u}_{ik}\left(r,\frac{p}{c_1}\right)\,\chi_{imk}(p)\;=\;U_{imk}(r,t)\;:,w_i^m\;=\;\delta_{im}(t)\;:\;(i\;=\;1,\;2;\;m,k\;=\;1\,,\;2,\;3\,,\;...)$ *Here transformation from elastic solution to viscoelastic one is exact.*

If $\delta_{ij}\,(t)=0$, the solution for constant Poisson coefficient is obtained. In addition, if the functions γ_k and W_k are the Dirac-delta function [1], then the elastic solution is obtained.

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ON AN EXISTENCE OF PERIODIC SOLUTIONS OF THE KDV TYPE EQUATION

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In this paper we investigate the existence of periodic solutions with respect to the variable t of the KDV type of equation

$$u_t(t,x) + u(t,x)u_x(t,x) + u_{xxx}(t,x) = f(t,x,u(t,x)), \tag{1}$$

where $f(t,x,u) \in \bar{C}(R_+,R,R)$ is the known periodic function, $f(t,x,u) = f(t+T,x,u)$, T=const.

We use the substitution

$$u(t,x) = \frac{1}{2\pi}\int_t^{t+T}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{-iw(x-s)-(w^2+s^2+1)(t-\nu)}}{e^{(w^2+s^2+1)T}-1}Q(\nu,s)dsdwd\nu, \tag{2}$$

with the new unknown function $Q(t+T,x) = Q(t,x)$, $\|Q(t,x)\| = \sup\limits_{\substack{0\le t\le T\\ -\infty<x<\infty}} |Q(t,x)|$.

To determine $Q(t,x)$we obtain the equation

$$Q(t,x) = f\left(t, x, \frac{1}{2\pi}\int_t^{t+T}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{-iw(x-s)-(w^2+s^2+1)(t-\nu)}}{e^{(w^2+s^2+1)T}-1}Q(\nu,s)dsdwd\nu\right)+$$

$$+\frac{1}{2\pi}\int_t^{t+T}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{-iw(x-s)-(w^2+s^2+1)(t-\nu)}(w^2+s^2+1)}{e^{(w^2+s^2+1)T}-1}Q(\nu,s)dsdwd\nu+$$

$$+\frac{1}{4\pi^2}\int_t^{t+T}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{-iw(x-s)-(w^2+s^2+1)(t-\nu)}}{e^{(w^2+s^2+1)T}-1}Q(\nu,s)dsdwd\nu\cdot$$

$$\cdot\int_t^{t+T}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{-iw(x-s)-(w^2+s^2+1)(t-\nu)}}{e^{(w^2+s^2+1)T}-1}iwQ(\nu,s)dsdwd\nu.$$

This operator equation proved to be contractive mapping. Therefore according to the principle of contracting mappings it has the unique solution. Then equation (1) poses the unique solution in the form (2).

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DEFINITION OF THE RIGHT-HAND SIDE OF HIGH ORDER NON-HOMOGENEOUS LINEAR DIFFERENTIAL EQUATIONS ON ADDITIONAL INFORMATION

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The problem of reconstructing of solutions and special right sides is solved for inhomogeneous linear differential equations of higher orders with the redefined two- point boundary conditions. It is possible to perform a full descriptions of all possible available right sides.

The following problem arises in various areas of mathematics:

$$y^n(t) + p_1(t)y^{(n-1)}(t) + \ldots + p_n(t)y(t) = \mu(t) + b(t)\nu(t), 0 \le t \le 1 \tag{1}$$

under conditions of

$$U_j(y) = \sum_{\nu=1}^{n-1} \left[\alpha_{j\nu} y^{(\nu)}(0) + \beta_{j\nu} y^{(\nu)}(1)\right] = b_j, j = \overline{1,n} \tag{2}$$

$$V_i(y) = \sum_{\nu=1}^{n-1} \left[\gamma_{i\nu} y^{(\nu)}(0) + \theta_{i\nu} y^{(\nu)}(1)\right] = a_i, i = \overline{1,n} \tag{3}$$

where $p_k(t)$, $k = \overline{1,n}$; $\mu(t)$ – given piecewise continuous functions, $\nu(t)$ – unknown function.

We assume, that

1)The matrix rank $(\alpha_{j\nu}, \beta_{j\nu})$ is equal n,

2)Homogeneous task $\dfrac{dy}{dt} = A(t)y(t)$,

$$y^n(t) + p_1(t)y^{(n-1)}(t) + \ldots + p_n(t)y(t) = 0,$$
$$U_j(y) = 0, j = \overline{1,n}$$

has only the trivial decision.

The considered problem (1) – (3) can be interpreted:

1) as the problem of definition of the right -hand of the non-homogeneous equation (1) on additional information. Then it represents a so-called return task,

2) as overdetermined boundary value problems for ordinary differential equations;

3) as the initial step in the development of algorithms for solving optimal control In this paper, a theorem which gives a complete description of the controls for the problem (1) – (3). Stated that the family of such control depends on an arbitrary square-integrable function

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HOMOTOPY ANALYSIS METHOD FOR SOLVING OF THE FRACTIONAL KDV EQUATION

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In this paper, we obtain approximate solution for soliton solution of the fractional KdV equation by homotopy analysis method. Numerical results which obtained by homotopy analysis method (HAM) are compared with other solutions which obtained with Adomian decomposition (ADM) and variational iteration method (VIM). The numerical results show that the only few terms are sufficiented to obtain accurate solutions. Also, the results given by tables and figures.

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EXISTENCE AND UNIQUENESS OF SOLUTION OF A B.V.P FOR SECOND ORDER O.D.E WITH GENERAL LINEAR NONLOCAL BOUNDARY CONDITIONS

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According to the classic text books of functional analysis and boundary value problems, existence of solution of B.V.P is uniqueness of its adjoint problem and vice versa. By making use of this mathematical fact, in this paper for the existence and uniqueness of B.V.P including second order ordinary differential equation with non-local boundary conditions, we prove the uniqueness of solutions for the main problem and its adjoint problem.

ON THE INVERSE SCATTERING TRANSFORM IN 2 + 1 DIMENSIONS FOR THE NONLINEAR EVOLUTION EQUATION RELATED TO NONSTATIONARY DIRAC-TYPE SYSTEMS ON THE PLANE

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The inverse scattering transform (IST) method for the nonlinear evolution equations with 1+1 dimensions (one space and one time dimensions) have been described in various reviews and monographs. The generalization of the IST method to nonlinear evolution equations with 2+1 dimensions (two space and one time dimension) has been developed in monograph [1]. The main step in the IST method is to represent the mentioned nonlinear evolution equation in the form of Lax equation

$$\frac{\partial \mathbf{L}}{\partial t} = [\mathbf{L}, \mathbf{A}]. \tag{1}$$

Let $\mathbf{L} = \frac{\partial}{\partial y} - \mathbf{M}$, where $\mathbf{M}$ is an ordinary differential operator in x. Then equation (1) is rewritten as (see [2])

$$\left[\frac{\partial}{\partial y} - \mathbf{M}, \frac{\partial}{\partial t} - \mathbf{A}\right] = 0. \tag{2}$$

Some nonlinear evolution equations in 2+1 dimension are presented in [3] by using commutativity condition (2) when $\mathbf{M}$ is an scalar coefficients ordinary differential operator in x.

Let $\mathbf{M} = \sigma\frac{\partial}{\partial x} + Q$ and $\mathbf{A} = \tau\frac{\partial}{\partial x} + P$ in (2). Here σ, τ, Q and P are square matrices of the order n $(n \geq 3)$. Let the matrices σ and τ be real and diagonal: $\sigma = \begin{bmatrix} I_{n_1} & 0 \\ 0 & -I_{n_2} \end{bmatrix}$, $\tau = \begin{bmatrix} 2B_1 & 0 \\ 0 & 2B_2 \end{bmatrix}$, where I_{n_i} $(i = 1, 2)$ is identity matrix of the order n_i $(n_1 + n_2 = n)$, $B_1 = diag(b_1, \ldots, b_{n_1})$, $B_2 = diag(b_{n_1+1}, \ldots, b_n)$ with $b_i \neq b_j$ $(i \neq j)$ and the matrices Q and P obey the relation: $[\sigma, P] = [\tau, Q]$. Let $Q = \begin{bmatrix} 0 & q_{12} \\ q_{21} & 0 \end{bmatrix}$. Take $P = \begin{bmatrix} p_{11} & B_1 q_{12} - q_{12} B_2 \\ q_{21} B_1 - B_2 q_{21} & p_{22} \end{bmatrix}$, where the functions p_{11} and p_{22} are solutions of the equations $\partial_\eta p_{11} = [q_{12}q_{21}, B_1]$ and $\partial_\xi p_{22} = [B_2, q_{21}q_{12}]$, respectively. Then, the condition (2) yields to the system of equations

$$\begin{aligned} &\partial_t q_{12} - B_1 \partial_\xi q_{12} + \partial_\eta q_{12} B_2 = p_{11} q_{12} - q_{12} p_{22}, \\ &\partial_t q_{21} - \partial_\xi q_{21} B_1 + B_2 \partial_\eta q_{21} = p_{22} q_{21} - q_{21} p_{11}, \end{aligned} \tag{3}$$

where $\partial_t = \frac{\partial}{\partial t}, \partial_\xi = \frac{\partial}{\partial y} + \frac{\partial}{\partial x}, \partial_\eta = \frac{\partial}{\partial y} - \frac{\partial}{\partial x}$. We recall the necessary results on the inverse scattering problem for the equation $\frac{\partial}{\partial y}\psi - \mathbf{M}\psi = \mathbf{0}$, with $\mathbf{M} = \sigma\frac{\partial}{\partial x} + Q$ on the whole plane, where the matrix coefficient Q decrease quite fast with respect to variables x and y at infinity (see [4]). Then this results are applied to the integration of the nonlinear 2+1 dimensional systems of equations (3).

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THE CAUCHY PROBLEM FOR DEGENERATE EQUATIONS OF MAGNETIC GAS DYNAMICS IN POROUS MEDIUM

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The equations of magnetic gas dynamics in porous medium in Lagrangian coordinates have the form:

$$\frac{\partial v}{\partial t} - \frac{\partial u}{\partial x} = 0, v = \frac{\rho^0}{\rho},$$
$$\rho^0 \frac{\partial u}{\partial t} = \mu \frac{\partial}{\partial x}(\frac{1}{v}\frac{\partial u}{\partial x}) - \frac{\partial p}{\partial x} - \mu_\ell H \frac{\partial H}{\partial x} - \beta(x)|u|^\alpha u, p = k\rho^0 \frac{\theta}{v},$$
$$\rho^0 \frac{\partial \theta}{\partial t} = \lambda \frac{\partial}{\partial x}(\frac{1}{v}\frac{\partial \theta}{\partial x}) - p\frac{\partial u}{\partial x} + \frac{\mu}{v}(\frac{\partial u}{\partial x})^2 + \frac{\mu_\ell \mu_H}{v}(\frac{\partial H}{\partial x})^2,$$
$$\frac{\partial}{\partial t}(vH) = \mu_H \frac{\partial}{\partial x}(\frac{1}{v}\frac{\partial H}{\partial x}). \qquad (1)$$

We study the system of educations under the initial conditions.

$$\rho|_{t=0} = \rho^0(x), u|_{t=0} = u^0(x), \theta|_{t=0} = \theta^0(x), H|_{t=0} = H^0(x), v|_{t=0} = 1, \qquad (2)$$

where $(\rho^0, u^0, \theta^0, H^0)$ are continuous, (ρ^0, u^0, θ^0) are bounded non-negative functions and have finite limits at infinity:

$$\lim_{x\to-\infty} \rho^0(x) = {\rho_1}^0, \ \lim_{x\to+\infty} \rho^0(x) = 0,$$
$$\lim_{x\to-\infty} u^0(x) = {u_1}^0, \ \lim_{x\to+\infty} u^0(x) = {u_2}^0, {u_1}^0 < {u_2}^0,$$
$$\lim_{x\to-\infty} \theta^0(x) = {\theta_1}^0, \ \lim_{x\to+\infty} \theta^0(x) = {\theta_2}^0, {\theta_1}^0 \neq {\theta_2}^0,$$
$$\lim_{x\to-\infty} H^0(x) = {H_1}^0, \ \lim_{x\to+\infty} H^0(x) = {H_2}^0, {H_1}^0 \neq {H_2}^0. \qquad (3)$$

Global a priori estimates are derived and global in time existence of a unique generalized solution is proved.

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A NEW NON STANDARD METHOD OF REDUCTION TO THE SYSTEM AND STABILITY OF SOLUTIONS OF LINEAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATION OF FOURTH ORDER

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All appearing functions and their derivatives are continuous at $t \geq t_0,\ t \geq \tau \geq t_0$; $J = [t_0, \infty)$; IDE - integro-differential equation; by the stability of solutions of a linear fourth-order IDE means boundedness on the interval J all its solutions and their derivatives up to third order inclusive.

The sufficient conditions for stability of solutions of linear IDE fourth-order Volterra type:

$$x^{(4)}(t) + \sum_{k=0}^{3} [a_k(t)x^{(k)}(t) + \int_{t_0}^{t} Q_k(t,\tau)x^{(k)}(\tau)d\tau] = f(t), t \geq t_0 \tag{1}$$

are established. To do this, proceed as follows. First, the IDE (1) make the following change:

$$x'''(t) + px''(t) + qx'(t) + rx(t) = W(t)y(t), \tag{2}$$

where p, q, r - some auxiliary parameters, and $p > 0,\ q > 0,\ r > 0; 0 < W(t)$ - some weighting function, $y(t)$ - new unknown function. We find from (2):

$$\begin{array}{l} x^{(4)}(t) = -px'''(t) - qx''(t) - rx'(t) + W(t)y'(t) + W'(t)y(t) = -p[-px''(t) - qx'(t) - \\ -rx(t) + W(t)y(t)] - qx''(t) - rx'(t) + W(t)y'(t) + W'(t)y(t) = [p^2 - q]x''(t) + \\ +[pq - r]x'(t) + prx(t) + W(t)y'(t) + [W'(t) - pW(t)]y(t). \end{array} \tag{3}$$

Substituting (2),(3) to (1), we obtain the first order IDE for the $y(t)$. Combining this with the IDE (1), we obtain the following system:

$$\begin{array}{l} x'''(t) + px''(t) + qx'(t) + rx(t) = W(t)y(t), \\ y'(t) + b_3(t)y(t) + b_2(t)x''(t) + b_1(t)x'(t) + b_0(t)x(t) + \\ + \int_{t_0}^{t} [P_0(t,\tau)x(\tau) + + P_1(t,\tau)x'(\tau) + P_2(t,\tau)x''(\tau) + K(t,\tau)y(\tau)]d\tau = F(t), \quad t \geq t_0. \end{array} \tag{4}$$

equivalent to the IDE (1), where $b_3(t) \equiv a_3(t) - p + W'(t)(W(t))^{-1}$, $b_2(t) \equiv [a_2(t) - pa_3(t) + p^2 - q](W(t))^{-1}$, $b_1(t) \equiv [a_1(t) - qa_3(t) + pq - r](W(t))^{-1}$, $b_0(t) \equiv [a_0(t) - ra_3(t) + pr](W(t))^{-1}$, $P_0(t,\tau) \equiv (W(t))^{-1}[Q_0(t,\tau) - rQ_3(t,\tau)]$, $P_1(t,\tau) \equiv (W(t))^{-1}[Q_1(t,\tau) - qQ_3(t,\tau)]$, $P_2(t,\tau) \equiv (W(t))^{-1}[Q_2(t,\tau) - pQ_3(t,\tau)]$, $K(t,\tau) \equiv (W(t))^{-1}Q_3(t,\tau)W(\tau)$, $F(t) \equiv (W(t))^{-1}f(t)$.

Now the first equation (4) squaring [1, p. 28], the second equation multiplied by $y(t)$ add the resulting relations, we integrate from t_0 to t, including parts, while to the integrals with $K(t,\tau)$ and $F(t)$ develop a method of cutting functions [1, p. 41]. We turn to an integral inequality and we obtain estimates for $x^{(k)}(t)\ \ (k = 0, 1, 2)$, $y(t)$, from which we have $x^{(k)}(t) = O(1)\ \ (k = 0, 1, 2)$, $y(t) = O(1)$. Finally, from (2) $W(t) = O(1)$ should and $x'''(t) = O(1)$

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THE BETHE-SALPETER TYPE EQUATION FOR THREE-QUARK BOUND STATES

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In this report we present some our results of the investigation of multi-quark equations in the Nambu–Jona-Lasinio (NJL)[1] model with chiral symmetry of $SU(2)-$group in the mean-field expansion. To formulate the mean-field expansion we have used an iteration scheme [2] of solution of the Schwinger-Dyson equations with the fermion multi-local source.

The NJL model has been applied by many authors to study properties of hadrons. Baryons can be constructed either by directly building a bound state of three quarks by solving Fadeev equations or by considering chiral solitons, and possible via solving other equations (see [2]).

The unique connected function in leading order (LO) is quark propagator S. First step of iterations starts via $G_1 = P_1G_0$, $P_1 = \frac{1}{2}S_2\eta^2 + S^{(1)}\eta + \bar{\xi}G_3^{(0)}\xi_2$, where η and ξ are bilocal and triply-local sources, accordingly. S_2 is LO two-particle, and $G_3^{(0)}$ is LO three-particle functions, $S^{(1)}$-next-to-leading order(NLO) single-particle function, accordingly. The solution of corresponding equation

$$(i\hat{\partial} - m_0)G_3^{(0)}\xi + igG_3^{(0)}\xi \cdot trS = 3iS \cdot S \star \xi$$

is: $G_3^{(0)} = -3iS \cdot S \cdot S$.

Second step of iterations is: $G_2 = P_2G_0$, $P_2 = \frac{1}{4!}S_4\eta^4 + \frac{1}{3!}S_3\eta^3 + \frac{1}{2}S_2^{(1)}\eta^2 + S^{(2)}\eta + \frac{1}{2}\bar{\xi}H_5\eta^2\xi + \bar{\xi}H_4\eta\xi + \frac{1}{2}\bar{\xi}^2G_6\xi^2 + \bar{\xi}G_3^{(1)}\xi$, where G_6, H_5, H_4, S_4, S_3 are LO, six-particle, five-particle, four-particle, three-particle, $S_3^{(1)}, G_3^{(1)}$ and $S_2^{(1)}$ - NLO, three and two-particle, $S^{(2)}$ next-to-next-to-leading order single-particle Green's functions, accordingly.

From Schwinger-Dayson equation [2] we have

$$-S^{-1}[\frac{1}{2}H_5\eta^2\xi + H_4\eta\xi + \bar{\xi}G_6\xi^2 + G_3^{(1)}\xi] + ig[trH_5\eta + i\gamma_5\tau^a tri\gamma_5\tau^a H_5\eta]\xi + ig[trH_4 + i\gamma_5\tau^a tri\gamma_5\tau^a H_4]\xi =$$

$$= \eta \star G_3^{(1)}\xi + 3i\{S_2 + [S_2\eta + S^{(1)}] \cdot S + S \cdot [S_2\eta + S^{(1)}] + S \cdot S[\frac{1}{2}S_2\eta^2 + S^{(1)}\eta + \bar{\xi}G_3^{(1)}\xi]\} \cdot \xi,$$

Consequently the three-particle Bethe-Salpeter equation is:

$$G_3 = G_3^{(0)} + G_3^{(1)} = -3i[S \cdot S \cdot S + S \cdot S_2 + S \cdot S \cdot S^{(1)} + S \cdot S^{(1)} \cdot S] + ig[S \cdot trH_4 + S \cdot i\gamma_5\tau^a \cdot tri\gamma_5\tau^a H_4].$$

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DIRECT AND ITERATIVE METHODS OF SOLVING INVERSE HYPERBOLIC PROBLEMS[1]

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Methods for solving inverse and ill-posed problems can be divided into two groups: direct and iterative methods [1].

First we describe direct methods, i.e., method are linearization, finite-difference scheme inversion, Gel'fand-Levitan-Krein method, boundary control method, singular value decomposition and time-domain migration. Direct methods allow one to determine unknown coefficients in a fixed point of medium when additional information is given by the trace of the direct problem solution on a (usually time-like) surface of the domain. Direct methods in multidimensional inverse problems seem to be very promising trend of investigation because in iteration algorithms (method of steepest descent, Landweber iteration, Newton-Kantorovich method and so on) we have to solve the corresponding direct (forward) and adjoint (or linear inverse) problems on every step of the iterative process while in multidimensional case solving of a direct problem is hard enough.

In the second part we describe iterative methods. The usual way of formulation of inverse coefficient problem is the operator form [2]

$$A(q) = f,$$

where q is the vector-function of desired coefficients, f is the inverse problem data.

The Newton-Kantorovich method

$$q_{n+1} = q_n - \left[A'(q_n)\right]^{-1} (A(q_n) - f)$$

is very sensitive to the initial guess q_0, and include the inversion of the compact operator $A'(q_n)$.

The Landweber iteration

$$q_{n+1} = q_n - \alpha \left[A'(q_n)\right]^{*} (A(q_n) - f)$$

include the solution of the direct and adjoint problem.

The comparative analysis of direct and iterative methods is presented and discussed as well as the results of numerical experiments.

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ON A SOLVABILITY OF THE SEMIPERIODICAL BOUNDARY VALUE PROBLEM FOR LINEAR HYPERBOLIC EQUOTION

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We consider on $\bar{\Omega} = [0, \omega] \times [0, T]$ a semiperiodical boundary value problem for linear hyperbolic equation with two independet variables

$$\frac{\partial^2 u}{\partial t \partial x} = A(x,t)\frac{\partial u}{\partial x} + B(x,t)\frac{\partial u}{\partial t} + C(x,t)u + f(x,t), \tag{1}$$

$$u(0,t) = \psi(t), \quad t \in [0,T], \tag{2}$$

$$u(x,0) = u(x,T), \quad x \in [0,\omega], \tag{3}$$

where $A(x,t)$, $B(x,t)$, $C(x,t)$, $f(x,t)$ are continuous functions on $\bar{\Omega}$, $\psi(t)$ is continuously- differentiable function on $[0,T]$ and satisfies the condition $\psi(0) = \psi(T)$.

Let's $C(\overline{\Omega})$ be a space of continuous function $u : \overline{\Omega} \to R$ on $\overline{\Omega}$ with norm $\|u\|_C = \max_{\overline{\Omega}} |u(x,t)|$.

We study the question of solvability of the problem (1)-(3). Necessary and sufficient conditions of correct solvability of semiperiodical boundary value problem are received for linear hyperbolic equation with two independent variables in the term coefficient $A(x,t)$ and T.

Definition. *Boundary value problem (1)-(3) is called well posed, if for any $f \in C(\overline{\Omega})$ and cotinuously- differentiable function $\psi(t)$ on $[0,T]$, it has unique solution $u(x,t)$ and the following estimate holds*

$$\max\left\{\|u\|_C, \|u_x\|_C, \|u_t\|_C\right\} \le K \max\left\{\max_{t\in[0,T]} |\psi|, \|f\|_C\right\},$$

where K is constant, not depending on $f(x,t), \psi(t)$.

Thorem. *The boundary value problem (1)-(3) is well posed if and only if, for some $\delta > 0$ the following inequality holds $\left|\int_0^T A(x,\tau)d\tau\right| \ge \delta$ for any $x \in [0,\omega]$.*

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ON AN UNIQUE SOLVABILITY OF NON-LOCAL BOUNDARY VALUE PROBLEM FOR THE SYSTEM OF LOADED HYPERBOLIC EQUATIONS

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The boundary value problem for the following system of the loaded hyperbolic equations

$$\frac{\partial^2 u}{\partial t \partial x} = A_0(x,t)\frac{\partial u}{\partial x} + B_0(x,t)\frac{\partial u}{\partial t} + C_0(x,t)u + \sum_{i=1}^{m+1} A_i(x,t)\frac{\partial u(x,\theta_{i-1})}{\partial x} + \sum_{i=1}^{m+1} B_i(x,t)\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=\theta_{i-1}} +$$

$$+\sum_{i=1}^{m+1} C_i(x,t)u(x,\theta_{i-1}) + f(x,t), \quad (x,t) \in \Omega = (0,\omega)\times(0,T), \quad u \in \mathbb{R}^n, \tag{1}$$

$$u(0,t) = \varphi(t), \quad t \in [0,T], \tag{2}$$

$$P_2(x)\frac{\partial u(x,0)}{\partial x} + P_1(x)\frac{\partial u(x,0)}{\partial t} + P_0(x)u(x,0) +$$

$$+S_2(x)\frac{\partial u(x,T)}{\partial x} + S_1(x)\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=T} + S_0(x)u(x,T) = \psi(x), \quad x \in [0,\omega], \tag{3}$$

is considered on $\bar{\Omega} = [0,\omega]\times[0,T]$, where $A_j(x,t)$, $B_j(x,t)$, $C_j(x,t)$, $j = \overline{0,m+1}$, $P_k(x)$, $S_k(x)$, $k = \overline{0,2}$ are $(n\times n)$ matrices, $f(x,t)$, $\psi(x)$ are n - vector functions that are continuous on $\bar{\Omega}$, $[0,\omega]$ respectively and $\varphi(t)$ is n - vector function continuously differentiable on $[0,T]$.

Following [1] we find conditions that guarantee unique solvability of the problem (1)-(3).

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ON A ABOUT NEW ASYMPTOTIC REPRESENTATION OF THE SOLUTION IT IS SINGULAR THE PERTURBED CAUCHY PROBLEM FOR THE ORDINARY DIFFERENTIAL SECOND-KIND EQUATION

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To study of problems for the is singular perturbed differential equations-equations with small parameter at the higher derivatives-are devoted numerous works. A.N.Tihonova, V.I.Vishika, L.A.Ljusternika, S.A.Lomova, A.B.Vasilevoj's works are basic. The review and classification of these and subsequent works are reduced in [1]. In the majority of existing works asymptotic expansions of a solution of the equations with small parametre in the form of the sum of regular and frontier layer parts [2-4] are considered.

In the given work, using a spectral resolution of a solution of a Cauchy problem for an operator with deviating argument, new effective recurrence formulas of representation of a solution of a Cauchy problem for an ordinary differential second-kind equation with small parametre are received at the higher derivative:

$$L_\varepsilon y = \varepsilon y''(x) + a(x)y'(x) + b(x)y(x) = f(x), x \in (0,1), \tag{1}$$

$$y(0) = 0, y'(0) = 0. \tag{2}$$

Here, and - set real and smooth enough on functions, and small parameter. The basic outcome of work is

Theorem. *Let,*$a(x) \in C^{n+1}[0,1], b(x) \in C^n[0,1]$, *satisfies to conditions*

$$a(x) > \alpha > 0, b'(x) < 0, b(1) \geq . \tag{3}$$

Then for any the Cauchy problem (1) - (2) *has a unique solution. This decision will be in a kind*

$$y(x,\varepsilon,f) = \sum_{k=0}^{n-1}(-1)^k[B^{-1}D^k f(x)]\varepsilon^k - \psi(x)\sum_{k=0}^{n-1}[(B^{-1}D^k f)'(0)](-1)^k\varepsilon^k + (-1)^n\varepsilon^n y(x,\varepsilon,D^n f). \tag{4}$$

With an error term, satisfying to an inequality:

$$\|y(x,\varepsilon,D^n f)\|_{W_2'(0,1)} \leq \frac{c}{\alpha}\|D^n f\|_{L_2(0,1)}. \tag{5}$$

Here $D^0 = I, Df(x) = \frac{d^2}{dx^2}B^{-1}f, B^{-1}f = \int\limits_0^x \frac{f(t)}{a(t)}\exp\left\{\int\limits_t^x \frac{b(\xi)}{a(\xi)}d(\xi)\right\}dt$, and $\psi(x)$ *problem solution:*

$$\varepsilon\psi''(x) + a(x)\psi'(x) + b(x)\psi(x) = 0, \tag{6}$$

$$\psi(0) = 0, \psi'(0) = 0. \tag{7}$$

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SOLUTIONS BEHAVIOUR OF THE SINGULAR PERTURBED SYSTEM OF THE ORDINARY DIFFERENTIAL EQUATIONS IN PARTICULARLY CRITICAL CASES

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The problem under consideration is as follows:

$$\varepsilon x'(t,\varepsilon) = D(t)x(t,\varepsilon) + \varepsilon[f(t) + B(t)x(t,\varepsilon)] + g(t, x(t,\varepsilon)), \quad (1)$$

$$x(-t_0,\varepsilon) = x^0(\varepsilon), \|x^0(\varepsilon)\| = O(\varepsilon), \quad (2)$$

where $D(t)$- simple classical form of certain initially given matrix $A(t)$, that has its own values $\lambda_k(t)$, $k = 1, 2$, $f(t) - colon(f_1(t), f_1(t))$; $B(t) = (b_{kj}(t))_1^2$; $\varepsilon > 0$ small parameter; $[-T_0, T_0]$- segment of actual axis, $t_0 < T_0$; $g(t, x(t,\varepsilon)) = colon(g_1(t,x), g_2(t,x)), g(t,0) \equiv 0$, $[t_0, T_0 \subset S_r]$ - open radius circle $r \geq \frac{T_0 - t_0}{2} + d \quad (d \in R, d > 0)$ with a centre at the point of $((T_0 + t_0)/2, 0)$; $t \in S_r, \Delta(t,x) = (t, x_1, x_2) : t \in S_r, |x_j| < \delta(j = 1,2), 0 < \delta - const,$
$\Phi(S_r)$-space of analytical functions in S_r,
$\Phi_0(S_r)$-space of analytical functions in S_r.

For the solution $x(t,\varepsilon) = colon(x_1(t,\varepsilon), x_2(t,\varepsilon))$ we will look in the class $x_k(t,\varepsilon) \in \Phi(S_r)$ $(k = 1,2)$ at t.

We will demand the solution of the following conditions:

I. Let $\lambda_k(t) \in \Phi(S_r); f_k(t) \in \Phi(S_r); b_{kj}(t) \in \Phi(S_r); g_k(t,x)\Phi(\Delta(t,x))(k,j = 1,2)$; in the sphere of $\Delta(t,x)$ there is inequality of $\|g(t,x) - g(t,\widetilde{x})\| \leq M\|x - \widetilde{x}\| max\{\|x\|, \|\widetilde{x}\|\}^\beta$, where $0 < M, \beta$.

II. $\lambda_1(t) = \alpha(t) + i\beta(t)$; $\lambda_2(t) = \alpha(t) - i\beta(t)$; where $\alpha(t), \beta(t)$-actual functions, at that $a(t) < 0$ at $t_0 \leq t \leq \alpha_0; \alpha(t) > 0 at \alpha_0 < t < T_0; \alpha(a_0) = 0, but \beta(a_0) \neq 0$

We will put a question on closeness of the solution of perturbed and unperturbed problems in the case of exchange of stabilities at the segment $[-T_0, T_0]$ at sufficiently small values ε.

III. We will think, that if $(t_1) + t_2$ the inner point of the range H then the harmonic function is $Jm\lambda_1(t_1, t_1, t_2) > 0, where H \subset H_0$.

The following is correct.

Theorem. *Let conditions I, II,III be fulfilled. In that case for the problem (1), (2) at* $t_0 \leq t < \widetilde{T_0} - \widetilde{\delta}(\varepsilon)$ *there is the only solution and the value* $\|x(t,\varepsilon)\| \leq cw(t,\varepsilon)$, *where* $0 < c - const$

$$w(t,\varepsilon) = \begin{cases} \varepsilon \quad by \quad t_0 \leq t \leq \widetilde{T_0} - a \quad (0 < a - const), \\ \varepsilon^{1-\lambda} \quad by \quad \widetilde{T_0} - a \leq t \leq \widetilde{T_0} - \widetilde{\delta}(\varepsilon) \quad (0 < \lambda < 1), \end{cases}$$

$\widetilde{\delta}(\varepsilon) \geq 0$ *continuous function at* $0 < \varepsilon \leq \varepsilon_0 \quad (\varepsilon_0 - const.)$, *at that* $\lim_{\varepsilon \to 0} \widetilde{\delta}(\varepsilon) = 0$.

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A SCHEME OF FRIEDRICHS FOR THE REGULARIZED HYPERBOLIC EQUATION

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V.M. Shalov [1] in terms of the Hilbert space shows the possibility of extending the Friedrich's scheme for a wide class of non-selfadjoint equations. In the domain $Q = \Omega \times (0, T)$, $\Omega \subset R^n$ – with boundary Γ, $\Sigma = \Gamma \times (0, T)$ consider the problem bounded

$$\begin{cases} u_{tt} - Au = f \quad on \ \ Q, \\ u(x, 0) = u_0, \\ u_t(x, 0) = u_1 \quad on \ \ \Omega, \end{cases} \tag{1}$$

where $A = -\dfrac{\partial^2}{\partial x^2}$, $D(-A) = H^2(\Omega) \cap H_0^1(\Omega)$, $u_0 \in H_0^1(\Omega)$, $u_1 \in L^2(\Omega)$, $f \in H^{-1}(\Omega)$.

Applying "parabolic regularization" the original problem can be written as:

$$L(t)\overrightarrow{u} \equiv \Lambda\overrightarrow{u} + M_\varepsilon\overrightarrow{u} = \{0, g\}, \ \ \overrightarrow{u}(0) = \{u_0, u_1\}, \tag{2}$$

where

$$\overrightarrow{u} = \left\{u, \frac{\partial u}{\partial t}\right\}, \quad g = \exp(-kt)f, \quad \Lambda = \frac{d}{dt} + \mu I,$$

$$M_\varepsilon = \begin{pmatrix} \lambda I & -I \\ A(t) & \varepsilon(A(t) + \lambda I) + \lambda I \end{pmatrix},$$

$k, \ \lambda, \ \mu$ – some numbers [2]. This formulation of the problem enables the use of variational principle. Where

Assumption 1. *a) for almost all t there exists an inverse operator $M_\varepsilon^{-1} : H \to H$ is selfadjoint and nonnegative, and; b) there exists a variety $\Phi \subset Y$ and a positive constant $\beta_1 : M_\varepsilon(t)\vec{u} \in H$, $(M_\varepsilon\vec{u}, \vec{u}) \geq \beta_1\|\vec{u}\|_Y^2$ for almost all $u \in \Phi$ and almost all $t \in [0, T]$.*

Assumption 2. *a) for almost all t there exists a family of operators $M_\varepsilon^{-1/2} : H \to H$ uniformly bounded, and; b) there are many Ψ maps $\vec{u} : [0, T] \to \Phi$ is a subspace $C^1([0, T]; H^*)$: $M_\varepsilon(t)\vec{u}, M_\varepsilon^{-1}\vec{u'}, K(t)\vec{u} \in L_2(0, T; H^*)$ at $\vec{u} \in \Psi$ where $K(t) = M_\varepsilon^{-1/2}(t)L(t)$. Notation and description of the spaces Y, H, H^* see [2].* The following theorem is proved

Theorem. *By virtue of assumptions 1 and 2, there is expansion of C_e is the Friedrichs operator $C\overrightarrow{u} \equiv \{L\overrightarrow{u}, \overrightarrow{u}(x, 0)\}$ problem (2) for which the equation $C_e\overrightarrow{u} = q$, $q \equiv \{f, \overrightarrow{u}(x, 0)\}$ is Euler's equation for the functional $\mathcal{J}(u) = \|\overrightarrow{u}\|^2 - 2(((q, B\overrightarrow{u})))$, where B – symmetrical operator* [2].

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ON THE INVERSE PROBLEM FOR SINGULAR STURM-LIOUVILLE OPERATOR

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In this work, we give the solution of the inverse problem on two spectras for Sturm-Liouville operator with Coulomb Potential. In particularly, we obtain theorem concerning the structure of the difference potentials. Before formulating the main result of this work, we must mention that the analogous inverse problems were examined in the work [1].

We consider the Sturm-Liouville equation with Coulomb Potential [2]

$$Ly \equiv -y'' + \left[\frac{A}{x} + q(x)\right] y = \lambda y,\ \lambda = s^2, 0 < x \leq \pi, \tag{1}$$

with boundary conditions

$$y(0) = 0,$$
$$y'(\pi) - Hy(\pi) = 0, \tag{2}$$

where the function $q(x)$ is real-valued and A and H real constants and $\frac{y(x)}{x} \in C[0,\pi]$. Let $\{\lambda_n, \alpha_n\}_{n=0}^{\infty}$ be spectral characteristics of L with conditions (2).

Consider a new operator

$$\widetilde{L}y \equiv -y'' + \left[\frac{A}{x} + \widetilde{q}(x)\right] y = \mu y,\ \mu = s_1^2, 0 < x \leq \pi. \tag{3}$$

Let $\{\mu_n, \widetilde{\alpha}_n\}_{n=0}^{\infty}$ be spectral characteristics of $\widetilde{L}$ with conditions (2). Under the assumption, it follows that

$$\max_{0<x\leq\pi} |\widetilde{q}(x) - q(x)| \leq C.B$$

where $B = \sum_{n=0}^{\infty} \{|\widetilde{\alpha}_n - \alpha_n| + |\mu_n - \lambda_n|\}$ and C is constant.

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OSCILLATIONAL SOLUTIONS OF DIFFERENTIAL EQUATIONS DETERMINATING ON GIVEN VECTOR FIELD

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Let (θ, ω)-periodic on (τ, t) vector field be given

$$\frac{dt}{d\tau} = a(\tau, t), \tag{1}$$

where $a(\tau, t) = (a_1(\tau, t), ..., a_m(\tau, t))$ – vector-function of variables $\tau \in (-\infty, +\infty) = R$, $t = (t_1, ..., t_m) \in R \times ... \times R = R^m$, satisfing the condition of periodicity and of smoothness

$$a(\tau + \theta, t + k\omega) = a(\tau, t) \in C^{(0,1)}_{\tau,t}(R \times R^m) \tag{2}$$

with integer number vector $k = (k_1, ..., k_m) \in Z \times ... \times Z = Z^m$, Z – set of integer number, $\omega_0 = \theta, \omega_1, ..., \omega_m$ – periods, $k\omega = (k_1\omega_1, ..., k_m\omega_m)$.

At condition (2) system (1) has characteristic $t = \varphi(\tau, \tau_0, t_0)$ with any initial condition $(\tau_0, t_0) \in R \times R^m$, have possessing properties

$$\varphi(s, \tau_0, \varphi(\tau_0, \tau, t)) = \varphi(s, \tau, t)) \in C^{(1,1,1)}_{s,\tau,t}(R \times R \times R^m),$$

$$\varphi(s + \theta, \tau + \theta, t + k\omega) = \varphi(s, \tau, t) + k\omega, k \in Z^m.$$

We it could be that oscillational way discribes (θ, ω, ω)-periodic on (τ, t, φ) solutions of the system differential equations of type

$$D_a x = f(\tau, t, \varphi, x), \tag{3}$$

determinating on vector field (1), where D_a – differential operator on direct field. Consequently,

$$D_a = \frac{\partial}{\partial \tau} + \sum_{j=1}^{m} a_j(\tau, t)\frac{\partial}{\partial t_j}.$$

The vector-function $f = (f_1, ..., f_n)$ of variables $\tau \in R$, $t \in R^m$, $\varphi(\tau_0, \tau, t) = \varphi \in R^m$, $x = (x_1, ..., x_n)$ satisfy property

$$f(\tau + \theta, t + k\omega, \varphi + k\omega, x) = f(\tau, t, \varphi, x) \in C^{(0,1,1,1)}_{\tau,t,\varphi,x}(R \times R^m \times R^m \times R^n), \quad k \in Z^m. \tag{4}$$

We shall study problem about existence of solutions $x(\tau, t, \varphi)$ of the system (3), satisfing property

$$x(\tau + \theta, t + k\omega, \varphi + k\omega) = x(\tau, t, \varphi), \quad k \in Z^m \tag{3_*}$$

under conditions (2) and (4),

$$x(\tau_0, , t, t) = u(t) = u(t + k\omega) \in C^(1)_t(R^m), \quad k \in Z. \tag{3_0}$$

In the report there are some properties of solutions of the problem $(3) - (3_0)$, nasessary for solution main problem $(3) - (3_*)$. The sufficient condition of existence solutions of the problem $(3) - (3_*)$ is obtained and in linear case is being the integral type.

This problem to study when linear system is given constant matrix on characteristic of vector field (1).

In report and uniqueness of system (1) .

SIMULATION OF THE DYNAMIC STABILITY OF BORING COLUMNS BY THE AT NONLINEAR COMPLICATING FACTORS

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The problem of stability of movement the boring columns taking into account nonlinear complicating factors is considered. Final deformations of boring columns concern as such factors. They take place in case of action on a column big variable axial forces $N(x,t)$ and turning moments $M(x,t)$ and have nonlinear oscillatory character:

$$\begin{aligned} &EJ_V \tfrac{\partial^2}{\partial x^2}\left[\tfrac{\partial^2 V}{\partial x^2}\left(1-\tfrac{3}{2}\left(\tfrac{\partial V}{\partial x}\right)^2\right)\right]+\tfrac{\partial^2}{\partial x^2}\left[M(x,t)\tfrac{\partial U}{\partial x}\right]+\tfrac{\partial}{\partial x}\left[N(x,t)\tfrac{\partial V}{\partial x}\right]+K_1 V = -\tfrac{\gamma F}{g}\tfrac{\partial^2 V}{\partial t^2},\\ &EJ_U \tfrac{\partial^2}{\partial x^2}\left[\tfrac{\partial^2 U}{\partial x^2}\left(1-\tfrac{3}{2}\left(\tfrac{\partial U}{\partial x}\right)^2\right)\right]+\tfrac{\partial^2}{\partial x^2}\left[M(x,t)\tfrac{\partial V}{\partial x}\right]+\tfrac{\partial}{\partial x}\left[N(x,t)\tfrac{\partial U}{\partial x}\right]+K_1 U = -\tfrac{\gamma F}{g}\tfrac{\partial^2 U}{\partial t^2}, \end{aligned} \tag{1}$$

where U, V are motions an elastic line of a column in planes XOY and XOZ, accordingly. The boundary conditions in case of hinge leaning columns are set as equality to zero of motions and the bending moment on the ends:

$$V = EJ_V\frac{\partial^2 V}{\partial x^2}=0, \quad U = EJ_U\frac{\partial^2 U}{\partial x^2}=0, \qquad (x=0,\ x=l). \tag{2}$$

Decisions $V(x,t)=\sum\limits_{k=1}^{\infty} f_k(t)\sin\frac{k\pi x}{l}$, $U(x,t)=\sum\limits_{k=1}^{\infty} g_k(t)\sin\frac{k\pi x}{l}$ satisfy to these conditions. The stability of boring column, rotating with a speed ω under the influence of variable longitudinal force $t = N_0 + N_t(t)$, when the equation of its bent axis (1) is led to the nonlinear parametrical equation of a kind:

$$\ddot f + C_k^2(1-2\nu\cos\Omega t)f+\alpha f^3=0 \tag{3}$$

is investigated. The case of the basic resonance of a boring column is considered. The equation of its excited state is led to the equation Mate-Hills type:

$$\frac{d^2\delta f}{dt^2}+\delta f\left[C_k^2+1,5\alpha r_1^2-2C_k^2\nu\cos\Omega t+1,5\alpha r_1^2\cos 2\phi_1\cos 2\Omega t+1,5\alpha r_1^2\sin 2\phi_1\sin 2\Omega t\right]=0 \tag{4}$$

Character of behaviour of the solution of (4) allows to judge about stability or instability of the basic resonance. It is identical to criterion of stability by Lyapunov. Borders of zones of instability of resonant oscillations of a boring column depending on forms of oscillations and system parameters are defined. Another way the analysis of behavior the decision of the equation (4) is the method of its partial discretization. It allows to receive the analytical decision of the equation Hills type, character is ting behavi our of small indignation δf in time t. For this purpose, the second component of the equations (4) is is discrete in a class of the generalised functions:

$$\begin{aligned} &\tfrac{d^2\delta f}{dt^2}+\tfrac{1}{2}\sum_{i=1}^{n}(t_i+t_{i-1})[(C_k^2+1,5\alpha r_1^2-2C_k^2\nu\cos\Omega t_i+1,5\alpha r_1^2\cos 2\phi_1\cos 2\Omega t_i+\\ &+1,5\alpha r_1^2\sin 2\phi_1\sin 2\Omega t_i)\times\delta f(t_i)\delta(t-t_i)-(C_k^2+1,5\alpha r_1^2-2C_k^2\nu\cos\Omega t_{i-1}+\\ &+1,5\alpha r_1^2\cos 2\phi_1\cos 2\Omega t_{i-1}+1,5\alpha r_1^2\sin 2\phi_1\sin 2\Omega t_{i-1})\times\delta f(t_{i-1})\delta(t-t_{i-1})]=0 \end{aligned} \tag{5}$$

where $i=\overline{1,n}$ - the number of arguments t discrete; $\delta(t-t_k)$- delta-function of Dirac. The decision of the equation (5) does not represent work. The results of researches received by two methods will be coordinated among them selves.

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ON THE CONVERGENCE OF THE METHOD OF LINES AS APPLIED TO THE SOLUTION OF THE PROBLEM FOR THE LINEAR LOADED DIFFERENTIAL EQUATION OF PARABOLIC TYPE INVOLVING NON-LOCAL BOUNDARY CONDITIONS

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Assume that it is necessary to find a continuous function $u = u(x,t)$ in the closed set $\bar{D} = \{0 \le x \le l\,,\ 0 \le t \le T\}$ -i.e. the solution to the following boundary problem:

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}(k(x,t)\frac{\partial u}{\partial x}) + b(x,t)u(x,t) + \sum_{k=1}^{m} b_k(x,t)u(x,t_k) + f(x,t)\,,\ 0 < x < l\,,\ 0 < t \le T\,, \tag{1}$$

$$\frac{\partial u(0,t)}{\partial t} + \alpha_1 u(0,t) + \beta_1 u(l,t) = \mu_1(t)\,,\ \ \frac{\partial u(l,t)}{\partial t} + \alpha_2 u(0,t) + \beta_2 u(l,t) = \mu_2(t)\,,\ 0 \le t \le T \tag{2}$$

$$u(x,0) = \varphi(x)\,,\ 0 \le x \le l. \tag{3}$$

Here $k(x,t) > 0\,,\ b(x,t)\,,\ b_k(x,t)\,,\ k = 1\,,2\,,\ldots,m\,,\ f(x,t),\ \mu_1(t),\ \mu_2(t),\ \varphi(x)$ are known continuous functions of their arguments; $\alpha_1,\ \beta_1,\ \alpha_2,\ \beta_2$ the real numbers; $t_1\,,t_2\,,\ldots,t_m \in (0,T]$ the fixed points.

Partitioning the segment into N equal parts by points $x_n = nh\,,\quad n = 0\,,1\,,\ldots,N\,,\ Nh = l$, and considering equation (1) on the lines $x = x_n,\ n = 1,2,...,N-1$, we juxtapose problem (1)-(3) with the following problem

$$\frac{dy_n(t)}{dt} = \frac{1}{h}\left[\frac{k(x_{n+1},t)+k(x_n,t)}{2}\frac{y_{n+1}(t)-y_n(t)}{h} - \frac{k(x_n,t)+k(x_{n-1},t)}{2}\frac{y_n(t)-y_{n-1}(t)}{h}\right] +$$

$$+ b(x_n,t)y_n(t) + \sum_{k=1}^{m} b_k(x_n,t)y_n(t_k) + f(x_n,t),\ \ n = 1,2,...,N-1,\ 0 < t \le T, \tag{4}$$

$$\frac{dy_0(t)}{dt} + \alpha_1 y_0(t) + \beta_1 y_N(t) = \mu_1(t),\quad \frac{dy_N(t)}{dt} + \alpha_2 y_0(t) + \beta_2 y_N(t) = \mu_2(t),\ 0 \le t \le T, \tag{5}$$

$$y_n(0) = \varphi(x_n),\quad n = 0,1,2,...,N. \tag{6}$$

Here $y_n(t)$ is approximate value of the solution $u = u(x,t)$ of problem (1)-(3) on the line$x = x_n$.

We derive the conditions under which there takes place principle of maximum for problem (4)-(6).

Theorem. *Let coefficient $k(x,t)$ of equation (1) satisfy the condition $k'_x(x,t) \ge k_0 > 0$, and coefficients $b(x,t)$,$b_k(x,t),\ k = 1,2,...,m,\ \alpha_1\,,\alpha_2\,,\beta_1\,,\beta_2$ - the conditions*

$$b_k(x,t) \ge 0,\ k = 1,2,...,m,\quad b(x,t) + \sum_{k=1}^{m} b_k(x,t) \le 0,\quad \beta_1 \le 0,\ \alpha_1+\beta_1 > 0,\ \alpha_2 \le 0,\ \alpha_2+\beta_2 > \varepsilon > 0.$$

If the conditions $\sigma \ge -\xi\alpha_2 l/\varepsilon,\ \xi \ge 1/k_0$ hold true, then the solution to problem (4)-(6) is reduced to the solution to problem (1)-(3). At that there takes place the estimation

$$|y_n(t) - u(x_n,t)| \le Lh^2(\sigma + \xi\, l),\ \ n = 0,1,...,N.$$

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SOLUTION OF THE CAUCHY PROBLEM FOR THE VOLTER CHAIN WITH UNBOUNDED INITIAL CONDITION

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Consider the Cauchy problem for semi-infinite Volter chain

$$\dot{c} = c_n(c_{n=1} - c_{n-1}), n = 0, 1, \ldots, c_{-1},\ \ c_n = c_n\left(t\right) > 0, \tag{1}$$

$$c_n\left(0\right) = \hat{c}_n,\ \ n = 0, 1, \ldots,\ \ \cdot = \frac{d}{dt}. \tag{2}$$

We'll seek the solution in the class of sequences $c_n = c_n\left(t\right)$ such that difference expression

$$\left(ly\right)_n = \sqrt{c_{n-1}}y_{n-1} + \sqrt{c_n}y_{n+1},\ \ n = 0, 1, \ldots$$

together with boundary condition $y_{-1} = 0$ generates self-adjoint operator $L = L\left(t\right)$ in $l_2\left(0, \infty\right)$.

Let $d\rho\left(t, \lambda\right)$ be spectral measure of the operator L as known [1], the following formulae is true

$$d\rho\left(\lambda, f\right) = \left(\int_{-\infty}^{\infty} e^{\lambda^2 t} d\rho\left(\lambda, 0\right)\right)^{-1} e^{\lambda^2 t} d\rho\left(\lambda, 0\right). \tag{3}$$

Suppose $S_n = S_n\left(t\right) = \int_{-\infty}^{\infty} \lambda^n d\rho\left(\lambda, t\right)$. Then c_n can be found on by the formulae [1]

$$c_n = D_{n-1}D_{n+1}D_n^{-2}, \tag{4}$$

where $D_{n-1} \equiv 1,\ \ D_n = \left|S_{j+k}\right|_{j,k=0}^{n}$ is Gankel's determinant.

It proves to be that Gankel's determinants satisfy the differential equations

$$\dot{D}_n = \left(n-1\right)D_1 D_n + D_{n-1}D_{n+1}D_n' + 2D_n \sum_{k=1}^{n-1} D_{k-1}D_{k+1}D_k^{-2},$$

It shows that constructed by the formulas (3), (4) function $c_n = c_n\left(t\right)$ is vally the solution of the problem (1), (2).

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ON THE HIGHT ORDER NONLINEAR SCHRODINGER EQUATION WITH A SELF-CONSISTENT SOURCE

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We consider the problem of integration of the following system of equations

$$iu_t - 4\sum_{k=0}^{m}\left(\frac{1}{2i}\right)^{k+1} D_x^k(u\Omega_{m-k}) = 2i\sum_{n=1}^{N}\sum_{j=0}^{m_n-1} C_{m_n-1}^j\left(\varphi_{1,n}^j\psi_{2,n}^{m_n-j-1} - (\varphi_{2,n}^j)^*(\psi_{1,n}^{m_n-j-1})^*\right), \quad (1)$$

$$\frac{\partial}{\partial x}\varphi_{1,n}^j + i\xi_n\varphi_{1,n}^j + ij\varphi_{1,n}^{j-1} - u(x,t)\varphi_{2,n}^j = \frac{\partial}{\partial x}\varphi_{2,n}^j - i\xi_n\varphi_{2,n}^j - ij\varphi_{2,n}^{j-1} + u^*(x,t)\varphi_{1,n}^j = 0, \quad (2)$$

$$\frac{\partial}{\partial x}\psi_{1,n}^j - i\xi_n\psi_{1,n}^j - ij\psi_{1,n}^{j-1} + u^*(x,t)\psi_{2,n}^j = \frac{\partial}{\partial x}\psi_{2,n}^j + i\xi_n\psi_{2,n}^j + ij\psi_{2,n}^{j-1} - u(x,t)\psi_{1,n}^j = 0, \quad (3)$$

$$n = 1,\ 2,\ \dots,\ N,\ \ j = 0,\ 1,\ \dots,\ m_n - 1,$$

with the initial conditions

$$u(x,0) = u_0(x), \quad x \in R, \quad (4)$$

where vector-functions $\Phi_n^0 = (\varphi_{1,n}^0, \varphi_{2,n}^0)^T$ and $\Psi_n^0 = (\psi_{1,n}^0, \psi_{2,n}^0)^T$ are eigenfunctions of the system (2) and (3) corresponding to eigenvalues ξ_n $(Im\ \xi_n > 0)$, with multiplicities m_n, $n = 1, 2, ..., N$. Here

$$\Omega_0(x,t) = -2i,$$

$$\Omega_j(x,t) = -2D_x^{-1}\sum_{k=0}^{j-1}\left\{\left(\frac{1}{2i}\right)^{k+1} u^* D_x^k(u\Omega_{j-k-1}) + \left(-\frac{1}{2i}\right)^{k+1} uD_x^k(u^*\Omega_{j-k-1})\right\},\ D_x = \frac{\partial}{\partial x},$$

$$j = 1,\ 2,\ \dots,\ m.$$

The function $u_0(x)$ $(-\infty < x < \infty)$ has the following properties:

1) $\int\limits_{-\infty}^{\infty} (1 + |x|)|u_0(x)|dx < \infty$;

2) If $t = 0$, then systems (2) and (3) has no spectral singularities and in $Im\xi > 0$ it has exactly N eigenvalues $\xi_1(0), \xi_2(0), ..., \xi_N(0)$ with multiplicities $m_1(0),\ m_2(0), ..., m_N(0)$.

We assume that

$$\frac{1}{(m_n - 1 - l)!}\int\limits_{-\infty}^{\infty} \tilde{\Psi}_n^{m_n-1}(x,t)\Phi_n^{m_n-1-l}(x,t)dx = A_{m_n-1-l}^n(t),\ n = 1, 2, ..., N, \quad (5)$$

where $A_{m_n-1-l}^n(t)$ is a continuous function, $n = 1, 2, ..., N$, $l = 0, 1, ..., m_n - 1$.

We look for complex-valued solution $u(x,t)$ of the problem (1)-(5), that is satisfy the following assumption:

$$\int\limits_{-\infty}^{\infty}\left((1 + |x|)|u(x,t)| + \sum_{k=1}^{m}\left|D_x^k u(x,t)\right|\right)dx < \infty. \quad (6)$$

In this work the method of inverse scattering problem is applied to the integration of the problem (1)-(6).

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SOLUTIONS ESTIMATIONS FOR THE LINEAR SYSTEM OF NEUTRAL TYPE DIFFERENCE DIFFERENTIAL EQUATIONS

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The important place in modeling processes in economy, biology, population dynamics is occupied with the delay differential equations. They allow to make making more adequate models which can calculate system status in previous moments of time. A system of linear difference differential equations of the neutral type with constant coefficients is considered

$$\frac{d}{dt}\left[x(t) - Dx(t-\tau)\right] = Ax(t) + Bx(t-\tau), t \geq 0. \tag{1}$$

The second Lyapunov method is used with a next type of functional

$$V_0\left[x(t), t\right] = x^T(t)\, Hx(t) + \int_{t-\tau}^{t} e^{-\beta(t-s)} \left\{ x^T(s)\, G_1 x(s) + \overset{\bullet}{x}{}^T(s)\, G_2 \overset{\bullet}{x}(s) \right\} ds. \tag{2}$$

We denote

$$S\left[\beta, H, G_1, G_2\right] = \begin{bmatrix} -A^T H - HA - G_1 - A^T G_2 A & -HB - A^T G_2 B & -HD - A^T G_2 D \\ -B^T H - B^T G_2 A & e^{-\beta\tau} G_1 - B^T G_2 B & -B^T G_2 D \\ -D^T H - D^T G_2 A & -D^T G_2 B & e^{-\beta\tau} G_2 - D^T G_2 D \end{bmatrix},$$

$$\varphi(H) = \frac{\lambda_{\max}(H)}{\lambda_{\min}(H)},\ \varphi_1(H, G_1) = \frac{\lambda_{\max}(G_1)}{\lambda_{\min}(H)},\ \varphi_2(H, G_2) = \frac{\lambda_{\max}(G_2)}{\lambda_{\min}(H)}. \tag{3}$$

We can formulate a next statement on stability of a zero solution of the system (1) and convergence estimations of the solution.

Theorem 1. *Let $|D| < 1$ and there exist positively definite matrixes H, G_1, G_2 and a parameter $\beta > 0$, in which the matrix $S[\beta, H, G_1, G_2]$ is positively definite matrix. Then the zero solution of the system (1) is exponentially stable in the C^1 metrics.*

And for any solution $x(t)$, $t > 0$ the following convergence estimation is held

$$|x(t)| \leq \left[\sqrt{\varphi(H)}\, |x(0)| + \tau\sqrt{\varphi_1(H, G_1)}\, \|x(0)\|_\tau + \tau\sqrt{\varphi_2(H, G_2)} \left\|\overset{\bullet}{x}(0)\right\|_\tau\right] e^{-\frac{1}{2}\gamma t}, \tag{4}$$

$$\begin{aligned} \left|\overset{\bullet}{x}(t)\right| \leq \Bigg[\left(\frac{|B|}{|D|} + M\left(\sqrt{\varphi(H)} + \tau\sqrt{\varphi_1(H, G_1)}\right)\right) \|x(0)\|_\tau + \\ + \left(\sqrt{\varphi(H)} + M\tau\sqrt{\varphi_2(H, G_2)}\right) \left\|\overset{\bullet}{x}(0)\right\| \Bigg] e^{-\frac{1}{2}\varsigma t}, \end{aligned} \tag{5}$$

$$M = |A| + |DA + B|\, e^{\frac{1}{2}\gamma\tau}\left[1 - |D|^{m-1} e^{\frac{m-1}{2}\gamma\tau}\right],$$

$$\gamma = \min\left\{ \frac{\lambda_{\min}(S[\beta, H, G_1, G_2])}{\lambda_{\max}(H)} \quad \beta \right\},\ \varsigma = \min\left\{ \frac{2}{\tau}\ln\frac{1}{|D|}, \quad \gamma \right\}.$$

An optimization method for finding functionals of the type (2), based on methods of nonsmooth optimization, is proposed.

INVERSE PROBLEMS FOR THE LIQUID FILTRATION IN ELASTO-PLASTIC REGIMES

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In this paper we solve an inverse coefficient problem with additional data, given on two points of the earea $[0, L]$, $L = \text{const} > 0$, for elasto-plastic filtration equations

$$\downarrow \frac{\partial p_1}{\partial t_1} = a_1 \frac{\partial^2 p_1}{\partial x^2}, \quad \uparrow \frac{\partial p_2}{\partial t_2} = a_2 \frac{\partial^2 p_2}{\partial x^2}, \tag{1}$$

$p_1,\ p_2$ - current pressure, $t_1,\ t_2$ - time, x - linear coordinate, $a_1,\ a_2 = \text{const}$ - pressure conductivity coefficients, $a_2 \geq a_1$, the signs $\downarrow$ and $\uparrow$ corresponds to pressure decreasing and ingreasing (restoring) regimes, respectively.

In the direct problem for (1) we have the following initial and boundary conditions

$$p_1(0, x) = p_0 = \text{const}, \quad w(t_1, 0) = w_0 = \text{const}, \quad p_1(t_1, L) = p_0, \tag{2}$$

$$p_2(0, x) = p_1(x), \quad \frac{\partial p_2(t_2, 0)}{\partial x} = 0, \quad p_2(t_2, L) = p_0, \tag{3}$$

where $p_1(x)$ - the pressure distribution at last moment $t_1 = T_1$ of pressure decreasing regime - $\downarrow$, w - filtration velocity. Time scale t_2 starts from $t_1 = T_1$ as $t_2 = 0$.

In inverse problem to determine $a_1,\ a_2,\ k$ we pose conditions in two points of $[0, L]$

$$\downarrow w(t_1, 0) = -\frac{k}{\mu} \left.\frac{\partial p_1}{\partial x}\right|_{x=0} = z_0(t_1), \quad \downarrow p_1(t_1, x_1) = z_1(t_1),$$

$$\uparrow p_2(t_2, 0) = f_0(t_2), \quad \uparrow p_2(t_2, x_1) = f_1(t_2),$$

where k - permeability, μ - viscosity of liquid, $z_0(t_1),\ z_1(t_1),\ f_0(t_2),\ f_1(t_2)$ - given functions, $x_1 \in [0, L]$.

To find a_1 and k we minimize the following functional

$$J_1\left(\overset{m}{a}_1, \overset{m}{k}\right) = \int_0^{T_1} \left[-\frac{\overset{m}{k}}{\mu} \frac{\partial \overset{m}{p}_1(\xi, 0)}{\partial x} - z_0(\xi)\right]^2 d\xi + \frac{\overset{m}{k}}{\mu L} \int_0^{T_1} \left[\overset{m}{p}_1(\xi, 0) - z_1(\xi)\right]^2 d\xi,$$

where overscript m denotes that all values are calculated with $a_1 = \overset{m}{a}_1,\ k = \overset{m}{k}$. For test examples functions $z_0(t_1)$, $z_1(t_1)$, $f_0(t_2)$ and $f_1(t_2)$ are determined by solving corresponding direct problems (1)-(3) with given values of a_1, k and a_2. To minimize $J_1(a_1, k)$ we used the coordinate descent method. Comparatively small iterations are used to restore a_1 and k with precision $\varepsilon = 10^{-4}$. The gradient descent method gives larger number of iterations. To accelerate iteration procedure we used so called "ravine" method.

Coefficient a_2 is determined by minimizing the functional with respect $(p_2 - f_j(t_2))^2,\ j = 0, 1$.

The following iteration procedure is obtained

$$\overset{m+1}{a_2} = \left\{\sum_{j=0}^{1} \int_0^{T_2} \left[\overset{m}{a_2} \overset{m}{w}(\xi, x_j) - \overset{m}{p_2}(\xi, x_j) + f_j(\xi)\right] \overset{m}{w}(\xi, x_j)\, d\xi\right\} \left\{\sum_{j=0}^{1} \int_0^{T_2} \overset{m}{w^2}(\xi, x_j)\, d\xi\right\}^{-1},$$

where $w(x, t)$ is determined from the corresponding auxiliary problem.

In computational experiments the value of a_2 is restored for only three iterations.

NECESSARY AND SUFFICIENT CONDITIONS OF SOLVABILITY OF THE BOUNDARY VALUE PROBLEMS FOR THE NONHOMOGENEOUS POLYHARMONIC EQUATIONS IN A SPHERE

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Let m be natural number and in the n - dimensional unit ball $\Omega = \{x : |x| < 1\} \subseteq R^n$ we consider the nonhomogeneous polyharmonic equation [1]

$$\Delta_x^m u(x) = f(x), \quad x \in \Omega, \tag{1}$$

with boundary conditions

$$\begin{aligned} &\frac{\partial^{k_1}}{\partial n_x^{k_1}} u \Big|_{x\in\partial\Omega} = \varphi_1(x), \quad x \in S = \partial\Omega, \\ &\frac{\partial^{k_2}}{\partial n_x^{k_2}} u \Big|_{x\in\partial\Omega} = \varphi_2(x), \quad x \in S, \\ &\dots \\ &\frac{\partial^{k_m}}{\partial n_x^{k_m}} u \Big|_{x\in\partial\Omega} = \varphi_m(x), \quad x \in S, \end{aligned} \tag{2}$$

where $0 \le k_1 < k_2 < ... < k_m \le 2m - 1$.

Theorem 1. *Let $f(x) \in C^{\alpha}(\overline{\Omega})$, $\varphi_s(x) \in C^{2m-k_s+\alpha}(S)$, $s = 1, 2, ..., m$. Then a necessary and sufficient condition for the solvability of problem (1)-(2) in the class $u(x) \in C^{2m+\alpha}(\bar{\Omega})$ for arbitrary m and any $0 \le k_1 < k_2 < ... < k_m \le 2m - 1$ is the condition*:

$$rang\, A(k_1, k_2, ..., k_m) = rang\, \left(A(k_1, k_2, ..., k_m), \vec{F}\right), \tag{3}$$

where $A(k_1, k_2, ..., k_m)$ denotes a matrix of dimension $m \times m$, which stores the rows of matrix A (dimension $2m \times m$) with numbers equal to $k_1, k_2, ..., k_m$:

$$A = \begin{bmatrix} 1=\frac{0!}{01} & 1=\frac{2!}{2!} & 1=\frac{4!}{4!} & 1=\frac{6!}{6!} & \cdots & (2m-4)!/(2m-4)! & (2m-2)!/(2m-2)! \\ 0 & 2!/1! & 4!/3! & 6!/5! & \cdots & (2m-4)!/(2m-5)! & (2m-2)!/(2m-3)! \\ 0 & 2!/0! & 4!/2! & 6!/4! & \cdots & (2m-4)!/(2m-6)! & (2m-2)!/(2m-4)! \\ 0 & 0 & 4!/1! & 6!/3! & \cdots & (2m-4)!/(2m-7)! & (2m-2)!/(2m-5)! \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & (2m-4)!/0! & (2m-2)!/2! \\ 0 & 0 & 0 & 0 & \cdots & 0 & (2m-2)!/1! \\ 0 & 0 & 0 & 0 & \cdots & 0 & (2m-2)!/0! \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix},$$

also a vector - columns $\vec{U}, \vec{F}$ (dimension m) *with elements $\vec{U} = (u_0(0), u_1(0), ..., u_{m-1}(0))^T$,*
$\vec{F} = (\frac{1}{\omega_n}\int\limits_S \left[\varphi_1(x) - \frac{\partial^{k_1}}{\partial n_x^{k_1}}\varepsilon_{2m,n} * f\right] dS_x,\ ...,\ \frac{1}{\omega_n}\int\limits_S \left[\varphi_m(x) - \frac{\partial^{k_m}}{\partial n_x^{k_m}}\varepsilon_{2m,n} * f\right] dS_x)^T,$
i.e. rang of matrix $A(k_1, k_2, ..., k_m)$ complies with the rang of extended matrix systems:

$$A(k_1, k_2, ..., k_m)\vec{U} = \vec{F}. \tag{4}$$

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THE OSCILLATION PROPERTIES FOR THE SOLUTIONS OF HALF-LINEAR SECOND ORDER AND HIGHER ORDER DIFFERENTIAL EQUATIONS

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On the interval $I = (a, b)$, $-\infty \le a < b \le +\infty$ we consider the following differential equation:

$$(-1)^n(\rho(t)|y^{(n)}(t)|^{p-2}y^{(n)}(t))^{(n)} - v(t)|y(t)|^{p-2}y(t) = 0, \tag{1}$$

where $1 < p < \infty$, v and ρ are continuous and n-times continuously differentiable functions on I, respectively. Moreover, $\rho(t) > 0$ for any $t \in I$.

If $n = 1$, then equation (1) becomes a second order differential equation. Here, different criteria of oscillation of the second order differential equation have been obtained by depending on the integral behavior of the function $\rho^{1-p'}$ at the end points of I.

If $n = 1$ and $p = 2$, then equation (1) becomes a second order linear differential equation. We have obtained necessary and sufficient conditions of strong oscillation and of strong nonoscillation of the higher order differential equation.

Next, some spectral properties of the operators in the following form

$$l(y) = (-1)^n \frac{1}{v(t)}(\rho(t)y^{(n)})^{(n)},$$

we have obtained by the applications of our new results.

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REPRESENTATION OF THE SOLUTION OF DISTRIBUTED SYSTEMS WITH AFTEREFFECT

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We consider a system of two linear homogeneous second order partial differential equations with constant coefficients and constant delay:

$$\begin{aligned} \frac{\partial u(x,t)}{\partial t} &= a_{11}\frac{\partial u^2(x,t-\tau)}{\partial x^2} + a_{12}\frac{\partial v^2(x,t-\tau)}{\partial x^2}, \\ \frac{\partial v(x,t)}{\partial} &= a_{21}\frac{\partial u^2(x,t-\tau)}{\partial x^2} + a_{22}\frac{\partial v^2(x,t-\tau)}{\partial x^2}. \end{aligned} \tag{1}$$

Functions $u(x,t)$, $v(x,t)$ are assumed to be defined for $t \geq -\tau$ and $0 \leq x \leq l$. Initial and boundary conditions are defined:

$$\begin{aligned} & u(0,t) = \mu_1(t), u(l,t) = \mu_2(t), \\ & v(0,t) = \theta_1(t), v(l,t) = \theta_2(t), t \geq -\tau, \\ & u(x,t) = \varphi(x,t), v(x,t) = \psi(x,t), 0 \leq x \leq l, -\tau \leq t \leq 0. \end{aligned} \tag{2}$$

And "matching conditions" are fulfilled:

$$\begin{aligned} & \mu_1(t) = \varphi(0,t), \mu_2(t) = \varphi(l,t), \\ & \theta_1(t) = \psi(0,t), \theta_2(t) = \psi(l,t), -\tau \leq t \leq 0. \end{aligned}$$

The first boundary value problem for the system (1) has been solved using a special function called the delay exponential function. A solution is presented in an analytical form of formal series for the case, when matrixes of coefficients are commutative and their eigenvalues are real and different.

PERIODIC SOLUTION OF QUASI-LINEAR SYSTEM WITH CONSTANT COEFFICIENTS ON DIAGONAL

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In the report is considered the question of existence of the periodic on arguments $\tau \in (-\infty, +\infty) = R$, $t = (t_1, ..., t_m) \in R \times ... \times R = R^m$ and $\sigma = t - e\tau$ solution $x(\tau, t, \sigma)$ of systems of the type

$$D_e x = A(\sigma)x + f(\tau, t, \sigma, x) \tag{1}$$

with operator $D_e = \frac{\partial}{\partial \tau} + \sum\limits_{j=1}^{m} \frac{\partial}{\partial t_j}$, where $e = (1, ..., 1)$ – m-vector, $A(\sigma)$ – $n \times n$-matrix, $f(\tau, t, \sigma, x)$ – n-vector-function, $x = (x_1, ..., x_n)$ – vector.

The condition of periodicity and of smoothness is obtained:

$$A(\sigma + k\omega) = A(\sigma) \in C_{\sigma}^{(1)}(R^m), \tag{2}$$

$$f(\tau + \theta, t + k\omega, \sigma + k\omega, x) = f(\tau, t, \sigma, x) \in C_{\tau,t,\sigma,x}^{(0,1,1,1)}(R \times R^m \times R^m \times R^n) \tag{3}$$

for all $k \in Z^m$, where Z – set of integer number, $k\omega = (k_1\omega_1, ..., k_m\omega_m)$.

It is possible to show that with the help of linear ω-periodic and continuously differential transformation $x = B(\sigma)y$ system (1) happens to system with jordanian matrix

$$D_e y = J(\sigma)y, \quad J(\sigma) = diag\,[J_1(\lambda_1), ..., J_m(\lambda_m)], \tag{4}$$

where $J_s(\lambda_s)$ – jordanian matrix, corresponding roots $\lambda_s(\sigma) = \alpha_s(\sigma) + i\beta_s(\sigma)$, $s = \overline{1, m}$, possessing properties periodicity, continuous differentiability, certainty of sign and separatiability.

The condition is obtained

$$Re\,\lambda_s(\sigma) = \alpha_s(\sigma) < 0, \quad s = \overline{1, d}, \quad \sigma \in R^m. \tag{5}$$

In the space S_Δ (θ, ω, ω)-periodic unceasing function $x(\tau, t, \sigma)$ with rate $\|x\| = \sup\limits_{R \times R^m \times R^m} |x(\tau, t, \sigma)| \leq \Delta = const > 0$ shall define the operator Q:

$$(Qx)(\tau, t, \sigma) = \int\limits_{(-\infty, -\infty)}^{(\tau, t)} X(\tau - s, \sigma) f(s, æ, \sigma, x(s, æ, \sigma)) d_e(s, æ). \tag{6}$$

At condition (3) and volumes

$$|X(\tau, \sigma)| \leq \Gamma e^{-\gamma\tau}, \quad \tau > 0, \quad \Gamma > 0, \quad \gamma > 0,$$

$X(\tau, \sigma)$ – matricant of linear system,

$$|f(\tau, t, \sigma, x)| \leq H + N|x|,$$

where $H = \sup |f(\tau, t, \sigma, 0)|$ under $(\tau, t, \sigma) \in R \times R^m \times R^m$, $N > 0$ – of Lipshic constant,

$$\frac{\Gamma H}{\gamma - \Gamma N} < \Delta, \quad \frac{\Gamma N}{\gamma} < 1 \tag{7}$$

easy make sure that operator Q maps S_Δ in itself and is compressional. Consequently, the unique still point $x^*(\tau, t, \sigma) = (Qx^*)(\tau, t, \sigma)$ exists.

The still point $x^*(\tau, t, \sigma)$ is continuously differentity on own arguments and the solution of system (1) is proved.

In the report sufficient condition of existence and uniqueness of (θ, ω, ω)-periodic solution of system (1) is obtained.

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ON OSCILLATION AND SPECTRUM OF THE GENERAL DIFFERENTIAL EQUATION

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Let

$$Ly = (-1)^n y^{(2n)} + \sum_{k=0}^{n-1} (-1)^k \left(\rho_k(t) y^{(k)}\right)^{(k)} - \sum_{k=0}^{n-1} (-1)^k \left(r_k(t) y^{(k)}\right)^{(k)} = 0. \tag{1}$$

Here $\{\rho_k\}$ and $\{r_k\}$ are systems of non-negative smooth functions defined on $I = [0;\infty)$ such that

$$\sup_{x>0} \sum_{k=0}^{n-1} r_k(x) < \infty, \quad \lim_{x\to+\infty} \sum_{k=0}^{n-1} \rho_k(x) > 0.$$

The aim of this paper is the study of oscillatory properties of the equation (1) and spectrum of the operator L in (1). Lower are given part of results. Let

$$S(x, h|\{\rho_k\}) = \sum_{k=0}^{n-1} h_x^{-2k-1x+h} \rho_k(t)dt, \quad (h > 0, x \geq 0).$$

Denote

$$h^*(x) = \inf \left\{ h^{-2n} : h^{2n} S(x, h|\{\rho_k\}) \leq 1 \right\}.$$

For $h = h^*(x)$ the following equation holds

$$\frac{S(x, h^*(x)^{-1/2n}|\{\rho_k\})}{h^*(x)} = 1 \quad (\forall x > 0).$$

Theorem. *There exist constants $c > 1$, δ, $0 < \delta < 1$, independent on $\{\rho_k\}$ and $\{r_k\}$ such that if*

$$\frac{1}{c} \lim_{R\to\infty} \sup_{x\geq R} \frac{S(x + 2^{-1}\delta h^*(x)^{-1/2n}, (1-\delta) h^*(x)^{-1/2n}|\{r_k\})}{h^*(x)} > 1$$

then the equality (1) is oscillatory.

We have also obtained that (1) is non-oscillatory if

$$16c \sup_{x>0} \frac{S(x, h^{\#}(x)^{-1/2n}|\{r_k\})}{h^{\#}(x)} < \eta < 1.$$

Denotes see in [1].

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ON THE NUMBER OF EIGENVALUES OF THE DISCRETE SCHRÖDINGER OPERATORS

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Let $\mathbb{Z}^d$ be the $d = 1, 2, ...-$ dimensional hypercubic lattice and $\mathbb{T}^d = (\mathbb{R}/2\pi\mathbb{Z})^d = (-\pi, \pi]^d$ be the d - dimensional torus (Brillouin zone), the dual group of $\mathbb{Z}^d$. Let $L_2^e(\mathbb{T}^d)-$ be Hilbert space of square-integrable even functions on $\mathbb{T}^d$.

In the momentum representation the discrete Schrödinger operator $h_{\mu\lambda}(k), k \in \mathbb{T}^d$ being the two-particle quasi-momentum, is of the form [1, 2]:

$$h_{\mu\lambda}(k) = h_0(k) - \mu v.$$

The non-perturbed operator $h_0(k)$ on $L_2^e(\mathbb{T}^d)$ is multiplication operator by the function

$$\mathcal{E}_K(q) = \sum_{j=1}^{d} \left[2 - 2\cos\left(\frac{K_j}{2}\right)\cos q_j \right].$$

The perturbation $v_{\mu\lambda}$ is an integral operator of rank $d+1$:

$$(v_{\mu\lambda} f)(p) = \frac{1}{(2\pi)^{d/2}} \int\limits_{\mathbb{T}^d} \Big(\mu + \lambda \sum_{i=1}^{d} \cos p^{(i)} \cos t^{(i)}\Big) f(t)dt, \quad f \in L_2^e(\mathbb{T}^d).$$

The essential spectrum $\sigma_{ess}(h_{\mu\lambda}(k))$ of $h_{\mu\lambda}(k)$ fills the segment $[\mathcal{E}_{\min}(k), \mathcal{E}_{\max}(k)]$, where

$$\mathcal{E}_{\min}(k) = \min_{q\in\mathbb{T}^d} \mathcal{E}_k(q), \ \mathcal{E}_{\max}(k) = \max_{q\in\mathbb{T}^d} \mathcal{E}_k(q).$$

Note that according the Birman-Schwinger principle the operator $h_{\mu\lambda}(k)$ has no more than $d+1$ eigenvalues lying outside of the essential spectrum $\sigma_{ess}(h_{\mu\lambda}(k))$.

For the discrete Schrödinger operators $h_{\mu\lambda}(k)$ on $d-$dimensional lattice $\mathbb{Z}^d$, $d = 1, 2, ...$ the following results have been established.

(i) Let $d = 1$ or 2. Then for any $\mu, \lambda \geq 0$ and $\mu + \lambda > 0$ the operator $h_{\mu\lambda}(k), k \in \mathbb{T}^d$ has an eigenvalue below the bottom $\mathcal{E}_{\min}(k)$ of the essential spectrum $\sigma_{ess}(h_{\mu\lambda}(k))$.

(ii) Let the operator $h_{\mu\lambda}(0), 0 \in \mathbb{T}^d$ has $1 \leq n \leq d+1$ eigenvalues (counting multiplicities) below the bottom $\mathcal{E}_{\min}(0)$ of the essential spectrum $\sigma_{ess}(h_{\mu\lambda}(0))$. Then for any non-zero $k \in \mathbb{T}^d$ the operator $h_{\mu\lambda}(k), k \in \mathbb{T}^d$ has at least n eigenvalues (counting multiplicities) lying below the bottom $\mathcal{E}_{\min}(k)$ of the essential spectrum $\sigma_{ess}(h_{\mu\lambda}(k))$.

Remark that in [3] the number of eigenvalues of the discrete Schrödinger operator $h_{\mu\lambda}$ associated to an one particle system on $\mathbb{T}^3$ have been studied.

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ON AN ASYMPTOTICS OF EIGENVALUES OF THE DISCRETE SCHRÖDINGER OPERATORS

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Let $\mathbb{Z}^d$ be the d - dimensional hybercubic lattice and $\mathbb{T}^d = (\mathbb{R}/2\pi\mathbb{Z})^d = (-\pi,\pi]^d$ be the d - dimensional torus (Brillouin zone), the dual group of $\mathbb{Z}^d$. Let $L_2^e(\mathbb{T}^d)-$ the Hilbert space of square-integrable even functions on $\mathbb{T}^d$.

Let $h_\mu(k), k \in \mathbb{T}^d$ be the discrete Schrödinger operators, associated to the Hamiltonian of system of two identical particles (bosons), moving on d - dimensional lattice $\mathbb{Z}^d$, interacting via zero-range attractive potentials $\mu > 0$.

The operator $h_\mu(k), k = (k_1,\dots,k_d) \in \mathbb{T}^d$ acts on $L_2^e(\mathbb{T}^d)$ as follows [1, 2]:

$$h_\mu(k) = h_0(k) - \mu v.$$

The non-perturbed operator $h_0(k)$ on $L_2^e(\mathbb{T}^d)$ is multiplication operator by the function

$$\mathcal{E}_k(q) = \sum_{j=1}^{d} \left[2 - 2\cos\frac{k_j}{2}\cos q_j\right].$$

The perturbation $v_{\mu\lambda}$ is an integral operator of rank one

$$(v_\mu f)(p) = \frac{\mu}{(2\pi)^{d/2}} \int\limits_{\mathbb{T}^d} f(t)dt, \quad f \in L_2^e(\mathbb{T}^d).$$

The essential spectrum $\sigma_{ess}(h_\mu(k))$ of $h_\mu(k)$ fills the segment $[\mathcal{E}_{\min}(k), \mathcal{E}_{\max}(k)]$, where

$$\mathcal{E}_{\min}(k) = \min_{q\in\mathbb{T}^d} \mathcal{E}_k(q) \geq 0, \quad \mathcal{E}_{\max}(k) = \max_{q\in\mathbb{T}^d} \mathcal{E}_k(q) \leq 4d.$$

For the discrete Schrödinger operators $h_\mu(k), k \in \mathbb{T}^d$, $d = 1, 2$, the following results are obtained.

(i) For any $\mu > 0$ and $k \in \mathbb{T}^d$ the operator $h_\mu(k)$ has a unique eigenvalue $z(\mu, K)$ lying below the bottom $\mathcal{E}_{\min}(k)$ of the essential spectrum of $h_\mu(k)$. The function $z(\cdot, k)$ is monotonously decreasing on $(0, +\infty)$ and $z(\mu, \cdot)$ is even real-analytic function on $\mathbb{T}^d$. Moreover, for any $k \in \mathbb{T}^d \setminus \{\mathbf{0}\}$ the eigenvalue $z(\mu, k)$ satisfies the relation

$$z(\mu, k) > z(\mu, \mathbf{0}), \mathbf{0} = (0, ..., 0) \in \mathbb{T}^d.$$

(ii) For any $k = (k_1, ..., k_d) \in (-\pi, \pi)^d$ the function $z(\mu, k)$ has the following asymptotics as $\mu \to 0$:

$$\text{if } d = 1 \text{ then } z(\mu, k) = \mathcal{E}_{\min}(k) - \frac{\mu^2}{4\cos\frac{k}{2}} + O(\mu^3),$$

$$\text{if } d = 2 \text{ then } z(\mu, k) = \mathcal{E}_{\min}(k) - b(k)\exp\{-\frac{c(k)}{\mu}\} + o(\exp\{-\frac{c(k)}{\mu}\}),$$

where $b(k) > 0,\ c(k) = 4\pi\sqrt{\cos\frac{k_1}{2}\cos\frac{k_2}{2}} > 0.$

In [3], analogously results for the continuous Schrödinger operators have been established.

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THE CAUCHY PROBLEM FOR LANGMUIR CHAIN WITH STEPWISE INITIAL PROBLEM

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The Langmuir chain has important applications in plasma physics and in biology (see [1]). The applications of the method of the universe spectral problem to integration of the Langmuir chain in the case of quick-decreasing initial conditions in the periodic case as well are known well.

The case when the scattering associated with Langmuir chain of difference operator exists only on one side is also of great interest.

Consider the following Cauchy problem for the Langmuir chain

$$\dot{c_n} = c_n(c_{n+1} - c_{n-1}), \quad n = 0, 1, \dots \ c_n = c_n(t) > 0, \tag{1}$$

$$c_n(0) = {c_n}^{\wedge}, \ n = 0, 1, \dots \tag{2}$$

where the sequence satisfies the conditions

$$\sum_{n=1}^{\infty} |n| \left| {c_n}^{\wedge} - 1 \right| < \infty \,, \ \ {c_n}^{\wedge} \to 0 \ as \ n \to -\infty.$$

We'll look for the solution of problem (1)-(2) that for each satisfies the following conditions

$$\left\| \sum_{n=1}^{\infty} n \left| c_n(t) - 1 \right| \right\|_{C[0,T]} < \infty, \tag{3}$$

$$\|c_n(t)\|_{C[0,T]} < \infty.$$

In the paper the formulas allowing to find the solution of problem (1)—(2) are obtained by the inverse scattering problem method

Theorem. *The solution of problem* (1)-(2) *exists and is unique of* (3) *is fulfilled.*

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ON THE PROPERTIES OF THE GENERALIZED SOLUTION OF THE CAUCHY PROBLEM FOR HYPERBOLIC EQUATION[1]

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Let's consider the equation

$$L_\alpha u \equiv y u_{yy} + u_{xx} + \alpha u_y = 0 \tag{1}$$

in domain D of the half plane $y < 0$, bonded by the characteristics of the equation (1) the modified Cauchy problem with initial conditions [1]

$$u_\alpha(x, 0) = \tau(x), \quad \lim_{y \to -0} (-y)^\alpha \left[u_\alpha - A_n^-(\tau)\right]_y' = \nu(x). \tag{2}$$

As you see that solution is inconveniently in use, when we solve the boundary value problems of the mixed type. Because it is difficult to get the relation between $\tau(x)$, $\nu(x)$ on the type changing line. Thereupon Karol define new class of generalized functions for the solution of the Tricomi problem for the equation of the elliptic-hyperbolic type.

The author of this paper find the comfortable form of the solution in the class of generalized solutions for the defined above problem (1), (2):

$$u(\xi, \eta) = \int_0^\xi (\eta - \zeta)^{-\beta} (\xi - \zeta)^{-\beta} T(\zeta) d\zeta + \int_\xi^\eta (\eta - \zeta)^{-\beta} (\zeta - \xi)^{-\beta} N(\zeta) d\zeta, \tag{3}$$

where

$$N(\zeta) = \frac{1}{2 \cos \pi\beta} T(\zeta) - (-1)^n \cdot 2^{4\beta - 2} \gamma_2 \nu(\zeta).$$

It let to prove the unique solvability of the boundary value problems as for the equation of the elliptic-hyperbolic type, as for the equation of the parabolic-hyperbolic type.

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ON THE BEHAVIOR OF THE SOLUTION OF THE PSEUDO-HYPERBOLIC EQUATION WITH NONLINEAR BOUNDARY CONDITIONS

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Consider the problem

$$u_{tt} - \sum_{i=1}^{n} D_i(|D_i u|^{p-2} D_i u) - \alpha \Delta u_t + f(u) = 0, (x,t) \in \Omega x[0,T](1) \tag{1}$$

$$u(x,0) = u_0(x), u_t(x,0) = u_0(x), x \in \Omega, \tag{2}$$

$$\sum_{i=1}^{n}(|D_i u|^{p-2} D_i u)\cos(x_i, v) + \alpha \frac{\partial u_t}{\partial n} = g(u), (x,t) \in \partial\Omega x[0,T] \tag{3}$$

where $\Omega \in R^n$, $n \geq 2$ is boundary domain with boundary $\partial\Omega$, $u_0(x) \in W_2^1(\Omega)$, $u_1(x) \in L_2(\Omega)$ - given functions, $f(u)$ and g(u)- some nonlinear functions, $\alpha > 0$-some number, $p \geq 2$, $D_i = \frac{\partial}{\partial x_i}$, $i = 1, 2, ..., n$, $\frac{\partial}{\partial n}$ - is the derivative on the outward normal to $\partial\Omega$.

The problems with linear and homogeneous Dirihlet and Neumann boundary conditions for different pseudo-parabolic and pseudo-hyperbolic equation have been investigated enough (see, for instance [1]-[3]). There exist recent results for the cases when the boundary condition in linear (for example, in [4] the blowing up of the solutions of the Aller equation for $p = 2$).

In this research we study the question on blow up solutions of the non-linear pseudo-hyperbolic equation when $p \geq 2$. The following theorem is proved for the problem (1)-(3).

Theorem. *Let for any $u \in R^1$ and some $\alpha > 0$ the conditions*

$$2(2\alpha + 1)F(u) - uf(u) \geq 0, F(u) = \int_0^u f(s)ds,$$

$$ug(u) - 2(2\alpha + 1)G(u) \geq 0, G(u) = \int_0^u g(\tau)d\tau, \alpha \geq \frac{p-2}{4}$$

be valid. Then if $\int_\Omega F(u_0)dx - \int_{\partial\Omega} G(u_0)ds + \frac{1}{p}\int_\Omega \sum_{i=1}^{n} |D_i u_o|^p dx \leq 0$, $(u_0, u_1) > 0$ and $u(x,t) \in W_2^1(0,T; W_2^2(\Omega)) \bigcap W_2^2(0,T; L_2(\Omega))$ is a solution for the problem (1)-(3), then there exist $t_0 < \infty$ such, that the relation

$$\lim_{t \to t_0}\left[\|u(x,t)\|^2 + \alpha \int_0^t \|\nabla(x,t)\|^2 d\tau\right] = \infty$$

satisfied.

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CONSTRUCTION OF THE GENERAL SOLUTION TO LOADED EQUATION OF PARABOLIC TYPE

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Assume that a controlled process is described by function $u\,(t,x)$ that satisfies the equation

$$u_t = a^2 u_{xx} + \sum_{k=1}^{m} b_k\,(t,x)\,u\,(t,x_k) + f\,(t,x) \tag{1}$$

inside the domain $\overline{Q} = [0,T] \times [0,1]$. At the boundary Q it satisfies the following initial and boundary conditions

$$\begin{cases} u\,(0,x) = u^0\,(x)\,, \\ u_x\,(t,0) = 0, \;\; u_x\,(t,1) + \alpha u\,(t,1) = 0, \;\; \alpha = const > 0, \end{cases} \tag{2}$$

where $b_k\,(t,x)$, $f\,(t,x)$, and $u^0\,(x)$ are given quadratically summable functions.

As is well known, the solution to problem (1), (2) can be represented in the form

$$u\,(t,x) = \int_0^1 G\,(x,\xi,t)\,u^0\,(\xi)\,d\xi + \int_0^t \int_0^1 G\,(x,\xi,t-\tau)\left[\sum_{k=1}^{m} b_k\,(\tau,\xi)\,u\,(\tau,x_k) + f\,(\tau,\xi)\right] d\xi d\tau \tag{3}$$

with the use of source function $G\,(x,\xi,t)$. Entering $x = x_i$, $i = 1,2,...,m$ into both sides of this equation, we obtain a system of integral equations to determine functions $u_i\,(t) = u\,(t,x_i)$:

$$u_i\,(t) = \int_0^t \sum_{k=1}^{m} R_{ik}\,(t,\tau)\,u_k\,(\tau)\,d\tau + \varphi_i\,(t), \;\; i = 1,2,...,m,$$

$$R_{ik}\,(t,\tau) = \int_0^t G\,(x_i,\xi,t-\tau)\,b_k\,(\tau,\xi)\,d\xi, \;\; \varphi_i\,(t) = \int_0^t G\,(x_i,\xi,t)\,u^0\,(\xi)\,d\xi + \int_0^t \int_0^1 G\,(x_i,\xi,t-\tau)\,f\,(\tau,\xi)\,d\xi d\tau. \tag{4}$$

Introducing the following notations

$$u\,(t) = \{u_i\,(t)\}_{i=1,2,...,n}\,, \quad \varphi\,(t) = \{\varphi_i\,(t)\}_{i=1,2,...,n}\,, \quad , \quad R\,(t,\tau) = \{R_{ik}\,(t,\tau)\}_{i=1,2,...,n}^{k=1,2,...,m}\,,$$

system (4) can be rewritten in the form of integral equation

$$u\,(t) = \int_0^t R\,(t,\tau)\,u\,(\tau)\,d\tau + \varphi\,(t)\,. \tag{5}$$

Solution to this equation can be rewritten in the form

$$u\,(t) = \sum_{n=1}^{\infty} v_n\,(t), \tag{6}$$

where $v_0\,(t) = \varphi\,(t)$, $v_n\,(t) = \int_0^t R\,(t,\tau)\,v_{n-1}\,(\tau)\,d\tau$, $n = 1,2,....$

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THE VOLTERRNESS OF THE BITSADZE-SAMARSKII TYPE PROBLEM FOR THE MIXED PARABOLIC-HYPERBOLIC EQUATION OF THE THIRD ORDER

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Let $\Omega \subset R^2$ be the finite domain limited at $y > 0$ by segments AA_0,A_0B_0,B_0B of the straight lines $x = 0$, $y = 1$, $x = 1$, respectively, and at $y < 0$ by characteristics AC:$x + y = 0$ and BC:$x - y = 1$ of the mixed parabolic-hyperbolic equation of the third order

$$Lu = f\left(x, y\right), \tag{1}$$

where

$$Lu = \frac{\partial}{\partial x} lu = \frac{\partial}{\partial x} \begin{cases} u_x - u_{yy}, \ y > 0 \\ u_{xx} - u_{yy}, \ y < 0. \end{cases}$$

Let a smooth curve AD:$y = -\gamma\left(x\right)$,$0 \leq x \leq 1$, where $0,5 < l < 1$;$\gamma\left(0\right) = 0$,$l + \gamma\left(l\right) = 1$ be located in a characteristic triangle $0 \leq x+y \leq x-y \leq 1$. Further, concerning a curve AD we will assume that $\gamma\left(x\right)$ is twice continuously differentiable function and functions $x - \gamma\left(x\right)$ and $x + \gamma\left(x\right)$ monotonously increase, moreover,

$$0 < \gamma'\left(0\right) < 1, \ \gamma\left(x\right) > 0, \ x > 0.$$

In this work we investigate a nonlocal problem for the equation (1) in the domain Ω, in a hyperbolic part of it, the condition of Bitsadze-Samarskii connects values of a tangent of a derivative of the searched solution on the characteristic AC with derivatives along a direction of the characteristic BC of that function on any curve AD, lying in a characteristic triangle ABC, with the endpoints in the origin and on the characteristic BC (in a point D).

Problem A. *To find a solution of the equation (1), satisfying to boundary conditions*

$$u|_{AA_0 \cup A_0B_0} = 0,$$

$$\left.\frac{\partial u}{\partial n}\right|_{AA_0 \cup AC} = 0,$$

$$\left[u_x - u_y\right]\left[\theta_0\left(t\right)\right] + \mu\left(t\right)\left[u_x + u_y\right]\left[\theta^*\left(t\right)\right] = 0,$$

where n is an internal normal, $\theta_0(t)$ $(\theta^*\left(t\right))$ is an affix of a point of intersection of characteristics AC (curve AD) with the characteristic outgoing from the point $\left(t, 0\right)$, $0 < t < 1$, $\mu\left(t\right)$ is a given function.

The main aim of the present work is to show that for a correctness and Volterrness of the problem A, crucial importance has a parity between "compression" coefficient $\mu(0)$ in the origin of the derivative in a direction of the characteristic BC and a polar corner α, generated by a curve AD an axis of abscissas.

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THE COEFFICIENT OF THE INVERSE PROBLEM FOR NONLINEAR FORTH-ORDER PARTIAL INTEGRO-DIFFERENTIAL EQUATION

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Let's consider in domain $G = \{(t,x) : 0 \le t \le T,\ 0 \le x \le 1\}$ the following equation

$$u_t(t,x) = a_0(Au)_t + a_1(Au) + \sum_{i=0}^{3} b_i(t,x)\frac{\partial^{3-i}u(t,x)}{\partial x^{3-i}} + \int_0^t F(t,x,u(s,x))\lambda(t-s)ds + f(t,x) \qquad (1)$$

satisfying to the initial, boundary and additional conditions

$$u(0,x) = u_0(x),\ x \in [0,1], \qquad (2)$$

$$u(t,0) = u(t,1) = u_x(t,0) = 0,\ t \in [0,T], \qquad (3)$$

$$u(t,x_0) = g(t),\ t \in [0,T],\ x_0 \in (0,1), \qquad (4)$$

where $a_0 \neq 0$, a_1 are given constants, $b_i(t,x)$ $(i = \bar{0,3})$, $f(t,x) \in C^{1,0}(G)$, $F(t,x,u)$ are known functions, at that F satisfies a Lipschitz condition with respect to u. It is required to find functions $u(t,x)$ and $\lambda(t)$. In equation (1) A is a differential operator as

$$Au = \frac{\partial^3 u}{\partial x^3} + \alpha_1\frac{\partial^2 u}{\partial x^2} + \alpha_2\frac{\partial u}{\partial x} + \alpha_3 u,$$

where $\alpha_1, \alpha_2, \alpha_3$ are given constants.
The inverse problem (1) - (4) reduces to the system of nonlinear Volterra operator equations of the second kind using integral equations method and Green function. The sufficient conditions of existence and uniqueness solution of problem are received.

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THE PROBLEM OF CONTROLLABILITY FOR THE VIBRATIONS OF BAR

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Recently controllability problems are intensively studied for the various partial differential equations. In the present work the problem of controllability for the equation for the vibration of the bar is studies.

Assume that the controlled process is described by the equation

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial^4 u}{\partial x^4} = 0,\ 0 < x < 1,\ 0 < t < T, \tag{1}$$

$$u(x,0) = \varphi(x),\ \ \frac{\partial u(x,0)}{\partial t} = \psi(x), \quad 0 \le x \le 1(2) u(0,t) = 0,\ \ u(1,t) = \mu(t)\,, \quad 0 \le t \le T \tag{2}$$

$$\frac{\partial^2 u(0,t)}{\partial x^2} = \nu(t),\ \ \frac{\partial^2 u(0,t)}{\partial x^2} = 0, \quad 0 \le t \le T, \tag{3}$$

where $\mu(t),\ \nu(t)$ are control functions.

It needs to determine the time $T > 0$ and the corresponding controls functions $\mu(t),\ \nu(t)$ such that corresponding solution $u(x,t)$ of the problem (1)-(3) satisfies

$$u(x,T) = 0,\ \frac{\partial u(x,T)}{\partial t} = 0, 0 \le x \le 1. \tag{4}$$

In this work the explicit form of boundary control, which ensures the conditions (4) is a obtained.

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DIFFERENTIAL EQUATIONS OF THE TWO-DIMENSIONAL PARABOLIC RESTRICTED THREE-BODY PROBLEM

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The present work is devoted to both of the perturbing function development and differential equations of the two-dimensional parabolic restricted three-body problem construction. The above-mentioned differential equations of motion are presented in rectangular coordinates.

The existence of Lagrangian and Eulerian partial solutions of these equations in polar coordinates are considered. Lagrangian points dispositions are determined from the nonnegative condition of finite mass relation. Further the differential equations of perturbed motion in osculating elements are presented. Then accepting as independent variable the true anomaly υ' of the perturbing body instead of time the latter equations are transformed to a view which is more convenient for the further examination.

At the end of work it is derived the expression of perturbing function for the case when two first members of its development on ascending powers of the distance relation are taken into account. It has the view:

$$R = \frac{\sqrt{2}}{32}\beta\sqrt{a}\left\{\sum_{n=1}^{3}\left\{\sum_{i=0}^{3}A_{i,n}^{0}(e)\cos\left[iM+(2-n)\upsilon'\right]+\right.\right.$$
$$\left.+\sum_{i=-1}^{5}A_{i,n}^{2}(e)\cos\left(iM+2\omega-n\upsilon'\right)\right\}+\frac{3}{16}\frac{a}{q'}\sum_{n=1}^{5}\left\{\sum_{i=-2}^{4}A_{i,n}^{1}(e)\times\right.$$
$$\left.\left.\times\cos\left[iM+\omega+(2-n)\upsilon'\right]+\sum_{i=0}^{6}A_{i,n}^{3}(e)\cos\left(iM+3\omega-n\upsilon'\right)\right\}\right\}$$

Here a, e, ω, M are the grand semi-axis, the eccentricity, the argument of pericenter and the mean anomaly of perturbing body accordingly, β is a small parameter proportional to the a/q' relation, where q' is the distance from the perturbing body pericenter up to the central body. Coefficients $A_{i,n}^{j}(e)$, $(j=0,\ 1,\ 2,\ 3)$ are found up to members proportional to cube of eccentricity. At last, calculating partial derivatives of R on elements a, e, ω, M, the differential equations of perturbed motion in these elements are derived.

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ON NECESSARY AND SUFFICIENT CONDITIONS OF EXISTENCE OF ISOLATED SOLUTION FOR NON-LINEAR TWO-POINT BOUNDARY VALUE PROBLEM WITH PARAMETER

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We consider non-linear two-point boundary-value problem

$$\frac{dx}{dt} = f(t, x, \mu), \qquad t \in [0, T], \qquad x \in R^n, \qquad \mu \in R^m \tag{1}$$

$$g(\mu, x(0), x(T)) = 0, \tag{2}$$

where $f : [0, T] \times R^{n+m} \to R^n$, $\quad g : R^{m+2n} \to R^{n+m}$ are continuous functions.

A solution of the problem (1), (2) is the pair $(\mu^*, x^*(t))$, where continuous differentiable on $[0, T]$ function $x^*(t)$ at $\mu = \mu^*$ satisfies to differential equation (1) and boundary condition (2).

Linear boundary value problems with parameter for systems of ordinary differential equations were investigated in [1, 2]. The algorithm to finding the approximate solution were offered and the criteria of its unique solvability were obtained.

In the communication non-linear boundary value problems with parameter (1), (2) are investigated by parametrization method [3]. The definition of isolated solution for two-point boundary value problem with continuous differentiable data are introduced. The nesessary and sufficient conditions for existence of isolated solution are obtained in terms of functions of right hand side of differential equation and boundary condition.

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ON THE PROBLEM OF THREE-BODY-POINTS WITH VARIABLE MASSES

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The problem of three body-points under Newton interaction is considered. Masses of the points change isotropically by the different laws $m_0 = m_0(t)$, $m_1 = m_1(t)$, $m_2 = m_2(t)$. Equations of motion in the Jacobi coordinates are given by

$$\mu_1 \ddot{\vec{r}}_1 = \mathrm{grad}_{\vec{r}_1} U, \quad \mu_2 \ddot{\vec{r}}_2 = \mathrm{grad}_{\vec{r}_2} U - \mu_2 (2\dot{\nu}_1 \dot{\vec{r}}_1 + \ddot{\nu}_1 \vec{r}_1), \tag{1}$$

$$\frac{\dot{m}_i}{m_i} \neq \frac{\dot{m}_j}{m_j}, \ i \neq j, \quad \mu_1 = \frac{m_1 m_0}{m_0 + m_1} \neq const, \quad \mu_2 = \frac{m_2(m_1 + m_0)}{m_0 + m_1 + m_2} \neq const,$$

where μ_i are the reduced masses, $\nu_1 = m_1/(m_0 + m_1)$, U is the power function [1]. In general, in contrast to the classical problem of three bodies with constant masses, non-autonomous differential equations (1) do not have any integral. The problem is investigated by the perturbation methods with use of the system of analytical calculations MATHEMATICA [2].

A periodical motion on quasionic section is used as an initial unperturped intermediate motion. Equations of secular perturbations in the analogues of the second system of the Poincare elements [1, 3] have the form

$$\begin{aligned} \dot{\xi}_i &= \frac{\partial R_{i\,\mathrm{sec}}}{\partial \eta_i}, \quad \dot{\eta}_i = -\frac{\partial R_{i\,\mathrm{sec}}}{\partial \xi_i}, \\ \dot{p}_i &= \frac{\partial R_{i\,\mathrm{sec}}}{\partial q_i}, \quad \dot{q}_i = -\frac{\partial R_{i\,\mathrm{sec}}}{\partial p_i}, \ i = 1, 2 \end{aligned} \tag{2}$$

here $R_{i\,\mathrm{sec}}$ are the corresponding expressions of the secular disturbing functions.

Masses of bodies are assumed to be comparable, but the laws of mass changing are arbitrary. Eccentricities and inclinations of the orbits of bodies are the small quantities. In expression of the perturbing function the members till the second degree inclusively concerning small quantities are preserved. And even in this case analytical calculations are very cumbersome and difficult foreseeable.

Under these assumptions the expression of the disturbing function in analogues of the second system of the Poincare elements is obtained. Equations of secular perturbations are obtained and solutions in the first approximation by the Picard method are analyzed.

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ON THE BOUNDARY VALUE PROBLEM FOR THE OPERATOR-DIFFERENTIAL EQUATION OF THE THIRD ORDER

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Let H is separable Hilbert space, A is the self- adjoined positive-defined operator in H, $H_\gamma-$ a scale of Hilbert spaces, generated by operator A, i.e. $H_\gamma = D(A^\gamma)$, $(x,y)_\gamma = (A^\gamma x,\ A^\gamma y)$, $x, y \in H_\gamma$ $(\gamma \geq 0)$, $H_0 = H$.

Let's determine Hilbert spaces

$$L_2(R_+;H) = \left\{ \|f\|_{L_2(R_+;H)} = \left(\int_0^{+\infty} \|f\left(t\right)\|_H^2 \, dt \right)^{1/2} < \infty \right\},$$
$$W_2^3\left(R_+;H\right) = \left\{ u\left(t\right) : u'''(t), A^3u(t) \in L_2\left(R_+;H\right), \right.$$
$$\left. |||u||| = \left(||u'''||_{L_2(R_+;H)} + \left\|A^3u\right\|^2_{L_2(R_+;H)} + \|u'(0)\|^2_{3/2} \right)^{1/2} \right\}.$$

In this paper the conditions are obtained for resolvability of the boundary value problem

$$\frac{d^3u}{dt^3} + A^3u + \sum_{j=1}^{2} A_j u^{(3-j)} = f, \tag{1}$$

$$u'\left(0\right) = Ku = 0, \tag{2}$$

where $f(t) \in L_2(R_+;H)$, $u(t) \in W_2^3(R_+;H)$, A_jA^{-j}, $j = \overline{1,3}$ are bounded operators in H, and $K : W_2^3\left(R_+;H\right) \to\ H_{3/2}$.

As we see, the boundary condition and the equation are perturbed by some operator.

Theorem. Let A be the self-adjoined positive-defined operator, $B_j = A_jA^{-j}$, $j = \overline{1,2}$, are bounded, $\kappa = \|K\|_{W_2^3(R_+;H)\to\ H_{3/2}} < 2^{-1/6}$ and

$$q = \sum_{j=1}^{2} \|B_j\| < 2^{1/6} \left(1 - 2^{1/6}\kappa\right)^{-1/2}.$$

Then for the $f \in L_2(R_+;H)$, there is existed unique $u \in W_2^3(R_+;H)$, such that it satisfies the boundary condition as $\lim\limits_{t\to 0} \|\, u'\left(t\right) - Ku \,\|_{H_{3/2}} = 0$ and the equation (1) almost everywhere.

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ON THE DISCRETE SPECTRUM OF TWO-PARTICLE DISCRETE SCHRÖDINGER OPERATORS

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The fundamental difference between multiparticle discrete and continuous Schrödinger operators is that in the discrete case the analogue of the Laplasian $-\Delta$ is not rotationally invariant.

Due to this fact, the Hamiltonian of a system does not separate into two parts, one relating to the center-of-motion and other one to the internal degrees of freedom. In particular, such a handy characteristics of inertia as mass is not available. Moreover, such a natural local substituter as the effective mass-tensor (of a ground state) depends on the quasi-momentum of the system and, in addition, it is only semi-additive (with respect to the partial order on the set of positive definite matrices). This is the so-called *excess mass* phenomenon for lattice systems (see, e.g., [4] and [5]): the effective mass of the bound state of an N-particle system is greater than (but, in general, not equal to) the sum of the effective masses of the constituent quasi-particles.

The two-particle problem on lattices can be reduced to an effective one-particle problem by using the Gelfand transform instead: the underlying Hilbert space $\ell^2((\mathbb{Z}^{\mathrm{d}})^2)$ is decomposed to a direct von Neumann integral, associated with the representation of the discrete group $\mathbb{Z}^{\mathrm{d}}$ by the shift operators on the lattice and then the total two-body Hamiltonian appears to be decomposable as well. In contrast to the continuous case, the corresponding fiber Hamiltonians $h^{\mathrm{d}}(k)$ associated with the direct decomposition depend parametrically on the internal binding k, the quasi-momentum, which ranges over a cell of the dual lattice. As a consequence, due to the loss of the spherical symmetry of the problem, the spectra of the family $h^{\mathrm{d}}(k)$ turn out to be rather sensitive to the variation of the quasi-momentum k, $k \in \mathbb{T}^{\mathrm{d}} \equiv (-\pi, \pi]^{\mathrm{d}}$.

For example, in [1, 3] it is established that the discrete Schrödinger operator $h^3(k)$, with a zero-range attractive potential, for all non-zero values of the quasi-momentum $0 \neq k \in \mathbb{T}^3$, has a unique eigenvalue below the essential spectrum if $h^{\mathrm{d}}(0)$ has a virtual level (see [1]) or an eigenvalue below its essential spectrum (see [3]).

The similar result is obtained in [2], which is the (variational) proof of existence of the discrete spectrum below the bottom of the essential spectrum of the fiber Hamiltonians $h^{\mathrm{d}}(k)$ for all non-zero values of the quasi-momentum $0 \neq k \in \mathbb{T}^{\mathrm{d}}$, provided that the Hamiltonian $h^{\mathrm{d}}(0)$ has either a virtual level (in dimensions of three and four) or a threshold eigenvalue (in all dimensions $\mathrm{d} \geq 5$). We emphasize that the authors of [2] considered more general class of two-particle discrete Hamiltonians interacting via short-range pair potentials.

In this paper, we explore the some spectral properties of some d - dimensional two-particle discrete Schrödinger operator $h^{\mathrm{d}}(k) = h_0^{\mathrm{d}}(k) + \mathbf{v}$, $k \in \mathbb{T}^{\mathrm{d}}$, k being the two-particle quasi-momentum, corresponding to the energy operator H^{d} of the system of two quantum particles moving on the d-dimensional lattice $\mathbb{Z}^{\mathrm{d}}$ with the short-range potential $\mathbf{v}$. In the case $k \in \mathbb{T}^{\mathrm{d}} \setminus (-\pi, \pi)^{\mathrm{d}}$, we establish necessary and sufficient conditions for the existence of infinite number eigenvalues of the operator $h^{\mathrm{d}}(k)$.

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CRITERION OF THE SPECTRUM DISCREETNESS OF A CLASS OF HYPERBOLIC TYPE DIFFERENTIAL OPERATORS

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Let $\Omega = \{(x, y) : -\pi < x < \pi, -\infty < y < \infty\}$. In the space $L_2(\Omega)$ we consider the differential operator of hyperbolic type

$$(L + \mu E)u = u_{xx} - u_{yy} + a(y)u_x + c(y)u + \mu \mathrm{u} \tag{1}$$

with the domain $D(L)$ of infinitely differentiable functions satisfying the conditions $u(-\pi, y) = u(\pi, y)$, $u_x(-\pi, y) = u_x(\pi, y)$ and compactly supported with respect the variable y.

Everywhere, we assume that the coefficients $a(y)$, $c(y)$ satisfy the condition

$i)|a(y)| \geq \delta_0 > 0$, $c(y) \geq \delta_0 > 0$ are continuous functions in $R(-\infty, \infty)$.

Note that the operator L admits closure in the space $L_2(\Omega)$ which will be again denoted by L.

Theorem 1. *Let the condition $i)$ be fulfilled. Then the operator $L + \mu E$ is continuously invertible in the space $L_2(\Omega)$ for sufficiently large $\mu > 0$.*

This result covers a case when the coefficients are unbounded. The questions of existence of the resolvent in case of pseudodifferential operators have been well studied and ascertained typical problems connected with behaviour of coefficients [1,2].

Theorem 2. *Let the condition $i)$ be fulfilled. Then the resolvent of the operator L is compact in the space $L_2(\Omega)$ if and only if for any $\omega > 0$*

$$\lim_{|y|\to\infty} \int_y^{y+\omega} c(t)dt = \infty.$$

Before, such a theorem was obtained by A.M.Molchanov [3] for the Sturm-Liouville operator.

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ON AN INVERSE BOUNDARY VALUE PROBLEM FOR THE THIRD ORDER PSEUDOHYPERBOLIC EQUATION WITH INTEGRAL CONDITION

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For the equation

$$u_{tt}(x,t) - \alpha u_{txx}(x,t) - \beta u_{xx}(x,t) = a(t)u(x,t) + f(x,t) \tag{1}$$

in the domain $D_T = \{(x,t) : 0 \le x \le 1,\ 0 \le t \le T\}$ consider an inverse boundary value problem under the conditions:

$$u(x,0) = \varphi(x),\ u_t(x,T) = \psi(x) \quad (0 \le x \le 1), \tag{2}$$

$$u_x(0,t) = 0,\ \int_0^1 u(x,t)dx = 0 \ \ (0 \le t \le T), \tag{3}$$

$$u(0,t) = h(t) \quad (0 \le t \le T), \tag{4}$$

where $\alpha > 0, \beta > 0$ are the given numbers, $f(x,t), \varphi(x), \psi(x), h(t)$ are the given functions, $u(x,t)$and $a(t)$ are the sought functions.

Definition. *We call the pair $\{u(x,t), a(t)\}$ of the functions $u(x,t)$ and $a(t)$ possessing the following properties the classic solution of the inverse boundary value problem* (1)-(4).

(1) the function $u(x,t)$ is continuous in D_T together with all its derivatives contained in equation (1):
(2) the function $a(t)$ is continuous on $[0,T]$;
(3) all the conditions (1)-(4) are satisfied in the ordinary sense.

In paper at first the given problem is reduced to the equivalent problem (in definite sense) and a theorem on the existence and uniqueness of the solution is proved for it. Further, using this fact, the existence and uniqueness of the classic solution of the input problem is proved.

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A NON-LINEAR GROUND STATE PROBLEM

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Using standard compactness and symmetrization methods (see. e.g., [L1] and also [L2]) one proves
Lemma. *The infimum*

$$\inf \left\{ \frac{\int_{R^n} |\nabla u|^2 \, dx}{\frac{1}{2} \iint_{R^n \times R^n} \frac{|u(x)|^2 |u(y)|^2}{|x-y|^2} dxdy} : u \in H^1(R^n), \|u\| = 1 \right\}$$

is strictly positive and there exists a symmetric decreasing $\psi \in H^1(R^1)$*with* $\|\psi\| = 1$*which minimizes the ratio.*

Theorem. *One has*

$$\int_{R^n} |\nabla \psi_0|^2 \, dx = \min \left\{ \int_{R^n} |\nabla u|^2 \, dx : \frac{1}{2} \iint_{R^n \times R^n} \frac{|u(x)|^2 \, |u(y)|^2}{|x-y|^2} dxdy = \|u\|^2 \right\},$$

where ψ_0*-the ground state of the following equation*

$$-\Delta \psi_0 - \int_{R^n} \frac{|\psi_0(y)|^2}{|x-y|^2} dy \psi_0 = -\psi_0$$

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ELLIPTIC EQUATION RELATIVELY TO THE DOMAIN EVOLUTION

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Let's define the speed of the evolution of the domain $U(t)$ as

$$\frac{\partial P_{U(t)}(x)}{\partial t} = \lim_{\Delta t \to 0} \frac{P_{U(t+\Delta t)}(x) - P_{U(t)}(x)}{\Delta t}, \quad x \in S_B. \tag{1}$$

If there exist such domains $V_1(t),\ V_2(t) \in M,\ \ t \in [0,T]$, that

$$\frac{\partial P_{U(t)}(x)}{\partial t} = P_{V_{1(t)}}(x) - P_{V_{2(t)}}(x),$$

then we write $v(t) = \dot{U}(t) = (V_1(t), V_2(t)) \in M \times M$.

Let $D \subset R^n$ a given by bounded domain with smooth boundary S and convex domain $U \subset R^m$ depends on the parameter $y = (y_1, y_2, ..., y_n) \in D$, i.e. . We'll write $U \in C(D)$, if support function $P_{U(y)}(x)$ of the domain $U(y)$ continues on y in D. Analogically we can define $U \in C^1(D)$.

Consider the boundary problem

$$\Delta U = -F(y), \quad y \in D, \tag{2}$$

$$U(\xi) = G(\xi), \quad \xi \in S. \tag{3}$$

Let $F(y) = (F_1(y), F_2(y)) \in M \times M,\ y \in D,\ G(\xi) = (G_1(\xi), G_2(\xi)) \in M \times M,\ \xi \in S$.

In the difference of traditional problems, here solution of the problem (2) is convex domain $U = U(y) \in M$or pair of the convex domains $U(y) = (U_1(y), U_2(y)) \in M \times M$. For the of simplicity, this type functions we'll call domain function. Equation (2) and boundary condition we understand as equality pair of the domains.

Theorem 1. *Let $F_i \in C^1(D) \bigcap C(\bar{D})$ and $G_i \in C(S),\ i = 1, 2$. Then there exists unequal solution $U(y) = (U_1(y), U_2(y)) \in M \times M,\ y \in D$ of the problem (2), (3).*

It is interesting to investigate problem (2), (3), when $F(y),\ G(\xi)$ are convex domains from R^m. There is, that in this case solution of the problem (2), (3) also is the convex set from.

Theorem 2. *Let for any $y \in D$ and $\xi \in S$ $F(y),\ G(\xi)$ be convex closed set and $F \in C^1(D) \bigcap C(\bar{D})$,$G \in C(S)$. Then there exists domain function $U = U(y) \subset R^m$ solution of the problem (2), (3) and this solution is convex closed set.*

Theorem 3. *(Maximum principle). Let $U = U(y)$ be harmonic in$D$, $U \in C(\bar{D})$ and $U(\xi),\ \xi \in S$be convex close bounded domains. If there exists domains$G_0,\ G_1 \subset R^m$, such that*

$$G_0 \subset U(\xi) \subset\ G_1,\ \ \forall \xi \in S, \tag{4}$$

Then, for any $y \in D$

$$G_0 \subset U(y) \subset\ G_1.$$

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SPECTRAL ANALYSIS OF NON-SELFADJOINT MATRIX STURM-LIOUVILLE EQUATIONS

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Let E be an n-dimensional $(n < \infty)$ Euclidian space with the norm $\|.\|$. We denote by $L^2(\mathbb{R}_+, E)$ the Hilbert space of vector-valued functions with the values in E. In the space $L^2(\mathbb{R}_+, E)$ we consider the BVP

$$-y'' + Q(x)\,y = \lambda^2 y\ , \qquad x \in \mathbb{R}_+\ , \tag{1}$$

$$y(0) = 0\ , \tag{2}$$

where Q is a non-selfadjoint matrix-valued function (i.e.$Q \neq Q^*$). It is clear that, the BVP (1)-(2) is a non-selfadjoint.In this work we investigate the principal functions corresponding to the eigenvalues and the spectral singularities of the boundary value problem (1)-(2).

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ABOUT THE METHOD FOR THE STUDY OF CONDITIONALLY WELL-POSED BOUNDARY VALUE PROBLEM FOR DIFFERENTIAL EQUATIONS

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The method consists two ways: direct and inverse. Each way consists of problems of three equations. The first level of a forward way is a problem for the equations partial private derivatives (an initial problem), which after applying the integral Laplace-Fourier transform becomes a problem of the second level – the boundary value problem for a parameterized system of ordinary differential equations (ODU), and the latter moves to the third level – to the system of linear algebraic equations (S.L.A.E.).

The problem is considered incorrect, when determinant S.L.A.E. $\Delta \equiv= \Delta(b, p, \omega) = 0$ and $Rep << 1$ – is arbitrarily large, thus arguments specify dependence Δ accordingly from factors of an initial problem and from parameters of Laplace and Fourier transformation. Depending on structure of set P_0 (zero $\Delta = 0$) a condition of incorrectness of H, we will divide into three types: "the coefficient" – when P_0 is defined by an element b, and parameters p and ω - are free; "root" – when p and ω_j are fastened; " coefficient – root" - some part from parameters of integrated transformations is fixed, as there is a parity between elements b. "Coefficient" condition defines "incorrectness area O". So, as a result of a forward way tasks in view are allocated incorrectly. In reverse motion these problems are solved.

Find the solution of the degenerate S.L.A.E. – The first level of reverse, with his help recovering solution for the ODE, and then by calling (by using the inverse transformations) the latter is determined by the original problem.

By virtue of the degeneracy of S.L.A.E. there is normal solution C_H with the conditions of solvability or pseudonormalization decision C_H – without him. Value problem for ODEs, if defined with C_H, then it satisfies the system of equations and boundary equalities are satisfied only where C_H is defined. These two decisions: the decision S.L.A.E. C_H and the corresponding solution V_H – the boundary value problem is called conditional fine or normal. If the basis is C_H, then its approximate solution with accuracy of the method of least squares.

When the original problem is solved, there is difficulty encountered in accessing the condition of solvability and the meaning C_H, therefore proposed a non-standard way of treatment. The resulting solution satisfies the system of differential equations and the homogeneous initial condition, and the equality of the boundary condition may have the right parts, mismatched with the original.

In addition to constructability, the advantage of the method in the fact that obviously written solvability conditions. And the kind of decision, from which can be directly obtained estimates of stability.

The method is implemented by the problem of heat transfer and mass transfer for generalized Moise-Teoderesku and for Maxwell.

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COERSIVE ESTIMATE FOR THE SOLUTION OF THE DIFFERENTIAL SECOND-ORDER EQUATION AND ITS APPLICATIONS

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Let $1 \leq p < \infty$. Consider the second order differential equation

$$ly = -y^{''} + r(x)y^{'} + q(x)y = f(x), \tag{1}$$

where $f \in L_p \equiv L_p(R)$, $R = (-\infty, +\infty)$, the function r is continuously differentiable and q is a continuous function.

A solution of the equation (1) be called the function $y \in L_p$ if there exist a sequence $\{y_n\}_{n=1}^{\infty}$ of continuous and continuously differentiable up to second order finite functions such that $\|y_n - y\|_p \to 0$, $\|ly_n - f\|_p \to 0$ as $n \to \infty$.

In the report sufficient conditions under which the estimate of the form

$$\|y^{''}\|_p + \|ry^{'}\|_p + \|qy\|_p \leq c\|f\|_p \tag{2}$$

holds for the solution y are discussed. Such estimates are called coercive estimates of a solution. If (2) holds then we say that the operator l, corresponding to the equation (1), is separated in space L_p.

In a case when $r = 0$ various sufficient conditions of the separability of the operator l in the space L_p have been obtained in [1-5]. Results of [1-5] can be extended to the case $r \neq 0$ when the operator rd/dx is a weak perturbation of the operator $d^2/dx^2 + qE$ (E is the identity operator). In particular, our results imply the separability of the operator l when rd/dx is a not necessarily weak perturbation of the operator $d^2/dx^2 + qE$.

Some consequences of estimate (2) for the solvability of quasilinear second order equation are resulted.

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OSCILLATION FOR CERTAIN IMPULSIVE PARTIAL DIFFERENCE EQUATIONS

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In this paper, we obtain some sufficient criteria for the oscillation of the solutions of linear impulsive partial difference equations with continuous variables.

Let $0 = x_0 < x_1 < \ldots < x_n < x_{n+1} < \ldots$, be fixed points with $\lim_{n\to\infty} x_n = \infty$, and let for $n \in \mathbb{N}$ $x_{n+r} = x_n + \tau$, where r is a fixed natural number, and $\tau > 0$ is a constant. Define $J_{imp} = \{x_n\}_{n=1}^{\infty}$, $\mathbb{R}^+ = [0, \infty)$, $J = \{(x, y) : \ x \in J_{imp}, y \in \mathbb{R}^+\}$.

In this paper we shall consider the impulsive partial difference equations with continuous variables of the form

$$\begin{aligned} &c_1u(x+a, y+b) + c_2u(x+a, y) + c_3u(x, y+b) - c_4u(x, y) \\ &+ F(x, y)u(x-\tau, y) + G(x, y)u(x, y-\sigma) + H(x, y)u(x-\tau, y-\sigma) = 0, \ (x, y) \in (\mathbb{R}^+ \times \mathbb{R}^+) \setminus J, \end{aligned} \quad (1)$$

$$u(x_n^+, y) - u(x_n^-, y) = L_n u(x_n^-, y), \qquad (x_n, y) \in J, \quad (2)$$

where $u(x^+, y) = \lim_{\substack{(q,s)\to(x,y) \\ q>x}} u(q, s)$, $u(x_n^-, y) = \lim_{\substack{(q,s)\to(x,y) \\ q<x}} u(q, s)$ and $F, G, H \in C(\mathbb{R}^+\times\mathbb{R}^+, \mathbb{R}^+-\{0\})$, a, b, τ, σ positive constants, c_1, c_2, c_3 and c_4 are nonnegative constants.

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THE ISOSPECTRALITY PROBLEM FOR THE DIRAC SYSTEM

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We call even a Dirac problem

$$By' + \Omega(x)\, y = \lambda y,\ 0 \le x \le \pi, \tag{1}$$

$$y_2(0) - hy_1(0) = 0, \tag{2}$$

$$y_2(\pi) + Hy_1(\pi) = 0, \tag{3}$$

with

$$B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\quad \Omega(x) = \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix},\quad y(x) = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}$$

in which $H = h$ and $\Omega(\pi - x) = \Omega(x)$ on $[0, \pi]$. Here $p(x)$ and $q(x)$ are real functions which are defined and continuous on the interval $[0, \pi]$, h and H are real numbers, and λ is an eigenvalue.

The isospectrality problem is that of describing all problems of the form (1), (2) and (3) that have the same spectrum. This problem has been studied by Trubowitz. Then Max Jodeit and Levitan have studied this problem by using Gelfand-Levitan integral equation and transmutation operator. In this work we describe some results for Dirac operator.

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INVERSE NODAL PROBLEM FOR DIRAC OPERATOR

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Inverse nodal problems consist in constructing operators from the given zeros of their eigenfunctions. In this study, we have estimated nodal points and nodal lengths for Dirac operator. Furthermore, by using nodal points (zeros of eigenfunctions), we have shown that the potential functions of Dirac operator can be established uniquely.

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CONDITIONS FOR THE EXISTENCE OF THE SOLUTION OF ONE CLASS OF NONLINEAR EQUATIONS

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Let H be a separable Hilbert space and $A(\cdot)$ some continuous transformation from H to itself. We will consider the following equation:

$$Au = f, \qquad u, f \in H. \tag{1}$$

Let assume, that if $\varepsilon \leq 1$ small number and $\omega, u \in H$, then

$$A(u + \varepsilon\omega) = A(u) + \varepsilon B(u)\omega + \varepsilon D(u, \omega, \varepsilon). \tag{2}$$

Where $B(u)$-is a linear operator for every $u \in H$, such that

$$|B(u)\omega|_H \leq C(|u|)|\omega|, \tag{3}$$

where $C(\cdot)$ is continuous function on $[0, \infty)$ and $D(u, \omega, \varepsilon)$ is nonlinear transformation on the pair $\omega, u \in H$ such that

$$\|D(u, \omega, \varepsilon\| \leq [C_1(|u|) + C_2(|\omega|)]^2|\omega|\varphi(\varepsilon). \tag{4}$$

Here $C_1(\cdot)$, $C_2(\cdot)$ and $\varphi(\cdot)$ are continuous functions defined on $[0, \infty)$ and $\lim\limits_{\varepsilon \to 0} \varphi(\varepsilon) = 0$. For the solution of equation (1) we will make functional from $u \in H$:

$$J(u) = |A(u) - f|^2. \tag{5}$$

To us it is necessary to impose some conditions on operator $A(\cdot)$:

$$A(0) = 0. \tag{6}$$

If $|A(u)| \leq C_0 \neq 0$, then

$$|u| \leq \varphi_0(C_0). \tag{7}$$

And operator $B^*(u)$ for every $u \in H$, satisfying (7) has limited inverse operator $B^{*-1}(u_n)$, such that

$$|B^{*-1}(u)| \leq \varphi_1(C_0) \tag{8}$$

Theorem 1. *Let the conditions* (3), (4), (6), (7) *and* (8) holds. If $\|f\|_H < \frac{C_0}{2}$, *then there exist the sequence* $u_0, u_1, ..., u_n$, *such that* $\lim\limits_{n \to \infty} J(u_n) = 0$. *Where* C_0 *is from* (7).

Theorem 2. *Let the conditions of theorem* 1 *is satisfied. We suppose that, if* $\lim\limits_{m,n \to \infty} |A(u_n - A(u_m))| = 0$, *then* $\lim\limits_{m,n \to \infty} |u_n - u_m| = 0$. *And then the solution of equation (1) exist, and hold the following correct estimations for the sequence which was constructed in the theorem 1:*

$$|u_n - u| \leq C2^{-n}, \quad |A(u_n) - f| \leq C2^{-n}.$$

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ON THE HIGHT ORDER NONLINEAR SCHRODINGER EQUATION WITH A SELF-CONSISTENT SOURCE

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We consider the problem of integration of the following system equations

$$iu_t - 4\sum_{k=0}^{m}\left(\frac{1}{2i}\right)^{k+1} D_x^k(u\Omega_{m-k}) = 2i\sum_{n=1}^{N}\sum_{j=0}^{m_n-1} C_{m_n-1}^{j}\left(\varphi_{1,n}^{j}\psi_{2,n}^{m_n-j-1} - (\varphi_{2,n}^{j})^*(\psi_{1,n}^{m_n-j-1})^*\right), \qquad (1)$$

$$\frac{\partial}{\partial x}\varphi_{1,n}^{j} + i\xi_n\varphi_{1,n}^{j} + ij\varphi_{1,n}^{j-1} - u(x,t)\varphi_{2,n}^{j} = \frac{\partial}{\partial x}\varphi_{2,n}^{j} - i\xi_n\varphi_{2,n}^{j} - ij\varphi_{2,n}^{j-1} + u^*(x,t)\varphi_{1,n}^{j} = 0, \qquad (2)$$

$$\frac{\partial}{\partial x}\psi_{1,n}^{j} - i\xi_n\psi_{1,n}^{j} - ij\psi_{1,n}^{j-1} + u^*(x,t)\psi_{2,n}^{j} = \frac{\partial}{\partial x}\psi_{2,n}^{j} + i\xi_n\psi_{2,n}^{j} + ij\psi_{2,n}^{j-1} - u(x,t)\psi_{1,n}^{j} = 0, \qquad (3)$$

$$n = 1,\, 2,\, \dots,\, N,\ j = 0,\, 1,\, \dots,\, m_n - 1,$$

with the initial conditions

$$u(x,0) = u_0(x), \quad x \in R, \qquad (4)$$

where vector-functions $\Phi_n^0 = (\varphi_{1,n}^0, \varphi_{2,n}^0)^T$ and $\Psi_n^0 = (\psi_{1,n}^0, \psi_{2,n}^0)^T$ are eigenfunctions of the system (2) and (3) corresponding to eigenvalues ξ_n $(Im\,\xi_n > 0)$, with multiplicities m_n, $n = 1, 2, ..., N$. Here

$$\Omega_0(x,t) = -2i,$$

$$\Omega_j(x,t) = -2D_x^{-1}\sum_{k=0}^{j-1}\left\{\left(\frac{1}{2i}\right)^{k+1} u^* D_x^k(u\Omega_{j-k-1}) + \left(-\frac{1}{2i}\right)^{k+1} uD_x^k(u^*\Omega_{j-k-1})\right\},\ D_x = \frac{\partial}{\partial x},$$

$$j = 1,\, 2,\, \dots,\, m.$$

The function $u_0(x)$ $(-\infty < x < \infty)$ has the following properties:

1) $\int\limits_{-\infty}^{\infty}(1+|x|)|u_0(x)|dx < \infty$;

2) If $t = 0$, then systems (2) and (3) has no spectral singularities and in $Im\xi > 0$ it has exactly N eigenvalues $\xi_1(0), \xi_2(0), ..., \xi_N(0)$ with multiplicities $m_1(0),\, m_2(0), ..., m_N(0)$.

We assume that

$$\frac{1}{(m_n-1-l)!}\int\limits_{-\infty}^{\infty}\tilde{\Psi}_n^{m_n-1}(x,t)\Phi_n^{m_n-1-l}(x,t)dx = A_{m_n-1-l}^{n}(t),\ n = 1, 2, ..., N, \qquad (5)$$

where $A_{m_n-1-l}^{n}(t)$ is a continuous function, $n = 1, 2, ..., N$, $l = 0, 1, ..., m_n - 1$.

We look for complex-valued solution $u(x,t)$ of the problem (1)-(5), that is satisfy the following assumption:

$$\int\limits_{-\infty}^{\infty}\left((1+|x|)|u(x,t)| + \sum_{k=1}^{m}\left|D_x^k u(x,t)\right|\right)dx < \infty. \qquad (6)$$

In this work the method of inverse scattering problem is applied to the integration of the problem (1)-(6).

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BEHAVIOUR OF SOLUTION DEGENERATE ELLIPTIC EQUATIONS

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For linear elliptic and parabolic equations the questions on behavior of solutions near the boundary were studied on the papers of O.A.Oleinik and his followers [1]. For quasilinear equations, similar result were obtained in the T.S.Gadjiev [2]. S.Bonafade [3] and others studied quality properties of solutions for degenerate equations.

We obtained some estimations that are analogies of Saint-Venaut's principle known in theory of elasticity. By means of these estimations we obtained estimations on behavior of solutions and their derivative on bounded domains up to boundary.

In the cilindric domain $Q = \Omega \times (0,T)\,, T > 0$, where $\Omega \subset R^n, n \geq 2$ bounded domain, a generalized solution from the Sobolev space $\dot{W}^{m,1}_{p,\omega}(\Omega)$ of the mixed problem for the equation

$$\frac{\partial u}{\partial t} - \sum_{|\alpha|\leq m} (-1)^{|\alpha|} D^\alpha A_\alpha(x, u, \nabla u, ..., \nabla^m u) = \sum_{|\alpha|\leq m} (-1)^{|\alpha|} D^\alpha F_\alpha(x), \tag{1}$$

$$u/_{t=0} = 0\,, \tag{2}$$

where $D^\alpha = \frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1} \partial x_2^{\alpha_2} ... \partial x_n^{\alpha_n}}$, $|\alpha| = \alpha_1 + \alpha_2 + ... + \alpha_n$, $m \geq 1$ is considered. Also we suppose Dirichlet conditions on boundary satisfying.

Our main goal is to obtain estimations of behavior of the integral of energy $I_\rho = \int_{\Omega_\rho} \omega(x) \left|\nabla^m u\right|^p dxdt$, for small ρ, dependent on Ω_ρ geometry of Ω in the vicinity of the point 0.

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INITIAL-BOUNDARY PROBLEM FOR THE HEAT EQUATION WITH THE NOT-STRONG REGULAR BOUNDARY CONDITIONS

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The report is dedicated to a problem on finding solving of the heat equation in $\Omega = \{(x,t) : 0 < < x < 1,\ 0 < t < T\}$

$$u_t(x,t) = u_{xx}(x,t) - q(x)u(x,t) + f(x,t), \tag{1}$$

satisfying the initial condition:

$$u(x,0) = \varphi(x), 0 \le x \le 1 \tag{2}$$

and the boundary conditions as

$$\begin{cases} a_1 u_x(0,t) + b_1 u_x(1,t) + a_0 u(0,t) + b_0 u(1,t) = 0, \\ c_1 u_x(0,t) + d_1 u_x(1,t) + c_0 u(0,t) + d_0 u(1,t) = 0, \end{cases} \tag{3}$$

where a_k, b_k, c_k, d_k are complex numbers, $k = 1, 2$.

Parabolic type problems with the two-point boundary general form conditions (3) studied earlier in N.I.Ionkin's and E.I.Moiseev's works [1] where the solving of the problems (1)-(3) is constructed in the strong regularity assumption of the conditions (3) by a modified method of variables separation, the solution's uniqueness and the stability to the initial data in various norms are proved.

The question on the eigen and adjoint functions' basis of an ordinary differential operator is still not fully resolved in case when the boundary conditions are regular but not strongly regular. In this case the problem (1)-(3) may not always be solved by the method of variables separation. The modified method of variables separation in solving the problem (1)-(3) for one case of the boundary conditions called nowadays as Samarskiy-Ionkin's conditions was justified in [2].

However, even in the simplest case $q(x) \equiv 0$ the solution's and problem correctness proof's unique way independent on the basis properties of the corresponding ordinary differential operator was not until today.

In this report we suggest a new method of problem solution (1)-(3) for the case when the boundary conditions are regular but not strongly regular. The suggested solution method can be applied for forming both classical and generalized solutions.

Theorem. *The problem solution (1)-(3) in case of regular but not regular conditions when $q(x) = q(1-x)$ can always be equivalently reduced to the sequential solution of two boundary problems with strong regular boundary Sturm conditions.*

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DEVELOPING OF THE ALGORITHMS TO REALIZATION OF THE PROBLEM SCHEDULER

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Let $0 < \varphi_0 \leq 2\pi,\ 0 < \varphi_1,\, \varphi_2 < \varphi_0$, $G = \left\{ z = re^{i\varphi} : 0 \leq r < \infty,\ \ 0 \leq \varphi \leq \varphi_0 \right\}$. We consider the equation

$$2\overline{z}a_1(\varphi)\frac{\partial w}{\partial \overline{z}} + 2za_2(\varphi)\frac{\partial w}{\partial z} + a_3(\varphi)w + \frac{r^{\alpha}b(\varphi)\overline{w}}{|y-k_1x|^{\alpha}} = \frac{f(\varphi)r^{\nu+\alpha}}{|y-k_2x|^{\alpha}} \tag{1}$$

in G, where $a_1(\varphi),\ a_2(\varphi),\ a_3(\varphi),\ b(\varphi),\ f(\varphi) \in C[0,\varphi_0]$; $k_1 = \tan\varphi_1$, $k_2 = \tan\varphi_2$, $0 < \alpha < 1$, $\nu > 0$ are real numbers. Let $p > 1$ if $\nu \geq 1$ and $1 < p < \frac{1}{1-\nu}$ if $\nu < 1$. The solutions of equation (1) we will find in the class

$$W_p^1(G) \cap C(G). \tag{2}$$

Here $W_p^1\,(G)$ is the Sobolev space. For $\alpha = 0$ and $a_2(\varphi) \equiv 0$ such equations are studied in the papers [1] to [5]. Our goal is to study equation (1) for $\alpha \neq 0$. We proved the following theorem.

Theorem 1. *If $a_1(\varphi) \neq a_2(\varphi)$ on the interval $[0,\varphi_0]$, then the equation (1) has a variety of solutions from the class (2).*

We consider the following Dirichlet and Neuman boundary value problems.

Problem D. *Find a solution of equation (1) from the class (2) satisfying the half Dirichlet condition*

$$w(r,0) = \beta_1 r^{\nu}, \tag{3}$$

where β_1 is a given real number.

Problem N. *Find a solution of the equation (1) from the class (2), satisfying the half Neumann condition*

$$\frac{\partial w}{\partial \varphi}(r,0) = \beta_2 r^{\nu}, \tag{4}$$

where β_2 is a given real number.

Let $\delta = |A_\nu(0)|^2 - |b_\alpha(0)|^2$, $\quad A_\nu(\varphi) = -\frac{i(\nu a_1(\varphi)) + \nu a_2(\varphi) + a_3(\varphi))}{a_1(\varphi) - a_2(\varphi)}, b_\alpha(\varphi) = \frac{b(\varphi)}{|sin\varphi - k_1 cos\varphi|^{\alpha}(\alpha(\varphi) - a_2(\varphi))}$, $a_1(\varphi) \neq a_2(\varphi)$

We proved the following theorems.

Theorem 2. *The half Dirichlet problem D has a unique solution from class (2).*

Theorem 3. *If $\delta \neq 0$, then the half Neumann problem N has a unique solution from (2) and if $\delta = 0$, then the half Neumann problem N has an infinite number of solutions from (2) having the form $w = r^{\nu}\psi(\varphi)$.*

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INVERSE PROBLEM OF THE KIND OF TIKHONOV-LAVRENTEV INCLUDING THE CAUCHY-RIEMANN EQUATION ON A BOUNDED REGION

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In this paper we consider an inverse problem which contains the Cauchy-Riemann equation with two non-local boundary conditions.

Besides the unknown function, the right-hand side of the second boundary condition is unknown. For this, at first by fundamental solution of the Cauchy-Riemann equation, necessary conditions are obtained and then some sufficient conditions to reduce this problem to a second kind Fredholm integral equations are presented. Finally singularities in the kernel of integrals are regularized.

At first, we consider the following inverse problem

$$\frac{\partial u(x)}{\partial x_2} + i\,\frac{\partial u(x)}{\partial x_1} = 0, \qquad x \in \Omega, \tag{1}$$

$$\alpha_j(x_1)\,u(x_1,\gamma_1(x_1)) + \beta_j(x_1)\,u(x_1,\gamma_2(x_1)) = \phi_j(x_1), \quad j=1,2,\ x_1 \in (a,b), \tag{2}$$

where $\Omega = \{\, (x_1,x_2) \in \mathbb{R}^2 |\ x_1 \in (a,b),\ x_2 \in (\gamma_1(x_1),\gamma_2(x_1) \,\}$ is a bounded region in $\mathbb{R}^2$ with the known smooth boundaries γ_1,γ_2 and the $\alpha_j,\ \beta_j\ (j=1,2),\ \phi_1$ and $\phi_2,\ u$ are known continuous and unknown functions, respectively.

By finding the necessary conditions of equation (1) and inserting them in a linear expansions of boundary values of the unknown functions $u(x_1,x_2)$ and considering the relations (2), the unknown function ϕ_2 appears in the form of an integral equation that by solving this integral equation with respect to ϕ_2 and inserting it in the other relation, we can calculate the boundary values of unknown function $u(x_1,x_2)$. Finally by inserting them in the relation, the analytic solution of inverse problem (1)-(2) are obtained.

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INVESTIGATION OF BOUNDARY LAYERS IN SINGULAR PERTURBATION PROBLEMS WITH GENERAL LINEAR NON-LOCAL BOUNDARY CONDITIONS

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Singular perturbation problems have been studied by many mathematicians. Because of the approximate solutions of these problems are as the sum of internal solution (boundary layer area) and external ones, therefore the formation or non-formation of boundary layers should be specified.In this paper,we investigated this subject for a singular perturbation problem inc luding a first order differential equation with general non-local boundary condition. It is necessary to say that for local boundary conditions it is simple and there is no difficulty.However, for non-local case the formation of boundary layers is not as stright forward as local case. To tackle this problem we make use of generalized solution of differential equation and some necessary conditions. At the end, some unsolved problems were presented for second and fourth order differential equations and matrix form of first order differential equations with general linear non-local boundary conditions.

One of the important subjects in applied mathematics is the theory of singular perturbation problems.The mathematical models for this kind of problems usually are in the form of either ordinary differential equation (O.D.E) or partial differential equation(P.D.E) in which the highest derivative is multiplied by some powers of " as a positive small parameter. The object theory of singular perturbation problems is to solve differential equation with some initial or boundary conditions with small parameter ". These problems are divided into 2 types i.e regular and singular cases.

We consider the following perturbation problem,

$$lx \equiv \varepsilon\dot{x}(t) + x(t) = t, \ t \in (0,1), \tag{1}$$

$$\alpha_0 x(0) + \alpha_1 x(1) = \alpha. \tag{2}$$

Where ε is the small parameter, and α_0, α_1 and α are real constants. $x(t)$ is real unknown function on [0,1].

Theorem 1. *In singular perturbation problem* (1)-(2), *for any arbitrary values of* α_0, α_1 and α *there is no boundary layer at the point* $t = 1$; *provided* α_0, α_1 *are not zero together.*

Theorem 2. U*nder conditions and hypothesis of theorem 1, there is no boundary layer at the point* $t = 0$, *provided* $\alpha = \alpha_1$.

Theorem 3. *Underconditions and hypothesis of theorem 1, there is a boundary layer at the point* $t = 0$, *provided* $\alpha \neq \alpha_1$.

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A HOCHSTADT-LIEBERMAN THEOREM FOR SINGULAR STURM-LIOUVILLE OPERATOR

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In this paper we shall be concerned with an inverse problem for singular Sturm-Liouville equation. We shall consider the equation

$$L_i y \equiv -y'' + \left[\frac{A}{x} + q_i(x)\right] y = \lambda y \quad [(i = 1, 2), \ (0 < x \leq \pi)] \tag{1}$$

subject to the boundary conditions

$$y(0) = 0, \quad y'(\pi, \lambda) - Hy(\pi, \lambda) = 0. \tag{2}$$

As is well known, the operator L_i subject to 2 has a discete spectrum consisting of simple eigenvalues $\{\lambda_n\}$. If second condition in (2) is replaced by

$$y'(\pi, \lambda) - H_1 y(\pi, \lambda) = 0, \tag{3}$$

we obtain a second spectrum $\{\lambda'_n\}$. It is shown that here that if $q_i(r)$ is prescribed over the interval $\left(\frac{\pi}{2}, \pi\right)$, then a single spectrum suffices to determine $q_i(r)$ on the interval $\left(0, \frac{\pi}{2}\right)$. Consequently $q_1(r) = = q_2(r)$ almost everwhere on $(0, \pi)$ [1], [2].

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TRAVELING WAVE SOLUTIONS OF THE RLW AND BOUSSINESQ EQUATION

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In this study, we use the generalized tanh function method for the traveling wave solutions of the generalized regularized long-wave (gRLW) equation and Boussinesq equation system.

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ON A ABOUT CLASSICAL SOLVABILITY OF SEVERAL PROBLEMS OF THE MAGNETIC HYDRODYNAMICS

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In [1] 3 nonlinear initial-boundary problems are studied in the limited cylinder domain $Q_T = \Omega \times [0,\ T]$ for the following time-dependent combined equations of the magnetic hydrodynamics

$$\overrightarrow{v}_t - \nu\Delta\overrightarrow{v} + \sum_{k=1}^{3}\left(v_k\overrightarrow{v}_{x_k} - \mu H_k\overrightarrow{H}_{x_k}\right) + \nabla\left(p + \frac{\mu\overrightarrow{H}^2}{2}\right) = f\left(\overrightarrow{x}, t\right), \tag{1}$$

$$rot\overrightarrow{E} = -\mu\overrightarrow{H}_t, \tag{2}$$

$$rot\overrightarrow{H} - \sigma\left(\overrightarrow{E} + \mu\left[\overrightarrow{v} \times \overrightarrow{H}\right]\right) = \overrightarrow{j}\,(x,\ t), \tag{3}$$

$$div\overrightarrow{v}\,(x,\ t) = 0, \quad div\overrightarrow{H}\,(x,\ t) = 0, \tag{4}$$

in which $\overrightarrow{v}\,(x,\ t)$ - vector of the flow fluid; $\overrightarrow{H}\,(x,\ t)$, $\overrightarrow{E}\,(x,\ t)$- vectors of the intensity of the magnetic and electrical field; $\overrightarrow{f}\,(x,\ t)$, $\overrightarrow{j}\,(x,\ t)$ – given outside fore of flow; μ - magnetic penetrability, σ – conductivity; ν – kinematic viscosity of fluid; the liquid density is equal to 1.

First problem: we need to find $\overrightarrow{v}\,(x,\ t)$, $\overrightarrow{H}\,(x,\ t)$, $\overrightarrow{E}\,(x,\ t)$ from (1)-(4) and p in a limited cylinder region $Q_T = \Omega \times [0,\ T] \quad \left(\Omega \in R^3,\ \ t \in [0,\ T]\right)$ under the following conditions:

$$\overrightarrow{v}\,(x,\ t)\big|_{t=0} = \overrightarrow{v}_0\,(x)\,,\ \ \overrightarrow{H}\,(x,\ t)\Big|_{t=0} = \overrightarrow{H}_0\,(x) \quad \overrightarrow{v}\,(x,\ t)\big|_{x\in S} = \overrightarrow{a}\,(x,\ t)\,,\ \ \overrightarrow{H}_n\Big|_S = 0,\ \ \overrightarrow{E}_\tau\Big|_S = 0.$$

The second problem: vessel Ω_1 with liquid is covered by dielectric or vacuum, what takes limited domain Ω_2, outer limit S_2 which is an ideal conductor. Magnetic penetrability of the liquid μ_1, dielectric μ_2, we must find $\overrightarrow{v}\,(x,\ t)$, $\overrightarrow{H}\,(x,\ t)$, $\overrightarrow{E}\,(x,\ t)$ and p from the system (1)-(4) in the domain Ω_1, from the following combined equations

$$rot\overrightarrow{H} = 0,\ div\overrightarrow{H} = 0, \quad rot\overrightarrow{E} + \mu\overrightarrow{H}_t = 0,\ div\overrightarrow{E} = 0.$$

In the domain Ω_2 by following boundary conditions

$$\overrightarrow{v}\,(x,\ t)\big|_{x\in S} = \overrightarrow{a}\,(x,\ t)\big|_{x\in S_1},\ \ \left(a_n\big|_{S_1} = 0\right),$$

$$\overrightarrow{H}^{(1)}_\tau\Big|_{x\in S_1} = \overrightarrow{H}^{(2)}_\tau\Big|_{x\in S_1},\ \ \mu_1 H^{(1)}_n\Big|_{x\in S_1} = \mu_2 H^{(2)}_n\Big|_{x\in S_1}$$

on the surface $S_1 \equiv \partial\Omega_1$ and on the surface

$$S_2 \equiv \partial\Omega_2, \quad H_n\big|_{x\in S_2} = 0,\ \ \overrightarrow{E}_r\Big|_{x\in S_2} = 0$$

and also by the conditions in the region Ω_1.

The results of this work are devoted to the expansion of the results O. A. Ladizhinskaya and V. A. Solonnikov for the functional space norms $W_p^{2,1}\,(Q_T)$ with $p > 1 \quad C^{2+\alpha,\ 1+\alpha/2}\,(Q_T)$, $\alpha < 1$.

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INVERSE PROBLEMS FOR PARABOLIC EQUATIONS ON UNLIMITED TIME INTERVALS[1]

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Let consider heat conduction equation in the following form

$$u_t = u_{xx} + f(x,t),\ \ 0 < x < L,\ 0 < t < T. \tag{1}$$

$$u|_{t=0} = \varphi(x). \tag{2}$$

$$u_x|_{x=0} = b(t). \tag{3}$$

$$u_x|_{x=0} = y(t). \tag{4}$$

The right boundary restriction plays a role of parameter. It's also given that $u(0,t) = a(t)$. On the problem (1) – (4) we need to minimize the functional:

$$I(y) = \int_0^T \Big[u(0,t;y) - a(t)\Big]^2 dt \to \min.$$

We use gradient method to reach the goal. The minimizing sequence is built by recurrence relation:

$$y_{n+1}(t) = y_n(t) - \alpha_n I(y_n(t)),\ \alpha_n > 0.$$

The first challenge is to find the exact expression for gradient of functional, $I(y(t))$. It can be determined by adjoint problem, which is stated below:

$$\begin{aligned} \psi_t + \psi_{xx} &= 0, \\ \psi(x,T) &= 0, \\ \psi_x(L,t) &= 0, \\ \psi_x(0,t) &= -2\big(u(0,t;y) - a(t)\big). \end{aligned}$$

The exact expression for gradient is $I'(y(t)) = \psi(L,t)$. Having in hand the value of gradient, the iteration process can be launched. The values of α_n–coefficients are calculated according to steepest descent method.

If the algorithm is launched you will have several problems. The first, algorithm doesn't make the values of subsequent approximations more precise. The second, this problem is ill-conditioned problem, that is, if you have an oscillating influence on the right boundary, it doesn't seem to be feasible to restore the values of $y(t)$. These obstacles should be considered first.

The series of numerical experiments with known solutions were implemented. The information was collected and analyzed. The corresponding inferences were made about the solution on unlimited time interval, about nonlinear cases of problem, about the algorithm efficiency with different values of main parameters.

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A METHOD FOR THE SOLUTIONS OF THE MIXED PROBLEM FOR THE HEAT EQUATION

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Let $\Omega \subset R^2$ be a square, bounded by segments: $AB : 0 \leq t \leq 1, x = 0$; $BC : 0 \leq x \leq 1, t = 1$; $CD : 0 \leq t \leq 1, x = 1$; $DA : 0 \leq x \leq 1, t = 0$. The set of functions $u(x,t)$ that are twice continuously differentiable with respect to x and once continuously differentiable with respect to t in the domain Ω is denoted by $C^{2,1}(\Omega)$. The boundary of the region Ω means a series of segments $\partial\Omega = AB \bigcup AD \bigcup CD$.

Let's consider in the Hilbert space $L^2(\Omega)$ the mixed problem for the heat equation:

$$Tu = u_t(x,t) - u_{xx}(x,t) = f(x,t), (x,t) \in \Omega \tag{1}$$

$$u|_{t=0} = 0; \tag{2}$$

$$u|_{x=0} = 0, u|_{x=1} = 0, \tag{3}$$

where $f(x,t) \in L^2(\Omega)$. This boundary-value problem corresponds to the linear operator

$$Tu = u_t(x,t) - u_{xx}(x,t) \tag{4}$$

with domain

$$D(T) = \left\{ u(x,t) \in C^{2,1}(\Omega) \bigcap C\left(\overline{\Omega}\right),\ u|_{t=0} = 0,\ u|_{x=0} = 0,\ u|_{x=1} = 0 \right\} \tag{5}$$

and range $R(T) \subset L^2(\Omega)$. It is known that the inverse operator T^{-1} exists and is Volterra type, i.e. it is a compact operator with non-zero eigenvalues. If $T^{-1}u = 0$, then applying the operator T on both sides of this equation, we obtain $u = 0$, i.e. the point $\lambda = 0$ is not an eigenvalue of the operator T^{-1}. The function $u \in C^{2,1}(\Omega) \bigcap C\left(\overline{\Omega}\right)$, identifying equation (1) and boundary conditions (2) - (3) is called *a regular solution* of the problem (1)-(3). Function $u \in L^2(\Omega)$ is called a *strong solution* of the problem, if there is a sequence of functions $u_n(x,t) \in C^{2,1}(\Omega) \bigcap C\left(\overline{\Omega}\right)$, $n = 1, 2, ...$ and satisfying the boundary conditions such that $\{u_n\}$ and $\{Tu_n\}$, $n = 1, 2, ...$ converges in m$L^2(\Omega)$, respectively, to $u(x,t)$ and $f(x,t)$ by $n \to \infty$. Boundary-value problem (1)-(3) is called *strongly solvable* if there exists a strong solution for any right hand side $f(x,t) \in L^2(\Omega)$ and is unique.

Theorem. (a) *The mixed problem* (1) - (3) *for the heat equation is strongly solvable in the space* $L^2(\Omega)$; (b) *The inverse operator* $\left(\overline{T}\right)^{-1}$ *is completely continuous on this space and is Volterra type;* (c) *There is a "spectral" decomposition*

$$u(x,t) = T^{-1}f = \sum_{n,m=1}^{\infty} \frac{(Sf, \upsilon_{nm})}{\lambda_{nm}} \upsilon_{nm}, \tag{6}$$

where

$$S(x,t) = f(x, 1-t), \frac{\partial}{\partial t}\upsilon_{nm}(x,t) - \frac{\partial^2}{\partial x^2}\upsilon_{nm}(x,t) = \lambda_{nm}(x, 1-t), \tag{7}$$

$$\upsilon_{nm}|_{t=0} = 0, \upsilon_{nm}|_{x=0} = 0, \upsilon_{nm}|_{x=1} = 0, \upsilon_{nm}(x,t) = \sqrt{2}\sin n\pi x \cdot \frac{B_{nm}\sin\nu_{nm}t}{\nu_{nm}}, n, m = 1, 2, ..., \tag{8}$$

- roots of equation

$$\cos\nu_{nm} + \frac{n^2 \sin\nu_{nm}}{\nu_{nm}} = 0,\ n, m = 1, 2, ... \tag{9}$$

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ON AN INVERSE CAUCHY PROBLEM

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Let's consider the Cauchy problem in space $H = L^2(0,1)$

$$y'(x) = f(x), x \in [0,1] \tag{1}$$

$$y(0) = 0, \tag{2}$$

where $f(x) \in L^2(0,1)$. As is known, the solution of this problem has the following form:

$$y(x) = \int_0^x f(t)dt, x \in [0,1]. \tag{3}$$

Let's assume, that the left hand side of (3) is known, whether it is possible to find the functions $f(x) \in L^2(0,1)$, i.e. to find the solution of the integral equation

$$\int_0^x f(t)dt = y(x). \tag{4}$$

It seems, that is easier to differentiate the left part of this formula then under the theorem of Lebesgue [1] we will get $f(x) = y'(x)$. But a differentiation operator an unbounded in space $L^2(0,1)$, therefore this operation, generally speaking, cannot be applied, if a right hand side of the equation (4) is known only approximately, that often happens in practice. Hence, the problem is not always solvable and is among, so-called, incorrect problems [2]. The problem essence is reduced to search of result algorithm. In the present work the algorithm based on ideas of the work [3] is offered. The basic result is

Theorem. a) *If λ is any real number not equal to zero, solution of the equation:*

$$(SC^{-1} - ial)f_\alpha = Sy \tag{5}$$

exists for any $y \in L^2(0,1)$ and has a form:

$$f_\alpha(x) = \sum\nolimits_{n=0}^{\infty} \frac{(S_y, u_n)}{\mu_n + i\alpha} u_n(x), \tag{6}$$

where

$$C^{-1}f(x) = \int_0^x f(x)dt = y(x), Sy(x) = y(1-x), \tag{7}$$

$$\mu_n = \frac{\cos n\pi}{n\pi + \frac{\pi}{2}}, u_n(x) = \sqrt{2}\cos(n\pi + \frac{\pi}{2})x, n = 0, 1, ... \tag{8}$$

b) *For any $y \in R(C^{-1})$ the estimation holds*

$$\left\|C^{-1}f_\alpha - y\right\| \leq |\alpha| \left\|f\right\|, \tag{9}$$

where $y = C^{-1}f$, which shows a velocity of approach of elements $C^{-1}f_\alpha$ to y at $\alpha \to 0$.

c) *If $y \in R(C^{-1})$ and $\alpha \to 0$, then $\|f_\alpha - f\|$ tends to zero.*

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AN INVERSE PROBLEM FOR THE HEAT EQUATION

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It is known that the mixed problem

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \, (0 \le x \le \infty, t \ge 0) \tag{1}$$

$$u\,|_{x=0} = \varphi(t), u\,|_{t=0} = 0 \tag{2}$$

for the heat equation has the form

$$u(x,t) = \frac{x}{2\sqrt{\pi}} \int\limits_0^t \frac{\varphi(\tau)\, e^{-\frac{x^2}{4(t-\tau)}}}{(t-\tau)^{\frac{3}{2}}} d\tau \tag{3}$$

where $\varphi(t)$ - continuous and bounded on the semiaxis $t \ge 0$, satisfying $\varphi(0) = 0$. The question, is whether one can find this function $\varphi(t)$, if the function $u(x,t)$ is known ie solve the inverse problem [1], [2]. The main result of this work is

Theorem. *If the solution $u(x,t)$ of the mixed problem (1)-(2) satisfies to the following two conditions*

$$b) \lim_{t \to 0} \int_0^{-\infty} u(x,t) dx = 0, \tag{4}$$

$$b) \frac{1}{\sqrt{\pi}} \int_0^t \frac{\int_0^{+\infty} u(x,t)\, dx}{\sqrt{t-\tau}} d\tau \tag{5}$$

this function is absolutely continuous on each finite segment $[0,T]$, then solution $\varphi(t)$ of inverse problem exists and has the form

$$\varphi(t) = \frac{1}{\sqrt{\pi}} \frac{d}{dt} \int_0^t \frac{\int_0^{+\infty} u(x,t)\, dx}{\sqrt{t-\tau}} d\tau, \tag{6}$$

where $0 \le t \le T$, and $T > 0$ - arbitrary positive value.

Thus, the problem reduces to the calculation of the derivative or to the integration of the functions $u(x,t)$ with respect to the variable x [2].

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AN INVERSE BOUNDARY VALUE PROBLEM FOR A PSEUDOPARABOLIC EQUATION OF THIRD ORDER WITH INTEGRAL OVERDETERMINATION CONDITION

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For the equation

$$u_t(x,t) - bu_{txx}(x,t) - a(t)u_{xx}(x,t) = p(t)u(x,t) + f(x,t) \tag{1}$$

in the domain $D_T = \{(x,t) : 0 \le x \le 1,\ 0 \le t \le T\}$ consider an inverse boundary value problem under the initial condition:

$$u(x,0) = \varphi(x) \quad (0 \le x \le 1), \tag{2}$$

the boundary conditions

$$u(0,t) = u(1,t),\ u_x(1,t) = 0\ \ (0 \le t \le T), \tag{3}$$

and the overdetermination condition

$$\int_0^1 u(x,t)dx = h(t) \quad (0 \le t \le T), \tag{4}$$

where $b > 0$ the given number, $a(t) > 0, f(x,t), \varphi(x), h(t)$ are the given functions, $u(x,t)$ and $p(t)$ are the desired functions.

Definition. *We call the pair $\{u(x,t), p(t)\}$ of the functions $u(x,t)$* and *$p(t)$ possessing the following properties the classic solution of the inverse boundary value problem* (1)-(4).

(1) the function $u(x,t)$ is continuous in D_T together with all its derivatives contained in equation (1):
(2) the function $p(t)$ is continuous on $[0,T]$;
(3) all the conditions (1)-(4) are satisfied in the ordinary sense.

In paper at first the given problem is reduced to the equivalent problem (in definite sense) and a theorem on the existence and uniqueness of the solution is proved for it. Further, using this fact, the existence and uniqueness of the classic solution of the input problem is proved.

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EXISTENCE AND UNIQUENESS THEOREM FOR THE SOLUTION OF THE IMPULSIVE DIFFERENTIAL EQUATIONS WITH NON-LOCAL CONDITIONS

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In this report our purpose is to study the existence and uniqueness of solutions for the following impulsive differential equation with non-local conditions,

$$\begin{cases} x'(t) = f(t, x(t)), 0 \le t \le T, t \ne t_i, \\ Ax(0) + Bx(T) = C, \\ \Delta x(t_i) = I_i(x(t_i)), i = 1, 2, ..., p; 0 < t_1 < t_2 < ... < t_p < T \end{cases} \tag{1}$$

in R^n, where, f is a continuous function; $A, B \in R^{n \times n}; C \in R^n, \Delta x(t_i) = x(t_i^+) - x(t_i^-)$, and L_i are some functions.

Definition. *A solution of the equation (1) is a function*

$$x(.) \in PC([0, T], R^n) \cap C^1([0, T] \setminus \{t_1, t_2, ..., t_p\}, R^n)$$

which satisfies (1) on $[0, T]$.

Theorem 1. *A function* $x \in PC([0, T], R^n)$ *is a solution of the equation (1) if and only if*

$$x(t) = (A + B)^{-1} C + \int_0^T K(t, \tau) f(\tau, x(\tau)) d\tau + \sum_{i=1}^{P} M(t, t_i) I_i(x(t_i)), \tag{2}$$

$$K(t, \tau) = \begin{cases} (A + B)^{-1} A, 0 \le \tau \le t \\ -(A + B)^{-1} B, 0 \le \tau \le T \end{cases},$$

$$M(t, t_i) = \begin{cases} (A + B)^{-1} A, 0 \le t_i \le t \\ -(A + B)^{-1} B, 0 \le t_i \le T \end{cases}$$

We list out the following hypotheses:

$(H1)$ $\det(A + B) \ne 0$

$(H2)$ $f : [0, T] \times R^n \to R^n, I_i : R^n \to R^n, i = 1, 2, ..., p$ *are continuous and there exists constants* $K > 0, L_i > 0, i = 1, 2, ..., p$, *such that.*

$$|f(t, u) - f(t, v)| \le K |u - v|, t \in [0, T], u, v \in R^n$$

$$|I_i(u) - I_i(v)| \le L_i |u - v|, u, v \notin R^n.$$

$(H3)$ $L = \max\left\{ \left\| (A + B)^{-1} A \right\|, \left\| (A + B)^{-1} B \right\| \right\} \left[KT + \sum_{i=1}^{P} L_i \right] < 1.$

Theorem 2. *Let assumptions (H1)-(H3) are satisfied. Then for every* $C \in R^n$ *a equation (1) has a unique solution on* $[0, T]$, *satisfying (2).*

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PROBLEMS OF INTERFACE FOR THE LINEAR PSEUDO-PARABOLIC EQUATIONS OF THE THIRD ORDER

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In the domain D, bounded by lines $x=0$, $y=-h_1$, $x=\ell$, $y=h$, $x=-\ell_1$, $y=0\,(h,\, h_1,\, \ell_1>0)$, it is considered the solution of the boundary value problem for the linear pseudo-parabolic equations of the third order

$$u_{xxx}-u_{xy}+a_1u_x+d_1u=0,\quad (x,y)\in D_1, \tag{1}$$

$$u_{xxy}+a_2u_{xx}+b_2u_{xy}+c_2u_x+d_2u_y+e_2u=0,\quad (x,y)\in D_2, \tag{2}$$

$$u_{xyy}+a_3u_{xy}+b_3u_{yy}+c_3u_x+d_3u_y+e_3u=0,\quad (x,y)\in D_3, \tag{3}$$

where $a_i,\, d_i,\, b_j,\, c_j,\, e_j\,(i=\overline{1,3},\; j=2,3)$ - functions , in $\mathrm{D_1=D\cap(x>0,y>0)}$, $\mathrm{D_2=D\cap(x>0,y<0)}$, $\mathrm{D_3=D\cap(x<0,y>0)}$.

The equations (1) - (3) is represented equations of the third order on classification of work [1]. Such equations often are called as pseudo-parabolic on character of properties of slutions [2, 3]. Special cases of the considered equations meet at studying of absorption of a soil moisture by plants [4].

Let C^{n+m} be a class of the functions having derivatives $\partial^{r+s}/\partial\, x^r \partial\, y^s\,(r=0,1,...,n;\; s=0,1,...,m)$. It is considered the problem.

Problem. *To find function $u(x,y)\in C(\bar{D})\cap[C^{1+1}(D_1)\cup C^{2+1}(D_2)\cup C^{1+2}(D_3)]$, $u_{xxx}\in C(D_1)$, satisfying the equations (1), (2) and (3) in the domains D_1, D_2 and D_3 respectively, to initial value conditions*

$$u(-\ell_1,y)=\varphi_1(y),\;\; u(\ell,y)=\varphi_2(y),\;\; 0\le y\le h,$$

$$u(0,y)=\chi_1(y),\;\; u_x(0,y)=\chi_2(y),\;\; -h_1\le y\le 0,$$

$$u(x,0)=\psi_1(x),\;\; u_y(x,0)=\psi_2(x),\;\; -\ell_1\le x\le 0$$

$$u(x,-0)=u(x,+0),\;\; u_y(x,-0)=u_y(x,+0),\;\; 0\le x\le \ell,$$

$$u(-0,y)=u(+0,y),\;\; u_x(-0,y)=u_x(+0,y),\;\; 0\le y\le h,$$

where $\varphi_i(y),\; \chi_i(y),\; \psi_i(x)\,(i=1,2)$ -- are smooth functions.

By the method of Rieman function of, is proved the existence of uniqueness of the solution of this problem.

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FAITHFUL TRANSFER OF SOMMERFELD'S RADIATION CONDITIONS TO AN BOUNDARY OF A DOMAIN[1]

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Consider the following problem

$$\Delta u + q^2(x)u = f,\ x \in R^d, d > 1, \tag{1}$$

$$\lim_{r \longrightarrow \infty} r^{(d-1)/2}\left(\frac{\partial u}{\partial r} + iku\right) = 0, \tag{2}$$

where $f, q \in C^1(R^n)$, $suppf, supp(q^2 - k^2) \subseteq \overline{\Omega}$, $\Omega \subset R^d$ is a bounded domain with sufficiently smooth boundary $\partial\Omega$, r is a radial coordinate and k is a wave number.

Theorem 1. *Exists an unique classical solution of the problem* (1)-(2) *and satisfies the following boundary condition*

$$-\frac{u(x)}{2} + \int_{\partial\Omega} \frac{\partial \varepsilon_n(x-y,\lambda)}{\partial n_y} u(y)dS_y - \int_{\partial\Omega} \varepsilon_n(x-y,\lambda)\frac{\partial u(y)}{\partial n_y} dS_y = 0, x \in \partial\Omega. \tag{3}$$

Conversely, if a function $u \in C^2(\Omega) \cap C^1(\overline{\Omega})$ satisfies the equation (1) and the boundary condition (3) then it coincides with the solution of the problem (1)-(2) in $\overline{\Omega}$.

Where $\varepsilon_n(x)$ is a fundamental solution of the Helmholtz operator $(\Delta + k^2), k \equiv const$, which satisfies the Sommerfeld's radiation condition (2).

Example 1. *Let be* $q^2 \equiv k^2$ *then the boundary value problem* (1),(3) *has the following unique solution*

$$u = \int_{\Omega} \varepsilon_n(x-y,\lambda) f(y)dy, x \in \Omega. \tag{4}$$

On the other hand, since $q^2 \equiv k^2$, it is known that the volume potential (4) is an unique solution of the problem (1)-(2) in R^d. Here we can verify that the solution of (1),(3) coincides with the solution of (1)-(2) in Ω.

It should be noted that Theorem 1 gives complete answer to Bezmenov's problem [see 1] for transfer of Sommerfeld's radiation condition to a boundary of a bounded domain and our method can be used more general elliptic problems and other type of PDE problems. In addition, obtained Theorems have many applications in numerical theory [see 2] and in spectral theory of volume potentials [see 3 and 4].

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DEFINITION OF THE SPEED AND DOMINANT FREQUENCY AHEAD SEISMIC WAVES IN THE MONODISPERSIVE SYSPENSIYA

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The proposed model describes the process of nonlinear wave propagation in the two-phase medium (One of these phases is solid and other is liquid) considering thermodynamic powers. The system of equations which describes the low of conserve of the every phase is complained by the reology equation of solid phase and thermodynamic relations. The problem of one-dimensional dynamics has been solver bu the method of small parameter. The dispersion type relations and evolutionary equation with the Kortevege de-Frise and diffusion type nonlinearities have been obtained. The interval of changes of the dominant frequencies in the layers saturated by water and gaz. There fore the essence of the theory of waves is contained in the construction of the transport equations and in explanation of secondary effects connected with the change in the change in the forms and amplitudes of the waves in medium with complicated properties. In this work the next modified partial differential equation in high order with nonlinearity Kortevege de-Frise, which form depends on dissipation, dispersion and interphase dissipation of resistance:

$$\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial T} - R_2 v\left(1 + \frac{b}{|R_1|}\left|1 - \frac{\alpha_1^{(0)}\rho_1^{(0)} - \frac{a_0}{b_0 c^2}}{\rho_1^{(0)}(\alpha_1^{(0)} - \gamma)}\right| |\overrightarrow{v}|\right) + R_3\sum_{l=1}^{n}(-1)^{l+1}A_{l+1}\frac{\partial^{l+1} v}{\partial T^{l+1}} = 0,$$

where

$$A_{l+1} = \Gamma_{m-l}\frac{a_0 b_l}{b_0} - \Gamma_{n-l}a_l, \Gamma_{n-e} = 1, n \geq l, \Gamma_{n-l} = 0, n < l.$$

On the basis of the suggested model there are discovered new effects holding at analysis and regulation of development of oil-and-gas deposits as well as for forecasting consequences of accepted technological measures on influence upon stratum and stimulation of oil production.

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ONE- DIMENSIONAL MOTION OF A COMPESSIBLE MEDIUM IN PIPES OF CONSTANT CROSS SECTION

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In technology and industry processes are often found, contact the unsteady flows in liquids and gazes in cylindrical tubes. When considering the corresponding problems in one-dimensional formulation for the case of compressible continuous medium the closed-loop system of equation of hidrodynamics includes the equation of state:

$$P = Z\rho gRT \tag{1}$$

the first low of thermodynamics in the form of the energy equation

$$dQ = du + Pd(\frac{1}{\rho}) \tag{2}$$

continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho w)}{\partial x} = 0 \tag{3}$$

and the equation of quantity of movement

$$\frac{\partial(\rho w)}{\partial t} = -\frac{\partial P}{\partial x} + \rho F_S + \rho X, \tag{4}$$

where P-pressure, Z-function introduced to account for real gas properties , ρ-density, g-acceleration due to gravity, R- gas constant, T- gas temperature , w- averaging over the cross rate of gas flow, X-mass force. The friction in turbulent flow is proportional to the square of velocity we take care,

$$F_S = -\frac{\lambda}{2d} w|w|, \tag{5}$$

where, d- diameter of the pipe, λ- coefficient of resistance. We consider the isothermal case, which is characteristic for piping lines, which have reached thermal equilibrium with the environment.

Combining equation (1)-(4), we obtain the following equation

$$C^2 \frac{\partial^2 P}{\partial x^2} = 2a\frac{\partial P}{\partial t} + \frac{\partial^2 P}{\partial t^2} \tag{6}$$

This hyperbolic equation is solved by the initial

$$P(x,0) = \varphi_1(x) \ \ and \ \ \frac{\partial P(x,0)}{\partial t} = \varphi_2(x), \tag{7}$$

and boundary conditions:

$$P(0,t) = P1(t), \ \ P(l,t) = P_2(t). \tag{8}$$

To solve the problem (6)-(8) use the finite difference method or the net method. The influence of physical and mechanical parameters on the wave characteristics.

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ON A NUMERICAL METHOD FOR THE LINEAR FREDHOLM PARTIAL INTEGRO-DIFFERENTIAL EQUATIONS

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In this paper, a method based on the matrix Tau method with standard base is developed to obtain the numerical solution of two dimensional linear Fredholm integro-differential equations of the form

$$a_1\frac{\partial^2\phi(x,t)}{\partial x^2}+a_2\frac{\partial^2\phi(x,t)}{\partial x\partial t}+a_3\frac{\partial^2\phi(x,t)}{\partial t^2}+a_4\frac{\partial\phi(x,t)}{\partial x}+a_5\frac{\partial\phi(x,t)}{\partial t}+a_6\phi(x,t)$$
$$-\lambda\int_c^d\int_a^b K(x,t,y,z)\phi(y,z)dydz=f(x,t),\quad x\epsilon[a,b],\quad t\epsilon[c,d].$$

with conditions

$$\sum_{p=1}^{2}\sum_{j=0}^{1}\sum_{k=0}^{1-j}d_{ijk}^{(p)}\frac{\partial^{j+k}\phi(x,t)}{\partial x^k\partial t^j}|_{x=x_p}=\alpha_i(t),\qquad i=1,2$$

and

$$\sum_{k=0}^{1}e_{jk}\frac{\partial^k\phi(x,t)}{\partial t^k}|_{t=t_0}=\beta_j(x),\qquad j=0,1$$

where $K(x,t,y,z)$, $f(x,t)$, $\alpha_i(t)$, $\beta_j(x)$ and $a_1(x,t),\dots,a_6(x,t)$ are continuous functions and λ is a constant.

Here some theoretical results to simplify the application of the Tau method are given. Then by proving some lemmas and theorems we convert TDLFIDE to a system of linear algebraic equations. Finally, some numerical examples are given to demonstrate efficiency and accuracy of the presented method.

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THE CONSTRUCTION OF THE NORMAL-REGULAR SOLUTIONS FOR THE GENERAL SYSTEM OF SECOND ORDER PARTIAL DIFFERENTIAL EQUATIONS

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In this paper, we consider the following general system of second order partial differential equations:

$$\begin{cases} P^{(0)}\cdot Z_{xx}+P^{(1)}\cdot Z_{xy}+P^{(2)}\cdot Z_{yy}+P^{(3)}\cdot Z_x+P^{(4)}\cdot Z_y+P^{(5)}\cdot Z=0, \\ Q^{(0)}\cdot Z_{xx}+Q^{(1)}\cdot Z_{xy}+Q^{(2)}\cdot Z_{yy}+Q^{(3)}\cdot Z_x+Q^{(4)}\cdot Z_y+Q^{(5)}\cdot Z=0, \end{cases} \tag{1}$$

where coefficients $P^{(i)}=P^{(i)}(x,y)$ and $Q^{(i)}=Q^{(i)}(x,y)$ $(i=\overline{0,5})$ are analytical functions or polynomials of two variables; and $Z=Z(x,y)$ is common unknown.

We need to define abilities of construction of normal-regular solutions for

$$Z=\exp Q(x,y)\cdot x^{\rho}\cdot y^{\sigma}\cdot\sum_{\mu,\,\nu=0}^{\infty}A_{\mu,\,\nu}\cdot x^{\mu}\cdot y^{\nu}\qquad(A_{0,0}\neq 0) \tag{2}$$

where $\rho,\sigma,A_{\mu,\,\nu},$ $(\mu,\nu=0,1,2,...)$ are unknown constants. Coefficients $\alpha_{p\,0},\alpha_{0p},...,\alpha_{11},\alpha_{10},\alpha_{01}$ of a polynomial

$$Q(x,y)=\frac{\alpha_{p\,0}}{p}\cdot x^{p}+\frac{\alpha_{0p}}{p}\cdot y^{p}+...+\alpha_{11}\cdot xy+\alpha_{10}\cdot x+\alpha_{01}\cdot y \tag{3}$$

are also unknowns. We find degree of polynomial $Q(x,y)$ using rank $p=1+k$ (here, k is a subrank); rank was introduced by Poincare.

The solution of (2) exists if rank of a system $p\leq 0$ and antirank $m\leq 0$ ($m=-1-\lambda$ (λ − *antisubrank*)). An antirank was introduced by L.Tome. Suppose system (1) is joint [1]. Singular curves of a system are determined by equating to zero coefficients at higher derivatives Z_{xx},Z_{xy} and Z_{yy}, that is $P^{(j)}(x,y)\equiv 0$ and $Q^{(j)}(x,y)\equiv 0$ $(j=0,1,2)$.

For a construction of solutions for (2) the Frobenius-Latysheva method is used [2]. In general, normal-regular solutions are associated with $(0;0)$ singularity.

In this paper:

1. there was performed for the various cases of coefficients' setting the classification of system (1) singular curves. Let us remark that *behavior* of solutions in the neighborhood of points where at least two singular curves are intersected *is still little known.*

2. The first and the second necessary conditions for existence of (2) solutions were established.

3. A theorem on necessary and sufficient condition for existence of normal-regular solutions of (2) is proved.

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OPTIMIZATION METHOD TO THE SOLVING THE GELFAND-LEVITAN EQUATION OF THE FIRST KIND

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Direct problem with a point source is to find a generalized solution $u(x,t)$ of the Cauchy problem

$$u_{tt} = u_{xx} - q(x)u, \quad x \in R,\ t > 0, \tag{1}$$

$$u|_{t=0} = 0,\ ,\ u_t|_{t=0} = \delta(x). \tag{2}$$

In the inverse problem it is required to recover a continuous function $q(x)$ on the following for more information about solving the direct problem (1)-(2)

$$u(0,t) = f(t),\ \ u_x|_{x=0} = 0,\ \ t \geq 0, \tag{3}$$

where f is twice continuously differentiable function and $f(+0) = 1/2$. We extend $u(x,t)$ and $f(t)$ an odd way to negative t: $u(x,t) = -u(x,-t)$, $f(t) = -f(-t)$, where $f(-0) = -1/2$. Function f has a point $t = 0$ rupture of the first kind , and when $t \neq 0$ twice continuously differentiable. Then the extended function $u(x,t)$ satisfies for all $(x,t) \in R^2$ equation

$$u_{tt} = u_{xx} - q(x)u, \tag{4}$$

$$u(x,t) = 0,\ \ t < |x|. \tag{5}$$

Assume, that the inverse problem (1)-(3) exists. Then function $u(x,t)$ satisfies to the additional condition

$$u(0,t) = f(t),\ \ u_x|_{x=0} = 0,\ \ t \geq 0. \tag{6}$$

In [1] it is shown, that a solution (4)-(6) is reduced to solving an integral equation of Gelfan-Levitan of the first kind

$$\frac{1}{2}[f(t+x) + f(t-x)] + \int_{-x}^{x} f(t-s)q(s)ds = 0,\ \ t \in (-x,x). \tag{7}$$

Equation (7) we write the operator form

$$Aq = h, \tag{8}$$

where

$$Aq = \int_{-x}^{x} f(t-s)q(s)ds, \quad h = \frac{1}{2}[f(t+x) + f(t-x)],\ \ t \in (-x,x).$$

For the numerical solution of the equation (8) we apply the optimization method. Consider the entire functional $J(q) = \|Aq - h\|^2$. To minimize it using a simple iteration method $q_{n+1} = q_n - \alpha J'q_n$. We calculate the adjoint operator A^*: $\langle Aq, \varphi\rangle = \langle q, A^*\varphi\rangle$, $\ A^*\varphi = \int_{-x}^{x} f(t-\xi)\varphi(\xi)d\xi$. As

$$J'q(s) = 2A^*(Aq-h)(s) = \int_{-x}^{x} f(\xi - s)[\int_{-x}^{x} f(\xi-\beta)q(\beta)d\beta - h(\xi)]d\xi,$$

the method of simple iteration can be written $q_{n+1}(s) = q_n(s) - 2\alpha \int_{-x}^{x} f(\xi - s)[\int_{-x}^{x} f(\xi-\beta)q(\beta)d\beta - h(\xi)]d\xi$. Parameter $\alpha \in (0, \|A^{-2}\|)$.

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THE EFFECT OF DELAYS ON THE BOUNDEDNESS OF SOLUTIONS OF LINEAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATION OF SECOND ORDER

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All appearing functions and their derivatives are continuous and the relations hold under $t \geq t_0,\ t \geq \tau \geq t_0; |x_j|\ ,\ |y_j| < \infty\ \ (j = 1..n); J = [t_0, \infty)$; IDE - integro-differential equation.

This work is devoted to the problem of establishing sufficient conditions for boundedness of all solutions on the interval J of weakly nonlinear IDE second-order Volterra type with delays:

$$x''(t) + a_1(t)x'(t) + a_0(t)x(t) + \int\limits_{t_0}^{t} K(t,\tau)x'(\tau)d\tau = f(t) + \sum_{j=1}^{m} F_j(t, x(\alpha_j(t)),\ x'(\beta_j(t))), \quad t \geq t_0, \quad (1)$$

where

$$t_0 \leq \alpha_j(t) \leq t, \qquad t_0 \leq \beta_j(t) \leq t,$$

$$|F_j(t, x_j, y_j)| \leq g_{0j}(t)\,|x_j| + g_{1j}(t)\,|y_j| \quad (g_{kj}(t) \geq 0,\ k = 0, 1;\ j = 1..m),$$

when all the solutions corresponding IDE without delays:

$$x''(t) + a_1(t)x'(t) + a_0(t)x(t) + \int\limits_{t_0}^{t} K(t,\tau)x'(\tau)d\tau = f(t) + \sum_{j=1}^{m} F_j(t, x(t),\ x'(t)), \quad t \geq t_0 \qquad (2)$$

can be unbounded in J. Note, that this task is achieved by the integral term and the introduction of a weighting function $\varphi(t) > 0$. For example, we show that all solutions IDE with delays:

$x''(t) + x(t) + \int\limits_0^t 24e^{-\,t+\tau}x'(\tau)d\tau = 7[x(\frac{t}{2}) + x'(\frac{t}{3})]\ ,\ \ t \geq 0$

bounded on the half-axis $R_+ = [0, \infty)$ and at the same time, all non-zero solutions $x(t) = (3e^t - 2e^{3t})c_1 + (9e^{2t} - 8e^{3t})c_2\ (c_1, c_2 - \forall\ const)$ IDE without delays:

$x''(t) - 7x'(t) - 6x(t) + \int\limits_0^t 24e^{-\,t+\tau}x'(\tau)d\tau = 0,\ \ t \geq 0$

are unbounded on the half-axis R_+. The method of weighting and cutting functions [1] and the method of integral inequalities with delays [2, 3] are applied.

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BOUNDARY PROBLEM FOR THE GENERALIZED SYSTEM MOISILA-TEODORESKU

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The known system Moisila-Teodoresku is a special case of next system

$$\begin{array}{ll} u_x+v_y+b_1w_z-b_2s_z=0, & v_x-u_y+b_1s_z+b_2w_z=0, \\ w_x-s_y-kb_1u_z-kb_2v_z=0, & s_x+w_y-kb_1v_z+kb_2u_z=0, \end{array} \tag{1}$$

where $b=b_1+ib_2$ – any complex number, $k=(b_1^2+b_2^2)^{-2}\neq 0$ – a real number. When the $b_1=1$, $b_2=0$ from system (1) follows Moisila-Teodoresku's system.

The common decision of system (1) is expressed through two any harmonious functions σ and τ under the formula

$$u=b_1\sigma_z-b_2\tau_z,\quad v=b_1\tau_z+b_2\sigma_z,\quad w=-\sigma_x-\tau_y,\quad s=-\tau_x+\sigma_y. \tag{2}$$

By using of (2) an analogue of the Riemann-Hilbert's problem for system (1) about finding of the regular decision in the field $D\equiv\{z>0\}$ of the system (1), satisfying on border $G\equiv\{z=0\}$ to conditions $\alpha_j u+\beta_j v+\gamma_j w+\delta_j s=f_j,\ j=1,2,$ ($\alpha_j,\ \beta_j\ \gamma_j,\ \delta_j\ f_j$ – set of continuous on Hölder functions on G) it is reduced to a problem of search of two regular in the field of D harmonious functions σ and τ, satisfying on border G of area D to conditions

$$(\overline{A}_j,\nabla\sigma)+(\overline{B}_j,\nabla\tau)=f_j,\quad j=1,2, \tag{3}$$

where $\overline{A}_j(-\gamma_j,\delta_j,\alpha_j b_1+\beta_j b_2)$, $\overline{B}_j=(-\delta_j,-\gamma_j,\beta_j b_1-\alpha_j b_2)$, $\nabla\equiv\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)$.

By method of Buligan-Zhiro we will search the decision of a problem (3) in a form

$$\sigma(X)=\int_G[G_1(X,P)\mu_1(P)+G_2(X,P)\mu_2(P)]dpG,\ \tau(X)=\int_G[H_1(X,P)\mu_1(P)+H_2(X,P)\mu_2(P)]dpG,$$

here $X=(x,y,z)\in D$, $P=(a,b,c)\in G$.

The following theorem is proved: if harmonious Ω function satisfies the equation

$$v_1\Omega_{xz}+v_2\Omega_{yz}+v_3\Omega_{zz}=\frac{\partial}{\partial n}\left(\frac{1}{r}\right),$$

$(v_1=-m_2\delta_1+n_2\gamma_1-m_1\delta_2+n_1\gamma_2),\ v_2=-m_1\gamma_2+n_1\delta_2+m_2\gamma_1+n_2\delta_1),\ v_3=m_1n_2+n_1m_2-\gamma_1\delta_2-\delta_1\gamma_2),\ m_j=\alpha_jb_1+\beta_jb_2,\ n_j=\beta_jb_1-\alpha_jb_2,\ j=1,2)$ that

$$G_1(X,P)=(\overline{B}_2,\nabla\Omega),\ H_1(X,P)=-(\overline{A}_2,\nabla\Omega),\ G_2(X,P)=-(\overline{B}_1,\nabla\Omega),\ H_2(X,P)=-(\overline{A}_1,\nabla\Omega)$$

satisfies system

$$(\overline{A}_j,\nabla G_i)+(\overline{B}_j,\nabla H_i)=\delta_{ij}\frac{\partial}{\partial n}\left(\frac{1}{r}\right),\ \ i,j=1,2,$$

(here δ_{ij} – a symbol of Kronekera) and densities μ_1, μ_2 are decisions of system of the Fredholm integrated equations

$$\mu_j(Q)+\frac{1}{2\pi}\int_G\frac{\partial}{\partial n}\left(\frac{1}{r}\right)\mu_j(P)dpG=\frac{1}{2\pi}f_i(Q),\ \ i,j=1,2,\ \ Q\in D.$$

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ON PROPERTIES OF THE HEAT POTENTIAL FOR THE HIGH-ORDER HEAT EQUATION

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When computing the solution of a partial differential equation in an unbounded domain, one often introduces artificial boundaries. In order to limit the computational cost, these boundaries must be chosen not too far from the domain of interest. Therefore, the boundary conditions must be good approximations to the so-called transparent boundary condition (i.e., such that the solution of the problem in the bounded domain is equal to the solution in the original domain). This question is of crucial interest in such different areas as geophysics, plasma physics, fluid dynamics [2,3,5]. In view of the mentioned shortcomings of the methods described, the need for practical transparent (artificial) boundary conditions combining efficiency and simplicity is evident. Such conditions must satisfy several criteria: (i) The resulting initial boundary value problem should be unique and stable; (ii) the solution to the initial boundary value problem should coincide or closely approximate the solution of the infinite problem on; and (iii) the conditions must allow for an analytical solution or an efficient numerical implementation. In this work we consider artificial boundary conditions for the high-order Cauchy problem for the heat equation. The conditions satisfy the above-mentioned criteria (i), (ii) and (iii). Similar results were taken for the Laplace equation in [4]. The transparent boundary condition is usually an integral relation in time and space between u and its normal derivative on the boundary, which makes it impractical from a numerical point of view. Alternatively, the requirement for boundary conditions can be avoided when the solution of a partial differential equation is approximated in the form of convolution in space and time with the fundamental solution. An efficient approximation of this type for the heat equation is proposed in [1].

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BOUNDARY CONDITIONS OF THE WAVE POTENTIAL

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In the bounded domain $\Omega \equiv \{(x,t) : (0,l) \times (0,T)\}$, we consider the following wave potential

$$u(x,t) = \int_{\Omega} \varepsilon(x-\xi, t-\tau) f(\xi,\tau) d\xi d\tau \tag{1}$$

where $\varepsilon(x-\xi, t-\tau) = \frac{1}{2}\theta(t-\tau-|x-\xi|)$ is a fundamental solution of Cauchy problem for the wave equation [1], i.e.

$$\frac{\partial^2 \varepsilon(x-\xi,t-\tau)}{\partial t^2} - \frac{\partial^2 \varepsilon(x-\xi,t-\tau)}{\partial x^2} = \delta(x-\xi,t-\tau), \tag{2}$$

$$\frac{\partial^2 \varepsilon(x-\xi,t-\tau)}{\partial \tau^2} - \frac{\partial^2 \varepsilon(x-\xi,t-\tau)}{\partial \xi^2} = \delta(x-\xi,t-\tau). \tag{3}$$

$$\varepsilon(x-\xi,t-\tau) \mid_{\tau=t} = \frac{\partial \varepsilon(x-\xi,t-\tau)}{\partial \tau} \mid_{\tau=t} = \frac{\partial \varepsilon(x-\xi,t-\tau)}{\partial t} \mid_{\tau=t} = 0. \tag{4}$$

If $f(x,t) \in L_2(\Omega)$, then $u(x,t) \in W_2^1(\Omega) \cap W_2^1(\partial\Omega)$ and the wave potential (1) satisfies to the following equation [see 1]

$$\frac{\partial^2 u(x,t)}{\partial t^2} - \frac{\partial^2 u(x,t)}{\partial x^2} = f(x,t), (x,t) \in \Omega, \tag{5}$$

with initial conditions

$$u(x,0) = u_t(x,0) = 0, 0 < x < l. \tag{6}$$

The wave potential (1) is widely used at solutions of various boundary problems for the wave equation. Below we find the lateral boundary conditions of the wave potential (1).

Theorem 1. *If $f(x,t) \in L_2(\Omega)$, then $u(x,t)$ the wave potential* (1) *satisfies lateral boundary conditions*

$$(u_x - u_t)(0,t) = 0, x = 0, 0 < t < T. \tag{7}$$

$$(u_x + u_t)(l,t) = 0, x = l, 0 < t < T. \tag{8}$$

Conversely, if a function $u(x,t) \in W_2^1(\Omega) \cap W_2^1(\partial\Omega)$ satisfies to the equation (5) and initial conditions (6), also the lateral boundary conditions (7)-(8) then the function $u(x,t)$ uniquely defines the wave potential (1).

Here we use the technique of [3].

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ON OSCILLATORY PROPERTY OF THE ELLIPTIC EQUATIONS

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We consider the following elliptic equation in R^n

$$-\sum_{i,j=1}^{n}(a_{ij}(x)u_{x_i})_{x_j}+a(x)u=0,\quad x\in R^n, \tag{1}$$

where $a_{ij}(x), i,j=1,...,n,$ and $a(x)$ is the locally bounded measurable functions in R^n. Assume $a_{ij}(x)=a_{ji}(x)$ and

$$\sum_{i,j=1}^{n}a_{ij}(x)\xi_i\xi_j\geq C|\xi|^2,\quad C=const>0,\xi\in R^n. \tag{2}$$

Let

$$V_i=x_1^{2p_1}+x_2^{2p_2}+...+x_i^{2p_i},\ p_i>0(i=1,...,n),\ \sum_{i=1}^{n}\frac{1}{2p_i}=\frac{n}{2},$$

$$R_\alpha^n=\{x\in R^n:\quad V_1>\alpha\}.$$

In this article we give a new oscillatory and nonoscillatory conditions for the equation (1) in R^n.

Definition. *We say that the equation (1) is oscillatory in R^n, if for any bounded domain $G\subset R_\alpha^n$, $\alpha>0$, the equation (1) have nontrivial solution $u(x)\in W_0^{1,2}(G)$ and is nonoscillatory in the contrary case.*

Theorem. *Let $a_{ij}(x)=\delta_{ij}, p_k=\max\{p_1,p_2,...,p_n\}, p_i\neq p_j$ if $i\neq j$. If*

$$a(x)\geq-\left[\left(\frac{n}{2}-\frac{1}{p_k}\right)^2+\frac{1}{\ln^2 V_1}\right]\frac{p_k^2(2p_k-1)}{(n+2)p_k-2}V_1^{p_k^{-1}}(V_2V_3...V_{k-1}(1-V_k))^{1-p_k^{-1}},$$

then the equation (1) is oscillatory in R^n. If for some $\epsilon>0$

$$a(x)\leq-\left[\left(\frac{n}{2}-\frac{1}{p_k}\right)^2+\frac{1+\epsilon}{\ln^2 V_1}\right]\frac{p_k^2(2p_k-1)}{(n+2)p_k-2}V_1^{p_k^{-1}}(V_2V_3...V_{k-1}(1-V_k))^{1-p_k^{-1}},$$

then the equation (1) is nonoscillatory in R^n.

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ROBIN BOUNDARY VALUE PROBLEM OF n-ORDER ELLIPTIC SYSTEMS IN THE PLANE WITH ARE SINGULAR LINES

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Let $0 < \varphi_0 \leq 2\pi,\ G = \{z = re^{i\varphi} : 0 \leq r < \infty,\ 0 \leq \varphi \leq \varphi_0\}$, $0 < \varphi_j < \varphi_0$, $(j = 1, 2, ..., m)$. We consider the equation

$$\sum_{j=1}^{n} f_j(\varphi)(2\bar{z}\frac{\partial}{\partial \bar{z}})^j V + f_{n+1}(\varphi)V + r^{\alpha}\frac{f_{n+2}(\varphi)}{\prod_{j=1}^{m}(y-k_jx)^{\alpha_j}}\overline{V} = r^{\nu}\frac{f_{n+3}(\varphi)}{\prod_{j=1}^{m}(y-k_jx)^{\alpha_j}} \qquad (1)$$

in G, where
$f_j(\varphi) \in C[0, \varphi_0]$, $(j = 1, 2, ..., n + 3)$, $f_n(\varphi) \neq 0$ for all $\varphi \in [0, \varphi_0]$, $k_j = tan\varphi_j$, $0 < \alpha_j < 1$, $\nu > 2 + \alpha$, $\alpha = \sum_{j=1}^{m} \alpha_j$,

$$\frac{\partial}{\partial \bar{z}} = \frac{1}{2}(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}),\ \frac{\partial^j V}{\partial \bar{z}^j} = \frac{\partial}{\partial \bar{z}}(\frac{\partial^{j-1}V}{\partial \bar{z}^{j-1}}),\ (j = 2, 3, .., n).$$

Equation (1) is investigated for $\alpha = 0,\ f_j(\varphi) \equiv 0,\ (j = 2, 3, ..., n),\ f_1(\varphi) \equiv const \neq 0$ in [1]. For $\alpha = 0$, there is no singular line. This case is investigated for $f_j(\varphi) \equiv 0$, $(j = 3, 4, ..., n)$, $f_2(\varphi) \equiv const \neq 0$ in [2], and for $f_j(\varphi) \equiv 0$, $(j = 4, 5, ..., n)$, $f_3(\varphi) \equiv const \neq 0$ in [3]. In [4] the Cauchy problem and in [5] the Robin problem for (1) with $\alpha = 0$, $f_j(\varphi) \equiv 0$, $(j = 5, 6, ..., n)$, $f_4(\varphi) \equiv const \neq 0$ are solved. We consider the Robin problem for system (1).

Problem R. *Find a solution of equation (1) from the class* $W_p^n(G) \cap C^{n-1}(G)$ *, satisfying the conditions* $\sum\limits_{k=1}^{n} \alpha_{j,k}\frac{\partial^{k-1}V(r,0)}{\partial\varphi^{k-1}} = \beta_j r^{\nu-\alpha}$, $(j = 1, 2, ..., n)$, *where* $\alpha_{j,k}, \beta_j$, $(j = 1, 2, ..., n)$, $(k = 1, 2, ..., n)$ *are given real numbers.*

Here $W_p^n(G), p > 1$, *is the Sobolev space.*
We proved the following theorem.

Theorem. *If* $|A| \neq 0$ *then problem R has a unique solution.*
Here $|A|$*-is determinant of the matrix* $A = \{a_{i,j}\}$, $(i, j = 1, 2, ..., n)$.

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OF ELLIPTIC SYSTEMS IN THE PLANE WITH A SINGULAR POINT

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Let G be bounded domain of the complex plane with the inner point $z = a$. Let $S(G)$ be the set of measurable, essentially bounded functions $f(z)$ in G with the norm $||f||_{S(G)} = \lim_{p\to\infty} ||f||_{L_p(G)}$. Now the spaces used bellow are defined: $S_\nu(G, a)$ is the class of functions $f(z)$, for which $f(z)|z-a|^\nu \in S(G)$. The norm in $S_\nu(G, a)$ is defined by formula $||f||_{S_\nu(G,a)} = ||f(z)|z-a|^\nu||_{S(G)}$, where ν is a real number. $C_\nu(\bar{G}, a)$ is the class of functions $f(z)$, for which $f(z)|z-a|^\nu \in C(\bar{G})$. The norm in $C_\nu(\bar{G}, a)$ is defined by the formula $||f||_{C_\nu(\bar{G},a)} = ||f(z)|z-a|^\nu||_{C(\bar{G})}$. $W_p^1(G), p > 1$ is the Sobolev space [1]. Let us consider in G the equation

$$\partial_{\bar{z}} V + A(z)V + B(z)\overline{V} = F(z), \tag{1}$$

where $A(z), B(z) \in S_1(G); F(z) \in S_\beta(G, a), 0 < \beta < \frac{2}{q}$, $q > 2$.

For $a = 0$ we receive the equation

$$\partial_{\bar{z}} V + \frac{A_0(z)}{|z|} V + \frac{B_0(z)}{|z|} \overline{V} = F(z), \tag{2}$$

where $A_0(z), B_0(z) \in S(G); F(z) \in S_\beta(G, 0), 0 < \beta < \frac{2}{q}$, $q > 2$.

Equation of the form (2) with $F(z) = 0$ arise in the theory of infinitesimal deformations of surfaces of positive curvature with a flat point [2]. There it is required to prove the existence of continuous of equation (2) in the neighborhood of a singular point $z = 0$. In this connection the equation (2) is studied in many works of L.G.Mikhailov, Z.D.Usmanov [3,4], etc. In all results usually a sufficient smallness of the coefficients $A_0(z)$ and $B_0(z)$ or the smallness of the domain G is supposed. We proved existence of solutions $V(z)$ of the equation (1), satisfying to the condition $V(a) = 0$, without any conditions on the smallness of the coefficients or on the smallness of the domain G in the class

$$W_q^1(G) \cap C_{\beta-1}(\overline{G}, a),\ 0 < \beta < \frac{2}{q},\ q > 2. \tag{3}$$

The Riemann-Hilbert problem for the equation (2) in the class (3) is solved without any smallness conditions on the coefficients assumed in [3,4]. The structures of the zeros and poles of the solutions of the equation (1) in the class (3) are investigated.

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A NONLOCAL FREE BOUNDARY PROBLEM FOR QUASILINIEAR PARABOLIC EQUATION

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The present paper is devoted to the study of a non-classical free boundary problem for nonlinear parabolic equation in one-space dimension. Free boundary problems arise naturally in a namber of physical phenomena with change of state (such as melting of ice and recrystallization of metals) and have been studied by many authors.

The precise statement of the problem is as follows:

Let $T > 0$ and $D = \{(t,x) : 0 < t \le T, 0 < x < s(t)\}$.

Find a pair of functions, $u(t,x)$ and $s(t)$, with are defined on $\bar{D}$ and $[0,T]$, respectively, and with satisfy

$$u_{xx}(t,x) = a(u)u_t(t,x), \quad (t,x) \in D \tag{1}$$

$$u(0,x) = \varphi(x), \quad 0 \le x \le s_0, \tag{2}$$

$$u(t,0) = mu(t,x_0), \quad 0 \le t \le T, \quad 0 < x_0 \le s_0 \tag{3}$$

$$u(t,s(t)) = 0, \quad 0 \le t \le T, \tag{4}$$

$$\dot{s}(t) = -cu_x(t,s(t)), \quad 0 \le t \le T, \tag{5}$$

where $s_0, 0 < m < 1, c$- are positive constants, $a(u) \ge a_0 > 0$ is a function defined for $u(t,x) \ge 0$ and $a'(u) > 0$.

This kind of problems for a certain class of nonlinear parabolic equations was treated earlier by Douglas [1] and Kyner [2] in with they showed the existence and uniqueness of solutions by using a strong maximum principle for parabolic equations with variable coefficients.

Problems for linear equations with nonlocal (initial or boundary) conditions have been studied in many papers. But fue boundary value problems for non-linear equations with non-local conditions are almost investigated yet.

The principal difficulty under constructing the theory of nonlocal boundary value problems is the obtaining of apriori estimations for the absolute value of the derivative u_x and its Holder's constant.

We will first study the local solvability.

Then, some apriori estimates are derived to establish the global existence of the solution.

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UNIFORM ASYMPTOTIC SOLUTIONS OF THE CAUCHY PROBLEM FOR A GENERALIZED MODEL EQUATION OF L.S.PONTRYAGIN IN THE CASE OF VIOLATION OF CONDITIONS OF ASYMPTOTIC STABILITY

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In this paper, we construct a uniform asymptotic solution of the Cauchy problem of the small parameter for the inhomogeneous differential equation with small parameter at the derivative, when the linear part of the equation is a pure complex and his real part changes from negative to positive when one going from the left half on the right half plane.

Consider the Cauchy problem for a singularly perturbed equation

$$\varepsilon x'(t) = (t + i)x(t) + f(t), x(t_0) = x^0, \quad (1)$$

where $\varepsilon > 0$ - small parameter, $f(t)$ - analytic function in D,

$D = \{(|t| \leq \rho > 1\}, x^0 - const, t \in D$, $x(t)$ - unknown function.

This problem was first considered by a graduate student [1] of academician L.S.Pontryagin and restrictions solutions on the interval $t \in [-1, 1]$.

Here it is constructed a uniform asymptotic expansion of (1) on the interval $t \in D$. **Theorem.** *Asymptotical of the solution of the problem (1)is represented in the form:*

$x(t,\varepsilon) = \mu^{-1}\pi_{-1}(t) + \pi_0 + x_0(t) + ... + (\pi_n(t) + x_n(t))\mu^n + R_{n+1}(t,\mu)$, ***where*** $\mu = \sqrt{\varepsilon}, x_k(t)$*- analytic functions in* D, $\pi_k(t,\varepsilon) = O(1)e^{\frac{t^2-1}{\varepsilon}}, k = \overline{0,n}$,

$|R_{n+1}(t,\mu)| \leq M, M - const.$

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THE SYSTEM OF KAUP'S EQUATION WITH A SELF-CONSISTENT SOURCE OF INTEGRAL TYPE IN THE CLASS OF PERIODIC FUNCTIONS

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We study the following system of Kaup's equations ([1])with self-consistent source

$$p_t = -6pp_x - q_x + \int_{-\infty}^{\infty} \beta(\lambda,t)(\psi_+\psi_-)_x d\lambda, \tag{1}$$

$$q_t = p_{xxx} - 4qp_x - 2pq_x + 2\int_{-\infty}^{\infty} \beta(\lambda,t)\{-p_x\cdot(\psi_+\psi_-) + (\lambda-2p)\cdot(\psi_+\psi_-)_x\}d\lambda \tag{2}$$

coupled with the initial conditions

$$p(x,t)|_{t=0} = p_0(x)\,, \qquad q(x,t)|_{t=0} = q_0(x). \tag{3}$$

We look for real-valued solution p and q, that is π-periodic on the partial variable x and satisfy the following regularity assumptions: 1)$p \in C^{3,1}_{x,\,t}(t>0)\cap C(t\geq 0)$, $q \in C^1(t>0)\cap C(t\geq 0)$; 2) For any nontrivial function $y \in W_2^2[0,\pi]$, satisfying the equality $y'(0)\bar{y}(0) - y'(\pi)\bar{y}(\pi) = 0$, the following inequality holds $\int_0^{\pi}\left\{|y'(x)|^2 + q(x,t)\,|y(x)|^2\right\}dx > 0$. In the previous expressions, β is a given real continuous function having a uniform asymptotic behavior $\beta(\lambda,t) = O(\lambda^{-3})$, $\lambda\to\pm\infty$ and $\psi_\pm(x,\lambda,t)$ are the Floquet's solutions (normalized by the condition $\psi_\pm(0,\lambda,t) = s(\pi,\lambda,t)$) of the quadratic pencil of Sturm-Liouville's equations:

$$-y'' + q(x,t)y + 2\lambda\, p(x,t)y - \lambda^2 y = 0\,, \quad x\in R^1. \tag{4}$$

We denote by $s(x,\lambda,t)$ the solution of (4) satisfying the initial conditions $s(0,\lambda,t)=0$, $s'(0,\lambda,t)=1$.

Theorem. *Let* $(p,\ q,\ \psi_+,\ \psi_-)$ *be the solution of* (1)-(4). *Then the eigenvalues* λ_n, $n\in Z$ *of the periodic and anti-periodic problem for the quadratic pencil of Sturm-Liouville's equations, with coefficients* $p(x+\tau,t)$ *and* $q(x+\tau,t)$, *do not depend on the parameters* τ *and* t, *and the eigenvalues* $\xi_n(\tau,t)$, $n\in Z\backslash\{0\}$ *of the Dirichlet problem satisfy the analogue of system of Dubrovin-Trubovitz's equations.* **Corollary 1.** *This theorem provides a method for solving the problem (1)-(4).*

Corollary 2. *Using the results of [2] we can conclude, that if* $p_0(x)$ *and* $q_0(x)$ *are real analytical functions, then the solution of* (1)-(4) $p(x,t)$ *and* $q(x,t)$ *are also real analytical functions on* x.

Corollary 3. *Using the results of [3] we can conclude, that if* $p_0(x)$ *and* $q_0(x)$ *are* $\pi/2$*-periodic functions, then the solution of problem (1)-(4)* $p(x,t)$ *and* $q(x,t)$ *are also* $\pi/2$*-periodic functions on* x.

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THE KORTEWEG-DE VRIES EQUATION WITH A SELF-CONSISTENT SOURCE OF INTEGRAL TYPE IN THE CLASS OF PERIODIC FUNCTIONS

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We study ([1]) the following Korteweg-de Vries equation with a self-consistent source

$$q_t = q_{xxx} - 6qq_x + 2\int\limits_{-\infty}^{\infty} \beta(\lambda,t)\,(\psi_+\psi_-)_x\,d\lambda, \quad t>0, \ \ x\in R^1 \tag{1}$$

coupled with the initial condition

$$q(x,t)|_{t=0} = q_0(x). \tag{2}$$

We look for real-valued solution that is π-periodic on the partial variable x and satisfy the following regularity assumptions: $q(x,t)\in C^{3,1}_{x,\,t}(t>0)\cap C(t\geq 0)$. Here $\beta(\lambda,t)$ is a given real continuous function having a uniform asymptotic behavior $\beta(\lambda,t)=O(\lambda^{-3})$, $\lambda\to\pm\infty$ and $\psi_\pm(x,\lambda,t)$ are the Floquet's solution (normalized by condition $\psi_\pm(0,\lambda,t)=s(\pi,\lambda,t)$) of the Sturm-Liouville equation

$$L(t)y \equiv -y'' + q(x,t)y = \lambda\, y, \quad x\in R^1. \tag{3}$$

We denote by $s(x,\lambda,t)$ the solution of (3) satisfying the initial conditions $s(0,\lambda,t)=0$, $s'(0,\lambda,t)=1$.

Theorem. *Let $(q,\ \psi_+,\ \psi_-)$ be the solution of the problem (1)-(3). Then the eigenvalues λ_n, $n\geq 0$ of the periodic and antiperiodic problem for Sturm-Liouville's equation with coefficient $q(x+\tau,t)$ do not depend on the parameters τ and t, and the eigenvalues $\xi_n(\tau,t)$, $n\geq 1$ of the Dirichlet problem satisfy the analogue of system Dubrovin-Trubowitz's equations.*

Corollary 1. *This theorem provides a method for solving the problem (1)-(3).* **Corollary 2.** *In* [2] *the generalization of Borg's inverse theorem for Sturm-Liouville's operator is proven. According to this result we can conclude, if $q_0(x)$ is π/n-periodic function, then the $q(x,t)$ is also π/n-periodic function on x. Here $n\geq 2$ is a integer number.* **Corollary 3.** *Using the results obtained in [3] we can conclude that, if $q_0(x)$ is real analytical function, then the $q(x,t)$ is also real analytical function on x.*

Remark. *If the KdV equation is given in the form $q_t = q_{xxx} - 6qq_x + f(t)$ then the spectrum of Sturm-Liouville operator (3) depends on the parameter t. More precisely, the spectrum moves along the axis while preserving its structure.*

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ON SOME CONSEQUENCES OF THEOREM I. AKBERGENOVA

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Murdered in 1938, Ibatolla Akbergenov was the author of the first scientific papers on mathematics among Kazakhs.

Let the Fredholm integral equation of 2-nd kind

$$\varphi(x) = f(x) + \lambda \int_a^b K(x,y)\,\varphi(y)\,dy, \tag{1}$$

be given $(a < b)$. Replace the kernel $K(x,y)$ by the kernel close to $\tilde{K}(x,y)$ and consider the integral equation

$$\tilde{\varphi}(x) = f(x) + \lambda \int_a^b \tilde{K}(x,y)\,\tilde{\varphi}(y)\,dy. \tag{2}$$

The proximity of the kernel to the kernel is characterized by the smallness of the value determined by norm $L_2[a,b]^2$. Let λ be not an eigenvalue of (2). Function $f(x)$ and $K(x,y)$ assumed to be continuous in the square $a \le x \le b$, $a \le y \le b$. Then following is valid.

The extension of Theorem (I. Akbergenov [1]) to the case $K(x,y) = g(u) \in \ell^\infty(D_{2s})$ estimate (see the notation in [2-3])

$$\left\| g(u) - \sum_{n \in V_q} \Lambda_{\Omega(\nu(n);q)}\left(g(\cdot)\,e^{-2\pi i(n,\cdot)}\right)\cdot e^{2\pi i(n,u)} \right\|_{L^2(0,1)^{2s}} \ll$$

$$\ll \left(\sum_{n \in V_q} \sum_{\substack{\mu=(\mu_1,\ldots,\mu_{2s}):\mu_j \ge \nu_j^{(0)} \\ \tau=(\tau_1,\ldots,\tau_{2s}):\tau_j \in Z \bigcup \left\{-\frac{1}{2}\right\}}} \left| D_{2s}^{-1}\left(n + (2\tau+1)\,2^{\mu}\right) \sum_{\substack{\nu=(\nu_1,\ldots,\nu_{2s}):\,\nu_j \ge \nu_j(n) \\ \nu_1+\ldots+\nu_{2s} > q \\ \tau_j=-\frac{1}{2},\,\nu_j=\nu_j^{(0)}, \\ \tau_j \in Z,\,\nu_j=\nu_j^{(0)}\ 8\ \nu_j=\mu_j+1}} \sum_{t \in Z^{2s}} (-1)^{\sum_{j=1}^{2s} sgn\left(\nu_j-\nu_j^{(0)}\right)} \right|^2 + \sum_{n \in Z^{2s}\setminus V_q} D_{2s}^{-2}(n) \right)^{\frac{1}{2}}$$

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RECONSTRUCTION OF THE POTENTIAL FUNCTION FOR SCHRODINGER EQUATION

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This abstract is about the inverse problem for Schrodinger operator and consists of reconstruction of this operator by its spectrum and norming constants. In this paper, we solve inverse problem for Schrodinger operator using the nodal set of eigenfunctions. We prove the uniqueness theorem and a reconstructing formula for the potential function. The technique that we use to obtain the results is an adaptation of the method discussed in the references [1] and [2].

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AN EXPANSION METHOD FOR FINDING TRAVELING WAVE SOLUTIONS TO NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

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In this study, we constructed an expansion method. We have implemented this method for finding traveling wave solutions of nonlinear KdV equation and Hirota-Satsuma equation. We also pointed out that this expansion method could implement the other nonlinear equations.

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ON ONE BOUNDARY VALUE PROBLEM FOR EVEN-ORDER EQUATION

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Let's consider the equation

$$Lu \equiv \frac{\partial^{2k}u}{\partial x^{2k}} - \frac{\partial^{2p}u}{\partial t^{2p}} \ = \ f(x,t), \tag{1}$$

in the domain $\Omega = \{ \ (x,t): \ 0 < x < l, \ 0 < t < T \ \}$, where k, p are fixed natural numbers.

Problem. Find solution $u(x,t) \in C^{2k-1,2p-1}_{x,\ t}(\Omega) \bigcap C^{2k,2p}_{x,\ t}(\Omega)$ of the equation (1) in the domain Ω satisfying conditions

$$\frac{\partial^{2i}u}{\partial x^{2i}}|_{x=0} \ = \frac{\partial^{2i}u}{\partial x^{2i}}|_{x=l}, \ 0 \le t \le T, \ i = \overline{0, k-1}, \tag{2}$$

$$\frac{\partial^{2j}u}{\partial t^{2j}}|_{t=0} \ = \frac{\partial^{2j}u}{\partial t^{2j}}|_{t=T}, \ 0 \le x \le l, \ j = \overline{0, p-1}. \tag{3}$$

In present work we study solvability of the problem (1)-(3) depending on values of the numbers k and p.

Lemma. *Let $(k-p)$ is odd. Then there exists a constant $C > 0$ that depends only on sizes of the domain and the numbers k, p and doesn't depend on function $u(x,t)$ such that*

$$\|u\|_{W_2^{k,p}(\Omega)} \le C\|Lu\|_{L_2(\Omega)}$$

Theorem 1. *Let $(k-p)$ is odd and $f \in C^{\alpha}(\Omega)$, $\alpha > \frac{1}{2}$, $f(0,t) = f(l,t) = 0, \ 0 \le t \le T$* and *$f(x,0) = f(x,T) = 0, \ 0 \le x \le l$. Then there exists a unique regular solution of the problem (1)-(3).*

Theorem 2. *Let $(k-p)$ is even. Then regular solution of the problem (1)-(3) is unique if and only if ratio $\frac{T}{l}$ of the seizes of the rectangle Ω is irrational.*

Theorem 3. *Let $(k-p)$ is even and $\frac{l}{T\pi^{k-p}}$ is irrational. If $f(x,t) \in C^{k+3,p+2}_{x,\ t}(\overline{\Omega})$ and*

$$\frac{\partial^{2m}u}{\partial x^{2m}}|_{x=0} \ = \frac{\partial^{2m}u}{\partial x^{2m}}|_{x=l}, \ 0 \le t \le T,$$

$$m = 0, 1, ..., \frac{k+1}{2}, \ if \ k+3 \ is \ even,$$

$$m = 0, 1, ..., \frac{k+2}{2}, \ if \ k+3 \ is \ odd,$$

$$\frac{\partial^{2n}u}{\partial t^{2n}}|_{t=0} \ = \frac{\partial^{2n}u}{\partial t^{2n}}|_{t=T}, \ 0 \le x \le l,$$

$$n = 0, 1, ..., \frac{p+1}{2}, \ if \ p+2 \ is \ even,$$

$$m = 0, 1, ..., \frac{p}{2}, \ if \ p+2 \ is \ odd.$$

Then there exists a regular solution of the problem (1)-(3).

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GENERALIZED FUNCTIONS METHODS IN THE DYNAMICS FOR THE PIEZOELASTIC MEDIUM

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Consider the system of hyperbolic -elliptical equations, which are typical for the dynamic of piezoelastic medium [1]:

$$C_{ij}^{ml(E)} u_{m,lj} + e_{lji}\varphi_{,lj} + G_i = \rho u_{i,tt} \quad i,j,m,l = \overline{1,N} \tag{1}$$

$$e_{jml}u_{m,lj} - \epsilon_{jl}\varphi_{,lj} = 0, \quad E_m = -\varphi_{,m} \tag{2}$$

where ρ is the density of medium, E is density of electric field, φ is the electric potential, $G(x,t)$ is the mass force which may be generalized vector-function with limited support; $C_{ij}^{ml(E)}$ is the elasticity tensor, measured at constant electric field; e_{lij} is the piezoelectric tensor; ϵ_{il} is the permittivity tensor, measured at constant strain. The material tensors exhibit the symmetries

$$C_{ij}^{ml(E)} = C_{ij}^{lm(E)} = C_{ji}^{ml(E)} = C_{ml}^{ij(E)}, \quad e_{lij} = e_{lji}, \quad \epsilon_{il} = \epsilon_{li} \tag{3}$$

and for general anisotropy they contain 21, 18 and 6 independent material constants respectively. Notice, constants $C_{ij}^{ml(E)}$ satisfy conditions of strong hyperbolicity: $C_{ij}^{ml(E)} n_m n_l v^i v^j > 0 \quad \forall n \neq 0, \quad v \neq 0$; $x \in S^- \subset R^N$, S^- is limited by boundary S which belongs to the class of Lyapunov's surfaces with exterior normal n, $\|n\| = 1$, $(x,t) \in D^-$, $D^- = S^- \times [0,\infty)$, $D_t^- = S^- \times [0,t]$, $D = S \times [0,\infty)$, $D_t = S \times [0,t]$. We suppose $u, \varphi \in C(D^- + D)$, $G \in C(D^- + D)$ $G \to 0$, $t \to +\infty$, $\forall x \in S^-$. There are initial conditions

$$u_i(x,0) = u_i^0(x), \quad x \in S^- + S; \quad \partial_t u_i(x,0) = u_i^1(x), \quad x \in S^-; \tag{4}$$

and boundary conditions for $x \in S, \quad t \geq 0$:

$$\sigma_i^j(x,t) n_j(x) = g_i(x,t), \quad \sigma_i^l = C_{ij}^{ml} u_{j,m} \tag{5}$$

$$D_j n_j = q^s(x,t), \quad \varphi(x,t) = \varphi^s(x,t) \tag{6}$$

here D_i are electric displacements.

The method of construction of conditions on wavefronts has been suggested [2,3]. The theorem of uniqueness of the solution (including the class of shockwaves) has been proved. Analogues of the Somigliana's formulae for the generalized solutions of nonstationary boundary value problems for piezoelastic media have been constructed. These formulae allow to find the displacements, tensions and electric field intensity if initial values and boundary displacements, surface forces, electric potential, charge flux density are known.

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SECTION V

Probability Theory, Mathematical Statistics and Fuzzy Systems

THE CONDITION OF NONPRECIPICING OF THE HOMOGENEOUS MARKOV PROCESSES BY FIRST COMPONENT

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Let $\vec{S}(t) = (S^1(t), S^2(t))$, $t \geq 0$, is the homogeneous stochastic process with independent growing in $R^2 = (-\infty, \infty) \times (-\infty, \infty)$ such that

$$M \cdot \exp\{s(S^1(t) - S^1(0)) - \sigma(S^2(t) - S^2(0))\} = \exp\{tk(s, \sigma)\}, s \geq 0,\ \sigma \geq 0,$$

where

$$K(s, \sigma) = as + b\sigma + \lambda[\varphi(s, \sigma) - 1],\ 0 < b < a < +\infty,\ \lambda > 0,$$

are some constants, and

$$\varphi(s, \sigma) = M \cdot e^{s\xi - \sigma\eta},$$

where ξ, η are some nonnegative chance values and M is the mathematical expectation.

For $t \geq 0$ we set

$$\tau[t] = \inf\{u \geq 0 : S^1(u) \wedge S^2(u) = -t,\ if\ \exists u : S^1(u) \wedge S^2(u) = -t, \},$$

$$\tau[t] = \inf\{u \geq 0 : S^1(u) \wedge S^2(u) = +\infty,\ if\ \exists u : S^1(u) \wedge S^2(u) \neq -t, \},$$

$$\tau = \inf\{t > 0 :\ \tau(t) = +\infty\},$$

$$\sigma[t] = S^1(\tau[t]) - S^2(\tau[t]),\ \tau > t;\ \tau[t] = 0,\ \tau \leq t,$$

where $x \wedge y = \min\{x, y\}$.

In this paper is proved that the Markov process $(\tau[t], \sigma[t],\ 0 \leq t < T)$ is nonparticipating if and only if

$$\min\{\lambda M\xi - a,\ \lambda M\eta - b\} \leq 0.$$

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ABOUT FUZZY RECOGNITION SYSTEM OF AZERBAIJANI HAND-PRINTED TEXTS[1]

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There a most of computer systems for solving the problem of recognition of handwritten characters. There are a lot of both theoretical and practical difficulties associated with a huge variety of possible ways to write individual handwritten characters in this problem. The hand-printed recognition systems for increase of input reliability and information processing, as a rule, make rigid requirements to the filling of handwritten forms, for example, to use only constrained caps characters. However, in practice it is not always possible to follow this rule since during filling casual infringements of this restriction are probable. Besides that, distortion of recognition area and additional parts in the scanning area are the problems, which are the reasons of poor quality of recognition systems.

The presented system for recognition of handwritten characters uses fuzzy neural networks with smooth class of membership functions, structural features and error back - propagation method for training of neural networks. Here the main goal is showing the importance of usage of fuzzy numbers with smooth membership function in the process of neural network's training.

Multilayered fuzzy neural network is used in the recognition system. Input elements of the network consist of the values of calculated features. For each these features are constructed terms and membership functions.

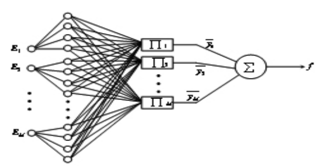

Fig. 1. Model of neural network

Membership degrees of terms for the features are calculated in the first layer (Fig.1). To distinguish classes of symbols IF and THEN type rules are constructed. Values of these rules (z_i, i=*1, 2, ...*, M, where $M$is the number of classes) are calculated in the next layer of neuronal network. Then defuzzification according to the weight center (1) is conducted and output of the network is calculated.

$$f = \sum_{i=1}^{M} z_i \bar{y}_i \Bigg/ \sum_{i=1}^{M} z_i. \tag{1}$$

Weights of the last layer consist of the peak points of membership functions constructed according to classes of symbols. Parameters of membership functions of terms of features and weights of the last layer are optimized during training process by the error back - propagation method.

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SMOOTH CLASS OF MEMBERSHIP FUNCTIONS FOR FUZZY SETS AND THEIR APPLICATION[1]

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In this work was offered a class of parametrical membership functions constructed by spline principle, which generalized known types of MFs (triangular, trapezoidal, Gaussian, bell, etc.) and have following properties [1]:

1) Membership functions everywhere are continuous with their derivatives on both arguments: $(\mu_A(x,P))'_x$, $(\mu_A(x,P))'_p$, and for expression of all derivatives there are simply calculated analytical formulas;

2) For any possible values of a vector of parameters P and argument values $x \in X^0$, $X^0 = supp\mu_A(x,\, p) = \{x \in X : 0 < \mu_A(x;P)\}$- support, MF are strictly unimodal functions;

3) Area X^0 generally can be broken into 5 parts: on the ends of area X^0 MF it is convex, in an average part it is bent, the convex and bent parts of membership functions separate with linear functions. In special cases any parts of MF can be absent;

4) For any numerical values of $x \in X^0$values of MF are various for various parameters P_1, P_2 from any arbitary small interval, i.e. from $\left\|P^2 - P^1\right\|_{R^{12}} < \varepsilon$, $P^1 \neq P^2$ follows $\mu_A(\bar{x}, P^1) \neq \mu_A(\bar{x}, P^2)$;

These properties have great importance in "training" problems, in which adjustment of parameters of MF is carried out, namely in computer recognition systems, constructed on the base of neural-fuzzy networks. The training level consists in using of iterative procedures in space of parameters of neural-fuzzy networks based on first-order optimization methods (gradient types), more exactly on back propagation method. Clearly that in case of using not differentiated MF it is impossible to prove application of these methods. For non-smooth MF it is necessary to use methods of not differentiated optimization which concede to "smooth" methods.

Formulas of definition of various characteristics of fuzzy numbers are defined, for example:

1) α- cut;

2) the weighed average and average representative;

3) Width.

and also arithmetical and logical operations.

The offered class fuzzy MF is used by authors in developed to recognition system hand-written texts of the Azerbaijan language.

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A NEW TYPE OF CONVERGENCE IN A FUZZY VALUED METRIC SPACE

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In this study, we introduce the concept of τ_F-convergence of a sequence of fuzzy numbers in the fuzzy metric space of fuzzy numbers defined by Guangquan. We compare τ_F-convergence with the convergence in the supremum metric. We also transfer some results from classical mathematical analysis, namely, we present τ_F-convergence analogues of these results.

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INVERSE SYSTEM OF FUZZY SOFT MODULES

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Molodtsov [1] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties. Later, work on the soft set theory is progressing rapidly. Maji et al. [2] have published a detailed theoretical study on soft sets. Aktaş and Çağman [3] introduced the basic properties of soft groups, and derived their basic properties. Gunduz and Bayramov [4] introduce fuzzy soft module and investigate some of fuzzy soft module basic properties.It is known that the inverse limit is not only an important concept in the category theory, but also plays an important role in topology, algebra, homology theory etc. To the date, inverse system and its limit was defined in the different categories. Using methods of homology algebra, we give inverse system of fuzzy soft modules. We introduce inverse system in the category of fuzzy soft modules and prove that its limit exists in this category. Later, we show that limit of direct system of exact sequences of fuzzy soft modules are exact. Generally, limit of inverse system of exact sequences of fuzzy soft modules is not exact. Then we define first derived functor of the inverse limit functor.

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ASYMPTOTIC BEHAVIOUR OF THE CONDITIONAL PROBABILITY OF INTERSECTION OF THE LEVEL OF MARKOV CHAIN TRAJECTORY

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Let $X = \{X_n = X(u,n),\ n \geq 0\}$ be real-valued Markov chain, homogeneous in time with initial value $u = X(u,0) = X_0$ and transitive probability

$$P(u,B) = P(X_1 \in B),$$

where $X_1 = X(u,1)$ and $B \in \beta(R)$ are σ algebra of Borel sets in $R = (-\infty, \infty)$.

Let's consider the moment of the first exit

$$\tau_c = \inf\{n \geq 1 : X_n > c\}$$

of the Markov chains X_n for the level $c \geq 0$. We will assume that $\inf\{\emptyset\} = \infty$.

The moment of the first exit τ_c is investigated in many works in which the boundary problems connected with intersection of boundary by a trajectory of a Markov chain are studied.

In the present work the conditional probability $P\left(\tau_c \geq n | \bar{X}_n = x\right)$ of intersection is considered $P\left(\tau_c \geq n | \bar{X}_n = x\right)$, by the trajectory of a chain X of level $c > 0$ under the conditions, that $\overline{X}_n = \frac{X_n}{n} = x$ asymptotic behaviour of this probability is studied at and $n \to \infty$ and $c \to \infty$, in such a manner that $\frac{c}{n} \sim x$.

Similar problems are studied completely for the case when the Markov chain X_n is the sum of the independent equally distributed random variables.

Let's notice that the specified conditional probability of intersection of boundary plays a key role at study of an asymptotical behaviour of local probabilities of intersection of boundary a random walk.

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INTUITIONISTIC FUZZY SOFT MATRIX THEORY AND ITS DECISION MAKING

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In this work, we first introduce the soft sets, fuzzy sets, soft matrices and their operations. We then define intutionistic fuzzy soft matrices and their operators which are more functional to make theoretical studies in the intuitionistic fuzzy soft set theory. By using the products of intuitionistic fuzzy soft matrices, we finally construct construct a decision making method which selects optimum alternative from the set of the alternatives. We finally give an example to show the method can be successfully applied to the problems that contain uncertainties.

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ON THE PROBABILITY FUNCTIONS OF ORDER STATISTICS FROM DISCRETE UNIFORM DISTRIBUTION

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Ahsanullah M. and Nevzorov V. B. (2001) are given the probability functions of sample extremes of order statistics from discrete uniform distributions. In this study, probability function of rth order statistics from discrete uniform distribution are obtained.

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STABILITY ESTIMATIONS OF SOLUTIONS OF STOCHASTIC SYSTEMS OF NEUTRAL TYPE

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Systems of linear stochastic difference differential equations of the neutral type are considered in this report. The second Lyapunov method with quadratic Lyapunov-Krasovsky functionals and an exponential multiplier is used. Stability conditions are obtained and decay coefficients for solutions are calculated.

The significant amount of practical problems erased in last years require researches of stability and obtaining estimations of dynamic processes described by functional differential equations. These equations consider an impact of a prehistory and widely used in a description of processes in ecology, population dynamics, medicine, economy. It should be noted that it is almost impossible to make a differential equation which discribes dynamics of real processes absolutely adequately. There are always small unpredictable perturbations which impact on dynamics of process. The mathematical apparatus, which considers these phenomena, is the theory of stochastic functional differential equations.

A system of linear stochastic difference differential equations with constant coefficients of the neutral type is considered

$$d\left[x(t) - Dx(t-\tau)\right] = \left[A_0 x\left(t\right) + A_1 x\left(t-\tau\right)\right]dt + \left[B_0 x\left(t\right) + B_1 x\left(t-\tau\right)\right]dw\left(t\right). \tag{1}$$

Here A_0, A_1, B_0, B_1, D - matrixes with constant coefficients, , $x(t) \in R^n$, $\tau > 0$ - constant delay, $w\left(t\right)$ - scalar standard Wiener process. By a solution of the system (1) we understand F_t- measurable stochastic process $\{x\left(t\right) \equiv x\left(t,\omega\right)\} \in R^n$ which satisfies the Ito integral equation with probability 1.

The Lyapunov-Krasovsky functional is used to study stability:

$$V[x(t),t] = e^{\gamma t}\left\{\left[x\left(t\right) - Dx\left(t-\tau\right)\right]^T H\left[x\left(t\right) - Dx\left(t-\tau\right)\right] + \int_{t-\tau}^{t} e^{-\beta(t-\xi)} x^T\left(\xi\right) G x\left(\xi\right) d\xi\right\}, \tag{2}$$

Denote

$$S\left[H,G,\beta,\gamma\right] =$$
$$= \begin{bmatrix} -A_0^T H - HA_0 - B_0^T HB_0 - G - \gamma H & -A_0^T HD - HA_1 - B_0^T HB_1 - \gamma H \\ -D^T HA_0 - A_1^T H - B_1^T HB_0 - \gamma D^T H & -A_1^T HD - D^T HA_1 - B_1^T HB_1 - \gamma D^T HD - e^{-\beta\tau} G \end{bmatrix}.$$

$\lambda_{\min}\left(S\left[H,G,\beta,\gamma\right]\right)$ - a minimal eigenvalue of the matrix $S\left[H,G,\beta,\gamma\right]$.

The following result takes place.

Theorem. *Let there exist constants $\beta > 0$, $\gamma > 0$ and positive definite matrixes H, in which the matrix $S\left[H,G,\beta,\gamma\right]$ is positive defined. Then a zero solution of the system (1) is exponentially $(\gamma,\beta)-$integrally mean-square stable. And for any solution $x(t)$ the following convergence estimation is held*

$$M\left\{\left\|x\left(t\right)\right\|_{\tau,\beta}^2\right\} \leq \left[\sqrt{\left(1+|D|^2\right)\frac{\lambda_{\max}\left(H\right)}{\lambda_{\min}\left(G\right)}}\left(|x(0)| + |x(-\tau)|\right) + \left\|x(0)\right\|_{\tau,\beta}\sqrt{\frac{\lambda_{\max}\left(G\right)}{\lambda_{\min}\left(G\right)}}\right] e^{-\theta(H,G,\beta,\gamma)t},$$

$$\theta\left(H,G,\beta,\gamma\right) = \min\left\{\ \beta,\quad \frac{\lambda_{\min}(S[H,G,\beta,\gamma])}{(1+|D|^2)\lambda_{\max}(H)} + \gamma\ \right\}, t \geq 0.$$

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ROUGH CONVERGENCE OF A SEQUENCE OF FUZZY NUMBERS[1]

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In this talk, first we define rough convergence of a sequence $\{X_i\}$ of fuzzy numbers–based on Phu's definition–as follows:

The sequence $\{X_i\}$ is said to be r-convergent to the fuzzy number X_*, denoted by $\{X_i\} \xrightarrow{r} X_*$, provided that

$$\forall \varepsilon > 0 \ \exists i_\epsilon \in \mathbb{N}: \ i > i_\epsilon \Rightarrow d(X_i, X_*) < r + \varepsilon,$$

where r is a nonnegative number, and d is the supremum metric on the set $L(\mathbb{R})$ of fuzzy numbers. The set

$$LIM^r X_i := \{X_* \in L(\mathbb{R}) : X_i \xrightarrow{r} X_*\}$$

is called the r-limit set of the sequence $\{X_i\}$. Then we characterize the set $LIM^r X_i$ when the sequence $\{X_i\}$ is convergent and when it is not. Later, we show that $\{X_i\}$ is bounded if, and only if, it is rough convergent. Finally, we investigate some topological properties of the set $LIM^r X_i$.

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EVALUATION OF FAULT INDICTION IN POWER ELECTRICAL CABLES USING FUZZY SYSTEMS

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Transmission networks and distribution systems have made great progress and more advances than the past decade and it is considerable in high voltage networks and the ground medium voltage network. Ground network have more reliability than air networks, but their Fault Indication are difficult and time consuming, and require expensive and advanced equipment and high expertise. Fuzzy systems are knowledge-based systems and the heart of a fuzzy system is knowledge base that has been formed if - then rules. In this paper, determination of fault type and location of cable fault using fuzzy system will be studied. In this study Phase voltage and current are as the input designed fuzzy system.

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COMPUTER SIMULATION OF THE MODIFICATED STOCHASTIC - CONTINUAL MODEL OF COMBAT OPERATIONS VIA SIMULINK

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Let us consider the following modificated model of combat operations between two opposite parties (so called Lanchester models of combat operations):

$$\begin{cases} \frac{dx(t)}{dt} = -a_{11}x(t) - a_{12}y(t) + P(t, \zeta_1), \\ \frac{dy(t)}{dt} = -a_{21}x(t) - a_{22}x(t)y(t) - a_{23}y^2(t) + Q(t, \zeta_2). \end{cases} \tag{1}$$

$$x(0) = x_0, \;\; y(0) = y_0. \tag{2}$$

where, t is the time, $x(t)$ and $y(t)$ - are the quantities of alive forces of the opposing combat parties respectively, $a_{11}, a_{12}, a_{21}, a_{22}$ and a_{23} are the specific constant factors related with firepower rate and interior discipline rate of corresponding parties; x_o and y_o - are initial conditions (initial alive forces of both sides). We have accepted participation of non-regular (guerilla) armed subdivisions in the party of y. $P(t, \zeta_1)$ and $Q(t, \zeta_2)$ - are the periodic reinforcements functions with the stochastic parameters 1 and 2. the system of nonlinear ODE-s (1)-(2) we have simulated with using of SIMULINK, the MATLAB subsystem for simulation and modeling of dynamical systems. To implement of stochastic parameters ζ_1 and ζ_2 into dynamical system (1), we have used specific SIMULINK blocks for generation of random signals. The results of simulations were successfully: we have achieved main research goal. The designed SIMULINK model really has an effective flexibility and very convenient graphical outputs (the graphics of numerical solutions depending and phase portraits for analyzing stability of the stochastic dynamical model). So, results of computer simulations have showed us that the internal competition in the party y and consideration of stochastic parameters really have a big effect: they drastically have influenced to the dynamics of combat operations. On the other hand, with specially designed GUI of this SIMULINK model the user can very easily evaluate parameters of dynamical system during the simulation procedure.

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NUMERICAL SOLUTION OF FUZZY FREDHOLM INTEGRAL EQUATIONS BY THE SPLINE INTERPOLATION[1]

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In this paper, we propose a new numerical method by fuzzy spline interpolation to approximate the solution of the following linear fuzzy Fredholm integral equation of the second kind: $\tilde{F}(t) = \tilde{f}(t) + + \lambda \int_a^b k(s,t)\tilde{F}(t)ds$ where $\lambda > 0$,$K(s,t)$ is an arbitrary kernel function over the square $a \leq s,t \leq b$ and $\tilde{f}(t)$ is a fuzzy function of t. We consider fuzzy Spline interpolation $\tilde{y}(x) \approx \sum_{i=0}^{n} s_i(x)\tilde{y}(x_i)$, where s_i is fuzzy polynomial spline that interpolates the (x_i, δ_{ij}),δ_{ij} is Kronecker delta and obtain an upper bound for the distance between the exact and numerical solution. For this purpose, we apply the extension principle.

We prove the following theorem and introduce some numerical examples for iterative procedure for numerical solution of Fredholm integral equation of the second kind.

Theorem: *Consider the following iterative procedure* $\tilde{y}_0 = \tilde{f}(t), \tilde{y}_k(t) = \tilde{f}(t) + \lambda \sum_{i=0}^{n}(S_i.K(t_i,t)) \otimes \otimes \tilde{y}_{k-1}(t_i)$, *where* $k \geq 1, n \in N$, $S_i = \int_{t_0}^{t_n} s_i(t)dt$ *and* $\Delta : a = t_0 < t_1 < ... < t_n = b$ *is a partition of the interval* $[a,b]$.

Then the upper bound for the distance between the exact $(\tilde{F})$ *and numerical solution* $(\tilde{y}_k)$ *for the Fredholm integral equation is* $D(\tilde{F}, \tilde{y}_k) \leq \frac{l^k}{1-l} D(\tilde{F_1}, \tilde{F_0}) + \frac{l}{1-l}(\omega(\tilde{y}_m, v(\Delta)) + m_l(1 + \frac{2\hat{S}}{b-a}))$, *where* $\hat{S} = = max_{i=0,\ldots,n} \mid S_i \mid$ *and* $m_l = max\{m_0, ..., m_{k-1}\}$, $l = \lambda M(b-a)$, $M = max \mid K(s,t) \mid$, $a \leq s,\ t \leq \leq b$, $m_i = sup_{a \leq t \leq b} \parallel \tilde{y_i(t)} \parallel$.

According to the relation in theorem we can observe that the convergence of the iterative procedure is satisfied if the second term of the relation is a small value.

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RESEARCH OF ONE-LINEAR SYSTEM OF MASS SERVICE FROM NOT ORDINARY ENTERING STREAM

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The one-linear system of mass service on which is considered acts markovs a stream of the moments of occurrence of requirements which parameter during the moment t is equal $\lambda_i$if during this moment the length of turn was equal i. If during the moment of receipt of group of requirements the length of turn was equal i, probability of that in group will be j requirements, is equal $a_j^{(i)} \sum_{j=1}^{\infty} a_j^{(i)} = 1$. Speed of service is equal α_i, if the length of turn is equal i. At arrival of group of requirements, for example in quantitquantity{amount}asses in λ_{i+j}, α_i- century α_{i+j}As well as earlier, for the requirement, during the moment of which beginning of service the length of turn was equal i, it is necessary to perform quantity of work, distribution under the law $H_i(x)$. the length of turn includes serving requirements. We shall designate in $\nu(t)$ length of turn during the moment t. It is necessary to find distribution length of turn the moment t.

Acting also as [1] we shall receive recurrent parities for $\tilde{P}_i(u)$ and $\tilde{\varphi}_i(s,u)$.

$$\tilde{P}_1(u) = \frac{1}{(u+\lambda_1)\tilde{h}_1\left(\frac{u+\lambda_1}{\alpha_1}\right)} \left[\tilde{P}_0(u)(u+\lambda_0)\left(1-\tilde{h}_1\left(\frac{u+\lambda_1}{\alpha_1}\right)\right) + P_1^{(0)}\tilde{h}_1\left(\frac{u+\lambda_1}{\alpha_1}\right) - \right.$$
$$\left. - P_0^{(0)}\left(1-\tilde{h}_1\left(\frac{u+\lambda_1}{\alpha_1}\right)\right) - \tilde{\varphi}_1^{(0)}\left(\frac{u+\lambda_1}{\alpha_1}\right)\right], \qquad (1)$$

$$\tilde{\varphi}_1(s,u) = \frac{1}{(u-s\alpha_1+\lambda_1)\tilde{h}_1\left(\frac{u+\lambda_1}{\alpha_1}\right)} \left[\left((u+\lambda_0)\tilde{P}_0(u) - P_0^{(0)}\right)\left(\tilde{h}_1(s) - \tilde{h}_1\left(\frac{u+\lambda_1}{\alpha_1}\right)\right) + \right.$$
$$\left. + \tilde{\varphi}_1^{(0)}(s)\tilde{h}_1\left(\frac{u+\lambda_1}{\alpha_1}\right) - \tilde{\varphi}_1^{(0)}\left(\frac{u+\lambda_1}{\alpha_1}\right)\tilde{h}_1(s)\right], \qquad (2)$$

$\delta_{ij} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases}$ Also $\tilde{P}_0(u)$ it is defined from a condition $\sum_{i=0}^{\infty} \tilde{P}_i(u) = \frac{1}{u}$.

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THE LIMIT OF SEQUENCE OF A NEARLY CRITICAL BRANCHING PROCESSES WITH IMMIGRATION

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Let for each $n \in N$ $\left\{\xi_{k,j}^{(n)},\ k,j \in N\right\}$ and $\left\{\xi_{k}^{(n)},\ k \in N\right\}$ be two independent a sequence of independent, nonnegative integer – valued and identically distributed random variables. We define for each $n \in N$ branching process with immigration $\left\{X_0^{(n)} = 0,\ k = 0, 1, ...\right\}$ recurrently

$$X_0^{(n)} = 0,\ X_k^{(n)} = \sum_{j=1}^{X_{k-1}^{(n)}} \xi_{k,j}^{(n)} + \varepsilon_k^{(n)},\ k \in N \tag{1}$$

Assume that

$$m_n = E\xi_{1,1}^{(n)},\ \lambda_n = E\varepsilon_1^{(n)},\ \sigma_n^2 = var\xi_{1,1}^{(n)},\ b_n^2 = var\varepsilon_1^{(n)},\ \tau_n = E\left(\varepsilon_{1,1}^{(n)}\right)^3,\ \gamma_n = E\left(\varepsilon_1^{(n)}\right)^3$$

are finite for all $n \in N$. The sequence (1) is said to be nearly critical, if $m_n \to 1$ as $n \to \infty$.

Let $\{d_n,\ n \in N\}$ be sequence of positive real a number such that $d_n \to \infty$ and $nd_n^{-1} \to \infty$ as $n \to \infty$.

In this paper we will prove next theorem.

Theorem. *Suppose that* $0 \le \gamma < 1$ *and*

1) $m_n = 1 + \alpha d_n^{-1} + o\left(d_n^{-1}\right),\ \alpha > 0,\ d_n\sigma_n^2 \to \sigma^2$ as $n \to \infty$,
2) $d_n^{\gamma}\lambda_n \to \lambda \ge 0,\ d_n^{\gamma} b_n^2 \to b^2 \ge 0$ as $n \to \infty$,
3) $\tau_n \le m_n^3,\ n \in N$ and $d_n^{\frac{3\gamma-1}{2}}\gamma_n \to 0$ as $n \to \infty$.

Then distribution of $\left(m_n^n d_n^{(1-\gamma)/2}\right)^{-1}\left(X_n^{(n)} - EX_n^{(n)}\right)$ weak convergence as $n \to \infty$ to the normal distribution with mean 0 and variance $(2\alpha)^{-1}\left(\alpha^{-1}\sigma^2\lambda + b^2\right)$.

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PROPERTIES OF THE ENTROPY OF EVENTOLOGICAL SUBDISTRIBUTIONS

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The term entropy is widely applied in various areas of knowledge. Probabilistic treatment of statistical entropy (or entropy of Boltzmann) and information entropy (or entropy of Shannon) are projected on entropy in eventology [1] – a new direction of probability theory. Eventology is based on the Kolmogorov's axiomatics which is added by two eventological axioms: a sufficiency and a simpliciality [2]. The basic objects of researches are random events, sets of random events and their eventological distributions (E-distributions). Eventology studies the structures of the dependences of the sets of events. It allows to include the mathematical model of a person, together with his/her persuasions, in a subject of the scientific research of in the form of E-distribution of the set of his/her own events, i.e. allows to consider any kinds of the set of events which are perceived and/or created by a person. New eventological language, new eventological methods and approaches allow to formulate and solve various tasks in socio-economic areas which did not manage to formulate and solve earlier within the limits of traditional approaches.

Let's consider the finite set of random events $\mathfrak{X} \subseteq F$, chosen of algebra of probability space $(\Omega, F, \mathbf{P})$. E-distribution of the set of random events $\mathfrak{X}$ of the power $N = |\mathfrak{X}|$ is a collection $\{p(X), X \subseteq \mathfrak{X}\}$ of 2^N probabilities of event-terraces [1] generated by this set of events in which $p(X) = \mathbf{P}\left(\mathrm{ter}(X)\right) = \mathbf{P}\left(\bigcap_{x\in X} x \bigcap_{x\in X^c} x^c\right)$, $X \subseteq \mathfrak{X}$ where $x^c = \Omega - x, X^c = \mathfrak{X} - X$. The event-terraces $\mathrm{ter}(X)$ form the partition Ω in all $X \subseteq \mathfrak{X}$ and provide of probabilistic normalization $\sum_{X\subseteq\mathfrak{X}} p(X) = 1$ for this sort of E-distribution of the set $\mathfrak{X}$. The more events contains a given set of events, the more complex structures of probabilistic dependencies of events it has.

In given work the definitions of eventological subdistributions (E-subdistributions) of the set of random events $\mathfrak{X}$ are introduced. It is dictated by necessity to analyze not all the set of events $\mathfrak{X}$ entirely, but only its part - a subset of events $\mathfrak{Y} \subseteq \mathfrak{X}$ from the original set of events $\mathfrak{X}$. Thus, E-subdistributions of the set of events $\mathfrak{X}$ can be interpreted as E-distribution of a subset of events $\mathfrak{Y} \subseteq \mathfrak{X}$ of the power $m = |\mathfrak{Y}|$, $m = 1, \ldots, |\mathfrak{X}|$, which are sets from 2^m probabilities and $\sum_{X\subseteq\mathfrak{Y}} p(X) = 1$.

Entropy of E-distribution $\{p(X), X \subseteq \mathfrak{X}\}$ of the set of events $\mathfrak{X}$ is defined under the classical formula just as entropy of probabilistic distribution or entropy of a random variable with the finite number of values is defined in probability theory and can be interpreted as a measure of uncertainty of E-distribution of the set of events [1], [3], [4]. Entropy of E-subdistribution of the set of events $\mathfrak{X}$ can be defined by analogy as the entropy of E-distribution and interpreted as a measure of uncertainty of E-distribution of the subset of events $\mathfrak{Y} \subseteq \mathfrak{X}$. In given work the properties of the entropy for E-subdistribution are formulated and proved. That expands a mathematical toolkit of eventology. Introduced definitions of E-subdistribution and properties of the entropy for them are illustrated on the simple example for any triplet of events, considering subsets of various powers.

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A NEW ALGORITHM TO FORECAST FUZZY TIME SERIES WITH C PROGRAMMING

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Neural network have been successfully used in nonlinear problems and relationships. To forecast fuzzy time series, this study aims to improve forecasting performance with different degrees of membership in establishing fuzzy relationships by using neural networks. Differing from previous studies, we used various degrees of membership in establishing fuzzy relationships with various numbers of hidden nodes and we performed different neural network models such as Generalized Regression Neural Networks, Multilayer Perceptron and Radial Basis Function to improve forecasting performance. We also used these models for forecasting without using fuzzy approach. We made a comparison study to improve forecasting performance of enrollment data for the University of Alabama and Istanbul Stock Exchange (ISE) national-100 index during 2006-2010 years with different number of hidden nodes and using an approach for determining the length of intervals proposed by Huarng[1]. Conventional time series refer to real numbers, but fuzzy time series are structured by fuzzy sets[2]. Song and Chissom first proposed the definitions of fuzzy time series [3]. A fuzzy set is a class of objects with a continuum of grade of membership. The some general definitions of fuzzy time series are given as follows: **1.** Let U be the universe of discourse, where $U = u_1, u_2, ..., u_n$ and u_i are possible linguistic values of U , then a fuzzy set of linguistic variables A_i of U is defined by

$$A_i = \mu_{A_i}(u_1)/u_1 + \mu_{A_i}(u_2)/u_2 + \cdots + \mu_{A_i}(u_n)/u_n \tag{1}$$

where μ_{A_i} is the membership function of the fuzzy set $A_i, \mu_{A_i} : U \to [0, 1]$. u_k is an element of fuzzy set A_i and $\mu_{A_i}(u_k)$ is the degree of belongingness of u_k to A_i. $\mu_{A_i}(u_k) \in [0, 1]$ and $1 \leq k \leq n$.
2. Let $Y(t)$ which is a subset of real numbers be the universe of discourse defined by the fuzzy set $f_i(t)$. If $F(t)$ consists of $f_i(t)$ $(i = 1, 2, ...)$, then $F(t)$ is defined as a fuzzy time series on $Y(t)$. **3.** If there exists a fuzzy relationship $R(t-1, t)$, such that $F(t) = F(t-1) \times R(t-1, t)$, where $\times$ is an operator, then $F(t)$ is said to be caused by $F(t-1)$. The relationship between $F(t)$ and $F(t-1)$ can be denoted by

$$F(t-1) \to F(t). \tag{2}$$

The novelty of this study is to apply a new algorithm based on Yu and Huarng's method(2010)[4] algorithm with some changes and modifications for improving forecasting performance of neural network-based fuzzy time series and use C programming language in some steps of algorithm. Also, differing from previous studies, this study decides the length of intervals by using an effective approach instead of using constant length of intervals as 100 or 1000 by following previous studies. After obtaining the best results for our algorithm with C programming, it can be said that this method outperforms other methods including Yu and Huarng's neural network-based fuzzy time series method proposed in the literature.

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WAVE PREDICTION BY REGRESSION SPLINES AND ADAPTIVE NEURO-FUZZY INFERENCE SYSTEM

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In this paper we made nonparametric analysis using P-splines and cubic regression spline model and we used adaptive neuro-fuzzy inference system. For this purpose we used significant wave height data set. The selection of optimal smoothing parameter is made by Generalized Cross Validation (GCV). In this study we concentrate on B-splines with penalties, known as P-splines ([1], [2]). Eilers and Marx (1996) showed how P-splines can be used in many different contexts and illustrate their remarks with examples on density estimation and nonparametric smoothing.
From a least-squares perspective, the coefficients are chosen to minimize

$$S = \sum_{i=1}^{m} \left\{ y_i - \sum_{j=1}^{n} a_j B_j(x_i) \right\}^2 + \lambda \sum_{j=k+1}^{n} (\Delta^k a_j)^2. \tag{1}$$

For least squares smoothing we have to minimize S in (1). The system of equations that follows from minimization of S can be written as:

$$B'y = (B'B + \lambda D_k' D_k)a. \tag{2}$$

Where D_k is a matrix representation of the difference operator Δ^k, and the elements of $\mathbf{B}$ are $b_{lj} = B_j(l)$. The basic idea of Generalized Cross Validation is to replace the denominators $1-(S_\lambda)_{ii}$ of $CV(\lambda)$ by their average $1-n^{-1}tr(S_\lambda)$, giving GCV score function

$$GCV(\lambda) = \frac{1}{n} \frac{\sum_{i=1}^{n} \left\{ y_i - \hat{f}_\lambda(x_i) \right\}^2}{\left\{1-n^{-1}tr(S_\lambda)\right\}^2}. \tag{3}$$

As for $GCV(\lambda), \lambda$ is chosen as the minimizer of $GCV(\lambda)$.

Adaptive neuro-fuzzy inference system(ANFIS) was first introduced by [3]. An ANFIS is a network structure consisting of a number of nodes connected through directional links. Each node is characterized by a node function with fixed or adjustable parameters. The basic learning rule is the well-known backpropagation method which seeks to minimize some measure of error [4].

Depending on the types of inference operations upon "if-then rules", most fuzzy inference systems can be classified into three types: Mamdani's system, Sugeno's system and Tsukamoto's system. In first-order Sugeno's system, a typical rule set with two fuzzy IF/THEN rules can be expressed as

Rule 1: If x is A_1 and y is B_1, then $f_1 = p_1x + q_1y + r_1$
Rule 2: If x is A_2 and y is B_2, then $f_2 = p_2x + q_2y + r_2$

More information for ANFIS can be found in related literature [3].

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ASYMPTOTICAL BEHAVIOUR OF THE JOINT DISTRIBUTION OF THE MOMENT OF THE FIRST EXIT AND THE RANDOM WALK OVER NONLINEAR BOUNDARY

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Let $\xi_n, n \geq 1$ be an independent equally distributed random variables with $E|\xi_1| < \infty$, defined in some probability space (Ω, F, P) and let $f_a(t), t > 0, a > 0$ be some set of functions from the increasing parameter a, $f_a(1) \uparrow \infty$ by $a \to \infty$.

Let's consider the moment of the first exit

$$\tau = \tau_a = \inf\{n \geq 1 : S_n > f_a(n)\}$$

of the random walk for $S_n = \xi_1 + ... + \xi_n, n \geq 1$ nonlinear boundary $f_a(t)$.

A lot of works (see [1]-[3]) have been devoted to the study of asymptotic properties of the distribution of the moment of the first exit τ_a.

In the present work for enough wide class of nonlinear boundaries $f_a(t)$ a theorem on asymptotic behavior of the joint distribution of the normalized moment of the first exit τ_a is proved and τ_a $\chi_a = S_\tau - f_a(\tau)$ at $a \to \infty$. In particular, from this theorem follows that normalized moment of the first exit τ_a and χ_a asymptotically are independent by $a \to \infty$.

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ASYMPTOTICAL BEHAVIOUR OF LOCAL PROBABILITIES INTERSECTIONS OF NONLINEAR BOUNDARIES BY THE RANDOM WALK

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Let $\xi_n,\ \ n \geq 1$ be a sequence of the independent equally distributed random variables defined in some probability space (Ω, F, P).

Let's suppose

$$S_n = \sum_{k=1}^{n} \xi_k,\ n \geq 1.$$

We consider the moment of the first exit

$$\tau_a = \inf \{n \geq 1 : S_n \geq f_a(n)\} \tag{1}$$

of the random walk for $S_n,\ n \geq 1$ nonlinear boundary $f_a(t),\ t > 0$, depending on some increasing parameter $a > 0$.

At usual let's consider that $\inf \{\emptyset\} = \infty$.

In work [1] the asymptotical behaviour of the conditional probability of intersection of boundaries $P\left(\tau_a \geq n \left| \bar{S}_n = x\right.\right)$ is studied by $a \to \infty$ for enough wide class of nonlinear boundaries and random walks $f_a(t)$.

Using the results of the work [1] in the present work an asymptotical behaviour of the density of joint distribution τ_a and χ_a by $a \to \infty$, also probability $P(\tau_a = n)$ and conditional density of exit χ_a under the condition $\tau_a = n$ are studied.

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ECONOMICAL MATHEMATICAL FUZZY MODEL OF PROFIT MAXIMIZATION IN E-SHOP

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It was considered economical mathematical model of profit maximization in e-shop with applying fuzzy logic in the work.

The profit depends on price and costs. The price for each kind of product is defined in the market.

But costs are regulated by e-shop. And costs depend on some quantity and quality factors. Usually, these factors are uncertain. The uncertain factors are working by different methods. One of them is fuzzy theory. That's why for defining the crisp value of the total costs for each product we'll use the expert values of these quantity and quality factors (Veb site, advertising and procedural costs and etc.), which are given linguistic. In order, on the base of expert values it was defined term sets and interval values of linguistic variables.

Then we use fuzzy logic inference method for defining each cost.

Fuzzy logic inference method consists of following steps:

(1) **Fuzzyfication**. In this step is created the membership function of the fuzzy sets.
(2) **Creating rules**. It is created logic rules on the base of expert reviews.
(3) **Composition-** is aggregation of the fuzzy inferences.
(4) **Defuzzyfication**. In this step is defined crisp value by centroid method.

So it was defined crisp value of the each cost. Then, on the base of crisp values of each cost we define total costs. So, by subtraction total costs from price was defined profit for each product.

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STOCHASTIC PROPERTIES OF THE DYNAMIC SERIES OF LINGUISTIC SIGNS

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As we know, any sign of a mass phenomenon is characterized not only by a single number, but as a rule, with the several quantitative traits. Numeral characteristic of any sign of a mass phenomenon, in general, is not constant, but varies with the time. For example, the origin of rare linguistic events are subject to Poisson probability theory with parameter λ and time-dependent, $\lambda(t)$.

As a result of changes in the numerical characteristics of the linguistic sign is formed of a sequence is called a dynamic series.

Dynamic range, which characterized the mass phenomenon usually characterized by a specific feature, consisting of the addition, $\eta_i,\ i = 1, ..., n$ that every member of this series is:

(1) From certain "level" γ;

(2) From a random deviation $\xi_k,\ \ k = 1, ..., n$: from this level

$$\eta_i = \gamma + \xi_i,\ \ i = 1, ..., n. \tag{1}$$

The first term γ on the right side of this oversight is essential typical features of this mass phenomenon, for example, the average length of word forms of language and the second $\xi_i, i = 1, ..., n$ random deviation from the level γ. In quantitative linguistics, where we study the quantitative regularities of language, there are such linguistic phenomens as: information weight of morphemes and words; analytic of language; frequency of words and phrases, etc, each of which is to some extent characterized by the amounts, thus allowed mathematical interpretation.

Information, stimulus, speech $\qquad$ Useful signal S and noise $\xi_t(\omega)$

$$\begin{array}{ccc} & \downarrow\leftarrow & \\ I = S & + & \zeta_t(\omega). \end{array} \tag{2}$$

Here under the random component $\xi_t(\omega), t = T, \omega = \Omega$ is meant a device or chain property, in particular the Markov property. Note that, such an expansion takes place in the theory of perturbation of operators. In modern Azerbaijan, compound components of the text and some relations according to this view also meet.

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ON THE RESTORATION PROBLEM WITH DEGENERATED DIFFUSION

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The sufficient conditions of restoration problem's solvability in a class of the Ito stochastic differential systems of the first order (with random disturbances from a class of Wiener processes and the diffusion degenerated with regard to a part of variables) on the given properties of a movement, when a control is included into the coefficient of drift, are obtained by separation method.

Let us give the system of the stochastic differential equations of first order of Ito type

$$\begin{cases} \dot{y} = f_1(y,z,v,w,t), \quad y \in R^{l_1}, \quad z \in R^{l_2}, \quad v \in R^{p_1}, \quad w \in R^{p_2}, \\ \dot{z} = f_2(y,z,v,w,t) + \sigma_1(y,z,v,w,t)\dot{\xi}, \quad \xi \in R^r, \\ \dot{v} = f_3(y,z,v,w,t) + L_1(y,z,v,w,t)u_1, \quad u_1 \in R^{k_1}, \quad u_2 \in R^{k_2}, \\ \dot{w} = f_4(y,z,v,w,t) + L_2(y,z,v,w,t)u_2 + \sigma_2(y,z,v,w,t)\dot{\xi}, \end{cases} \tag{1}$$

where $l_1 + l_2 + p_1 + p_2 = n$. It is required to define the vector-functions $u_1(y,z,v,w,t) \in R^r$ and $u_1(y,z,v,w,t) \in R^r$, entering into pulling down coefficient, by given integral manifold

$$\Lambda(t): \quad \lambda(y,z,v,w,t) = 0, \quad where \quad \lambda = \lambda(y,z,v,w,t) \in C^{12121}_{yzvwt}, \lambda \in R^m. \tag{2}$$

The posed problem is solved in [1] by other method, namely, by a quasi-inversion method.

The following theorem is valid and it is proved by separation method.

Theorem. *The set (2) is integral manifold of system of differential equations (1) if the following requirements are satisfied:* 1) *square submatrixes* D', G' *of matrixes* D, G *are nondegenerate* det $D' \neq$ $\neq 0$, *det* $G' \neq 0$; *2) under arbitrarily given* u_1, $u_2'' \in K$ *the first* m *coordinates* u_2' *of vector* u_2 *looks like* $u_2' = (D')^{-1}\big(N - \frac{\partial \lambda}{\partial v} L_1 u_1 - D'' u_2''\big)$; *3) under arbitrarily given* σ_1, $\sigma_2'' \in K$ *a submatrix* σ_2' *of matrix* σ_2 *looks like* $\sigma_2' = (G')^{-1}\big(B - \frac{\partial \lambda}{\partial z}\sigma_1 - G''\sigma_2''\big)$.

Here $D = \frac{\partial \lambda}{\partial w} L_2$, $\quad D = (D', D'')$, $\quad \frac{\partial \lambda}{\partial w} = (G', G'')$, $\quad \sigma_2 = \begin{pmatrix} \sigma_2' \\ \sigma_2'' \end{pmatrix}$, $\quad u_2 = \begin{pmatrix} u_2' \\ u_2'' \end{pmatrix}$, where D', G' are square matrixes of dimensionality $(m \times m)$, D'' - $(m \times (k_2 - m))$ matrix, G'' - $(m \times (p_2 - m))$ matrix, σ_2' - $(m \times r)$ matrix, σ_2'' - $((p_2 - m) \times r)$ matrix, u_2' - m-vector, u_2'' - $(k_2 - m)$- vector; $N = A -$ $- \left(\frac{\partial \lambda}{\partial t} + \frac{\partial \lambda}{\partial y} f_1 + \frac{\partial \lambda}{\partial z} f_2 + \frac{\partial \lambda}{\partial v} f_3 + \frac{\partial \lambda}{\partial w} f_4 + S_1 + S_2\right)$, $S_1 = \frac{1}{2}\left[\frac{\partial^2 \lambda}{\partial z \partial z} : \sigma_1 \sigma_1^T\right]$, $S_2 = \frac{1}{2}\left[\frac{\partial^2 \lambda}{\partial w \partial w} : \sigma_2 \sigma_2^T\right]$. The vector-function A and the matrix B possess property $A(0; y,z,v,w,t) \equiv 0$, $B(0; y,z,v,w,t) \equiv 0$ and an equality $\dot{\lambda} = A(\lambda; y,z,v,w,t) + B(\lambda; y,z,v,w,t)\dot{\xi}$ has place. K is a class of functions, which are continuous on t and Lipschitzian on y, z, v and w in a neighborhood of set $\Lambda(t)$.

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SECTION VI

Computational Mathematics

HIGH ACCURACY FACTORIZATION ALGORITHM FOR THE IRREGULAR POLYNOMIAL MATRIX RELATIVELY THE IMAGINARY AXIS

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The iterative factorization algorithm for the is offered irregular polynomial matrix relatively imaginary axis with use of procedure Symbolic Toolbox of package Matlab . Efficiency of this algorithm is illustrated by examples. In irregular polynomial matrix on is reduced for the regular polynomial matrix. For this purpose package Matlab procedure SVD.m is used(singular decomposition of a matrix) which supporting symbolical calculations. Reducing the unitary matrix into the diagonal matrix. Multiplying by the unitary matrix to given polynomial matrix on the left and right sides to until then while in result it will be received a regular polynomial matrix. Here the algorithm described in [1] is used for the multiplication of two polynomial matrix. Further algorithm [2,3] factorization a polynomial matrix will be used. It is known, that factorization of the polynomial matrix is reduced to the solution of algebraic Riccati equation (ARE). The for solution AER with use of procedure Symbolic Toolbox high accuracy software is created. It plays the important role at factorization polynomial matrix. For definition of a required polynomial matrix is used divisions of two a polynomial matrix. Thus is used algorithm Euclid supporting symbolical calculations. The software is created for factorization an irregular polynomial matrix concerning an imaginary axis in Matlab.

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DIFFERENCE SCHEME CORRECTNESS RESEARCH FOR NON-EQUILIBRIUM FILTRATION MODEL OF TWO-PHASE INCOMPRESSIBLE LIQUID[1]

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In this work difference scheme for numerical realization of the model of non-equilibrium filtration of two immiscible incompressible liquids is made up and scheme stability and convergence are studied.

Mathematical problem statement of non-equilibrium filtration of two incompressible liquids has the following form:

$$\bar{\tau}\nu m\frac{\partial\eta}{\partial t} = \bar{\tau}div\big(K_0 a\nabla\eta + \vec{F} - b\vec{v}\big) + m(1-\nu)\Big(s_0 e^{-\frac{t}{\bar{\tau}}} - \eta + \frac{1}{\bar{\tau}}\int\limits_0^t \eta e^{-\frac{(t-\xi)}{\bar{\tau}}}d\xi\Big), \tag{1}$$

$$div(K\nabla\rho + \overrightarrow{f}) = 0, \tag{2}$$

$$\tau\frac{\partial s}{\partial t} - \tau\nu(x)\frac{\partial\eta}{\partial t} = æ(x)(\eta - s), \tag{3}$$

here s, η are true and effective saturation, p is pressure, m is porosity, ν is non-equilibrium degree, K is permeability of the medium, k_{01}, k_{02} are phase permeability (of water and oil), μ_1, μ_2 are coefficients of viscosity of the corresponding phases, $\bar{\tau} = \frac{\tau}{\chi}$ is substitution time, $\vec{v} = \vec{v}_1 + \vec{v}_2$ is filtration speed, $a = -\frac{\partial p_c}{\partial\eta}\cdot\frac{k_{01}k_{02}}{k}$, $b = \frac{k_{01}}{k}$.

(1) -(3) equations were considered in the following initial and boundary conditions:

$$\begin{gathered} s|_{t=0} = s_0,\ \eta|_{t=0} = s_0,\ p|_{t=0} = P_0(x), \\ \eta|_\Gamma = \mu(x,t),\ p|_\Gamma = P_0(x) \end{gathered} \tag{4}$$

here, $x = (x_1, x_2)$, $x \in G$ is bidimentional final area with Γ border.

Difference scheme of (1)-(4) boundary problem has the following form:

$$\begin{gathered} m_{ij}\nu\frac{\eta_{ij}^{n+1} - \eta_{ij}^n}{\tau} = K_0 a_{ij}\Delta_h\eta_{ij}^n - m_{ij}(1-\nu)\tfrac{1}{\bar{\tau}}\big(1 - \tfrac{\tau}{2\bar{\tau}}\big)\eta_{ij}^{n+1} + \tfrac{m_{ij}(1-\nu)}{2}\tfrac{\tau}{\bar{\tau}^2}e^{-\frac{\tau}{\bar{\tau}}}\eta_{ij}^n + \\ +(div_h\vec{F}_{ij} - b_{ij}div_h\vec{v}_{ij}) + m_{ij}(1-\nu)\tfrac{1}{\bar{\tau}}\Big(s_0 e^{-\frac{t}{\bar{\tau}}} + \tfrac{1}{\bar{\tau}}e^{-\frac{(n+1)\tau}{\bar{\tau}}}\gamma_{ij}^n\Big), \end{gathered} \tag{5}$$

$$K_{ij}\Delta_h p_{ij}^{n+1} + div_{\bar{h}}\vec{f}_{ij} = 0, \tag{6}$$

$$s_{ij}^n = (1-\nu)s_0 e^{-\frac{(n+1)\tau}{\bar{\tau}}} + \nu\eta_{ij}^{n+1} + \frac{1}{\bar{\tau}}(1-\nu)e^{-\frac{(n+1)\tau}{\bar{\tau}}}\gamma_{ij}^{n+1}, \tag{7}$$

$$\begin{gathered} \eta_{ij}^0 = s_{ij}^0 = const,\ p_{ij}^0 = P_0(x_{ij}), \\ \eta|_{\gamma_h} = \mu(x_{ij}, (n+1)\tau),\ p|_{\gamma_h} = P_0(x_{ij}). \end{gathered} \tag{8}$$

The following theorem is proved.

Theorem. *Let us consider that* $m\nu - \frac{8\tau M_1}{h} \equiv \delta > 0$ *condition is true, then* (5)-(8) *difference scheme solution is stable and coincides with* (1)-(4) *problem solving.*

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WEIGHTED ESSENTIALLY NON OSCILLATORY SCHEMES FOR SUPERSONIC AIR FLOW

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In this work a planar supersonic turbulent multi-species flow is numerically simulated in the channel with perpendicular injection of hydrogen from slots located symmetrically on the lower and upper walls of the channel. Height of the channel is taken as 3cm, length 10cm. The slots of the width 0.1cm is located on distance 5cm from entrance. For the mathematical modelling of such flows the Favre-averaged Navier-Stokes equations for multi-species gas are used. For convenience of calculation the jet injection is considered from the bottom wall.

The WENO (Weighted Essentially Non oscillatory) scheme of 4th order has been used to approximate the inviscid fluxes of the system of the Favre-averaged Navier- Stokes equations. This scheme is based on the idea of ENO (Essentially Non oscillatory) - scheme developed in [1], which was constructed according to a principle of Godunov's scheme. Instead if choosing the smoothest stencil for interpolating polynomial for the ENO here was used a convex combination of all candidates to achieve the essentially non-oscillatory property. The algebraic Baldwin-Lomax's [2] turbulence model has been used to calculate the eddy viscosity coefficient. The boundary conditions are taken as: adiabatic no-slip walls at the bottom, supersonic inlet at the left boundary and sonic inlet on the slot; on the top condition of symmetry; on an outlet non- reflection condition [3]. In regions of large gradients, that is in the boundary layer, near the wall and at the slot level, the grid clustering is introduced. Then the system of the Navier- Stokes equations was formed in transformed generalized coordinates system. The computations were done on a staggered spatial grid with parameters: $2 \leq M_\infty \leq 4$, $M_0 = 1$, $Pr = 0.7$, $2 \leq n \leq 15$, here index "∞" concerns to values of parameters of a flow, index "0"- to values of parameters of a jet, $n = P_0/P_\infty$, where P_0-pressure in jet and P_∞-pressure in a flow.

For testing the method a following numerical experiment was performed: the sound hydrogen jet was injected perpendicularly to the main air flow through a slot of width 0.1 cm with parameters: $M_\infty = 3.75, M_0 = 1, Re = 62.73x10^6, n = 10.29, T_0 = 800K$, and $T_\infty = 629.34K$, the results are compared with solution obtained by using ENO scheme [1] and experimental work [4], it gave good agreements. The numerical experiment showed, that because of deceleration of the incoming flow, the pressure ahead of the jet increases, and a bow shock wave is formed. An oblique shock wave emanates upstream from the bow shock wave. In addition to the separation region, there is also a supersonic flow region behind the oblique shock wave; subsequent flow deceleration is accompanied by emergence of a second shock wave: barrel shock parallel to the jet axis. The detached shock wave, oblique shock wave, and barrel shock wave intersect at one point to form a complicated λ-shape structure of shock waves.

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THE PROBLEM OF NUMERICAL INTEGRATION COMPUTER IS NOT COMPUTABLE

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Computer as a technical device is constantly being improved, but in its modern version is nearing the limit of high-speed capabilities, which is due to the finite propagation of electrical impulse - "only" three hundred thousand kilometers per second. Moreover, as noted in [1, p.34] "*Given that the lifetime of the universe is approximately* 10^{10} *years* $<10^{18}$ *seconds, it is clear that none of the fantastic speed calculations do not provide the accuracy required by simple addition of the series*", where we mean the approximately sum of the series $S = \sum_{k=2}^{\infty} \frac{1}{k \ln^2 k}$ with accuracy 10^{-3} for this we need not less than $k,\ k+1 > e^{10^3} > 10^{300}$ members of the series. It follows that if all the people who have passed their way on earth, and living now and who should be born by the end of the twenty-first century (there are no more than a million billion) would have worked all the time as the immortal one on a million computers at a rate of a billion billion operations per second, that with all these impossible circumstances inflated will hold no more than $10^{18+15+6+18} = 10^{57}$elementary arithmetic operations. All of these rough calculations are given in order to dispel the prevailing misconceptions about brand "omnipotence of computers." In this connection, let us consider the practical implementation of algorithms in the method of Quasi Monte Carlo. In the paper [2-4] there are given algorithm of construction uniformly distributed in the unit cube $[0,1]^s$ $(s = 1, 2, ...)$ Korobov's grid

$$\eta_k(a) = \left(\frac{k}{p}, \left\{\frac{k}{p} a^{\frac{p-1}{l}}\right\}, ..., \left\{\frac{k}{p} a^{\frac{p-1}{l}(s-1)}\right\}\right) \quad (k = 1, ..., p)$$

for $\leq c(s)\, p \ln\ln p$ elementary arithmetic operations.

This algorithm is "almost" optimal in the sense that the p nodes of the grid is find on $<< p \ln\ln p$ elementary arithmetic operations.

We can show that the constant $c(s) \geq 72s6^s$ can not be significantly lowered, while the above upper bound is the number of elementary arithmetic operations of practical importance in financial mathematics $s = 365$ dimension even when the number of $p = 10$ nodes is of $c(s) \geq 6^{365} > 36^{180} > 10^{180}$.

Output is seen in transition on the basis of efficient algorithms for computational experiments [4]

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ON CONVERGENCE, ASYMPTOTIC BEHAVIOR AND COMPUTATIONAL ALGORITHM OF THE GENERALIZED EXPONENTIAL INTEQRAL FUNCTIONS

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In this paper, we investigate uniform convergence of the generalized exponential integral functions (GEI) arising in the study of quantum chemistry calculations of electronic structure of atomic and molecular systems. Using the uniform convergence, we study the properties and asymptotic behavior of the GEI functions. We also give an effcient algorithm for the computations of the values of the GEI functions.

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APPLICATION OF CUDA TECHNOLOGY FOR THE FILTRATION PROBLEM

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Filtering process requires very accurate calculation. The key factor of accuracy is the size of computing model's grid of the layer. More dense computational grid and bigger discrete areas provide more realistic results. This paper is important because it makes possible to conduct rapid calculation of large models of the layer, without the usage of expensive computing systems such as supercomputers. The novelty of this work is using GPU and CUDA technology to accelerate the calculations.

Nowadays, parallel computing is increasingly used in calculation in scientific applications. This paper considers the problem of filtering, in particular, the problem of oil displacement by water. It has been decided to use an efficient parallelization of Compute Unified Device Architecture (CUDA), which allows to speed up calculations to the tens- hundreds times. Choice of various methods for solving Poisson's equation is justified. Jacobi's method, the red-black Gauss-Seidel relaxation, which are suitable for GPU architecture, were selected. It is well known that the weakness of CUDA is working with memory that requires careful programming. In our calculation program there are huge amount of input data including a description of a complex non-uniform grid, and values of permeability, porosity, temperature, initial pressure, the saturation, capillary pressure for the reservoir, etc. The processing with such volume of data, unloading and loading operations significantly reduce the performance of CUDA. This article discusses effective techniques of working with memory in CUDA.

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ITERATIVE METHOD FOR DEFINITION OF THERMO GRADIENT FACTOR OF SOIL MOISTURE

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The first studies of permafrost migration revealed its important consequences for the economy. Its main positive values are replenishing of moisture content in the upper root layer of soil. At the same time, the redistribution of moisture in freezing soils and grounds can bring huge damage to national economy. In terms of agriculture it is very important to take into account negative impact of the moisture migration in soil on wintering conditions of winter cultures. Formation of ice layers, lenses and large crystals can lead to damage to root systems and a massive loss of winter cultures. Extremely negative consequence of moisture migration in freezing layer is a frost heaves leading to destruction of road and airfield pavements, foundations and structures foundation (Bastamov S.D. (1946), Goldstein, M.N. (1948), Puzakov N.A. (1950)). Participation of the liquid phase in the movement of moisture to the freezing front was proved experimentally by researches of Eisentadt R. (1955), Bakulin F.G. (1957), Globus, A.M. (1962), Jumikis A. (1962) Biersmans M., Dijkema K.M., de Vries D.A. (1978). Under the influence of the experimental works of Bouyoucos GJ (1915) there arose theories of film migration of moisture and the freezing front, based on the idea of a liquid film of unfrozen water covering the ice crystals and connecting with a capillary-film moisture of thickness (Fletcher NH, 1966). More clearly the theory of membrane migration was formulated by Taber S. (1954), whereby freezing of water wetting the ice crystals, causes movement of other molecules on its place from the water film that does not lose continuity due to the "tensile strength". Forces causing the migration of water, are called by Bouyoucos G.J. and Taber S. as "the forces of crystallization." In above studies it was unclear relation between permafrost moisture migration with its thermal transfer mechanisms that acts in the thawed soil - with a temperature gradient and the absolute temperature. In this paper the equation of heat propagation in a heterogeneous environment is studied. An approximate method for calculating the coefficient thermogradient multilayered soil is developed. Aprioristic estimations for the decision direct and interfaced adjoint problem are obtained. Limitation of the approached value for thermogradient factor is proved. Changes in temperature of soil and moisture on the surface of the earth are measured by special instrument. Under laboratory conditions, thermal characteristics of multilayered soil are defined. Numerical calculations with the data on a surface of the earth are carried out and thermogradient factor of a multilayered ground is defined. Calculated data were compared with measured data.

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INVESTIGATION OF BLOOD FLOW AS A NON-NEWTONIAN FLUID IN THE STENOTIC VESSELS

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Arterial stenosis is the consolidation of the entima layer of the vessel in which the arterial wall becomes sclerotic as a result of formation of a thin layer on it, which distorts the vessel (some researchers attribute this to overuse of lipids). Arterial stenosis leads to reduction of the blood flow in a proportion that sometimes blood clots and restricts the blood flow to the heart and causes unstable angina and cardiac muscle injury. It has been demonstrated that vessel fluid movement has an important role in progression of atherosclerosis. In recent years, vast investigation has been done on bio fluid movement in magnetic field. Magnetic systems have been established to separate the cells and magnetic article transfer. The most characteristic biofluid is the blood which behaves like a magnetic fluid. Oxygenated blood has magnetic properties while deoxygenated blood has paramagnetic properties. In this article non-Newtonian behavior of blood flow in stenotic blood vessels is studied and magnetic field effect on this flow in the stenotic sites will be investigated. It is an attempt to study the shape and radius of distortions in the stenotic sites on the fluid flow by mathematical modeling, In this study a proper finite difference scheme is used to solve momentum and continuity equations for non-Newtonian fluids in cylindrical coordinate systems. With numerical methods anyone can estimate effects of Heartman number (M), stenosis severity , Reynolds number(Re), force rule(n) on viscous flow, and wall shear stress. The results show that by increasing the Hartmann number, the axial velocity profile decreases. One can assume a mean velocity of the axial velocity to investigate the trend of the graph. According to the results high changes occur in the flow rate due to the severity of the stenosis in the vessel.

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METHODS OF THE DECISION OF MATHEMATICAL MODEL

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Usually mathematical models are constructed in the form of the differential or integrated equations. Fitting and identification of models are follows further. Often, by virtue of step-type behavior of the information, at a stage of identification it is necessary to pass from the differential equations to certainly - different, from integrated - to the sums, etc. Then on the basis of known data it is necessary to estimate unknown parameters the equation of model. The present work is devoted to some questions of identification of model, development of corresponding methods and the software. At identification of model it is possible to pass from variables standing in the equations of model, to other variables. It is possible to establish steady parities on available numbers of supervision between these variables. For example, in economic-mathematical models for exception of action of inflation in variables of model, believes:

$$x(t) = x^n(t)/d^f(t),$$

where $x^n(t)$ - observable a number of some variable, $d^f(t)$ - deflator. If a number of supervision $x^n(t)$ does not cause the big trust in the equations of model instead of $x^n(t)$ it is possible to use

$$x(t) = \mu x^n(t) + (1-\mu)x^e(t),$$

where $0 \leq \mu \leq 1$, μ - a degree of trust to a number supervision $x^n(t)$ $x^n(t)$, $x^e(t)$ - an expert estimation. It is possible to use smoothing. Linear smoothing:

$$x_p(t) = \sum_{s=-p}^{p} \alpha_s x^n(t+s), \ \sum_{s=-p}^{p} \alpha = 1, \ \alpha_s > 0.$$

Exponential smoothing:

$$x(t) = (x^n(t) - x^n(t-1))/(\ln x^n(t) - \ln x^n(t-1)).$$

Differential smoothing:

$$\frac{d^s x(t)}{dt^s} = \sum_{\theta=0}^{s} \alpha_\theta x^n(t-\theta).$$

Integrated smoothing:

$$\int_{t-\theta}^{t} x(t-s)q(s)ds = \sum_{i=0}^{\theta} \alpha_i x^n(t-i).$$

The smoothed derivative:

$$\frac{dx(t)}{dt_{(q)}} = \frac{\sum\limits_{\theta=-q}^{q} \theta x^n(t+\theta)}{\sum\limits_{\theta=-q}^{q} \theta^2}.$$

The smoothed trend:

$$\frac{1}{x(t)}\frac{dx(t)}{dt_{(q,s)}} = \frac{\sum\limits_{k=0}^{q-1} x^n(t-k)}{\sum\limits_{k=0}^{q-1} x^n(t-s-k)} - 1.$$

APPLICATION OF THE HOMOTOPY PERTURBATION METHOD FOR NONLINEAR (3+1)-DIMENSIONAL BREAKING SOLITON EQUATION

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In this paper we studied to in order to obtain numerical solution of the nonlinear (3+1)-dimensional breaking soliton equation (BSE) by Homotopy Perturbation Method (HPM) and then formed a tablo which is include numerical conclusion for this equation by drawing graphics of the this equation by means of Mathematica.

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NUMERICAL SOLUTION OF KLEIN-GORDON-ZAKHAROV EQUATION USING CHEBYSHEV CARDINAL FUNCTIONS

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In this article, a numerical method is used for solving Klein-Gordon-Zakharov equations. The approximate solutions can be found by using the basis of Chebyshev cardinal functions. The partial differential equations are reduced to nonlinear algebraic equations by using operational matrices of derivatives. The computational results are compared with those obtained in previous work and found that the method is efficient and accurate.

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THE SOLUTION OF LATERAL HEAT LOSS PROBLEM USING COLLOCATION METHOD WITH CUBIC B-SPLINES FINITE ELEMENT

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This work deals with the solutions of lateral heat loss equation by using collocation method with cubic B-splines finite elements. The stability analysis of this method is investigated by considering Fourier stability method. The comparison of the numerical solutions obtained by using this method with the analytic solutions is given by the tables and the figure.

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THE NUMERICAL MODELING OF THE THREE-DIMENSIONAL VISCOSITY PLASTICS MOVING SURFACE [1]

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We consider the unsteady flow regime of oil in two-ply beds. They have stacked by muck with equal porosity, but they have different conductivity. The bed with less conductivity has a greater power than well conductivited bed. The setting of mathematical problem is a nonlinear parabolic equation system. The flow rate and bottom pressure on exploitation bore are given. The condition of front definition of perturbed viscosity plastics area is given. The filtration current and pressure function on butt two beds are continuous. The metathetical process with environment are absent. We have found the law of spatial movement of perturbed viscosity plastics area in case of hydrodynamic communication of two-ply oil beds. [1]

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ON THE DEVELOPMENT OF PARALLEL ALGORITHMS FOR DISCRETE MODELS OF FLUID DYNAMICS

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This goal of this paper is the applying the parallel algorithms to construct the discrete models of Boltzmann equation.

In classical kinetic theory, the macroscopic description of a gas needs the knowledge of the macroscopic variables: density, momentum and temperature (or energy). For a discrete gas, the set of macroscopic variables can be different, and it depends on the model [1]. A models of Boltzmann equation for a simple discrete velocity were introduced by Carleman, Broadwell [2], Godunov, Sultangazin [3]. They are described in the one dimensional case by the system of nonlinear hyperbolic equations. The most general discrete model of the Boltzmann equation has been written by Cabannes [4].

We construct 32-velocity 3-dimensional model relative to the dodecahedron [5]. We consider only the binary collisions, which can be written in the following form:

$$\frac{\partial f}{\partial t} + \mathbf{u_i}\nabla f_i = \frac{1}{2}\sum_{j,k,l} A_{ij}^{kl}(f_k f_l - f_i f_j),\ (i,j,k,l = 1,...,32),$$

where the vectors $\mathbf{u_i}$ are constant vectors (discrete velocities); the coefficients A_{ij}^{kl} are non-negative constants (transition probabilities) and the unknowns $f_i(x,y,z,t)$ denote the densities of the particles with the velocity $\mathbf{u_i}$.

We have 270 nontrivial collisions between particles in this models.

In this way we can construct 64-velocity model, where other 32 velocities are equal $\frac{\sqrt{5}+1}{2}\mathbf{u_i}$. That model has 1080 nontrivial collisions.

It is easy to see that if the number of velocities increases than calculation of nontrivial collisions becomes more complecated. Parallel computing allows to simplify the process of constructing the discrete models with any number of velocities.The collisions in each group and between groups can be calculate in different threads independently. As the result we obtain the system (1) and apply to this system the numerical schemes with parallel algorithms.

Parallel computations were implemented on a supercomputer with the use of the MPI technology.

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APPROXIMATION OF FUZZY INTEGRALS USING FUZZY BERNSTEIN POLYNOMIALS

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In this work we approximate the integration of continuous fuzzy real number valued function of one and two variables on $[0, 1]$ using monotone and bivariate Bernstein-type fuzzy polynomials. Moreover, the error estimates are obtained for this approximation in terms of the modulus of continuity.

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MODELING OF THE AGENT TECHNOLOGY FOR THE COMPUTING OPTION OF FLEXIBLE MANUFACTURE SYSTEM ELEMENT

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Investigation of the problems of computing design of flexible manufacture systems (FMS), especially in the beginning stages, it is necessary to do purpose-directly's option of elements, information data and parameters of the FMS Computing option of FMS. As a complex iteration process, is performed with many level organization of design procedures from design demands at first level to design solution of the demanded degree of detailing at the next levels. At computing option of FMS, for creation its informing, mathematical, program supports, the levels of logical and parametric designing application are demanded. In this connection, for design process advancement of efficiency and accuracy, in the work the problem of computing option and definition of FMS elements parameters on logical and parametric designing levels with agent's technology application in considered.

As well as now FMS posses a great number of industrial robots, grippers and specials manipulators of different purpose, installed on manufacture units, technological equipment, used in productions making, elements of control system. At logical sublevels of designing, criterions of efficiency variants of FMS and its elements option are heuristically and their levels - values are corrected. On each logical sublevel by means of the search models the set of suitable variants are chosen. Option on each sublevel is divided on complex of interacted designing problems (logical and mathematical procedures), which are suited aspects of designing.

Solution of the individual sub problems of computing option of FMS elements demands application of the knowledge, executed by agents (expert or software component), which are responsible for its solution. For formal description of subject's area of elements the expert knowledge defining them basis parameters, working out kinematical and dynamical schemes of active elements and manufacturing modules of FMS, defining commonly coordinates, speeds of industrial robots served technical units of manufacture modules, developing their control algorithm are applied. Process of FMS element option is excised by coordination use with traditional components of computing design and intelligence systems, passing between agents, worked out units of informing support (knowledge and data bases of the designed objects) .

At elements and manufacturing modules of FMS manufacture appointment and technical characteristics (number of movement degree, load lifting, linear and revolving relocations, speed of linear and revolving relocations), maximal radius of service are necessary to define. In this case, each agent suit by some knowledge of subject area, knowledge about limitation and local problems of option an the base of common informing area of FMS active elements and composes structure, which is excised by means of the term conditions.

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AN ESTIMATE OF THE ABSOLUTE VALUE AND WIDTH OF THE SOLUTION OF A LINEAR ALGEBRAIC SYSTEM OF EQUATIONS BY THE INTERVAL PENTAPOINT SWEEP METHOD

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We consider linear systems of algebraic equations

$$\boldsymbol{S}u = \boldsymbol{f}, \tag{1}$$

where $\boldsymbol{S} \in \mathbb{IR}^{(n+1)\times(n+1)}$ – pentadiagonal interval matrix, $\boldsymbol{f} \in \mathbb{IR}^{(n+1)}$ – interval vector of the right part.

As a solution set for the system, we'll consider so called *united set of solutions*:

$$\Xi_{uni}(\boldsymbol{S}, \boldsymbol{f}) = \tilde{\boldsymbol{u}} = \{\tilde{u} \in \mathbb{R}^{(n+1)} \,|\, (\exists S \in \boldsymbol{S})\,(\exists f \in \boldsymbol{f})\,(S\tilde{u} = f)\}.$$

In the attitude of $\Xi_{uni}(\boldsymbol{S}, \boldsymbol{f})$ we give the problem of out interval estimate:

find an interval vector $\boldsymbol{u} \in \mathbb{IR}^{(n+1)}$, *appropriate to* $\Xi_{uni}(\boldsymbol{S}, \boldsymbol{f}) \subset \boldsymbol{u}$.

In the given problem are interested in $\boldsymbol{u}$ (Cartesian multiply of n intervals), where there is an estimation Ξ_{uni} according to out attitude is in some sense much more exactly or closer to Ξ_{uni}.

Supposing, that $\tilde{u}_i \in \tilde{\boldsymbol{u}}_i$, we can identify the system (1) in a more definite form:

$$\left.\begin{array}{cc} \boldsymbol{c}_0\tilde{u}_0 + \boldsymbol{d}_0\tilde{u}_1 + \boldsymbol{e}_0\tilde{u}_2 = \boldsymbol{f}_0 & \\ \boldsymbol{b}_1\tilde{u}_0 + \boldsymbol{c}_1\tilde{u}_1 + \boldsymbol{d}_1\tilde{u}_2 + \boldsymbol{e}_1\tilde{u}_3 = \boldsymbol{f}_1 & \\ \boldsymbol{a}_i\tilde{u}_{i-2} + \boldsymbol{b}_i\tilde{u}_{i-1} + \boldsymbol{c}_i\tilde{u}_i + \boldsymbol{d}_i\tilde{u}_{i+1} + \boldsymbol{e}_i\tilde{u}_{i+2} = \boldsymbol{f}_i & i = \overline{2,\, n-2}, \\ \boldsymbol{a}_{n-1}\tilde{u}_{n-3} + \boldsymbol{b}_{n-1}\tilde{u}_{n-2} + \boldsymbol{c}_{n-1}\tilde{u}_{n-1} + \boldsymbol{d}_{n-1}\tilde{u}_n = \boldsymbol{f}_{n-1} & \\ \boldsymbol{a}_n\tilde{u}_{n-2} + \boldsymbol{b}_n\tilde{u}_{n-1} + \boldsymbol{c}_n\tilde{u}_n = \boldsymbol{f}_n & \end{array}\right\},$$

where

$$\boldsymbol{a}_i,\ (i = \overline{2,n}); \quad \boldsymbol{b}_i,\ (i = \overline{1,n}); \quad \boldsymbol{c}_i,\ (i = \overline{0,n}); \quad \boldsymbol{d}_i,\ (i = \overline{1,n-1}); \quad \boldsymbol{e}_i,\ (i = \overline{0,n-2})$$

– corresponding to diagonal elements of the interval matrix $\boldsymbol{S}$.

On the formula for pentapoint sweep method [1] we can calculate intervals $\boldsymbol{u}_i$. The substantiation of the interval version of pentapoint sweep methods is supposed to be done the traditional condition of diagonal dominance.

In the given paper at some limits on the elements of the matrix $\boldsymbol{S}$, under certain assumptions of the condition for the diagonal dominance we obtained estimates for absolute value and width of the interval solutions.

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EFFECTIVE NUMERICAL METHOD FOR INVERSE PROBLEM FOR GEOELECTRIC EQUATION

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In this work the physical formulation of direct and inverse problems of subsurface radar are considered. We had used a mathematical model of direct and inverse which reduced and studied in the paper [1].

For solving inverse problems applied optimization method described in detail in the monograph [2]. The formulas for calculating the gradient of the functional and the dual problem are obtained on the discrete level.

The statement of the problem of subsurface radar is: a source of external current j^{cm}, which having at the time bell-shaped form $r(t)$ and length of about two nanoseconds, includes on the surface of the roadway. The electromagnetic field (the response of the medium) are measured, on which need to define ε (or σ) at depths ranging from zero to 10 meters.

It is assumed that the coefficients ε and σ depend on the depth x_3 and the source of the external current is sufficiently long cable, it is located in the center and stretched along the roadway. In this case, the system of Maxwell equations are remained regarding the components E_2 of electric intensity. Further, for variable x_1 we are applying the Fourier transform, get the problem:

$$\varepsilon U_{tt} + \sigma U_t = \frac{1}{\mu}(U_{x_3x_3} - \lambda^2 U) - P_\lambda q(x_3) r'(t), \tag{1}$$

$$U|_{t=0} = 0, \qquad U_t|_{t=0} = 0, \tag{2}$$

here: $U = F_{x_1}[E_2(x_1, x_2, t)]$, $P_\lambda = F_{x_1}[p(x_1)]$, λ - the parameter of the Fourier transform for variable x_1.

Direct problem: *The known values of piecewise continuous functions $\varepsilon(x_3)$, $\sigma(x_3)$ and the positive continuous μ function to define U as a solution to the generalized Cauchy problem of the relations* (1)-(2). *Let it be known additional information at the point $x_3 = 0$ (the response of the medium)*:

$$U^\lambda(0, t) = f(t). \tag{3}$$

The inverse problem: *To determine $\sigma(x_3)$, U^{λ_0} (either $\varepsilon(x_3)$, U^{λ_0}) from* (1)-(2), *from the known values $\varepsilon(x_3)$, μ (either $\sigma(x_3)$) and additional information* (3) *(where: $\lambda = \lambda_0$ is fixed).*

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APPLICATION OF NUMERICAL METHODS IN SOLUTION OF THE INCOMPRESSIBLE NAVIER-STOKES EQUATIONS USING A COMPACT FINITE DIFFERENCE SCHEME

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The viscous incompressible Navier-Stokes equations are the basic equations governing a wide range of fluid flows. The numerical solution of these equations to study the issues associated with the incompressible fluid flows is of great importance.Several computational methods for solving these equations have been developed in the past years. Researches show that the high-order upwind compact finite difference schemes are more accurate than the non-compact finite difference methods.This is unlike most of other upwind compact methods where three or five points are used in implicit parts.In this paper,a detailed discussion on the implementation of the flux-difference splitting based third-order upwind compact method to the convective terms of the Navier-Stokes equations is given.In this paper,a compact finite difference scheme is developed for time-accurate solution of the Navier-Stokes (N-S) equations governing the unsteady,incompressible, viscous fluid flows. The scheme is implemented along with the artificial compressibility method, which is widely used to solve the N-S equations. The convective terms are discretized using a flux-difference splitting-based third-order upwind compact method. In addition, the viscous terms are discretized with a fourth-order central compact method. Dual-time stepping along with the beam-warming approximate method is employed for time-accurate calculation of the equations. It is found that,the third-order upwind compact method is more accurate than its non-compact counterpart.

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THE EXTREMAL FUNCTION OF THE ERROR FUNCTIONAL OF QUADRATURE FORMULA AND ITS NORM IN $H_2^\mu(R)$ SPACE

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We consider the following quadrature formula

$$\int\limits_R P(x)f(x)dx \approx \sum_{\beta=0}^{N} C_\beta f(x_\beta), \tag{1}$$

with the error functional

$$\ell(x) = P(x)\varepsilon_R(x) - \sum_{\beta=0}^{N} C_\beta \delta(x - x_\beta), \tag{2}$$

on $H_2^\mu(R)$ space, where C_β and x_β are coefficients and nodes of the formula (1), $\delta(x)$ is Dirac's delta function, $\varepsilon_R(x)$ is the indicator of the domain R and $P(x)$ is a weight function. Definition. The space $H_2^\mu(R)$ is defined as closure infinite differentiable functions given in R and decreasing in infinity faster than any negative degree in the norm

$$\|f(x) \mid H_2^\mu(R)\| = \left\{ \int\limits_{-\infty}^{\infty} \left|F^{-1}[\mu(\xi)] \cdot F[f(x)](\xi)\right|^2 dx \right\}^{\frac{1}{2}}.$$

Here F and F^{-1} are the Fourier transformations [1]:

$$F[f(x)](y) = \int\limits_{-\infty}^{\infty} f(x)e^{2\pi iyx}dx, \quad F^{-1}[f(x)](y) = \int\limits_{-\infty}^{\infty} f(x)e^{-2\pi iyx}dx,$$

$\mu(\xi)$ is infinite differentiable and $\mu > 0$. The following holds Theorem. The extremal function of the error functional (2) of formula (1) has the form

$$\psi_\ell(x) = F^{-1}\left\{\mu(\xi) \cdot F[\ell_N(x)](y)\right\} = [p(x) \cdot \varepsilon_R(x)] * \nu_m(x) - \sum_{\chi=0}^{N} C_\beta \nu_\beta(x - h\beta),$$

where $\nu_m(x) = F^{-1}[\mu(\xi)](x)$. The square of the error functional on the space $H_2^\mu(R)$ has the form

$$\left\|\ell_N \left| H_2^{\mu*}(R) \right.\right\| = \int\limits_{-\infty}^{\infty} \left|F^{-1}\left\{\mu(\xi) \cdot F[\ell_N(x)](y)\right\}(x)\right|^2 dx.$$

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ON PARALLELIZATION OF PRIME FACTORIZATION ALGORITHMS

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Security of the public key encryption algorithms are based on the fact that the prime factorization problems is mathematically hard. RSA, which is one of the most reliable encryption algorithm, also relies on the principle that prime factorization of the big numbers can not be computed efficiently with computers.

Prime factorization problem is the problem of finding the numbers "p" and "q", given the number "n" which is consists of the product of two big prime numbers "p" and "q". The prime factorization problems is too complex when the numbers are big enough.

In this work, the prime factorization algorithms, the center of the attacks against the encryption algorithms, are emphasized, different prime factorization algorithms are studied and new prime factorization algorithms are proposed. The parallel versions of the algorithms are designed.

Proposed algorithms were implemented in C using MPI and GMP libraries and computational experiments were done. Experiments showed that proposed algorithms are efficient.

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CUBIC B-SPLINE COLLOCATION FOR NONLINEAR FREDHOLM INTEGRAL EQUATIONS

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The collocation method based on cubic B-spline, is developed to approximate the solution of second kind nonlinear Fredholm integral equations. First of all, we collocate the solution by B-spline collocation method then the Newton-Cotes formula use to approximate the integral. Convergence analysis has been investigated and proved that the quadrature rule is third order convergent. The presented method is tested with four examples, and the errors in the solution are compared with the existing methods [1-4] to verify the accuracy and convergent nature of proposed methods. The RMS errors in the solutions are tabulated in table 3 which shows that our method can be applied for large values of n, but the maximum n which has been used by the existing methods are only $n = 10$, moreover our method is accurate and stable for different values of n.

NUMERICAL SOLUTION OF FREDHOLM- HAMMERSTEIN INTEGRAL EQUATION BASED ON HERMITE-SPLINE INTERPOLATION

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In this work, we introduce a numerical method for solving Fredholm-Hammerstein integral equation based on linear and cubic Hermite-spline interpolation. In each case we prove a convergence theorem and illustrate the convergence theorem by numerical examples.

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THE NUMERICAL DECISION OF THE EQUATION OF A BOUNDARY LAYER OF AN ATMOSPHERE WITH USE CONDENSED CURVILINEAR NET POLLUTED AREAS OF CITY

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At numerical modelling a boundary layer of an atmosphere special complexity is represented with the account of orography of district and high-altitude a building in city buildings. For the decision of such problems for today there is a number of ways as a method of fictitious areas and construction orthogonal curvilinear nets. The construction of orthogonal curvilinear nets on physical area of research enables to receive a better picture and raises its accuracy even at the moderate quantity of units.

The basis of numerical model considered by us for city of Ust Kamenogorsk is made with spatial not hydrostatic numerical models of local atmospheric processes [1,2]. Thus hydrodynamical aspects of a problem - interaction of air weight with a spreading surface, formation of island of heat and local circulation on a background of an external stream first of all are considered. On development of atmospheric processes in considered industrial city, except for natural factors the wide spectrum of indignations of an anthropogenous origin influences. To consider their total effect, in numerical model the opportunity of change of its structure depending on characteristic existential scales of anthropogenous sources and the investigated phenomena has been incorporated.

In the given work as physical area the territory of city of Ust Kamenogorsk in the size 22000×20000 m^2 and height 2500 m is, the bottom border coincides with a lay of land. Condensations of units of a grid were spent in the central part of city where the basic street-high systems and high-altitude buildings of city [3] are located. In all physical area the orthogonal curvilinear grid which with height is straightened is constructed and on the top border is a plane.

At numerical realization of the given model the known method of splitting on physical processes which consists of two stages was used: Advective-diffusional carry of substances in view of force Koriolice; the coordination of meso-meteorological fields. The system of the equations at the first stage of a method of splitting for nonlinear composed is used the monotonous scheme in view of a sign.

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ANALYTICAL DESCRIPTION OF THE $3D$- MULTIDIMENSIONAL NONLINEAR MODULAR DYNAMIC SYSTEMS

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In the work the analytical description of the $3D$ - multidimensional nonlinear modular dynamic systems $(3D - MNMDS)$ [1] with n_0 memory and $P = P_1 \times P_2$ bounded relation is considered. This system over the field GF(2) is described by the following functional relation

$$y[n, c_1, c_2] = G\{u[\sigma, c_1 + \rho_1, c_2 + \rho_2]\,|n - n_0 \le \sigma \le n,\, \rho_1 \in P_1, \rho_2 \in P_2\}, GF(2), \tag{1}$$

where $n \in Z_0, c_i \in Z, i = \overline{1,2}, Z$ discreet space; $y[n, c_1, c_2] \in GF^k(2)$, $u[n, c_1, c_2] \in GF^r(2)$, $P_i = \{p_i(1), ..., p_i(r_i)\}$, $p_i(1) < ... < p_i(r_i)$, besides $p_i(1)$, and $p_i(r_i)$, $(i = \overline{1,2})$ - finite integers. The following theorem is proved.

Theorem. *The full reaction of the $3D - MNMDS$ characterized by (1) is possible to present in the following form of two-digit analog of the Volterra polynomial*

$$y_\nu[n, c_1, c_2] = \sum_{i=0}^{(n_0+1)rr_1r_2} \sum_{\bar{\eta}\in\Lambda(i)} \sum_{(\gamma_1,\gamma_2,\bar{\bar{m}})\in F(\bar{\eta})} \sum_{\bar{\bar{j}}\in L_1} \sum_{\bar{\bar{\tau}}\in L_2} \sum_{\bar{\bar{n}}_2\in\Gamma(\gamma_1,\gamma_2,\bar{\bar{m}})} h_{i,\bar{\eta},\gamma_1,\gamma_2,\bar{\bar{m}}}[\bar{\bar{j}}, \bar{\bar{\tau}}, \bar{\bar{n}}_2] \prod_{\ell\in Q_0(\bar{\eta})} \times$$
$$\times \prod_{(\alpha_\ell,\beta_\ell)\in Q'_\ell(\eta_\ell,\gamma_1(\ell),\gamma_2(\ell),\bar{m}_\ell)} \prod_{\sigma=1}^{m_{\ell,\alpha_\ell,\beta_\ell}} u[n - n_1(\alpha_\ell, \beta_\ell, \sigma), c_1 + p_1(j_{\alpha_\ell}(\ell)), c_2 + p_2(\tau_{\beta_\ell}(\ell))], GF(2), \nu = \overline{1,k}, \tag{2}$$

where $\Lambda(i)$, $Q_0(\bar{\eta})$; $F(\bar{\eta})$, L_1, L_2, $\Gamma(\gamma_1, \gamma_2, \bar{\bar{m}})$ are the characteristic neighborhoods or the templates of the neighborhood

Also the question of a finding of unknown coefficients of the polynomial (2) by known input and output sequences is investigated.

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NUMERICAL SIMULATION OF FLOWS AROUND TWO CIRCULAR CYLINDERS IN THE SIDE-BY-SIDE AND TANDEM BY COMPRESSIBLE GAS

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In many areas of engineering, circular cylinders form the basic part of structures, for example, cooling systems for nuclear power stations, chimney stacks, transmission cables, heat exchange tubes, etc. Named engineering structures are exposed to either air or water flow, and therefore they experience flow- induced vibration, which could lead to destruction. To avoid the situations and to improve the structure design, it is necessary for engineers to understand the details of the flow- structure interaction and possess the ability to predict the force and response of the cylinder-like structures. Flow past one cylinder has been well studied and it is now considered as a classical case for a validating new numerical schemes. It is known that the flow field around one cylinder shows a wide class of models. From the point of view of a geometrical configuration flow around two cylinders can be considered as the expanded case of one isolated cylinder. Despite it, the corresponding problem is much more complicated. That is because the dynamic interaction between the shed vortices, shear layers and vortex sheets appears in the wake of the cylinders. Consequently, the wake behaves quite differently from one isolated cylinder. Interest to characteristics of flow past a pair cylinders has attracted many researches. The early experimental studies of flow around to circular cylinders were reviewed by Zdravkovich [1]. With fast development of computer technology, the flow around two circular cylinders is also numerically investigated [2, 3]. At numerical modeling of a flow of obstacles there are the difficulties connected with satisfies of boundary conditions on an obstacle. Approaches to the permission of these problems are known, the most effective among them is the immersed-boundary method [4], a method of the fictitious areas, different simple realization. This paper is provided numerical solutions for the flows past two cylinders in the side-by-side and tandem arrangements by compressible turbulent gas in the field of the gravity, described by the non-stationary Navier-Stokes. For an exception of the difficulties arising at numerical integration of initial system of the equations for small Mach numbers, the model of hyposound flows is used .

The exact satisfaction of boundary conditions on an obstacle influences definition of the forces operating from a liquid on a body. Linear and bilinear interpolations are applied to increase of an order of approximation of dynamic characteristics on obstacles. It is already known that the wake interference behind two cylinders is influenced by the gap between the cylinders and there exists a so-called critical gap, which is used to classify the flow patterns behind the cylinder into several categories. To illustrate the detailed the structure of a stream near the wake region behind two circular cylinders, numerical visualization are presented in the form of vorticity contours and streamlines. To describe the flow pattern quantitatively, mean values of lift and drag coefficients are computed and compared with the results of other authors.

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ABOUT NUMERICAL SOLUTION OF THE INVERSE PROBLEM OF THE TWO-PHASE FLUID FLOW IN A POROUS MEDIUM

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Let's consider the Rapoport-Leas Model describing the Linear Two-Phase Fluid Flow in a Porous Medium [1]

$$\frac{\partial s}{\partial t} = \frac{\partial}{\partial x}\left(a(s)\frac{\partial s}{\partial x} - c(s)\right), \tag{1}$$

$$\frac{\partial s}{\partial x} = \frac{B\mu_0}{f'(s)f_2(s)k_0} \ \ on \ \ x = 0, \tag{2}$$

$$s(1,t) = \eta(t) \ \ on \ \ x = 1, \tag{3}$$

$$s(x,0) = s_0(x) \ \ on \ \ t = 0, \tag{4}$$

$$a(s) = \frac{k_0}{B}\frac{f_1(s)f_2(s)f'(s)}{f_2(s) + u_0 f_1(s)}; c(s) = \frac{f_2(s)}{f_2(s) + \mu_0 f_1(s)}; B = \frac{\mu_1 LV}{\sigma cos\theta\sqrt{nk_0}}.$$

Here s is the saturation, the fractional pore volume occupied by the non-wetting fluid, k_0 is the permeability, $f_i(s)$ are the relative permeabilities, σ is the surface tension at the water-oil interface, n is porosity, V is the bulk flux, $f(s)$ is the Leverett function, $x = \frac{\bar{x}}{L}, t = \frac{V\bar{t}}{Lm}$ are the dimensionless variables, L is the medium length.

We consider the following inverse boundary problem: find the condition on the right border (the function $\eta(t)$ is unknown). We also have the following additional condition on the left border

$$s(0,t) = \varphi(t). \tag{5}$$

By minimize the functional $J(\eta^{n+1}) = (s^{n+1}(0) - \varphi^{n+1})^2 + \alpha(\eta^{n+1})^2$ the numerical method of the identification the right boundary condition for the Rapoport-Leas Model is elaborated . On the base of numerical experiments, the performance capabilities of this method are analysing.

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ON A NUMERICAL METHOD FOR THE SOLUTION OF THE PROBLEM DESCRIBED BY THE SYSTEM OF HYPERBOLIC TYPE DIFFERENTIAL EQUATIONS[1]

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In [1] a model is developed for the gas-lift well, described by the system of hyperbolic type differential equations

$$\begin{cases} \frac{\partial P}{\partial t} = -\frac{c^2}{F}\frac{\partial Q}{\partial x}, & x \in (0,l) \cup (l, 2l), \\ \frac{\partial Q}{\partial t} = -F\frac{\partial P}{\partial x} - 2aQ, & t \in (0,T), \end{cases} \tag{1}$$

where $P(x,t)$ - superfluous pressure on its stationary value, $Q(x,t)$- the volumetric charge hardening gas and gas liquid mixture in the lift; F, c and a some parameters of the well. For definition of $P(x,t)$, $Q(x,t)$ we use the following initial

$$P(x,0) = P^0(x), \quad Q(x,0) = Q^0(x), \tag{2}$$

and boundary conditions

$$P(0,t) = P_0(t), \quad Q(2l,t) = Q_{2l}(t), \tag{3}$$

also joining conditions

$$P(l+0,t) = \alpha P(l-0,t) + P_{pl}(t), Q(l+0,t) = \beta Q(l-0,t) + Q_{pl}(t), \tag{4}$$

where α, β-the parameters; $P^0(x)$, $Q^0(x)$, $P_0(t)$, $Q_{2l}(t)$ given functions, $P_{pl}(t)$, $Q_{pl}(t)$-functions describing feedbacks of a layer. For the solution of the problem (1)-(4) it is used difference scheme. Denoting $\Delta t = T/M$, $\Delta x = l/N$ we receive the following grid:

$$x_0 = 0,\ x_i = x_0 + i\Delta x,\ i = \overline{1,N}, x_N = l-0,\ x_{N+1} = l-0,\ x_{N+i} = x_{N+1} + (i-1)\Delta x,\ i = \overline{2, N+1},$$
$$t_0 = 0,\ t_j = t_0 + j\Delta t,\ j = \overline{1,M}.$$

Then initial and boundary conditions turn to conditions:

$$\begin{array}{l} P_i^0 = P(x_i,0) = P^0(x_i),\ Q_i^0 = Q(x_i,0) = Q^0(x_i),\ i = \overline{0, 2N+1}, \\ P_0^j = P(0,t_j) = P_0(t_j),\ Q_0^j(2l,t_j) = Q_{2l}(t_j) = Q_{2l}^j,\ j = \overline{0,M}. \end{array} \tag{5}$$

Further, for the finding of value required functions on $j+1$ layer we obtain the following relations

$$\begin{array}{l} P_i^{j+1} = P_i^j - \frac{c_1^2}{F_1}\frac{\Delta t}{\Delta x}\left(Q_i^j - Q_{i-1}^j\right),\ i = \overline{1,N}, \\ Q_i^{j+1} = Q_i^j - F_1\frac{\Delta t}{\Delta x}\left(P_{i+1}^j - P_i^j\right) - 2a_1\Delta t Q_i^j,\ i = \overline{0,N}, \quad j = \overline{0, M-1}, \\ P_{N+1}^{j+1} = \alpha P_N^{j+1} + P_{pl}^{j+1},\ Q_{N+1}^{j+1} = \beta Q_N^{j+1} + Q_{pl}^{j+1},\ j = 0, \overline{M-1}, \\ P_i^{j+1} = P_i^j - \frac{c_2^2}{F_2}\frac{\Delta t}{\Delta x}\left(Q_i^j - Q_{i-1}^j\right),\ i = \overline{N+2, 2N+1}, \\ Q_i^{j+1} = Q_i^j - F_2\frac{\Delta t}{\Delta x}\left(P_{i+1}^j - P_i^j\right) - 2a_2\Delta t Q_i^j,\ i = \overline{0,N}, \quad i = \overline{N+2, 2N}. \end{array} \tag{6}$$

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NUMERICAL SOLUTION OF THE KINEMATIC ANALYSIS OF LINKAGES

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High class linkages are commonly present as the consisting components of complex industrial mechanisms. Their geometric and kinematic description can not be done in a standard way. The choice of settlement model describing of linkages motion is defined by the circuit of the mechanism. Due to the extensive use of computers and modeling program, creation of efficient models without analytical relations of required parameters in the explicit form gains more importance. In this thesis a model for describing four class mechanisms (fig. 1) is proposed.

Modeling of work of the investigated lifting mechanism [1] is executed in a program MVS. Position of output link PLQ has to be determined. Position of the hydraulic cylinder AD is known and is described equation (1) and (2) of equation system which is presented in fig. 2 in the program.

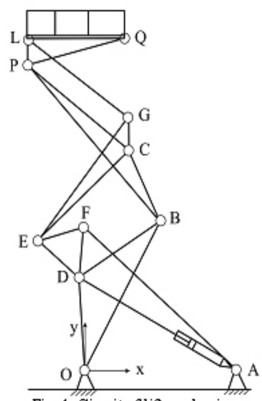

Fig. 1. Circuit of lift mechanism

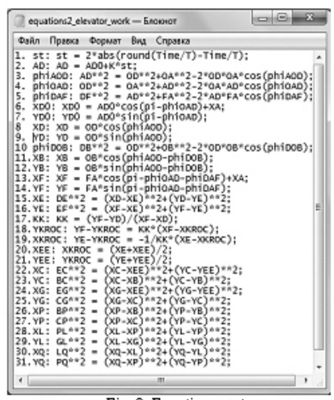

Fig. 2. Equations system

At fist, values for angles of the ODA dyad is calculated using equations (3) and (4) and coordinates of external joints O and A. Next, position angle of the ADF dyad and position of joint D are determined from kinematic group ADF using equations from (5) to (9). Then unknown position of joint F is received as a projection of the party of the triangular link ADF (13-14). Position of joint B of the link ODB is founded from the equals (10), (11) and (12). Coordinates of joint E of the link DEF are calculated from the system of quadratics (15-16). The solution of its system is given two possible configurations of the link DEF that why the equations (17-21) are determined the decision when the point E is located above or at left concerning the points D and F. The equals (22-31) for calculation positions of the moving joints C, G, P, L and Q are received similarly as is described above.

For the method convergence, the starting values of variables must be close to exact solutions, which find from the kinematic synthesis of the lifting mechanism. Presented model is developed primarily for position analysis; nevertheless it can be also used for velocity and acceleration analysis, but using expressions for velocity and acceleration kinematical parameters instead.

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2D HYBRID NUMERICAL SIMULATION OF A GLOW DISCHARGE PLASMAS[1]

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Numerical modelling approaches for gas discharges can be classified as fluid methods, kinetic (particle) methods and their combinations, known as hybrid methods [1, 2]. Here we developed and tested 2D hybrid model [3], which includes the continuity equations for the three plasma species, namely, the electrons, positive ions, and metastable atoms with particle densities n_e, n_i, and n_m,

$$\frac{\partial n_j}{\partial t} + \nabla \cdot \mathbf{\Gamma}_j = S_j, \tag{1}$$

with the particle flux $\mathbf{\Gamma}$ in the drift-diffusion approximation,

$$\mathbf{\Gamma}_j = sgn(q_j)\, n_j \mu_j \mathbf{E} - D_j \nabla n_j, \tag{2}$$

the Poisson's equation for the electric field in electrostatic approximation,

$$\epsilon_0 \nabla \cdot \mathbf{E} = \sum_j q_j n_j, \;\; \mathbf{E} = -\nabla \phi, \tag{3}$$

and the electron energy equation

$$\frac{3}{2}\frac{\partial w}{\partial t} + \frac{5}{2}\nabla \cdot (-D_e \nabla w - \mu_e \mathbf{E}\, w) = -e\mathbf{\Gamma}_e \cdot \mathbf{E} - \frac{3}{2}\frac{m_e}{m_g} K_{el}\, k_B\, (T_e - T_g) - \sum_k \Delta E_k R_k. \tag{4}$$

Here, subscripts j, i, e, and g indicate the jth species, ions, electrons, and background argon gas, respectively. $\mathbf{E}$ and ϕ are the electric field and potential, q is the charge, ϵ_0 is the dielectric constant, μ and D are the particle mobility and diffusion coefficients, S is the particle creation rate, $w = 2/3\,\varepsilon n_e$, where ε is the mean electron energy. ΔE and R are the energy loss due to inelastic collision and the corresponding reaction rate, m is the particle mass, K_{el} is the momentum transfer rate between electrons and background gas, k_B is the Boltzmann constant, and the electron and gas temperatures are defined as $T_e = 2\varepsilon/3k_b$ and $T_g = 300$ K.

A separate solver for the electron Boltzmann equation is used to relate mean electron energy to the electron transport coefficients (diffusion, D_e, and mobility, μ_e) as well as the rates S of electron-induced plasma-chemical reactions for excitation and ionization.

We used COMSOL Multiphysics^{TM} finite element simulation software to develop the model. Calculation results have been compared with those obtained from the CFD-ACE+^{TM} package for the identical discharge model under the same operating conditions. Although these models are sufficiently detailed and predict correctly the basic properties of the dc glow discharges, calculations show that the models are very approximate and cannot provide accurate quantitative results.

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CONNECTIVE HEAT TRANSFER FACTOR CALCULATION SITES OF A CUT OF OPEN-CAST MINING 'EKIBASTUZSKY'

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Physical and chemical processes and mathematical models of heat distribution was thoroughly studied in works of Martynov G.A. (1959). Methods of the some identification solving problems of heat distribution parametres were studied in the works of Rysbaiyly B., Baimankulov A., Ismailov A., Makhambetova G., (2008-2010). Theoretical problems of the developed approached method for a ground heat transfer factor definition to environment without a wind speed were studied in works Rysbaiyly B., Adamov ., Akishev T.B. (2009-2010). It is experimentally proved (Chudnovskij A.F., 1976) that with the increasing of ground wind speed heat transfer factor to environment increases. The iterative method of wind influence equation definition of connective heat transfer factor is considered. The connective heat transfer we search in a kind: $\alpha = a+b\omega^k$ here ω - a wind speed of m/s, a-exponent, $k > 0$, a and b - constants.

The Cut 'Ekibastuzsky' depending on quantity of homogeneous layers have broken into three sites And, In and And the site A (the top horizon) consists of 4 homogeneous layers: a damp ground - a stony ground-quartz sand-coal; the site In (on 8-10 m. below the top horizon) consists of two layers: quartz sand-coal, and at last a site C (on 15-20 m. below the top horizon) consists only of one layer - coal. From each homogeneous layer of the 'Ekibastuzsky' cut rock's samples are taken for research. Using devices and giving to the removed tests the cylindrical form, values of factors of a thermal capacity and heat conductivity have measured, and also have established relative density of each kind of rocks. Calculation was done with the use of the 'TOTEM' program complex created by us. Entrance data for each site are: a thickness of each layer, heat physical characteristics of each layer, air and ground temperature of surface.

And also set time has begun also the measurement end, an interval of measurement of temperature of air and a ground $dt = 1/12$ hour, a step on a spatial variable $dz = 0.001$ meter. Settlement data were compared to the measured data. The method error makes 3-4 percentage.

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BICHARACTERISTICS METHOD IN NONSTATIONARY CONTACT PROBLEMS OF THE DYNAMICS OF ELASTIC MEDIA

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Consider the inhomogeneous composite elastic medium: homogeneous, isotropic half-space $x_1 > 0$ (D_1 with the density ρ_1 and Lame coefficients λ_1 and μ_1) and a rectangular surface inclusion ($D_2 : \rho_2, \lambda_2, \mu_2$) in plane strain conditions at relieving the stress on the horizontal fissure $S \in D_1$. To describe the motion of an elastic medium using two equations:

$$\sigma^{(k)}_{i\beta,\beta} + F^{(k)}_i = \rho_k \frac{\partial^2 u^{(k)}_i}{\partial t^2}, \qquad i, k = 1,\ 2, \tag{1}$$

and the ratio of the generalized Hooke's law :

$$\sigma^{(k)}_{ij} = \lambda_k\, u^{(k)}_{\beta,\beta}\delta_{ij} + \mu_k(u^{(k)}_{i,j} + u^{(k)}_{j,i}), \quad (i, j, k = 1,\ 2). \tag{2}$$

Here k is the medium number, $F^{(k)}_i$ are the components of body forces, δ_{ij} is Kronecker symbol. Over repeated Greek indices summation is implied from 1 to 2 (tensor contraction).

To simulate the stress-relief on the crack in the half-space we put the body force $F^{(1)}$, whose components are defined by a singular generalized function such as a simple layer on the horizontal crack S [1].

At the contact surface the conditions of hard connection are required:

$$\nu^{(1)}_i = \nu^{(2)}_i, \quad \sigma^{(1)}_{1i} = \sigma^{(2)}_{1i}, \quad i = 1, 2. \tag{3}$$

At infinity the damping conditions are performed:

$$u_j \to 0, \quad \sigma_{ij} \to 0 \qquad i, j = 1, 2 \quad \text{for } \|x\| \to \infty.$$

To solve this problem the bicharacteristics method is used with the ideas of the splitting method, which was proposed by G.T.Tarabrin [2]. The modified method for the solution of contact problems of interaction of elastic bodies with corners in plane strain conditions [3]. The difference equations for the internal, boundary, corner, and the special pair of contact points of the band and half are obtained. Numerical experiments on the stress-strain state of an elastic half-space and an elastic body when the vertical and horizontal stresses faulting on the horizontal crack by using physical and mechanical parameters, which are typical for rocks and building structures. Waveforms of displacement rates and the diffraction patterns of the velocity fields and the stress of shock waves are constructed. The effect of array parameters, crack depth and the nature of shock waves on the stress-strain state of the environment and the elastic body are studied.

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DEAD BEAT CONTROL OF DC-DC CONVERTOR AND SIMILATION WITH MATLAB

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In discrete-time control theory, the dead beat control problem consists of finding what input signal must be applied to a system in order to bring the output to the steady state in the smallest number of time steps. In this work, deat-beat control of DC-DC convertor is done and simulation of it realized.

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ANALYSIS OF OIL DEPOSIT EXPLORATION STATE ON THE BASE OF MULTIFRACTAL APPROACH

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On the base of multifractal approach the method is suggested in order to diagnost the current state of lay system. Thisn method allow to bring decision over regulation and control on line regime by deposit exploration. In limits on method of multifractal fluctuation analysis to field data the dynamic change of generalization fractal dimension to be considered for time series values of current oil production. Carring dynamic analysis give estimation of self organization for exploration process in purpose to select most effective method of deposit action and its aim direct correction at different deposit exploration stages.

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NUMERICAL SOLUTION OF THE BOUNDARY INVERSE PROBLEM OF THERMAL CONDUCTION BY THE METHOD OF THERMALLY THIN LAYER

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For a unrestricted plate there can be put the problems of thermal conduction with the various equivalent edge conditions generating same temperature field. On surfaces of a plate temperature (the first boundary value problem) or heat flow (the second boundary value problem) are set. By selection near to surfaces of a plate of the thin fictitious areas, the second boundary value problem is led to a boundary value problem of thermal conduction with small parameter at a derivative on time from temperature in boundary conditions. It stipulates generation of family of decisions $T_\delta(x,t)$ depend from small parameter δ (widths of a frontier thin layer). By $\delta \to 0$ the boundary condition with a derivative on time again passes in a boundary condition of the second type. The decision of the perturbed problem $T_\delta(x,t)$ tends to the decision of an initial problem. If on the given thermal flow $q_2(t)$ set on a surface $x = l$ and the measured temperature $T_2(t)$ it is required to define the temperature $T_1(t)$ of a surface $x = 0$ then boundary inverse problem of thermal conduction (IPT) is solved. For its decision application of one of known special methods of the decision, for example a quasi-inversion method is required. However in this case for evaluation of the decision $T_1(t^{(k)})$ in the instant $t = t_k$ it is required knowledge of the measured temperature in all previous and the subsequent instants if time down to completion of investigated heat process. In case of research and control of process of heat in real time such methods are unsuitable.

For the decision boundary IPT the method of thermally thin layer [1,2], which allows on each step on time to realize direct count of the measured temperature $T_2(t^{(k)})$ to required temperature $T_1(t^{(k)})$, is in real time used. At one-sided monotonic heat initial boundary IPT is reduced to a Cauchy problem for the ordinary differential equation of the first order. By means of a contraction mapping principle parameters of a regularization δ and τ are find. The existence, uniqueness and conditional stability of the received decision on a small interval of time is proved. Process of electro contact heat is considered. According to data of imitational simulation of the measuring information the temperature inaccessible for direct temperature measure of electrode-detail contact surfaces is calculated. The results of the numerical experiment are presented which confirm efficiency of the offered method of a numerical IPT solution boundary.

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ON ONE ALGORITHM OF FINDING SOLUTION OF A NONLINEAR NONLOCAL BOUNDARY VALUE PROBLEM FOR ONE CLASS THE SYSTEMS OF HYPERBOLIC EQUATIONS

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Consider the nonlinear boundary value problem

$$u_{xt} = f(x,t,u,u_x), \quad (x,t) \in \Omega = (0,\omega) \times (0,T), \tag{1}$$

$$u(0,t) = 0, \quad t \in [0,T], \tag{2}$$

$$g\big(x, u_x(x,0), u_x(x,T)\big) = 0, \quad x \in [0,\omega], \tag{3}$$

where $f : \overline{\Omega} \times \mathbb{R}^{2n} \to \mathbb{R}^n$, $g : [0,\omega] \times \mathbb{R}^{2n} \to \mathbb{R}^n$ are continuous functions.

The step size is chosed as $h > 0$, where $Nh = T$ $(N \in \mathbb{N})$. The partition $[0,\omega]\times[0,T) = \bigcup\limits_{r=1}^{N} \Omega_r$, $\Omega_r = [0,\omega]\times[(r-1)h, rh)$, and the replacement $\lambda_r(x) = u_x(x,(r-1)h)$, $v_r(x,t) = u_x(x,t) - u_x(x,(r-1)h)$, $(x,t) \in \Omega_r$, $r = 1 : N$ are made. The algorithm of finding of unknown quantity $\lambda_r(x)$, $v_r(x,t)$, $(x,t) \in \Omega_r$, $r = 1 : N$ is offered. On each step of algorithm sequential approximations of functional parametres $\lambda_r(x)$, $r = 1 : N$ are found from system following equations

$$h \cdot g\Big(x, \lambda_1(x), \lambda_N(x) + \int\limits_{T-h}^{T} f\Big(x, t, \int\limits_0^x \lambda_N(\xi)d\xi + \int\limits_0^x v_N(\xi,t)d\xi, \lambda_N(x) + v_N(x,t)\Big)dt\Big) = 0, \tag{4}$$

$$\lambda_s(x) + \int\limits_{(s-1)h}^{sh} f\Big(x, t, \int\limits_0^x \lambda_s(\xi)d\xi + \int\limits_0^x v_s(\xi,t)d\xi, \lambda_s(x) + v_s(x,t)\Big)dt - \lambda_{s+1}(x) = 0, \tag{5}$$

$s = 1 : (N-1)$, $x \in [0,\omega]$, at known $v_r(x,t)$, $(x,t) \in \Omega_r$, $r = 1 : N$. Unknown functions $v_r(x,t)$, $(x,t) \in \Omega_r$, $r = 1 : N$, are defined from the following integral equations

$$v_r(x,t) = \int\limits_{(r-1)h}^{t} f\Big(x, \tau, \int\limits_0^x \lambda_s(\xi)d\xi + \int\limits_0^x v_s(\xi,\tau)d\xi, \lambda_s(x) + v_s(x,\tau)\Big)d\tau, \ (x,t) \in \Omega_r, \ r = 1 : N, \tag{6}$$

at finding $\lambda_r(x)$, where $x \in [0,\omega]$.

Algorithm begins with solving of a set of equations (4), (5) with respect to parametres $\lambda_r(x)$, $r = 1 : N$, at $v_r(x,t) = 0$, $(x,t) \in \Omega_r$, $r = 1 : N$.

On the basis of the parametrization method [1] sufficient conditions of convergence of algorithm and existence of the isolated solution of a problem (1)-(3) are received.

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ALGORITHM FOR NUMERICAL REALIZATION OF THE "LOGARITHMIC" DIFFERENCE SCHEMES

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The most compact differential equations of gas dynamics appear in the Lagrangian mass variables (s,t) [1-2]:

$$\begin{cases} \dfrac{\partial}{\partial t_L}\left(\dfrac{1}{\rho}\right) = \dfrac{\partial u}{\partial s} \\ \dfrac{\partial u}{\partial t_L} + \dfrac{\partial p}{\partial s} = \dfrac{\partial}{\partial s}\left(\dfrac{\mu}{\mathrm{v}} \cdot \dfrac{\partial u}{\partial s}\right) \\ \dfrac{\partial \varepsilon}{\partial t_L} = -p\dfrac{\partial u}{\partial s} \end{cases} . \tag{1}$$

Algorithm for the numerical solution of equations of gas dynamics in Lagrange variables (1) using the "logarithmic" difference scheme is written as

First step:

$$\begin{aligned} U_{i+1/2} &= \tfrac{u_{i+1}^n + u_i^n}{2} - \left(p_{i+1}^n + p_i^n\right) + \\ &+ \left[\mu_{i+3/2} \cdot \tfrac{\ln \mathrm{v}_{i+1}^n - \ln \mathrm{v}_{i+1}^{n-1}}{\mathrm{v}_{i+1}^n - \mathrm{v}_{i+1}^{n-1}} \cdot \left(u_{i+1}^n - u_i^n\right) - \mu_{i+1/2} \cdot \tfrac{\ln \mathrm{v}_i^n - \ln \mathrm{v}_i^{n-1}}{\mathrm{v}_i^n - \mathrm{v}_i^{n-1}} \cdot \left(u_i^n - u_{i-1}^n\right)\right], \end{aligned} \tag{2}$$

$$P_{i+1/2} = \frac{p_{i+1}^n + p_i^n}{2} - (\gamma - 1) \cdot \varepsilon^n \cdot \left(\rho^2\right)^n \left(u_{i+1}^n - u_i^n\right). \tag{3}$$

Second stage:

$$\frac{\mathrm{v}_i^{n+1} - \mathrm{v}_i^n}{\tau} - \frac{1}{h} \cdot \left(U_{i+1/2} - U_{i-1/2}\right) = 0, \frac{u_i^{n+1} - u_i^n}{\tau} + \frac{1}{h} \cdot \left(P_{i+1/2} - P_{i-1/2}\right) = 0, \tag{4}$$

$$\frac{e_i^{n+1} - e_i^n}{\tau} + \frac{1}{h} \cdot \left(P_{i+1/2} \cdot U_{i+1/2} - P_{i-1/2} \cdot U_{i-1/2}\right) = 0, p_i^{n+1} = (\gamma - 1) \cdot \left(e_i^{n+1} - \frac{\left(u^2\right)_i^{n+1}}{2 \cdot \mathrm{v}_i^{n+1}}\right). \tag{5}$$

Calculations were performed up to time, in the field $(-1 \le x \le 1)$ has a flat layer of a viscous gas. To the left of break point $x = 0$, the state of the gas as follows: $\rho = 1$, $p = 1$, and right – $\rho = 0,125$, $p = 0,1$. The initial condition for velocity is given as $u = (1 - x)/31$.

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NUMERICAL METHODS TO THE FACTORIZATION OF THE MATRIX POLYNOMIAL WITH RESPECT TO UNIT CIRCLE

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It is known that one of the key procedures of the frequency methods for the synthesis problems is factorization of the matrix polynomials and separation of the fractional-rational matrices[1].

In this research we offer a calculation high accuracy algorithm to the factorization of the matrix polynomial with respect to unit circle. This algorithm doesn't contain the procedure of finding of the roots of the polynomial. The considered factorization is reduced to the solution of the discrete algebraic Riccati equation (DARE).Solution of these problems may be reduced to the solution of the matrix algebraic Riccati equation [2, 3]. To do this using the matrix sign-function method high accuracy algorithm is offered. The results are illustrated by examples. On a concrete example the analytical solution is compared with the results obtained by usual MATLAB and symbolical approaches.

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ALGORITHM OF NUMERICAL SOLVING OF EQUATIONS FOR NON-STATIONARY MOTION OF THE FLEXIBLE MASSIVE THREAD IN AN ARBITRARY FORCE FIELD

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In present mechanisms are used widely, where connection between the links is realized by a flexible element in the form of tapes, thread, wires, ropes, etc. For example, such mechanisms are winding mechanisms, clutch with centrifugal flexible connections.

In many mechanisms non-stationary motion and masses of the flexible elements are so large that transient regimes of these mechanisms can not be fully studied, and we can be limited by consideration of small vibrations only. For study of non-stationary motion it is necessary to find the form and tension of the flexible element in the general case. It is reduced to study of non-stationary motion of a massive thread in the arbitrary force field. As non-stationary motion is poorly understood this problem is of definite interest.

In the paper the algorithm for numerical solving of equations of a plane non-stationary motion of a flexible thread in arbitrary force field is proposed [1],[2]

$$\frac{\partial}{\partial s}\left(\frac{1}{\gamma(s)}\frac{\partial T}{\partial s}\right)-\frac{T}{\gamma(s)}\left(\frac{\partial\varphi}{\partial s}\right)^2=-\left(\frac{\partial\varphi}{\partial t}\right)^2-\frac{\partial F_1}{\partial s}+F_2\frac{\partial\varphi}{\partial s}, \tag{1}$$

$$\frac{1}{T}\frac{\partial}{\partial s}\left(\frac{T^2}{\gamma(s)}\frac{\partial\varphi}{\partial s}\right)=\frac{\partial^2\varphi}{\partial t^2}-\frac{\partial F_2}{\partial s}-F_1\frac{\partial\varphi}{\partial s}. \tag{2}$$

Here s is coordinate of the point on the thread, measured along the thread from the arbitrarily chosen beginning; t is the time; $\varphi=\varphi(s,t)$ an angle between tangent to the thread and some fixed direction, for example, horizon; $T=T(s,t)$ is thread tension; $F_1=F_1(s,t), F_2=F_2(s,t)$ are the projection of external forces $F=F(s,t)$ on tangent and normal directions correspondingly; $\gamma=\gamma(s)$ is mass of thread per unit of its length (in general case it may be not constant and be some given function of s, while it is taken continuously). Forces are considered per unit mass of the thread element (in particular the forces of weight, forces of aerodynamic resistance and so on). In general case $F=F(s,t)$ is nonlinear.

In paper the method is offered for obtaining of stable solutions of these equations on a computer by implicit finite-difference scheme with using of an iterative process.

As an example of the proposed algorithm, large fluctuations of the flexible thread in the gravitational field are considered, when one end of the thread is fixed and the other is free. Since aerodynamic resistance is not taken into account, control of the solution accuracy at each step in time is realized by condition of energy conservation. Forms of the thread, founded theoretically, were compared with photographs of various forms of real vibrating metal chain.

The proposed method can solve various problems of non-stationary motion of a massive thread by the corresponding boundary and initial conditions.

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SECTION VII

Control Theory and Optimization

GLOBAL BEHAVIOR OF SOLUTIONS TO DYNAMICAL PROBLEMS OF MATHEMATICAL BIOLOGY

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We are going to discuss the problems of global existence uniqueness, global stability and structural stability of the Cauchy problem and initial boundary value problems for FitzHugh - Nagumo equations, Arima-Hasegawa equation and Goodwin-Trainor system of nonlinear partial differential equations that model qualitative behavior of neurons and cellular morphogenesis.

SOME RESULTS ABOUT OPTIMAL CONTROL PROBLEMS UNDER WEAK HYPOTHESES

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In this paper, by considering vector-valued maximum-minimum type functions satisfying Lipschitz condition, and optimal control systems with continuous-time which is governed by systems of ordinary differential equation, we derive necessary conditions for obtimality under weak hypotheses and properties concerning the generalized Jacobian set for optimal control problems of these systems.

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ON A MINIMIZATION OF THE FIRST EIGENFREQUENCY OF THE PLATE WITH RESPECT TO ITS DOMAIN[1]

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Let's consider the problem

$$\Delta^2 u = \lambda u, \quad x \in D, \tag{1}$$

$$\Delta u = 0, \qquad x \in S_D, \tag{2}$$

$$\frac{\partial \Delta u}{\partial n} = 0, \quad x \in S_D, \tag{3}$$

$$\lambda_1(D) \to \min, \quad D \in K, \tag{4}$$

where Δ- Laplace operator, $K = \left\{D \subset R^2 : \bar{D} \in K_0, \quad S_D \in C^2\right\}$, K_0 is some subset of all bounded convex domains from E^2.

The problem (1)-(3) is indeed mathematical description of the across vibrations of the free plate and eigenvalues of this problem λ_j, $j = 1, 2, ...$ indeed are eigenfrequencies of the plate [1].

The following condition of optimality is proved for this problem using previous results [1], [2] .

Theorem. *If the domain $D^* \in K$ is a solution of the problem* (1)-(4) *then for any $D \in K$ is valid*

$$\int_{S_{D^*}} u_1^2(x) \left[P_D(n(x)) - P_{D^*}(n(x))\right] ds \leq 0. \tag{5}$$

Here $u_1(x)$ is the first eigenfunction (indeed eigenvibration of the plate) corresponding to the first eigenfrequency, $P_D(x) = \max\limits_{l \in D} (l, x)$, $x \in R^2$- support function of the domain D.

If to define K_0 by the condition

$$K_0 = \left\{D \subset R^2 : D_1 \subset D \subset D_2\right\},$$

where $D_1, D_2 \subset R^2$ are given bounded convex domains, then it will be equivalent to the condition

$$P_{D_1}(x) \leq P_D(x) \leq P_{D_2}(x), \; x \in R^2.$$

In this case as one can see from (5) D_2 will be an optimal domain minimizing the functional (4), i.e. the first eigenfrequency of the free plate.

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INVESTIGATION OF DYNAMIC ECONOMIC MODEL

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In this work necessary extremum conditions are obtained for the optimal control problem of discrete inclusions.

Let $a_t : R^{n(t)} \to 2^{R^{n(t+1)}}$, t=0,1,2, ...,T-1 be a multivalued mapping, where 2^X denotes the set of all subsets of X, $n(t) \in N$. Let $k = n(1) + n(2) + \cdots + n(T)$ and $q : R^k \to R \bigcup \{+\infty\}$.

Consider the minimization of the function $q(x_1, \cdots, x_T)$ on the trajectories of the problem

$$x_{t+1} \in a_t(x_t), \quad t = 0, 1, ..., \ T - 1, \tag{1}$$

$$x_0 = \bar{x}_0. \tag{2}$$

We denote the set of solutions $(x_1, \cdots, x_T)$ of problem (1),(2) by M. $(\bar{x}_1, \cdots, \bar{x}_T) \in \in M$ is called optimal, if $q(\bar{x}_1, \cdots, \bar{x}_T) \leq \ q(x_1, \cdots, x_T)$ for $(x_1, \cdots x_T) \in M$. If we denote the sets in R^k as

$$M_0 = \left\{ (x_1, ..., x_{\mathrm{T}}) \in R^k : x_1 \in a_0(\bar{x}_0) \right\},$$

$$M_t = \left\{ (x_1, ..., x_{\mathrm{T}}) \in R^k : (x_t, x_{t+1}) \in gra_t \right\}, \quad t = 1, ..., T - 1,$$

then the formulated problem will be reduced the minimization of the function $q(x_1, \cdots, x_T)$ on the set $M = \bigcap_{i=0}^{T-1} M_t$. Let $grb_t = intT_{gra_t}(\bar{x}_t, \bar{x}_{t+1})$, where $gra_t = \left\{ (x, y) \in R^{n(t)} \times R^{n(t+1)} : y \in a_t(x) \right\}$, $T_C(\bar{x})$ tangent Clarke cone.

Theorem 1. *Let $(\bar{x}_1, \cdots, \bar{x}_T) \in M$ minimizes the function q on the set M, gra_t $t = \overline{1, T-1}$, are closed sets, $grb_t = intT_{gra_t}(\bar{x}_t, \bar{x}_{t+1}) \neq \emptyset$ for $t = \overline{1, T-1}$ and function q satisfies to Lipschitz condition in the neighborhood of $(\bar{x}_1, \cdots, \bar{x}_T)$, then there exist vectors $x_1^*(0) \in N_{a_0(\bar{x}_0)}(\bar{x}_1)$, $(x_t^*(t), x_{t+1}^*(t) \in N_{gra_t}(\bar{x}_t, \bar{x}_{t+1})$, $t = \overline{1, T-1}$, and $\lambda \in \{0, -1\}$ that $(x_1^*(0) + \ x_1^*(1), \cdots, x_T^*(T-1) + \ x_T^*(T)) \in \in \lambda\partial q(\bar{x}_1, \cdots, \bar{x}_T)$, where $x_T^*(T) = 0$; $x_t^*(t-1)$, $t = \overline{1, \mathrm{T}}$ and λ non zero simultaneously, $N_C(\bar{x})$ normal Clarke cone. Let us consider the discrete inclusions*

$$z_{t+1} \in b_t(z_t), \quad t = 1, ..., \ T - 1, \tag{3}$$

$$z_0 \in T_{a_0(\bar{x}_0)}(\bar{x}_1). \tag{4}$$

Theorem 2. *Let $(\bar{x}_1, \cdots, \bar{x}_T) \in M$ minimizes the function q on the set M, gra_t, $t = \overline{1, T-1}$, closed sets, exist solution of problem (3),(4) and function q satisfy to Lipschitz condition in the neighborhood of $(\bar{x}_1, \cdots, \bar{x}_T)$, then there exist vectors $x_1^*(0) \in N_{a_0(\bar{x}_0)}(\bar{x}_1)$, $(x_t^*(t), x_{t+1}^*(t) \in N_{gra_t}(\bar{x}_t, \bar{x}_{t+1})$, $t = \overline{1, T-1}$,* and $x_T^*(T) = 0$, that

$$(x_1^*(0) + \ x_1^*(1), \cdots, x_T^*(T-1) + \ x_T^*(T)) \in -\partial q(\bar{x}_1, \cdots, \bar{x}_T).$$

Theorem 3 . *Let us suppose that $(\bar{x}_1, \cdots, \bar{x}_T) \in M$ minimizes the function q on the set M, gra_t are the convex sets, q is convex and continuous function of the point $(\bar{x}_1, \cdots, \bar{x}_T,)$ then there exist the vectors $x_1^*(0) \in N_{a_0(\bar{x}_0)}(\bar{x}_1)$, $(x_t^*(t), x_{t+1}^*(t) \in N_{gra_t}(\bar{x}_t, \bar{x}_{t+1})$, and $\lambda \in \{0, -1\}$* that

$$(x_1^*(0) + \ x_1^*(1), \cdots, x_T^*(T-1) + \ x_T^*(T)) \in \lambda\partial q(\bar{x}_1, \cdots, \bar{x}_T),$$

where $x_T^*(T) = 0$; $x_t^*(t-1)$, $t = \overline{1, \mathrm{T}}$ and λ are non zero simultaneously.

OPTIMIZATION OF FLOW ON STREAM DATA PROCESSING IN REAL TIME SYSTEMS

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Let's imagine data stream as workflow of arbitrary particles. In that case we will get an assembly job on continuously arriving parts which is divided into several work items.

$$\mathrm{J} = (\mathrm{w}_1^{\mathrm{j}}, \mathrm{w}_2^{\mathrm{j}}, \dots, \mathrm{w}_{n_{\mathrm{j}}}^{\mathrm{j}}), \mathrm{w}_{\mathrm{i}}^{\mathrm{j}} : 1 \le i \le n_j,$$

there are J is complete Job which consists of n_j work items. w_{i+1}^j cannot be started before w_i^j is completed. One or more work items may be carried out by an agent.

Thus, the entire system will have the following hierarchy: each Job is divided among several agents, each agent consists of number of work items (performs several works). Our goal is optimization of given workflow: to process each part as fast as possible (minimal overall processing time) with use of minimal agents. Of course, we assume that number of spare agents as many as needed.

Example: J1=$\{w_1^1, w_2^1, w_3^1, w_4^1, w_5^1\}$. Q_1^1, Q_3^1 – sizes of queues of arriving parts before each agent, Q_2^1, Q_4^1, Q_5^1 – sizes of internal queues before each work items inside appropriate agent. $t_1^1, t_2^1, t_3^1, t_4^1, t_5^1$ – times of processing of parts in each work item. L_1 – total time spent by system to process single part. See Figure 1. B_1^1 : %of time A_1 is busy doing Job 1. B_2^1: % of time A_2 is busy doing Job 2. Ideally must be $B_1^1 \approx 100\%, B_2^1 \approx 100\%$. Job parts arrival rate λ_1 may be too slow or too fast.

In case of small λ_1:

$B_1^1$and B_2^1 may be well below 100%. In that case , is is better to free either A_1 or A_2 and use a single Agent. Thereby, transportation cost is also saved.

In case of large λ_1:

When Q_1^1 is full, object drop to backup queue BQ_1^1 of infinite length. $\theta-$ number of object movements between Q_1^1 and BQ_1^1.The time for shuffling one object from Q_1^1 to BQ_1^1 is t_s. Increase in per object queuing delay (waiting in queue + actual processing time) $\approx \theta \cdot t_s$. Ideally θ must be kept close to 0.

Here we consider two forms of optimization of given workflow: load balancing (vertical partitioning) and parallelization (horizontal partitioning). In case of load balancing queue size is doubled, B_1^1 will decrease, additional transportation cost is introduced.

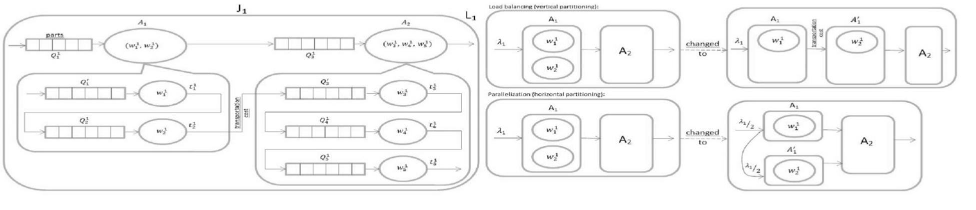

FIGURE 1. Example of continuously part processing Picture and two forms of workflow optimization.

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CONTROL OF HEATING PROCESS WITH FEEDBACK AT LUMPED POINTS AND AT INSTANTS OF TIME[1]

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Let some homogenous rods of length l be sequentially heated in a heating stove at the expense of temperature $\vartheta(t)$ created in it by an external source. The temperature is assumed to be the same all over the heating stove. Then the heating process of each rod is described by the following differential equation of parabolic type and including boundary conditions:

$$u_t(x,t) = a^2 u_{xx}(x,t) + \alpha\left[\vartheta(t) - u(x,t)\right], \ (x,t) \in \Omega = (0,l)\times[0,T], \tag{1}$$

$$u_x(0,t) = \lambda\left[u(0,t) - \vartheta(t)\right], \quad t \in (0,T], \quad u_x(l,t) = -\lambda\left[u(l,t) - \vartheta(t)\right], \quad t \in (0,T], \tag{2}$$

where a, α, λ are given parameters of the process. For simplicity, we assume that the initial temperature of the rods is constant along their entire length, but different for each rod. Assume that we have an admissible set (interval) of possible initial values of temperatures $B = [\underline{B}, \overline{B}]$:

$$u(x,0) = b = const, \ b \in B, \ x \in [0,l], \tag{3}$$

at that we know of the density function of initial temperatures $\rho_B(b)$ such that

$$\int_B \rho_B(b)db = 1, \quad \rho_B(b) \geq 0, \quad b \in B. \tag{4}$$

Feedback control is sought in the form:

$$\vartheta(t) = K_j \sum_{i=1}^{L_x} \gamma_i u(\bar{x}_i, \bar{t}_{j-1}) = const, \ K_j = const, \ t \in [\bar{t}_{j-1}, \bar{t}_j), \ j = 1,2,...,L_t. \tag{5}$$

where $K(t)$ is an optimized parameter of regulations; γ_i, $i = 1,2,...,L$weighting coefficients, $u(\bar{x}_i, \bar{t}_{j-1})$, $i = 1,2...,L_x$, $j = 1,2,...,L_t$ current temperature at given discrete instants of time $\bar{t}_j \in [0,T]$, $j = 0,1,...,L_t$, $t_0 = 0$, $\bar{t}_{L_t} = T$ and at given L points $x_i \in [0,l]$, which is measured with the use of sensors, depending on the values of which the current temperature $\vartheta(t)$ is set in the heating stove.

Performance criterion of control of the heating process is set as follows:

$$\begin{gathered} J(K,\bar{\gamma}) = \int_B \left[\int_0^l \mu(x)\left[u(x,T;K,\gamma,b) - U(x)\right]^2 dx\right]\rho_B(b)db + \\ +\varepsilon_1 \left\|K(t) - K_0\right\|^2_{L_2[0,T]} + \varepsilon_2 \left\|\gamma - \gamma_0\right\|^2_{R^L}, \end{gathered} \tag{6}$$

where $U(x)$, $\mu(x) \geq 0$ are given functions, $\varepsilon_1 > 0, \varepsilon_2 > 0$, $K_0 \in R^1$, $\gamma_0 \in R^L$.

We obtain necessary optimality conditions, which contain formulas for the components of the gradient of the target functional on $K(t)$. For numerical solution to the problem using iterative first order optimization methods, problem (1)-(6) is reduced using method of lines to a problem of control of a loaded system of ordinary differential equations involving non-local conditions.

Results of carried out numerical experiments are given in the work.

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ANALYSIS OF OPTIMAL TRANSIENT CONDITIONS IN OIL PIPELINE SYSTEMS[1]

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In the paper, we state the formulation and analysis of the results of numerical investigation of the problem of optimization of transient conditions in oil pipeline systems that arise during the transition from one steady-state condition of transportation of hydrocarbons to another.

We obtained the following main results of the qualitative analysis of transient conditions.

What is most important is the fact that the period of the transition process depends essentially on the chosen class of control actions and constraints imposed on the control and on phase state.

Minimal period of the transition process under piecewise-continuous control actions without any constraints imposed on the control process does not depend on the pipeline's diameter, the hydraulic resistance coefficient, oil viscosity and the values of the initial and final steady-state conditions, but does depend only on the length of the pipeline. But for all that obtained optimal regimes of pumping stations are practically not realizable.

If there are technological constraints imposed on control actions from the class of piecewise continuous functions, the following facts take place:
a) if the dissipation coefficient increases, the transition period decreases; at that, if the range of the set of admissible controls increases, the influence of the dissipation coefficient decreases;
b) the difference between the values of the parameters of the initial and final steady-state conditions affects the period of the transition process. Namely, the more we need to increase consumption, the longer the period of the transition process (the effect of this difference on the transient period decreases as the range of the set of admissible values of control actions increases);
c) if the pipe run increases, the period of the transition process increases much more slowly when narrowing the interval of admissible values of the control; this change becomes directly proportional to the increase of the pipe run when expanding the range of admissible values of the control actions;
d) minimal period of the transition process under the transition from a lower value of the condition to a greater one and, vice versa, from larger to smaller is the same; at that, optimal regimes of transition are symmetrical.

When controlling the transition process on the class of piecewise constant functions under technological constraints imposed on the control regimes, all the quality characteristics which piecewise continuous regimes posses and listed above are also observed in this class. In addition to these, the following properties are established:
a) in comparison with the piecewise-continuous controls, in case of using piecewise constant actions we need more time for the transition process under the same initial values of the technological parameters and constraints;
b) the increase in the number of intervals of constancy and in the range of admissible values of the control results in the reduction of the period of the transition process;
c) in case when we also optimize the switching moments of the controls, the narrowing of the range of admissible values of the control results in significant reduction of the number of intervals of constancy for the controls.

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DETERMINATION OF BREAKPOINTS IN OIL PIPELINE NETWORKS UNDER NON-STATIONARY REGIMES[1]

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We investigate a problem of the determination of breakpoints and of volume of oil leakages under non-stationary regimes of transportation over a linear part of a main pipeline. With this purpose, we rewrite the linearized system of differential equations of unsteady laminar flow of incompressible fluid with constant density in the following form:

$$\begin{cases} -\frac{\partial P(x,t)}{\partial x} = \frac{\partial Q(x,t)}{\partial t} + 2a[Q(x,t) - \sum\limits_{i=1}^{m} q_i^{loss}(t)\delta(x-\xi_i)], \\ -\frac{\partial P(x,t)}{\partial t} = c^2 \frac{\partial Q(x,t)}{\partial x}, \end{cases} \tag{1}$$

where $P(x,t)$, $Q(x,t)$ are pressure and flow rate, respectively, at the point $x \in (0,l)$ of the pipeline at point of time $t > t_0$; $c$sound speed in the environment; $2a$ linearized friction coefficient; $q_i^{loss}(t)$ defines the unknown volume of oil leakage at unknown breakpoint $\xi_i \in (0,l)$, $i = 1,...,m$ of the pipeline; m the number of breakpoints.

We assume that there is permanent long-term observation over the expenditure and pressure at the ends of the pipeline, i.e. we have

$$\{P(0,t) = p_0(t),\ P(l,t) = p_l(t), \} \quad \{Q(0,t) = \bar{q}_0(t),\ Q(l,t) = \bar{q}_l(t)\}, \quad t \in [0,T]. \tag{2}$$

It is evident that under known points $\xi_i$and expenditures of leakages $q_i^{loss}, i = 1,..,m$to calculate the regimes of flow in pipelines, it is necessary to have one of the conditions (2).

The problem consists in solving for the values $q^{loss}(t) = (q_1^{loss}(t),...,q_m^{loss}(t))$ $\xi = (\xi_1,...,\xi_m)$, under which the functional

$$\begin{aligned} J(\xi, q^{loss}(t)) = \int\limits_{t_0}^{T} [Q(0,t;\xi,q^{loss}(t)) - \bar{q}_0(t)]^2 + \\ +[Q(l,t;\xi,q^{loss}(t)) - \bar{q}_l(t)]^2 dt + \varepsilon_1 \left\| q^{loss}(t) - \tilde{q}^0 \right\|^2_{L_2^m[0,T]} + \varepsilon_2 \left\| \xi - \bar{\xi} \right\|^2_{R^m} \to \min \end{aligned} \tag{3}$$

takes on its minimal value, where $t_0 > 0$ is the estimated time when a leakage occurred; ε_1, ε_2, $\bar{\xi}, \tilde{q}^0$ regularization parameters.

A numerical approach to the solution to problem (1)-(3) is proposed in the work. Results of numerical experiments are given.

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DETERMINATION OF RESISTANCE COEFFICIENT FOR PIPELINE SECTION UNDER NON-STATIONARY REGIME[1]

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In the paper, we consider the following process of non-stationary isothermal flow of incompressible fluid at linear section of main pipeline [1]:

$$\begin{cases} -\frac{\partial p(x,t)}{\partial x} = \frac{\partial \omega(x,t)}{\partial t} + \lambda \frac{\omega(x,t)}{2d}, \\ -\frac{\partial p(x,t)}{\partial t} = c^2 \frac{\partial \omega(x,t)}{\partial x}, \quad (x,t) \in \Omega = \{(x,t) : 0 < x < l, \ 0 < t \le T\}, \end{cases} \tag{1}$$

where $\lambda = \lambda(Re, \varepsilon/d)$ is the coefficient of hydraulic resistance of a pipeline section; ε/d roughness of the interior surface of the pipeline of diameter d. Then, for a particular pipeline, taking into account that $Re = \omega\beta d/\mu$, we have $\lambda = \lambda(\omega)$. To obtain this dependence, we put the following inverse problem. Assume that we have the results of long-term observations over the regimes at the ends of the pipeline of length l. Initial conditions may be given or not (in this case we have a problem without initial conditions):

$$p(0,t) = \varphi_1^p(t), \ \ p(l,t) = \varphi_2^p(t), \ \omega(0,t) = \varphi_1^\omega(t), \ \ \omega(l,t) = \varphi_2^\omega(t), \quad t \in [0,T], \tag{2}$$

where $\varphi_1^p, \ \varphi_2^p, \ \varphi_1^\omega, \ \varphi_2^\omega$ are given functions. It is possible to consider other types of observations; particularly, observations may be carried out at internal points of the pipeline $x_s \in (0,l)$, for example, over the pressure $p_s(t)$ and velocity $\omega_s(t)$, $s = \overline{1, N-1}$, $x_0 = 0$, $x_N = l$.

Introduce the following criterion of identification of the coefficient:

$$I(\lambda) = \sum_{s=0}^{N} \int_0^T \left[p(x_s,t;\lambda) - \overline{p_s}(t)\right]^2 dx + \int_0^T \left[\omega(x_s,t;\lambda) - \overline{\omega_s}(t)\right]^2 dx \to \min_{\lambda(\omega)\in A}, \tag{3}$$

where A is given convex closed domain of admissible values of λ; $p(x,t;\lambda), \ \omega(x,t;\lambda)$ solution to problem (1), (2). To solve problem (1)-(3) and obtain the function λ we use an approach proposed in [2].

We obtain formulas for the components of the gradient of the target functional in the space of identified values of the parameters $\lambda \in R^L$, which allow to use first-order optimization methods to solve the problem (1) numerically. The obtained values of the optimized vector are then used to build the function $\lambda(\omega)$ from any class of functions using interpolation and approximation methods.

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ON AN IMPLEMENTATION OF CRITERIA TRUNCATION METHOD IN VECTOR OPTIMIZATION PROBLEMS[1]

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We consider the finite-dimensional problem of vector optimization:

$$F(x) = (f_1(x), ..., f_l(x)) \to \min_{x \in D \subset R^n}, \tag{1}$$

$$D = \{x \in R^n : g_i(x) \leq 0, i = 1, ..., m\}, \tag{2}$$

where the given functions $f_j(x), g_j(x)$ are continuously differentiable, convex, $D \subset R^n$ is a non-empty bounded admissible set.

The approach in which the solution of the problem (1), (2) is carried out iteratively in an interactive mode is as follows. At each s-th iteration the user sets the desired values of criterion functions f_i^s, $i = 1, ..., l$. To find $x^s \in D$, satisfying conditions

$$f_i(x) \leq f_i^s, i = 1, ..., l, \ x \in D, \tag{3}$$

the sequence $\{x_q^s\}$, $q = 0, 1, ...,$ $x_0^s = x^{s\ 1}$- the solution taken at the previous iteration, x_0^0 - an initial admissible point of the admissible set is constructed:

$$F_q(x_q^s; x_{x-1}^s) = \min_{x \in D} F_q(x; x_{q-1}^s), \quad F_q(x; x_{q-1}^s) = \sum_{i=1}^{l} \frac{1}{f_i^s - f_i(x) + \xi_i^q} + \sum_{i=1}^{m} \frac{1}{g_i(x) + \varepsilon}.$$

Here $\varepsilon > 0$ is the sufficiently small given value,

$$\xi_i^q = \left\{ \begin{array}{ll} f_i(x_q^s) - f_i^s + \varepsilon & f_i(x_q^s) \geq f_{i_s}^s, \\ 0 & f_i(x_q^s) < f_i \end{array} \right\}.$$

Theorem. *The point$x^s = \lim\limits_{q \to \infty} x_q^s$ belongs to the admissible set D, while it satisfies the constraints (3) if they are compatible, otherwise it is a point of the Pareto set of problem (1), (2).*

Based on the theorem, an algorithm for consistent implementation of criterion truncations was developed. Correlation properties of the objective functions $f_i(x), i = 1, ..., l$ and constraint functions $g_j(x)$, $j = 1, .., m$ are used in the developed algorithm. For this purpose, an extended function $F(x) = (f_1(x), ..., f_l(x), g_1(x), ..., g_m(x))$ and symmetric $(m + n)$ dimensional matrix are entered. Elements of the matrices are:

$$r_{ij}(x) = (\nabla F_i(x), \nabla F_j(x)) / \|\nabla F_i(x)\| \cdot \|\nabla F_j(x)\|), \quad i, j = 1, ..., m + n,$$

where $r_{ij} \in [-1; 1]$. Elements of the matrices characterize the degree of antagonism (inconsistency) of functions at the current point $x^s \in D$. Depending on the values of $r_{ij}(x^s)$ and proximity of x^s to the boundaries of D, i.e. to $g_j(x) = 0, j = 1, ..., m$ new desirable values of criteria $f_i(x), i = 1, ..., l$ are assigned.

The results of test problems are presented in the report.

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IDENTIFICATION OF DOMAIN'S BOUNDARY WITH THE APPLICATION OF SPLINE TYPE FUNCTIONS[1]

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Application of spline type functions for numerical solution to problems of identification of two-dimensional domain's boundary of distributed systems described by evolutionary and non-evolutionary equations are considered in the work. In particular, we give results obtained in case when the system with distributed parameters is described by the following boundary problem with respect to elliptic equation:

$$\Delta u = f(x,y), \quad (x,y) \in D, \qquad u(x,y)|_{(x,y)\in\Gamma} = \varphi(x,y). \tag{1}$$

Here $\Gamma = \Gamma_1 \cup \Gamma_2$, $\Gamma_1 \cap \Gamma_2 = \emptyset$, is the boundary of the domain$D \subset E^2$; Γ_1 known part of the whole boundary Γ; Γ_2 unknown part of Γ; $\varphi(x,y)$ sufficiently smooth function defining boundary conditions of the problem. We assume that Γ_2 is determined by an unknown function$y(x)$, $x \in \left[\tilde{x}, \tilde{\tilde{x}}\right]$.

Assume that we have results of observations over the state of the process at given L points $\left(x^k, y^k\right)$:

$$u\left(x^k, y^k\right) = u^k, \quad \left(x^k, y^k\right) \in D, \quad k = \overline{1, L}. \tag{2}$$

Assume that the identified part of the boundary Γ_2 is sought in piecewise-given form of spline type functions:

$$y_i(x,\alpha) = \left(\frac{x-\bar{x}_{i-1}}{\bar{x}_i-\bar{x}_{i-1}}\bar{y}_i + \frac{\bar{x}_i - x}{\bar{x}_i-\bar{x}_{i-1}}\bar{y}_{i-1}\right) + (x-\bar{x}_{i-1})(x-\bar{x}_i)\sum_{j=1}^{m}\alpha_{ij}\psi_j(x), \quad x \in [\bar{x}_{i-1}, \bar{x}_i], \ i = \overline{1,n}. \tag{3}$$

Function (3) satisfies continuity conditions on the whole in $[\bar{x}_1, \bar{x}_n]$. Here $\bar{x}_i$, $i = \overline{1,n}$, $\bar{x}_1 = \tilde{x}$, $\bar{x}_n = \tilde{\tilde{x}}$ are fixed points; $\psi_j(x)$, $j = \overline{1,m}$ given linearly independent functions.

The problem consists in finding vector $z = (\bar{y}_1, \bar{y}_2, ..., \bar{y}_n, \alpha_{11}, ..., \alpha_{1m}, ..., \alpha_{nm})$ defining the identified boundary Γ_2, for which the functional

$$J(\Gamma_2) = \sum_{k=1}^{L}\left[u\left(x^k, y^k, \Gamma_2\right) - u^k\right]^2 + \varepsilon\left\|z - \bar{z}\right\|^2_{R^{n(m+1)}} \to \min_{\Gamma_2} \tag{4}$$

takes on its minimal value, where ε, $\bar{z}$ are regularization parameters.

For numerical solution to problem (1)-(4), we use the schemes proposed in [1]. In the work, formulas for the gradient of the objective functional on the optimized parameters are obtained, thus providing a possibility of applying efficient first order optimization methods.

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OPTIMAL CONTROL IMPULSIVE SYTEMS WITH NONLOCAL CONDITIONS DESCRIBED BY THE SECOND ORDER DIFFERENTIAL EQUATIONS

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In this talk we consider the following optimal control problem, which described by the second order impulsive differential equation with non-local conditions: to find a control $(u(\cdot),[v]) \subset U \times V$ that minimizes the cost functional

$$J(u,[v]) = \Phi(x(0), x(T)) \tag{1}$$

on the solutions of the following impulsive differential equations with nonlocal conditions

$$\begin{cases} x''(t) = f(t, x(t), u(t)), 0 \leq t \leq T, t \neq t_i, \\ x(0) = x(T) = \int_0^T n(t) x(t) dt, \\ \Delta x(t_i) = I_i(x(t_i), v_i), i = 1, 2, ..., p; 0 < t_1 < t_2 < ... < t_p < T \end{cases} \tag{2}$$

$$(u(\cdot),[v]) \in U \times V \subseteq L_2[0,T] \times L_{2p} \tag{3}$$

where f is a continuous function, $n(t) \in R^{n\times n}, \Delta x(t_i) = x(t_i^+) - x(t_i^-)$, and I_i are some functions, $x(t) \in R^n, (u(\cdot),[v])-$ are state and control of the system, respectively.

We list out the following hypotheses:

$(H1)$ $f : [0,T] \times R^n \times R^r \to R^n, I_i : R^n \times R^{r_i} \to R^n, i = 1,2,...,p$ are continuous and there exists constants $K > 0, L_i > 0, i = 1,2,...,p$, such that.

$$|f(t,x,u) - f(t,y,u)| \leq K|x-y|, t \in [0,T], x,y \in R^n$$

$$|I_i(x,v) - I_i(y,v)| \leq L_i |x-y|, x,y \in R^n.$$

$$(H2) \left[KT + \sum_{i=1}^{P} L_i\right] < 1.$$

(H3) $n(t) \in L_1[0,T]$ is nonnegative and $\mu \in [0,1)$, where $\mu = \int_0^T n(t) dt$.

If assumptions $(H1)$-$(H3)$ are satisfied, then for every $C \in R^n$, an equation (1) has a unique solution on $[0,T]$ is proved. The formula of gradient of the functional was derived. Necessary conditions of optimality were obtained.

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OPTIMIZATION OF MINERAL EXTRACTION PROCESS BY CONTROLLING OF REAGENT SUPPLY INTENSITY

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In the present work optimization of in-situ leach mineral mining process is considered. Preliminary performed numerical calculations show that recovery degree of layer strongly depends on the input reagent concentration and the arrangement of wells at the in-situ leaching (ISL) of minerals. Therefore, solution of optimization problem of mineral leaching process is reduced to solve two problems, one of which is a problem of optimal control the intensity of reagent supply, and another - optimal arrangement of wells. The process of mineral extraction by the ISL method is described by the equations of mineral dissolution, transport liquid solution and dissolved minerals, on condition that the distribution of mineral in the reservoir is known and concentration of solution and dissolved useful element doesnt exist at initial time. During the leaching, when the arrangement of wells is already known, current process in layer can be controlled only by the input concentration of reagent on the injection well. Therefore, reagent concentration is taken as a control in the considering problem. Control of reagent concentration value on the injection well leads to the changing of mineral value in liquid phase. On the value of input reagent concentration imposed two constraints: 1) in the inequality form (inequality constraint) constraining amount of reagent concentration on each injection well, 2) an equality of the total reagent concentration to all wells to a definite value at each time station. Computational program for calculation of optimization problem is elaborated. Based on received results influence of reagent supply on excavation rate, mining efficiency and time of deposit excavation has been investigated and optimal value of reagent concentration on injection well is obtained.

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ON THE OPTIMAL GUILLOTINE CUTTING OF A RECTANGLE INTO RECTANGLES WITH TWO HEIGHTS

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The problems of packing and cutting are difficult problems of discrete optimization. For example, the computational complexity of a problem of packing in a rectangular sheet (A, B) the maximum number of equal small rectangles $(c, d), (d, c)$ (pallet loading problem (PLP)) is unknown. There exist fast polynomial algorithms for a problem of optimum guillotine cutting a rectangle (A, B) into rectangles $(a, b), (c, d)$ [1].

Here we study a problem of optimum guillotine cutting a rectangular sheet (A, B) into rectangles $(a_i, b_1), i \in I_1, (a_i, b_2), i \in I_2$, without a restriction on the number of copies of each small rectangle. It is required to find a cutting pattern with a minimum trim loss. We consider the set of feasible solutions to the knapsack problem for the side B:

$$S(B) = \{(x_1, x_2)|b_1x_1 + b_2x_2 \leq B,\ x_1, x_2 \in \mathbf{Z}_+ = \{0, 1, 2...\}\}.$$

It is easy to show, that the convex hull of this set $P(B) = \mathrm{co}(S(B))$ is equal to the Minkowski sum of rectangular triangles:

$$P(B) = \oplus_{j=1}^{m} T(c_j, d_j),$$

where $T(c_j, d_j)$ is a rectangular triangle with vertecies $(0, 0), (c_j, 0), (0, d_j), c_j, d_j \in \mathbf{Z}_+$. It is easy to show, that fractions $\frac{c_j}{d_j}$ are non-principal convergents of $\frac{b_1}{b_2}$. There are exist polynomial algorithms for calculating $P(B) = \mathrm{co}(S(B))$. The following theorem on the structure of the optimum solution of considered problem is fair.

Theorem 1. *A problem of optimum guillotine cutting a rectangular sheet (A, B) into rectangles $(a_i, b_1), i \in I_1, (a_i, b_2), i \in I_2$ without a restriction on the number of copies of each small rectangle is decomposed into m integer knapsack problems*

$$Z_j = \max \left\{ \sum_{i \in I_1} c_j a_i x_i + \sum_{i \in I_2} d_j a_i x_i \right\},$$

$$s.t.\ \sum_{i \in I_1} c_j a_i x_i + \sum_{i \in I_2} d_j a_i x_i \leq A, x_i \in Z_+, j = 1, 2, ..., m,$$

so that $Z_1 + Z_2 + ... + Z_m$ is equal to maximum cutting area of a rectangle (A, B).

This theorem and known polynomial algorithm for an integer linear programming imply the existence of a polynomial algorithm for the considered problem.

Theorem 2. *There exists a polynomial algorithm for problem of optimum guillotine cutting a rectangular sheet (A, B) into rectangles $(a_i, b_1), i \in I_1, (a_i, b_2), i \in I_2$ without a restriction on the number of copies of each small rectangle for any fixed set of indexes I_1, I_2.*

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OPTIMIZATION OF CONTROL AND COMPLETION TIME OF NON-STATIONARY PROCESSES IN HYPERBOLIC TYPE SYSTEMS[1]

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We consider a problem of optimization of boundary control and of completion time of non-stationary processes described by hyperbolic type equations. To solve the problem, we propose to use first order optimization methods based on analytical formulas for the gradient of the functional obtained in the work.

Consider a problem of optimal boundary control of a non-stationary process in an oscillating system the state of which is described by the following initial and boundary problem for telegraph equation:

$$y_{tt} = a^2 y_{xx} - b y_t, \quad t \in (0, T], x \in (0, l),$$

$$y(x, 0) = \varphi_0(x), \;\; y_t(x, 0) = \varphi_1(x), \;\; x \in (0, l),$$

$$y(0, t) = v_1(t), \;\; y(l, t) = v_2(t), \; t \in (0, T],$$

where $y(x, t)$ is phase function of the system's state, a speed of perturbation propagation in environment, b friction or dissipation coefficient, $\varphi_0(x) \in H^1[0, l], \varphi_1(x) \in L_2[0, l]$ given functions, $v_i(t) \in L_2[0, T]$, $i = 1, 2$, control functions, T completion time of the process that is an optimized parameter.

We assume that the control actions satisfy the inequalities $\underline{v} \le v_i(t) \le \overline{v}$, $i = 1, 2$ nearly everywhere on $[0, T]$.

The problem of time-optimal boundary control consists in solving for optimal values of piecewise continuous functions $v_i(t)$, $i = 1, 2$ and of parameter T that minimize the functional

$$J(u, T) = \alpha_1 \int_0^T \int_0^l f^0(y(x, t)) dx dt + \alpha_2 \int_0^l f^1(y(x, T), y_t(x, T)) dx + \Phi(T) \to \min,$$

where α_1, α_2 are positive constants; given functions f^0, f^1, Φ are assumed to be continuous together with their derivatives on all the arguments.

To solve the problem, two approaches are applied with the purpose of comparison. According to the first one T is considered as a parameter and double-level optimization is used: at the upper level, we apply some one-dimensional optimization method to determine optimal time T^* of the transient process; at the lower level, given the current values to find $J_T^* = J(u_T^*, T) = \min_u J(u, T)$, we solve a problem of optimal control of a distributed system with fixed time.

According to the second approach, T is considered as a component of the control, and to solve for its optimal value, we apply a procedure of simultaneous optimization of T and $u(t)$.

Necessary optimality conditions are obtained. They contain formulas for the components of the gradient of the functional on the control actions and on completion time of the process.

Results of numerical experiments are given. They allow to compare the approaches described above.

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OPTIMAL CONTROL BY BOUNDARY OF A DOMAIN VARYING IN TIME AND BY COMPLETION TIME OF CONTROL PROCESSES[1]

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The problem of the optimization of the domain boundary and the completion time of the process of heating is considered in the given work, where boundary of the domain can vary in time [1,2]. Let us consider the problem of optimal control on the heating process describing by the next one-dimensional boundary problem of Dirichlet:

$$v_t = av_{xx} + u(x,t),\ (x,t) \in \Omega(t) \times (0,T], \tag{1}$$

$$v(x,0) = \varphi(x),\ \ x \in (0, l(0)), \tag{2}$$

$$v(0,t) = \mu_1(t),\ t \in (0,T], \tag{3}$$

$$v(l(t),t) = \mu_2(t),\ t \in (0,T], \tag{4}$$

$$\underline{L} \le l(t) \le \overline{L}, \quad t \in (0,T], \tag{5}$$

where $\Omega(t) = (0, l(t)) \subset R^1, t \in (0,T]$, R^n is n-dimensional Euclidian space; $v = v(x,t)$ is a phase position of the object; $a, \underline{L}, \overline{L}$ are given positive quantities; $\varphi(x), \mu_1(t), \mu_2(t)$ are given continuous functions satisfying adjoint conditions $\varphi(0) = \mu_1(0)$, $\varphi(l) = \mu_2(0)$; $l(t)$is the position of the right border of the area, $u(x,t)$ is the function of inside sources; T is a completion time of the process – are optimized parameters.

The problem consists in finding piecewise-continuous values of functions $l(t)$, $u(x,t)$ and parameter T, at which the given functional

$$J(u,l,T) = \alpha_1 \int_0^T \int_0^{l(t)} f^0(u(x,t), v(x,t))dxdt + \alpha_0 \Phi(T) \to \min \tag{6}$$

takes on its minimum value, where α_0, α_1 are positive constants; the given functions $f^0(u,v),\ f^1(u,v),\ \Phi(T)$ are considered continuous with derivatives on their arguments.

First order optimization methods based on the received analytical formulas for the gradient of the functional are proposed to use for solving the problem. The results and analysis of the numerical experiments will be given in the lecture time.

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AN ASYMPTOTIC METHOD TO THE CONSTRUCTION OF THE OPTIMAL REGULATOR IN GASLIFT WELLS [1]

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The aim of the work is the solution of the stabilization problem with respect to the control and program trajectory which depends on small parameter in the gas-lift problem. Optimal control problem on time interval $[0,\ T]$ has been considered. Let's consider the stabilization problem in the infinite time interval $[0,\ \infty)$ [1].

The stabilization problem consists of finding K that makes the closed-loop system stable when the equation of the object $\nu = K\beta$ is written as a system of linear differential equations

$$\dot{\beta} = (A_0 + A_1\varepsilon)\beta + B\varepsilon\nu \tag{1}$$

and the functional

$$J = \frac{1}{2}\int_0^T (\beta^T Q\beta + v^T R\nu)dx, \tag{2}$$

takes its minimum value.

Here

$$\beta(t) = x(t) - x_{np}(t), \nu(t) = u(t) - u_{np}(t), K = -R^{-1}B'S, \tag{3}$$

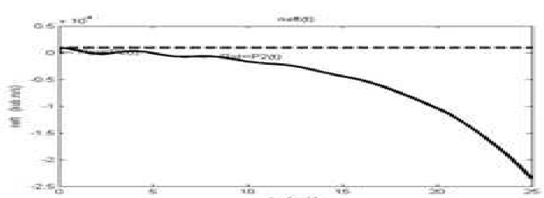
S is a positively defined solution of the following Riccati equation

$$S(A_0 + A_1\varepsilon) + (A_0 + A_1\varepsilon)^T S - \varepsilon SBR^{-1}B'S + Q = 0. \tag{4}$$

Note that *Shur* and *Sign* function methods do not work to solve this problem for enough large n. Therefore to find the solution of the problem asymptotic methods are suitable. Let us seek the solution of the equation (4) in the form $S = S_0 + \varepsilon S_1 + \dots$. Then we got two Lyapunov equations with respect to S_0 and S_1

$$\begin{gathered} S_0A_0 + A_0'S_0 + Q = 0, \\ A_0'S_1 + S_1A_0 + S_0A_1 + A_1'S_0 - S_0BR^{-1}B'S_0 = 0. \end{gathered} \tag{5}$$

If consider solution of these equations in the expression of matrix K we can find its expression in the following form $K \approx -R^{-1}B'S_0 - \varepsilon R^{-1}B'S_1$. As a concrete example we give the following figures



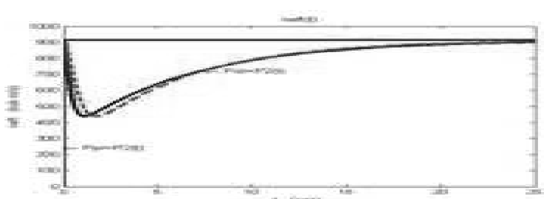


Figure 1. shows that for $n > 8$ the results of the known methods Q_{deb} for infinite time t is not stabilized;

Figure 2. we see that the result of the asymptotic calculation Q_{deb} is stabilized.

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ON AN EXTREMAL PROBLEM FOR THE DIFFERENTIAL INCLUSIONS

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In this work the optimal control problem is studied described by Goursat-Darboux equation by using of the extremal problem for Goursat-Darboux type two dimensional differential inclusions [1].

We assume that $u(\cdot) \in A^n([0,1] \times [0,1])$ [1,2].

Let $f : [0,1]^2 \times R^{3n} \times Q \to R$; $g : [0,1]^2 \times R^{3n} \times Q \to R$; $\xi : R^n \to R$; $\varphi_1 : [0,1] \to R^n$; $\varphi_2 : [0,1] \to R^n$, where $Q \subset R^m$ be a compact set.

The following problem is considered

$$J(\nu) = \int_0^1 \int_0^1 g(t, s, u(t,s), u_t(t,s), u_s(t,s)) dt\, ds + \xi(u(1,1)) \to \min \tag{1}$$

with the conditions

$$u_{ts}(t,s) = f(t, s, u(t,s), u_1(t,s), u_s(t,s), \nu(t,s)) \tag{2}$$

$$u(t,0) = \varphi_1(t), u(0,s) = \varphi_2(s), \varphi_1(0) = \varphi_2(0),$$

where $\varphi_1(\cdot)$, $\varphi_2(\cdot)$ absolutely continuous functions $\nu : [0,1] \times [0,1] \to Q$ measurable function.

The necessary condition of optimality for the problem (1), (2) is obtained.

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ON AN APPROACH AND NUMERICAL SOLUTION TO CONTROL PROBLEMS FOR OBJECTS DESCRIBED BY LOADED HYPERBOLIC TYPE EQUATIONS

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It is known that a number of biological thermal and of underground fluid flow processes are described by loaded equations of hyperbolic type. The controlled process that we consider is described by the following integral and differential equation of hyperbolic type:

$$\begin{aligned} &p_{tt}\left(x,t\right)=ap_{xx}\left(x,t\right)+\int\limits_{t-\tau}^{t} K(x,t,\nu)p(x,\nu)d\nu+\varphi_0(x,t), \\ &(x,t)\in\Omega=(0,l)\times(\tau,\mathrm{T}) \end{aligned} \tag{1}$$

with boundary and initial conditions:

$$p(x,t)=\varphi_1(x,t),\quad 0\le t\le\tau,\ \ x\in[0,l], \tag{2}$$

$$p(0,t)=\varphi_2(t),\ \ \tau\le t\le T, \tag{3}$$

$$\begin{aligned} &p(l,t)=\varphi_3(t),\quad \tau\le t\le T,\\ &\tau=const>0,\ 2\tau<T,\ a=const>0. \end{aligned} \tag{4}$$

Here the function $p=p(x,t)$ defines the phase state of the object; function $K(x,t,\nu)$ is piecewise continuous on the first two arguments, and continuously differentiable on the third argument; functions $\varphi_0(x,t)$, $\varphi_1(x,t)$, $\varphi_2(t)$, $\varphi_3(t)$ are control actions.

The target functional is as follows:

$$J(\varphi)=\int\limits_0^1\left[p(x,t;\varphi-P(x)\right]^2dx+\sum_{i=0}^{3}\varepsilon_i\left\|\varphi_1\right\|_{L_2}^2, \tag{5}$$

where $P\left(x\right)$ is given function;ε_i,$i=1,2.3$ regularization parameters. Controlling function $\varphi(x,t)$ is sought on such class of functions that loaded boundary problem (1)-(4) would have unique solution. Moreover, they satisfy additional conditions resulting from technical and technological processes, for instance, in the following form:

$$\left\|\varphi_i(x,t)\right\|_{L_2}\le M_i,\quad i=0,1,2.3.$$

In the work, we propose an approach to the numerical solution to the problem of optimal control of the object. The main problem is approximated by finite difference method and thus reduced to finite dimensional optimization problem and written in the form of difference equations. To calculate the gradient of the functional, we derive an analytical formula. Numerical experiments have been carried out by the example of the solution to test problems. The results obtain show the efficiency of the derived formula. To find the minimal value of the functional, we use first order optimization methods.

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THE ALGORITHM OF POWER SYSTEMS REGIME OPTIMIZATION THROUGH SHORT-LINEAR APPROXIMATION OF NONLINEAR DEPENDENCES

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In the given work the algorithm of Power Systems regime optimization based on short-linear approximation of nonlinear dependences and linear programming methods application is offered.

Let's consider the problem of Power Systems regime optimization which means to minimize the summary fuel expenses of n Thermal Power Stations (TPS)

$$B = \sum_{i=0}^{n} B_i(P_i) \tag{1}$$

taking into account the limitations on active power balance in Power System

$$\sum_{i=1}^{n} W_i(P_i) = 0, \tag{2}$$

on limit values of TPS active power Pi and its some functions

$$P_{i\min} \le P_i \le P_{i\max} \quad i = \overline{1,n}, h_l = \sum_{i=1}^{n} h_{li}(P_i) \le 0, \quad l = \overline{1,L},. \tag{3}$$

Short-linear approximation of (1)-(3) provides by a choice in the set interval $[P_{i\min},\ P_{i\max}]$ m_i points with coordinates $(P_{ik},\ B_{ik})$ and its replacement in each piece $[P_{ik},\ P_{ik+1}]$ for linear functions.

Having presented thus values P_i laying in piece $[P_{ik},\ P_{ik+1}]$ as

$$P_i = \lambda_{ik} P_{ik} + \lambda_{ik+1} P_{ik+1} \tag{4}$$

choosing λ_{ik} and λ_{ik+1} according to condition $\lambda_{ik} + \lambda_{ik+1} = 1$, receive the following task of linear programming for (1) - (3): to minimize the function

$$B = \sum_{i=1}^{n} \sum_{k=1}^{m_1} \lambda_{ik} \bar{B}_{ik}(P_i) \tag{5}$$

taking into account the limitations

$$\left.\begin{array}{l} \sum_{i=1}^{n} \sum_{k=1}^{mi} \lambda_{ik} \bar{W}_{ik}(P_i) = 0, \quad h_l = \sum_{i=1}^{n} \sum_{k=1}^{mi} \lambda_{ik} \bar{h}_{lik}(P_i) \le 0, \quad l = \overline{1,L}, \\ \sum_{k=1}^{mi} \lambda_{ik} = 1; \quad i = \overline{1,n}, \quad \lambda_{ik} \ge 0; \quad i = \overline{1,n}; \quad k = \overline{1,m_i} \end{array}\right\}, \tag{6}$$

where λ_{ik} - found parameters.

The problem (5) - (6) is solved by the simplex method taking into account the limitations on a choice of the basis consisting in inadmissibility to include in basis two vectors which correspond to two not next λ_{ik}. The results of experimental calculations approved that the offered algorithm is high effectively.

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ZONAL FEEDBACK CONTROL IN SYSTEMS WITH LUMPED PARAMETERS[1]

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In the work, we propose an approach to numerical solution to a feedback control system for a process described by a system of non-linear ordinary differential equations:

$$\dot{x}(t) = f(x(t), u(t), p), \quad t \in (0, T], \tag{1}$$

$$x(0) = x_0 \in X^0, \quad p \in P \subset R^m, \tag{2}$$

where $x(t)$ is n-dimensional vector-function defining the state of the process; $u(t) \in U$ r-dimensional vector-function of control, $U \subset R^r$ the closed set of admissible values of control actions; p m-dimensional vector of parameters of the process, P the set of possible values of the parameters; $X^0 \subset R^n$ the set of possible initial states of the process with its density (weighting) function. In these equations, we have inaccurately given parameters defined in some domain with their density functions.

It is assumed that observations are carried on over the phase state of the process only at given discrete points of time $\tau_i \in [0, T]$, $i = 0, 1, ..., N$. The values of the control $u(t)$, $t \in [0, T]$ are assigned subject to the value of the last observed current state of the process, to put it more precisely, subject to the set (zone) of the phase space to which the measured (observed) current state belongs:

$$u(t) = v^i = const, \quad x(\tau_j) \in X^i, \quad t \in [\tau_j, \tau_{j+1}),$$

$$v^i \in U, \qquad i = 1, 2, ...L, \qquad j = 1, 2, ...N. \tag{3}$$

The problem consists in finding such values of the control actions v^i, $i = 1, 2, ..., L$ that minimize some given functional defining the quality of the control. Particularly, the functional is taken as follows:

$$J(v) = \int\limits_{X^0} \int\limits_{P} I(v; x_0, p) \cdot \rho_P(p) \cdot \rho_{X^0}(x_0)\, dP dX^0, \tag{4}$$

$$I(v; x_0, p) = \int\limits_0^T f^0(x(t), u(t))\, dt + \Phi(x(T)). \tag{5}$$

Necessary optimality conditions are obtained with respect to piecewise constant values of the feedback control. These conditions allow to apply finite-dimensional first-order optimization methods to find optimal values of the control. Results of numerical experiments obtained by the example of solution to several test problems are given.

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AN ALGORITHM FOR CALCULATION OF THE ATTAINABLE SETS OF THE NONLINEAR CONTROL SYSTEMS WITH INTEGRAL CONSTRAINT ON CONTROLS[1]

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Attainable set of the controllable system is the set of all points to which the system can be steered at the instant of given time and it is very useful tool in the study of various problems of control systems, dynamical systems and differential game theory. The approximate construction of the attainable sets, allows to specify most properties of the given controlllable evolution systems in advance.

The control systems with integral constraint on controls arise in mathematical models of the systems having limited energy resources which are exhausted by using, such as fuel or financial recources. For example, the motion of flying apparatus with variable mass is described in the form of controllable system, where the control functions have an integral constraint (see., e.g. [3]).

Approximation method for the construction of attainable sets of affine control systems with integral constraint on the controls is given in [1]. Algorithm presented here is based on the results obtained in the paper [2] and is designed for approximate calculation of the attainable sets of nonlinear control systems with integral constraint on controls. The attainable set of given 3 dimensional control system with integral constraint on controls is calculated. This system describes the evolution of the controllable biological system which consists of 3 species.

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ON ONE APPROACH FOR SOLVING OF SOME CLASS OF MULTICRITERIA OPTIMIZATION PROBLEMS WITH INTERVAL COEFFICIENTS[1]

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In present paper it a multicriteria linear programming problem under is investigated interval indeterminacy, when the significant coefficients of the model's constraints can take any value from the specified intervals, at that significance of criteria are not known a' priori. Such problems arise under a solving of some applied problems of physical and techno-economic processes and phenomenon, when the modeling process often reflects the subjective representation of experts and researchers about target processes and phenomenon, when it is revealed the interdependence between key factors and their influence to the constructible elements of the model. As a rule, used hypotheses, empirical dependencies and initial data in the modeling process lead to indeterminacy of conditions of the constructed mathematical model. It should be emphasized that the class of interval multicriteria programming problems essentially differs from the class of stochastic programming problems, in which we can always assume that there is additional information about the distribution of indeterminate values of functions within the intervals.

In this paper we use the point weights' estimation method (for instance, see [1] and respective references given inside) to reduce the original multicriteria problem to single-criterion interval problem, in which the criterion is a weighted sum of the criteria of the original problem with the normalized weight coefficients (i.e. strictly convex combination of the original criteria). Received interval single-criterion linear programming problem is intrinsically a parametric family of deterministic linear programming problems. Proposed approach consists in finding such common solution (so-called an universal solution, for instance, see [2] and respective references given inside) to all family of problems that would satisfy the constraints of the problem accurate within the minimal residuals norm.

In this paper with the help of the duality theory it is proved a solvability of the reduced interval single-criterion problem provided that the total length of the intervals within which the coefficients vary is not zero for each constraint, which is quite a weak condition/requirement of a rather general nature, that does not narrow the class of interval multicriteria optimization problems.

For practical application of the foregoing approach for solving of the reduced interval single-criterion problem the Big M Simplex method (see [1], [3]) is used. In additional, we develop the appropriate software for computer implementation of the obtained theoretical results of this paper.

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NUMERICAL METHOD OF SOLUTION TO A BOUNDARY INVERSE PROBLEM OF TWO-PHASE FLOW[1]

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We consider a process of water-oil displacement in a rectangular two-phase stratum. As a mathematical model of the process we take a model of two-phase flow of incompressible and immissible fluids in deformable stratum:

$$\frac{\partial m S_w}{\partial t} + \frac{\partial u_w}{\partial x} = 0, \quad \frac{\partial m S_o}{\partial t} + \frac{\partial u_o}{\partial x} = 0,$$
$$u_w = -\frac{k k_w}{\mu_w}\frac{\partial P}{\partial x}, \quad u_o = -\frac{k k_o}{\mu_o}\frac{\partial P}{\partial x}, \quad S_w + S_o = 1. \tag{1}$$

where S_w, S_o are water and oil saturations, respectively; P pressure; u_w, u_o velocities of water and oil filtration, respectively; m porosity coefficient; k absolute permeability; k_w, k_o relative phase water and oil permeability.

We assume that the movement of each phase obeys the generalized Darcy's law, and capillary pressure is taken equal to 0. By means of transformation, system (1) is reduced to the following system of non-linearly linked partial differential equations with respect to functions $P(x,t)$ and $S_w(x,t)$:

$$\phi\frac{\partial P}{\partial t} = \frac{\partial}{\partial x}\left(\lambda(S_w)\frac{\partial P}{\partial x}\right), \quad \frac{\partial m S_w}{\partial t} = \frac{\partial}{\partial x}\left(\lambda_w(S_w)\frac{\partial P}{\partial x}\right). \tag{2}$$

Here $\phi = \frac{dm}{dP}$. For system (2), we give the following initial conditions:

$$S_w\,|_{t=0} = s_0(x), \quad P\,|_{t=0} = p_0(x), \tag{3}$$

along with boundary conditions

$$\begin{aligned} S_w\,|_{x=0} &= s_w(t), \\ -\frac{k k_w}{\mu_w}\frac{\partial P}{\partial x}\,|_{x=0} &= q_w(t), \\ -\frac{k k_o}{\mu_o}\frac{\partial P}{\partial x}\,|_{x=L} &= q_o(t) \end{aligned} \tag{4}$$

where function $q_o(t)$ is also not given. To formulate a correct problem we give additional conditions:

$$P\,|_{x=0} = p_w(t). \tag{5}$$

Problem (2)-(5) relates to a class of boundary inverse problems. For numerical solution to the stated problem we use difference schemes: implicit on pressure and explicit on saturation and propose a method of solution to the obtained system of difference equations.

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TIME-OPTIMAL CONTROL PROBLEM FOR THE OBJECT DESCRIBING BY THE HEAT-CONDUCTING EQUATION WITH NON-CLASSIC BOUNDARY CONDITION

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Let the controlled process be described by the function $y(x,t)$ that satisfies to the equation

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2} + u(x,t), \tag{1}$$

with initial

$$y(x,0) = y^0(x), \tag{2}$$

and boundary conditions

$$y(0,t) = 0,\ y_x(1,t) = y_x(0,t), \tag{3}$$

in the domain $D_T = \{(x,t),\ 0 < x < 1,\ 0 < t < T\}$.

The class of admissible controls U_∂ assumed to be convex, closed and bounded subset of $L_2(D_T)$.

It is proved that there exists almost everywhere unique solution $y(x,t)$ of the problem (1)-(3) under conditions by $y^0(x) \in W_2^1(0,1)$, $y^0(0) = 0$, $u(x,t) \in L_2(D_T)$.

For the given weak-closed subset K of $L_2(0,1)$ it needs to find the control $u(x,t) \in U_\partial$ such that corresponding solution $y(x,t)$ satisfy to the condition

$$y(x,\tau_0) \in K,$$

moreover $\tau_0 \in [0,T]$ take possible minimal value.

The existence of the optimal control is proved for this problem. In the case when $u(x,t)$ is of the form $u(x,t) = p(x)v(t)$, where $p(x) \in L_2(0,1)$ is a given function, $v(t) \in U_\partial = \{v(t) \in L_2(0,T)\}$, $|v(t)| \leq 1$ almost everywhere .

The following theorem is proved:

Theorem. *Let $u^* \in U_\partial$, $t \in (0,\tau_0)$ be an optimal control in the sense of time optimality. Then $|v^*(t)| = 1$ almost every where in $(0,\tau_0)$.*

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ON THE SOLVABILITY OF THE CAUCHY INTEGRO-DIFFERENTIAL EQUATIONS IN PARTIAL DERIVATIVES

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Partial integro-differential equations, which enable mathematical representation of processes with delay, occurring in space and time play an important role in mathematics and its applications. Still scantily explored area of the problem determine the solvability of the Cauchy problem for integro-differential equations in partial derivatives. Consider the equation of the form

$$u_{txx} + \alpha u_{xx} + 2\beta u_{tx} + 2\alpha\beta u_x + \left(\beta^2+1\right)u_t + \alpha\left(\beta^2+1\right)u = f(t,x,u(t,x)) + \int\limits_0^t K(t,s,x,u(s))ds \quad (1)$$

with initial condition

$$u(0,x) = \varphi(x), \quad (2)$$

where $\alpha,\ \beta$ - some positive constants, $f(t,x,u(t,x)), K(t,s,x,u), \varphi(x)$ - are known continuous functions. In the region $R = \{0 \le t < T,\ -\infty < x, u < \infty\}$, the functions $f(t,x,u), K(t,s,x,u), \varphi(x)$ are continuous and bounded:

$$\|f(t,x,u)\| \le M_1 = const, \|K(t,s,x,u(s))\| \le M_2 = const.$$

In addition:

$$\|f(t,x,u_2) - f(t,x,u_1)\| \le L\,\|u_2-u_1\|, \|K(t,s,x,u_2) - K(t,s,x,u_1)\| \le L_1\,\|u_2-u_1\|, \frac{L+TL_1}{\alpha\beta} \le \frac{1}{2}.$$

Propose a method for making the initial transformation of the Cauchy problem in an equivalent Volterra integral equation to which we apply the principle of contraction mapping. Cauchy problem (1)-(2) in the form

$$u(t,x) = c(t,x) + \int\limits_0^t e^{-\alpha(t-s)} \int\limits_{-\infty}^x e^{-\beta(x-\rho)} \sin(x-\rho) Q(s,\rho) d\rho ds$$

where $c(t,x)$ - known continuous function, with

$$c(o,x) = \varphi(x),$$

$Q(t,x)$ - a new unknown function, which is defined as the solution of nonlinear Volterra integral equation. It is assumed that the function $c(t,x)$ is uniformly bounded together with their derivatives appearing in (1).

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REGULAR SOLUTIONS OF OPTIMAL CONTROL PROBLEMS WITH STATE CONSTRAINTS

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The optimal control problems with state constraints are recognized as an important and difficult class of the similar problems, since the maximum principle for such problems [2] contains an unknown infinite-dimensional Lagrange multiplier of the complex nature-bounded regular Borel measure which has a rather complicated relationship with the optimal trajectory. Therefore, the optimal control problems with state constraints are outside the scope of the effective application of the Pontryagin maximum principle [1]. Questions arise: Are there any solutions of an optimal control problem for which the corresponding conjugate function is non-trivial and absolutely continuous, and if so, how to find them? Applying the method of similar solutions [3] try to answer these questions for non–autonomous systems with phase and endpoint constraints. Consider the time optimal control problem with state and endpoint constraints for non–autonomous system

$$\dot{x} = f\left(x, u, t\right),$$

$$x\left(0\right) \in C_{\alpha}, x\left(T\right) \in C_{\beta},$$

$$u\left(t\right) \in U\left(t\right), \;\; a.a. \;\; t \in \left[0, T\right],$$

$$x\left(t\right) \in X, \; t \in \left[0, T\right],$$

$$T \to \min .$$

Here, $x \in E^n$-state variable, $u \in E^m$-control parameter. Let $\Omega\left(E^n\right)$ be the set of all nonempty compact and *conv* $\Omega\left(E^n\right)$ the set of all nonempty compact convex subset of E^n. Functions f, $\frac{\partial f}{\partial x}$ are continuous in (x, u) and measurable in t. Let the set valued map $U: \; E^1 \to \Omega\left(E^m\right)$ be measurable and satisfy the estimate $\left|U\left(t\right)\right| \le k\left(t\right)$, where $k\left(t\right)$ is a scalar function, Lebesgue integrable on any finite time interval $[0, T]$. $f\left(x, U\left(t\right), t\right) \in conv\ \Omega\left(E^n\right), \; t \in [0, T]$. $C_{\alpha}, C_{\beta} \in conv\ \Omega\left(E^n\right), \; X$ is a closed convex subset of E^n.

This work is dedicated by using of the method of similar solutions [3] to deriving the maximum principle for which the optimal solution is regular. Optimal solution is called regular, if the corresponding conjugate function is a nontrivial absolutely continuous function. The case of terminal cost function is also considered by using of this method.

The results are illustrated by the examples.

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REGULAR SOLUTIONS OF OPTIMAL CONTROL PROBLEMS WITH DELAY AND STATE CONSTRAINTS

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In the presence of the phase constraint optimal control problem turned out to be much more complicated due to the fact that the conjugate function to obtain the necessary conditions, in general, is a function of bounded variation, which was a rather complicated relationship with the optimal trajectory [3]. Therefore, the optimal control problem with state constraints are outside the scope of the effective application of the Pontryagin maximum principle [1], [2]. His language was complicated and more difficult to form an analytical study in specific situations. Questions arise: Are there any solutions of the optimal control problem for which the corresponding conjugate function is non-trivial and absolutely continuous, and if so, how to find them? Applying a similar technique in [4] we try to answer these questions for systems with delays and phase constraints.

This work is dedicated to eliminating the deficiencies to a great extent by means of which maximum principle for systems with delays and state constraints will be extracted in such a form where the corresponding optimal solution is regular. The optimal solution is called regular if the corresponding conjugate function is a nontrivial absolutely continuous function.

Consider the optimal control problem

$$\dot{x} = f\left(x, x\left(t-\tau\right), u\right),\ t \in \left[0, T\right],\ T \in \left(\tau, +\infty\right),\ \tau > 0,$$

$$x\left(t\right) = l\left(t\right),\ t \in \left[-\tau, 0\right),\ x\left(0\right) = x_{\alpha},\ x\left(T\right) = x_{\beta}, x_{\alpha} \neq x_{\beta},$$

$$u\left(t\right) \in U,\ a.\ a.\ \ t \in \left[0, T\right],$$

$$x\left(t\right) \in X,\ t \in \left[0, T\right],$$

$$T \to \min.$$

Here, $x \in E^n$ is state variable, $u \in E^m$ is control parameter. Let $\Omega\left(E^n\right)$ be the set of all nonempty compact and $conv\ \Omega\left(E^n\right)$ the set of all nonempty compact convex subsets of E^n.

Given function $l\left(t\right),\ t \in \left[-\tau, 0\right)$ is bounded and measurable. The functions $f\left(x, y, u\right), \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ are continuous. $U \in \Omega\left(E^m\right)$ and $f\left(x, y, U\right) \in conv\ \Omega\left(E^n\right), \quad x, y \in E^n$. X is a closed convex subset of E^n.

The case of terminal cost criterion is also considered by using of this method.

The results are illustrated by the examples.

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IDENTIFICATION OF DISCONTINUITY CONDITIONS IN DYNAMIC SYSTEMS[1]

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Assume that the dynamic of the investigated object is described by a discontinuous system of non-linear differential equations with variable structure [1]:

$$\dot{x}(t) = f^l\left(x(t), p^l(t)\right), \quad x(t) \in X^l(t), \quad t \in (0, T], \quad l = 1, 2, ..., L, \tag{1}$$

where $x(t) \in R^n$ is the state vector of the process; $p^l(t) \in R^{r_l}$ the values of the parameters when the state of the process belongs to $\mathrm{X}^l(t)$ - the sub-domain (zone) of the phase space of all possible states X. The sub-domains of phase space

$$\begin{array}{l} \mathrm{X}^1(t) = \left\{x \in R^n : g^1(x,t) \le 0\right\}, \ \mathrm{X}^L(t) = \left\{x \in R^n : g^{L-1}(x,t) > 0\right\}, \\ \mathrm{X}^l(t) = \left\{x \in R^n : g^{l-1}(x,t) > 0, \quad g^l(x,t) \le 0\right\}, \quad l = 2, 3, ..., L-1, \end{array} \tag{2}$$

are simply connected and defined by their boundaries with the use of identified continuously differentiable functions $g(x,t) = \left(g^1(x,t), g^2(x,t), ..., g^{L-1}(x,t)\right)$.

Assume that in the aim of identifying unknown parameters, we have the results of observations over the dynamics of the object at different initial conditions:

$$x^i(0) = x_0^i, \quad i = 1, 2, ..., N. \tag{3}$$

We also have observations over the state of the object at different moments of time:

$$x^i\left(t_{ij}; x_0^i, p, g\right) = x^{ij}, \quad t_{ij} \in (0, T], \quad j = 1, 2, ..., N_i^1, \quad i = 1, 2, ..., N. \tag{4}$$

The investigated problem then consists in identifying $(L-1)$-dimensional vector-function $g(x,t)$ and piecewise constant vector-function $p(t)$.

In the aim of identifying the function $g(x,t)$, we propose to use some known finite system of linearly independent continuously differentiable functions $\left\{\varphi^i(x,t)\right\}, \quad i = 1, 2, ..., \overline{v}$, and search for functions $g^l(x,t), \quad l = 1, 2, ..., L-1$ in the form

$$g^l(x,t) = g^l\left(x, t; \alpha^l\right) = \sum_{i=1}^{\nu^l} \alpha_i^l \varphi^i(x,t), \quad l = 1, 2, ..., L-1, \quad \alpha^l \in R^{\nu^l}, \quad \nu = \sum_{l=1}^{L-1} \nu^l,$$

$$\overline{\nu} = \max_{1 \le l \le L-1} \nu^l, \quad \alpha = \left(\alpha_1^1, ..., \alpha_{\upsilon^1}^1, \alpha_1^2, ..., \alpha_{\nu^{L-1}}^{L-1}\right) \in R^\nu.$$

Thus, the initial problem is reduced to identification of finite-dimensional vector α.

Formulas for the components of the gradient on the identified parameters are obtained. They allow to use first order optimization methods to solve the problem numerically. The results of numerical experiments are given.

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ANALYTICAL CONSTRUCTION OF THE OUTPUT REGULATOR IN THE SINGULARLY PERTURBUTED SYSTEM

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This work is devoted to development of the approximate methods of construction of the solution of singulary perturbuted output regulator problem. This problem is characterized by the free right end of trajektory and quadratic functional. Such problems are considered in [1,2,3] and reduced to investigation to Koshas problem for the system of singularly-perturbuted nonlinear differential equations. In [2,3] an asymptotical approximation to optimal control problem is formed by the boundary function. The considered problem is reduced to solution of three independent linear matrix differential equations relatively supmatix Gramm system. On the base of the obtained algorithms it is easy to get an asymptotical approximate solution of the considered problem.

Lets consider the problem

$$\dot{y} = A(t,\mu)y + B(t,\mu)u, y(t_0) = y_0, s = S(t)y, \tag{1}$$

$$J = \int_{t_0}^{t_1} u'(t)u(t)dt + S'(t_1)FS(t_1) \to \min, \tag{2}$$

where $A(t,\mu) = diag\,(A_1(t),\ (1/\mu)A_2(t))\,, y = col\,(x,\ z)\,,\quad y_0 = (x_0,\ z_0)\,, x \in R^n\ ,\quad y \in R^m\ , u \in R^r$, $B(t,\mu) = col\,(B_1(t),\ (1/\mu)B_2(t))\,,\ S(t) = \begin{pmatrix} S_1(t) & S_2(t) \end{pmatrix},\ \mu > 0,\ S \in R^k$.

It is required to estimate the quality of the transformation process not by errors of the process, but by the errors of the output variables of the system (1) comparing with the given values. In such formulation (2) has a form

$$J = \int_{t_0}^{t_1} u'(t)u(t)dt + \vec{S}'(t_1)F\vec{S}(t_1) \to \min, \tag{3}$$

where F is positive defined, constant matrix of $(k \times k)$ dimension,

$$\vec{S}(t_1) = diag(S_1(t_1), S_2(t_1)).$$

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WEIGHT OPTIMAL BY ORDER OF CONVERGENCE CUBATURE FORMULAS IN SOBOLEV'S FACTOR SPACE

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The problem of construction of weight optimal quadrature, cubature formulas is the general problem of optimal approximation of linear functionals at the present time, which is at the center of attention.

In the present work the following cubature formula is considered

$$\int\limits_{S_{n-1}} P(\theta)f(\theta)ds \cong \sum_{\lambda=1}^{N} C_\lambda f(\theta^{(\lambda)}), \qquad (1)$$

in the space $L_2^{(m)}(S^{n-1})$ on sphere, where $\theta = (\theta_1, \theta_2, ..., \theta_{n-1})$, $\|\theta\| = 1$ *and* $P(\theta)$ is summable function on the sphere S^{n-1}, i.e. $P(\theta) \in L(S^{n-1})$, $\sum\limits_{\lambda=1}^{N} C_\lambda = \frac{2\pi^{\frac{n}{2}}}{F(\frac{n}{2})} P_{0,0}$ and $P_{k,\ell} = \int\limits_{S_{n-1}} P(\theta)Y_{k,\ell}(\theta)d\theta$ the error functional of the cubature formula (1) has the form

$$\ell^N(\theta) = P(\theta) - \sum_{\lambda=1}^{N} C_\lambda \delta(\theta - \theta^{(\lambda)}),$$

where $\delta(\theta)$ is Dirac's delta function.

The following is holds.

Theorem. *The cubature formula (1) for*

$$D^N(\theta) = \prod_{i=1}^{n-1/2} D_{np}^{N_i}(\theta_i) \cdot \prod_{i=\frac{n-1}{2}+1}^{n-1} \widehat{D}^{N_i}(\theta_i),$$

where $D(\theta) = \sum\limits_{\lambda=1}^{N} C_\lambda \delta(\theta - \theta^{(\lambda)})$, $\left\|P(\theta_i) - D^{N_i}(\theta_i)/L_2^{m*}(\Omega_i)\right\| \leq K_i N_i^{-m}$, $N_1 = N_2 = ... = N_{n-1}$, $\prod\limits_{i=1}^{n-1} N_i = N$ *is optimal by order on the space* $L_2^m(S^{n-1})$, i.e. $\left\|\ell^N(\theta)/L_2^{m*}(S^{(n-1)})\right\| = O(N^{-m/n-1})$.

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ON AN OPTIMAL CUBATURE FORMULA IN $\tilde{\mathrm{H}}_p^{\mu}(\mathrm{T}_n)$ SPACE

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Consider cubature formula

$$\int_{T_n} f(x)dx \approx \sum_{\lambda=1}^{N} C_\lambda f(x^{(\lambda)}), \tag{1}$$

with the error functional

$$\ell(x) = \varepsilon_{T_n}(x) - \sum_{\lambda=1}^{N} C_\lambda \delta(x - x^{(\lambda)}), \tag{2}$$

in $\tilde{\mathrm{H}}_p^{\mu}(\mathrm{T}_n)$ space, where C_λ and $x^{(\lambda)}$ the coefficients and the nodes of the formula (1), $\delta(x)$ is Dirac's delta function, $\varepsilon_{T_n}(x)$ is the indicator of the domain T_n and T_n is n dimensional torus.

Definition 1. *The set* $T_n = \{x = (x_1, x_2, ..., x_n); \quad x_k = \{t_k\},\ t_k \in R$ *is called* n *dimensional torus, where* $\{t_k\} = t_k - [t_k]$ [1].

Definition 2. *The space* $\tilde{\mathrm{H}}_p^{\mu}(\mathrm{T}_{n)})$ *is defined as the space of periodic functions with matrix of period* H *of the form* $f(x) = \sum\limits_{\gamma} \hat{f}[\gamma] e^{-2\pi i \gamma^* \mathrm{H}^{-1} x}$ *for which the sum* $\sum\limits_{\gamma} \left| \hat{f}[\gamma] \right|^p \mu^p(\gamma \mathrm{H}^{-1})$ *is finite. The norm in* $\tilde{\mathrm{H}}_p^{\mu}$ *space is defined by following formula*

$$\left\| f(x) \big/ \tilde{\mathrm{H}}_p^{\mu} \right\| = \left\{ \sum_{\gamma} \hat{f}[\gamma] \,|^p \ \mu^p(\dot{\gamma} \mathrm{H}^{-1}) \right\}^{\frac{1}{p}}, \quad 1 \le p \le \infty, \tag{3}$$

and $\left\| f(x) \big/ \tilde{\mathrm{H}}_\infty^{\mu} \right\| = \underset{\gamma}{Sup} \left\{ \left| \hat{f}[\gamma] \right| \dot{\mu}(\gamma * \mathrm{H}^{-1}) \right\}, \quad p = \infty.$

Definition 3. *The cubature formula*

$$\int_{T_n} f(x)dx \approx \sum_{\lambda=1}^{N} c_\lambda f\,(h\lambda), \lambda = (\lambda_1, \lambda_2, \lambda_3, ..., \lambda_\mathrm{n}) \tag{4}$$

is called exact on some set M *if (4) is exact equality for any* $f(x) \in M$*. In particular, for* $M = R$ *the following holds* $\sum\limits_{\lambda=1}^{N} c_\lambda = 1.$

Theorem 1. *The rectangular cubature formula is optimal among cubature formulas of the form (4), which are exact on constants in* $\tilde{H}_p^{\mu}(T_n)$ *space.*

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ON SPECIFIC ASYMPTOTIC STABILITY OF SOLUTIONS OF VOLTERRA FOURTH ORDER LINEAR HOMOGENEOUS INTEGRO-DIFFERENTIAL EQUATIONS

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IDE - integro-differential equation; under the asymptotic stability of solutions of a linear fourth-order IDE understand the desire to zero $t \to \infty$ all its solutions and their derivatives up to third order inclusive.
Solve the following problem.

Establish sufficient conditions for asymptotic stability of solutions of linear homogeneous IDE Volterra type:

$$x^{(4)}(t) - a_3(t)\,x'''(t) + a_2(t)\,x''(t) + a_1(t)\,x'(t) + a_0(t)\,x(t) +$$
$$+\int_{t_0}^{t}[Q_0(t,\tau)\,x(\tau) + Q_1(t,\tau)\,x'(\tau) + Q_2(t,\tau)\,x''(\tau) + Q_3(t,\tau)\,x'''(\tau)]d\tau = 0, \quad t \geq t_0 \qquad (1)$$

in case $a_3(t) \geq 0$, i.e. in the case of asymptotic instability of the non-trivial solutions of the corresponding linear homogeneous differential equation of the fourth order:

$$x^{(4)}(t) - a_3(t)\,x'''(t) + a_2(t)\,x''(t) + a_1(t)\,x'(t) + a_0(t)\,x(t) = 0, \; t \geq t_0,$$

that follows from formula Ostrogradskii - Liouville.

In the IDE (1) all functions $a_k(t)$, $Q_k(t,\tau)$ $(k = 0,1,2,3)$ are continuous at $t \geq t_0$, $t \geq \tau \geq t_0$. We are talking about the solutions $x(t) \in C^4(J,R)$ IDE (1) with any initial Cauchy data $x^{(k)}(t_0)$ $(k = 0,1,2,3)$. Each such solution exists and is unique.

To solve this problem are applied: non-standard method of reduction to the system [1] (in the IDE (1) made the following changes:
$x'(t) + \lambda_1 x(t) = W_1(t)y(t)$,
$y'(t) + \lambda_2 y(t) = W_2(t)u(t)$,
where $0 < \lambda_k$ $(k = 1,2)$ - some auxiliary parameters, $0 < W_k(t)$ $(k = 1,2)$ - some weighting functions, $y(t)$, $u(t)$ - the new unknown function); the method of weighting and cutting functions with transformations [2], similar, the transformations in scheme A) → B) → C) [2, pp.113-162], [3] and the method for estimating solutions and their derivatives [4].

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DYNAMIC DESIGN OF THE CONTROL DEVICES FOR THE NON-STATIONARY SYSTEMS

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In this research we consider the non-stationary controlling object described by the vector deviation equation:

$$\begin{aligned} &\dot{x}\,(t) = \widehat{\mathrm{A}}\,(t)x\,(t) + \widehat{\mathrm{B}}\,(t)u\,(t)\ , \\ &x\,(t_0) = x^0, \quad t \in [t_0, t_k]\ , \end{aligned} \tag{1}$$

where $x(t)-$ is the state vector; $u\,(t)^T$- a control vector; $\widehat{A}\,(t)$, $\widehat{B}\,(t)-$ real matrices, t_0, t_k – the initial and final moments of control.
Let the structure of the control law be given

$$u(t) = Ke(t) = -\,K\,x(t)\,, \tag{2}$$

where $K - m \times n-$ dimension matrix of a regulator.

Theorem 1. *Let* $e_i(t_0)\ \in E_i(t_0)$ *in the initial time moment. Then in order to* $e_i(t)\ \in E_i(t)$ by $t > t_0$ *it is sufficient fulfillment of the condition*

$$\int_{\mathrm{t_0}}^{\mathrm{t}} \mathrm{e_i}\,(\tau)\ \ \dot{\mathrm{e}}_{\mathrm{i}}\,(\tau)\ \mathrm{d}\tau \quad \le \int_{\mathrm{t_0}}^{\mathrm{t}} 1_{\mathrm{i}}\,(\mathrm{t})\ \ \dot{1}_{\mathrm{i}}\,(\mathrm{t})\ \mathrm{d}\tau\ , \qquad \mathrm{t} \in [\mathrm{t_0}, \mathrm{t_k}]\ . \tag{3}$$

For all $t \in [t_0, t_k]$.
On the basis of conditions (3) *one may to formulate the following theorem [2].*

Theorem 2. *Let* $e(t_0)\ \in E(t_0)$. Then $e(t)\ \in E(t)$ *is true for the vector of the control error if the conditions*

$$\begin{aligned} \dot{\mathrm{e}}_{\mathrm{i}}^{+}(\mathrm{t}) &\le \dot{1}_{\mathrm{i}}(\mathrm{t}), \\ -\dot{\mathrm{e}}_{\mathrm{i}}^{-}(\mathrm{t}) &\le \dot{1}_{\mathrm{i}}(\mathrm{t}), \qquad \mathrm{i} = \overline{1, \mathrm{N}}, \quad t \in [t_0, t_k]\ , \end{aligned} \tag{4}$$

are satisfied . Here

$$\dot{e}_i^{+}\,(t) = \dot{5}_i\,(t)\ \big|_{5_i(t)\,=\,1_i(t)}\,;\dot{e}_i^{-}\,(t) = \dot{5}_i\,(t)\ \big|_{5_i(t)\,=\,-\,1_i(t)}.$$

Conclusion. *The description of an admissible subset* P *in space of parameters of the regulator, guaranteeing to the closed multidimensional control system given experimental (engineering indicators) of quality is obtained. The analysis of this subset allows one to define required vector-parameter* $p\ \in\ P$ *of the regulator for the linear non-stationary system.*

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EXTREME PROBLEM FOR PARTIAL INCLUSIONS

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In the paper (see [1,2]) the problems of a minimum for extreme problems of multidimensional differential inclusions are considered. The work consists of three parts. In p.1 extremal problem for multidimensional differential inclusions hyperbolic type are investigated. In p.2 extremal problem for multidimensional differential inclusions parabolic type are investigated. In p.3 extremal problem for multidimensional differential inclusions elliptic type are investigated.

Extremal problem for inclusions hyperbolic type consists of four parts. In p.1 continuous dependence of decisions of inclusions hyperbolic type on the right part and on boundary conditions(see [1,2]) are investigated. In p.2 of extremum for nonsmooth variational promlems are studied. In p.3 extreme problems for inclusions hyperbolic type are considered, necessary and sufficient conditions of extremum are obtained. In p.4 the necessary conditions of the second order of problems a minimum for inclusions hyperbolic type are obtained.

Let D be a bounded domain in R^k, $compR^n = \{C \subset R^n : C \neq \emptyset\, compact\ set\}$, $T > 0$, $Q_T = (0,T) \times D$, $M_1 : D \to compR^n$, $F : [0,T] \times D \times R^{(k+2)n} \to \to compR^n$, $x = (x_1, ..., x_k)$, $u_x(t,x) = (u_{x_1}(t,x), ..., u_{x_k}(t,x))$, $f : [0,T] \times D \times \times R^{(k+3)n} \to \bar{R}$ and $g : D \times R^{(k+2)n} \to \bar{R}$ be normal integrants.

Let $1 \leq p < +\infty$, $k(\cdot) \in C^1(\bar{D})^n$, $a(\cdot) \in C(\bar{D})^n$, $k_j^0 > 0$ for $j \in \overline{1,n}$, $k(x) = = (k_1(x), ..., k_n(x)) \geq (k_1^0, ..., k_n^0)$. We put $a(x)u(t,x) = (a_1(x)u_1(t,x), ..., a_n(x)u_n(t,x))$,
$div(k_i(x)\nabla u_i)(t,x) = \sum_{j=1}^k \frac{\partial}{\partial x_j}(k_i(x)\frac{\partial}{\partial x_j}u_i)(t,x)$, $(u_{tt} - div(k(x)\nabla u)(t,x) =$
$= ((u_{1_{tt}} - div(k_1(x)\nabla u_1))(t,x), ..., (u_{n_{tt}} - div(k_n(x)\nabla u_n))(t,x))$,

$$\dot{V}_p = \{u \in L_p^n(Q_T) : (u_{tt} - div(k(\cdot)\nabla u)) + a(\cdot)u \in L_p^n(Q_T),\ u(0,\cdot) \in \dot{W}_{p,1}^n(D),\ u_t(0,\cdot) \in L_p^n(D)\}.$$

We consider the minimization of the functional

$$J(u) = \int_0^T \int_D f(t,x,u(t,x),u_x(t,x),u_t(t,x),(u_{tt} - div(k(x)\nabla u))(t,x)+$$
$$+a(x)u(t,x))dtdx + \int_D g(x,u(0,x),u_x(0,x),u_t(0,x))dx$$

in space $\dot{V}_p$, where $1 \leq p < +\infty$. The necessary and sufficient conditions of extremum for nonsmooth variational problems are studied.

The function $\bar{u} \in \dot{V}_2$, satisfying problem

$$(u_{tt} - div(k(x)\nabla u)(t,x) + a(x)u(t,x) \in F(t,x,u(t,x),u_x(t,x),u_t(t,x)),(t,x) \in Q_T,$$
$$u(0,x) = 0, \quad u_t(0,x) \in M_1(x), \quad x \in D, \ \ u \in \dot{V}_2$$

and minimizing of function J(u) among all solution problem is called an optimal. It is required to find necessary extremum conditions of the optimality.

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NONSMOOTH STOCHASTIC OPTIMAL CONTROL PROBLEM

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It is well known that the maximum principle, the necessary condition of the optimal control, which is a milestone-like result in the optimal control theory was established for the deterministic control system by Pontryakin's group[1] in the 19500s and 1960s. Since then, a lot of work has been done on the forward stochastic control system such as Bensoussan A[2], Bismut J. M [3], Kushner H J[4], Peng S[5] etc. Peng [6] firstly studied one kind of forward-backward stochastic control system which had the economic background and could be used to study the recursive utility problem in the mathematical finance. He obtained the maximum principle for this kind of control system with the control domain being convex. And then Xu [7] studied the non-convex control domain case and obtained the corresponding maximum principle. But all in these article authors investigate stochastic optimal control in smooth case (diffusion coefficient is smooth). In this paper we study the stochastic maximum principle for the optimal control problem. The diffusion coefficient is nonsmooth. More pricesely, we consider following stochastic optimal control problem.

Let (Ω, F, F_t, P) be a filtered probability space, satisfying the usual conditions, on which a d-dimensional Brownian motion (B_t) is defined with the filtration (F_t). Let T be a strictly positive real number, A_1 is a nonempty subset of $R_n$and $A_2 = ([0;\infty))^m$. $U_1$is the class of measurable, adapted processes $u : [t_0, t_1]\times\Omega \to A_1$ and U_2 is the class of measurable, adapted processes $\xi : [t_0, t_1]\times\Omega \to A_2$.

The problem is to find the couple $(\bar{u}, \bar{\xi})$, which is satysfing the the relation

$$dx_t = b(t, x_t, u_t)dt + \sigma(t, x_t)dB_t + M_t d\xi, \ \ \mathrm{t} \in [\mathrm{t}_0, \ \mathrm{t}_1] \tag{1}$$

and takes minimum value of the expected cost

$$J(u, \xi) = E\left[\int_{t_0}^{t_1} f(t, x_t, u_t)dt + \int_{t_0}^{t_1} k_t d\xi_t + g(x\,(t_1)\right] \tag{2}$$

Assume that $b,\ \sigma, f, M$ and g are Borel measurable, bounded functions. By using some useful lemmas and theorems we got necessary optimality conditions for the problem (1) and (2) in the case of diffusion coefficient is nonsmooth in equation (1).

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FIRST AND SECOND ORDER NECESSARY OPTIMALITY CONDITIONS IN CONTROL PROBLEM DESCRIBED BY A SYSTEM OF VOLTERRA DIFFERENCE EQUATIONS

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The report is devoted to derivation of necessary optimality conditions of first and second orders in the processes described by a system of Volterra type difference equations under assumption that the control domain is open.

It is required to minimize the functional

$$S(u) = \varphi(x(t_1)), \tag{1}$$

under restrictions

$$u(t) \in U \subset R^r, t \in T = \{t_0, t_0 + 1, ..., t_1\}, \tag{2}$$

$$x(t) = \sum_{\tau=t_0}^{t} f(t, \tau, x(\tau), u(\tau)), t \in T. \tag{3}$$

Here $x(t)$ is n-dimensional vector of phase variables, $u(t)$ is r- dimensional vector of control actions, U is the given non-empty, bounded and open set, t_0, t_1 are the given numbers.

By means of the method that is a modification of the increment method, at first the analogue of the Euler equation is established.

Then different necessary optimality conditions of second order are obtained.

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NECESSARY AND SUFFICIENT CONDITIONS OF MINIMALITY OF THE IMAGE OF THE GIVEN SURFACE IN THE MAPPING OF P-DIMENSIONAL SURFACES OF N-DIMENSIONAL EUCLIDEAN SPACE

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It is considered two p-dimensional surfaces F, F' of Euclidean space E_n.

$$\Re = (X, \overrightarrow{e}_i, \overrightarrow{e}_\alpha) \qquad (X \in F, i, j, k = 1, ..., p;\ \ \alpha = p+1, ..., n)$$

is moving frame on F, where $\overrightarrow{e}_i \in T_p(X)$ - tangent vector space of F, $\overrightarrow{e}_\alpha \in N_{n-p}(X)$ - orthogonally complementary vector space to $T_p(X)$. Derivation formulas of the frame $\Re$ are following:

$$d\overrightarrow{x} = \omega^i \overrightarrow{e}_i, \qquad d\overrightarrow{e}_i = \omega_i^j \overrightarrow{e}_j + \omega_i^\alpha \overrightarrow{e}_\alpha, \qquad d\overrightarrow{e}_\alpha = \omega_\alpha^i \overrightarrow{e}_i + \omega_\alpha^\beta \overrightarrow{e}_\beta$$

differential forms ω^i, ω_i^j, ω_i^α are satisfied structure equations of Euclidean space.

It is defined the differential mapping $f : F \to F'$ so that

$$f(X) = Y \in F',\ Y = F' \cap (X, \overrightarrow{e}_{p+1}).\Re' = (Y, \overrightarrow{a}_i,\ \overrightarrow{a}_\alpha)$$

moving frame on F', where $\overrightarrow{a}_i = p_i^j \overrightarrow{e}_j + p_i^\beta \overrightarrow{e}_\beta,\ \overrightarrow{a}_\alpha = \overrightarrow{e}_\alpha$.

Derivation formulas of the frame $\Re'$ are following:

$$d\overrightarrow{y} = \omega^j \overrightarrow{a}_j,\ d\overrightarrow{a}_i = \overline{\omega}_i^j \overrightarrow{a}_j + \overline{\omega}_i^\alpha \overrightarrow{a}_\alpha,\ d\overrightarrow{a}_\alpha = d\overrightarrow{e}_\alpha = \overline{\omega}_\alpha^i \overrightarrow{a}_i + \overline{\omega}_\alpha^\beta \overrightarrow{a}_\beta.$$

$$\overrightarrow{M'}_p = \frac{1}{p} \sum_{i=1}^{p} \overline{a}_{ii}^\alpha \overrightarrow{e}_\alpha$$

mean curvature vector of the surface F'.

It is proved that surface is minimal if ant only if when

$$\overrightarrow{K}_{N(i)} \overrightarrow{e}_\alpha = h(\overrightarrow{K}_{T(i)}) \overrightarrow{e}_\alpha,$$

where $\overrightarrow{K}_{N(i)}$ - forced curvature vector, $\overrightarrow{K}_{T(i)}$ - tangent curvature vector of coordinate line ω^i,

$$h : T_p(X) \to N_p(X),$$

$$\forall \overrightarrow{m} = m^i \overrightarrow{e}_i \in T_p(X) : h(\overrightarrow{m}) = p_k^\alpha \widetilde{p}_i^k m^i \overrightarrow{e}_\alpha.$$

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ON A RELATION BETWEEN THE SETS OF SOLUTIONS OF MAIN AND CONVEX BOUNDARY VALUE PROBLEMS IN A CONTROL SYSTEM

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In the work the relation between the sets of solutions of the following equations

$$\beta z_{tt} + z_t - \varepsilon \frac{\partial^2}{\partial x \partial t} F_1(z_x) - \frac{\partial}{\partial x} F(z_x) = f_1(x,t,u), \tag{1}$$

$$\beta z_{tt} + z_t - \varepsilon \frac{\partial^2}{\partial x \partial t} F_1(z_x) - \frac{\partial}{\partial x} F(z_x) = \langle f_1(x,t,u), \mu_{xt} \rangle \tag{2}$$

with the initial boundary conditions

$$z(0,t) = z(1,t) = 0, \;\; z(x,0) = z_0(x), \;\; z_t(x,0) = z_1(x), \tag{3}$$

is established.
Here $(x,t) \in Q(0,1) \times (0,T)$, $u(x,t)$- measurable on Q r-dimensional control vector –function with values from $U \subset R^r$("ordinary" control), μ_{xt}- weakly measurable finit set of the probability Radon measures ("generalized" control),

$$\langle \cdot, \mu_{xt} \rangle = \int_{R^r} (\,\cdot\,) d\mu_{xt}.$$

The set of solutions of the problem (1)-(3) we define by G_0, the sets (2), (3) by G correspondingly.

In the work the relation between the sets G_0 and G is established.

MAXIMUM PRINCIPLE ON SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS OF ELLIPTIC TYPE

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On this paper we establish a maximum principle which applies on solutions of some partial differential equations of elliptic type.

This maximum principle related to some harmonic and biharmonic functions.

INTEGRAL EQUATIONS METHOD IN NON-HOMOGENEITY DETECTION IN RESISTIVITY SOUNDING PROBLEM

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The problem of the vertical electrical sounding above the local heterogeneity or the buried relief is considered. The surrounded medium is assumed to be homogeneous. The form of the inclusion need to be detected. For a stationary field the electrostatic potential can be defined by the integral equation method [1]. This method is considered as the most accurate and appropriate one in numerical solution of the BVP for considered model of medium. The corresponding inverse problem is written as a minimization problem of the residual functional with respect to the parametrization of the inclusion boundary[2]:

Let the Γ be an inclusion boundary and given by the following parametrization:

$$x_3 = q(x_1, x_2) \in C^2(\Omega), (x_1, x_2) \in \Omega \subset R^2, x_1 \in [a, b], x_2 \in [c, d]$$

Consider the mathematical model of the resistivity sounding problem. The corresponding minimization problem can be reduced to a following statement:

$$J(q) = \int_\alpha^\beta \Phi(s, u(x(s)), u'(x(s)))ds \to min \text{ with respect to } q(x_1, x_2), \tag{1}$$

$$u(\mathbf{x}) = \int\int_\Gamma G(\mathbf{x}, \mathbf{x}')d\Gamma \tag{2}$$

$$\nu(\mathbf{x}) = \int\int_\Gamma F(\mathbf{x}, \mathbf{x}')d\Gamma + f(\mathbf{x}, \nabla q) \equiv A\nu + f \tag{3}$$

It was proved that the Frechet derivative of the residual functional (1) has the form:

$$\nabla J = \int_\alpha^\beta [\Phi_u(G_q\nu I + G\nu I_q - (G\nu)_{x_1})I - (G\nu)_{x_2}I +$$
$$B(E + R)(F\nu I_q + F_q\nu I - (FI(\nu + f))_{x_1} - (FI(\nu + f))_{x_2}]ds + \int_\alpha^\beta \sum_{i=1}^3 \frac{\partial\Phi}{\partial u_{x_i}} \frac{dx_i}{ds},$$

here $R = (I - A)^{-1}$ is the resolvent of the operator (3) and B is the notation of the integral operator

$$Bf = \int\int_\Omega G(...)\nu(...)I(q, \nabla q)fd\Omega,$$

I is the Jackobian of the boundary parametrization.

In numerical simulation the gradient was calculated by the empirical formula. Some numerical results were obtained. The efficiency of the method was improved by using a multi processor parallelization technology MPI. It turns out that the parallelization considerably increases calculations efficiency.

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STABILIZATION OF ONE CLASS OF THE NONLINEAR SYSTEMS WITH THE LIMITED CONTROL

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Let's consider one of the class nonlinear operated systems which is described by the differential equation of a following kind

$$\dot{x}(t) = Ax(t) + b\varphi(\sigma) + u(t),\ \ t \in (t_0, \infty), x(t_0) = x_0,\ \ x(\infty) = x_1, \tag{1}$$

$$\varphi(\sigma) = \sigma^{\alpha},\ \ \sigma = c^* x,\ \ 0 < \alpha < 1,\ \ \beta_0\varphi(\sigma_0) \le u(t) \le \beta_1\varphi(\sigma_1)\ , \tag{2}$$

where $x(t)$ - a vector of state of dimension $n \times 1$; $u = u(\varphi(\sigma), t)$ - control vector – $n \times 1$; A - gurvisev matrix of dimension $n \times n$; $b,\ c,\ \beta_0,\ \beta_1$ - vectors of dimension $n \times 1$; $x_0,\ x_1$ - the given vectors.

In work [1] considered the three-sector model of branch. The three-sector model is dynamic and described by the differential equations. It is nonlinear because releases of sectors are given by nonlinear production functions. For distribution of labor and investment resources was used control which provides growth of economics sectors. Considered model (1) - (2) can be applied at studying of the transients occurring at change of one variant of economic policy by another.

Let

$$J(u) = \frac{1}{2}\int_{t_0}^{\infty} [(x(t) - x_1)^* Q(x_0, x_1)(x(t) - x_1) + u^* R(x, \varphi)u]dt, \tag{3}$$

where $Q(x_0,\, x_1) = diag\{(x_{i0} - x_{i1})^{-2}\}$ - a symmetric positively definite matrix,
$R(x, \varphi) = diag\{\frac{(x_{i1} - x_i)\varphi^{-1}(\sigma)}{2\lambda_i^2 (x_{i0} - x_{i1})^2 (x_{i1}\varphi_1^{-1} - x_{i0}\varphi_0^{-1})}\}$- a symmetric positively definite matrix.

Problem. *To find stabilizing control $u(\varphi(\sigma), x_0, x_1)$ such that admissible pair corresponding to it $\{x(t),\ \ u(t)\}$ satisfied to the differential equation (1) with conditions (2) and set the minimal value to functional (3). In the given work considered problems of optimum stabilization when boundary conditions are set, i.e. it is necessary to result operated object from some initial condition x_0 in the set desirable condition x_1. The new approach of construction of the stabilizing control based on a principle of feedback taking into account restrictions on controls in a following kind is offered*

$$u(x, \varphi) = K(x_1\varphi_1^{-1} - x_0\varphi_0^{-1})\varphi(\sigma). \tag{4}$$

The matrix K and boundary conditions are used control to (4) satisfy to conditions of optimality [2] and to set restrictions (2). At the received control (4) movement of system $x(t)$ satisfy to a conditions $x(t_0) = x_0$ and $\lim x(t) = x_1$ when $t \to \infty$.

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PROBLEM OF CONTROLLABILITY FOR SOME LINEAR HYPERBOLIC EQUATIONS

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Let controlled process is described by the following mixed problem for the hyperbolic equation

$$\frac{\partial^2 u}{\partial t^2} - \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial u}{\partial x_j}\right) + a(x)u = f(x,t),$$
$$(x,t) \in Q_T = \Omega \times (0,T), n = 1,2, \tag{1}$$

$$u(x,0) = u_0(x), \ \frac{\partial u(x,0)}{\partial t} = u_1(x), \ x \in \Omega, \tag{2}$$

$$\left(\frac{\partial u}{\partial \nu} + \sigma(x)u\right)|_{S_T} = g(x)\,\upsilon(t), S_T = \partial\Omega \times (0,T), x \in \partial\Omega, \ t \in (0,T), \tag{3}$$

where the coefficients $a_{ij}(x)$, $i = \overline{1,n}$ are measurable bounded functions; $a_{ij}(x) = a_{ji}(x)$, $\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \geq \alpha^2 \sum_{i,j=1}^{n} a_{ij}(x)\xi_i^2$, $\alpha = const > 0$; $a(x) \geq 0$, $\sigma(x)$ is measurable bounded functions; $f(x,t) \in L_2(Q_T)$, $u_0(x) \in W_2^1(\Omega)$, $u_1(x) \in L_2(\Omega)$, $g(x) \in L_2(\Omega)$ are given functions, the function $\upsilon(t)$ is considered as a control from $L_2(0,T)$.

The problem is: to find a control $\upsilon(t)$ from $L_2(0,T)$ that for the corresponding solution $u(x,t)$ of the boundary value problem (1)- (3) be satisfied the conditions

$$u(x,T) = \psi_0(x), \ \frac{\partial u(x,T)}{\partial x} = \psi_1(x). \tag{4}$$

It is proved that if sequence $\left\{\frac{c_n^1}{g_n}\sqrt{\lambda_n}\right\}$or $\left\{\frac{c_n^2}{g_n}\sqrt{\lambda_n}\right\}$ is unbounded, then the equation $Ma = c$ has no solution in some class of sequences, where M- infinite matrix, $c = \left\{\begin{pmatrix} c_n^1 \\ c_n^2 \end{pmatrix}\right\}_{n=1}^{\infty}$,which expressed by the problem data, $\{\lambda_n\}$ are eigenvalues of some spectral problem associated with an initial mixed problem, $\{g_n\}$- factors Fourier coefficients of function $g(x)$.

OF THE ASYMPTOTICS OF OPTIMAL CONTROL DESCRIBED BY SINGULARLY PERTURBED PARABOLIC EQUATION

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Let's see the following problem of the optimal control:

$$\partial_t u(x,t,\varepsilon) = \varepsilon^2 a(x)\partial_x^2 u(x,t,\varepsilon) + b(x,t)u(x,t,\varepsilon) + p(x,t,\varepsilon) + f(x,t), (x,t) \in \Omega,$$
$$u(x,t,\varepsilon)|_{t=0} = 0,\ \partial_x u(x,t,\varepsilon)|_{x=0} = 0,\ (\partial_x u(x,t,\varepsilon) + \alpha u(x,t,\varepsilon))|_{x=1} = 0, \qquad (1)$$

with $\alpha = const > 0$, $\varepsilon > 0$- the small parameter, $\bar{\Omega} = \{(x,t) : x \in [0,1],\ t \in [0,T]\}$, $f(x,t) \in C^\infty(\bar{\Omega})$, $0 < a(x) \in C^\infty[0,1]$, $\partial_t = \partial/\partial t$, $\partial_x^2 = \partial^2/\partial x^2$, $p(x,t,\varepsilon)$ - the control function from $L_2(\bar{\Omega})$.

The allowable control of $p^0(x,t,\varepsilon)$ shall be found as well as it's relevant solution of $u^0(x,t,\varepsilon)$ of problems (1), so that the functional

$$I = \int_0^1 [u(x,T,\varepsilon) - \Phi(x)]^2 dx + \beta \int_0^1 \int_0^T p^2(x,t,\varepsilon)dtdx, \beta = const > 0$$

takes over the smallest possible value with $p(x,t,\varepsilon) = p^0(x,t,\varepsilon), u(x,t,\varepsilon) = u^0(x,t,\varepsilon)$. Here $\Phi(x)$ - is given function from $L_2[0,1]$.

Based on the maximum principle on the analogy of [1], the problem presented is brought to determination of p^0, u^0 and ψ^0 from (1) and correlations

$$p(x,t,\varepsilon) = \frac{1}{2\beta}\psi(x,t,\varepsilon), \qquad \partial_t\psi(x,t,\varepsilon) + \partial_x^2(a(x)\psi(x,t,\varepsilon)) - b(x,t)\psi(x,t,\varepsilon) = 0, \qquad (2)$$
$$\psi(x,t,\varepsilon)|_{t=T} = -2[u(x,T,\varepsilon) - \Phi(x)],\ \partial_x(a(x)\psi)|_{x=0} = 0,\ [\partial_x(a(x)\psi) + \alpha a(x)\psi(x,t,\varepsilon)]|_{x=1} = 0.$$

As opposed to the work [1], here there is only one regularizing variable for both problems (1) and (2)

$$\xi_l = \frac{\varphi_l(x)}{\varepsilon} = \frac{(-1)^{l-1}}{\varepsilon}\int_{l-1}^x \frac{ds}{\sqrt{a(s)}}, \qquad l = 1,2.$$

The solutions of the extended problems found are looked as series with powers of the small parameter ε. The iterative problems received are solved within special spaces, namely, the iterative problems relevant to problems (2) within the space

$$Y = \left\{ y(x,t,\xi) : y = q(x,t) + \sum_{l=1}^2 \left[X_l(x,t,\xi_l) + c_l(x,t)erfc(\frac{\xi_l}{2\sqrt{T-t}}) \right], \right.$$
$$\left. |X_l(x,t,\xi_l)| < c\exp\left(-\frac{\xi_l^2}{8(T-t)}\right),\ c_l(x,t) \in C^\infty(\bar{\Omega}),\ erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty \exp(-s^2)ds \right\},$$

and the iterative problems for (1) within the space

$$U = \left\{ u(x,t,\xi) : u = v(x,t) + \sum_{l=1}^2 \left[Z_l(x,t,\xi_l) + d_l(x,t)erfc\left(\frac{\xi_l}{2\sqrt{t}}\right) \right], |Z_l(x,t,\xi_l)| < c\exp\left(-\frac{\xi_l^2}{8t}\right) \right\}.$$

So the asymptotic nature of the solutions found is proved.

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CONTROL WITH MINIMAL ENERGY IN THE ILL-POSED LINEAR INTEGRO-DIFFERENTIAL SYSTEM

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We assume that controlled process is described by the integro-differential equation of the Fredholm type

$$L\left[x\right] \equiv x^{(n)} + \sum_{i=1}^{n} p_i(t)x^{(n-i)} = f(t)u(t) + \lambda \int_0^1 \sum_{i=0}^{n} M_i(t,s)x^{(i)}(s)ds \tag{1}$$

subject to the boundary conditions

$$x(t_1) = a_1, x(t_2) = a_2, ..., x(t_n) = a_n. \tag{2}$$

Here $0 \le t \le 1$; $0 \le s \le 1; 0 \le t_1 < t_2 < ... < t_{n-1} < t_n \le 1$, $u(t) \in L^2\left[0,1\right]$, λ is a parameter and functions $p_i(t), f(t), M_i(t,s)$ are given. Function $u(t) \in L^2\left[0,1\right]$ can be interpreted as a controllable parameter.

Problem of control with the minimal energy is formulated in the following form. Assume that different set of points $\left\{\tau_i\right\}, i = 1,...,n,$ $0 \le \tau_1 < \tau_2 < ... < \tau_{n-1} < \tau_n = 1$ is specified in the interval $[0\ ,\ 1]$ and the conditions

$$x(\tau_1) = b_1, x(\tau_2) = b_2, ..., x(\tau_n) = b_n \tag{3}$$

are satisfied. The problem is to find the function $u = u^0(t)$ from the set of permissible controllable parameters $u(t) \in L^2\left[0,1\right]$ such that the corresponding solution $x = x(t)$ of the problem (1), (2) will satisfy the condition (3) and provide the minimum possible value of the quadratic integral functional

$$J\left[u\right] = \left\|u\right\|^2_{L^2[0,1]} = \int\limits_0^1 u^2(t)dt = \min. \tag{4}$$

The formulated problem has been considered in [1] under certain conditions on the kernel of Fredholm type integral equation, which provided uniqueness of the solution. In the present work, the problem is studied in the case that these conditions are violated, i.e., when the problem is ill-posed.

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OPTIMIZATION PROBLEM FOR A SINGULAR NONSMOOTH SYSTEM WITH POINTWISE STATE CONSTRAINTS AND NONCONVEX SET OF ADMISSIBLE CONTROL

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Consider the homogeneous Dirichlet problem

$$\Delta y + a(y) = v + f$$

in the bounded three dimensional set Ω, where continuous function a has a bounded velocity of the increment, $f \in H^{-1}(\Omega)$. The control v is a point of the nonconvex closed subset V of $H_0^1(\Omega)$. The state function y includes to the convex closed subset Y of the space $H_0^1(\Omega)$. The pair $(v, y) \in V \times Y$ is called *admissible* if it satisfies the given equation. The *optimization problem* is the minimization of the functional

$$I(v, y) = \|y - z\|^2 + \nu \|v\|^2$$

on the set M of admissible pairs, where $\nu > 0$, $z \in H_0^1(\Omega)$ with the norms of the space $H_0^1(\Omega)$. This problem is solvable if the set M is not empty.

Our problem is difficult enough because of the singularity of the boundary problem (existence and uniqueness of its solution for all control are not guarantee), nonsmoothness of the function a, nonconvexity of the set V, and state constraints. We propose to use the penalty method with smooth and convex regularization for analysis of this problem. Let us determine the functional

$$I_k(v, y) = I(v, y) + (\varepsilon_k)^{-1} \int_\Omega [\Delta y + a_k(y) - v - f]^2 dx + (\varepsilon_k)^{-1} \Phi(v)$$

on the set $U = V \times Y$. The set U is the convex closed span of V, $\varepsilon_k > 0$ and $\varepsilon_k > 0$ for $k \to \infty$, $\{a_k\}$ is a sequence of smooth functions, which converges uniformly to a, Φ is no negative Frechet differentiable functional on U that equals to zero only on the set V. The problem of the minimization of the functional I_k on the set W has a solution w_k.

The pair w is an *approximate solution* of the initial problem if it includes in a small enough neighborhood of the set M and satisfies the inequality

$$I(w) \leq \inf I(M) + \varepsilon$$

for small enough positive value ε.

Theorem 1. *If $k \to \infty$ then $I(w_k) \to \inf I(M)$, besides the sequence $\{w_k\}$ has a weak accumulation point from the set M.*

Thus the solution of the approximate problem for large enough number k can be chosen as an approximate solution of the initial optimization problem.

Theorem 2. *The point w_k of minimum of the functional I_k on the set w satisfies the variational inequality*

$$I'_k(w_k)(w - w_k) \geq 0 \ \forall w \in W.$$

Last formula is in really the system of too variational inequalities with respect to control and state function and a corresponding adjoint system. This problem can be solved by standard iterative methods.

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ON EXISTENCE OF THE BEST CUBATURE FORMULAS OF THE GENERAL FORM ON SOBOLEV SPACE $W_2^{(m)}(T_n)$

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In investigation of the best formulas of approximate integration first occurs the question about existence of such type formulas. This problem is investigated completely even if is sufficiently complicated (see, for example [1]). Up to present time investigation of the best quadrature formulas based on investigation of splines, which connected with formulas under consideration. So far these methods of investigation is not applied effectively in investigation of cubature formulas in $W_2^{(m)}(T_1)$.

We consider the following cubature formula in $W_2^{(m)}(T_n)$ space

$$\int\limits_{T_n} P(x)\, f(x)\, dx \approx \sum_{\lambda=1}^{N} \sum_{|\alpha|\le r} c_\lambda^\alpha f^{(\alpha)}(x_\lambda), \tag{1}$$

where $c_1^\alpha, ..., c_N^\alpha$, $|\alpha| \le r$, are constants, $x_1, ..., x_N$ are knots and T_n is torus.

$$< \ell, f >= \int\limits_{T_n} P(x)\, f(x) - \sum_{\lambda=1}^{N} \sum_{|\alpha|\le r} c_\lambda^\alpha f^{(\alpha)}(x_\lambda) \tag{2}$$

is the error functional of the formula (1).

Definition 1. The space $W_2^{(m)}(T_n)$ is defined as linear normed space consisting of functions f with the norm [2]

$$\left\| f \Big/ W_2^{(m)}(T_n) \right\|^2 = \left(\int\limits_{T_n} |f(x)|\, dx \right)^2 + \left\| f \Big/ L_2^{(m)}(T_n) \right\|^2 .$$

Theorem 1. *There exists the best cubature formulas of the form (1) on the space* $W_2^{m}(T_n)$ if $\frac{n}{2}$ *is not integer number.* **Definition 2.** *The cubature formula with the error functional* $\ell \in \tilde{L}(N)$ *is called the best periodic if* $\left\| \ell \Big/ \tilde{W}_2^{(m)^*} \right\| = J(N)$, where $\tilde{J}(N) = \inf_{\ell \in \tilde{L}(N)} \left\{ \left\| \ell \Big/ \tilde{W}_2^{(m)^*} \right\| \right\}$.

Theorem 2. *The best periodic cubature formulas of the form* (1) in $\tilde{W}_2^{(m)}$ exist if $n/2$ *is not integer.*

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THE SOLUTION OF THE PARAMETRIC INVERSE PROBLEM OF STOCHASTIC OPTIMAL CONTROL

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We consider PDF-parameters estimating a problem. The problem has additional control parameter. This parameter perhaps can be randomly chosen. Let $f(y|\,x,z)$ is PDF relative to measure function $\mu(dy)$ and depends on two parameters: x, z, where $x \in X \subset E_l$, respectively $y \subset E_k$, $z \in Z \subset E_k$, E_l and E_k is l - dimensional and k - dimensional Euclidean spaces, the parameter $x = x_0$ is unknown and estimated by random samples with f PDF. The parameter z is playing role of control. The problem is in determining a random sample of admissible control (plan) $Z_1, Z_2, ...$, providing quality test $\mathbf{E}\big(x_n - x_0\big)^2 \to \min$.

NECESSARY CONDITIONS OF OPTIMALITY IN THE SYSTEMS WITH IMPULSE INFLUENCES

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In this research we consider minimization of the functional

$$S(u_1, v_1.....u_n, v_n) = G_n(x_n(\theta_n^{m_n-1}), x_n(\theta_n))$$

with restrictions

$$\dot{x}_i(t) = f_i(t, x_i(t), u_i(t), v_i(t)), t \in [\theta_{i-1}, \theta_i], i = \overline{1, n},$$

$$x_{i+1}(\theta_i) = G_i(x_i, (\theta_i^1), ...x_i(\theta_i^{m_i-1}), x_i(\theta_i)), i = \overline{1, n-1},$$

$$x_i(\theta_0) = x_0.$$

Here $f_i(t, x_i(t), u_i(t), v_i(t))$ are given n_i-dimensional vector functions, continuous in $[\theta_{i-1}, \theta_i] \times R^{n_i} \times \times R^{r_i} \times R^{k_i}$ together with its partial derivatives with respect to x_i, u_i, v_i up to second order; - given n_{i+1}-dimensional two times continuously differentiable vector-functions; $G_n(z_1, ..., z_{m_n})$- given two-times continuously differentiable function, $x_0 \in R^{n_i}$-given point;
$\theta_0, \theta_i, ..., \theta_n (t_0 = \theta_0 < \theta_i <, , , \theta_{n-1} < \theta_n = T)$-fixed time moments;
$\theta_i^1 \in (\theta_{i-1}, \theta_i]$, $i = \overline{1, n}$, $j = \overline{1, m_i} (\theta_{i-1} < \theta_i^1 < ... < \theta_i^{m_i-1_i} < \theta_i^{m_i} = \theta_i)$-given points, $u_i(t), v_i(t)) - (r, Tk,)$- dimensional piece-wise vector functions of the controlling influences with values from given sets $U_i \times V \subset R^{r_1} \times R^{k_2}$, moreover $u_i(t) \in U \subset R^{r_i}$, $v_i(t) \in V_i \subset R^{k_i}$, $t \in [\theta_{i-1}, \theta_i]$, $i = \overline{1, n}$-given points, $U-$bounded, V- bounded convex sets. Various type first and second order conditions of optimality are obtained for the considered problem.

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OPTIMAL PROBLEM FOR THE CONTROL IMPULSIVE SYSTEMS WITH NONLOCAL CONDITIONS

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Impulsive equations arise from many different real processes and phenomena which appeared in physics, chemical technology, population dynamics, medicine and economics.

On the other hand, boundary problems with nonlocal conditions are appropriate models for describing a lot of natural phenomena, which cannot be described using classical Cauchy or boundary problems. In this report we consider the following optimal control problem, which described by impulsive differential equation with non-local conditions: to find a control$(u(\cdot), [v]) \subset U \times V$ that minimizes the cost functional

$$J(u, [v]) = \Phi(x(0), x(T)) \tag{1}$$

on the solutions of the following impulsive differential equations with nonlocal conditions

$$\begin{cases} x'(t) = f(t, x(t), u(t)), 0 \le t \le T, t \ne t_i, \\ Ax(0) + Bx(T) = C, \\ \Delta x(t_i) = I_i(x(t_i), v_i), i = 1, 2, ..., p; 0 < t_1 < t_2 < ... < t_p < T, \end{cases} \tag{2}$$

$$(u(\cdot), [v]) \in U \times V \subseteq L_2[0, T] \times L_{2p}, \tag{3}$$

where f is a continuous function;$A, B \in R^{n\times n}, C \in R^n, \Delta x(t_i) = x(t_i^+) - x(t_i^-)$, I_i are some functions, $x(t) \in R^n, (u(\cdot), [v])$ -are state and control of the system, correspondingly.

We list out the following hypotheses:

(H1) $\det(A + B) \ne 0$

(H2) $f : [0, T] \times R^n \times R^r \to R^n, I_i : R^n \times R^{r_i} \to R^n, i = 1, 2, ..., p$-are continuous and there exists constants $K > 0, L_i > 0, i = 1, 2, ..., p$, such that

$$|f(t, x, u) - f(t, y, u)| \le K|x - y|, t \in [0, T], x, y \in R^n$$

$$|I_i(x, v) - I_i(y, v)| \le L_i|x - y|, x, y \in R^n$$

(H3) $L = \max\left\{\left\|(A + B)^{-1} A\right\|, \left\|(A + B)^{-1} B\right\|\right\} \left[KT + \sum_{i=1}^{P} L_i\right] < 1.$

It is proved that if assumptions (H1)-(H3) are satisfied, then for every $C \in R^n$, the equation (1) has a unique solution on $[0, T]$. The formula of gradient for the functional was derived. Necessary conditions of optimality have been obtained.

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ON NEW EFFECTS OF CONSTANT DELAYS IN VARIATION FORMULAS AND OPTIMALITY CONDITIONS[1]

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Let E_f be the space of functions $f(t,x,p,z)$ continuous on $[a,b]\times R^n_x\times R^k_p\times R^m_z$ and continuously differentiable with respect to (x,p,z), where $k+m=n$; let $0<\tau_1<\tau_2, 0<\sigma_1<\sigma_2$ be given numbers, E_φ and E_g be spaces of continuously differentiable functions $\varphi:[\hat\tau,b]\to R^k_p$ and $g:[\hat\tau,b]\to R^m_z$, where $\hat\tau=a-\max\{\tau_2,\sigma_2\}$. To any element $\mu=(t_0,\tau,\sigma,p_0,\varphi,g,f)\in E_\mu=(a,b)\times(\tau_1,\tau_2)\times(\sigma_1,\sigma_2)\times R^k_p\times E_\varphi\times E_g\times E_f$, we assign the delay functional-differential equation

$$\dot x(t)=(\dot p(t),\dot z(t))^T=f(t,x(t),p(t-\tau),z(t-\sigma)), \tag{1}$$

with the initial condition

$$x(t)=(\varphi(t),g(t))^T,t\in[\hat\tau,t_0),x(t_0)=(p_0,g(t_0))^T. \tag{2}$$

A function $x(t;\mu),t\in[\hat\tau,t_1],t_1\in(t_0,b]$, is called a solution of equation (1) with the condition (2) or a solution corresponding to μ, if it satisfies condition (2) and on $[t_0,t_1]$ satisfies equation (1). Let $\mu_0=(t_{00},\tau_0,\sigma_0,p_{00},\varphi_0,g_0,f_0)\in E_\mu$ and $x(t;\mu_0)$ be solution defined on $[\hat\tau,t_{10}]$. We introduce the set of variations $V=\{\delta\mu=(\delta t_0,\delta\tau,\delta\sigma,\delta p_0,\delta\varphi,\delta g,\delta f)\in E_\mu-\mu_0:\mid\delta t_0\mid\le\alpha,\mid\delta\tau\mid\le\alpha,\mid\delta\sigma\mid\le\alpha,\mid\delta p_0\mid\le\alpha,$ $\delta\varphi=\sum_{i=1}^{\nu}\lambda_i\delta\varphi_i,\delta g=\sum_{i=1}^{\nu}\lambda_i\delta g_i,\delta f=\sum_{i=1}^{\nu}\lambda_i\delta f_i,\mid\lambda_i\mid\le\alpha,i=\overline{1,\nu}\}$, where $\delta\varphi_i\in E_\varphi-\varphi_0,\delta g_i\in E_g-g_0,\delta f_i\in E_f-f_0,i=\overline{1,\nu}$ are fixed functions, $\alpha>0$ is a fixed number.

Theorem. *Let $t_{00}+\tau_0<t_{10}$, then there exist $\varepsilon_1>0$ and $\delta>0$ such that $x(t;\mu_0+\varepsilon\delta\mu)=x(t;\mu_0)+\varepsilon\delta x(t;\delta\mu)+o(t;\varepsilon\delta\mu),\forall(t,\varepsilon,\delta\mu)\in[t_{10}-\delta,t_{10}]\times[0,\varepsilon_1]\times V$, where*

$$\delta x(t;\delta\mu)=\Big[Y(t_{00};t)\{(\Theta_{k\times1},\dot g_0(t_{00}))^T-f_0[t_{00}]\}-Y(t_{00}+\tau_0;t)f_{01}\Big]\delta t_0+Y(t_{00};t)(\delta p_0,\delta g(t_{00}))^T$$
$$-\Big\{Y(t_{00}+\tau_0;t)f_{01}+\int_{t_{00}}^{t}Y(s;t)f_{0p}[s]\dot p_0(s-\tau_0)ds\Big\}^{*)}\delta\tau-\Big\{\int_{t_{00}}^{t}Y(s;t)f_{0z}[s]\dot z_0(s-\sigma_0)ds\Big\}^{**)}\delta\sigma$$
$$+\int_{t_{00}}^{t}Y(s;t)\delta f[s]ds+\int_{t_{00}-\tau_0}^{t_{00}}Y(s+\tau_0;t)f_{0p}[s+\tau_0]\delta\varphi(s)ds+\int_{t_{00}-\sigma_0}^{t_{00}}Y(s+\sigma_0;t)f_{0z}[s+\sigma_0]\delta g(s)ds, \tag{3}$$

$f_0[s]=f_0(s,x_0(s),p_0(s-\tau_0),z_0(s-\sigma_0)),f_{01}=f_0[t_{00}+\tau_0]-f_0(t_{00}+\tau_0,x_0(t_{00}+\tau_0),\varphi_0(t_{00}),z_0(t_{00}+\tau_0-\sigma_0));\Theta_{k\times1}$ *is the $k\times1$- zero matrix, $Y(s;t)$ is a matrix function satisfying the equation $Y_s(s;t)=-Y(s;t)f_{0x}[s]-(Y(s+\tau_0;t)f_{0p}[s+\tau_0],Y(s+\sigma_0;t)f_{0z}[s+\sigma_0]),s\in[t_{00},t]$ and the conditions $Y(t;t)=I,Y(s;t)=\Theta_{n\times n},s>t;I$ is the identity matrix.*

The expression (3) is called the variation formula. The expressions $*)$ and $**)$ in (3) are the effects of perturbation of delays τ_0 and σ_0.Variation formulas for various classes of functional-differential equations without perturbation of delay are given in [1]. Finally, we note that the necessary optimality conditions of delays $\tau\in[\tau_1,\tau_2]$ and $\sigma\in[\sigma_1,\sigma_2]$ are obtained on the basis of variation formula (3).

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CALCULATION OF THE GRADIENT IN THE PROBLEM OF OPTIMAL CONTROL FOR ONE NON-LOCAL PROBLEM FOR WEAK-LINEAR WAVE EQUATION

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Let the controlled process be described in the cylinder $Q = \Omega x(0,T)$ by the equation

$$\frac{\partial^2 u}{\partial t^2} - \Delta u = f(x,t,u(x,t),\vartheta(x,t)), \tag{1}$$

with initial conditions

$$u(x,0) = \varphi(x), \frac{\partial u(x,0)}{\partial t} = \psi(x) \tag{2}$$

and non-local condition

$$\frac{\partial u}{\partial \nu}|_S = \int_\Omega K(x,y)u(y,t)dy, \tag{3}$$

where $u(x,t)$ is a state of the process, $\vartheta(x,t)-$ controlling function, $\Omega-$ bounded domain in R^n with smooth boundary, $\partial\Omega, S = \partial\Omega x(0,T)-$ lateral surface of the cylinder Q. As a class of admissible controls we take the given set from $L_2(Q)$.

The problem is: to find a control from U that together with the corresponding solution of the problem (1)-(3) gives minimum to the functional

$$I(\vartheta) = \int_Q f_0(x,t,u(x,t),\vartheta(x,t))dxdt. \tag{4}$$

In the work under natural conditions on the problem data, a formula for the gradient of the functional (4) is obtained.

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ON LIMITING ERROR OF INACCURATE INFORMATION AT THE OPTIMUM DISCRETIZATION SOLUTIONS OF THE HEAT OF THE TRIGONOMETRIC FOURIER COEFFICIENTS

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The reconstruction problem of inaccurate information is put in different formulations (see, eg., [1-4]). In formulation of [3] studied the Cauchy problem for the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x_1^2} + ... + \frac{\partial^2 u}{\partial x_s^2} \ (t \geq 0, x \in R^s)$$

with initial conditions

$$u(0,x) = f(x), \ x \in R^s.$$

Indicate a computational aggregate D_N defined as following. Let $\varphi_N(z_1, ..., z_N; x)$ is an arbitrary measurable function on $L^2(0,1)^s$ At any fixed $z_1, ..., z_N$ (and, of course, $\varphi_N(0, ..., 0; x) \equiv 0$). Let

$$D_N = \left\{ l_1(f) = \hat{f}(m^{(1)}), ..., l_N(f) = \hat{f}(m^{(N)}) : m^{(1)} \in Z^s, ..., m^{(N)} \in Z^s \right\} \times \{\varphi_N\},$$

where $\hat{f}(m)$ - trigonometric Fourier coefficients.

Theorem. *Let $s(s = 1, 2, ...)$be given and $r > 3$,* numerical sequence $\tilde{\varepsilon}_N = \frac{(lnN)^{r(s-1)}}{N^r}$ $(N = 1, 2, 3, ...)$. *Then the following relation*

$$\delta_N(0) \equiv \delta_N(D_N; \frac{\partial u}{\partial t} = \Delta u, u(x,0) = f(x); E_s^r; 0)_{L^2} \succ\prec$$

$$\succ\prec \delta_N(D_N; \frac{\partial u}{\partial t} = \Delta u, u(x,0) = f(x); E_s^r; \tilde{\varepsilon}_N = \frac{(lnN)^{r(s-1)}}{N^r})_{L^2} \succ\prec \tilde{\varepsilon}_N \sqrt{N} = \frac{(lnN)^{r(s-1)}}{N^{r-\frac{1}{2}}}$$

valid, for any tending to $+\infty$ positive sequence $\{\eta_N\}_{N=1}^{\infty}$ the equality

$$\varlimsup_{N\to\infty} \frac{\delta_N\left(D_N; \frac{\partial u}{\partial t} = \Delta u, u(x,0) = f(x); E_s^r; \tilde{\varepsilon}_N \eta_N = \frac{(lnN)^{r(s-1)}}{N^r}\eta_N\right)_{L^2}}{\delta_N\left(D_N; \frac{\partial u}{\partial t} = \Delta u, u(x,0) = f(x); E_s^r; \tilde{\varepsilon}_N = \frac{(lnN)^{r(s-1)}}{N^r}\right)_{L^2}} = +\infty.$$

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OPTIMAL CONTROL OF A CLASS OF DISTRIBUTED PARAMETER SYSTEMS WITH MOVING SOURCES

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Let the condition operated object is described by functions $u(x,t)$ and $s(t)$, moreover functions $u(x,t)$ in domain $\Omega = \{0 < x < l, 0 < t \le T\}$ satisfies to the equation of parabolic type

$$u_t = a^2 u_{xx} + \sum_{k=1}^{n} p_k(t)\delta(x - s_k(t)), \tag{1}$$

with initial and boundary conditions

$$u_x\,|_{x=0} = 0,\ u_x\,|_{x=l} = 0,\ 0 < t \le T, \tag{2}$$

$$u(x,o) = \varphi(x), 0 \le x \le l, \tag{3}$$

where $a, l, T > 0$ are given numbers; $\varphi(x) \in L_2(0,l)$ -is the known function; $\delta(\cdot)$- Dirac's function; $p(t) = (p_1(t), p_2(t), ..., p_n(t)) \in L_2(0,T)$ is the control function. The function $s(t) = (s_1(t), s_2(t), ...$ $..., s_n(t)) \in H^1(0,T)$ is supposed the solution of the following problem of Kosher

$$\dot{s}(t) = f(s, \vartheta, t), 0 < t \le T, s(o) = s_0, \tag{4}$$

where s_0-is the given real number; $\vartheta = \vartheta(t) = (\vartheta_1(t), \vartheta_2(t), ..., \vartheta_r(t)) \in L_2(0,T)$- is the control function; function $f(s,\vartheta,t) = (f_1(s,\vartheta,t), f_2(s,\vartheta,t), ..., f_n(s,\vartheta,t))$ is continuous, has continuous derivatives on s and ϑ at $(s,\vartheta,t) \in E^n \times E^r \times [o,T](s,\vartheta,t) \in E^n \times E^r \times [o,T]$ and they are limited: $|f_\vartheta(s,\vartheta,t)| \le M_\vartheta, |f_s(s,\vartheta,t)| \le M_s$.

It is required to find such controls $\bar{\vartheta} = (p(t), \vartheta(t))$ from following set V

$$\begin{gathered}\bar{\vartheta} \in V = \left\{\overline{\vartheta} = (p,\vartheta) : p = p(t) = (p_1(t), ..., p_n(t)), \vartheta = \vartheta(t) = (\vartheta_1(t), ..., \vartheta_r(t)),\right. \\ \left. p_i(t) \in L_2(0,T), \vartheta_j(t) \in L_2(o,T), 0 \le p_i(t) \le A_i, 0 \le \vartheta_j(t) \le B_j, i = \overline{1,n}\,, j = \overline{1,r}\right\},\end{gathered} \tag{5}$$

that the functional

$$J(\overline{\vartheta}) = \int_0^l \int_0^T [u(x,t) - \tilde{u}(x,t)]^2 dxdt + \alpha_1 \sum_{k=1}^{n} \int_0^T [p_k(t) - \tilde{p}_k(t)]^2 dt + \tag{6}$$

$$+\alpha_2 \sum_{m=1}^{r} \int_0^T [\vartheta_m(t) - \tilde{\vartheta}_m(t)]^2 dt,$$

accepted the least possible value at conditions (1)-(4). Here $\alpha_1, \alpha_2 \ge 0, \alpha_1 + \alpha_2 > 0, A_i > 0, i = \overline{1,n}, B_j > 0, j = \overline{1,r}-$ are given numbers; $\tilde{u}(x,t)) \in L_2(Q)$, $\omega = (\tilde{p}(t), \tilde{\vartheta}(t)) \in H$, $\tilde{p}(t) = (\tilde{p}_1(t), \tilde{p}_2(t), ...$ $..., \tilde{p}_n(t)) \in L_2(0,T)$, $\tilde{\vartheta}(t) = (\tilde{\vartheta}_1(t), \tilde{\vartheta}_2(t), ..., \tilde{\vartheta}_r(t) \in L_2(0,T))$ - are the known functions.

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STATISTICAL MODELLING OF OIL TRANSPORTATION THROUGH THE MAIN PIPELINE

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Statistical modeling of processes for oil transportation through the pipelines taking into account casual parameters is considered in this paper. As casual parameters of ambient temperature, factor of the heat transfer from oil in environment and the empirical factors depending on a mode of a current of oil is considered. Casual parameters are played by the method of Monte-Carlo, then they are used for calculation of model of oil transportation through the pipeline.

Oil movement through the pipeline with diameter D and length L is described by differential equations [1-4]

$$\frac{\partial \theta}{\partial t} + v\frac{\partial \theta}{\partial x} = \frac{4k}{\rho c D}(\theta_e - \theta) + \frac{4W}{\rho c \pi D^2}, \tag{1}$$

$$\frac{\partial P}{\partial x} = -\rho g a \frac{\nu^m}{D^{m+1}}\left(\frac{\pi v}{4}\right)^{2-m} - \rho g\frac{dH_b}{dx},\ (t,x) \in Q \tag{2}$$

with the initial

$$\theta(0,x) = \theta_0(x),\ x \in (0,L) \tag{3}$$

and boundary conditions

$$\theta(t,0) = \alpha(t),\ P(t,0) = \beta(t),\ t \in (0,T), \tag{4}$$

where t - time; x - spatial coordinate; *θ(t,x), P(t,x)* – temperature and pressure of oil; *W=W(t,x)* – capacity of electroheating sources.

The basic symbols in the system (1)-(4) are correspondent accepted in [1-4].

The task for choosing the function *W(t, x)* that minimizes the functionce is set up here:

$$J(W) = \omega_\theta \int_0^T \int_0^L (\theta(t,x) - \theta_*(x))^2 dxdt + \omega_p \int_0^T \int_0^L (P(t,x) - P_*(x))^2 dxdt +$$

$$+\omega \int_0^T \int_0^L W^2(t,x) dxdt,$$

where $\omega_\theta, \omega_P, \omega > 0$ - weight indicators; $\theta_*(x)$, $P_*(x)$ - desirable modes of oil transportation. The method of successive approximations and its convergence is applied to the solving of the task for optimal control.

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ON O-MINIMAL LATTICIES

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Since 1984 o-minimality of totally ordered structures is one of the main branches of model theory. This notion was introduced by A. Pillay and Ch. Steinhorn in [1]. Now this branh of model theory is deeply investigated and has various application to other fields of mathematics. In this work we study o-minimality for partial ordered sets, mainly for lattices. The aim of this work is to describe o-minimal lattices.

Let $a < b$. By an interval in a lattice L we mean the following sets: $\{c \in L : a < c < b\}$.

Definition. A lattice L is called *o-minimal* if any parametrically definable subset of L is a finite union of intervals and points.

Define the following formulae:

$$\begin{aligned}
\varphi_1(x,y) &:= x < y \wedge \forall t_1 \forall t_2 [x \le t_1 \le y \wedge x \le t_2 \le y \to t_1 \le t_2 \vee t_2 \le t_1] \wedge \\
&\quad \wedge \forall t_1 [x \le t_1 \le y \to \forall t_2 (t_2 > t_1 \to t_2 \le y \vee t_2 \ge y) \wedge \\
&\qquad \wedge \forall t_3 (t_3 < t_1 \to t_3 \ge x \vee t_3 \le x)] \\
\varphi_2(x,y) &:= \varphi_1(x,y) \wedge \exists x_1 \exists y_1 (x_1 < x \wedge y < y_1 \wedge \varphi_1(x_1,y_1)) \\
E_1(x,y) &:= x = y \vee \varphi_1(x,y) \vee \varphi_1(y,x) \\
E_2(x,y) &:= x = y \vee \varphi_2(x,y) \vee \varphi_2(y,x)
\end{aligned}$$

Theorem 1. *Each relation E_1 and E_2 is an equivalence relation.*

Theorem 2. *If L is an o-minimal lattice then the number of finite, but not one-element classes of the equivalence E_i is finite, for $i = 1, 2$.*

Theorem 3. *If L is an o-minimal lattice then so is the quotion lattice L/E_1 as well as L/E_2.*

Corollary 4. *The number of non-one-element E_2-classes is finite.*

Lemma 5. *The number of E_1-classes which have a minimal (maximal) element is finite.*

Lemma 6. *Let (a, b) be an interval such that for any $c, d \in (a, b)$ with $\neg E_1(c, d)$ it holds that $a = c \wedge b$ and $b = c \vee b$. Then the number of E_1-classes in the interval (a, b) is finite.*

Theorem 7. *The number of non-one-element E_1-classes is finite.*

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THE PARAMETRICAL SYNTHESIS TASK OF CONTROL BY THE OBJECTS IN A FUZZY ENVIRONMENT ON THE BASE OF COMMON PARAMETER METHOD

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The application of computer facilities and creation of automatic control systems of a various direction is necessary and urgent for development of every possible spheres of human activity at the present stage. Almost in all cases the parametrical uncertainty is characterized by an inhering of real values of parameters of technical object to some intervals, which boundaries are a priori known. Their mathematical models can be presented by systems of interval differential or difference equations with use of rules and notations of the interval analysis. Such object can be considered as family of objects, which is determined by an inhering of parameters of this object with the given interval. It means, that there is a problem of control not only by unique object, but of whole family of objects. The theory of the interval analysis is applied to a solution of the problem also of the analysis of dynamic properties and synthesis of control by a similar class of objects. In considered operation on the basis of common parameter method [1] with multiplicate parametrical tunable and device of the interval analysis the task of parametrical synthesis of control of the interval-given object is solved.

Let's present a required interval vector of tunable parameters $\mathbf{K}$ from control actions $U(t) = \mathbf{K}^T X(t)$ as follows:

$$U(t) = \beta K_0^T X(t), \tag{1}$$

where $\beta = \underline{\beta}, \overline{\beta}$ - multiplicate common interval tunable parameter; $K_0 \subseteq R^n$- pointwise vector of tunable parameters of system of the algebraic equations of the following form:

$$mid\mathbf{P}K_0 = mid\mathbf{H}.$$

Consider algebraic inclusion to which solution of the problem of parametrical synthesis is reduced. An interval system of such algebraic inclusion for the algorithm of control (1) is represented as:

$$\mathbf{P}\beta K_0 \subseteq \mathrm{H}, \tag{2}$$

or in scalar form:

$$\sum_{j=1}^{n} \mathbf{p_{ij}} k_{0j} \left[\underline{\beta}, \overline{\beta}\right] \subseteq \mathrm{h_i}, \qquad i = \overline{1, n} \tag{3}$$

We find the solution of the equation (2) in the class of allowable set of solutions. To solve the problem it is necessary to determine such a parameter β which provide the inclusion (2). The following theorem conditions ensure the solution of formulated problem:

Theorem 1. *The interval vector* $\mathbf{K} = \beta K_0$ *provides the following inclusion* $\mathbf{P}\beta K_0 \subseteq \mathrm{H}$*, if the vector* K_0 *represents the solution of system of equations* $mid\,[P]\,K_0 = mid\,[H]$ *and satisfies to the inequality* $\sum_{j=1}^{n} rad\,(\mathbf{p_{ij}})\,.k_{0j} \leq rad\mathbf{h_i}$*, and the common tunable parameter* β *is determined by the following expression:*

$$\beta = \min_{1 \leq i \leq n} \left\{ \frac{rad\mathbf{h}_i - \sum_{j=1}^{n} rad\mathbf{p}_{ij}\,|k_{0j}|}{\sum_{j=1}^{n} mid\,|\mathbf{p}_{ij}\,|k_{0j}| + rad\mathbf{p}_{ij}\,|k_{0j}||} \right\}.$$

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TRANSFORMATION OF PROGRAM MOTION'S DEGENERATE AUTOMATIC CONTROL SYSTEMS

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Let's consider the differential equations' system unsolved with respect to high derivative, arising at constructing of control's systems by given program manifold Ω :

$$H(t,x(t))\dot{x} = -A(t)x - q(t) - b\xi,\ \xi = \varphi(\sigma),\ \sigma = c^T\omega,\ t \in I = (\alpha,\beta), \quad (1)$$

where $H(t,x(t)) = \partial\omega/\partial x,\ x \in R^n,\ \omega \in R^s,\ H \in R^{s\times n},\ \alpha,\beta$- are finite or infinite numbers, b, c - are vectors, nonlinearity ξ satisfies conditions of local quadratic connections. We examine the case when $s = n$ and matrix H has r zero roots, Erugin's function $F(t,x,\omega) = -A_1(t)\omega,\ \omega \equiv H_1(t)x + g(t) = 0,\ A(t) = -A_1(t)H_1(t)$, where $A_1(t)$ and $H_1(t)$ are given continuous matrices, $g(t)$ is given continuous function, $q(t) = \partial\omega/\partial t + A_1(t)g(t)$.

We consider the problem of finding transformation matrices for the system (1) permissive to reduce to equivalent system.

Theorem 1. *Let $H(t)$ and $A(t)$ are absolute continuous matrices bounded together with their first derivatives in the interval I, $rankH(t) = k\ \forall t \in I$, for all k, $1 < k < s$ and there is submatrix $H_0(t) \in R^{k\times k}$ of matrix $H(t)$ satisfying the following conditions*

$$\inf[det(H_0(t))] > 0 \quad t \in I, \quad \inf[\partial^r/\partial\lambda^r det(H(t)\lambda + A(t))] > 0 \quad t \in I. \quad (2)$$

Then for all $t \in I = (\alpha,\beta)$ nonsingular matrices T and S exist such that multiply by T the left hand side and replacing by $x = S(t)z$ the system (1) is reduced to equivalent system, where the matrices $T(t)$ and $S(t)$ are of the form:

$$T(t) = \left\| \begin{array}{cc} E_1 & -C_1(t) \\ O_1 & H_0^{-1}(t) \end{array} \right\|,\ S(t) = \left\| \begin{array}{cc} E_1 & O_2 \\ -C_3(t) & E_0(t) \end{array} \right\|. \quad (3)$$

We find absolute continuous matrices $C_1(t)$ and $C_3(t)$ bounded together with their derivatives. Hence the matrix H will be represented in bloch form

$$H(t) = \left\| \begin{array}{cc} C_1(t)H_0(t)C_3(t) & C_1(t)H_0(t) \\ H_0(t)C_3(t) & H_0(t) \end{array} \right\|. \quad (4)$$

Consequently the system (1) is equivalent to the following system

$$\begin{array}{l} F_1(t)z_1 + F_2(t)z_2 = q_1(t), \\ \dot{z}_2 = F_3(t)z_1 + F_2(t)z_2 - H_0^{-1}(t)q_2(t) - B_2\xi_2,\ \xi = \varphi(\sigma), \\ \sigma = P^TA_1(t)S(t)z + P^Tg(t), \end{array} \quad (5)$$

Here matrix F has the following structure

$$F(t) = T(t)(H(t)\dot{S}(t) + A(t)S(t)). \quad (6)$$

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METHOD OF SOLUTION OF OPTIMAL CONTROL PROBLEM WITH MULTIPOINT BOUNDARY CONDITIONS

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Assume that the considered process is described by the equation

$$\dot{x}(t) = F(t)x(t) + G(t)U(t) + v(t), t \in [0, T], \tag{1}$$

with multipoint boundary conditions

$$\sum_{i=0}^{p} \Phi_i x(t_i) = q, 0 = t_0 < t_1 < ... < t_i < ... < t_{p-1} < t_p = T. \tag{2}$$

It needs to minimize the functional

$$J = \frac{1}{2}\int_0^T \left[x'(t)Q(t)x(t) + u'(t)C(t)u(t)\right] dt. \tag{3}$$

For this purpose, we find the corresponding Euler-Lagrange equations by constructing the expanded functional [1]:

$$\begin{array}{l} \dot{x}(t) = F(t)x(t) - M(t)\lambda(t) + v(t), \quad \dot{\lambda}(t) = -Q(t)x(t) - F'(t)\lambda(t), \\ \lambda(t_0) = \lambda(0) = -\Phi_0'\gamma, \\ \lambda(t_i + 0) = \lambda(t_i - 0) - \Phi_i'\gamma, \qquad i = \overline{1, p-1}, \\ \lambda(t_p) = \lambda(T) = \Phi_p'\gamma, \end{array} \tag{4}$$

where $M(t) = -C^{-1}(t)\lambda'(t)G(i)$, $\lambda(t)$ and the constant γ are Lagrangian multipliers. The unknown function $\lambda(t)$ in (4) is being searched for the form

$$\lambda(t) = S(t)x(t) + N(t)\gamma + \omega(t), \quad t \in [0, p], \tag{5}$$

where $S(t), N(t)$ and $\omega(t)$ are the discrete functions to be found. By substituting the expression (5) into the equation (6) and after some transformations we obtain the system of equations

$$\begin{array}{l} \dot{S}(t) = -F'(t)S(t) - S(t)F(t) + S(t)M(t)S(t) - Q(t) \\ \dot{N}(t) = [S(t)M(t) - F'(t)]N(t) \\ \dot{\omega}(t) = [S(t)M(t) - F'(t)]\omega(t) - S(t)v(t). \end{array} \tag{6}$$

Further $n(i)$ and $W(i)$ are determined from the differential equations

$$\begin{array}{l} \dot{n}(t) = N'(t)M(t)N(t) \\ \dot{W}(t) = N'(t)[M(t)\omega(t) - v(t)]. \end{array} \tag{7}$$

To find the unknowns $x(0)$ and γ we obtain the system of equations:

$$\begin{bmatrix} S(t_0) & N(t_0 + 0) + \Phi_0' \\ \Phi_0 + N'(t_0 + 0) & \sum_{i=1}^{p-1} n(t_i + 0) \end{bmatrix} \begin{bmatrix} x(t_0) \\ \gamma \end{bmatrix} = \begin{bmatrix} -\omega(t_0) \\ q - \sum_{i=0}^{p-1} W(t_i + 0) \end{bmatrix}. \tag{8}$$

Then the control $u(t)$ is determined in the form $u(t) = C^{-1}(t)G'(t)(S(t)x(t) + N(t)\gamma + \omega(t))$ and $x(i)$ as the solution of the problem $\dot{x}(t) = (F(x) + M(t)S(t))x(t) + M(t)N(t)\gamma + M(t)\omega(t) + v(t)$, with the initial condition $x(0) = x_0$.

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SECTION VIII
Mathematical Modeling

COMPUTATIONAL ISSUES IN THE SYNTHESIS OF PARALLEL MECHANISMS FOR THE MODELING OF ARTICULAR HUMAN JOINTS

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Mathematical models of human articular joints proved to be of great theoretical and practical relevance. They play, indeed, a fundamental role in many important issues such as preplanning surgical operations, diagnosis procedures and prosthesis design. Many planar and spatial models have been proposed in the literature. Planar models are generally simpler but they provide very limited information, especially for the design of advanced (sophisticated) prostheses. On the other hand, spatial models are very involved and quite often the model results are not easy to be used practically. Recently, we developed a procedure which aims at defining human joint models that can take into account the main (and many) anatomical structures of the joint such as ligaments and articular bone surfaces. The models are based on the use of equivalent spatial parallel mechanisms that mimic both the joint anatomical structures and the joint motion. The procedure to synthesize these mechanisms has however some critical points which deserve a great attention in order to be successfully overcome. This study focuses on the main mathematical issues involved in the procedure and on the strategies the authors have devised in order to overcome these problems and let the algorithms to be stable and efficient. In particular, the treated issues are the avoidance of singularity problems in the solution of non-linear systems for the synthesis of the equivalent mechanisms, and the definition of efficient algorithms to converge to a satisfactory solution by recursive optimization procedures. Examples are reported for the definition of joint models of the lower limb (knee and ankle) that show the efficiency of the adopted strategies.

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MATHEMATICAL MODELING OF THE PROBLEMS RELATED TO THE EFFECTS OF THE MULTI-PARAMETRIC, MULTI-SCALE AND SINGULARITY IN PHYSICS, CLIMATOLOGY AND ECONOMY

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The formation of the multi-parametric, multi-scale and singularity effects with respect to the problems related to the behavior of the spherical bubble cluster in acoustical field, in ocean flow and economical distribution of the consumption of the energy and cash income are investigated.

The enlargement (in the case where the acoustical pressure is negative) and collapse (in the case where the acoustical pressure is positive) processes of the steam cavitations cluster are characterized with the huge change of the problem parameters: the pressure changes from 0 up to 10^{11} bar; temperature T changes from 10^2 up to 10^8 K, velocity changes from 1 up to 10^5 m/c. Micro-shock waves, heat transfer, evaporation, condensation, dissociation and ionization of steam accompany these processes. The plasma is formed at moment of convergence of micro-shock waves in center of the bubble, according to which, the corresponding conditions for appearing of the thermonuclear reaction may be realized. Within this scope, the bubble can be taken as hydrogen bomb.

Analyzes show that in the cases where the maximal radius of the bubble is about 0.7 mm the conditions for the thermonuclear reaction realize in the central nucleus of the imploding bubble the radius of which is about 100 nm during the 100 sec. In this case the liquid which is on the surface of the phase interface has a high pressure (about $10^5 - 10^6$ bar) during, approximately, 10^{-9} sec. The characteristic scale of a time for the one extension-compression cycle is equal, approximately, to $5 - 10^{-5}$ sec from which the multi-scale character of the considered phenomenon follows.

Numerical results show that within the scope of the foregoing conditions 10 neutrons appear because of the collapse of the micro-bubble. In experiments with deuterated acetone (C_3D_6O) in the ultrasonic acoustic field it can be provide the appearing about 10^4 collapses during one second by the use of the light flashes, at the same time, there are about $10 - 10^2$ collapsed micro-bubble in the cluster. Therefore, theoretical results agree with the flux measurements of the fusion neutrons and with the intensity of the appearance of the tritium nuclei (5×10^5 sec^{-1}).

Analyzes show that significant resources preservation of the spherically symmetric collapse of the micro-bubbles is caused by the viscosity of fluid and by the high density of the steam in the compressed bubble.

Further, it is considered the multi-scale processes with singularity in the two-phase systems consisting of ocean and atmosphere which influences on the climatologically changes, in particular, on the global warming.

Finally, it is shown that as singularity or abnormally concentration of the cash income in the hand of a small proportion ($0.1 - 1\%$) of the population leads not only to obstruction of the development of productive forces, but also to degradation of their.

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GENERAL PROBLEM OF MODELING AND CONTROL OF GAS-LIFT PROCESS

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As is known [1], when exploitation of the oil way by gushing way finishes - the second stage of the oil well exploitation gas-lift method is used. Despite of wide application of this method, till now their precise mathematical model and solution methods of the corresponding problems (control and stabilization) are absent. However some problems have been considered. For example, in [2] the problems of control of the movement of gas and gas-liquid mixtures in ring and elevating pipes are investigated at replacement of properties of the layer by the corresponding impulse equation, in [3], the properties and characteristics of the layer are studied. All these and others approaches do not allow one to represent the real description of the gas-lift process. In our opinion it is connected by the fact that the real factors of the layer are not taken into account.

In the present work the movement in the layer which is described by the essentially nonlinear partial differential equations is identified by the ordinary linear differential equation [4]. It carried out by the methods of least squares and unconditional minimization [5]. Further instead of the impulse equations which are included in the well+layer system, the identified equation is taken that allows one to develop complete mathematical model of gas-lift process. As such system has a variable structure, the application of known methods meet some serious difficulties [6]. Therefore describing the condition of compatibility in a top and bottom, and also in the exit the problems of choice of the optimal rejime, i.e. the problems of control of gas-lift are put. Further the algorithm to the solution of the given problem, i.e. the problem of linear - square-law control is offered. The results are illustrated on the concrete example.

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THE CALCULATION OF THE LONGITUDINAL-TRANSVERSE SPECTRUM BASED ON NUMERICAL SOLUTIONS OF THE NAVIER-STOKES EQUATION

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In this work the influence of the impact of the strain flow associated with a change of the Reynolds number on the characteristics of isotropic turbulence, and the process of degeneration of isotropic turbulence is considered. Characteristics of turbulence of gas flow or liquid have a great importance for many technical devices [1].

The calculation of the longitudinal-transverse three dimensional energetic spectrum of degeneracy of homogeneous isotropic turbulence is usually produced on the basis of spectral equations and the von Karman-Howarth's equation, for the closure of which it is necessary to determine the correlation functions. When calculating the energy spectrum, is reduced to computational complexity.

In this paper, this problem is solved on the basis of the numerical solution of three-dimensional Navier-Stokes equation by large eddy method on the high-performance cluster. The idea was concluded in specify in the phase space of initial conditions for the velocity field, which satisfies the condition of continuity. At the same time main spectral equation is not solved, and given initial condition with phase space translates into the physical space using a Fourier transform. Obtained velocity field is used as an initial condition for the filtered Navier-Stokes equation. Then we solve three-dimensional nonstationary Navier-Stokes equation for modeling of the degeneration of isotropic turbulence.

To solve the Navier-Stokes equation splitting scheme is used by physical parameters, which consists of three stages. In the first stage Navier-Stokes equation without pressure is solved. To approximate the convective and diffusion terms of this equation compact scheme of high order accuracy is use. In the second stage is solved Poisson's equation, obtained from the continuity equation, taking into account the velocity field from the first stage. For solving the three-dimensional Poisson equation an original algorithm is developed for solving - the spectral transformation in combination with a matrix factorization [2, 3]. The resulting pressure field in the third stage is used to recalculate the final velocity field.

Proposed numerical algorithm allows to calculate changing in the characteristics of isotropic turbulence with high Reynolds numbers. Analyzing the results of modeling can make the following conclusion: One dimensional spectra of the fields were non-negative and monotone, which corresponds to the requirements of the Khinchin's theorem. The viscosity of the flow have a significant effect on the turbulence and therefore can be used for control the turbulence. The obtained results allow us to calculate precisely changing in the characteristics of isotropic turbulence in time, at high Reynolds numbers.

Thus, based on the solution of the Navier-Stokes equations has been constructed a numerical algorithm, allowed to accurately calculate changing in the longitudinal-transverse energy spectrum.

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FORECASTING OF THE DEVELOPMENT ACTIVITIES OF PERSPECTIVE QAS CONDENSATE STRUCTURES WITH THE HELP OF HYDRODYNAMICAL MODEL

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Both operations of exploitation of old gas condensate deposits, and research and development of the new perspective gas condensate structures is very important branch of the modern oil-gas industry. But, introduction to exploitation of the new structures, drilling of wells, development and introduction of various methods, etc. are accompanied by colossal financial costs. Therefore, the creation of the development project, forecasting of multiple prospective parameters (including technical and economic) development of such structures before the industrial development is of great importance. The purpose of the development projecting of the oil deposits is the formation and the choice of variants of developing of the productive layers, providing the efficiency of the exploitation of the deposits. One of the basic directions oriented to achievement of this purpose, is the modeling of the deposits, allowing to imitate the exploitation at the various variants. The modeling allows the engineer to understand the geology of a layer and to predict its behavior, quantitatively and qualitatively to estimate the reaction of layers on various technological decisions at various scripts of development. Forecasting of behavior of the layer is used for the solution of the problems connected to planning, exploitation and diagnostics at all stages of the deposits development. Therefore, the modeling and forecasting of various activities with the help of modern software products, is the basic part of the projecting of the development of oil and gas deposits.

In the given work, by using of the software package "VIP", hydro dynamical models of three new gas-condensate structures - A, D and B, situated in the Azerbaijan sector of Caspian Sea and having the huge hydrocarbon stocks are constructed. Analyzed the various variants of development and are predicted parameters of development.

By using of the interpreted seismic data the geological models of structures A, D and B have been constructed, and then on the basis of these models hydro dynamical models have been constructed.

In the future, using the results of physic-chemical and thermodynamic researches spent on samples taken from the received extraction, we can precise the calculations and give more precise forecast of development and exploitation of deposits.

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RATIONALE OF THE MODEL OF TOXIC SUBSTANCES SPREAD IN LOWER LAYER OF ATMOSPHERE

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In the layer of air above the earth's surface with a thickness of 10 to 100 m-called Ground-layer, all the meteorological elements have maximum gradients and the relative constancy of the height of the turbulent potokov. Pri certain meteorological conditions, even small emissions can create unfavorable environmental conditions in human settlements. Even more dangerous is the natural and manmade disasters, which may result in large-scale pollution of the environment[1-3]. A pilot study of these phenomena is expensive and in some cases it is not possible to conduct a complete physical simulation of interest theoretical research methods - methods of mathematical,modeling.Mathematical models can be divided into two classes: deterministic and stochastic (probabilistic). In this paper, the model only the first type. Mathematical modeling using a deterministic approach that includes the following steps [3]: 1. Physical analysis of the phenomenon under study and the establishment of a physical model of the object. 2. Determination of the reaction medium properties, transport coefficients and structural parameters of the medium and the withdrawal of the basic system of equations with appropriate initial and boundary conditions. 3. The choice of method of numerical or analytical method for solving boundary-value problem. 4. Obtaining a discrete analogue for the corresponding system of equations, if you want a numerical solution. 5. The choice of method used to obtain solutions for the discrete analogue. 6. Develop a program for calculating computer. Test validation program for calculation. Obtaining a numerical solution of differential equations. 7. Comparing the results with known experimental data, their physical interpretation. Parametric study of the object.The main requirement for a mathematical model - the consistency of the results of numerical analysis with experimental data. To perform this sufficient condition requires that:− in the mathematical model were carried out fundamental laws of conservation of mass, energy and momentum;− mathematical model correctly captures the essence of the phenomenon under study.In general, we have to solve time-dependent spatial tasks that require considerable effort in preparing the programs for calculating and sufficiently powerful computers. To overcome the above problems in setting targets used reasonable assumptions, no significant effect on the result of calculations for the task.Thus, by constructing a mathematical model (in the surface layer of the atmosphere in the aquatic environment, etc.) you can explore the dynamics of the pollution under the influence of various external conditions (air temperature, wind speed, temperature stratification in the atmosphere, etc.) as well as the parameters of the source of contamination. Comparing these data with the established maximum - allowable concentrations (MACs), we can analyze the pollution levels of various components at different time points and suggest ways to reduce the concentration of air pollution.

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MATHEMATICAL MODELLING HEAT AND MASS EXCHANGE PROCESSES IN POROUS MEDIA

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Very many researches of the world in the field of the theory of a filtration with a sufficient set of those or other mathematical models and various approaches of their decision, but unfortunately in a reality by development of oil fields and gas there are more complex variants of course of processes of a filtration in view of kinetics heat exchange, mass exchange, etc., that is natural on a straight line influences the technological scheme of object operation. The basic moments are reduction of problems of a nonequilibrium filtration problems with free (unknown) boundaries of Stefan and Verigin type's. Last fact is justified by the fact that, having information on well-boring to restore boundary of considered area. It is known, that the boundary or a part of boundary can vary gradient of temperature or gradient of pressure [1,2].

The problem of a nonequilibrium filtration is considered, following statement.In the bounded domain Ω with piece - smooth boundary $\equiv \partial\Omega$, consider

$$\vec{u}_i = -\frac{K_0 f_i}{\mu_i}\nabla p\,(i=1,2)\,,\ \ mH\frac{\partial s}{\partial t} + \operatorname{div}(H\vec{u}_1) = 0,\ \ -mH\frac{\partial s}{\partial t} + \operatorname{div}(H\vec{u}_2) = 0,\ \ s + s_1 = 1,$$

$$mH\frac{\partial}{\partial t}\left[c_1 s + (1-s)\,c_2\right] + H\frac{\partial a}{\partial t} + \operatorname{div}(Hc_1\vec{u}_1) + \operatorname{div}(Hc_2\vec{u}_2) = r, \tag{1}$$

$$H\tfrac{\partial((1-m)\rho_0 C_0 T_p)}{\partial t} + H\tfrac{\partial(m(\rho_1 C_1 s\tilde{c}_1 + \rho_2 C_2 s_1 \tilde{c}_2)T)}{\partial t} +$$
$$\operatorname{div}(H(\rho_1 C_1 v_1 \tilde{c}_1 + \rho_2 C_2 v_2 \tilde{c}_2)\cdot T) + H(\rho_1 C_1 v_1 \varepsilon_1 + \rho_2 C_2 v_2 \varepsilon_2)\cdot\nabla(p)$$
$$= \operatorname{div}(H\left(\overline{\lambda}(s,T,\tilde{c}_1,\tilde{c}_2)\nabla T + \overline{\lambda}_0(T_p)\nabla T_p\right)) - 2\sqrt{\tfrac{\lambda_3 C_3}{\pi\cdot t}}\rho_3(T_p + t\tfrac{\partial T_p}{\partial t}),$$

with the equations of heat and mass exchange kinetics in the porous medium:

$$\alpha_T\frac{\partial T_p}{\partial t} = \eta(T) - T_p,\quad \frac{\partial a}{\partial t} = \frac{1}{\tau}\cdot(G(c) - a)\,, \tag{2}$$

where taking $c_1 = c$ and $c_2 = \varphi(c)$ the functions $\eta(T)$ and $G(c)$ are defined as,

$$\eta(T) \equiv T,\quad G(c) = \begin{cases} 1, & c > c* \\ [0,1], & c = c* \\ 0, & c < c* \end{cases} \tag{3}$$

$$s\big|_{t=0} = s_0(x),\ \ c\big|_{t=0} = c_0(x),\ \ a\big|_{t=0} = a_0(x),\ \ T\big|_{t=0} = T_0(x), \tag{4}$$

$$(P,S,\theta) = (P_0,S_0,\theta_0),\ \ -D\cdot\frac{\partial c}{\partial n} + \vec{v}_{1n}\cdot c = q_n\cdot c^*,\ \ (x,t)\in\Sigma^1 = \Gamma^1\times[0,\bar{t}], \tag{5}$$

where q_n - set of charge on unit of the area, c^* – known value of concentration of an impurity. For the given problem the computing algorithm is constructed and the analysis of distribution of the basic technological parameters of oil extracting is obtained.

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SIR EPIDEMIC MODEL WITH FUZZY INITIAL VALUES

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Infectious diseases such as mumps, rubella, measless are modeled by classifying individuals in the population according to their status with respect to the disease: healthy, infected and immune. The disease states, S,I and R are defined as follows: S = susceptible; individuals not affected but who are capable of contracting the disease and becoming infective I = infected; individuals who are infected and infectious, capable of transmitting the disease to others R = removed; individuals who have had the disease and have recovered, and who are permanently immune, or are isolated until recovery and permanent immunity occur. The progress of individuals is schemetically described by

$$S \to I \to R.$$

Such models are often called SIR models. Differential equations corresponding to an SIR model is as follows:

$$\frac{dS}{dt} = -\alpha SI,$$

$$\frac{dI}{dt} = -\alpha SI - \beta I,$$

$$\frac{dR}{dt} = \beta I,$$

$\alpha > 0$ is the infection rate
$\beta > 0$ is the rate of recovery.
N = total population size (we assume N is constant and there are no births and deaths in the population). We will investigate graphical solutions of the SIR model with fuzzy initial values which are fuzzy triangular numbers.

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MATHEMATICAL MODELS OF GENE ACTIVITY REGULATION PROCESSES: MECHANICAL PERTURBATION OF DNA STRUCTURE[1]

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Binding of proteins and low molecular weight ligands to DNA play a fundamental role in processes of gene activity regulation. Binding of a protein to DNA can be coupled with DNA confirmation changes which can be transmitted over great distance extending outside the DNA region immediately covered by a bound ligand. The perturbed DNA structures and even DNA cleavage can induced by mechanical tensions. Electromechanical perturbation of DNA is a wide-spread phenomenon which may play a fundamental role in processes of gene activity regulation.

In present paper a simple mathematical approach is developed to describe electromechanical perturbations of DNA in complexes with large ligands, as revealed from recent studies (for instance, see [1]–[6] and the broad list of corresponding references in them) on effects of ultrasound irradiation on DNA complexes with sequence specific binding ligands.

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DYNAMICS OF ELASTIC MEDIUM AT THE ACTION OF TRANSPORT LOADS ON STRIP CONCENTRATED

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Mathematical simulation of the dynamic processes, connected with the transport moving, brings about decision of the problems dynamics of elastic mediums in class of functions, parametric and automodel on row variables. The parameter of the problem - a velocity of the moving the source of loadings in medium - greatly influences at type of the motion equations, which depends on velocities of the spreading the waves in medium, so named as sonic velocities. In isotropic elastic medium their two, characterizing velocity of the spreading transverse and longitude of the waves. The type of the differential equations, describing moving the medium, is changed depending on relation of the velocities of the source of the loadings to sound velocities (the Mah's numbers).

Here for describing of the moving of elastic mediums at action of the transport loads $G(x_1, x_2, x_3+$ $+ct)$, moving with constant velocity, are used equations of the Lame's equation in mobile coordinate system $x' = (x'_1, \dots, x'_N) = (x_1, \dots, x_3 + ct)$, [1]:

$$\left\{ \left(M_1^{-2} - M_2^{-2}\right) \frac{\partial^2}{\partial x'_i \partial' x_j} + \left(M_2^{-2}\Delta - \frac{\partial^2}{\partial x'^2_3}\right)\delta^i_j \right\} u_i + c^{-2}G_j = 0, \quad j = 1, 2, 3, \tag{1}$$

where u_j - components of the moving medium, $M_k = c/c_k$ – Mah's numbers, $M_1 < M_2$ (on of the same name indexes is tensor convolution).

At $M_k < 1\,(k = 1, 2)$ load is subsonic, system (1) of the elliptical type; if load supersonic, then $M_k > 1$ $(k = 1, 2)$ and system becomes hyperbolic; if velocity transonic, $M_1 < 1$, $M_2 > 1$ and the type of the equations hyperbolic-elliptical. Under sound velocity of the equation parabolic-elliptical if $M_2 = 1$, but under $M_1 = 1$ become parabolic-hyperbolic.

Under $M_j > 1$ appear the shock waves [2,3]. Conditions on fronts (F) are of the form of:

$$[u]_F = 0, \quad [n_3\, u_{i,j} - n_j\, u_{i,3}]_F = 0, \quad \left[\,\sigma_{ij}n_j - \rho\, c^2\, n_3\, u_{i,3}\right]_F = 0, \quad i, j = 1, 2, 3;$$

n_j - normal's components to front of the wave F,σ_{ij} - stress tensor, ρ - density of the medium.

Generalised decisions have built (1) at action vertical and horizontal transport loads concentrated on strip for difficult motion velocities.

Results of numerical experiment have reduced, describing dynamic elastic medium for subsonic, transonic and supersonic velocities. It is studied influence of motion velocities on character stress-strain state of medium.

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INTERACTION OF TWO-PHASE GAS-DROPLET TRANSVERSE JET WITH THE ONCOMING SUPERSONIC FLOW

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In this paper we numerically investigate the interaction of the jet containing dispersed liquid droplets, with a supersonic flow of gas. It was believed that the jet flows from the transverse to the main flow of a plane gap with a velocity of sound. Modeling of flow was based on assumptions and equations of mechanics of multiphase media [1]. Numerical simulation was based on the method of large particles [2] using the algorithm developed in an environment MATLAB. In this case, the size of the computational domain were chosen such that the disturbance caused by gas injection, did not reach the free boundaries. The calculations were performed to obtain the steady flow pattern. Optimal step account was set sustainability criteria and the required accuracy of calculation. As the characteristic length of problem, was used the minimum characteristic length $L_* = \min(L_v\ ,\ L_T\ ,\ \ell)$ where L_v and L_T are the relaxation lengths of the velocity and temperature of phases, ℓ - width of the split. As a typical time convenient to take $\tau_* = L_*/a_\infty$ (a_∞ is velocity of sound in the oncoming flow). Determining influence on the flow conditions in the interaction of the two-phase jet with the oncoming stream usually have the effects of interfacial friction. Therefore, the main criteria for the approximate similarity of such flows can be considered the following parameters: ratio of specific heats of gas, the Mach number, pressure and temperature ratio of the incoming stream and injected gas, the ratio of the characteristic length of relaxation rates of the phases to the width of the split, as well as the relative mass content of the dispersed phase in the injected mixture.

Calculations showed that a jet is formed in front of a wave of compression. It should be noted that in contrast to the injection of a pure gas jet, in the case of injection of gas-drop jet at some distance from the slit wave compaction, how-to forks, taking "lambda"-shaped. And near the site of branching wave front seal occurs a narrow zone of strong dilution of the gas. Apparently, this is due to heating of the gas due to the work of friction forces on the interfacial surface of the droplets and its subsequent expansion. Another feature of the flow produced when blowing a two-phase gas-droplet jet is the presence of two zones of rarefaction wave for sealing. This is because the jet is blown out of gas after a gap of almost immediately turns to the direction of motion of the incident flow, and "jet" drops out of inertia rises to a considerable height, and thus, before the incident flow there are two "obstacles" in the interaction with which There are two areas of rarefaction.

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MODELING OF TURBULENT CURRENTS IN CHANNELS WITH SMALL CURVATURE

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The solution of the fluid flow problem plays an important role in the practical engineering design. There is a very wide range of applications from a supersonic flow over a space ship to blood flow in the circulatory system of a human being. The equations of fluid mechanics - which have been known for over a century - have analytical solutions for only a limited number of flows. Numerical simulations represent an alternative or at least a complementary method to the experimental approach. This field is known as computational fluid dynamics (CFD). CFD is able to avoid some of the most annoying problems of the experimental approach such as the blockage effect caused by the boundary layers along the walls of a wind tunnel. Boundary conditions are easily prescribed in the computations.

This paper presents a new procedure for the simulation of unsteady incompressible turbulent flows using Galerkin finite element and adaptive grids. The adaptive grids are generated during the simulation using a new mesh generation technique. This technique is fast, produces a quad-dominant mesh while preserving the quality of the elements without the need to move any grid points. The new points are nested to the old mesh to avoid hanging nodes. Interpolation operators are used to map the differrent variables from one grid to the next one. Refinement zones are defined using the gradient of the vorticity from the previous time step.

A Galerkin finite element method is implemented to simulate unsteady incompressible turbulent flows in channels with small curvature using the primary variables with mixed elements for the velocity components and the pressure. The resulting linearized system is solved using Krylov subspace iterative methods and multigrid. The least-squares commutator is implemented as a preconditioner of the indefinite linear system. The turbulence model to close the RANS equation is based on method by Kolmogorov-Rotta [1,2] and the solver is coupled with the adaptive grid generator in order to produce a new solution resolved grid for everytime step.

The technique was implemented through an object oriented platform written in C++ and used the Message Passing Interface (MPI) for parallel programing.

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THE MODEL FOR OPTIMAL MANAGEMENT OF THE ECONOMY BASED ON R. LUCAS PRODUCTION FUNCTION

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Problems of economic growth currently occupy a central place in economic discussion of leading countries. The growing volume of real production rather allows to resolve the problem faced by any economical system: the limited resources with the growth of human needs.

Many models are known, among them are R. Lucas model [1] of endogenous economic growth,which that takes into account the factor of human capital accumulation described by the following production function:

$$Y = \bar{\bar{A}} K^{\beta} (whL)^{1-\beta} h_a^{\psi},$$

where h – human capital, L – labor force, wL – amount of unskilled labor force and whL – amount of labor in efficiency units, ψ – elasticity of Y output to the average for the whole economy level of human capital.

This production function is used in the economic optimization models and for its optimality criterion is assumed to maximize the discounted sum of a finite (non-industrial) consumption for the forecasting (planning) duration [0; T]:

$$W = \int_0^T e^{-\delta\, t} \frac{C}{L} dt \to \max_D,$$

where δ - discount factor, $0 < \delta < 1$.

To find the optimal $v = (K(t), X(t), u(t))$ - process of model, Krotov V.M. theorem was used [2]. According to this and from the condition

$$\bar{k}(t), \bar{x}(t) \to \max_{0 \le x \le f(k,t)} R(t, k, x)$$

optimal $\hat{k}(t) = A_1^{\frac{1}{1-\beta}} e^{\frac{\gamma t}{1-\beta}}$, $A_1 = \frac{\beta(1-a)A}{\mu+n+\delta}$ and $\hat{u}\,(t) = 1 - \frac{(\mu+n+\frac{\gamma}{1-\beta})\beta}{\mu+n+\delta}$,were found
where $R(t, k, x) = e^{-\delta\, t}[(1-a)x - (\mu + n + \delta)k]$.

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ANALYSIS OF THE CONVERGENCE OF ITERATIVE ALGORITHMS TO SOLVE GRID EQUATIONS OF INCOMPRESSIBLE FLUID IN THE VARIABLES "STREAM FUNCTION-VORTICES OF VELOCITY

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The report discusses the convergence of some iterative algorithms to solve grid Navier-Stokes equations for incompressible fluid variables stream function and vortices of velocity with the boundary conditions for a vortex in the form of Thomass formula. Estimations of convergence for the considered relatively stable iteration schemes were obtained in the case of one-dimensional boundary value problem. The grid equations for Navier-Stokes with homogeneous boundary conditions for the stream function and vortices of velocity were considered in the nonlinear two-dimensional case by introducing an auxiliary function vortices of velocity. Under certain conditions equivalent to the uniqueness conditions of the nonlinear difference problem, the theorems on convergence, stability have been proved by using energy inequalities, and an estimation of the convergence rate is provided.

BY VARIATIONS OF THE ELEMENTS IN MATHEMATICAL MODELS OF THE DYNAMICAL SYSTEMS

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Real observations dynamical systems to be realized from many elements which have different initial value and parameter. Then observations, theory and numerical simulation shows that the control strategy needs to select right error valuation. The mathematical model an research this system to admit as predicting evolution system and as to find out all interactions structural: last time, the present and the future tense. This be dependent on the interactions and evolution systems elements.

Evolution of the nonlinear dynamical systems, in arbitrary time t, can be represent in the form

$$X_t = \left\{x_{t,}^1 x_t^2, x_t^3, \ ..., x_t^N\right\}, \tag{1}$$

where N-number of the interactions elements, which jointly represent dynamical systems. In particular when $N = 5:$ x_t^1-time; x_t^2-space; x_t^3-matter; x_t^4-energy; x_t^5 - law and order evolution elements of the dynamical systems.

Mathematical modeling state variables of the evolutions dynamical system are important in practical result estimating observations, which can be represent values of the elements in different moments time t. After appropriate cutting statistical results observations evolutions dynamical system can be represent formula

$$X_t^i = \left\{x_t^i, \ x_{t+\tau}^i, \ ... , \ x_{t+(m-1)\tau}\right\}, \tag{2}$$

where τ- time intervals between observations, m- necessary number observations and measuring τ.

Effectiveness interpretations estimates results work request important attention to selection values τ and m. This connection which, that particularly to keep key to open interactions and evolution compound structures elements estimates models (1) and (2).

MATHEMATICAL MODELING AND NUMERICAL CALCULATION BY THE METHOD OF LARGE PARTICLES OF THE IMPACT OF ACTIVE LAYER OF SOIL TO THE CONTAMINATION IN THE REGION TAKING INTO ACCOUNT PROCESSES IN THE LOWER ATMOSPHERE

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One of the central problems of physics of atmosphere is the study of the variability of atmospheric composition and the effects of atmospheric pollution of the environment [1]. In many numerical models of atmospheric dynamics the soil temperature is found using the heat balance equation. Thermophysical characteristics of the soil are very diverse, which leads to the fact that temperatures above different soil types can differ dramatically on even small distances from each other, and thus influencing the dynamics of the lower layers of the atmosphere [1]. Additionally, due to their mutual dependence, the heat and humidity conditions in the soil-air system influence the formation and development of plants and thus affect the environment [2].

Currently, the solution of this complex problem can be obtained through a number of simplifications. Based on a joint review processes of molecular and turbulent thermal conductivity of the soil conductivity in the atmospheric boundary layer, the thermal regime of the soil is described in more detail.

Modeling of the regional atmospheric processes is implemented according to the field of meteorological variables in a limited area. Time-dependent boundary conditions are formulated on the basis of data obtained for the bordering areas. The proposed mathematical model differs from the known and widely implemented models by the use of a vertical coordinate σ. The solution of the problem gives the Y(n, t) distribution along the axis n and time t. Mean values of substance Y at each time t were found using the Reynolds's integral averaging.

The diurnal cycle of temperature was modeled in a single cell in the summer, cloudless, windless day for the latitude and declination values of $55,7^0$ and $23,4^0$ respectively. The surface was considered sufficiently hydrated ($q_0 = q_{HAC}(T_0)$) with the reflection coefficient (albedo) r = 0,2. Solid and gaseous impurities were not taken into account. The initial state was set to: $T_{-1} = 287$, $T_0 = 283$, $\theta_1 = 285$, $\theta_2 = 282$, $\theta_3 = 260$, $q_1 = 0,0054$, $q_2 = 0,0045$, $q_3 = 0,0014$. The system of equations integrated by the Runge-Kutta with the time step t=1 h. The calculation results were compared with the expeditionary observations.

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MATHEMATICAL MODELING OF SYSTEM OF OIL LAYER – GASLIFT WELL

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The work of the oil layer - gaslift well is mathematically modeled as an association of two problems: problem of oil filtration to central well and problem on motion of the qas-liquid mixture in the well tube.

1. A filtration of incompressible oil to the central well in a circular layer. This process is mathematically described by the equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right)+\frac{\partial^2 P}{\partial z^2}=\chi\frac{\partial P}{\partial t}, \tag{1}$$

where P is a pressure, r - radial coordinate, z - vertical coordinate, t - time, $\chi=\frac{k}{m\mu\beta}$ - pieseconductivity factor, k - factor permeability of a layer, m - factor of porosity of a layer, μ - factor of dynamic viscosity of oil, $\beta=m\beta_o+\beta_\tau$ - factor of volume elasticity of the layer, β_o,β_τ - factors comeressity oil and rock of a layer, correspondingly.

The equation (1) is solved under following conditions:

$$\begin{cases} P=P_b=\rho_H gL, & \texttt{at } t=0, \quad r_c\le r\le r_k, \\ P=P_k(t), & \texttt{at } r=r_k, \quad 0\le z\le h, \\ \frac{\partial P}{\partial z}=0, & \texttt{at } z=0, \quad r_c\le r\le r_k, \\ r\frac{\partial P}{\partial r}=\frac{\mu}{2k\pi h}q, & \texttt{at } z=r_c. \end{cases} \tag{2}$$

Solving this problem we find a condition $P_b=P(t)$ on a botton of the well that is a boundary condition for the problem on motion of the mixture in the pipe.

2. Motion of the mixture in elevating pipe. Nonstasionary Motion is modelled of the mixture accepting it as a viscous liquid, mathematically in the following way it is necessary to solve following differential equations:

$$\begin{cases} -\frac{\partial P}{\partial x}=\frac{\partial(\rho\vartheta)}{\partial t}+\frac{\lambda}{8R_n}\rho\vartheta^2+\rho g\cos\alpha+\frac{\partial}{\partial x}\left[(1+\beta)\,\rho\,\vartheta^2\right], \\ -\frac{\partial P}{\partial t}=c^2\frac{\partial(\rho\vartheta)}{\partial z}, \qquad 0<x<l,\ t>0\ , \end{cases} \tag{3}$$

under the conditions:

$$\rho\upsilon=q(z,t)=0 \texttt{ at } t=0,\ \ z\in[0,L]$$

$$P(x,t)=0;\ \ P_b(0,t)=P_1(t) \texttt{ at } z=0; \tag{4}$$

$$P(z,t)=P_y(L,t)\ =P_2\,(t) \texttt{ at } z=L,$$

Solving the problem (3), (4) the considered process -i.e. the working of the qas-lift well is investigated.

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MATHEMATICAL MODELLING OF PROCESS OF A NONEQUILIBRIUM FILTRATION OF A VISCOUS LIQUID TO AN INCOMPLETE SLIT

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In many cases the consideration of the filtrations is of great importance in the explotation oil of the layers .

The classical model of the elastic rejime describes the process of the equilibrium filtration. As the experiments show, it is not suitable for research of the nonequilibrium filtration when defining movement, conditions change fast. It takes place at definition of parametres of the layer, at layer opening etc.

The isothermal nonequilibrium filtration week-compresseble a viscous liquid in isotropic weak defermorable to a porous medium, mathematically is described by the equation of continuity of the elastic rejime

$$\beta\frac{\partial \mathrm{P}}{\partial t}+\nabla\cdot\bar{\vartheta}=0 \tag{1}$$

and by the law of a filtration of the viscous liquid, taking into account a velocity and pressure relaxation

$$\bar{\vartheta}+\tau_{\vartheta}\frac{\partial\bar{\vartheta}}{\partial t}=-\frac{k}{\mu}\nabla(P+\tau_{p}\frac{\partial P}{\partial t}), \tag{2}$$

where β– factor elastic values of the a layer, and τ_{ϑ}– $\tau_p$time of a relaxation of the velocity and pressure, ϑ- filtration velocity, p- pressure.

The solution of the system (1), (2) is searched with following initial and boundary conditions:

$P = P_0 = const$and $\vartheta = 0$ at $t = 0 r_c \leq r \leq R_k$;

$\vartheta = \frac{q}{2\pi r_c h}$ at $r = r_c h_1 \leq z \leq h$;

$\vartheta = 0$,at$r = r_c$, $0 \leq z \leq h_1$;

$\vartheta = 0$at $z = 0$and h$r_c \leq r \leq R_k$; $P = P_0$at$r = R_k$,

where q- the liquid production rate expense, h- thickness of the a layer, $h - h_1$- discover of the layer by the well, r_c- well radius, R_k- radius of a contour of the well.

The problem is gloved by known methods of mathematical physics.

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MATHEMATICAL MODEL OF DYNAMICS THE EARTH'S CORE

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The oscillatory mode of movement of an internal core of the Earth is studied. Numerical parameters are received at various initial deviations. The period of fluctuations is found at the account and without forces of viscous resistance. The problem is considered in statement about progress of two bodies. Numerical calculation is spent for systems Earth - Moon, Earth - Sun. The equation of movement of an internal core in own field is received in a kind:

$$m\ddot{x} + n\dot{x} + cx = 0, \tag{1}$$

where

$$n = \pi\mu_2 \left[16ab^3(b^3 - a^3) + 16a^4b(b^2 - a^2) + 8ab(b^5 - a^5)\right] / \left[9(b-a)(b^5 - a^5) \ -5(b^3 - a^3)^2\right] \ , \quad A = (4\pi/3)Gm\rho(1 - \rho/\rho_0). \tag{2}$$

Here μ_2- dynamic factor of viscosity of an external core, x - forward displacement of an internal core, a- radius of an internal core, b- radius of an external core, G- a gravitational constant, m- weight of a firm internal core, ρ- density of an external viscous core, ρ_0- density of an internal core. Forces of viscous resistance of an external core at forward displacement of a firm internal core from balance position are found by statement and the decision of a corresponding hydrodynamic problem of Nave-Stoksa, and on own gravitational field of a core of the Earth returning force is defined. At the geophysical data the parameters defining the decision of a problem, and also the period of fluctuations of an internal core are found at the account of forces of viscous resistance and in the absence of these forces. The oscillative motion of an internal core is studied at various initial deviations, depending on change of values of factor of dynamic viscosity of an external core, and also sizes of density both internal, and an external core of the Earth. The oscillative motion of an internal core of the Earth in a Newtonian field of the external drawing center is investigated. Thus in the movement equations, except forces of viscous resistance and forces of own gravitational field, the force defined as a difference of gravitational force operating from the drawing center and force of inertia is considered. The equation of movement of an internal core in a gravitational field of the external drawing center is found in a kind:

$$m\ddot{x} + n\dot{x} + \left(c - 2G\,\frac{\mathrm{Mm}}{\mathrm{R}^3}\right)x = 0; \tag{3}$$

where $n, c$are defined by the above-stated formulas (2). Influence of mechanical properties of a core and the external drawing center on its oscillatory mode is investigated. As an example the field of an attraction of the Moon and a field of an attraction of the Sun are considered.

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ON THE WELL POSED PROBLEM FOR HYDROGEN ATOM EQUATION

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We consider the following singular Sturm-Liouville Problem:

$$Ly = -y'' + \left[\frac{\ell(\ell+1)}{r^2} - \frac{a}{r} + q_0(r)\right] y = \lambda y, \quad (0 < r \le \pi), \tag{1}$$

$$y(0) = 0, \tag{2}$$

$$y(\pi,\lambda)\cos\beta + y'(\pi,\lambda)\sin\beta = 0, \tag{3}$$

where $q(r) = \frac{\ell(\ell+1)}{r^2} - \frac{a}{r} + q_0(r)$.

Let the second singular Sturm-Liouville Problem be given as:

$$\tilde{L}y = -y'' + \left[\frac{\ell(\ell+1)}{r^2} - \frac{\tilde{a}}{r} + \tilde{q}_0(r)\right] y = \tilde{\lambda} y, \qquad (0 < r \le \pi) \tag{4}$$

with the same boundary conditions above.

We denote the spectrum of the two Sturm-Liouville Problems above by $\{\lambda_n\}_{n=0}^{\infty}$ and $\left\{\tilde{\lambda}_n\right\}_{n=0}^{\infty}$ respectively.

In this research, we prove that the difference between two potential functions $q(r)$ and $\tilde{q}(r)$ becomes sufficiently small whenever the spectral datas $\{\lambda_n, \alpha_n\}$ and $\{\tilde{\lambda_n}, \tilde{\alpha_n}\}$ for $q(r)$ and $\tilde{q}(r)$ respectively are chosen sufficiently close to each other (see [1, 2]).

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NUMERICAL SIMULATION OF DISPROPORTIONATE PERMEABILITY REDUCTION TO CONTROL WATER PRODUCTION[1]

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The MLT model [1] of the two-phase immiscible fluids' (water and oil) flow through a porous medium can be written in the form (indexes $i = 1, 2, 3$ correspond respectively water, oil and porous medium below):

$$m\frac{\partial s}{\partial t} = \operatorname{div}\left[a_1\nabla s + \vec{f_1} + a_2\nabla\theta\right], \quad \operatorname{div}\left(K\nabla p + \vec{f_2} + a_3\nabla\theta\right) = 0,$$

$$-\vec{v} = K\nabla p + \vec{f_2} + a_3\nabla\theta, \quad \frac{\partial\theta}{\partial t} = \operatorname{div}[\lambda(x,s,\theta)\nabla\theta - \vec{v}\theta],$$

$m = m_0(1-s_1^0-s_2^0)$; m_0–porosity; s_i^0, $(i = 1,2)$ – residual water and oil saturations respectively; $s = \dfrac{s_1 - s_1^0}{1 - s_1^0 - s_2^0} \in [0,1]$; s_i–saturation; $a_1 = -K_0\dfrac{k_1 k_2}{k}\dfrac{\partial p_c}{\partial s}$; K_0 – symmetrical flow tensor of an anisotropic porous medium. In the case of anisotropic porous medium K_0 can be expressed via k_h – horizontal permeability and k_v – vertical permeability components of symmetrical flow tensor.

$k_i = \dfrac{\bar{k}_i}{\mu_i}$; $0 \le \bar{k}_i \le 1$ – relative permeabilities; μ_i – dynamic viscosity coefficient; $\mu_1 = const$; $\mu_2 = \mu_{2\max} + (\mu_{2\min} - \mu_{2\max})\dfrac{\theta - \theta_{\min}}{\theta_{\max} - \theta_{\min}}$; θ– temperature; $k = k_1 + k_2$;

$$p_2 - p_1 = p_c(x,s,\theta) \ge 0;\ p_c(x,s,\theta) = \bar{p}_c(x,\theta)J(s);\ \bar{p}_c = \sigma\cos\vartheta\left(\frac{m_0}{|K_0|}\right)^{1/2} = \gamma(\theta)\left(\frac{m_0}{|K_0|}\right)^{1/2},$$

where σ -interfacial tension coefficient; ϑ -wetting angle; $|K_0|$ -determinant of matrix $\{k_{i,j}\}$; $\gamma = \gamma_{\max} + (\gamma_{\min} - \gamma_{\max})\dfrac{\theta - \theta_{\min}}{\theta_{\max} - \theta_{\min}}$; $J(s)$ -Leverett function. ρ_i - density (both fluids are assumed to be incompressible, i.e. $\rho_i = const$); g – acceleration of gravity; h - height;

$$\vec{f_1} = K_1\left(\nabla p + \int_s^1 \nabla\frac{\partial p_c}{\partial s}\frac{k_2}{k}ds\right);\ K_i = K_0(x)k_i(s) = K_0(x)\frac{\bar{k}_i(s)}{\mu_i}; \quad \vec{v} = \vec{v}_1 + \vec{v}_2;$$

$$p = p_1 - \int_s^1 \frac{\partial p_c}{\partial s}\frac{k_2}{k}ds + \rho_1 gh;\ a_2 = K_1\int_s^1 \frac{\partial}{\partial\theta}\left(\frac{\partial p_c}{\partial s}\frac{k_2}{k}\right)ds; \quad \vec{f_2} = K\int_s^1 \nabla\frac{\partial p_c}{\partial s}\frac{k_2}{k}ds + K_2\left(\nabla p_c + (\rho_2 - \rho_1)\vec{g}\right)$$

$$a_3 = K\int_s^1 \frac{\partial}{\partial\theta}\left(\frac{\partial p_c}{\partial s}\frac{k_2}{k}\right)ds + K_2\frac{\partial p_c}{\partial\theta}; \quad \lambda(s,\theta) = \sum_{i=1}^3 \frac{\alpha_i\lambda_i}{\rho_i c_{pi}}; \quad \alpha_1 = m_0 s_1; \quad \alpha_2 = m_0(1-s_1); \quad \alpha_3 = 1-m_0.$$

λ_i – thermal conductivity; c_{pi} – heat capacity coefficient at constant pressure.

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SIMULATION OF ACID HYDRAULIC FRACTURING PROCESS

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Influence of regime parameters such as geometric structure, rock conductivity and fracturing pressure and acid solution composition on efficiency of acid hydraulic fracturing process is studied. Hydraulic fracturing is a complicated process to model, as it involves the coupling of at least three processes: the mechanical deformation induced by the fluid pressure on the fracture surfaces; the flow of fluid within the fracture; and the fracture propagation.

For study of hydraulic fracturing process the fracture growth and propagation is modeled by KGD model for plane strain assumptions applicated to horizontal sections. The fracture volume is zero at initial time. The volume of fracture would be equal to the total volume of fracturing fluid in term of fracturing fluid leak-off at the fracture surface. The basic equations governing a hydraulic fracturing model are: the fluid flow equation (expressing the conservation of fluid mass, which yields the velocity field of the fluid inside the fracture); the leak-off term (describing the history-dependent loss of the injected fluid from the fracture into the porous reservoir, due to a positive pressure gradient between the fluid-filled fracture and the reservoir); the proppant transport equation (describing the time-dependent distribution of the concentration of proppant in the fracture) and the fracture growth condition. The fracture size and propagation is defined by solving of given governing equations.

Modulus of elasticity is an important parameter describing geomechanical properties of formation. Received results shows that modulus of elasticity has influence on fracture propagation and thereby on average velocity of flow in fracture. Hydraulic fracturing leads to low width fracture formation and low average velocity of flow in fracture correspondingly in the soil with high modulus of elasticity value.

Length of acid propagation in fracture depends on liquid leak-off of fracturing fluid from the fracture to the surrounding rock and acid reaction rate with formation. High acid reaction rate reduce to acid leak-off therefore retardation of acid reaction rate is important part of projection of acid hydraulic fracturing processes.

Distribution of acid in a fracture for different values of acid reaction rate is received. Increasing of acid reaction rate reduce acid content in fracture deals with acid leak-off. Porosity also affects on acid distribution in fracture.

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TURBULENCE MODELING IN SIMULATION OF SUPERSONIC FLOW WITH PERPENDICULAR INJECTION OF THE GAS

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Numerical study of perpendicular injection jets in supersonic flow were conducted by many researchers. In practice, the problem of interaction of the gas jet with a supersonic flow is a major problem in the simulation of supersonic combustion chambers of scramjet engines. The flow field in such devices is very complex: the turbulent fuel-air mixing, chemical reactions, shock waves, separation region ahead of the jet and behind of it. One of the important problems in the correct physical description and in numerical realization of such tasks is the turbulence modeling, namely, definition of influence of turbulence on shock wave, supersonic and subsonic regions of the flow. The given characteristics are well described with the Jones-Launder k-e turbulent model with and without compressibility correction, where Navier-Stokes equation for k and e consist of low-Reynolds number terms for near-wall turbulent boundary layer modeling in subsonic region and compressibility correction in turbulent kinetic energy equation for supersonic region.

In this work the numerical simulation of supersonic air flow with perpendicular injection of sonic hydrogen jet in 2D channel is investigated. A mathematical model of this process is described by two-dimensional Favre-averaged Navier-Stokes equations for multi-component gas flow. Turbulence effects are modeled with algebraic Baldwin-Lomax's model, Jones-Launder k-e model with and without compressibility correction. The boundary conditions are taken as: supersonic inlet at the left boundary, adiabatic no-slip walls at the bottom, sonic inlet on the slot; on the top condition of symmetry; on an outlet non-reflection condition. The following parameters were taken at the left boundary and on the slot: $M_0 = 1, T_0 = 642, M_\infty = 3,75, T_\infty = 800, n = 10,26$. The space grid was taken as 241x181, the height of the channel - 7,62 cm, and length - 15 cm, width of slot 0,0559 cm. The slot was located on distance of 10 cm from entrance.

The comparative analyses of influence of turbulence models (algebraic Baldwin-Lomax's model, Jones-Launder k-e model with and without compressibility effect) on the surface pressure profiles, supersonic and subsonic zones, recirculation zones was made. The qualitative agreement of results is obtained, namely, boundary layer separation regions ahead of the jet and behind of it, and barrel structure of the expanding jet. The quantitative difference in results is seen in near-jet region.

Numerical experiments show, the constructed numerical model and computer code for studied the turbulent supersonic multi-species flow with different turbulent models allows to study influence of parameters to the shock wave structure and character of jet-supersonic flow interaction.

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2D SIMULATION OF INITIAL STAGE OF HYDROGEN COMBUSTION IN A TUBE

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The focus of this study is the role of the turbulence and pressure waves in the acceleration of stoichiometric hydrogen-oxygen flame propagating in a tube and how it affects to the transition processes. The interaction between the wall and the flow behind the leading shock wave generates turbulence in the whole flow field, starting at the wall and propagating toward the centre. Correspondingly, as known from experimental data, the leading point of the turbulent flame is in the vicinity of the wall and this determines the flame velocity.

The first necessary and obvious mechanism of flame acceleration and its transition to detonation is increase of turbulence intensity before flame. Turbulence intensity before flame is defined by local velocity and distance to the lead wave of pressure, behind which gas comes to movement. At the moment of igniting rather weak first pressure wave is generated, carries away motionless in the beginning gas and creates a turbulent flow before flame. The flame, extending on turbulence field, accelerates and in turn generates pressure waves which moves on the perturbed field with higher speed than previous and catch up with them. Intensity of pressure wave before a flame becomes more. Gas motion velocity is increased and turbulence before a flame also increased.

The predictions are based on numerical solutions of two-dimensional, non-steady set of the conservation equations for individual species, mass, and total energy and applied to the analysis of hydrogen-air gaseous mixture combustion in a tube ignited on closed end. For modelling of mixture and combustion kinetics of hydrogen combustion in air is realized by mechanism, which has seven reversible reactions and seven species - H2, O2, H2O, OH, H, O, N2. Molecular nitrogen is presented as an inert body.

The numerical procedure for solving the set of equations is based on a two-step, time splitting technique for the different physical processes. In the first step, the shock wave propagation is found by solving Euler equations using an explicit TVD scheme with Roes average. In the second step, the diffusion processes and the kinetic terms are solved fully explicitly.

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MATHEMATICAL MODELING OF THE PHENOMENON SEPARATION BEAM WITH INSTANTANEOUS POINT DYNAMIC IMPACT BASED DISCHARGE BASE

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Assuming that at some point in time, the rate of deflection of the beam axis begins to decrease. In this case, the rod base will cover the unloading wave [1-3]. Assuming that the increments of stress and strain is linear, define the character of the front discharge in geometric plane, which separates the field of active and passive deformation. In our case, the unloading wave will not be the front of strong discontinuity, as the transition from loading to unloading at the end of the rod is smooth. Unloading wave front will be weak discontinuity whose speed, in general, variable, in contrast to the speed of the unloading wave of strong discontinuity, which spread the unloading waves of weak discontinuity following estimate. The problem of determining traffic in the area of discharge is not reduced to classical problems for hyperbolic equations. Assuming that the unloading wave propagates to determine the speed end of the rod at the time and released from this point of the characteristic negative direction before crossing the line of discharge. As is known, the relationship between stress and strain in the discharge is given and it is assumed that the relationship between the beam and the base is one-sided [2]. Then, for sufficiently large loads, properly distributed along the beam length, beam separation can occur from the ground. Accounting separation phenomenon is significant, particularly when moving loads at high velocities, when considering the impacts, etc. with flexible and extended beams on elastic foundation. Not accounting separation phenomenon in some cases, the cause of discrepancies of experimental results with theoretical calculations[3]. If in a certain section of the beam deflection of the beam axis becomes negative, there is a separation of the beam from the base. With increasing load, the size of the separation zone will increase. In the separation zone is reasonable to assume that the reaction of the base is missing. Consequently, the right side of the equation will be zero. Accounting separation phenomenon has shown that its influence is most significant in the separation zone than at the point of force application. For example, the deflections of the beam axis in the separation zone increases almost in half.

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RESEARCH OF STABILITY OF FLUCTUATIONS OF THE OIL PIPELINE SPENT IN SEA WATER UNDER THE INFLUENCE OF PULSE LOADINGS

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Oil pipeline fluctuations can be caused by the reasons of technogenic or natural character: artificial explosions in a vicinity of the oil pipeline or earthquakes [1-5]. It is supposed that the oil flows on the oil pipeline laminarly and is modelled by a viscous incompressible liquid flows. The external forces operating on an internal surface of the oil pipeline are forces of a friction of constant intensity and normal pressure to this surface, linearly changing on length of the oil pipeline, caused by current oil are. Uniformly distributed pressure from the side of sea water operates on an external surface of horizontally spent oil pipeline. The oil pipeline is modelled by a circular cylindrical pipe of the set length with set internal and external radiuses, the module of shift and the factor of Poisson. Physical properties of oil are characterized by dynamic viscosity. The problem about meridional deformations of a pipe under the influence of the forces caused by action on an internal surface of a pipe by oil flowing in it and also pressure upon an external surface of a pipe from the side of sea water is solved. Axial and radial movings are functions of cylindrical coordinates. The decision of this problem as the basic precritical condition for research of a problem of stability of fluctuations of the oil pipeline on a method of Leibenzon - Ishlinsky is expressed through axisymmetric harmonious functions of Papkovich - Neiber. Arbitrary constant of integration are defined thus by means of boundary conditions taking into account expressions for pressure. Thus, the basic precritical condition is defined. Free azimuthal elastic fluctuations of a cylindrical pipe are investigated. Internal and external regional problems for elastic indignations in cylindrical system of coordinates taking into account the inertial composed are solved. The equation in movings, presented in the form of Tedone, is solved for a compressed elastic material of a cylindrical solid. Relative volume expansion or dilation submits to Helmholtz equation. The ordinary differential equations are received by the method of division of variables from the differential equations in private derivatives. Decisions of the homogeneous equations are found in bessel functions and heterogeneous - a method of uncertain factors. The characteristic equation for definition of frequencies of free azimuthal fluctuations is received by means of boundary conditions for free fluctuations. According to the method of Leibenzon - Ishlinsk, the bifurcational disturbances connected with dynamic forms of loss of stability differ from disturbances at free fluctuations by only arbitrary constants of integration. The boundary stability conditions containing sizes of the basic precritical condition are deduced. The characteristic equation for definition of frequencies bifurcational fluctuations is received. In process of the dynamic susceptibility depending on frequencies compelled bifurcational and free fluctuations, resonant effects which can lead to oil pipeline destruction are defined.

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ONE MODEL FOR DEFINITION THE DIRECTION AND AN AREA OF POLLUTION IN A COASTAL ZONE

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One of the most polluted sites of the Azerbaijan shelf of Caspian Sea is the Absheron coast. For definition the direction and an area of pollution in a coastal zone is lead model calculation distribution of mineral oil. The modeling-settlement complex consists of two parts: the module of carrying over of stains of oil and the hydrodynamic block. At the heart of model of carrying over of substance the method of wandering particles lies. The essence of this method consists in direct modeling of movement of the particles arriving from sources in the investigated environment. The hydrodynamic block is constructed on the basis of modified Princeton Ocean Model (POM) (Mellor, Yamada, 1982) . Numerical modeling has shown, that the basic direction distribution of the oil pollution will be east, i.e. along northern coast Absheron peninsula.

STATISTICAL MODELING OF EXPERIMENT APPLICATIONS IN THE SOLUTION OF THE LEARNING SCENARIOS BASED ON PROBLEM

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In education, especially scientific, new attractive techniques based on problem are most important instrument for the success of the students. The techniques based on problem are significant when using computer programs like Matlab, SPSS. But they do not say to researcher what one should do. The programs say correlations between modelling and inevitable results. Data, which even not be original, may say some results. But if data is not original, the modelling is not to be the new one. It will be repatition of whatever. So, data must be original.

In this study, we use original data with the new thecniques about secondary school in Turkey. Data have been analyzed by means of correlation-regression, general linear model and cluster analysis methods. We obtain regression equations and cluster dendograms.

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ANALYSIS OF MATHEMATICAL MODEL WITH MULTI-PARAMETRIC BINARY LINEAR DYNAMICAL SYSTEM

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The system of multiparametric dual linear difference equations

$$\xi_v s(c) = \Phi_v(c)s(c) \oplus \Psi_v(c)x(c),\ c \in G_d, \quad v = 1,2,...,k\ [GF(2)] \tag{1}$$

is analysed and exact solution is given. In this paper,

$$c = (c_1, c_2, ..., c_k) \in G_d = \{c|c \in Z^k,\ c_1^0 \leq c_1 \leq c_1^{L_1}, ..., c_k^0 \leq c_k \leq c_k^{L_k}\ c_i \in Z\}$$

are the points in Z^k. $L_i,\ i = 1,2,...,k$ are integers. $Z = \{..., -1, 0, 1, ...\}$ is a set of integers. $S = [GF(2)]^m$ and $X = [GF(2)]^r$ are state and input alphabets respectively. But $s(c)$ and $x(c)$ are m and r size state and input vectors which are described in set Z^k. $\xi_v s(c)$ is a shift operator defined as follows:

$$\xi_v s(c) = s(c + e_v),\ e_v = (0, ..., 0, 1, 0, ..., 0),\ v = 1,2,...,k$$

$\{\Phi_v(c), v = 1,2,...,k\}$, $\{\Psi_v(c), v = 1,2,...,k\}$ are characteristic Boolean transition matrices of size $m \times m$ and $m \times r$ respectively over the Galois field $[GF(2)]$.

Suppose that c^0 and c are points from Z^k, N is a discrete curve, connecting these points and $s_N(c)$ is solution of system (1), along to the discrete curve at the point c. It is clear that there is a unique solution to system (1) if and only if the solution of $s_N(c)$ is independent from the form of the discrete curve N. In this paper the following theorem shows that these phenomene is analytically expressed.

Theorem: *There is an exact solution to the system of equations* (1) *if and only if the following equality is satisfied:*

$$\begin{aligned} &\phi_v(c + e_\mu)\psi_\mu(c)x(c) \oplus \psi_v(c + e_\mu)x(c + e_\mu) = \\ &= \phi_\mu(c + e_v)\psi_v(c)x(c) \oplus \psi_\mu(c + e_v)x(c + e_v), \quad (v, \mu = 1,2,...,k), \quad [GF(2)] \end{aligned}$$

for fixed $x(c)$ *and all* $(c, s) \in Z^k \times [GF(2)]^m$.

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INVESTIGATION OF EFFECT OF LOW PRESSURE ON THE PHYSIOLOGICAL ASPECT ON HUMAN BODY BY USING ANALYTICAL METHODS

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One of most fascinating and amazing aspect of the human body is to all the adverse conditions and its functioning part; be it in desert bound regions having temperature ranges of 40-50^{o}C or be at glacier kind whether having temperatures as low as -40 to -50^{o}C. Though body is capable of sustaining of most of adverse conditions however, not all the human body behaves alike; some have a capability to sustain on the higher side and some on very low side of adverse weather conditions. The cold bite injuries (frost bite) are very dangerous. If adequate precautions are not taken, one may end up losing the affected tissue altogether. Though precautionary measurements have been already available [acclimatization] however, it has been observed that just acclimatization is not mere sufficient to cope with the adversaries but enough care has to be taken by understanding the physiological aspect of the individual [heart rate, blood pressure variation, flow rates of heart, heart beat etc] with respect to higher altitude. In the present studies physiological aspect of the blood flow has been modeled with a view to estimate the physiological flow parameters such as pressure, flow rates, and shear stress against adverse conditions. Mathematical model for blood flow has been developed in the present model with a view to correlate the finding with its impact on higher altitude [for the benefit of mountaineer or service, personnel getting inducted onto higher altitudes].It is assumed that the flow is steady and laminar and turbulence effects in the body are neglected

$$\eta\nabla^4 u - \mu\nabla^2 u + \delta B_0^2 u = -\frac{\partial p}{\partial z} + \rho G,$$

where $u(r)$ is the velocity in the axial direction, ρ and μ are the density and viscosity of blood, η is the couple stress parameter, σ is the electrical conductivity, B_0 is the external magnetic field and r is the radial coordinate.

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MATHEMATICAL MODELLING AND CALCULATION FROM POLYDISPERSE MATERIALS

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In the given work the technique of mathematical modeling and calculation a target component depending on the sizes of polydisperse raw materials and time of stay of particles of fractions in is developed. The raw materials structure, according to classical models and adsorptions from porous environments, is presented in the form of enclosed in each other channels and the sizes. The modeling equations of impregnation of a skeleton of raw materials and a target component in and micropores are written down in a following kind:

$$\frac{\partial^2\psi_a}{\partial u^2} + \sigma\gamma\frac{\partial\psi_i}{\partial\omega}\,|_{\omega=0} = \gamma\frac{\partial\psi_a}{\partial\tau}, \tag{1}$$

$$\frac{\partial^2\psi_i}{\partial\omega^2} = (1+\Gamma)\frac{\partial\psi_i}{\partial\tau}, \tag{2}$$

where functions $\psi_i,\ \psi_a$ are functions of coordinates and time: $\psi_i = \psi_i(u,\omega,\tau),\ \psi_a = \psi_a(u,\omega,\tau)$ also represent concentration of a target component in disperse units.
By approximation of derivatives in the differential equations (1), (2) the issue is brought to the system of finite-difference equations:

$$\frac{P_{k+1,l,m} - 2P_{k,l,m} + P_{k-1,l,m}}{h_u^2} + \sigma\gamma\frac{S_{k,l+1,m} - S_{k,l,m}}{h_\omega}\,|_{l=0} = \gamma\frac{P_{k,l,m+1} - P_{k,l,m}}{h_\tau}, \tag{3}$$

$$\frac{S_{k,l+1,m} - 2S_{k,l,m} + S_{k,l-1,m}}{h_\omega^2} = (1+\Gamma)\frac{S_{k,l,m+1} - S_{k,l,m}}{h_\tau}. \tag{4}$$

The scheme (3), (4) is obvious, the stability condition is provided with a choice of steps on time and coordinate. Realization of the presented problem has allowed to find dependence of distribution of concentration on time and a settlement way to define time of stay of fractions of polydisperse raw materials in :

$$T = \frac{1}{(1-g^*)Kf}\ln\left(\frac{1-g(C_0)/C_0}{1-\eta\,(1-g^*)}\right), \tag{5}$$

where
C_0 – initial concentration a target component in a firm phase, kmol/kmol;
f – size of an interphase surface, m^2;
K - kinetic factor, / (m^2s);
$g(C)$ – quilibrium concentration on the interphase surface, defined under the thermodynamic diagram, kmol/kmol;
η - degree of extraction of a target component.

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COMPARISON OF RESULTS OF RECOGNITION SYSTEM FOR AZERBAIJANI TEXTS WITH VARIOUS FEATURES CLASSES[1]

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Various approaches to a problem of image recognition were generated. Those are the discriminated analysis, neuro-fuzzy approach, construction of the decision trees, application of fuzzy qualifiers, etc. Each approach offers the advantages and can be applied with updating to the majority of practical problems. Using various approaches, it is possible to receive similar on their structure algorithms. The complex approach at creation of qualifiers will allow connecting of advantages for each of the chosen approaches.

The data analysis is an essential part at the decision of all problems. Therefore at the decision of problems of image recognition synthesized on final samples of precedents, always it is necessary preliminary data analysis. The result of such analysis defines a way of the decision of a problem. In work the method of recognition with using of hybrid neural network is offered. Results for various classes of structural features which are calculated by various formulas according to a skeleton of the set symbol are received:

To define first class features the symbol is placed into the rectangle of defined size and divided into 9 parts. Following features are defined in each part (Fig.1):

Number of pixels- “empty” (N1), “less full”(N2), “full”(N3) and “more full”(N4);

Angle coefficient-“vertical”(A1), “less right slopping”(A2), “right sloping”(A3), “horizontal”(A4), “left sloping”(A5) and “less left sloping”(A6);

Maximal lengths for x coordinat-“short” (L1), “middle” (L2), “long” (L3) and “more long”

Maximal lengths for x coordinat-“short” (L1), “middle” (L2), “long” (L3) and “more long” (L4). (L4).

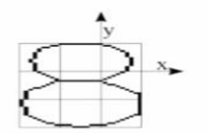

Fig. 1.

To define second class features, the area of symbol is transformed into a square and then is divided into 9 parts. Number of pixels in each part is calculated. Then diagonals of the main square and of 9 smaller squares are drawn and number of pixels in each resulting part is calculated. For each of part the terms “empty”, “little full”, “full”, “very full”. Membership functions had been constructed for these terms.

After calculating features are being grouped, then are presenting to the input of neural network and results are defining.

The various features, their groups influence the time of training and quality of recognition. Recognition accuracy of the system with the first class of features is high, but there are many undefined characters and the accuracy of recognition, time of learning and the number of undefined characters of the system with the second class of features is getting decreased. By this way after such experiments we can define an optimal group of features.

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MATHEMATICAL MODELLING OF SELF- ORGANIZED STATES IN LAMINAR FLAME

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Recently, the problem of self-organization in non-equilibrium systems has intensively been discussed [1]. The major role in the process of successive generation and formation of more complicated temporal and spatial structures is played by fluctuations. In the present paper some possible bifurcations in combustion systems are considered, namely, the onset of self-sustained oscillations, formation of new temporal and spatial structures. Approximate analytical prediction of the region of bifurcations is based upon the models of concentrated (zero-dimensional [2]) systems.

The problem is mathematically formulated by a system of equations for diffusion of the heat and reagents with chemical conversions for flat laminar flame. Two examples are considered: lean methane-oxygen-nitrogen mixture (simple kinetics) and rich hydrogen-oxygen-nitrogen mixture (two-step kinetics and four-step kinetics).

The concentrated system is obtained from the distributed one, the spatial derivatives being substituted by three-node finite-differences including boundary nodes. The analysis of stability of combustion regimes is reduced to analysis of motion stability in the phase plane (the first example) and phase space (second example) by applying the first Lyapunov's method. It has been found, that when some characteristic parameters are varied (e.g. Peclet number), the motion can become unstable, and new types of motion (structures) may result. In the plane of external parameters the parametric boundaries of stability have revealed the regions of nonsingular stationary states, steady and damped oscillations.

Numerically solutions of initial one - dimensional systems show that the stationary values of functions, the period of oscillations and possible parametric regions, can be predicted from analysis of zero-dimensional systems. The fact that the non-unique stationary states are available depending on the burner temperature, was established by the zero-dimensional and was confirmed by numerical solutions. The hysteresis in the dependence on the gas flow velocity was obtained numerically. The theoretically predicted stationary profiles of temperature and concentration are conform with experiment. In the second example the system exhibits self-oscillations and damped oscillations in the domain of non-unique regimes for high temperature stationary states.

If the process is periodically disturbed, then both regimes of undamped oscillations and combustion cut-off can occur in the system, depending on the amplitude and frequency of disturbance, i.e. different temporal and spatial structures are formed and stabilized.

Thus, the approximate analysis based upon a zero-dimensional model and one-dimensional numerical experiments can reveal the phenomena of self-organization in combustion.

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APPLYING EXPERT SYSTEM DECISION THEORY IN ALLOCATING QUOTAS FOR CASPIAN SEA BIORESOURCES

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In order to achieve the second goal and to determine the weights to be assigned to each, the factors are arranged according to significance by acknowledged experts. These experts clearly need to know the system and the resources, to have no obvious allegiance to any country's specific political view, and to be acceptable to all countries no matter which state nominates them for the task. They do not necessarily need to hold the nationality of any Caspian littoral state, though if they are acceptable to all states for the purpose of ranking performance by factor and parameter, they can be. Once accredited as experts for the process, they need to rank the factors in order of significance, as a rule ranking them using integer numbers from 1 to the total number of factors, p. Using an established method for processing expert opinion, all factors can then be ranked and their weight in the allocation process assigned. The sum of all weight coefficients should be equal to 1 (a condition of normalization).

To evaluate the correspondence of expert opinion, a function of collective ranking (FCR) is applied. This is defined as follows. Let$A = \{ a_1, a_2, \dots, a_p\}$ be the set of factors and $N = \{ 1, 2, \dots, n\}$ the group of experts. The total number of factors is$p$, and the number of experts is$n$. $L(A)$ is the set of all linear ordering of the elements of set A. In this case, this is the set of all possible rearrangements of factors $a_1, a_2, \dots, a_p$. Then, the ranking of factors by each expert is equal to the selection of one element of set $L(A)$. Element $u_i \in L(A), \quad i = 1, 2, ..., n$ is referred to as the preference or ranking of the i-expert, and vector $u = (u_1, u_2, \dots, u_n) \in L(A)^n$ is the profile of preferences (rankings) of all experts.

The FCR, φ, should ensure the correspondence of each ranking profile u with some optimal ranking from set $L(A)$: $\varphi : \quad L(A)^n \to L(A)$, which is a reflection of the ranking profile u by φ.

In order for φ actually to reflect the collective opinion of experts and to meet the reasonable expectations of fairness, rationality, and stability of choice, it should have the following properties (Gurvich V. A., and Menshikov, placeI. S. 1989. Institutes of the Agreement. Mathematics and Cybernetics, 6. Znanie, CityplaceMoscow. 48 pp.): Without these properties, the FCR cannot be put into practice. For the collective preference mechanism to work, one needs to put into φ the preferences of experts $u = (u_1, u_2, \dots, u_n)$ the resultant ranking will be $\overline{u}$.

Lemma: *Let Mij be the rank of the i- factor in the opinion of the j-expert, and let Fij be the number of ranked factors in the opinion of the j-expert. Then, MFij,j = i, and FMij,j = i $\forall i$, j. The values Mij and Fij are integer numbers from 1 to p.*

φ may be defined in several ways. As shown, sums of r_i (the last column) are found for each factor. If all sums of r_i are different, then the optimal ranking will be the ascendant order of factors in terms of the sums of r_i. If there are equal sums among r_i, then the optimal will be the ranking $\overline{u}$satisfying the condition:

$Arg \min\limits_k \sum_j \rho(F_j, u_k) = \overline{u}$, where $\rho(u_i, u_j) = \frac{p(p+1)(2p+1)}{6} - \sum_{k=1}^{p} b_{ki} \cdot b_{kj}$.

Theorem. *φ as defined as above is anonymous, neutral, and efficient if it is defined in the form of a vector ri = $\sum M_{ij} + \sum M_{ij}^2$, where i =1, 2,…,p.*

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THE PULSATION ACCOUNT AT MODELLING OF THE GASLIFT PROCESS

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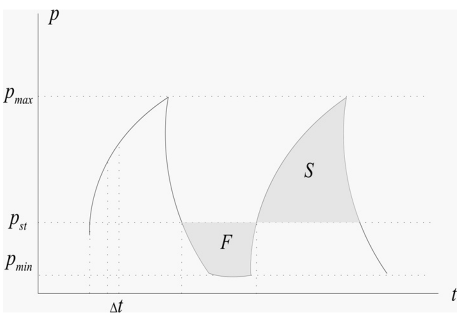
As it is known, mathematical modeling of process at oil recovery is connected with serious difficulties. It is connected with specificity of the process, its discrete character. It is necessary to notice that the action principle of gaslift consists in periodic emission of certain quantity of a liquid from the well. Thus, in each interval of such emissions the pressure in the lift goes down and the condition for inflow of the next portion to the bottom-hole is created. Thus the liquid level in the lift rises and the dynamic level is formed. And so all repeats anew. The frequency of these repetitions depends on a gas injection regime and reservoir performance. And so, it is visible that gaslift is the oscillatory process consisting of several phases



In work the algorithm for the forecasting of the gaslift process is offered. The idea, the offered approach is based on the account of noted phases which consist from the discrete conditions of enough small time interval.

The phase 1. The bottom-hole pressure restoration. *The liquid inflow to the well should form the liquid column in the lift of height* H_{st} wich equates the bottom-hole pressure to the reservoir pressure:

$p_w = \gamma_l \sum \Delta h_{st} + p_i$, *where* p_i *is the overpressure created by injected gas;* Δh_{st} *is the incremental value of the liquid level in the lift at the* Δt *time interval. For the definition of the current liquid level value in the lift we will write (without gas weight):* $h_{st} = \frac{1}{f} \sum Q_o \Delta t$, *where* Q_o *is the reservoir flow rate at the time interval* Δt*. It is defined by the numerical solution of the equations of movement of fluids in the formation [1]. And* p_i *is defined by the numerical solution of the following system of the equations [2]*:

$$-\frac{\partial P_i}{\partial x} = \frac{1}{F}\frac{\partial Q_i}{\partial t} + \frac{2a}{F} Q_i, 2a = \frac{g}{w_c} + \frac{\lambda_c w_c}{2D}$$

$-\frac{\partial P_i}{\partial t} = \frac{c^2}{f_i}\frac{\partial Q_i}{\partial x}$, where physical sense of all designations same as well as in work [2].

And so, h_{st} raises to the value $H_{st} = \frac{p_r}{\gamma_6}$. As h_{st} will reach to value H_{st} the second phase - the emission phase begins.

The phase 2. The emission phase. *The volume of the lifted liquid is defined by empirical expression of type* $Q_l = f(Q_i, d, \bar{\varepsilon})$, $V_l = Q_l \cdot \Delta t$. *And as a result a new liquid level and a new value of the bottom-hole pressure is formed:*

$h_{st}^1 = h_{st}^0 - \frac{V_l}{f}$ $p_w^1 = \gamma_l h_{st}^1 + p_h$. *Thus the condition for inflow of a liquid from the reservoir is created and the process repeats until as the reservoir pressure can't provide depression for inflow. The described approach allows to consider real features of the gaslift process and can be a basis at modeling.*

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DIFFERENT FORMS AND EXACT SOLUTIONS FOR THE EVOLUTION EQUATION OF A PLASTIC LAYER SPREADING ON A PLANE

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The problem of plastic layer spreading between the surfaces of converging external bodies [1] has been analyzed. Earlier [2] the nonlinear evolution differential equation for determining the contour $y = \varphi(x, t_1)$ of free spreading plastic layer on a plane was deduced:

$$\frac{\partial \varphi}{\partial t_1} - \varphi \left(1 + \left(\frac{\partial \varphi}{\partial x} \right)^2 \right) - \frac{1}{2} \varphi^2 \frac{\partial^2 \varphi}{\partial x^2} = 0, \tag{1}$$

where $t_1 = \ln(h(t_0)/h(t))$- the dimensionless time; $h(t)$- the thickness of plastic layer. In [3] the evolution equation for $y = \varphi_1(x_1, t_1)$, where $x_1 = x/h(t), y_1 = y/h(t)$,

$$\frac{\partial \varphi_1}{\partial t_1} + x_1 \frac{\partial \varphi_1}{\partial x_1} - \varphi_1 \left(2 + \left(\frac{\partial \varphi_1}{\partial x_1} \right)^2 \right) - \frac{1}{2} \varphi_1^2 \frac{\partial^2 \varphi_1}{\partial x_1^2} = 0 \tag{2}$$

was deduced with the help of another original method. In [3] it is also showed that the equation (2) passes to particular form of thermal conductivity equation

$$\frac{\partial w}{\partial \tau} = \frac{1}{2} \frac{\partial}{\partial x} \left(w^2 \frac{\partial w}{\partial x} \right) \tag{3}$$

The exact solutions for evolution equations (1)-(3), the well-known and some new ones, are described.

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A METHOD OF PAIRWISE RECOGNITION OF SOUNDS COMBINING FREQUENCY ANALYSIS WITH ANALYSIS OF FILTERED SIGNAL IN THE TIME DOMAIN

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A sound and a word are acoustically different phonetic objects, in essence. A sound has got comparatively homogeneous spectrum, while a word consists of a few parts having heterogeneous spectra. For this reason, when recognizing a whole word we should use a feature *vector* of some kind. While it is reasonable to use a suitable *scalar* feature or a set of independent scalar features to recognize pairs of sounds (or pairs of their classes). Each scalar feature must provide its own result of recognition. As a rule, it is possible to make a few tests and determine the range of values every feature can take, for each class of the two. Values out of the range should be interpreted as recognition failure.

Suppose that we have detected a speech segment with a single vowel, by using the segmentation algorithm described in [1], [2]. Now, how can we determine what exactly vowel is it? To solve the problem we use a complex approach, which combines frequency analysis (that is, processing the signal by a set of band-pass filters) with analysis of the filtered signal in the time domain.

So after we have selected a segment in the speech signal, we split it into windows n samples each. Suppose we have got m windows in number. In every window we calculate the value of

$$V = \sum_{i=1}^{n-1} |x_{i+1} - x_i| \tag{1}$$

- the numerical analogue of total variation for the discrete case. Then we count the arithmetic mean of m values of (1) calculated in m sequenced windows. We call the new value *variation V of selected segment.*

We also use the numerical analogue V of total variation with variable upper bound to define a new quantity - variation measure of signal. We consider an auxiliary function W that increases along with V, but is “thrown down” to 0 on reaching some specified value A. Suppose, it reaches A and gets thrown down to 0 at consecutive points N_1, N_2, ... We continue the series of points until the selected segment with a vowel is over. We count distances between neighboring points N_k, and get a series of numbers N_1, $N_2 - N_1$, $N_3 - N_2$, ...

Then we count the arithmetic mean of these numbers in the selected segment of the signal. We call the new value *variation measure* or *measure M of selected segment.*

V/M ratio calculated for a segment of the signal pre-processed by a set of digital band-pass filters will be eventually used as a feature for scalar recognizers allowing of discrimination between pairs of sounds.

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NUMERICAL MODELING OF MULTIPHASE FLOW IN POROUS MEDIUM

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The mathematical model is presented for unsteady two-dimensional percolation of two-phase multicomponent liquid in porous medium .The practical problem was considered of oil recovery from the system of wells by using chemical reagents during injections of water. Numerical results are presented. The micellar-polymer water flooding is investigated under the following assumptions: the fluids following through the porous medium are incompressible, the porosity is a piecewise-constant function of the Space coordinates, the skeleton of the porous medium is rigid, the flow is equilibrium and isothermal, and the pressures in the liquid phases are equal. The motion is assumed to be two-dimensional and obeys the Darcy's law. The polymer absorption by the skeleton is assumed to proceed in accordance with Henry law. We considered the flow for a five point well location diagram. The hyperbolic transport equations were approximated using an implicit upwind first-order scheme with minimization of the numerical dispersion. The elliptic equation for the pressure was solved by the stabilization method with sweep in variable directions. In order to take the features of the pressure distribution near the wells into account, in the neighborhood of the wells we distinguish zones in which the motion is assumed to be plane-radial. In the case of a five-point spacing scheme, for micellar-polymer flooding we found the two-dimensional pressure, saturation, and surfactant and polymer concentration distributions. On the basis of these distributions we can draw the following conclusions: the injection well pressure changes sharply with variation of the viscosity of the injected fluid and the mixture flow velocity is significantly different over the flow region, having a maximum in the neighborhood of the injection well and a minimum along a line equidistant from the wells. The efficiency of the process of improving oil recovery is analyzed: both the absolute permeability range over which the time of break-through of the micellar solution into the producing well is greater than for homogeneous reservoir.

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MATHEMATICAL MODEL OF PHENOMENA AT CLOSURE OF ELECTRICAL CONTACTS IN VACUUM

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Phenomena occurring at closure of electrical contacts in vacuum circuit breakers are very important for understanding of arc evolution, mechanism of contact erosion and welding.

Electrical breakdown, micro-arcing, melting and evaporation of micro-asperities are described by axisymmetric Stefan problem with two free boundaries corresponding to melting and evaporation interfaces. Three zones in contact area should be considered to find temperature dynamics and pressure of metal vapours in contact gap, including the zone of evaporated micro-asperity $D_0[-l \leq z \leq 0]$ with adjoining evaporated area inside contact $D_1[0 \leq z \leq \sigma_b(r,t)]$, melted zone $D_2[\sigma_b(r,t) \leq z \leq \sigma_m(r,t)]$ and solid zone $D_3[\sigma_m(r,t) \leq z < \infty]$.

The heat equation for the temperature field for each domain D_i , $i = 0,1,2,3$ should be written in the form

$$c_0\gamma_0\frac{\partial T_0}{\partial t} = \frac{1}{S(z)}\frac{\partial}{\partial z}[S(z)\lambda_0\frac{\partial T_0}{\partial z}] + \frac{I^2\rho_0}{S^2(z)}, c_i\gamma_i\frac{\partial T_i}{\partial t} = \Delta T_i + j^2\rho_i \quad i = 1,2,3.$$

The Stefan's conditions on free boundary surfaces are

$$-\lambda_2\frac{\partial T_2}{\partial n} = -\lambda_3\frac{\partial T_3}{\partial n} + L_m\gamma_m\frac{\partial \sigma_m}{\partial t} - \lambda_1\frac{\partial T_1}{\partial n} = -\lambda_2\frac{\partial T_2}{\partial n} + L_b\gamma_2\frac{\partial \sigma_b}{\partial t}.$$

This problem is solved using method of majorant functions [1]. Results of solution are verified by experimental data [2]. It was found that metallic vapor pressure plays very important role at contact bouncing and welding.

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MODELING UNSTABLE STRATIFIED TURBULENT FLOW IN OPEN CHANNELS

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This work considered an unstable stratified turbulent flow in the open channel. Constructed mathematical model allows to simulate the unstable stratified flows and define averages and pulsation characteristics of turbulent flow. Developed the algorithm to solve of the problem and receive results of calculations which well coordinated with the known experiment data.

Unstable stratified turbulent flow is a common type of geophysical flows. The main property of unstable stratified flow is the process of turbulence generation. The basic part of the turbulent energy in stratified flows is generated by the Archimedean buoyancy force. The mechanism of this process is one of the weak-studied problems of atmospheric and ocean science, as it differs from natural convection. In this paper we consider the problem of practical meaning, when the chilled liquid on the surface interacts with the main traffic flow and changes its temperature. In this case, the temperature can not be considered passive, since there is a complex correlation of velocity and temperature.

To study the interaction of velocity and temperature fields, we consider the turbulent flow in the three-dimensional open channel. In order to the problem we use the three-dimensional unsteady Reynolds equation for the motion and the turbulent heat transfer [1].

The system of the equations dares numerically. The splitting method is applied to the decision of the equation of movement on physical parameters where the method of rhythmic steps is applied to a finding components of speed. The equations for temperature dare a method of rhythmic steps by means of the three-step-by-step scheme where on each coordinate implicit difference the equations dares a method of scalar prorace.Vertical speeds is from the indissolubility equation.

The obtained numerical simulation results are compared with experimental data [2]. The results of simulation satisfy to the experimental data. The experiment was conducted in a rectangular tray for different values of Reynolds and Richardson. Unstable stratification was generated by cooling of the free surface, where measurements of the following correlations were made:

$$R_{uv} = \frac{\overline{u_1 u_3}}{-\sqrt{u_1^2}\sqrt{u_3^2}}, \quad R_{ut} = \frac{\overline{u_3 t}}{-\sqrt{u_3^2}\sqrt{t^2}}.$$

Thus, a mathematical model was built that allows simulating the unstable stratified flows to determine average and pulsation characteristics of turbulent flow.

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MATHEMATICAL MODELING INSTALLED THERMO OF THE MECHANICAL PROCESSES IN DEFORMED HARD SUBJECTS AT PRESENCE OF THE LOCAL TEMPERATURE, HEAT FLOW, HEAT EXCHANGE AND HEAT TO INSULATION

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The Carryinging elements of the engines of internal combustion and technological line processing industry work in complex heat and power field. In consequence of which in they appear the process of the heat expansion, which brings and thermo-tense-deformed to condition. For provision of toughness and reliability these element necessary to develop the special method allowing research the law of the sharing the temperature, displacement forming deformation and voltages in volume of the considered body. Herewith appears the corresponding to difficulties. These difficulties is motivated that that field of the sharing the temperature in volume of the under investigation body are described by equation in quotient of the derived parabolic type. In that time field distribution forming displacement, deformation and voltages are described by equation in quotient of the derived hyperbolic type. But in spite of these inconvenience possible to describe the under investigation phenomenas by means of using the laws of the conservation to energy. As a matter of convenience, the interpretations of the proposed methods shall consider the questions of mathematical modeling warm-up and thermo of the mechanical phenomenas in deformed hard body in peg of the limited length residing simultaneous under influence of the local temperature, heat flow, heat exchange and heat insulation. For this first form the expression a function to full energy. Minimizing her(it) on node importance of the temperature is got allowing system of the linear equations, which decision allows to build the field of the sharing the temperature in volume of the under investigation body. Whereupon using founded law of the sharing the temperature is formed function to potential energy springy deformation. Minimizing last on node importances of the displacement is built allowing system of the linear equations, which decision allows to build the field of the distribution, but on he is built field of the distribution all forming deformation and voltages. Here follows to note that proposed algorithm and method allows to take into account all existing heterogeneous border conditions and condition to usages.

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UNSTEADY VIBRATIONS OF THE ROTOR SYSTEM WITH TIME-VARYING MASS

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We consider the problem of unsteady vibrations of flexible rotor with variable mass on elastic supports having a linear characteristic of elasticity, with taking into account of internal and external friction and gyroscopic effect of the system caused by a simultaneous change of mass and angular velocity of the rotor. The rotor shell is motionless.

On the basis of an asymptotic single-frequency method of Bogoliubov-Mitropolsky the solutions are built of resulting systems of differential equations with variable coefficients, describing behavior of various schemes of rotor systems with time-varying masses. The obtained solutions allow to investigate both transient and stationary modes of operation of such systems.

For various schemes of the rotor systems with time-varying mass, it is shown that amplitude of the non-stationary oscillations depends on the "relative velocity" changing of its own and forced frequencies. If "relative velocity" of change of these frequencies is high enough, the system passes quickly the resonance zone and amplitude growth is avoided. Conversely, by small "relative velocity" of change of these frequencies, the system delays in the resonance zone and the amplitude reaches high values. By appropriate choice of velocity of mass changing and hence the own frequency, as well as the angular velocity of the system, it is possible to choose the best mode of passing through the zone of the main resonance of the rotor system with variable mass.

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MATHEMATICAL MODELLING OF THE PROBLEM OF HYDRODYNAMICS BY THE METHOD OF FICTITIOUS DOMAINS

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In this work justification of the fictitious domains method is given. For the first time obtained not improving mark of confergence rate of the solving the from auxiliary problem to solving the original problem at the time when the small parameter strive to zero.

Let's consider the boundary value problem for nonlinear elliptic equations in $\Omega \subset R^3$ with boundary S

$$\Delta v - v^3 = f, \tag{1}$$

$$v|_s = 0. \tag{2}$$

As method of fictitious domains for the continuation of the lower coefficient in the auxiliary area $D \supset \ \supset \Omega$ with boundary S_1, $S_1 \cap S = \emptyset$, solving equation with small parameter

$$\Delta v^\varepsilon - (v^\varepsilon)^3 - \frac{\xi(x)}{\varepsilon} v^\varepsilon = f, \tag{3}$$

$$v^\varepsilon|_{S_1} = 0, \tag{4}$$

where f - continued with zero out of Ω and $\xi(x) = \{ \begin{matrix} 0, \ x \in \Omega \\ 1, \ D_1 = D \backslash \Omega ... \end{matrix}$

Theorem 1. *Let's $f \in W_2^{-1}(D)$. Then there exists a unique weak solution of the problem (3)-(4) that satisfies to the estimate*

$$\begin{gathered} \|v_x^\varepsilon\|^2_{L_2(D_1)} + \|v^\varepsilon\|^4_{L_4(D)} + \tfrac{1}{\varepsilon} \|v^\varepsilon\|^2_{L_2(D_1)} \le C \|f\|^2_{W_2^{-1}(D)}, \\ \|f^\varepsilon\|_{W_2^{-1}(D)} = \sup_{\|\psi\|_{\overset{0}{W}{}_2^1(D)} = 1} (f, \psi)_{L_2(D)}. \end{gathered} \tag{5}$$

Moreover this solution converges to the generalized solution of the problem (1), (2) by $\varepsilon \longrightarrow \infty$.

Definition 1. *Strong solution of the problem (3)-(4) is called function $v^\varepsilon \in \overset{0}{W}{}_2^1(D) \cap W_2^2(D)$, that satisfies to the equation (3) almost everywhere.*

Theorem 2. *Let's $f \in L_2(\Omega)$, $S, S_1 \in C^2$. Then there a stronger solution of the problem (3)-(4) satisfying to the estimete*

$$\|v^\varepsilon\|_{W_2^2(D) \cap \overset{0}{W}{}_2^1(D)} \le C_\varepsilon \ \text{ where } C_\varepsilon \to \infty \ \text{ then } \varepsilon \to \infty. \tag{6}$$

Theorem 3. *Let's $f \in L_2(D)$, S, $S \in C^2$. Then*

$$\|v^\varepsilon - v\|_{L_2(\Omega)} \le C_0 \sqrt{\varepsilon}, \tag{7}$$

where C_0 *-positive constant that does not depend on ε.*

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OPTIMAL CONTROL OF THE INDUCTIVE HEATING PROCESS OF OIL-WELL CASING IN COLD, INTERMEDIATE AND HOT MODES

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Mathematical model of the inductive heating process of oil-well casing is represented as the equation of heat conduction that involved the control action with the boundary conditions and the criteria of heat quality. The performance criterion is energy functional that involves penalty function with penalty parameters governing heat quality during the process [1]. Control action satisfies the constraints according to the physical significance of the process of inductive heating. The optimal control action structure for the internal source heat intensity in cold, intermediate and hot modes is obtained in the paper. It is performed using the necessary condition of optimality for distributed systems [2] and application of optimization method with internal control actions [3]. Hot mode was carefully analyzed in this paper[4].

The practical problem of optimal inductive heating of oil-well casing in hot mode is solved numerically in this work. Numerical computation is carried out by means of integro-interpolation method using Crank-Nikolson scheme [5]. Numerous experiments and analysis allow tracing the dependence of minimized functional on the penalty parameters and get its three-dimensional plots. The analysis prompts to assign penalty parameters which provide optimal heat mode. According to the results of the carried research recommendations are given how to use them in practice.

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GEOINFORMATION SYSTEM IS BASED ON MATHEMATICAL MODEL MICROCLIMATE INDUSTRIAL CITY

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In recent years the problem of interaction between human society and the environment is an increasingly growing interest and the task of evaluating and abating air pollution in cities now bothers many people. Therefore, nowadays one of the most important tasks of science is to forecast changes in ecosystems caused by natural and anthropogenic factors. The first step in solving these problems is the study of pollution emissions of waste industrial enterprises, transport and the influence of anthropogenic heat on urban climate, the next step is to assess the impact of harmful contaminants on the biological environment. In this paper, to solve a mathematical model of the microclimate of an industrial city in computing the transition region from the Cartesian (x, y, z) to the curvilinear coordinate system (ξ, η, ζ). It was created a numerical algorithm for solving the finite - difference equations of the model of the microclimate of the industrial city by the splitting method by physical processes. Relevance of the work is determined by the needing to automate the modeling of air pollution in industrial centers, the establishment of geographic information system for the integrated study of the dynamics of pollutants in the air pool of a certain region, providing opportunities for computational experiments in the light of current information, and the study of various situations within a given situations. There are already developed in the CIS geographic information systems that is solving such problems, but they use only calculation method of emissions of AML-86 and to study the spread of harmful substances using a Gaussian plume model, which does not give the real picture of pollution, because it does not take into account the dynamics of city climate change, typical of the investigated area vortex flows, terrain. On the basis of the considered mathematical model of the microclimate of the industrial city was established GIS system which is implemented and used in the work of an engineer-ecologist in the field of air protection in the Regional Center of Ecological Monitoring of Ust-Kamenogorsk city.

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THE NETWORK MODEL USED IN DESIGNING INFORMATION SYSTEMS

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In designing of information system the stage of work can be presented in the type of network model.

Presenting of stages of work in the type of network model provides strict and obvious description of the structure elaborating information system, multileveled decomposition of the processes on the separate stages and types of work, presenting them on logical organization and ties, separate representation of inlet and outlet of managing information, also implementing methods and means of processes of life cycle software information system.

Main content of projecting technology includes of technological instructions composing of sequence description of technological operations, requirements which one or another operations and descriptions of the very operations would be depend on.

Information technology as one of the other system should meet the following demands: to provide high leveled partitioning of all process of informational implementation of the stage, operation and action; to include all kind of elements necessary for achieving the aims; to have regular character.

Stages, actions and operations of the technical processes could be standardized and unified; it would allow implementing goal-seeking agency on information system effectively.

Implementing technology for projecting of information system based on the structure-functional and object-oriented methods of analyses.

Object-oriented methods play an important role in the system analysis and projecting, based on decomposition in the domain object presenting in the kind of object package interacting by means of message passing.

This approach does not controversy the structure approach, moreover the fragments of structure analysis methodology (basic models are DFD, ERD, STD) is used on object-oriented methods of analyses for modeling the structures and conducts of the very objects.

In the implemented technology could be used combined methods of structure-functional and object-oriented methods of analyses. Depending on concrete requirements to projecting there could be offered the most optimal methods to create information system. The choice of methods are presenting as a pattern (of ready determination).

According to the projecting methodology were choose modeling methods and CASE-means for selected methods.

As a method were defined: IDEF0 - for modeling business processes "As is"; Use Case - for modeling business processes "What is"; DFD - for modeling dataflow "As is" and "What is"; Activite diagram, Sequence diagram - for modeling workflow "What is"; Class diagram - for modeling data "What is".

As CASE-means were selected: AllFusion Process Modeler BPwin 7.0 - for building IDEF0 and DFD models; Ration Rose 2003 - for building UML - chart.

In the implemented technology regardless selected methods of functional modeling IDEF0 USE CASE could be build network model of business-processes carrying out by information system.

Every functional block in IDEF0 or precedent in USE CASE can be presented as a type of sub-network of network model. Network model of business-processes shows connection between functions, and also allows to see the scheme of functional model system for further system decomposition on major components and functions.

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APPLICATION OF NEURAL NETWORKS TO THE PROBLEMS ON FINDING THE FORM IN DYNAMIC PROCESSES

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In this paper we considered the form identification problem in dynamic processes. According to the given situation of different forms of the initial and recent moments, we considered definition of recent situation with respect to another initial form application. Algorithm for applying artificial neural network to training of this problem is given. The prognostication problem related with evolution of the form is investigated and model examples are considered. For investigating the considered problems, at first the type of a neural network should be defined. The type of a neural network is chosen from the problem statement and data used in the training process. At the same time notice that there are no strict methods for chosing the structure of a neural network and in most cases each investigator choses the structure of this network a priori.

Now, the main problem is to perform the training process in the chosen neural network. For that we use the given n number data and n number outputs and define the weight coefficients of a neural network. But as it was noted, there is a difficulty in including (training) the form of a body to a network. For that we can use the following two rules. This method may be applied when the considered domains are convex. The continuous positive-homogeneous support functions $P_1(x), P_2(x), ..., P_n(x),\ x \in R^2$ and $\bar{P}_1(x), \bar{P}_2(x), ..., \bar{P}_n(x),\ x \in R^2$ are found for iput $D_1, D_2, ..., D_n$ and output $\bar{D}_1, \bar{D}_2, ..., \bar{D}_n$ domains, respectively. We try to construct such a neural network that associates the functions $P_1(x), P_2(x), ..., P_n(x),\ x \in R^2$ with the functions $\bar{P}_1(x), \bar{P}_2(x), ..., \bar{P}_n(x),\ x \in R^2$. For that we calculate the values of the functions $P_1(x), P_2(x), ..., P_n(x)$ and $\bar{P}_1(x), \bar{P}_2(x), ..., \bar{P}_n(x)$ at any k number points $x_1, x_2, ..., x_k \in R^2$.

Increasing the number of the k number points $x_1, x_2, ..., x_k \in R^2$, we can minimize the error in the training. To this network we train the values $P(x_1),\ P(x_2), ...,\ P(x_k)$of the support function $P(x)$ of a new domain D at corresponding points. In this case, the constructed neural network will give us the collection $(y_1\,, y_1\,, ...,\ y_k)$ as an output variable. These outputs are the values of the support function $\bar{P}(x)$ of the desired domain $\bar{D}$ at appriate points, i.e. $\bar{P}(x_1) = y_1,\ \bar{P}(x_2) = y_2, ...,\ \bar{P}(x_k) = y_k$.

According to these values, we can find the support function and its appropriate domain by the rule in [1, .12; 3, .32].

When the input $D_1, D_2, ..., D_n$ and output $\bar{D}_1, \bar{D}_2, ..., \bar{D}_n$ domains are not convex or it is difficult to find their appriate support functions, we can use the following rule.

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THE DEPENDENCE OF THE KAON ELEKTROMAGNETIC FORM FACTOR ON THE FACTORIZATION SCALE

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In this work the kaon electromagnetic form factor $F_K(Q^2)$ is calculated by means of the infrared matching scheme [1]. The dependence of the meson's distribution amplitude (DA) $\Phi_M(x, Q^2)$ on the factorization scale Q^2 is taken into account [2]. It is demonstrated that this method as the running coupling constant method allow one to evaluate power-suppressed corrections to $F_M(Q^2)$. In calculations the meson's model DA obtained in the context of QCD sum rules approach is used. We choose the renormalization scale μ_R^2 to be equal to the gluon virtuality

$$\mu_R^2 = xyQ^2, \mu_R^2 = (1-x)(1-y)Q^2 \tag{1}$$

depending on the Feynman diagram under consideration. For the factorization scale μ_F^2 a natural choice is $\mu_F^2 = Q^2$, which eliminates the logarithms of Q^2/μ_F^2.

The kaon DA have the following form

$$\Phi_K\left(x, \mu_F^2\right) = \Phi_K^{asy}(x)\left[\alpha + \gamma(2x-1) + \beta(2x-1)^2 + \delta(2x-1)^3\right], \tag{2}$$

where $\Phi_K^{asy}(x) = \sqrt{3} f_K x(1-x), \quad f_K = 0.112\, GeV$.

By freezing one of variables ($\langle y \rangle = 1/2$), we can express $Q^2 F_M(Q^2)$ in terms of moment integrals $f_p(Q)$ defined as

$$f_p(Q) = \frac{p}{Q^p} \int_0^Q dk k^{p-1} \alpha_S(k^2). \tag{3}$$

For $p \geq 4$ we have to use the running coupling constant method for their calculation by including the dependence of $\Phi_M(x, Q^2)$ on Q^2, in the region of small Q^2, are larger than the ones obtained neglecting this dependence and become smaller in the region of large values of Q^2. The size of these regions depends on the chosen renormalization scale (1 or 2 variables x, y) and on the meson under consideration.

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A NUMERICAL MODEL FOR STUDY OF STENOSES ARTERY AND UNSTEADY BLOOD FLOW

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Arterial stenosis is one of the most prevalent illnesses. Some of the scientists believe that damaging of the inner coating of the artery is the primary reason of stenosis. Moreover, some researchers offer angiography to distingue the irregularities in the blood flow. In numerical simulation of blood flow, the level of the irregular stenosis adds to the complexity of problem. Many researchers have been accomplished, studying the characteristics of blood flow along the artery stenosis. One of the most important parameters in this mathematical model is the blood viscosity and Reynolds number. The shape and irregularities of surface have an effect on the viscous flow. Also the stenosis of the cross-section in cosine-shaped one, leads to the more stenosis of the artery and as a result, causes high flow resistance. In this study, we have presented a mathematical model for observing characteristics of the Newtonian blood flow which passes through a part of the artery. Some of the characteristics of the Newtonian blood flow studied in this article are such as: flow instability, artery coating flexibility, comparison of numerical flow with different geometrical shapes of stenosis, the intensity of different stenosis and its effect on the blood flow characteristics. The aim of this study is to present a mathematical model for observing the characteristics of the blood flow in a part of an arterial which is committed to stenos. The obtained equations are solved with proper boundary and initial conditions with marker and cell (MAC) method. The accuracy and stability of this method are examined. In addition multi- irregular cosine-shaped and single stenosis are compared and investigate the effect of Reynolds number on them is investigated.

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MODELING OF SPECTRAL RADIANT THERMAL SOURCE AND BOUNDARY CONDITIONS FOR NEYMAN PROBLEM

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The analysis of radiant heat transfer for semitransparent media (SM) such as natural ones: (1) clean and polluted (by oil) seawater; (2) planetary and terrestrial glaciers and also artificial materials: (3) thermal barrier coatings (TBC) of aero-space apparatuses, (4) heat-insulating ones (SHIC) of combustion chamber (CC) of diesels, turbines shows how modeling of spectral radiant thermal source $F\lambda$ (at optical parameters $A, b, k^{\lambda}, \sigma^{\lambda}$ of SM in short wave length SWL) and boundary conditions for Neyman problem influences the solution of the inhomogeneous heat conductivity equations (with thermal physical c, ρ, K_T properties) and its physical interpretations. Modeled boundary conditions, determined by 1- absorption on exposed surface of long wavelength LWL radiation ε^{λ}; 2 - SWL one q_0^{IR}, q_0^{UV} for opaque media; 3 - characteristics α_T, T_A of natural or forced convection; 4- thermal losses at melting L_m or evaporisation L_v define formation of volumetric (surface) heating. There is appearance subsurface temperature maximum at intensive penetrating radiation q_0^{λ} through the media - environments interface. Similar processes take place for seawater and glacier at solar radiation, internal walls of CC at one of red-hot soot particle, rockets surface at boundary layer radiation, but there are difference effects. This phenomenon of radiant volumetric heating of SM dependences from their optical heterogeneity. The ice crystal or pores of snow, sintered ceramic micro- and nano-particles of TBC and SHIC, pollutions, oil emulsions, phytoplankton inside subsurface zone of seawater - are considered as ensembles of selective scattering σ^{λ} and absorbing k^{λ} particles in SWL and LWL. Also its could have essential spatial and temporal variability. Then heat conductivity equation for SM in one-dimensional have spectral radiant thermal source for SM layer with thickness H:

$$F^{\lambda} = q_0^{\lambda}(t) \cdot \frac{(1-A) \cdot b_-(x) \cdot}{1} - A^2 \exp(-2 \cdot b_-(x) \cdot H) \times$$

$$\times \left\{ e^{-b_-(x) \cdot x} - A \cdot e^{\cdot b_-(x) \cdot (\, x - 2 \cdot H)} + \frac{\tau_{se}(H) \cdot R_{sub}}{1 - r_{se}(H) \cdot R_{sub}} \cdot [e^{-b_-(x) \cdot (H-x)} - A e^{b\,(x) \cdot (H - x - 2 \cdot H}] \right\},$$

but for opaque media $\lambda = 0$. Temperature distributions $T(x,t)$ depend also from radiant flux transmitted by top SM layer r_{se}, τ se and reflected by substrate R_{sub} (soil bed of a glacier or an ocean; a metal wall of the CC or space rockets. Boundary conditions at front and back sides of SM are:

$$-K_T \cdot \frac{\partial T_{se}(x=0,t)}{\partial x} = \alpha_T \cdot (T_A - T_{se}(0,t)) + c_0(\varepsilon_A(t) \cdot T_A^(t)^4 -$$

$$-\varepsilon_W \cdot T_{se}(0,\ t)^4) + q_0^{IR} + q_0^{UV} - \rho\,(L_v\, x_v' \,+\, L_m\, x_m'),$$

$$-\frac{\partial T_{sub}}{\partial x}(x = H^+) = -\frac{\partial T_{se}}{\partial x}(x = H^-) = -\,q_0(t) \cdot \frac{\tau_{se}(H) \cdot (1 - R_{sub})}{1 - R_{sub} \cdot r_{se}(H)}.$$

The numerical simulations make possibility to regulate and predict thermal regimes of these SM by changing optical properties. For example, determination of subsurface melting of glaciers allows to forecast snowslip. In 2010 appearance of oil pollution (as scattering TBC) on the Atlantic Ocean surface would lead to cardinal changes in thermal regime of the ocean-atmosphere system (Zangari, 2010; Krass, Merzlikin, 2011). Application of semitransparent TBC (SHIC) for aero-space and auto car equipments is an opportunity to decrease overheating of these coatings, to increase efficiency and to improve ecological characteristics.

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ON THE 3-D PROBLEM OF MULTIPHASE FLUID FILTRATION IN THE OIL RESERVOIR

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With the rapid development of the petroleum industry, with an increase in the number of marginal fields, exhaustion recovery raises the question of the qualitative analysis of the effectiveness of oil facilities, using modern methods of mathematical tools for modeling the formation processes for the analysis of oil production [1-3]. Today, the possibility of common types of flooding is already exhausted and, therefore, apply additional ways to increase the effective impact on the oil reservoirs. For enhanced oil recovery using secondary, tertiary methods to improve the filtration-capacitive parameters of the reservoir. These methods include flooding with surface-active substances, steam-calorific method, gas injection, etc. Mentioned methods to improve the efficiency of development based on directional impact on the adjustable parameters of the process to ensure the best physical conditions of oil displacement from the natural reservoir. In turn, the reality is far impose conditions necessary to have an extensive bank of advanced mathematical models of multiphase flow in mining to identify ways to improve them and reduce capital and financial costs of their development.

In this paper the three-dimensional non-isothermal multiphase fluid filtration problem through a porous medium in the system of wells is considered. Simulated field consists of water, oil and gas zone and the network injection and production wells, respectively. The development of such deposits, taking into account the gas phase is very difficult.

Based on the theory of parallel programming computational algorithms for solving this problem were offered. Initial data and thermo physical parameters of the reservoir are taken from geological and production data, filtration-capacitive parameters of the oil fields of Western Kazakhstan. The results of calculations described in this paper can clarify the dynamics of development of these deposits from the use of a secondary method. Calculations reveal the regularities of the main technological parameters of development of oil and gas, as well as allow us to find the scheme of effective technologies to enhance and improve the pace of development of these fields.

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ON PROBLEMS OF LAND INVENTORY PERFORMING ON REMOTE SENSING DATA

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The materials of the report contains the experience of use of geoinformation systems of the National Company "Kazakhstan Gharysh Sapary". Experience in application of methods of Earth remote sensing in the practical problems of agriculture has a long history and currently this experience extends dynamically due to the new spacecrafts, techniques, methods and algorithms. However experience in solving these problems in Kazakhstan is insignificant - geoinformation technologies in the regional management underutilized. On the basis of the report - review of systems that provide the invert means and monitoring of agricultural areas on the basis of remote sensing data. The ultimate goal of systems of this type is the control of crop yield forecasting in the early stages and help farmers. In the report the problem is the development and implementation of a part of GIS agricultural sector at the regional level, designed for the needs of the Ministry of Agriculture. A method of pattern recognition of crops is represented with the use of interpreting functional.

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MATHEMATICAL MODELING AND NUMERICAL INVESTIGATION OF THE STRESS ANALYSIS IN THE RECTANGULAR THICK PLATE FROM COMPOSITE MATERIAL WITH SPATIALLY PERIODICALLY CURVED STRUCTURE

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In this work the stress distribution in a thick plate fabricated from the composite material with spatially periodically curved structure is studied. It is assumed that all the adges of the plate are simply supported and uniformly distributed normal forces act on its upper face.

Mathematical modeling of the considered boundary-value problem is formulated within the framework of continuum theory of Akbarov and Guz [1] and three dimensional exact equations of elasticity theory. We have system of partial differential equation with variable coefficients. Solution to the considered problem is made numerically by using 3D Finite Element Method. After the numerical solution to the problem the effects of some geometrical and material parameters on the stresses distribution in the plate are determined. All algorithm and programmed required for the numerical solution are made by authors.

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MATHEMATICAL MODELING OF THE INTERRUPTION DETONATION WAVE IN GAS SUSPENSIONS UNITARY FUEL

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In the proposed mathematical model of two-dimensional axisymmetric unsteady motion of the dispersed-phase two-component reactive mixture of gas, particles of unitary fuel and inert particles, numerical study of the interruption of detonation waves in gas suspensions of unitary fuel layers of homogeneous and inhomogeneous monodisperse inert dispersed particles.

As an example, numerically solved the following problem: the initial time $t = 0$ from the inert gas $(0 \leq z \leq z_f)$ in the direction of the clouds monopropellant $(z_f < z < \infty)$, enclosing a layer of homogeneous and inhomogeneous monodisperse inert particles $(z_L \leq z \leq z_R,\ z_L > z_f)$ shock wave moves with a triangular profile. Required to examine the influence of the distribution layer of inert particles to interrupt detonation waves in gas suspensions of unitary fuel.

The calculations were performed for mixtures of components: air, gaseous products of combustion of gunpowder, gunpowder particles and particles of quartz sand.

As a result of the numerical investigation found that, at a fixed total weight of the suspension is better interrupt effects of detonation waves in gas suspensions of unitary fuel layers are inhomogeneous monodisperse gas suspension with a positive gradient of the initial concentration of particles in comparison with the case of layers of gas suspensions with a negative or zero gradient of particle concentration. Obtained an integral dependence of the minimum initial relative mass content of inert particles from their initial diameter in a gas suspension of unitary fuel. It is shown, that for fixed particle size of inert and reactive phase dependence of the minimum relative weight content of inert particles required to suppress detonation process increases linearly with increasing the relative weight content of particles of the unitary fuel.

Comparison with experimental results showed the adequacy of the proposed model with the physical processes.

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MATHEMATICAL MODELS AND METHODS OF THE FINANCE INVESTMENT ANALYSIS

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A new view on formation of security portfolio in the conditions of uncertainty is suggested. The most recent methodology of investment is based on dynamics of development. The complete description of various statements and methods of problem solving about an optimum portfolio of securities is quite given. A duality in a problem of formation a new type portfolio is filled with economic content-richness.

Security portfolio means column vector $X = (x_i)$, $i = \overline{1,n}$, in which x_i is a share of cost of i-th security in the portfolio value. In this case $\sum\limits_{i=1}^{n} x_i = 1$, i.e. the whole portfolio value is taken as a unit. Let us introduce column vector I of dimensionality n, all components of which constitute 1, in this case the last clause will be written as $I^T X = 1$, where means transposition. Yield of portfolio X is a variate $\xi_X = \Xi^T X = \sum\limits_{i=1}^{n} x_i\xi_i$. Portfolio efficiency, i.e. its average yield, is $e_X = \sum\limits_{i=1}^{n} e_i x_i = E^T X$. Portfolio dispersion is $\nu_X = \sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} x_i \nu_{ij} x_j = X^T V X$, where V is a matrix of mutual yield variances. Quadratic mean of portfolio yield $\sigma X = \sqrt{v_X}$ is identified with portfolio risk and is sometimes designated r_X.

On efficient market the portfolio is formed with consideration of predictions actual for the moment of forming, and in any case this portfolio should have quite high. A competitor – the person who makes the decision – for preparation for coming events considers several variants $j = \overline{1,m}$. It is predicted that the market will be in one of these m conditions – variants, and we do not know the exact variant. Let j-th situation is characterized with vector Ξ_j of random quantities (ξ_{ij}) – yield of the same securities $(j = \overline{1,m}, i = \overline{1,n})$, but already in new conditions. Every variant reflects changes of efficiencies $E_1, E_2, \ldots, E_m$ predicted on financial market, and $E_j = (e_{ij})(j = \overline{1,m}, i = \overline{1,n})$.

We want the security portfolio X to have efficiency (average yield expected) not less that set level θ; and we would like to make set level θ as high as possible.

Let investor has non-risk assets with non-risk yield rate α and risk assets with average expected yield rate β, that can possess some value $\beta, j = 1, \ldots, m$ in future period, the risk r of these assets depend on β by the formula $r = \alpha + k\beta, k > 0$.

It is necessary to solve the question of forming the portfolio for the future period and to limit the risk of future assets and maximize their average expected yield. Exact statement is reflected by the following problem.

The new indicator of stock market - the *Ns*-factor of securities - is worked in. The meaning of security *Ns*-factor: *when capital K for the most rapid growth of capital should buy the amount of $K_s = sK$ of given security.*

Theorem 1. *$O(f)$ is a consistent estimate of Ns-factor.*

Theorem 2. *There is such $0 < z < t$ when $f(z) < 0$. This is because $f(s) \underset{s \to t}{\to} -\infty$.*

Theorem 3. $\Pr\left(\left|O\left(R, f\right) - O\left(f\right)\right| < \varepsilon\right) \underset{|R| \to \infty}{\to} 1$ *for any $\varepsilon > 0$.*

The considered model allows ample opportunities for the analysis of efficiency of enterprise activity directions in modern conditions and it is a component of problem solving for the active investor.

New statements of abstract problems with applied illustrations to economic problems of decision-making in the conditions of uncertainty are analyzed. Constructive algorithms of a finding of the best approximation to optimum decisions by means of linear programming are offered.

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THE FOREIGN INVESTMENT IN THE SYSTEM OF INDICATORS OF SUSTAINABILITY OF SOCIO- ECONOMIC DEVELOPMENT IN AZERBAIJAN REPUBLIC

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Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs. The goal of environmental sustainability is minimizing environmental degradation. Sustainable development does not focus solely on environmental issues. The classification of parameters offered by the commission of the United Nations on preservation of the environment is applied to the analysis of stability of social - economic development. According to this classification, 130 parameters considered more are divided on economic, social, ecological indicators. In article the role of foreign investments in the system of some indicators of sustainable development in Azerbaijan Republic is analyzed by the method of main components. This method is a statistical data reduction technique used to explain variability among observed random variables in terms of fewer unobserved random variables called factors. For period 2000-2009 year we considered 23 indicators and used program SPSS for our calculating. Application of a method of main component has allowed reducing set of the described indicators to three factors. The received results have shown, that the foreign investments have the largest factor loading. Then the regression equation between volume of foreign investments Y and these factors $(F1, F2, F3)$ have been constructed.

$$Y = 3340,141 + 1572,316F1 + 813,219F2 + 226,327F3,\ R^2 = 0,969,$$

$$DW = 1,843F = 41,172.\ t_0 = 22,238,\ t_1 = 9,792,\ t_2 = 5,064,\ t_3 = 1,409.$$

Econometric model shows that the first factor have more influence on the foreign investment. Increasing the first factor by1 unit leads to increasing of the foreign investment by $1572,3$ units. R squared coefficient is dood, show that 96% variation of the foreign investment is explained by variation of the factors. The t- statistic of the third factor shows weak influence on the volume of foreign investment. So $t_3 = 1,409 < t_{kr} = 2,776 (n - k - 1 = 4$ degree of freedom) and it means that factor $F3$ is statistically insignificant at the 5% level. T- test for other factor shows that they are significant. Durbin Watson (DW) coefficient is closed to 2. It means there is not autocorrelation. F-statistic also shows that econometric model is significant. The result of the researching describes the place and role foreign investments in the system of indicators of sustainability of sosio- economic development in Azerbaijan Republic. Using information about factors it is possible to forecast the tendencies of their development. Possible to forecast the tendencies of their development.

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THE FOAM DRAINAGE EQUATION: A REACTION-DIFFUSION PROBLEM WITH A FREE BOUNDARY

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The equation modeling the evolution of a foam (a complex porous medium consisting of a set of gas bubbles surrounded by liquid films) is solved numerically. We are interested in the process of drainage of water into edges of the lattice (Plateau borders) in which the foam is contained in a vertical column and is broken. The model considered is described by the reaction-diffusion problem with a free boundary. As a numerical method, namely the averaging in time and forward differences in space (the Crank-Nicolson scheme), in combination with Newtons method, is proposed for solving the governing equations and the results are presented graphically. The solution of the Burgerss equation is considered as a special case. The numerical solution of the foam drainage equation has been studied by many researchers, see [1-5] and [7]. We are interested in the case where the foam breaks due to the rupture of thin films during the drying process, i.e. the model considered is described by the reaction-diffusion problem with a free boundary, see [2], where they discuss the theoretical aspects of the convergence and well-posedness of the foam drainage equation. The following foam drainage equation is derived from assumptions which best apply to dry foam. The contribution of the films to drainage is not included in this model. On the basis of simple considerations of continuity and pressure balance, as well as the treatment of dissipation in a manner analogous to Darcys law in the theory of porous media, if we follow the steps given in [6], in dimensionless variables, and the foam drainage equation is obtained:

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} - \varepsilon \frac{\partial}{\partial x}\left(\sqrt{u}\frac{\partial u}{\partial x}\right) = 0, \tag{1}$$

with the boundary and initial conditions

$$\begin{aligned} &u(t,\gamma(t)) = \beta,\ \dot{\gamma}(t) = \beta - \tfrac{\varepsilon}{\sqrt{\beta}}\left[\tfrac{\partial u(t,x)}{\partial x}\right]_{x=\gamma(t)}, \\ &u(0,x) = u_0(x),\ x \in (\gamma(t), +\infty), \end{aligned} \tag{2}$$

where $0 < \beta < 1$ and $u_0(x) \geq \beta,\ \forall x \in (\gamma(0), +\infty)$ and $u(t,x)$ is the dimensionless cross section area of Plateau borders at time t and level x, i.e. $u(t,x)$ is a local measure of the foams liquid.

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ABOUT NEW FORM OF EQUATIONS IN THE PROBLEM OF A SOLID BODY ROTATIONAL MOTION[1]

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The problem of rotational motion of a solid body with fixed point is considered. The solid body is in gravitational field of forces. As is known [1] this motion is described by nonlinear ordinary differential equations of Euler. This system has three well-known first integrals: geometrical integral, energy integral and area integral. In paper with a help of original linear change of variables [2] the initial system is reduced to the normal type, i.e. to the system of the non-dimensional equations with explicitly expressed linear part:

$$\dot{\mathbf{u}} = \widetilde{A}\mathbf{u} + \mathbf{F}(\mathbf{u}), \tag{1}$$

where $\mathbf{u} = \{u_1, ..., u_6\}$ is the column vector of new variables; $\widetilde{A}$ is the constant square matrix of the 6^{th} order; $\mathbf{F}(\mathbf{u})$ is the 6^{th} order vector function, consisting of nonlinear members of equations. In new variables the known first integrals of the system take another form. Superposition of two of them gives us the new integral of the norm type:

$$u_1^2 + u_2^2 + u_3^2 + u_4^2 + u_5^2 + u_6^2 = C_1 = const. \tag{2}$$

As any first integral expression (2) can be obtained from the motion equations too. This integral provides the convergence in sense of norm of solution of the original system to solution of the transformed system. New form of equations is very convenient for studying of the motion stability. Solution of the transformed system was accepted as the non-perturbed motion and was investigated on stability on first approximation by Lyapunov.

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MATHEMATICAL MODELS OF SYNTACTICAL RULES OF THE KAZAKH LANGUAGE SUBJECT TO SEMANTICS OF PARTS OF THE SENTENCE

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There are two kinds of sentences in Kazakh: simple and composite. A composite sentence consists of a few simple sentences. For this reason, here we confine ourselves to mathematical modeling of syntactical rules of simple sentences only.

In Kazakh, a simple sentence has one or more parts (subject, predicate, object, attribute, adverbial modifier). Each part has its definite place [1]. There are 20 variants of simple sentence structure in all: sentence with only one part – 1 variant, two parts – 1, three parts – 2, four parts – 9, five parts – 7.

We consider principles of construction of simple sentence and ways of collocation of its parts subject to their morphological regularities and semantics. As a rule, there are used formal grammars and finite automata to generate (synthesis) and recognize (analysis) sentences of any natural language, respectively. However, they do not allow of describing sentence structure subject to semantics of their parts. Thus they are not sufficient for automatic processing of a natural language (machine translation or speech technology).

In the present work we suggest usage of semantic neural networks to generate Kazakh sentences subject to grammatical categories, such as part of speech, animate/inanimate object, number, voice, form, etc.

It is known that semantic neural network is equivalent to finite cellular automaton. Therefore the problem of syntactic analysis boils down to alteration of states of a sub-automaton that corresponds with the dictionary entry and its features [2].

Software implementation of the semantic neural network and the corresponding finite automaton have made it possible to automate analysis and synthesis of Kazakh sentences. This software will be used to develop new information technologies, including speech technologies for the Kazakh language.

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CONSTRUCTION OF THE MATHEMATICAL MODEL ON THE OLLENDORFF METHOD FOR FILTRATION OF WEAKLY COMPRESSIBLE CHEMICAL COMPOUND IN THE POROUS HETEROGENEOUS 3D MEDIUM[1]

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In this work the filtering process is considered, which occurs in a non-homogeneous porous curvilinear truncated pyramid with isotropic and spatially periodic (not compulsory with the constant period) structure. The general filtering model of fluids in non-homogeneous isotropic mediums with periodically changing permeability contains equations of filtering, where functional coefficients are rapidly oscillating functions and, generally speaking, are piecewise continuous functions. Therefore, finding of analytical solution of the corresponding initial-boundary problems for such equations is problematic (for instance, see fundamental monograph [1], as well as [2] and [3]). In this work a mathematical model is constructed, which describes basic filtering properties of isotropic mediums with a multiplicative coefficient of permeability $k(x) = \prod\limits_{i=1}^{3} K^{\{i\}}(\alpha_i(x))$, where $x = (x_1,\, x_2,\, x_3)$; auxiliary arguments-functions $\alpha_i = \alpha_i(x)$ $\left(i = \overline{1,\,3}\right)$ set the geometry of the periodic structure of the porous environment, and periods as to α_i $\left(i = \overline{1,\,3}\right)$ are the dimensions of the repeating structure elements – rectangular parallelepipeds, which altogether form a non-homogeneous porous curvilinear truncated pyramid. To construct required mathematical model the following assumptions are made: 1) level surfaces $\alpha_i(x) = \alpha_{0,\,i} \equiv const.$ $\left(i = \overline{1,\,3}\right)$ in the non-homogeneous porous curvilinear truncated pyramid form a triorthogonal system of surfaces; 2) anywhere in the filtering domain the length $L_{\omega,\,T_{per.}(\omega)} \overset{def}{\equiv} \int\limits_{\omega(xz)}^{\omega(x)+T_{per.}(\omega(x))} \left| \frac{\partial}{\partial\omega(x)} \sum\limits_{j=1}^{3} x_j(\alpha) \cdot \vec{i}_j \right| d\omega$ of arc of the $\omega = \left\{\alpha_i,\ i = \overline{1,\,3}\right\}$-th coordinate line, which corresponds to the period $T_{per.}(\omega)$, is infinitesimally small in comparison to its characteristic size L; 3) in a curvilinear parallelepiped, which is limited by coordinate planes $\alpha_i = \alpha_{0,\,i}$ and $\alpha_i = \alpha_{0,\,i} + T_{per.}(\alpha_i(x))$ $\left(i = \overline{1,\,3}\right)$, arc ends of length $L_{\omega,\,T_{per.}(\omega)}$ quite insignificantly deviate from the ends of the corresponding tangent lines. Such assumptions make it possible to divide the non-homogeneous porous curvilinear truncated pyramid into elementary approximately-averaged parallelepipeds, and this makes it possible to consider the elementary curvilinear parallelepiped as elementary rectangular parallelepiped with OA_i $\left(i = \overline{1,3}\right)$, lengths are equal to $L_{\omega,\,T_{per.}(\omega)}$, and such elementary rectangular parallelepiped is called approximately-averaged parallelepiped. Further, taking into account the made assumptions, Ollendorff method (see [4]) is applied to the studied filtering problem in order to construct a filtering model (consisting of fluid filtering equations and the corresponding initial-boundary conditions) in the porous curvilinear layered environment with periodically changing permeability in respect to space, the coefficient of permeability is multiplicative.

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MATHEMATICAL MODELS AND METHODS IN SPEECH UNDERSTANDING SYSTEMS[1]

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One of development parameters of information technologies is the achievement intellectual processing of an information. As the basic communicative means is a speech in society, automatic speech recognition and determining its meaning are urgent problems of the modern science. Speech understanding is the speech processing that involves the mapping of the acoustic signal, usually derived from some form of speech recognition system, to some form of abstract meaning of the speech.

The first stage of implementation of speech understanding is speech recognition process. Automatic speech recognition by computer is a process where speech signals are automatically converted into the corresponding sequence of words in text. The following mathematical methods are applied for speech recognition:

Speech processing. Speech signal is converted to the digital form, the digital signal cleaned from noise by passing through a special high pass filter, divided into frames and windowing in this stage. Different mathematical methods of spectral analysis are used for the carrying out these operations.

Calculation of speech features. For calculation of speech features is constructed speech generation model. The parameters of the vocal tract filter which necessary in speech recognition are calculated by the spectral analysis. The Fourier or Wavelet transformations, Levinson-Durbin Algorithm, Mel filtering and so on. operations are carried out in this stage.

Training and recognition of the speech recognition system. There are different mathematical models for training and recognition of the speech features. Most widely using models are Artificial neuron networks and Hidden Markov models. Artificial Neural Networks trained by Back Propagation method. But for training Hidden Markov Model used Baum-Welch algorithm.

The next stages in automatic speech understanding are syntactic, semantic and progmatic analysis.

Syntactic component. Syntax analysis is carried out after speech recognition for selected object in speech understanding system. Lexical and grammatical analysis of the language are applied for checking spelling of the words on the basis of vocabulary. Edit Distance method is used for the lexical analysis. Purpose of the syntactic analysis to find subject, predicate, verbal and object in sentences. The role of syntactic analysis is to check correctness of the syntax rules in sentences.

Semantic component. The Context-free grammar used for determining the meaning of the sentences and words in the speech understanding system. There is investigated the diapason of the words for selected field.

Pragmatic component. One of wide applicable fields of the speech understanding is spoken dialogue systems. The systems have been defined as computer systems with which humans interact on a turn-by-turn basis are called spoken dialogue systems. The main purpose of a spoken dialogue system is to provide an interface between a user and a computer-based application. System can be answered user's questions in different forms during dialogue process. The ultimate goal of pragmatic analysis to shortening dialogue by intellectual answering. The System has to choose so optimal strategies, that suggested assumption doesn't let user to repeat information and able to get the necessary information. There are used different heuristic methods.

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FLOW OF RELAXING COMPRESSIBLE FLUIDS IN A DEFORMABLE POROUS MEDIUM

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At resent time several experimental and theoretical investigations have been carried out with gased fluids in pre-transitional conditions (in the field of saturation pressure). It is shown that flow characteristics in such systems are determined by the content of small gas-bubbles [1, 2].

In the present paper flow of compressible gased relaxing fluid in a deformable porous medium is considered.

The equation of state of such systems is described as [3]:

$$P\left(\rho\right)=\frac{\alpha_{20}}{\alpha_{10}}\frac{P_0\rho}{\rho_{10}-\rho}, \tag{1}$$

where α_{10}, α_{20} – volume contents of liquid and gas phases P_0 – initial pressure; ρ_{10}^0, ρ – real and related densities the liquid phase.

Non-steady filtration equation with pressure relaxation is derived in the following form

$$\frac{\partial P}{\partial t}=\chi\sigma_1\left(P\right)\frac{\partial P}{\partial x}\left(\frac{\partial P}{\partial x}+\lambda_P\frac{\partial^2 P}{\partial x\partial t}\right)+\chi\sigma_2\left(P\right)\left(\frac{\partial^2 P}{\partial x^2}+\lambda_P\frac{\partial^3 P}{\partial x^2\partial t}\right), \tag{2}$$

where P – current pressure, x – linear coordinate, t – time, λ_P – relaxation time of the pressure gradient, $\chi=\frac{k}{\mu\alpha\beta_C}$, $\alpha=\frac{\alpha_{10}}{\alpha_{20}}$, k – permeability, μ – viscosity of liquid, β_C – coefficient of change in porosity, m_0 – initial (at $P=P_0=\text{const}$) porosity,

$$\sigma_1\left(P\right)=\frac{\alpha P_0}{P^2/\alpha+2P_0P+P_0\left(m_0/\beta_C-P_0\right)},\quad \sigma_2\left(P\right)=\frac{P^2+\alpha P_0P}{P^2/\alpha+2P_0P+P_0\left(m_0/\beta_C-P_0\right)}.$$

Equation (2) with (1) is numerically solved for semi-infinite beds with the following initial and boundary conditions

$$P(x,0)=P_0,\quad P(0,t)=P_C,\quad P\left(\infty,t\right)=P_0,\quad P_C=\text{const},(P_C<P_0). \tag{3}$$

Numerical experiments show that with increasing of relaxation time λ_P pressure decreasing is intensified.

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TEXT NORMALIZATION FOR AZERBAIJAN TTS SYSTEM[1]

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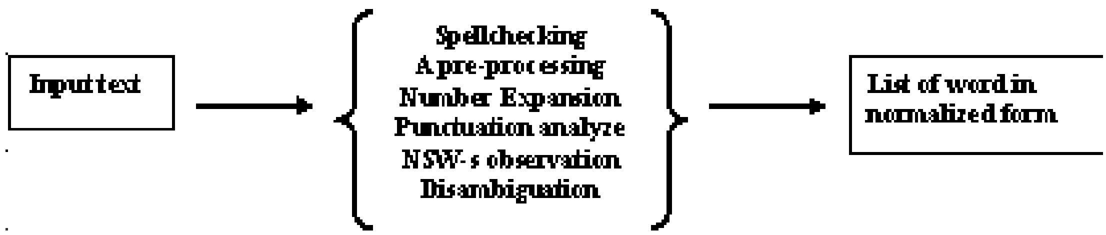
All written languages have special constructs that require a conversion of the written form (orthographic form) into the spoken form. Text normalization is an automated process that performs this conversion.

Text normalization - is a text transformation that makes the text consistent with some pre-defined rules. Text normalization module takes the raw text and converts entities such as numbers and abbreviations into their written-out word equivalents. Output is normalized text containing common orthographic transcription suitable for subsequent phonetic conversion.

The general structure of normalizer explained in Figure 1. Apparently from the figure this module has several stages [1].



Spellchecking of the text - The sounded text can be entered in any form. The size or font type is of no importance. The main requirement is that the text must be in Azerbaijani language.

A pre-processing module - a pre-processing module organizes the input sentences into manageable lists of words. First, text normalization isolates words in the text. For the most part this is as trivial as looking for a sequence of alphabetic characters, allowing for an occasional apostrophe and hyphen.

Number Expansion - there are many types of numeric representations, such as date, time, currency, identification number, range, percentage etc. Sentence analysis is performed to identify the type of the numbers. If the type is identified, the numbers can be normalized rules.

Punctuation analyze - normal writing, sentence boundaries are often signaled by terminal punctuation from the set: full stop, exclamation mark, question mark or comma {. ! ? ,} followed by white spaces. This block creates information base for the Voicing speech signal.

NSW-s observation - the aim of this stage is converting None Standard Words (NSWs) into their standard word pronunciations.

Disambiguation - in some system disambiguation module is generally handled by hand-crafted context-dependent rules [2].

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MATHEMATICAL MODELS OF DESIGNING AUTOMATIC TRANSCRIPTION

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Automatic transcription is the transfer program of any orthographic text into transcriptional record and vice versa on the basis of linguistic rules. Transcription of words into transcript record is present in all systems of natural language processing, such as translators, systems of recognition or speech synthesis. However depending on a system special-purpose designation this or that transcription model is chosen. So, for example, for systems of recognition of speech the simplified transcription can be used whereas for systems of speech synthesis it is necessary to use the most detailed transcription containing acoustic characteristics of sounds. Let's introduce the following designations for vowel sounds: G - set of vowel sounds of the Kazakh language, G_1 - set of hard vowel sounds, G_2 - set of soft vowel sounds, G_3 - set of not labial vowel sounds, G_4 - set of labial vowel sounds, G_5 - set of back tongue vowel sounds, G_6 - set of front tongue vowel sounds, G_7 - set of open vowel sounds, G_8 - set of the closed vowel sounds, G_9 - middle vowel sounds in pair distinguishable signs of sounds in each group, i.e. Also, we will introduce designations for consonants: S - set of consonants of the Kazakh language, S_1 - set of voiceless consonants, S_2 - set of sonorous consonants, S_3 - set sonorous consonants, S_4 - set of labial consonants, S_5 - set of front tongue consonants, S_6 - set of middle tongue, S_7 - set of back tongue consonants, S_8 - set occlusive consonants, S_9 - set round fricative consonants, S_{10} - set flat fricative consonants, S_{11} - set of shivering consonants, S_{12} - set of lateral consonants, i.e. Let A - the alphabet of the Kazakh language, i.e.

$$A = G \bigcup S \tag{1}$$

then the set of all. Now we will start the formalization of phonologic rules of the Kazakh language: Connection of two voiceless consonants. On a connection of two voiceless consonants in one word or in a case when one word ends to a voiceless consonant, and other word begins with a deaf consonant, both of these consonants remain voiceless:

$$\frac{\alpha \subseteq \gamma, \alpha = \alpha_1 x, \alpha_1 \neq \epsilon, x \in S_1, \beta \subseteq \gamma, \beta = y\beta_1, y \in S_1}{\gamma = (\alpha_1 x)(y\beta_1).} \tag{2}$$

The formal phonologic rules of the Kazakh language received thus will allow to construct transcription which automatically builds transcript word record which is the important element at system elaboration of the Kazakh speech recognition.

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MODELING RADIATION DEFECTS IN SEMICONDUCTORS IRRADIATED BY IONS

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The equation (1) is used for calculation the concentration of radiation defects at ion irradiation:

$$C_k(E_0, h) = \frac{E_d E_{2\max} - E_c}{E_c E_{2\max} - E_d} \sum_{n=n_0}^{n_1} \int\limits_{h-k\lambda_2}^{h} \psi_n(h') \exp\left(-\frac{h-h'}{\lambda_2}\right) \frac{dh'}{\lambda_1(h')\lambda_2}, \qquad (1)$$

where E_d is an average energy of the displacement, E_0 is the initial energy of the particle, E_c is the threshold energy, $E_{2\max}$ is the maximum energy, passing to atom and corresponding to head - on collision, $\psi_n(h')$ is the cascade and probabilistic function in modified types (1).

It is necessary to find the range of definition of the result at the calculations the concentration of radiation defects at ion irradiation. Finding this range of definition could allow us to reveal the following regularities:

1. With decreasing initial energy of the primary particle the range of the result is displaced to right and the values of concentration are increasing.

2. At the high atomic weight of collision particle and small target the time of a count is very strongly increasing and reaches of several hours.

3. In dependence from the depth of penetration the initial and final values of the number of interactions are increasing, the interval of the range of definition of the result (n_0 n_1) is also increasing and displaced to right.

4. With increasing atomic number of collision particle, the interval of the range of definition of the result is considerably displaced to right and increasing, the value of concentration at the point of the maximum and the concentrations values are strongly increasing. The results of calculations are presented on the figure 1.

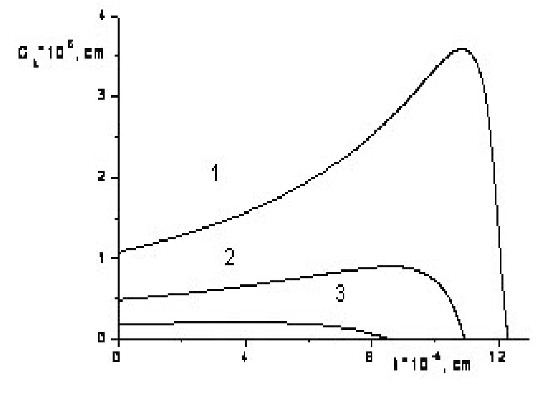

Figure 1. The dependence concentration of the radiation from the depht at irradiation silicon by silver ions at: $E_0 = 800\ keV,\ E_c = 50\ keV (1),\ 100\ keV\ (2),\ 200\, keV\ (3).$

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APPLICATION OF GEORADAR LOZA-B FOR THE STUDY OF CIRIC-RABAT AND MOUNDS IN THE KYZYL-ORDA REGION.

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The object of study - the city of Upland Ciric-Rabat and mounds in the Kyzyl-Orda region.
Problem – identifying the internal structure of the structure of Ciric-Rabat and mounds in Kyzylorda region. Comparison of the geophysical data and a priori information can reduce the work on archaeological excavations.

Justification of the method. The property is located in the wilderness, away from populated areas. Formulation of the problem requires the use of non-destructive, fast method, which gives an idea of the location of subsurface objects with an accuracy of $\pm$ 0.5 meters in plan. The method should be economically feasible, giving significant savings compared with available archaeologists by visual observation with the subsequent opening of the archaeological sites.
In this regard, was chosen as high-frequency pulse electromagnetic ground penetrating radar which allows to examine the structure of the soil within a depth of 0 to 15 meters. Georadar "Loza-B" allows us to study the geological structure of the soil to provide anomalous areas waterlogged, loosening, subsidence, karst, erosion, etc.

Technique. As a methodological procedure was chosen areal probe on a grid with increments of 0.5 meters. This observing system is adequate for the task. Total measurements were made at 8 stations. The overall plan of the measured sites 90 x 40 meters.
This sort of problem related to inverse. Georadiolocation inverse problem - the restoration of the structure of the underground environment of georadar data - a general statement, like all inverse problems, is incorrect. The possibility of its solution is determined by the specific wording of the problem, which includes a priori information about the environment, as well as the method of removal Georadars data.
Under the inverse problem we mean a further recovery in the stratified case geoelectric section: the permittivity and conductivity with the help of Georadars data. We used a mathematical model of direct and inverse of the reduced and studied in [1].To solve the inverse problem using optimization method, described in detail in the monograph [2].

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MATHEMATICAL MODEL OF DISTRIBUTION OF RESOURCES

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Problem 1. Let's network is set by the list of operation and technological communications between them. The ordered sequence Z is set technological, according to which operation of a network start to use resources. The problem will consist in definition of minimal time of performance of operation of a network, and also times of the beginning, the termination and intensity performance of all of operation of a network provided that statement of operation on service is carried out in conformity with technological allowable sequence Z at restrictions on resources and sequence of performance of operation. Statement of operation on performance in conformity with technological allowable sequence Z means, that the situation at which performance at present time of some operation blocks statement on performance of the operation, occupying in Z higher position is inadmissible. Let's assume, that technological the allowable sequence Z unequivocally defines the order of statement of operation on service [1]. Operation uses resources during the performance and returns back after the end. The resources which have been not used during some moment of time, do not increase cash quantity of resources during the subsequent moments of time, they do not accumulate. Intensity of performance of operation in given moment of time we shall name speed of increase of a point of a condition of operation during this moment of time. Let's designate u_i a condition of managing objects for operation i, $i \in I$. Management's, in a problem of calculation of performance of a complex of operation of a network, are intensity of the operation, appointed for performance at present time. We shall designate u a vector of intensity of performance of a complex of operation at present time $u = (u_1, u_2, ..., u_n)$, where u_i - intensity of performance of operation i at present time. The condition of operation i depends on a condition of operation v, $v \in I_i^+ \subset I$, management u_i, and also is defined by value of the parameters. Process of performance of a complex of operation of a network is controlled. There is certain freedom in a choice of intensity of performance of operation of a network. It is necessary to execute a complex of operation of a network for minimal time.

Problem 2. We shall make now changes to an above mentioned problem 1 (and we shall name its problem 2). We shall assume, that intensity of performance of each of operation are constant

$$u_i(t) = u_i^0, i \in I$$

and thus all restrictions of a problem carried out. In particular, it is carried out:

$$b_i \le u_i^0 \le B_i,$$

Let's designate Φ_1 - optimum value of efficiency function in a problem 1; Φ_2 - optimum value of efficiency function in problem 2. Rights the following theorem.

Theorem. *Optimum value of efficiency function in a problem 1 not more than optimum value of efficiency function in a problem 2:*

$$\Phi_1 \le \Phi_2.$$

Enough plenty of numerical experiments (on problems of various dimension, various structure of a network and various sets of resources) which have shown efficiency of the offered method for the decision of the specified problems has been lead.

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DETERMINATION OF GROSS EMISSION FROM VEHICLES ON THE ENVIRONMENT CITY ALMATY

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In this paper, we estimate the amount of energy and ecological impacts of road flows on a regular cell decomposition of the territory of Almaty. It is assumed that the flow involves the entire motor vehicle fleet, is uniformly distributed over the road network. Motorization is a progressive phenomenon and brings enormous benefit to society. However, along with many of the benefits of motorization is accompanied by a number of negative phenomena causing substantial damage to society and nature, which may manifest itself as directly and as wasting resources. Auto-road complex is one of environmental pollutants and makes it unfavorable changes.

To get started on a city map was overlaid uniform grid and used cellular functions of the road network in the city of Almaty. By this method was designed gross pollutant emissions from road transport.

Motorization is a progressive phenomenon and brings enormous benefit to society. However, along with many of the benefits of motorization is accompanied by a number of negative phenomena causing substantial damage to society and nature, which may manifest itself as directly and as wasting resources. Auto-road complex is one of environmental pollutants and makes it unfavorable changes [1].

Currently, all the more urgent becomes the problem of reducing the emissions of cars with exhaust gases, especially when driving in urban areas of complicated high-speed cyclic loading conditions and different modes of engine [2]. To begin with, we impose a uniform grid on the map of the city, and use the cellular functions of the road network in Almaty, obtained in each square of the partition. The considered algorithm was applied to the city of Moscow [3]. Impose on the region grid partition Δ increments of 2 km (Figure 1). Applying the algorithm for cellular function, we obtain the length of the road network in the bands.

In Almaty, there are three categories of roads: Three Way in one direction (Abay, Al-Farabi, etc.), two Way in the same direction (str Dzhandosov Str. Timiryazev etc); Plank in one direction.

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MOVEMENT OF THE PISTON IN A REACTING MIX OF GASES

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A one-dimensional motion of the piston mass "m" in a tube filled with a mixture of reacting gases. The piston moves through the voltage difference, the piston friction on the walls is neglected. We will investigate the challenge in the mass Lagrangian variables. We assume for simplicity that the ends of the tube are $x = 1$, the point $x = 0$ is the initial position of the piston. Introduce the notation:

$$\Omega_1 = (-1, 0), \quad \Omega_2 = (1, 0), \quad \Omega = \Omega_1 \cup \quad \Omega_2, \quad Q_i = \Omega_i \times (0, T), \quad i = 1, 2,$$

$$\Gamma = \{(0, t)|0 < t < T\}, \quad 0 < T < \infty,$$

$$v = \rho^{-1}, \quad \sigma = \mu\rho u_x - p, \quad F = \lambda_1\rho\theta_x, \quad H = \lambda_2\rho\theta\theta_x, \quad G = \chi\rho c_x, \quad p = R\rho\theta,$$

$[f] = f(+0, t) - f(-0, t)$- jump of function f. The system of the equations describing a condition of gas looks like:

$$\begin{gathered} v_t = u_x, \\ c_t = G_x - cg, \\ u_t = \sigma_x \\ \theta_t = F_x + H_x + u_x\sigma + \delta cg, \\ (x, t) \in (\Omega_1 \times (0, T)) \cup (\Omega_2 \times (0, T)), \end{gathered} \qquad (1),$$

The movement piston is defined(determined) by the second law of Newton:

$$mU_t = [\sigma], \quad U = u(0, t), \quad t \in (0, T). \qquad (2)$$

On a line$x = 0$ the conditions are carried out:

$$[u] = [\theta] = [c] = [H] = [F] = [G] = 0. \qquad (3)$$

The system (1)-(3) becomes isolated by addition of the boundary and initial data:

$$u = F = G = 0 \quad on \quad x = -1, x = 1, \quad t \in (0, T) \qquad (4)$$

$$u = u_0, \quad v = v_0, \quad \theta = \theta_0, \quad c = c_0 \quad on \quad t = 0 \qquad (5)$$

and the conditions of the coordination are executed. The functions u_0, v_0, θ_0, c_0 are assumed smooth at $x \neq 0$ and satisfying to conditions (2)-(3) at $x = 0$.

We proved an existence of a unique generalized solution of problem (1) - (5) by a method of a priori estimates. Our aim is to find global a priori bounds, in which the constants depend only on the data and the length of the time interval, but not on the interval of existence of the local solution. These estimates permit us to extend the local solution [1] to the whole time interval.

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MODELS AND ALGORITHMS OF TRANSLATION OF THE KAZAKH LANGUAGE SENTENCES INTO ENGLISH LANGUAGE WITH USE OF LINK GRAMMAR AND THE STATISTICAL APPROACH

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Creation of tools of the automated translation from the Kazakh language on English language and back is one of actual problems, in connection with an increasing role of Republic Kazakhstan on international scene. Creation of tools of the automated translation, which will allow increasing productivity and speed of translation is very much claimed now.

In the given work the machine translation system of the Kazakh language into English language and back with use of the link grammar and the statistical approach is considered. Feature of link grammar is the description of connections between words in the sentence that allows to make effective realization of the syntactic analysis of the sentence [1,2].

The link grammar defines connections between word pairs in the sentence, but as against the traditional approach to syntax does not try to construct them in a full tree of analysis. The basic element of link grammar is connector. A connector consists of a name of connection (for example, S - a subject, O - addition, etc.), which the analyzed word can enter, and a suffix determining a vector of a direction of connection ("+" the right - directed connector and "-" is left - directed connector). Left - directed and right - directed connectors of one type form a connection. To one word the whole formula of connectors, made with the help of the following operations can be attributed: & - asymmetrical conjunction, or - a disjunction, {} - optionality, @- limitlessness. For example, for the sentence " Marat jana kino kördi " is possible to use the following rules of connections:

<subject>: S+;
<adjective>: A+;
<object>: A- & O+;
<verb>: S- & O-.

They mean, that in sentences of the given type the element < `subject` > can have one connection such as S, directed to the right, < `object` > can have one connection such as A at the left and one connection such as O on the right, etc.

Parser of the link grammar consists of the grammatical file describing words of analyzed language, and directly the analyzer. At creation of a grammatical file the morphological analyzer is used. Efficiency of the syntactic analysis of sentences of language and translation thereof can be increased by using of statistical data, frequency of word use in Kazakh and English languages.

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THE VORTEX MODEL OF THE WINDTURBINE DARRIE

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In this work the developed vortex model for interaction research windturbine carousel type "Darrie" with a stationary air stream with use of the theory of complex variables on the basis of the equations of Bio-Savar is considered. The mathematical model of influence of a stationary air stream with a speed "V" on rotating windturbine "Darrie" with angular speed is described on the basis of the equations of Bio-Savar.

Under the influence of a wind stream on the blades executed in the form of a plane wing with a symmetric profile, there is a carrying power and forces of aerodynamic resistance. Carrying power operates on the blade positively, untwisting a rotor. It is known vertically-axial windturbine "Darrie", using carrying power of a wing for transformation of energy of an air stream to an electricity [1-3]. Advantage such windturbine was reflected in earlier published publications. In the given section calculation of scale characteristics windturbine devices "Darrie" is resulted. As it is above told the vortex model of the turbine is considered. All vortex models are based on this or that form of the equation of carrying over vorticity. One of advantages of this equation consists that pressure isn't included into it obviously and the field of speeds can be calculated, without knowing a field of pressure. Besides, in many currents it is possible to present area vortical veils or an even dot whirlwind that strongly facilitates reception of numerical decisions.

To calculation of a flow of vertically-axial rotors apply vortical models in the basic two types though there are some variations of that and another. Both types make models of "a free trace" and model of "a rigid trace". In the models of a rigid trace used for economy of time of the account, movement of whirlwinds in a trace set in advance while in models of a free trace movement of whirlwinds only slightly limit.

According to the theory each blade break into some segments on scope and model occurrence, a convection and interaction of the vortical systems descending from each segment. On these systems find "inductive speed" (or "speed of indignation") in various points of a field of speeds. Inductive speed it is simple speed which pluses to speed of not indignant wind stream in the given point owing to wind turbine work. Knowing inductive speeds, it is possible to find carrying power and resistance of a segment of the blade if to use aerodynamic characteristics of a profile or methods of the theory of a bearing surface.

In the report the problem statement, the received results and their analysis will be stated in detail.

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ANALYTICAL SOLUTION OF SOME NONLINEAR PROBLEMS FOR THE MOTION OF THE RIGID BODY WITH FIXED POINT DEPENDENCE

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Based on the method of partial discretization of nonlinear differential equations [1] constructed by one of the authors of the paper obtain new analytical solutions of the problems of motion with a fixed point of the solid and gyros described by a nonlinear system of ordinary differential equations of Euler and a system of ordinary differential equations with variable coefficients.

Great interest in classical and celestial mechanics and in other areas to this problem for over two centuries is due to the fact that this kind of movement in conjunction with the so-called progressive movement are done by the planets, stars, galaxies, metagalaxy, artificial satellites and many other systems and objects [2].

The report describes the analytical solutions of three problems.

1. In the first problem right sides of nonlinear differential equations of Euler reprezent disturbing external effects on the body, having the dimension of force moment:

$$\begin{aligned} A\frac{dp}{dt} + (C-B)qr &= -\lambda_1 p + f_1(t), \\ B\frac{dq}{dt} + (A-B)rp &= -\lambda_2 q + f_2(t), \\ C\frac{dr}{dt} + (B-A)pq &= -\lambda_3 r^n; \end{aligned} \tag{1}$$

$$t=0: \quad p=p_0, \quad q=q_0, \quad r=r_0, \tag{2}$$

where p, q, r - are unknown functions, $A, B, C, \lambda_1, \lambda_2, \lambda_3, n$- are constant parameters.

2. In the second problem right sides of differential equations, during some moment of time partially or completely distroy.

3. In the third, the parameters $A, B, C, \lambda_i, (i = \overline{1,3})$ are variables and can change almost any law.

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THE COMPARATIVE ANALYSIS OF THE DYNAMIC INFLUENCE OF MOVING TORQUE LOAD ON NON-SUPPORTED AND SUPPORTED CAVITES IN ELASTIC HALF-SPACE

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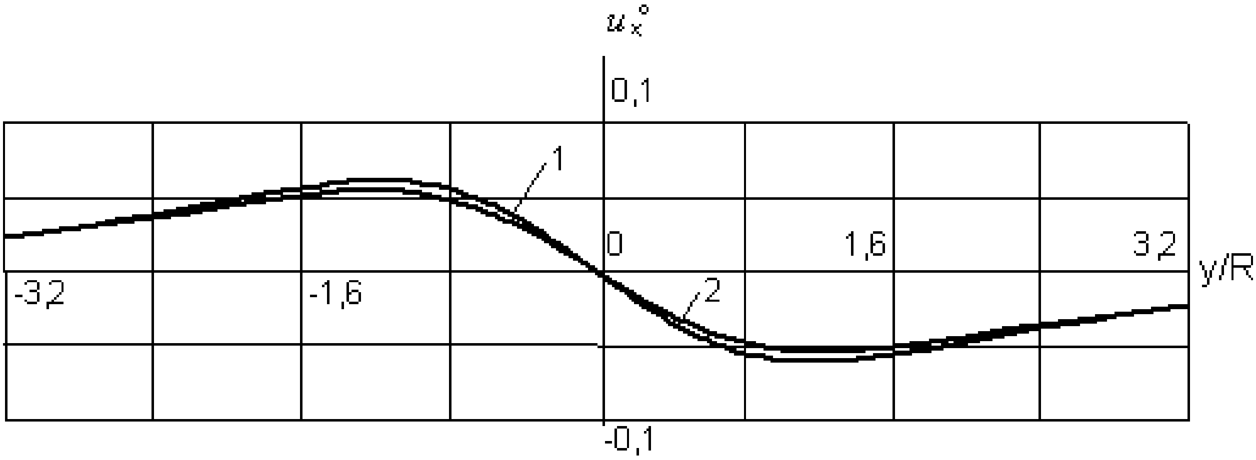
Using the solution in moving coordinates of the problem of the effect of torque load, attached at random to the surface of a circular cylindrical cavity (or to the internal surface of the elastic shell supporting it) and moving uniformly along the cavity's axis, on elastic half-space the mode of deformation of the earth surface in case of the shallow location of a tunnel under the influence of mobile axisymmetric load of the given kind distributed evenly along the finite length of the cavity (such influence takes place, for example, under the inequality of the dynamic loads transmitted to each rail tracked in a cylindrical tunnel) is investigated. It is believed that the load function can be expanded into Fourier line on angular coordinate and Fourier integral on axial coordinate. The motion of the half-space is described by the dynamic equations of the elasticity theory in Lam potentials for the solution of which the method of decomposition of potentials into plane waves and of re-decomposition of plane waves into series of cylindrical functions is applied, and the shells are described by the classical equations of the thin-shell theory.

Calculations were carried out given the following parameters: radius of the cavity in the siltstone solid - one meter, location depth - two meters, concrete shell thickness - 0,05 meters, load speed - 100 m/s. The intensity of the load was chosen so that the general load along the length of the 0.4 meter loading section was equal to the concentrated circular torque load P_0.

The figure shows the result of the load's influence on the earth surface where the deflection curves u_x are given in the cross plane passing the middle of the loading section (designation: $u_x^0 = u_x\mu/P_0$, where μ is the modulus of siltstone shift). Curve 1 is plotted for a non-supported tunnel, curve 2 - for a supported tunnel. It follows from the analysis of curve behaviour, that the shell reduces the surface deflections on sections $-2.4 \leq y/R \leq -0.2$ and $0.2 \leq y/R \leq 2.4$. The shell has similar impact also on horizontal displacement and normal stress.



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DRIVE WORK OF FRONTAL VARIATOR OF UNSTAGED TRANSMISSION

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To provide wide range of regulation in unstaged drives, frontal friction variators, having disks and rollers, are used.

To describe mathematical model of frontal variator work (schema 1), which corresponds to 2 set general coordinates, the second type Lagrange equation [1]

$$\begin{cases} \frac{d}{dt}(\frac{\partial T}{\partial \dot{\varphi}}) - \frac{\partial T}{\partial \varphi} = -\frac{\partial \Pi}{\partial \varphi} + Q_{\varphi}, \\ \frac{d}{dt}(\frac{\partial T}{\partial \dot{s}}) - \frac{\partial T}{\partial s} = -\frac{\partial \Pi}{\partial s} + Q_{s}. \end{cases} \tag{1}$$

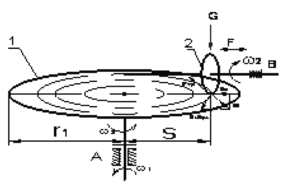

FIGURE 1. Diagram 2 - Scheme of transmission of frontal variator movement.

Differentiating the hand sides of equation (1) and using obtained results, we get following differential equations describing the motion

$$\begin{cases} \frac{J_2}{r_2^2}(2s\dot{s}\dot{\varphi}) + J_1\ddot{\varphi} = M, \\ m_2\ddot{s} - \frac{J_2}{r_2^2}s\dot{\varphi}^2 = F - fG\frac{s\varphi}{\sqrt{(\frac{\dot{s}}{\dot{\varphi}})^2 + s^2}}. \end{cases} \tag{2}$$

To get nonlinear differential equation of frontal variator motion (2), let's take primary data from [2]

$$\varphi(t=0) = \varphi_0, \;\; \dot{\varphi}(t=0) = \dot{\varphi}_0, \;\; s(t=0) = s_0, \;\; \dot{s}(t=0) = \dot{s}_0. \tag{3}$$

There is no analytical solution of the equation (2) with initial data (3). Because the system of differentiative equations is nonlinear.

After solving we'll get final system of differentiative equation of frontal variator motion

$$\begin{cases} \varphi^{n+1} = \dfrac{M\frac{J_2}{r_2^2}(2s^n\frac{s^n - s^{n-1}}{\Delta t}\frac{\varphi^n - \varphi^{n-1}}{\Delta t} + s^n\frac{-2\varphi^n + \varphi^{n-1}}{\Delta t^2}) - J_1\frac{-2\varphi^n + \varphi^{n-1}}{\Delta t^2}}{(\frac{J_2}{r_2^2}s^n + J_1)\frac{1}{\Delta t^2}}, \\ s^{n+1} = \dfrac{F + \frac{J_2}{r_2^2}2s^n\left(\frac{\varphi^n - \varphi^{n-1}}{\Delta t^2}\right)^2 + s^n\frac{-2s^n + s^{n-1}}{\Delta t^2} - m_2\frac{-2s^n + s^{n-1}}{\Delta t^2}}{m_1\frac{1}{\Delta t^2}} - fG\dfrac{s^n\varphi^n}{\sqrt{\left(\frac{s^n - s^{n-1}}{\varphi^n - \varphi^{n-1}}\right)^2 + (s^n)^2}}. \end{cases} \tag{4}$$

Mathematical model has been developed, which describes work of frontal variator drive of unstaged transmission. Control system of this mechanism with using of equation the second type Lagrange equation was done.

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CYLINDRICAL WAVES DIFFRACTION ON THE BOARDER OF ELASTIC AND LIQUID MEDIA

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Diffraction problem of various waves by rough surface was considered in plenty papers. The sound reflection from rough surface was considered in [1]. In this work the perfectly rigid body was considered, therefore the influence any spreading waves wasn't considered.

Although this problem has a strong practical interest [2], particularly as one of methods of nondestructive check. We consider a sound reflection from incident cylindrical axial-symmetric wave source deepened in the liquid on some middle distance h from the boarder of the liquid-solid, which is a random rough surface. At the average it is supposed that the surface is flat and average amplitude of roughness is smaller than normal component of wave length and length of distance the roughness noticeable change. Thus we can decompose the boundary conditions to serial expansion by the length of deviation, like as it use in the perturbation method [3].

In our approach we use a small parameter which is a ratio of the relative liquid impedance to solid body γ. The vector field image in the neighborhood of source and boundary surface liquid is presented for the displacement field.

In our paper we give explanation for experimental behavior of the reflection factor for the waves, falling from liquid to real boarder of solid body.

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MATHEMATICAL MODELS FOR INDEPENDENT COMPUTER PRESENTATION OF TURKIC LANGUAGES

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We give new definition of the notion "language" and the definitions of mathematical and computer models of a notion of a natural language suitable for computer presentation and offer the syntax of an auxiliary algorithmic language for such presentation with random choose of additional objects. We propose to create interactive presentations of foundations of Turkic languages with the following aims: to offer all computer users an acquaintance with Turkic languages without other languages as a media; to develop effective electronic text-books and complex examinations on Turkic languages; to fix their up-to-date state and to distinguish similar and same notions in them by comparing corresponding mathematical models.

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A MICROSCOPIC DESCRIPTION OF THE SPIN POLARIZATION EFFECTS ON THE MAGNETIC MOMENTS[1]

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In this study the effects of the spin-spin and spin-isospin interactions on magnetic moments were investigated by using a microscopic method called as Quasiparticle-phonon model (QPM). The aim of this approach is to take into account the interaction of quasi-particles with phonons of the even-even nucleus. The phonon states in doubly even nuclei are described in RPA and their structure is conserved in odd-mass nuclei when one takes into account the spin-spin interactions between phonons and quasiparticles. In this model the internal wave function for an odd-mass nucleus in a state with an angular momentum projection K is taken in the form $(K > 1/2)$

$$\Phi_K(\tau) = \left\{ N_K(\tau)\alpha^+_K(\tau) + \frac{1}{\sqrt{2}}\sum_{\tau'}\sum_{\nu} G_i^{KK_\nu}(\tau,\tau')\alpha^+_{K_\nu}(\tau)Q_i^+ \right\}\Psi_0. \tag{1}$$

Here $\alpha^+_{K_\nu}$ and Q_i^+ are the quasiparticle and phonon creation operators. Isospin index τ and τ' represent neutron and proton. The function Ψ_0 represents the phonon vacuum which corresponds to the even-even core of the nucleus. The quantities $N_K(\tau)$ and $G_i^{KK_\nu}$ determine the contribution of the one quasiparticle and the quasiparticle-phonon component in the wave function, respectively. In order to derive the secular equation that the energies η_K are calculated for an odd nucleus, we used the variational principle in the form

$$\delta\left\{ \langle \Phi_K(\tau) H \Phi_K(\tau)\rangle - \eta_K \left[N_K^2(\tau) + \sum_{i,\nu}[G_i^{KK_\nu}]^2 - 1 \right] \right\} = 0 \tag{2}$$

in which the normalization condition is expressed though $\langle \Phi_K(\tau) H \Phi_K(\tau)\rangle$; η_K acts as a Lagrange factor and the variations δN_K and $\delta G_i^{KK_\nu}$ are treated as independent variations. Finally, we derived an analytical expression for the effective spin gyromagnetic factor in the form of

$$\begin{aligned} g_s^{eff} - g_\ell^\tau &= (g_s^\tau - g_\ell^\tau)\left\{ 1 - 2N_K^2(\tau)\sum_i \frac{\kappa R_q^i(\tau,\tau')R_q^i}{(\varepsilon_K(\tau)+\omega_i(\tau)-\eta_K(\tau))} \right\} \\ &-(g_s^{\tau'} - g_\ell^{\tau'})2N_K^2(\tau)\sum_i \frac{\kappa R_q^i(\tau,\tau')R_q^i}{(\varepsilon_K(\tau)+\omega_i(\tau)-\eta_K(\tau))}. \end{aligned} \tag{3}$$

The detailed information about the equations represented above is in ref. [1].

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THE MATHEMATICAL DESCRIPTION OF KAZAKH PHONETICS

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One of the important research areas in information technology is speech recognition and synthesis, which is an attempt to advance human-machine interaction. The ability of a computer to understand speech and act accordingly would potentially reduce the human load and the risks in human-dependent applications. It is clear that the design and development of such speech-enabled systems mostly depend on the phonetic structure and the mathematical description of the underlying language.

However, if Kazakh language is concerned, there are no common standards regarding its phonetics. Current studies about the phonetics of Kazakh language were inherited from the Russian and do not precisely reflect the characteristics of the former what has been shown in [1, 2]. Moreover, with the bilingual status of the country, people frequently use words from both languages in their daily conversations and this adds more complication to processing of Kazakh speech [see Fig.]. Another issue is that there are no acoustic databases, which would include the utterances of Kazakh speech for speakers of different gender, age and region. The existence of such a database could help reveal physical properties of the Kazakh sounds and test the hypotheses stated by the linguists. Definitely, all these decelerate the process of the development of speech-enabled systems.

Therefore, to foster research in speech recognition and synthesis, we focus on Kazakh speech and its phonetic diversity. Here we aim to revise and understand the phonetical structure of Kazakh language and suggest a phonetic dictionary, which includes the mathematical description of unique physical characteristics of phonemes in time and frequency domains. Such characteristics are pitch, formants, frequency range, duration, periodicity and other acoustic parameters of the phonemes in speech signals of the various speakers. This study provides the classification of the phonemes, their variation among the speakers as well as the properties of phonetic transitions.

Our approach is based on the work done in [3] with the application to Kazakh speech. The results of the study can be beneficial to a broad audience that is doing research on Kazakh speech.

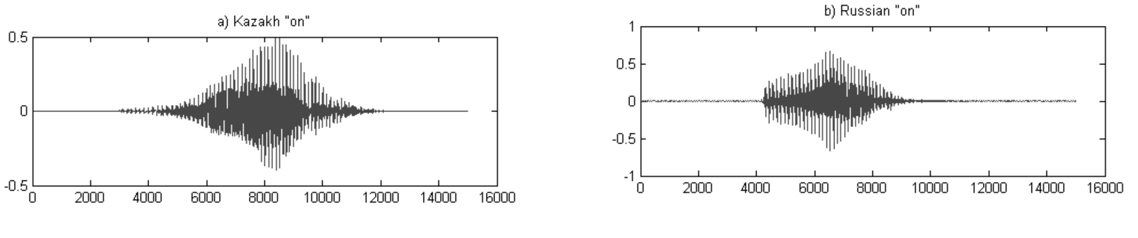

Figure. *Difference in starting sound o in Kazakh and Russian words:*
a) Kazakh on sounds like English on; b) Russian on sounds like English on

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N-FRACTIONAL CALCULUS OPERATOR N^{μ} METHOD TO A MODIFIED HYDROGEN ATOM EQUATION

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Let

$$q(r) = \frac{\ell(\ell+1)}{r^2} - \frac{n}{r} \quad (0 < r < \infty),$$

where ℓ is a positife integer or zero and $n > 0$. Then

$$\frac{d^2y}{dr^2} + \left[E + \frac{n}{r} - \frac{\ell(\ell+1)}{r^2}\right] y = 0. \tag{1}$$

In the quantum mechanics the study of the energy levels of the hydrogen atom leads to this equation [1].

Theorem. *Let* $y \in \{y : 0 \neq |y_\mu| < \infty; \mu \in \mathbb{R}\}$ *and* $f \in \{f : 0 \neq |f_\mu| < \infty; \mu \in \mathbb{R}\}$. *Then the non-homogeneous modified hydrogen atom equation (putting* $E = k^2$ *(k the corresponding wave number),* $m = \ell + (1/2)$ *in* (1)*):*

$$L[y, r, m, n] = y_2 + y\left[k^2 + \frac{n}{r} + \frac{(1/4) - m^2}{r^2}\right] = f \quad (r \neq 0) \tag{2}$$

has particular solutions of the forms:

$$y = r^{m+\frac{1}{2}} e^{-ikr} \left\{ \left[\left(f r^{\frac{1}{2}-m} e^{ikr} \right)_{-m-\frac{in}{2k}-\frac{1}{2}} e^{-2ikr} r^{m-\frac{in}{2k}-\frac{1}{2}} \right]_{-1} e^{2ikr} r^{-m+\frac{in}{2k}-\frac{1}{2}} \right\}_{m+\frac{in}{2k}-\frac{1}{2}}, \tag{3}$$

$$y = r^{-m+\frac{1}{2}} e^{-ikr} \left\{ \left[\left(f r^{\frac{1}{2}+m} e^{ikr} \right)_{m-\frac{in}{2k}-\frac{1}{2}} e^{-2ikr} r^{-m-\frac{in}{2k}-\frac{1}{2}} \right]_{-1} e^{2ikr} r^{m+\frac{in}{2k}-\frac{1}{2}} \right\}_{-m+\frac{in}{2k}-\frac{1}{2}}. \tag{4}$$

Here $y_2 = d^2y/dr^2$, $y = y(z)\,(z \in \mathbb{C})$, $f = f(z)$ *(an arbitrary given function) and* m, n *are given constants.*

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THE SIMULATION OF FORMATION PROCESS OF CERAMICS BY A HOT MOULDING METHOD

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The hydrodynamics of the thermoplastic slurry molding should be attributed to the class of physical processes of deformation and flow. The flow of the slurry after a feeder retains its configuration. In doing so, the flow velocity and pressure are constant. In the experiments it was found that in the range of possible casting speed the mode of the motion of the thermoplastic slurry in the molding form is laminar. The slurry comes in the form of injection molding at a temperature 75°C, cools it to a temperature at which the casting can be removed from the form without warping, i.e., (20-25)°C. The movement and the heat transfer of the slurry in the form molding is viewed as a set of steps: the slurry motion and the heat transfer in the liquid state; the motion and the heat transfer of the slurry in the light of crystallization; the movement of the casting and the heat transfer in the solid state [1-4].

The intensity of heat exchange between the hot slurry and the coolant liquid depends on the regime of the flow, the thermal and the physical characteristics of the slurry, the coolant temperature of the liquid, the wall material and the geometry dimensions of the bushing. In the process of the heat change flow the rheological properties, and a solidification of the slurry. The peculiarity of solidification is that the temperature of the external wall of bushing washed by cold water, will lower the temperature of its central part, this comes to uneven of the profile of the temperature and the rheological characteristics of the slurry. The solidification starts from that the temperature Tw of the outer wall, while the slurry will be in liquid state in the central part. In the result is a liquid makeup of the slurry to compensate for internal shrinkage during hardening in the cooled zone of the bushing.

The mathematical model includes a system of equations of motion of non-Newtonian Bingham's fluids, continuity and heat transfer, taking into account the speed of crystallization of the slurry. The thermal properties of the slurry (density, viscosity, limiting shear stress, thermal conductivity, heat capacity, heat of crystallization) are a function of the temperature.

Results of mathematical simulating of formation process of ceramics by a hot moulding method are resulted. The calculated data describes changes of modular condition thermoplastic slurry in process of cooling in a form-building cavity. The moulding characteristics providing homogeneous properties of thermoplastic slurry at formation of ceramics by the hot moulding method are received.

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ON EINSTEIN'S PRESCRIPTION

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The notation of the energy has been one of the most theory and important problem in Einstein's theory. There are many attempts to get a well defined expression for local or quasi-local energy and impulse [1-3]. However, there is still no generally accepted definitions known as result, different people have different points of view. Energy and impulse density are usually defined by the second rank tensor T_a^b. The conservation of energy and impulse are described by the requirement that the tensor divergence is zero. A contribution from the gravitational field must be add to obtain an energy-impulse expression with zero divergence. Einstein first obtained such an expression and many others such as Landau and Lifshits, Papapetrou, and Weinberg gave similar prescriptions [4-6]. These definitions only give meaningful results if the calculations are performed in "Cartesian" coordinates. Muller has shown that it is impossible to find a satisfactory expression for the energy-impulse complex in any coordinate system [5-6]. In a series of papers Cooperstock has propounded a hypothesis according to which, in curved space-time, energy impulse tensor T_a^b of the matter and all non-gravitation fields. Nashed and Mourad calculate the energy and impulse of the stationary beam of light. The results of Xulu and Bringley support this hypothesis. Sharif and Gad have applied some prescriptions to calculate energy-impulse densities of Godel space time [1-7]. The aim of this work is to calculate the energy-impulse densities of the stationary cylindrically symmetric of rotating perfect fluid with constant pressure, rigid rotation and the $r-z$ space is flat. By using Einstein's and Papapetrou's prescriptions for four classes these solutions explicitly as Godel type solutions by using Einstein's prescription and Papapetrou's prescription. The energy and impulse densities in Einstein's prescription are given by

$$\Theta_b^a = \frac{1}{16\pi} M_{a,c}^{bc},$$

where

$$M_a^{bc} = \frac{g_{ad}}{\sqrt{-g(-g(g^{bd}g_{ce} - g^{cd}g_{be})_e)}},$$

where $a, b, c, d, e = 0, 1, 2, 3$. Θ_0^0 is the energy density, Θ_0^a is the momentum density components, and Θ_a^0 is the energy current density. Einstein energy impulse satisfies the local conservation laws

$$\frac{\partial \Theta_a^b}{\partial x^b} = 0.$$

In order to evaluate the energy and impulse densities for Godel-type metrics, we need to calculate the non-vanishing components of M_a^{bc} and then we obtain the energy and impulse densities in Einstein's prescription. Then we calculate the non-vanshing components of the energy and impulse for the four cases.

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SECTION IX

Theoretical Bases of Information Technologies

QUALITY ANALYSIS OF THE INDUSTRIAL INFORMATION SYSTEMS

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The problem of modeling and quality analysis of the industrial information systems (IS) is considered. The variety of the project variants of IS obtained from the modules (M) of the system is presented with accuracy of isomorphism by means of the ortree $D\ (M,\ F)$, where M is the set of vertices corresponding to the functional structure of the information system; F are the mappings indicating the incidence of the vertices of the ortree [1].

According to the theory of additive utility the integrated criterion W is represented in the form: $W = \sum_{i=1}^{k} \lambda_i f_i(v_i)$ *(i=1,2,...,k)*, where λ_i is the weight of the *i-th* local quality criteria; λi=[0,1]; $\sum \lambda_i = 1,\ f_i(v_i)$is the normalized value of the *i-th* quality index.

To each ortree *D(T, Fi)*, *D(P, Fp)*, *D(I, Fi)* we correspond the ortree of the complex criteria *D(W,Q)*, and to each configuration on the ortree *D(W,Q)*, the total value of the integral quality criteria $X_T = \sum_{i=1}^{k} \omega_i$, where $\omega_1 \in W_1,\ \omega_2 \in W_2, ..., \omega_k \in W_k$.

To define the set of effective with respect to x_T configurations of the information system and to define the components of these configurations the notion of generalized graphs [2] of information system and module - graph of the subsystem of information system [3, 4] is introduced.

The proposed model defines enough universal and useful techniques for the modeling of the quality of information system. The important advantage of this model is that, it enables to obtain an integral estimate for the quality of the information system, differential quality estimate on the intersection of the different properties and also on the intersection of the different subsystems of information system.

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NETWORK MECHANISM DESIGN: AN OVERVIEW[1]

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Network mechanism design aims to achieve system level goals such as maximization of aggregate user performance on computer and communication networks, where users are independent and often selfish decision-makers with individual preferences. By imposing certain rules and pricing schemes, the mechanism designer aligns the system-wide objectives with those of the users, and achieves the targeted goals while ensuring user incentive-compatibility. The resulting mechanisms and games are useful in distributed control of a broad class of networks, including wired, optical, wireless, mesh, and heterogeneous. The methodology and algorithms developed are applicable to diverse network control problems such as interference and spectrum management, energy efficiency, as well as security, privacy, and adversarial user behavior.

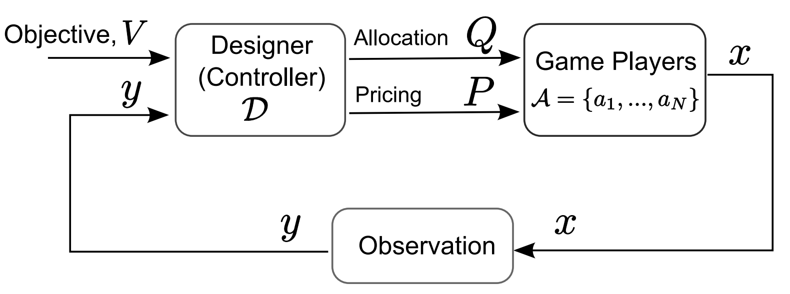
Let us define an N-player strategic (noncooperative) game, $\mathcal{G}$, where each player $i\mathcal{A}$ has a scalar decision variable x_i such that $x = [x_1, \ldots, x_N] \in \mathcal{X} \subset \mathbb{R}^N$, where $\mathcal{X}$ is the convex and compact decision space of all players. The player objective is to solve the following individual optimization problem

$$\min_{x_i} J_i(x),$$

under the given assumptions and constraints of the strategic game, and rules and prices imposed by the designer. The designer objective, e.g. maximization of aggregate user utilities or social welfare, can be formulated using a smooth objective function V for the designer:

$$\max_{x} V(x, U_i(x), c_i(x)),$$

where $c_i(x)$ and $U_i(x)$, $i \in \mathcal{A}$, are user-specific pricing terms and player utilities, respectively. It is important to note that the designer can only influence the outcome, e.g. Nash equilibrium of the game indirectly and cannot dictate actions of players, which would have immediately negated preference-compatibility. The goal is then to align these $N+1$ optimization problems, which can be formulated as a static or dynamic problem. An iterative and control-theoretic approach to network mechanism design is illustrated in the next figure.



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CONDITIONS OF IMAGE EXISTANCE IN AN INFRARED IMAGE CONVERTER[1]

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For infrared (IR) image converters following conditions of image existence can be formulated

$$j_c \geq \alpha j_T \tag{1}$$

$$j_c + j_T \geq j' \tag{2}$$

where j'- the threshold current for the visualizing unit of system, α- constant, determining 'minimal current' contrast. The following system of the inequalites defining (CIE) can be written as

$$q_c \geq \alpha q_T \tag{3}$$

$$q_c + q_T \geq q_m. \tag{4}$$

where q_m- minimum quantity of electricity. Considering that, $j = e\mu n\varepsilon$, CIE can be written as

$$n_c \geq \alpha n_T, \tag{5}$$

$$n_c + n_T \geq n' \tag{6}$$

where n_c and n_T- the concentration of nonequilibrium and equilibrium carriers, correspondingly. If in the limit of considering (5)and (6) as the equalities

$$n_c = \alpha n_T \tag{7}$$

$$n_c + n_T = n' \tag{8}$$

that this is the system $2 - x$ equations in two unknowns, that has the unique solution $n_c\left(\alpha, n'\right)$ and $n_T\left(\alpha, n'\right)$, that represents point on the plane $n_T\left(n_c\right)$

$$n_T = n' / \left(1 + \alpha\right) \tag{9}$$

$$n_c = \alpha n' / \left(1 + \alpha\right) \tag{10}$$

At $n_T > n'$ an inequality (6) is satisfied due to one only this condition, in this region the system ((5),(6) turns to system

$$n_c = \alpha n_T \tag{11}$$

$$n_c + n_T > n' \tag{12}$$

moreover the condition for exceeding threshold charge (12) is satisfied automatically; in the region $n_T < n' / \left(1 + \alpha\right)$ the system (5), (6) has the form

$$n_c > \alpha n_T \tag{13}$$

$$n_c + n_T = n' \tag{14}$$

and the condition of multiplicity (13) is satisfied automatically. It follows from above mentioned that full function $n_c\left(n_{\mathrm{T}}\right)$ it cannot be analytically expressed by one formula, but in individual ranges one of the inequalities of system turns to equality, whereas another inequality is fulfilled automatically.

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SLA BASED COMPUTING CLOUDS MERGER AND SEPARATION USING GAME THEORY

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High performance systems are offered considerable computing possibilities to enterprises and individuals. However, day-to-day application demand for computing resources has volatile nature, causing overconsumption at demand decreases and shortage at demand peaks. This problem has been addressed through numerous ways and the perspective solution is adopting cloud computing system [1].

The main idea behind the cloud computing is in dynamically increasing and decreasing processing power, depending on the executed application needs. As a result, resources are not overspent and, on the other hand, processes are supplied with required processing power on fly. Another conceptual characteristics of cloud computing is in perceiving it as a self managed distributed resources pool. This pool consists of numerous independent clouds, which are equipped with the quality-of-service (QoS) and service level agreements (SLA). Those are used in order to ensure adequate resources leverage and their fair utilization, as well as ensure that provided services do meet quality levels declared in contract agreements.

In this paper authors propose an approach to an SLA reconfiguration based cloud merger and separation framework. Particular attention is drawn to the secure physical and virtual resources leverage management using game theory. Main contribution of this paper is in applying alternative to existing methods - game approach to leveraging resources, and outlining its conceptual advantages.

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ON THE METHOD OF CALCULATION OF THE LIKELIHOOD-TIME INDICATORS OF OPTIMAL SERVICE IN THE DIGITAL NETWORK WITH INTEGRATION OF SERVICES

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In the given work the problem of calculation of the probabilistic time characteristics of a computer network functioning is investigated on the basis of an integrated digital communication network. At the present stage, this combination is the most perspective direction in the development of networks of electric communication. The developed analytical models of calculation of these characteristics of a computer network allows to adequately display interaction between working loading and resources of a network and to estimate its expected productivity, for preventing the peak situations which inevitably arise at an overload of networks. The given methods allow to supervise such undesirable situations at the expense of an effective utilization of circuitious directions of transfer of loading.

The system of equations determining values of node loading subnetworks of switching of channels with circuitious directions of its transfer is received. The uniqueness of solution to this system is proved. In the research a method allowing to calculate for each pair of nodes the current values of probabilities of node losses also is offered. The estimation of average time of a delay in transfer of a package subnetwork of switching of packages in structure of an integrated network is investigated and the dependence of function of a temporary delay subnetwork of switching of packages, in structure of an integrated COMPUTER network of the , from values of the missed flow subnetwork of switching of channels is shown.

The formalized mathematical problem of calculating the optimum probabilistic time characteristics for all computer network as a whole is given. The analytical approach to calculation of the optimum probabilistic time characteristics for a computer network is submitted, on the basis of a digital network with integration of services, with use of the concept of virtual connections care of multichannel calls on circuitious directions. Thus, a number of assumptions is made which with a rather exact degree allow to approach considered model to really functioning integrated networks of communication. The method of mathematical researches receives analytical representation of a considered problem. Further, through a method of Lagrange multipliers are received and the necessary and sufficient conditions of an optimality of the decision of a mathematical problem of calculating optimum probabilistic time characteristics for a digital network with integration of services are proved.

Decentralized algorithm of calculation probabilistic time characteristics for optimum performance of a digital network with integration of services is developed. On the basis of the received theoretical results the basic principle of functioning of algorithm is stated which consists in local procedure of performance of steps of algorithm on each node of an integrated network, that at the end results to the optimization of parameters of quality of service for all network as a whole.

The constructed algorithm of adaptive management of calculation probabilistic time characteristics of a digital network with integration of services is a decentralized algorithm. That is for its realization it is not required the additional information from all nodes of a network at correction of sizes of delays. For this purpose it is enough to carry out an exchange of such information only between the adjacent nodes. Thus the exchange is made by the strictly established rule according to order of nodes in relation to a direction of movement of a channel flow. With the help of consecutive transformations of algorithm the initial set variable is changed and approached to the optimum. At a final stage of algorithm the set of variables, being the decision of a problem of calculation optimum probabilistic time characteristics for a digital network with integration of services, is calculated.

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SIMULATION OF THE RADIAL DRILLING METHOD IN THE FIELD DEVELOPMENT BY WATERFLOODING FIELD

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Today, most oil fields in oil-producing countries of the world operate at a late stage of development, the recoverable reserves of such fields have been extracted, and the remaining oil is left unrecovered confined to low-permeability reservoirs. Moreover, the formations of these fields are highly watered out. Lately, in oil fields with complex geological structure at a late stage of development the method of drilling radial channels in the well by the hydro-monitored drilling is successfully applied. With the use of this method in 20 wells of EmbaMunaiGaz the average daily oil production rate from the sandstone formation has risen from 410 bbl/day to 1,161 bbl/day, which altogether comprised an additional oil production of 7,316 tons in 8 months. The experience of applying this technology in practice shows the need for a throughout analysis of the geophysical conditions and filtration capacity properties of the formation to determine the effectiveness of a use of this method in a given field. This is achieved through the development of 3D geological and hydrodynamic models of the reservoir. This report presents the results of calculations made on the 3D geological and hydrodynamic model of the radial drilling method and investigates the oil displacement efficiency by injected water through the radial channels. The results show 5-7the radial channels in the injection well, the additional production is 400 tones in case of the 5 spot injection pattern of the wells. The displacement front in the common waterflooding (Figure 1.a) is highly heterogeneous, which leads to a decrease in sweep efficiency. While with the use of the radial channels created by the hydro-monitored drilling method results in more homogeneous displacement front distribution through the formation (Figure 1.b).

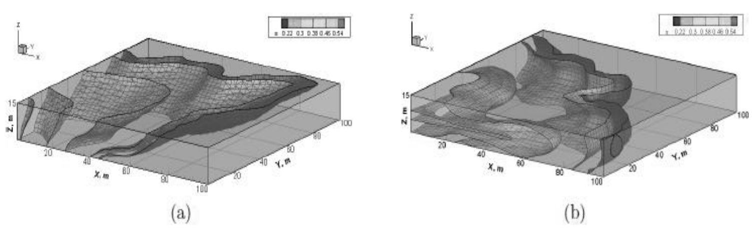

Figure 1. Distribution of the displacement front by a normal waterflooding (a) and by waterflooding through the radial channels (b)

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APPLICATIONS OF NEAR-RINGS TO SOFT SETS: SOFT PARTS OF SOFT INT AND SOFT UNI NEAR-RINGS

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In this study, we introduce soft parts of soft int and soft uni near-rings called soft zero-symmetric and soft constant parts and derive their basic properties. We give some applications of parts of nearrings to soft sets and study relations between parts of near-rings and soft parts of soft int and soft uni near-rings. We illustrate the results by giving several examples.

ON AN EMPLOYMENT PERIOD OF ONE CLASS OF SERVICE SYSTEMS

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The service system with expectation and the unreliable server not ordinary Poisson's stream of demands with intensity arrives, depending on some parameter $x \in R^+ = [0, \infty)$ is considered. This parameter characterizes out of work time the server or the time of processing of the demand (the period of repair of the server).

The state 0^+ or $k^{\pm}, k \geq 1$ of the system means that in the considered moment the system is free, the server is working or there exists a processing demand in the system, correspondingly. Such system is described by the homogeneous Markov process $\{\xi_t, \eta_t\}, t \geq 0$ with phase space $\{0^+, 1^{\pm}, 2^{\pm}, \ldots\} \times R^+$ and the set of transitive probabilities, containing arrival intensities $\lambda^{\pm}(x)$, service $\nu(x)$, failure (server restoration) $\mu^{\pm}(x)$, accordingly.

In the work the condition of the property of employment period is proved. For the considered service system this condition is equivalent to the condition of ergodicity of the same system.

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THE WAY OF CREATING AN INTEGRATED INFORMATION SPACE BASED ON THE INDEPENDENTLY DEVELOPED COMPONENTS

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Today the most commonly used technological approach to component integration systems within an information space is - SOA (Service-Oriented Architecture). It is a very popular approach to software development based on Services with standardized interfaces.

SOAP (Simple Object Access Protocol) is a standard data transfer protocol between SOA web services and client applications. A Web Service is an application that serves as a cross-point in a distributed system. The disadvantage of Web Services is a limited performance of information systems using this technology. This is because of a low text data transfer speed (using the XML format).

In our example another approach was used to create an integrated information space based on independently developed components, which is called - EAI (Enterprise Application Integration). This technology was used to create an integration bus that ensured the management system components' joint operation.

Not long ago, the most popular alternative to the EAI technology was through replacing system components with standardized modules, which required large-scale financial investments.

EAI-based integration systems serve as an interface between heterogeneous information systems and link poorly connected business processes. EAI- based systems eliminate the need to modify each application and improve internal work efficiency. The following tasks can be solved within the EAI technological framework: a) managing heterogeneous IT systems' cooperation; b) improving flexibility and operational efficiency of the utilized information systems and components; c) having the chance to use a centralized management system; d) viewing data from various systems using a single interface (we used web technologies).

Data could be transferred from one system to another using a model, in which an application sends messages to other applications, and they, in their turn, inform the message broker about the data request. This level was implemented using queue servers. Queue servers guarantee message delivery in both asynchronous and synchronous modes. Asynchronous operation capability is important, because it ensures that the infrastructure is independent of EAI systems.

EAI information systems organized access to various business systems through specialized interfaces "connectors", which interact with systems via API either by feeding data stored in databases, or by using special transfer formats, such as XML.

Different information systems store data in different formats. The most important function of EAI systems is data transformation into the format required by the receiving information system. These kinds of transformations are affected using a set of transformation guidelines, which establish data formats' compatibility between various information systems.

Nowadays many major software manufacturers have solutions that support EAI.

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DETECTION OF POTENTIAL REGULATORY SITES IN GENOMES OF PH.SOJAE AND PH.RAMORUM USING COMPARATIVE ANALYSIS

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Phytophtoras are members of Oomycetes clade, whose members causing enormousdamage to various crops worldwide. *Ph.sojae* has been developed as model speciesfor the genus, while *Ph.ramorum* is very vurulent species, destroying many oak forests in USA and Europe. We analysed 979 ortologous genes and corresponding promoterregions in 2 genomes in order to determine the potential conserved regulatory motifs. These sites may be found inside coding sequences where they have no sense. Therefore we should look for each site from tfrr.set both in coding and promoter regions. And sort out those ones which display much higher frequency in promoters than in genes and at the same time they are better preserved in both promoters than in both genes. That is, for each site from tfrr.set we count the number of its occurrences in all 979 promoter sequences and calculate the corresponding frequency (sum of occurrences divided by total length of all promoters) or, more exactly

$$N/Sum(L_i - L_{site} + 1),$$

where N – the number of sites detected, Li – length of the sequence (i = 1,979) and Lsite – length of the site. Along with the number of cases when given site was detected in both corresponding promoter sequences. We do the same with coding sequences. And select sites where frequency of occurrence in promoters is higher than in genes and at the same time these sites are preserved in promoters.

We searched both for known motifs from transcription factor binding sites (TFBS) database TRANSFAC (http:/www.generegulation.com/pub/databases.html), as well as for the de novo motifs using program MEME (Bailey and Elkan, 1994). For known sites we selected only motifs which were statistically more enriched in promoter regions relative to genic regions of the genomes and conserved in both genomes. Based on found motifs we developed a computer program for recognition of promoter regions using an algorithm of Boyer-Moore. This algorithm searches for patterns in the sequences from right to left using 2 methods: bad character heuristics and good suffix heuristics. In the worst-case scenario algorithm performs approximately 3n comparisons in the aperiodic sequences. We plan further to include into analysis another Oomycetes sequences, including the recently sequenced Phytophtora infestans genome.

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CREATION OF ELECTRONIC DATABASE OF TECHNICAL TERMS IN KYRGYZ LANGUAGE

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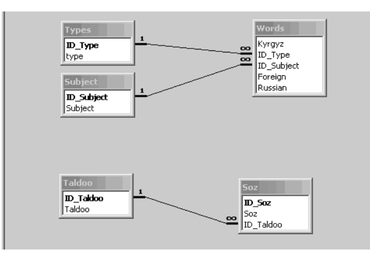
The present stage of development of science and technology is characterized by intensive enrichment of conceptual devices and occurrence of new terms designating advanced concepts. It is natural that new terms don't always designate well defined concepts, clear and accurate definitions which are available in corresponding area of knowledge. In this connection, the first and most significant to adopt new words is the area of computer technologies. Prominent features of technical style are its informative (pithiness), logicality (strict sequence, accurate communication between the basic idea and details), accuracy and objectivity, clearness and understandability.

The texts belonging to given style, can possess the specified lines to a greater or lesser extent. Primary use of language promotes and satisfaies requirements of a given sphere of dialogue, and is found in all such texts. In the field of lexicon it assumes use of scientific and technical terminology and special lexicons. Last year a new law was enacted in the Parliament, making it mandatory for all scientific and technological literature to be in the Kyrgyz language. This has forced scientists-teachers to create an electronical dictionaries of technical terms in various branches of science and technology for more unified and systematic use of technical terms. It is necessary to create automation in the field of lexicography of unique and original terminological dictionaries and automatic databases.

The purpose: To create an electronic dictionary of word indicators in the Kyrgyz language, the last achievements in programs using computer and information technology are directed to create electronic base of dictionaries of technical terms in the Kyrgyz language in object oriented language Delphi, to arrange all units entering into the dictionary. To arrange terms in alphabetic order, to carry out the morphological analysis of the terms, to define type (simple or difficult) to find the general and terminological rates in the Kyrgyz lexical units, in education, mathematics, physics and computer sciences.



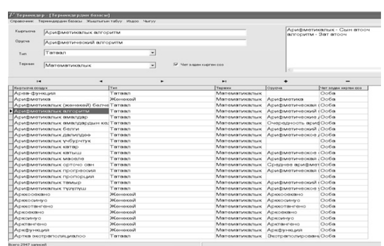


Fig.1.The Informational and logical scheme of a database

Fig.2.General view of electronic base of technical terms

The developed electronic base starts search and gives (sends) the report in Excel:

(1) Terms : simple and difficult terms, total amount;
(2) Search : terms (computer science, mathematics, physics), total amount;
(3) Search :international terms, total amount;
(4) Analysis : the main parts of sentence, total amount;
(5) Kyrgyz terms;
(6) Search the words.

This electronic base can be applied not only in the mathematics and computer science, but also in chemistry, biology and other sciences, it also completely supports technology Unicode .

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USE OF PARALLEL COMPUTING FOR IMAGE COMPRESSION

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Digital images take up an increasing part of the information world. Therefore, there is continued interest in improving the image compression algorithms. In this paper methods of wavelet transform for image compression use the redundancy in data representation is considered. The wavelet transform data is represented as a subtree that can be efficiently coded.

Wavelets are the foundation of a powerful new approach to image processing and analysis, called multiresolution theory. Basic idea of multiresolution theory is the process of decomposition of a discrete sequence of values in the middle and detailing values at different scales. This paper presents the multiresolution analysis based on Haar wavelets.

The Haar normalized scaling functions and wavelets are defined respectively as follows:

$$\phi_{kj}(t) \equiv \sqrt{2^k}\phi(2^k t - j), \quad j = 0, ..., 2^{n-1} - 1 \tag{1}$$

$$\psi_{kj}(t) \equiv \sqrt{2^k}\psi(2^k t - j), \quad j = 0, ..., 2^{n-1} - 1 \tag{2}$$

Wavelet compression scheme is as follows:

Performing the wavelet transform.

Wavelet coefficients are sorted.

Remembered only x% largest coefficients, while the remaining (100 - x)% of the coefficients are set equal to 0.

Decoding is performed by applying the inverse wavelet transform to a thinned array of coefficients.

Wavelet transform of images allows a very efficient image compression and restoration of low-loss information. However, its implementation is very time-consuming. Meantime, parallel computing technologies are an efficient method for image compression using wavelets.

In this paper, we propose a parallel compression algorithm based on Haar wavelets. We implement the algorithm using C++ and MPI technology. The experimental results confirm the efficiency of the algorithm.

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THE METHOD OF SPEECH SIGNAL INTONATION SYNTHESIS BASED ON SPLINE APPROXIMATION

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An important component part of the most text-to-speech synthesis systems is the intonation synthesis module. This module is intended for generation of an intonation contour and its subsequent applying to a synthesized signal. Naturalness of the synthesized signal is substantially determined by intonation contour definition quality. During the synthesis process it is important to realize the smooth control of speech signal parameters. Otherwise the synthesized speech will have unnatural sounding. Thus during the implementation of speech synthesis and recognition systems, the problem of smooth intonation processes modeling is actual. The given article describes the synthesis method of speech signal intonation component based on the splines - mathematically calculated curves which smoothly connect separate control points of an intonation contour. This method was used during the realization of speech signal compilative synthesis system which was developed at the Institute of Informatics and Control Problems of the Ministry of Education and Science of the Republic of Kazakhstan.

For the synthesis of a speech signal on the compilative principle, it is necessary to obtain preliminary the formalized description of its phonetic and intonation properties. As a part of the given description it is necessary to specify intonation characteristics for all phonemes. Their composition includes also a set of control points of parametrical curves. The parameters of the neighbor phonemes should be smoothly matched under these conditions. Thus the development of the specialized language is defined as a goal. This language will help to make the preliminary description of phonetic and intonation properties of a synthesized speech signal. Another necessary task is an algorithmization of the process of smooth parametrical curves calculation. The dynamics of change of the controlled parameters will be defined with their help. The ready-made and normalized by phonemes duration and by the common amplitude level, smoothly connected out of various fragments speech signal is submitted to the input of the parameters regulation system. Depending on required intonation characteristics the fundamental frequency contour is formed and applied to the initial speech signal. Then the amplitude envelopes are applied to the signal. The comparison of the method proposed in the given work with the method of linear interpolation which is used in the most of existing speech synthesis systems [1] has been conducted. The estimation was performed by the criterion of a deviation square sum minimum between calculated values according to the both methods and the natural etalon contour. As a result, for the method of linear interpolation, the average criterion is 0.25. For the proposed method the value of the average criterion is 0.07. Thus, the parametrical description of the synthesized speech signal in the UPL language in aggregate with the method of approximation of its intonational component by splines improve the quality of approximation in comparison with the existing methods.

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THE DEVICE OF RECOGNITION AND SCORING OF PRINTED TEXTS[1]

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The system is designed for reading and playing of Azerbaijani printed, hand-printed, handwritten texts. The proposed software-device system consists of mechanical device that fixes the document for reading, camera (mobile phone, scanner) and software application for printed text recognition and speech synthesis. The described system operates in a following order: Reading material is fixed in a special place of the device and captured by a special camera. The captured image is sent to the input of the text recognition system. After recognition the received text is scoring by means of a subsystem of speech synthesis.

Fig.1 - Recognition Steps

The recognition module. On Fig.1 the stages of recognition of the printing edition are shown. At the first stage there is a preliminary processing of the image (fig. 1)[1]:

1) converting in the B/W format; 2) noise removal; 3) removal of elements of the fixing device and edges of the printing edition; 4) normalizing page orientation; separate pages detection; 5) analysis of page structure and exception of non-text information for further processing; 6) text columns detection; 7) lines detection; 8) allocation of separate symbols; 9) recognize symbols; 10) post-recognition processes.

The module of speech synthesis. Module consist of two blocks: the block of linguistic processing and the voicing module. It is possible to allocate two main blocks in our synthesizer: the block of linguistic text processing and the block of voicing or actually formation of a speech signal. An acoustic signal database (ASD), which consists of fragments of a real acoustic signal elements of concatenation (EC), is the basis of our system of the speech synthesis based on concatenation a method. Dimension of these elements can be various depending on a concrete way of synthesis of speech ([2]).

Conclusion. *On the basis of a software-hardware complex the device is developed for recognition and scoring printing texts by means of the computer program focused on using of the Azerbaijan language. The given device will help to receive the information from printing sources for blind or visually impaired people.*

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RESEARCH OF DYNAMIC PROPERTIES OF INTELLECTUAL COMPLEX SYSTEMS

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Opportunities of immunocomputing - the new computing paradigm, shown at the decision of concrete applied problems in various subject domains, such as: monitoring of especially dangerous processes, information and ecological safety, estimating of dynamics of the fund markets, management, etc. allow to achieve a high level of efficiency, reliability, flexibility and speed of computing procedures [1].

The control systems based on traditional methods, as a rule, do not use modern information technologies, therefore application of these systems for control of complex dynamic objects results in decrease in quality of control. The control systems constructed on the basis of artificial neural networks, have no this lack and are intellectual control systems.

The procedure of research of asymptotic stability of the interval-given object with delay on the basis of interval analogue of the Lyapunov's direct method with use of the scalar-optimizing function and the Razumikhin's approach is offered.

The problem of parametrical synthesis is solved, results of experiment on modeling transients in a researched intellectual control system are given. Neurocontrol after the ending of training provides control in real time. Such systems are very flexibly adjusted on real conditions and do not contain restrictions which arise in construction of the given systems.

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ANALYSIS OF THE APPLICATION OF ATTRIBUTE GRAMMAR IN MACHINE TRANSLATION

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Attribute grammar – this is a context-free grammar with attributes that define the semantics of language constructs. The semantics of the program is determined by a set of attribute values, the root node. Attribute grammars were introduced by Donald Knuth in the late sixties as an extension of the idea of computing the semantics of the whole expression through the semantics of its subexpressions.

Attribute grammar is a quadruple $AG = (G, A_S, A_I, R)$, where

- $G = (N, T, P, S)$ – reduced CF grammar;
- A_S – a finite set of synthesized attributes;
- A_I – a finite set of inherited attributes, $A_S \bigcap A_I = \emptyset$;
- R – finite set of semantic rules.

In addition to the synthesized and inherited attributes, also appeared attributes-collection (collections), depending on the total parse tree, recursive attributes (circular attributes) and attributes of links (referenced attributes).

Attributes are two types of inherited and synthesized. Inherited attributes are used to pass semantic information down the parse tree program and synthesized attributes - upwards. The most widespread inherited attributes received in the implementation of multipass compilers.

Attribute grammars allow expanded rules on the existing syntax. As the analyzed program is subordinated to the set syntax we can enclose under each syntactic rule formulas of calculation of properties/attributes. Further we can recursively walk on a syntactic tree and calculate properties according to the set formulas.

Here is an example of possible record of attribute grammar:

```
 {{{ expression := literal { lhs.is
static := True } | function call { lhs.is static := $1.is static } |
type conversion { lhs.is static := $1.is static } ... type
conversion := type name (expression) { lhs.is static := $1.denote
static subtype and $2.is static } }}}.
```

Formulas for computing a lot - because many syntactic rules, but the formulas are simple, because part of each syntactic rules known in advance. Thus, attribute grammars allow to divide the computation complexity of semantics for a set of simple formulas organized by the syntax of the language.

Compilers are automatically created using attribute grammars are often inefficient in execution time and memory usage.

1. In attribute grammars are often lacking convenient modular tools. Thus, specification languages, created with attribute grammars are often extremely cumbersome.
2. Calculations in attribute grammars are quite strict - for example, it is impossible to calculate an attribute value, if not calculated all the values of attributes on which it depends.

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USAGE OF TERMINAL VOCABULARY IN THE PROCESS OF SPEECH SYNTHESIS BY CONCATENATIVE METHOD

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Concatenation synthesis comes down to the compilation of message from the pre-recorded vocabulary of basic elements of synthesis. The size of the synthesis elements is not more than a combination of two phonemes. The main issue in concatenation synthesis is the volume of memory for storing the dictionary of phoneme combinations.

Speakers read aloud sets of texts and isolated phrases. Then the relevant acoustic files are segmented exactly in the middle of each sound combination. In the combination of two phonemes the initial segment is in the middle of the first phoneme and the end of this segment is noted in the middle of the second phoneme. Then obtained audio file is saved under the name "[first phoneme][second phoneme].wav". It is quite clear that such amount of sound combinations is a finite number. It is ample to analyze a large vocabulary of the required language.

Then, each element of the database, which has quasi-periodic nature (vowels and voiced consonants) by definition, is subjected to automatic segmentation into quasiperiods of the basic one.

Simple example: the word "moloko" (milk) represents the following elements that should be stuck together: "ma" - "al" - "la" - "ak" - "ko".

By using this method, we can create new high-quality speech database. In collaboration with colleagues there was written program software, during which all the operations associated with the accumulation and segmentation of speech databases, were performed semi-automatically.

The technique of segmentation in the middle of the original sound allows us to "stick together" ready words, in accordance with transcription and to receive good quality synthesis, devoid of impairing the quality of synthesis traditional clicks – audible joints at different phonemes.

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DEVELOPMENT OF THE INTELLECTUAL STOCHASTIC CONTROL SYSTEMS ON THE BASIS OF ARTIFICIAL IMMUNE SYSTEMS

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The largest catastrophes and accidents in power sector, industry, on transport etc. show inability of the people to work in not regular situations because of the huge volume of the real information and the insufficient quality of the designing of the controlled complex systems. In this connection it is necessary to develop new non-traditional information technologies for automation of the complex nonlinear technological control objects with that or other kind of uncertainty. The majority of the complex control objects function in the stochastic environment. For the research of dynamic properties of the separated class of the control systems the quasi-splitting method is used [1].

Last achievement in cybernetics, computer science and artificial intelligence have resulted in formation and impetuous development of new area of research - artificial or intellectual control. The given area of researches is at the border of the classical theory of control and artificial intelligence. Now in the field of intellectual control the application of Artificial Immune Systems (AIS) is rather advanced. AIS are constructed on the principles of processing of the information by the proteins molecules. The processes taking place at information handling by human immune system and principles of their functioning amaze by the efficiency, economy and speed. The mathematical bases of the given approach consist in introduction of the concept of formal peptide [2], as mathematical abstraction of free energy of the protein molecules from its spatial form described in algebra of quaternion. AIS have ability of training, recognition and acceptance of the decisions in an unfamiliar situation.

The report is devoted to the development of intellectual technology of construction of stochastic quasi-splitting control systems on the basis of AIS.

The statement of the task is formulated as follows: it is necessary to develop the technology of the research of dynamic properties of quasi-splitting stochastic control systems on the basis of the biological approach of Artificial Immune Systems with the purpose of operative control of the current situation [3].

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APPLICATION OF CASE-BASED REASONING TO INFORMATION SECURITY MANAGEMENT

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Case-based reasoning (CBR) is now a mature subfield of artificial intelligence. The fundamental principles of case-based reasoning have been established, and numerous applications have demonstrated its role as a useful technology [1]. Recent progress has also revealed new opportunities and challenges for the field. For example, a hybrid model on the base of CBR and production rules is used for works in conditions of incompleteness and uncertainty of the information in complex dynamical domains [2]. A formal model of a combination of the script and case approaches for information security management is considered in this paper.

Definition 1. *A situation S_i is an estimation of values set of the system parameters and bonds between them at the time moment t_i:*

$$s_i = \left\langle \widetilde{C_{t_i}}, L, t_i \right\rangle, \tag{1}$$

where $\widetilde{C}_{t_i}$ - a system status evaluation (or parameters values), L - *a propositional language which is describing set of the bonds between the factors.*

A set of the language formulas makes a knowledge base about the data domain, which is received on the formal model base [3] of information security management systems (ISMS), on the basis of expert evaluations or by means of machine training methods [4].

Definition 2. *Precedent e is the pair*

$$\langle s, r \rangle \in \wp = S \times R, \tag{2}$$

consisting of the situation $s \in S$ and a decision $r \in R$ of the situation.

The decision for any precedent is a plan of the actions for influences compensating which have resulted to a deviation of the ISMS from the given goal status.

Definition 3. *A script $\sum$ is considered as a tuple*

$$\sum = \langle s_n, \{y_i, s_{k_i}, \mu_i\} | i \in 1..n \rangle, \tag{3}$$

where s_n - an actual state of system (the situation), s_{k_i} - a goal state of system (and-or values of goal factors), y_i -sets of the achievement plans of the goal state, μ_i - an estimation of possibility of the goal situation achievement at the plan realization.

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ON CYCLE PROBLEM IN LINEAR PROGRAMMING

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Simplex method to the solution of the problems of linear programming (LP), proposed by G. Dantzig in 1947 still remains to be the most popular. This popularity is connected with the fact that solution method harmonically combined with principles underlying at designing of the electronic computer-performance of the whole range of similar operations, connected with laborious arithmetic and logical actions. One could be contended with such state of affairs, if Beale E. [2] hadn't published the example of LP problem in 1955, the attempt for solving of which by simplex method resulted to cycling of computation process. After that immediately was started up the process for struggling with this phenomenon-anti-cyclene [3]. Introducing the notion of "non-degenerate basis" to the LP theory led to additional difficulties. Possible variations, connected with alteration of only one element is considered in simplex method. Exactly by this is defined appearance of "unclear" situations at realization of simplex method. In the row of LP problems, possible directions may be defined not only by one non-basis vector, but linear combination of several non-basis vectors. The quantity of non-basis vectors, required for definition of possible direction, may reach K quantity, whereas K-the level of degenerate basis, that is the number of zeroes in basis. The problem on finiteness of the numbers operation is completely solved considering notes above. In this aspect the situation it should be called the degenerate mode where the attempt to introducing one separate non-basis vector to basis (simplex-method) will not be a successful, this fact, will not contribute to increase the functional quality, and not situation, where basis include zero elements. Application of full arsenal of the non-basis vectors, provided by LP problem itself, exclude problem situations (cycles) at realization of computation process of successive improving the feasible solutions (basis). One of the algorithms to the solution of the LP problem, excluding cycling phenomenon is given in [4].

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REALIZATION OF SHORT LINEAR BLOCK CODES DECODER WITH PARITY-CHECK

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Decoder must take the following steps:

(1) compute the syndrome
(2) isolate error's model and
(3) modulo 2 addition of model error and the received vector, which leads to the elimination of errors.

Let's consider the short linear block code, for example code (6,3), which decoder can be implemented as simple scheme. Let the transmitted codeword U = 101110 and received vector r = 001110 [1].

Let e_j be a basic element of the coset or j-th coset's model error, then vector $U_i + e_j$ is n-tuple in this coset. Syndrome of the n-tuple is in the following form:

$$S = rH^T = (U_i + e_j)H^T = U_iH^T + e_jH^T. \tag{1}$$

As $U_iH^T = 0$ codes vector is U_i, then it can be written:

$$S = rH^T = (U_i + e_j)H^T = e_jH^T. \tag{2}$$

As we see from (2) each member of the coset has the same syndrome. Syndrome of each coset distinguishes from other cosets' syndromes; this particular syndrome is used to determine the model error.

From equation (2) let's compute all bits of syndrome through the digits of received code words:

$$S = rH^T,$$

$$S = (r_1, r_2, r_3, r_4, r_5, r_6) \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

and $s_1 = r_1 \oplus r_4 \oplus r_6$, $s_2 = r_2 \oplus r_4 \oplus r_5$, $s_3 = r_3 \oplus r_5 \oplus r_6$.

For longer codes, such realization is much more complicated; more preferred method of decoding is a series circuit. To correct the models with two erroneous bits requires additional circuit.

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MATHEMATICAL MODELLING AND STRESSED STATE OF THE COUPLED TONNELS IN THE ANISOTROPIC ENVIRONMENT

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In this work the questions of designing and development of the software of computer modeling to intense - isotropic environment with doubly periodic system of cracks is considered. The static distribution of movings and voltage near to pair extended transversally isotropic tunnels in anisotropic nonlinear file weakened doubly periodic system of a crack in conditions of small and finite motions is investigated. Such tasks do not give in to the decision by an analytical method, but are solved by methods. A method of finite elements is used in this case. In a method of finite elements the physical ratio between deformations and voltage are established in a number of finite element points volume. Let two parallel extended tunnels which are taking place on distance L from each other, identical size and vaulted structure passed along a line spreading of a isotropy file plane. A plane of cross section are in conditions of the flat generalized deformation [1-3]. The elastic condition which is described by the equation of the generalized Hooke's law transversally isotropic medium with an inclined plane isotropy[4].

For the solution of the considered problem the program complex Tonnel 3D in Delphi environment is developed. The program complex Tonnel 3D allows to automate the majority of stages of task modeling such as, definition is intense of deformed condition of two underground tunnels in a file with not continuous coupling of layers. The basic functions of a complex: creation and editing of geometrical parameters of considered area; automatic performance of settlement area splitting on finite of irregular grids; visual and tabulated definition of numerical results.

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SECTION X
History and Teaching Methods of Mathematics

MATHEMATICS AND INFORMATION TECHNOLOGIES AS A TOOL FOR REHABILITATION OF DISABLED PEOPLE

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In the present work the thesis about necessity of introduction of social policy in regards of disabled people on a new intellectual level it put forward and substantiated. Mathematics and information technologies are proposed as important tools in the achievement of this purpose.

The mathematics at auxiliary school solves one of the important objectives of education – **overcoming of objective difficulties arising during cognitive activities of students, development in them positive personal merits**. By developing elementary mathematical thinking and skills to solve the computing and logical tasks related with calculations, it establishes and corrects such forms of thinking as analysis, synthesis, comparison, develops ability to generalize and concretize, creates conditions for memory correction, attention and other mental functions.

Visual aids, mathematical games and competitions have a significant importance at education to mathematics. The important tool in formation of acoustical images of mathematical concepts are mathematical dictations when under dictation of teacher students write down mathematical expressions, in the beginning rather simple and then calculate their values. It should not be treated as an impossible goal the formation at students of auxiliary schools of skills of independent way of thinking, even at the solution of simple problems. Education to mathematics promotes also the solution of educational objectives. The regular solution of computational problems imparts to students correct perception of such concepts as duty, teaches discipline, provides skills of right orientation in world through an opportunity of correct evaluation of quantitative characteristics of people and subjects.

It is necessary to note that now it became essential to equip auxiliary schools with computer classes together with necessity to introduce in them the computer science lessons. First of all, it is crucial to utilize at these lessons the multimedia tools. This powerful factor of visualization of education is especially significant at auxiliary school. If even some portion of graduates will master the skills of computer work, even at a level of common user then it in a significant way will improve their position in modern society and also will raise chances of employment, make their life more protected, colorful and interesting.

The creative approach in choosing the methods of educational material for lessons of mathematics, introduction at auxiliary school the lessons of computer science, constant attention to successes of students, their individual problems, combination of learning with educational process, undoubtedly, will promote equipment of graduates of auxiliary schools with vital-practical skills of orientation in the society, their communion with useful activity, to reduction of feeling of impairment, to increase of self-respect and importance in modern complicated world.

THE APPLICATION OF THE RECURRING DECIMALS IN FINDING REMAINDERS

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The finding of the effective division signs for the division of the natural numbers has always been one of the important problems in arithmetic. We come across some different division signs in school textbook and in articles dealing with elementary mathematics/ this investigation is going on now/ the new division signs are found even in modern articles. But there is a lack in these division signs: these signs don't determine the rule for finding remainders if there is not the whole division. Some division signs determine rules for finding remainders. But the investigations show that these rules are not efficient. This lack even belongs to Pascal's sign. The reasons of the absence of the rules on finding remainders are very various. In this work these reasons are investigated in different variants. For instance in investigating the efficient variants of Pascal signs, it was known that this investigation was connected with recurring decimals. So the problem appeared about such an important connection between two great themes of school mathematics. That's why the recurring decimals theory taught in school mathematics is enriched and improved in the presenting article.

This investigation was carried out for many other division signs and satisfactory results have been received.

WHERE IS THE NON-EUCLIDIAN GEOMETRIES PLACE IN NON-HIGHER AND HIGHER CURRICULUM?

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This analysis begins with a historical view of Geometry. One presents the evolution of Geometry (Commonly known as Euclidean Geometry) since its beginning until Euclid's Postulates. Next, new geometric worlds beyond the Fifth Postulate are presented, discovered by the forerunners of the Non- Euclidean Geometries, as a result of the flaw that many mathematicians encountered when they attempted to prove Euclid's Fifth Postulate (the Parallel Postulate). After this brief perspective, a reflection is made on the presence of Non-Euclidean Geometries in Nature and Art and some philosophical implications. Then, we analyse the study of Geometry in Portuguese Secondary Education and the absence of Non-Euclidean Geometries in Higher Education curricula in Portugal. Finally, some suggestions will be made on the inclusion of Non-Euclidean Geometries in some details of the curricula of Geometry in Non Higher Education.

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PERFECTION OF THE THEORY AND MATHEMATICS TRAINING METHODS IN CONDITIONS OF INTEGRATION OF NATIONAL EDUCATIONAL SYSTEMS

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Preservation of unity of educational space in the CIS member countries is considered by us as a priority task. At the present time an education sphere is witnessing certain tendencies for integration of educational systems, rapprochement of educational programs and training cycles, formation of uniform approaches in an estimation and maintenance of quality of education. Practice of multilateral and mutually advantageous cooperation with these countries has confirmed presence of ample opportunities for its further development in all spheres including an education sphere.

Considering that during the Soviet period a considerable practical experience in mathematics has been reached, it is expedient within the borders of existing national educational systems to introduce corresponding corrective amendments with a goal of further perfection and rapprochement of educational programs, teaching materials, etc., both for schools and for other educational levels which would correspond to the advanced world standards. Without doubt, it is necessary to study more deeply and take on board an advanced practical experience of the theory and mathematics training methods which is available in a number of Turkic language countries.

FACTOR ANALYSIS OF THE EFFECT OF CLASS RULES ON THE BEHAVIOURS

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Intraclass behaviours effect on education is substantial. The behaviours are designated by the class rules.

In this study, the survey consists of 35 questions which are class rules' in regard to effect of behaviours are implemented to 400 secondary school students. Obtained data that eliminated by SPSS is analysed via factor analysis and is compared with other studying about the subject. Thereby is turned out statistical discussion substantial main components are gained.

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DIDACTICAL BACKGROUND FOR STUDYING THE ELECTIVE COURSE "ELEMENTS OF NUMERICAL MATHEMATICS" IN TECHNICAL SPECIAL INSTITUTES IN CREDIT SYSTEM OF TEACHING

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Though many kinds of computing techniques have appeared nowadays the necessity of manual, written and approximate calculation is still obvious. The use of a computer provides a quicker, more effective solution of the problem. The development of computer techniques on the one hand and the requirements of Mathematics on the other makes necessary to search more efficient ways of calculation teaching.

The term "Numerical mathematics" can't be considered as an established one because this field of science develops rapidly nowadays. Numerical mathematics as a branch of Mathematics settled at the beginning of the XX century. By that time different, rather effective and reliable algorithms of approximate calculation were worked out.

Progress in the development of numerical techniques led to the extended use of Mathematics in different spheres of science. Mathematical modeling became one of the most effective ways in natural sciences.

There is a very important question: what part of Mathematics is necessary to study in technical special institutes? The necessity to study Mathematics thoroughly on the one hand and peculiarities of the credit system of teaching on the other requires studying Numerical mathematics and to carry out laboratory works.

That's why working at the technical university for more than forty years I prepared out the elective course "Elements of Numerical Mathematics". The given course can attract students' attention; let them study the methods and ideas of the subject. This course deals with the questions and problems that were not worked out in the school program. The elective course "Elements of Numerical Mathematics" is studied for 1 semester and consists of 30 hours for the first year students of technical specialties. The elective course has an integrated character and provides intersubject connection between Mathematics and Computer science. The main theoretical aim of the course is a thorough study of some themes, stimulating students' interest in the sphere of Numerical Mathematics. The main practical aim is the improvement of students' skills in the use of different techniques for solution of a problem. Traditional forms of classes such as a lecture and a seminar can be practiced, but the first place in the course is taken by such forms as laboratory works, presentations of reports, individual and group work. The course allows to realize research and creative abilities of students. The teacher explains new material briefly and formulates the problem. The teacher is to offer consultations while students complete a practical task.

After studying the elective course "Elements of Numerical Mathematics" students should know:

- what Numerical Mathematics is, its tasks and methods;
- notions of precise and approximate calculations, measure of inaccuracy in calculations, their classifications and sources;
- basic numerical methods in solution of simple equation;
- elementary methods of numerical integration.

The contents and examples of laboratory-numerical works are presented in the publication "Mathematical laboratory course" (Part I, part II). It is necessary to use the above mentioned publication in teaching Mathematics by means of new innovative techniques. It is very important to intensify the teaching of Mathematics by means of new opportunities in credit system of teaching. Due to the reduction of Mathematics' lessons in technological universities the holding of the elective courses increasing students' training on Mathematics is suggested.

INNOVATIVE-DIDACTIC PROGRAM COMPLEX AND NEW FORMALIZED MODEL OF EDUCATION

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In this work, by the example of educational system of Uzbekistan, the new paradigm of education is substantiated in the following edition: education through out all life. It is demonstrated that the proposed in correspondence to this paradigm the formalized model reflects multi-component system, multi-variant approaches and cyclicity of educational process in a modern society. The examples of educational processes which are modelled within the framework of the proposed formalized model of education are presented.

The place of the concept of innovative - didactic program complex (IDPC) introduced earlier by authors in the new model of education is considered.

At that it is considered that IDPC is intended for providing effective educational process at studying of a specific subject. In the work the definition of IDPC components is provided, as well as their functional filling, the description of program interface which accompanies the process of education and also examples on specific university lecture course “Algebra and the theory of numbers”.

This message anticipates the received results on creation of optimization model of educational system of the higher education school of Uzbekistan which is intended for forecasting of optimal quantity of experts in different branches at conduction of admission into higher educational institutions.

USE OF ICT IN TEACHING OF THE REGIONAL PHYSICAL GEOGRAPHY

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A modern teacher has to control the teaching process and lead a pupil to get the knowledge by using of up-to-date technologies that contributes to their self learning and self realization. In this case the teacher becomes not only a source of knowledge and also coauthor, organizer of the research process, information processing and creative works. In the report, we offer a method of using of computer techniques to the teaching of the physical geography of the countries by on the example of Uzbekistan at in the faculty of geography of the Uzbekistan National University named after Mirzo Ulukbek. The results of the research have been successfully tested in other universities.

Teaching of the subject of physical geography of Uzbekistan by using of modeling different processes by computers essentially increased the interest of the students to the subject efficiency of the teaching and learning process, made it possible to independently learn some fields of the subject , consider a number of examples from real life, made easy caring out the main forms of teaching in the form of dialogue between teacher and students. Electronic teaching-methodological complex i.e. automatic learning system one have to accept as a learning informational media that is a logic continuation of the traditional methods based on the text books and which considering the specific intellectual properties of PC provides permanent improving of the teaching process: using of the innovative technologies, development of the teaching-methodological complexes on the base of using of the information and multi- media technologies.

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METHOD OF DECOMPOSITION FOR THE DECISION OF THE PROBLEM OF DISTRIBUTION OF RESOURCES

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Process of performance of a complex of operation of a network is controlled. There is certain freedom in operation select which will be carried out at present time, and intensity their performance. The problem consists in that for each moment of time, at existing restrictions on resources, should be named a vector of intensity u with which it is necessary for operation of a network to carry out. As the best are considered such u which deliver a minimum to the some functional.

All restrictions with which should satisfy intensity u, are broken into restrictions of two types.

Restrictions of the first type describe interdependence of performance of operation (the attitude of precedence). Generally restrictions of the first type are described by some function which to each condition x of a complex of operation of a network puts in conformity allowable values u operations intensity, from some set $U(x)$ with which operations of a complex in the given condition can be carried out. Thus, generally, restrictions of the first type look like:

$$u \in U(x).$$

Restrictions of the second type include restrictions on resources. Generally the vector of the resources spent on performance of a complex of operation at the moment of time t, depends on conditions $x\,(t)$ of a complex of operations and intensity $u(t)$ at the moment of time t. We shall designate $V(t)$ - a vector of the total resources available in system at the moment of time t. $D(x(t),\ u(t))$ - total of the resources spent on performance of operation of a complex during the moment t. Then, generally, restrictions of the second type look like:

$$D(x(t),\ u(t)) \leq V(t).$$

If the vector $V(t)$ during each moment of time is set, calculation of performance of a complex of operations is reduced to the following: for each moment the vector intensity $u(t)$with which it is necessary to carry out operations of a complex should be named. It is considered the best such $u(t)$at which some functional will be minimal. As functional it is possible to take, for example, a degree of satisfiability of complex of operation by some directive fixed moment of time T.

It is possible to replace restrictions of the first type the penalty for their infringement. As is known, computing methods of search of the optimum decision can be treated as use of penalties. In one cases the penalty is imposed without taking into account features of a problem, in other cases there is a communication between a deviation from the optimum decision and size of the penalty.

The penalty for infringement of restriction of the first type we shall take into account in functional. Then instead of functional F, it is optimized functional with the penalty.

The considered problem is solved a method offered in [1]. Use of this method allows to carry out decomposition of an initial problem on sequence of problems of linear programming not the big dimension. Thus, the problem of planning and distribution of resources of a network with directive terms of performance of a complex of operation of a network is reduced to sequence of problems of construction of the decision for set technological independent operations on the basis of methods of local optimization.

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"QUESTIONING AS A LEARNING TOOL" - MATHEMATICS TEACHING METHOD

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In this report I would like to talk about one mathematics teaching method in mathematical lecture. Teaching skills are important for lecturers at all levels, as well as the role of the lecturers and theirs mathematics teaching methods has an important role in guiding students' mathematical development. You should be ready to show some interest in your teaching and must know key of teaching methods. For example, you have a lecture, and you know how much time it will be long. You know as much about the topic as anyone in the lecture hall except, maybe, your advisor. You know topic of your talk and what do you need for talk? Is there a definition you need for comprehension? Firstly you must know that an illustrative example will usually give the audience more sense of the topic than proofs. Some students do better with practical examples while others do better with definitions.

One of the key teaching methods that is of great importance is using questioning as a learning tool. Questioning students not only allows the lecture to evaluate the level of understanding but also provides for feedback, fine tuning the levels of teaching, as well as improving the educational material presented. Teaching is learning. To teach is to learn. Good lecturers learn and adapt to their students, and expand or refine their teaching material as they learn about themselves as well. Questions allow the instructor to continually adapt the material and level of the lesson to better meet the students needs. By questioning students and seeing their strengths and weaknesses are one can provide better educational material. Fine tuning lessons can be very important.

To find the constitutive questions and those very questions are perhaps is one of the most important tools of teaching and Teaching Method.

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THE RELATIONSHIP BETWEEN THE ANXIETY LEVEL AND THE BELIEFS ABOUT TEACHING MATHEMATICS OF THE FRESHMAN ELEMANTARY TEACHER CANDIDATES

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One of the important subjects of mathematics education is the determination of the beliefs of the teachers/teacher candidates (Baydar&Bulut 2002). Beliefs are affecting the learning in their domain of expertise, anxiety and attitude of the students. (Krows 1999). The mathematics anxiety which have both emotional and cognitive structures is one of the most important affective factors that have influence on the attitude and achievement of the students (Bessant 1995). For this reason in this study, the relationship between the beliefs about teaching mathematics and anxiety level of the prospective elementary teachers was investigated. For data collection, two instruments were used: "Mathematics Anxiety Rating Scale: Short Version (MARS-SV)" of Suinn and Winston (2003) which was adapted by Baloğlu into Turkish (2010) and "Beliefs about the Teaching of Mathematics [BaToM]" which was developed by Baydar (2000). The BaToM reflects the child-centredness beliefs. The five factors of the MARS-SV are also labelled as the Mathematics Test Anxiety, Course Anxiety, Computation Anxiety, Application Anxiety, and Social Anxiety. The questionnaires were administered to 96 students of Department Primary Education in a state university in Central Anatolia. Data analysis involved descriptive and inferential statistics. A significance level of 0.05 was set for all inferential tests. The results of this study were: a) the freshman elementary teacher candidates' scores on the child-centredness beliefs were high in general whereas the mathematics anxiety levels were low in general, b) the correlations between the sub-scales of the MARS-SV and child-centredness beliefs are not significant.

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INTERSUBJECT CONNECTION PROBLEMS IN TEACHING OF THE SUBJECT ALGEBRA

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One of the important purposes set for ward in any mathematical course as well as Algebra is to deliver aquired knowledge to students consciously in a scientific way. To reach this target the material of the course must be delivered to students in a related way, systematically and in a unique in way. To realize this creating and fortifying of intersubject connections are of special importance. Study of creation technology of intersubject connections is the content of the presented work. Related with education refertus this problem is especially actual in teaching modern Algebra which is embodiment of the integration of subjects that used to be taught differently.

In the paper o number of definite teaching technologies has been determined:

(1) Using of symbols and theoretical–set language;
(2) Creation of intersubject connections with structure schemes, [1];
(3) Denoting mathematical suggestions in a lecture, [2];
(4) Eliminating of misunderstanding of expression with different terms of definitions with the same content, [3];
(5) Eliminating of confusion which arise by giving in different content of the same definition;
(6) Strengthening of coordination of learnt materials in lectures and practical works.
(7) Using of repetitions of various types;
(8) Over-emphasizing on the place and importance of algebraic definitions separately in the whole course.

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EDUCATIONAL DEVELOPMENTAL TRAINING OF MATHEMATICS AT SCHOOL

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The following provisions shall be put to present the given Thesis: The theoretical ground of Mathematics teachers educational evolutive activity is to be based on philosophical perception of personality development with system and activity approach to the educational system. It will help to present teachers training activity as the one aimed at the formation of subject knowledge and skills, and at the development of cognitive, affective and evaluative spheres of students activities, while the student is considered as a self-improving subject of learning process. Educational humanization is significantly changing the attitude toward the nature and essence of the pedagogical process, where teachers and students are the subjects of creative activity. Educational humanization is an important basis of the future teachers training improvement. It allows preparing teachers for implementation of educational and developmental functions of an academic discipline. The learning process model implements educational and developmental function of Mathematics in school. It should include the following components: didactic conditions updating educational and developmental functions of Mathematics teaching; learning objectives, making the prior educational and developmental functions of the educational process; learning content corresponding to the main types of mathematical activity, which describes the essence of the subject matter of Mathematics, its basic ideas and methods of the cognition of reality, revealing the structure of mathematical objects; teaching methods providing students incentives in motivational and cognitive spheres, and forming the basic mental operations and reflective learning activities; learning tools included in the structure of educational and developmental process, which provide search and creative activity of students in the educational process. The process of preparing future teachers of Mathematics should be based on interdisciplinary integration of the target, substantial, procedural and effective components of academic disciplines, among which the Theory and Methods of Mathematics Teaching course is the main. This will provide a more complete disclosure of educational and developmental functions of the special disciplines. Methodological system of training future teachers increases the didactic-educational potential of the subject, psychological, educational, teaching, special subjects and teaching practices, if the process of learning is build on the principles of fundamentality, binarity, main idea, continuity, students' independent work of research nature increase.

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OBJECTIVES AND TASKS OF EDUCATING IN THE XXI CENTURY

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Training of highly qualified personnel competent to the XXI century is one of the first-priority tasks of any state. These requirements are caused by the peculiarities of modern time.

The first of the peculiarities is a process of globalization. The mankind as never before is aware of open society building importance, i.e. humane society without national, religious and other boundaries. A citizen of this society should be free, tolerant and open for changes person. To educate this sort of person we need to search other approaches through analyzing the world know-how.

The another peculiarity is information explosion that is resulted by the uncontrollable information growth, by the prompt obsolescence due to new knowledge about the world, from the one side; and that currently the information can be easily found, from the another side. How can one orient in the existing information variety? Because books, manuals, mass media and worldwide Internet net contain too much knowledge and facts that are not applicable at the same time to human‘s mind. How to screen the information, how and on the basis of what a person can evaluate the information given to him? And at last, how can one chase the actual for today information, what does one have to do to utilize his knowledge to extend his knowledge?

These complex issues are in front of a modern human-being.

The third peculiarity is changes. They are swift and actually they touch upon the whole planet, all spheres and conditions of society‘s activity and human life. The character of a work varies as well it is becoming more and more intellectual. All these changes result another demand for qualified structures of the population‘s different categories; require professional and social mobility, constant education and permanent professional retraining not only at the beginning, but also at the middle of career.

How can one teach and learn? How can we prepare a schoolchild and a student to the life? Knowledge of what does have importance? How can we imagine a teacher of the XXI century? What is a teacher‘s purpose in new conditions and how can we achieve a compliance with a targeted ideal?

These are issues questioned by life and discussed all over the world. This article expounds appropriate treatment of the mentioned above problems.

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COMPUTERIZED ANALYZE OF MOTION PROBLEMS WITHOUT PARAMETERS BASED UPON THE GRAPHIC STRUCTURE[1]

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Problem solving is a very important concept not only as a teaching method but also as a matter which takes place in the mathematics curriculum, and needs to be focused on. NCTM standards also emphasize the priority of problem solving skills to include in the teaching of mathematics and teaching mathematics with problem-solving approach. Among the issues related to the curriculum of math problems; the movement problems, has a significant role because it includes different kinds of problems; and different solution ways and non-routine problems can be used. It is expressed in literature examination that the students have number of difficulties in learning concepts and the relationship between them, so difficulty in teaching mathematics is usually taken. Therefore, mathematics educators agree on the issue that , students should develop problem-solving skills and this should be the primary purpose of education. for better understanding of Problem solving by the students, test questions or open-ended problems are available. However, test questions cause not to evaluate the process of problem-solving, open-ended questions cause teachers in large classes not to find out deficiencies of their students learning. In this case, the students' individual weaknesses cannot be understood and cannot be achieved in the level of learning required. Therefore, in this study; students instead of memorizing the solutions with formulas are suggested to use a solution model based upon graphic structure in order to understand and achieve to format their meaningful learning. By programming this model both the weaknesses of students' learning will be corrected and heavy burden on teachers learning outcomes will be fully evaluated. In the study, made on the movement problems, firstly the problems asked about the issue are classified. Then in the generated classes 250 movement problems in different kinds are analyzed with the solution model based upon graphic structure prepared. Because the structure can solve the problem in both forward and backward chaining method; it provides different solutions of the problem, if any to be viewed.

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ABOUT CHARACTERIZATION OF INNOVATIVE EXPERIENCE TEACHERS OF MATHEMATICS IN MODERN CONDITIONS

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Innovative experience, we understand the range of tools, methods, forms and resources used by the teacher to introduce new innovative technologies in the learning process or solve a specific set educational and methodological problems. Formation of the innovative experience teachers include [4]: Subject innovative pedagogical experience, the source changes(contradictions, new learning tools, new conditions of education activities, etc.), the idea of change (the essence of an ICT: in the educational, communication and information, or other technologies to change the content of education, organization of educational or training process, the concept of change (processes, their advantages over similar products and innovation, restrictions, complexity, risk), the terms of the implementation of changes (including personal and professional qualities of the teacher and achieved by level of professionalism, the result changes, the publication of submitted innovative pedagogical experience. The main characteristics of the description of the innovative experience of teachers are similar in structure research on mathematical education should include the following aspects: relevance of the problem of an ICT and a working hypothesis, scientific novelty, theoretical value of an ICT, the practical significance of an ICT. For example, improving methods differentiation in the process of teaching mathematics is to develop diagnostic and methodological support in teaching pupils to create different groups of students to master the material and implement differentiated approach to learning through the use of "different levels of training tasks, reproduction - where problems are solved by students on the basis of the newly learned knowledge and ways of work that they reproduce from memory the general education-related tasks require students to use learned knowledge and methods of activity in atypical, but a familiar situation, which is accompanied by a transformative playback. Creative level, require students to creative activity in the selective application of learned knowledge and techniques solutions and design in a relatively new situation for him. This allows us to define the basic requirements to the skills of students at different levels. Research and practice has shown [1], [2] that the main characteristics of the innovative experience are: values and value orientations, goals and objectives, management education, the content of education technology, including ICT tools, methods, organizational forms, learning environment, organizational methodical environment informational, organizational, motivational, staffing, methodological, logistical resources, and regulatory resources to influence the most important aspects of training and education. Record and document them not only promote the dissemination of best innovative experience, but also systematically develop the use of innovative technologies in learning mathematics. It is necessary to navigate a wide range of modern innovative technologies [3], ideas, schools, directions, do not waste time on the opening of the already known and use the entire arsenal of experience. Teacher needs to adjust its innovative experience of the most characteristic of innovative technologies such as ICT, learner-oriented technologies, information and analytical support to the educational process and quality management education school, monitoring, educational technology, teaching technology as well as psychological and pedagogical support innovative technologies.

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STRENGTHENING OF APPLIED MATHEMATICS TEACHING ORIENTATION

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Mathematician's ability to conduct rational reasoning is the attribute of thinking. The success of using mathematic to solve practical problems is directly dependent on how the subject is armed in solving problems, by the skills of conducting rational reasoning. Therefore, to form in students initial ability using rational reasoning at the school bench must be considered as an important component of applied way of teaching mathematics. There are the following five most common types of rational arguments in applied mathematics: a) the use of statements that are valid in real cases, but allows to construct the artificial counterexamples; b) the involvement in the process of solving practical considerations; c) the arguments that are based on analogy or experiment; d) the proof based on the consideration of particular cases; e) using the results of the approximate calculation in the absence of a specific assessment of the resulting errors. Directly we can't form these types of rational reasoning in students. In this regard, we identified the following actions that are underlying the above types of reasoning: 1) discarding the redundant features, the choice of missing 2) the reasonable refinement and modification of problem's statement and intermediate results along the solutions, and 3) a reasonable involvement of physical experiments in solution with real content, and 4) replacement of the exact but complicated approximate formula but relatively simple formula, and 5) ensuring consistency between the degrees of precision stages of formalization and intramodel solutions; 6) the exclusion from consideration cases or subtasks that are inadequate to the initial actual situation, 7) model selection with taking into an ease of practical implementation of proposed solutions; 8) compared intramodel result with the original real situation. Connection between the types of rational reasoning and selected actions are shown in table 1.

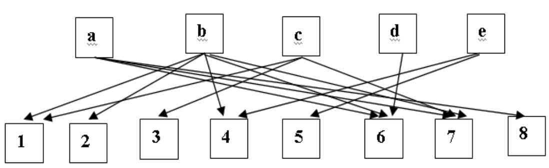

Table 1.

Coupling between FEM and FDM in one dimension. The interior nodes of the unstructured FEM grid are denoted by stars, while circles and crosses denote nodes, which are shared between the FEM and FDM grids. The circles are interior nodes of the FDM grid, while the crosses are interior nodes of the FEM grid. At each time iteration, FDM solution values at circles are copied to the corresponding FEM solution values, while simultaneously the FEM solution values are copied to the corresponding FDM solution values at cross nodes.

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ON THE MODULAR TRAINING FOR STUDENTS OF MATHEMATICAL DISCIPLINES

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At the present stage of development of the educational process of a gradual abandonment of the priority of forming ZUN (Knowledge, skills and abilities) in pure form. The center of attention is transferred to the formation of the ability to self-awareness, ability to solve everyday professional tasks based on the knowledge. These categories are included under the term "competence". Educating a competent person should be the primary ultimate objective of the educational process in schools and universities. Achieving this objective is facilitated if there is an improvement of the educational process of learning to build a modular approach that is based on the predicted order to optimize the workload. The modular approach is based on systematic, structured, dynamic and exible principles.

In general, modular training could be defined as an approach to training, which involves the following: - Student works independently with the curriculum provided to him in the form of modules; - Content and process of development of modules adapted to individual abilities and needs of the learners; - Management of the learning process occurs in a mode of feedback to the establishment of the initial, intermediate and final states of the learner to orient it on the learning objectives; - Interaction between teacher and student is based on a parity basis.

The main object of the modular training is creating exible educational structures, both in content and in providing training, which guarantees the satisfaction of needs, appropriate that time, and which defines a new vector of occurring interest. Tool of the modular training is a module. Usually, the module understands the autonomous organizational and methodological structure of the discipline, which includes teaching objectives, is logically complete unit of learning material (aligned with the inter-subjective and interdisciplinary connections), guidance (including didactic materials) and the control system.

Within a single discipline module is the targeted functional site that combines educational content and methods of training activities on the mastery of this content, i.e. instruction to achieve a goal of teaching and learning activities, individual program, which contains the target action plan, a block of information, guidance on the implementation of self-control, self-esteem, self-analysis. The module includes: 1) an action plan with specific goals; 2) block of information; 3) guidance to achieve these goals.

The module involves the following activities: - At the beginning of the module incoming inspection skills of students to determine their level of preparedness for future work is carried out. When necessary, the correction of knowledge through further explanation is taking place. - It is required to implement the current and intermediate control at the end of each training element. Most often it is self-control, checking the samples, etc. Its goal is to identify gaps in learning levels of educational elements and eliminate them. - After the completion of the module the output control is carried out. Its goal is to reveal the level of assimilation of the module with subsequent modification.

Technology of modular training creates a solid foundation for individual and group self-studies to the students and saves up to 30 percent of training time without sacrificing completeness and depth of the material under study. In addition, the achieved exibility and mobility in the formation of knowledge and skills of students develop their creative and critical thinking.

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ON HISTORY OF DEVELOPMENT OF MATHEMATICS

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It is impossible to imagine our present world without mathematics, application of mathematical knowledge in one or another field of humanity activity. That's why we offer to make a short excursus in dynamics of the mathematics development from ancient times till today, which continue with growing rate about 4-5 thousand years.

Development of mathematics, as well as our life is full of light up idea splashes and its stagnation, triumphs and tragical moments. It is enough to recollect Gipatiy Aleksandriyskiy, fallen a victim of religious fanaticism (V century), Ulubek Taragay , who was killed as a result of conspiracy against him (XV century), Galua Everist, who was lost in 21 years old in a duel for practical motives in days of French revolution in France (XIX century).

In XYIII-XIX centries the swift development of mathematics aroused high progress in many sciences, both scientific and applied plan. In connection with it in our work we made an attempt to show in short how was making contribution to a mathematical science by the countries ancient East, West Europe and Russia. The basic results and mathematics researches making by scientists of ancient times up to now also is briefly described. Then separately was shown directions in mathematics obtaining by those or other scientists and how were formed various sections of mathematics.

Dynamics of mathematics development in Azerbaijan and acquaintance the reader with representatives making contribution to mathematics science of republic are marked out. Then there was given some list of main mathematical organizations, societies and institutes, being world cradle of mathematical science.

Information on awards and winners in mathematics area on a world scale is of interest. It is quite a lot important the acquaintance of reader with mathematical symbolism close connecting with general development of concepts and methods of mathematics.

Certainly, the wide range of questions light up in given work aren't exhaustive and is given enough briefly, but for the first acquaintance is enough interesting. This book can became handbook for many mathematics and students.

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OF REVIEW ARTICLES ON THE RESULTS OF SCIENTIFIC SCHOOLS

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In many ways beneficial, as experience shows, reviews of scientific schools in the context of international mathematics - a quick introduction to the topic, determining the place of results in the development of themes, etc., and finally, the convenience of registration papers and theses. Overview of 2010 [1] is based on 613-page book [2] and materials [3] International Conference "Function theory and computational methods," in 2007 (and four other publications, 2010) and partly a continuation and development of those studies included in the survey [4] of 1997, set out on pages 5-261 edition [2]. The content of the review [1] are topics and subtopics.

Computer width. Functions classes. Algebraic number theory and tensor product of functionals (in combination with harmonic analysis) in the tasks of recovery. Uniformly distributed grid and the problem effectivization Monte Carlo method. Algebraic number theory and harmonic analysis in numerical integration. Application of tensor products of functionals in numerical integration. Reconstruction functions. Discretization of solutions of partial differential equations. Probability-theoretic approach to the problems of analysis. Embedding and approximation theory. Fourier series.

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MATHEMATIZATION OF EDUCATION IS THE PRIMARY CONCERN OF TECHNICAL EDUCATION QUALITY IMPROVEMENT

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These days higher school teachers face the issue of the quality of higher education as the main criterion of an expert's competence is his/her achievement of such a level that allows to be a claimed expert in any country of the world on receiving corresponding education and qualification. And exact and engineering sciences in the sphere of higher education are higher-priority at the new stage of education development. In this connection mathematical education plays an important part.

Considering the issue of fundamental education it is impossible to overlook basic mathematical knowledge. It is very important, because in engineering education the solution of technical problems in many respects depends on the relationship between mathematical, physical, technical and other knowledge.

In this connection mathematical preparation of engineers-to-be is considered on the one hand a necessary means for studying general technical disciplines, on the other, a device for solving professional tasks. In this aspect mathematical education should not stop with the end of the core course in Mathematics. We the need for optional courses in Mathematics, without which it is impossible to design mathematical models of technical processes.

Mathematical preparation of a future engineering should be continuous, i.e. the knowledge of the general course in Mathematics should be applied in special courses, the content of which is variable and is connected to students' specialization. Special courses in Higher Mathematics will provide mathematical apparatus for qualitative mathematical preparation of university graduates of various specializations within the framework of the selected specialty. Course and degree projects should naturally follow special courses, as they require mathematical knowledge. Such approach will allow to shorten the gap between the Mathematics studied as a general course, and the Mathematics used in practical activities for solving professional tasks.

The structure of mathematical preparation of a technical university student should be such that each subsequent level should be based on the previous one: 1- Higher Mathematics, 2 - special courses in Higher Mathematics, 3 - course project work together with major sub-departments, 4 - mathematical provision of degree projects.

Taking into account the role of Mathematics in technical universities and the objectives of the study of Mathematics, we have introduced the sequence of the study of Higher Mathematics sub-disciplines according to the State Standards. After the general course there come special courses in Mathematics, directly related to students' future profession.

Thus, the concept of professional applied orientation of training university students in Mathematics is implemented in practice.

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AN INVESTIGATION ABOUT HIGH SCHOOL STUDENTS' MATHEMATICS ANXIETY LEVEL ACCORDING TO SOME VARIABLES

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Mathematics anxiety is one of important components of mathematics classroom. The mathematics anxiety which have both emotional and cognitive structures is one of the most important affective factors that have influence on the attitude and achievement of the students (Bessant 1995). Negative attitudes and mathematics anxiety are identified as barriers in learning mathematics and attending courses and jobs/careers which require mathematics (Baloğlu, 2010). For this reason, to investigate stundents' mathematics anxiety level and reason of this is the one of the most important issues of mathematics education.

In this study, we investigated high school students' mathematitcs anxiety level according to school type, perceived primary and secondary mathematics teachers' attitude and gender. For data collection, "Mathematics Anxiety Questionnaire" which developed by Dede (2008) was used. The questionnaire has four foctors: Individual Anxiety, Peer Anxiety, Task anxiety, Test Anxiety. The questionnaire was administered to 76 high school students in the one of the city of Central Anatolia. Data analysis involved descriptive and inferential statistics. A significance level of 0.05 was set for all inferential tests. Data analyzing show that the anxiety level of the students is quite high.After analyzing data, there were considerable statistical differences among students' mathematics anxiety levels according to perceived primary school mathematics teachers' attitude at task anxiety factor. We also had the same results at school type variable. On the other hand, there was no significant difference among mathematics anxiety level of the students according to perceived middle school teachers' attitude and gender.

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"UNDERSTANDABLE" STRANGE NOTIONS AND OTHER "VERMIN" OF PROFESSIONAL MATHEMATICS IN SCHOOL MATHEMATICS

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As you know, all used in math concepts and terms must be made in strict definition, which is mainly done in the school mathematics course, taking into account the age factor of students.

Nevertheless, in our view, a number of important concepts in school mathematics is applied without any explanation for granted that can not pay its share of difficulties in learning this, is not related to an easily accessible, discipline.

Identified a number of mathematical concepts, which at the school level require explanation: the set, the quantity, variable, dependent variable, function as the dependent variable, compliance, dependence, formula, direct and inverse proportionality; scope; range.

There are things that should be left without the inherent professional mathematics (school math as a glow of understanding of the mathematics with its objectives and subordinate to its content rules) through the formalization of the definition.

With regard to some of them, such as *a set, match, dependence* can restrict ourselves to the explanation of examples, without any formalized: "*The concept of the set taken as the basic, ie not reducible to other concepts* "- we read in [1, p.5-6]. Meaning of the term "*set*" can be illustrated by examples, as well as, for example, under "*housing*" can be understood as the house, hut, cottage, villa, apartment, or as a "*stationery*" understand the pen, pencil, eraser, notebook, paper. The notion of "*correspondence*" used in school mathematics in the usual sense with the simultaneous involvement of the words "*relationship*", etc., as a synonym;

Others, such as *value* and *variable*, should be consumed with great caution, if only to "*do no harm*" And quite renounce the *dependent variable* and *independent variable or argument of function* name symbol, which denotes an arbitrary element of the function definition.

Thirdly, the type of *formula* - in the development of a particular situation. "*Formula - an expression of a formalized language designed for writing judgments. ... In practice, the mathematical formula known as meaningful combination of symbols that carry a variety of semantic meaning*" We read in [2, p. 637]. But such a definition is too difficult to understand students. Attempts to clarify what is meant by a "*formula*" we do not find in textbooks, operating in Kazakhstan.

Fourthly - type "*domain of definition*" is replaced by "*set of definition*" Better than in the definition of the term "*domain*" of the term "*set*"? The term "area" in a professional mathematics means "*open connected set*".

Fifth - such as "*directly proportional*" to apply only after a clear-cut definitions. Apparently, in the general case of *direct proportionality* should be defined as a function of the form $f(x) = kx$, and the *inverse proportionality* - as a function of the form$f(x) = \frac{k}{x}$ with a description f of the rules in the form of "*algorithm*", followed by comments by the identities of $\frac{y}{x} = \frac{f(x)}{x} \equiv k$ and $y \cdot x = f(x) \cdot x \equiv k$ (see [3, pp. 60,65]) .

And finally, We must confess that I do not quite understand is widely used in mathematics, the term "*concept*", which, apparently, it is necessary to devote a separate study.

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ON PRESENTATION OF THE FUNCTIONS IN SCHOOL MATHEMATICS

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The notion of function relates to the central concepts of mathematics in general, particularly for School Mathematics.

The history of the concept of functions pervades the whole history of mathematics from antiquity to the time of Dedekind (1887), Lobachevski (1834), Dirichlet (1837), and according to some views (e.g., A. Church (1960) This concept should be attributed to the initial or undetectable. All this once again underlines the complexity of learning the concept of function, especially in high school.

Secondary school teachers and university professors of Soviet and post-Soviet era have gained various experience in teaching this topic. It seems to us common way to determine the function of the sequence of the variable as something changing, the independent variable, the dependent variable and function as a definite relationship between the variables, on a plan designed to facilitate, in fact, creates a formidable obstacle to understanding.

In order to make it more comprehensible students is a complex concept of mathematics, discuss the main points that need to be, in our opinion, to consider the study of functions in high school.

(1) The introduction of the function is necessary to discuss each key word (x argument, the rule f, the value of the function $f(x)$, the set of definitions, a set of valid values) in the definition [1, p.24].
(2) "Rule" - is the same as a law, and an algorithm, and compliance.
(3) Teach students to write an analytic function to formulate a rule that a given function describes, "as well as" has to be taken out and vice versa, according to the verbal formulation of rules to be able to write it analytically.
(4) Algorithmic definition of the function should be used when setting studied in the school of numerical functions.
(5) With proper study of the concept of a function, students are free to give examples of functions from the surrounding reality.
(6) Students should recognize correspondences among those that are and are not functions.
(7) A visual representation of the function is its geometric interpretation - is a more detailed study of the coordinate system in conjunction number - the point.
(8) Need to develop students' skills of working with graphs of functions, build them with the transition.

Our goal is to clarify the definition of the rules, the algorithm of the law applied to the argument x as a symbolic designation of an arbitrary element of its set of tasks. At the same time refrain from the notion of "dependent variable", focuses on skill to clearly specify in each case the argument, the rule specifying the function, the set of values. To consolidate the concepts proposed by the students to make the problem

During the trainings on working with the definition of the functions of students to set clear and correct perception of a function.

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