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Chapter 5

Inverse conductivity problem for spherical particles

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5.1 Introduction

Inverse problems for random composites with nonoverlapping spherical inclusions are of considerable interest in a number of fields. Detection of locations of inclusions by means of the measurement data on the boundary of a sample can be performed by a method of integral equations and the far field patterns [2,3,6,10]. Application of conformal mappings yielded effective computational methods for two-dimensional problems [11]. The main attention was paid to determination of shapes of one or few inclusions. Various numerical reconstructions by the time reversal method [9] by acousto-electric tomography [13], thermoacoustic and photoacoustic tomography, etc., [1] were developed.

The considered inverse problem can be studied via a perturbation problem when the boundary field induced by a homogeneous body is compared to the field caused by inclusions embedded in the host material. Such a problem can be stated by the Dirichlet-to-Neumann operator for fields governed by Laplace's equation when the Dirichlet data (potential) is fixed on the boundary, and the difference of the normal fluxes called the perturbation term is used to determine the physical properties of inclusions and their location. Let the shapes, sizes of inclusions, and their physical properties be given. The inverse problem is reduced to determination of the location of inclusions by the perturbation term.

In the present paper, we propose a new statement of inverse problem when the shape of each inclusion is fixed as a sphere of radius r_k (k = 1, 2, ..., n). The centers of spheres \mathbf{a}_k (k = 1, 2, ..., n) have to be found during solution to the problem. The radii r_k and the number of spheres n may be also considered as unknown parameters. Here, we consider the case of given n and r_k , in particular, the case of the same radius $R = r_k$. This problem yields an approximate solution to the shape problem, since any shape can be arbitrarily approximated by packing spheres. Though the statement of the previously discussed inverse