Peshaya S.L.

OPTICS

Study Guide for universities

Almaty 2020

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The study guide is intended for laboratory works and brief lecturers for workshop on optics for university students. The Study guide was compiled in accordance with the current curriculum for the general course of physics for students of physical and technical specialties. The study guide contains set of fifteen brief lectures, descriptions of laboratory work on optics, it was considered the corresponding questions of optics and the principles of operation of various optical installations. Each laboratory work contains a brief theoretical description, an overview of the experimental device, a work assignment, and a methodology for carrying out an experiment. The methods of processing experimental and instrumental errors of the results are presented in the apex.

The study guide “Optics” is recommended to study for the 2nd and 3rd year full-time students of specialties 5B060400 - Physics, 5B071000 - Materials and the technology of new materials, 5B071800 - Electric Power Engineering, 5B071700 - Thermal power engineering, 5B072300 - Technical Physics, 5B061100 - Physics and astronomy, 5B060500 - Nuclear Physics and other technical and engineering specialties in educational programs of which the course of general physics is studied.

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Foreword for the Student

Study guide for universities “Optics" is dedicated to one of the sections of general physics -"Optics" and intended for students of technical and engineering specialties of higher educational institutions, contains guidelines for the implementation of laboratory work and brief theoretical lectures on optics. Optics occupies an important place in the system of training specialists in the profile. Being an integral part of the general course of physics, the workshop on optics plays a major role in familiarizing students with the experimental foundations of physical laws and phenomena and allows students to acquire the skills of independent preparation and carrying out a physical experiment.

The Study guide “Optics” includes lectures and laboratory works of photometry, geometric optics, interference, diffraction, polarization and optics of anisotropic media. Statement of the theory and practice of the study guide is given in accordance with the current curriculum.

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Glossary of Symbols

List of Physical Constants

Description	Symbol	Value	Unit
Gravitational constant	G	$6{,}6720(41)\cdot10^{-11}$	$N\cdot m^2/kg^2$
Boltzmann constant	k	$1{,}380662(44)\cdot10^{-23}$	J/K
Avogadro's number	N_A	$6{,}022045(31)\cdot10^{23}$	mol^{-1}
Speed of light in vacuum	c	$2{,}99792458(12)\cdot10^{8}$	m/s
Planck's constant	h	$6{,}626176(36)\cdot10^{-34}$	J·s
Faraday constant	F	$9{,}648456(27)\cdot10^{4}$	C/mol
Electron charge	e	$1{,}6021892(46)\cdot10^{-19}$	C
electron rest mass	m_e	$9{,}109534(47)\cdot10^{-31}$	kg
Electric constant	ε_o	$8{,}85418782\cdot10^{-12}$	F/m
Magnetic constant	μ_o	$1{,}256637061(44)\cdot10^{-6}$	H/m
Stefan-Boltzmann constant	σ	$5{,}67032(71)\cdot10^{-8}$	$W/(m^2\cdot K^4)$
Wien's displacement constant	b	$2{,}897790(90)\cdot10^{-3}$	m·K

International system of units

Quantity	Symbol	Name	Unit
Length	l	meter	m
Mass	m	kilogram	kg
Time	t	second	s
Electric current	I	ampere	A
Temperature	T	kelvin	K
Luminous intensity	J	candela	cd
Amount of substance	v, n	mole	mol

Contents

LECTURE 1. RADIOMETRIC QUANTITIES AND THE RELATIONSHIPS BETWEEN THEM

Optics is the branch of physics where the *behavior* and *properties* of electromagnetic radiation (EMR), including its *interactions* with matter and the *construction* of instruments that use or detect it. We consider in the lecture some units of measurement in optics and basic statements.

Radiometric quantity is a photometric quantity quantitatively expressed in units of energy or power and its derivatives. These values characterize light regardless of the properties of human vision. Energy photometric quantities are indicated by the subscript "e", for example, X_e.

1. Radiant energy - is the energy carried by electromagnetic (optical) radiation. It is measured in joule $[Q_e]$ = J. Serves as the basis for other energy photometric quantities.

2. Radiant flux or **radiant power** is a measure of the amount of energy by the field per unit time through the site. It is measured in watts $[\Phi_e]$ = W. **Radiant flux** is equal to the ratio of the energy carried by radiation through the surface to the time of transfer. (Figure 1).

$$\Phi_e = \frac{dQ_e}{dt}, \tag{1}$$

where dQ_e is a radiation energy transferred through the surface over time dt.

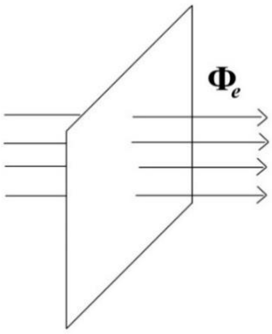


Figure 1 - Interpretation of the radiant flux

3. **Radiant intensity** is the ratio of the radiation flux propagating from the radiation source inside the small solid angle to the solid angle. It is measured in watts per steradian $[I_e]$ = W/sr.

$$I_e = \frac{\partial \Phi_e}{\partial \Omega}, \tag{2}$$

where Φ_e is radiation flux and Ω is a solid angle. (Figure 2)

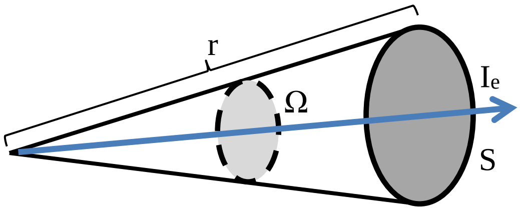


Figure 2 - Interpretation of the radiation power

4. **Radiant exitance** characterizes the radiant flux emitted by a surface per unit area. It is measured in watts per square meter $[M_e]$ = W/m^2. Equal to the ratio of the radiation flux emitted by a small portion of the surface of the radiation source to its area.

$$M_e = \frac{\partial \Phi_e}{\partial S}, \tag{3}$$

where Φ_e is the radiant flux and S - area.

5. Radiance - is the radiant flux emitted by a given surface per unit solid angle per unit area onto a plane perpendicular to the direction of propagation. It is measured in watt per square meter per steradian. $[L_e]$ = W/sr·m^2.

$$L_e = \frac{\partial^2 \Phi_e}{\partial \Omega \, \partial S \cos\theta} = \frac{\partial I_e}{\partial S \cos\theta}, \tag{4}$$

where Φ_e is the radiant flux, S is an area, Ω is solid angle and I_e is radiant intensity. (Figure 3)

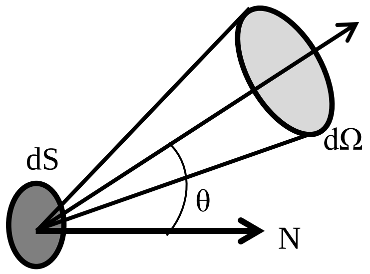


Figure 3 - Interpretation of the radiance

The solid angle is equal to the ratio of an area S of the surface cut out on the sphere by the cone, to the sphere radius *r* squared. The solid angle is measured in steradians [Ω]=sr.

$$\Omega = \frac{S}{r^2} \tag{5}$$

6. Irradiance - is the radiant flux received by a surface per unit area. It is measured in watt per square meter $[E_e]$ = W/m^2.

$$E_e = \frac{d\Phi_e}{dS}, \tag{6}$$

where Φ_e is the radiant flux and S is an area. If the surface is illuminated by a point source, for its irradiation

$$E_e = \frac{I_e}{r^2}\cos\theta, \tag{7}$$

where I_e is radiant intensity, r is the distance between this point and the source, and θ is the angle.

7. Transmittance of a surface - its effectiveness in transmitting radiant energy. (Figure 4).

$$\tau = \frac{\Phi_e'}{\Phi_e}, \tag{8}$$

where Φ_e' is the radiant flux transmitted by that surface, and Φ_e - is the radiant flux received by that surface. Transmittance is a dimensionless quantity. $0 < \tau < 1$.

8. Absorbance is the common logarithm of the ratio of incident to transmitted radiant flux through a material

$$A = \log\frac{\Phi_e'}{\Phi_e} = -\log\tau, \qquad (9)$$

where Φ_e' is the radiant flux transmitted by that material, and Φ_e - is the radiant flux received by that material. Absorbance is a dimensionless quantity.

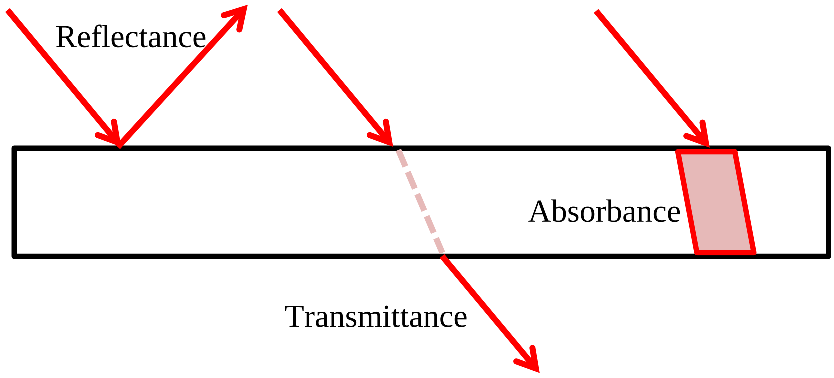


Figure 4 - Interpretation of the transmittance, absorbance and reflectance

LECTURE 2. PHOTOMETRY QUANTITIES AND THE RELATIONSHIPS BETWEEN THEM

Photometry quantities are values formed from radiometric quantities using the relative spectral sensitivity - luminosity function. Photometry quantities differ from radiometric ones in that they characterize light considering its ability to evoke visual sensations in a person. Photometry quantities are denoted by the same letters as the energy quantities from which they are formed, but are supplied with the index "v", for example, X_v.

1. Luminous energy (quantity of light) characterizes the ability of the energy carried by light to evoke visual sensations in a person. *The eye is most sensitive to light at a wavelength of 555 nm.* Luminous energy is measured in lumen second, which is unofficially known as the Talbot. $[Q_v] = lm \cdot s$.

$$Q_v = K_m \cdot \int_{380\,nm}^{780\,nm} Q_e(\lambda)\,V(\lambda)d\lambda = \int_0^T \Phi_v(t)dt \qquad (10)$$

where K_m maximum luminous efficiency of radiation equal to 683 lm/W, $Q_e(\lambda)$ - radiant energy, $V(\lambda)$ - dimensionless photopic luminosity function, representing spectral sensitivity of the average human eye, Φ_v - luminous flux, T - given period of time.

2. Luminous flux - is the measure of the perceived power of light (reflect the varying sensitivity of the human eye to different wavelengths of light). It is measured in lumen. $[\Phi_v]=lm=cd\cdot sr$.

$$\Phi_v = I_v \cdot \Omega \qquad (11)$$

where $I_v$is a luminous intensity, Ω is a solid angle

3. Luminous intensity - is the measure of the wavelength power emitted by a light source in a particular direction per unit solid angle, based on the luminosity function $K(\lambda)$. Luminous intensity is measured in candela - an SI base unit. $[I_v]$ = cd.

$$I_v = K(\lambda) \cdot I_e = K_m \cdot V(\lambda) \cdot I_e = \frac{\partial \Phi_v}{\partial \Omega} \qquad (12)$$

where K_m maximum luminous efficiency of radiation equal to 683 lm/W, $V(\lambda)$ - dimensionless photopic luminosity function, representing spectral sensitivity of the average human eye, I_e - radiant intensity, Φ_v - is luminous flux, Ω - is a solid angle.

4. Luminance - is the measure of the luminous intensity per unit area of light going through in a given direction (Figure 5). **Brightness** is the term for the subjective impression of the objective luminance measurement standard, broadly speaking it is the effect of luminance on the brain. Luminance is measured in candela per square meter. $[L_v]$ = cd/m^2.

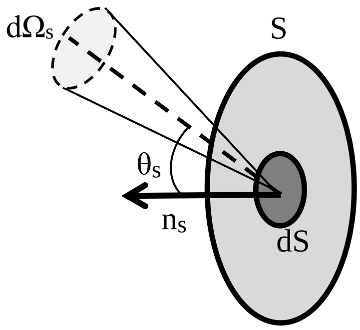


Figure 5 - Interpretation of the luminance

Luminance is a measure of how much power is emitted in a certain direction, radiance is a measure of how much of the luminance you'll actually see.

$$L_v = \frac{d^2\Phi_v}{dSd\Omega_s \cos\theta_s} = \frac{I_v}{dS \cos\theta_s} \tag{13}$$

5. Illuminance is the total luminous flux that incident on a certain surface, per unit area and measures in turn how the incident light illuminates the surface. It is measured in lux. [E_v] = *lx=cd· sr/m²*.

$$E_v = \frac{\partial\Phi_v}{\partial S}, \tag{14}$$

where Φ_v is luminous flux, S - certain area.

6. Luminous exposure is the amount of light per unit area. It is measured in lux seconds. [H_v] = *lx· s*.

$$H_v = E_v \cdot t, \tag{15}$$

Where E_v - illuminance, t - exposure time.

7. **Luminosity function -** describes the average spectral sensitivity of human visual perception of brightness. It is measured in lumen per watt. [$K(\lambda)$]=lm/W.

$$K(\lambda) = K_m \cdot V(\lambda) = 683 \cdot V(\lambda) \tag{16}$$

where K_m is a value of $K(\lambda)$ reached at the maximum of $K(\lambda)$, equals 683 lm/W at the wavelength 555 nm, where an eye is most sensitive to light (Figure 6), $V(\lambda)$ - dimensionless photopic luminosity function.

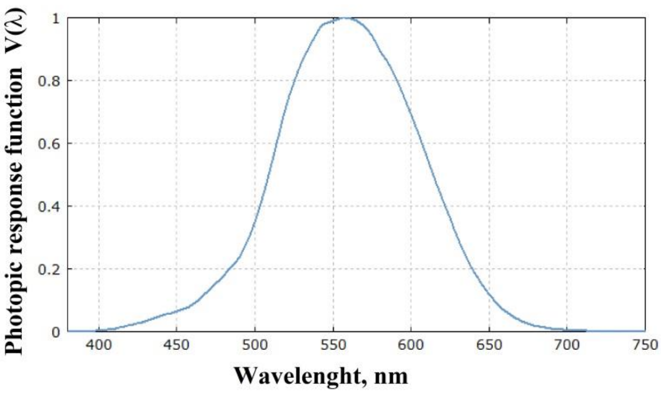


Figure 6 - The photopic response function according to the International Commission on Illumination (CIE)

Inverse-square law

In general, it means that a certain physical quantity could be inversely proportional with respect to the square of the distance from the source of it. In case of optics, this physical quantity is linear waves radiating from a point-like source (energy per unit of area is perpendicular to the source). Let’s make some calculations for Illuminance (E_v):

$$E_v = \frac{\partial \Phi_v}{\partial S} = \frac{I_v \partial \Omega}{\partial S}, \tag{17}$$

where I_v - luminous intensity, S - area, Ω - solid angle. According to Figure 7 and considering that:

$$\partial \Omega = \frac{\partial S'}{r^2};\ \partial S' = \partial S \cdot \cos\theta \tag{18}$$

we will have:

$$E_v = \frac{I_v \cdot \cancel{\partial S} \cdot \cos\theta}{r^2 \cdot \cancel{\partial S}} = \frac{I_v \cdot \cos\theta}{r^2} \tag{19}$$

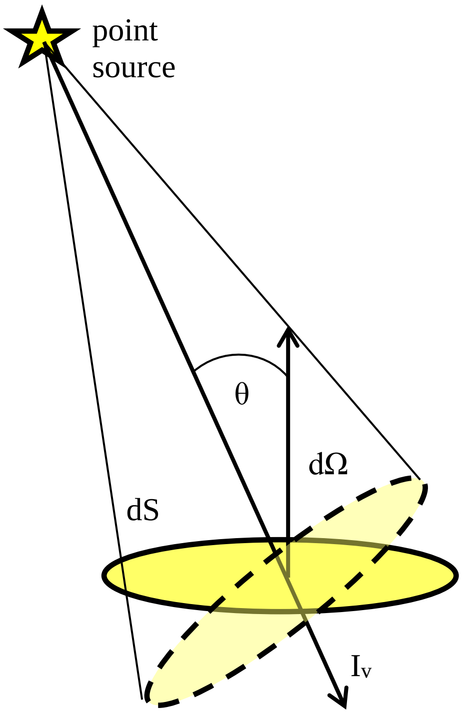


Figure 7 - Interpretation of inverse-square law for Illuminance

LECTURE 3. OPTICAL FIBERS

A waveguide is a structure that directs waves (electromagnetic or sound, etc.) with minimal energy loss. Waveguides have limited expansion to one or two dimensions and their geometry reflects its function. (Figure 8) Transmitted wave frequency also determines the waveguide shape: an optical fiber conducting high-frequency light will not conduct microwaves that are a much lower frequency. The waveguide width must be in the same order as the guided wavelength.

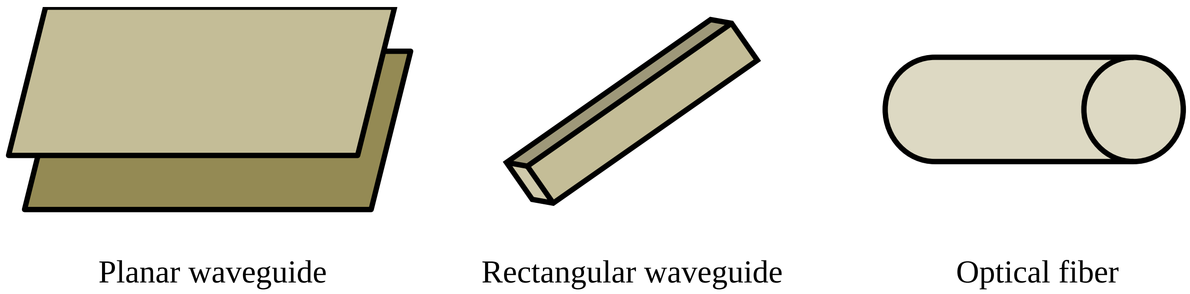


Figure 8 - Types of waveguides

Fiber is a sort of thin optical waveguides which is typically used to transmit information. Usually, fibers are made of quartz glass (with sore polymer dopants), they can be very long and fairly flexible [1].

Most optical fibers have a core with a refractive index which is somewhat higher than that of the surrounding medium (cladding). Due to differences in refractive indexes under a certain angle of incidence occurs a phenomenon, called total internal reflection on which is based propagation of light through the optical fiber. The certain angle is determined by a **numerical aperture**, which in turn is the sine of that maximum angle of an incident ray to the fiber axis. (Figure 9)

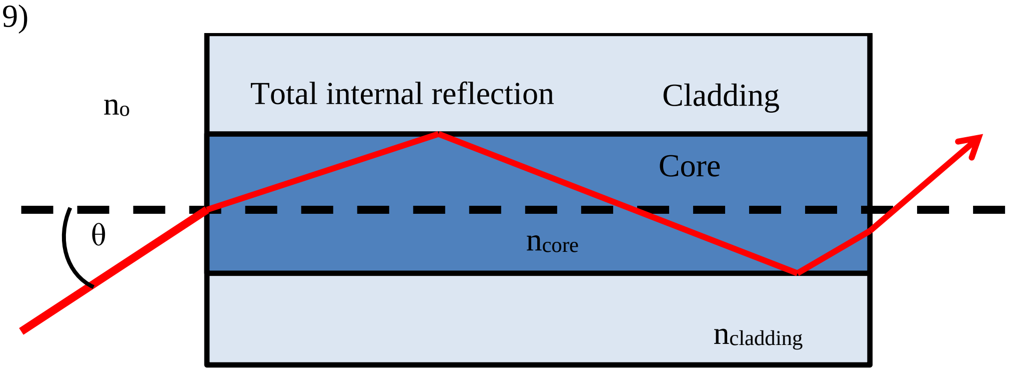


Figure 9 - Total internal reflection in the optical fiber at the core-cladding interface.

The numerical aperture ($\sin\theta_{max}$) can be calculated from the refractive index difference between core and cladding:

$$\sin\theta_{max} = \frac{1}{n_0}\sqrt{n_{core}^2 - n_{cladding}^2} \qquad (20)$$

where n_o - refractive index of an ambient atmosphere, n_{core} - refractive index of core and $n_{cladding}$ - refractive index of the cladding.

As with any transmission of information, the signal fades over time and can be weakened (or lost) for various reasons, which have in turn different nature. (Figure 10)

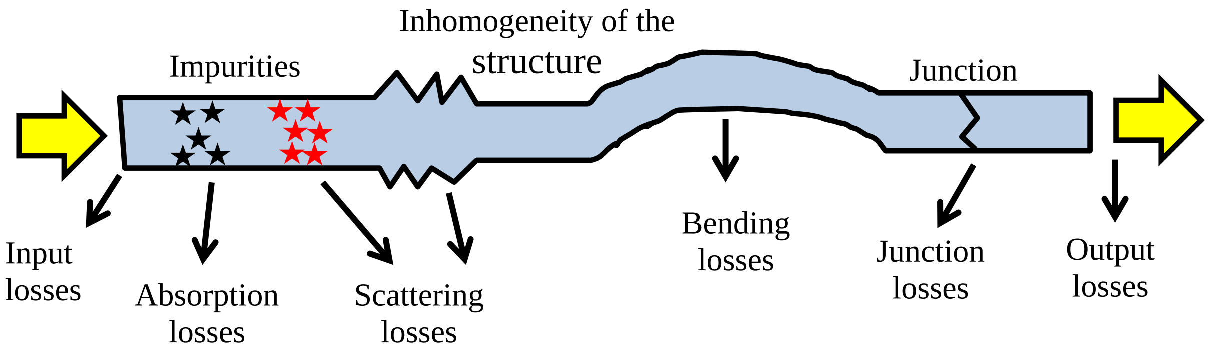


Figure 10 - Losses of nature examples in fibers

The attenuation (loses) of light in an optical fiber could cause by light absorption. Absorption can be defined as the conversion of light into heat, and is associated with resonance in the fiber material. Internal absorption is due to the properties of the fiber material and molecular resonance. External absorption is determined by the presence of impurities in the fiber material of the optical fiber. There are two types of losses due to bending: microbending and macrobending.

Microbending is the microscopic changes in the geometry of the fiber core that occur during production.

Macrobend is a large bend of an optical fiber that exceeds the minimum allowable radius and causes the light flux (or part of it) to leave the core of the optical fiber. A decrease in the bending radius leads to a significant increase in the scattering of the signal.

The attenuation index (measured in decibel dB) for a given wavelength is defined as the ratio of the optical power input (P_{input}) to the fiber to the power of the optical signal received from the fiber (P_{output}):

$$K(dB) = 10\log_{10}\frac{P_{output}}{P_{input}} \tag{21}$$

Typically, the attenuation (losses) index depends both on the parameters of the optical fiber and on the wavelength of the light. Communication through optical fibers is operated in a certain wavelength region, called “telecom windows” (Figure 11) [3].

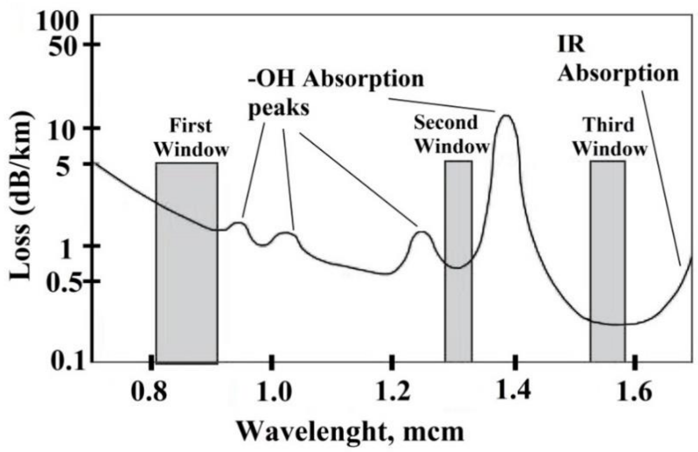


Figure 11 - Optical fiber transmission spectrum

• The first window is at 0.8-0.9 μm. The fiber losses are relatively high in this region, and fiber amplifiers are not well developed for this spectral region. Therefore, the first telecom window is suitable only for short-distance transmission.

• The second telecom window is around 1.3 μm, where **silica fibers** losses are much lower with respect to the first window. The window is originally used for long-haul transmission.

• The third telecom window, which is now very widely used is around 1.5 μm. The losses of silica fibers are the lowest in this region.

It is important to know that for the short wavelengths near the first telecom window Rayleigh scattering of inhomogeneities is important and moving to the UV wavelengths electronic absorption starts to begin. At 1.4 μm there is a strong absorption from OH- groups besides small ones that We can see from figure 11. For larger wavelengths (after the third telecom window) infrared absorption begins to increase.

LECTURE 5. CURVED MIRRORS

A curved mirror is a mirror with a curved reflecting surface There are 2 types of mirrors : convex and concave (Figure 12) [4].

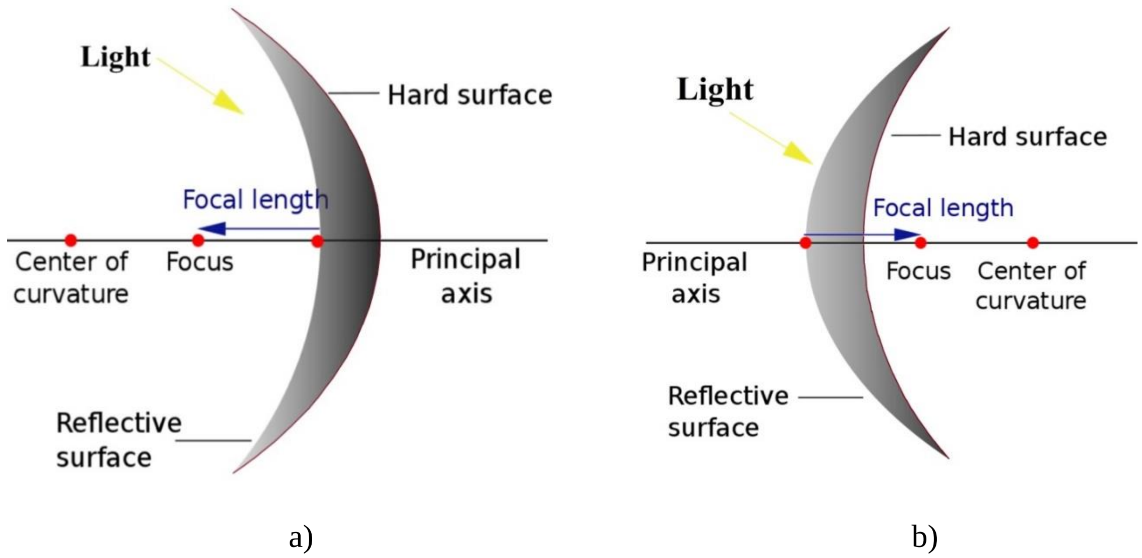


Figure 12 - Types of curved mirrors a) concave mirror, b) convex mirror

A concave mirror is a type of curved mirror, wherein the reflecting surface projects back towards the incoming light. A convex mirror is a type of curved mirror where the reflective surface sticks out towards the incoming light. Let's consider how does the focal length of the mirror is connected with the radius of curvature (Figure 13).

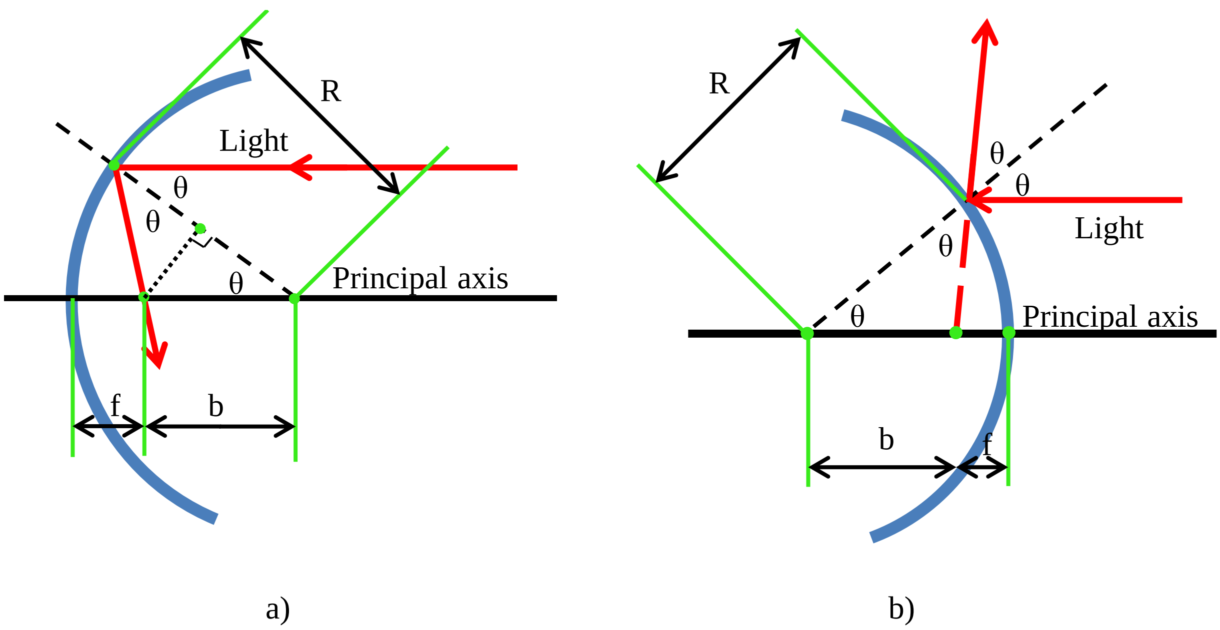


Figure 13 - Ray path in an a) concave mirror and b) convex mirror

The law of reflection tells us that the marked angles (θ) are equal, and the light crosses a principal axis at the focal point. If we drop a median from this point we have a right angle. Using the trigonometry one can conclude that:

$$\cos\theta = \frac{R}{2b} \rightarrow b = \frac{R}{2\cos\theta} \rightarrow f = R - \frac{R}{2\cos\theta} \quad (22)$$

If angles are very small $\cos\theta = 1$, then

$$f = \frac{R}{2} \quad (23)$$

The lens equation works for curved mirrors and shows relates the object distance (d_o), image distance (d_i) to the focal length (f):

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \quad (24)$$

Note, the focal length (because of sign convection) is positive for concave mirrors and negative - for convex mirrors. The magnification index for the mirror:

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \quad (25)$$

Where h_i - is image height, h_o- the object height, d_o - object distance and d_i - image distance.

Let's consider now image acquisition by paraxial rays in a convex mirror (Figure 14). Note, a paraxial ray is a ray that makes a small angle (θ) to the optical axis and lies propagates close to the axis. For paraxial rays we have the following approximation:

$$\sin\theta = \theta; \ \tan\theta = \theta; \quad (26)$$

We can see, that regardless of the distance to the mirror for paraxial rays we obtain upright, virtual reduced image.

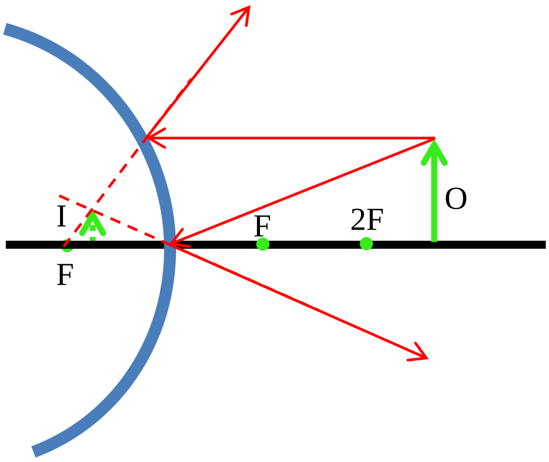


Figure 14 - Image acquisition in a convex mirror

Let's consider image acquisition by paraxial rays in a concave mirror (Figure 15). Here, we can see that type of image (size, virtual or real, inverse or upright) has a strong dependence on the distance to the mirror.

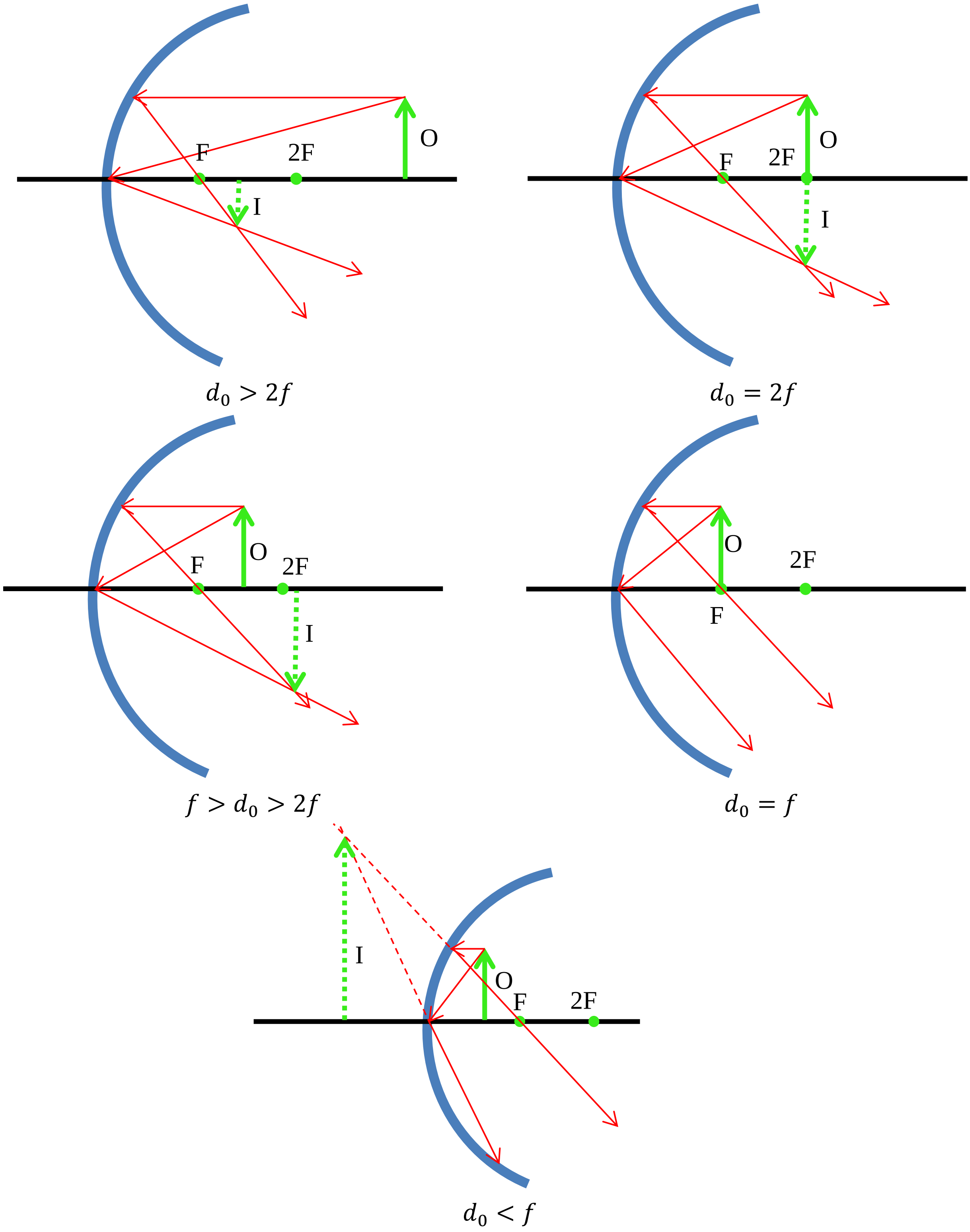


Figure 15 - Image acquisition in concave mirror depending on object distance to the mirror

However, we should consider that mirrors have aberrations (namely spherical aberrations) just like spherical lenses.

LECTURE 6. INVARIANTS FOR PARAXIAL RAYS

In general, invariant means that some physical quantity doesn't change under a transformation. For instance, the invariant is the transformation of coordinates and time during the transition from one inertial frame to another.

Lagrange-Helmholtz invariant

The Lagrange-Helmholtz invariant relates the linear size of an object and the angular size of the rays beam:

$$\alpha_1 \cdot h_o \cdot n_1 = \alpha_2 \cdot h_i \cdot n_2 \tag{27}$$

Here, h_i - the height of the image, h_o - the height of the object, n_1- refractive index of the first medium, n_2- refractive index of the second medium, α_1 and α_2 - angles between the principal axis and rays from the object and image correspondingly.

This invariant mathematically expresses the law of conservation of information in geometric optics (Figure 16).

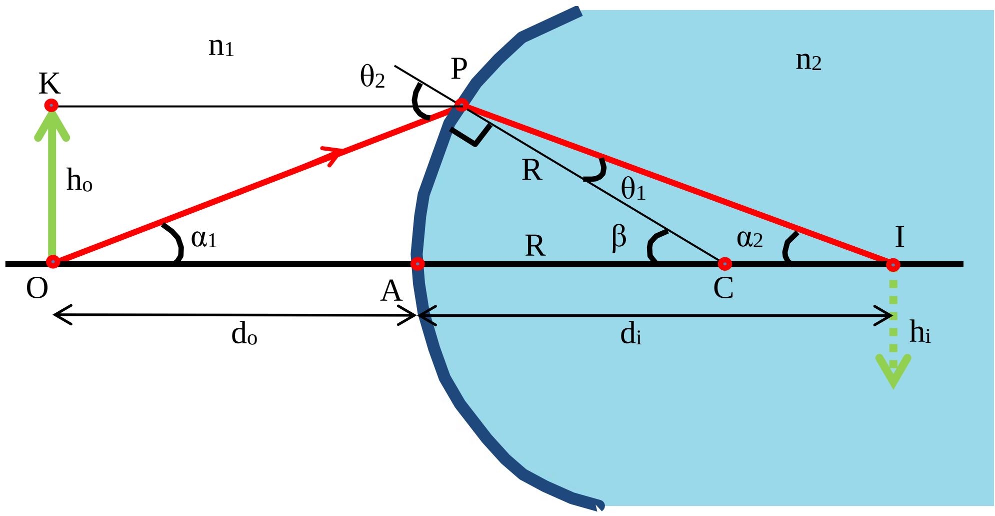


Figure 16 - Quantities that connect Lagrange-Helmholtz invariant

Considering that all rays are paraxial let's consider the angular increase using Figure 16. We can see from ΔOAP, that

$$AP \approx OK \approx h_0; \tan(-\alpha_1) \approx \frac{h_0}{-d_0} \approx -\alpha_1 \rightarrow \frac{h_0}{d_0} \approx \alpha_1 \tag{28}$$

We use the expression relating the angular and linear magnifications [5]:

$$\beta \cdot W = -\frac{f_1}{f_2} = \frac{n_2}{n_1} \tag{29}$$

Then, using the expressions $\beta = \frac{h_i}{h_o}$ and $W = \frac{\alpha_2}{\alpha_1}$ which determines the linear and angular increases, we obtain the following relation:

$$\frac{h_i \cdot \alpha_2}{h_o \cdot \alpha_1} = \frac{n_1}{n_2} \rightarrow \alpha_2 \cdot h_i \cdot n_2 = \alpha_1 \cdot h_o \cdot n_1 \tag{30}$$

The Lagrange-Helmholtz invariant characterizes the information capacity of the optical system, that is, the amount of space that can be displayed by the optical system.

Abbe Sine Condition

Since the elimination of spherical aberration made it possible to obtain practically non-aberrational images of axial points using wide beams of rays, it could be assumed that this will also happen for points located near the axis. Contrary to all expectations, this turned out to be wrong. Abbe showed that different areas of a simple lens form an image of a flat element with different magnifications (Figure 17). The corresponding points of individual images of an object formed by different zones, coincide only on the optical system axis, when superimposed in the image. On the contrary, they are located at a greater/lesser distance off the axis, and a sharp image cannot form on that place.

In order to obtain sharp images, along the spherical aberration correction for the axial point of the optical system. So, to ensure that all system parts give the same sized separate images of the object. Abbe showed that to do this, a specific condition must be satisfied, which he called in turn the "sines condition". The sines condition presents that for all rays illuminated from the axis point of the optical system and after refraction is guided to the conjugate point of the image by the optical system, the ratio between the angles sines of the conjugate rays should be constant:

$$h_o \cdot n_1 \cdot \sin\alpha_1 = h_i \cdot n_2 \cdot \sin\alpha_2 = const \qquad (31)$$

here n_1, n_2 are the refractive indices of the medium from the side of the object and image; respectively, $\beta = \frac{h_i}{h_o}$ is the increase in the optical system, which should remain constant for any pair of conjugate rays emanating from a point lying on the axis and bounded by angles α_1 and α_2 with the principal axis. Two points for which spherical aberration is eliminated and the condition of sines is met are called *aplanatic*. Abbe found that only one pair of aplanatic points is possible on the principal axis.

Abbe also pointed out a simple way to find out to what extent the sine condition is satisfied. For this purpose, Abbe made a special drawing (Figure 17), which is examined through the tested optical system.

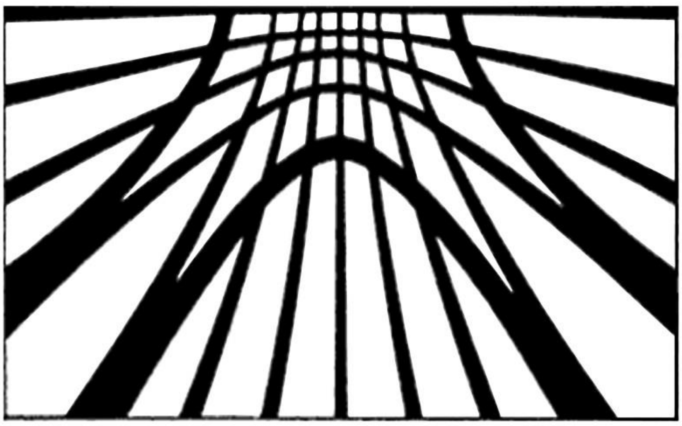


Figure 17 - Abbe drawing for testing of the optical system

If the sine condition is fulfilled, then it is possible to find a position of the figure in which the observer sees his image in the form of a rectangular grid. Having tested many microscope lenses made by the old masters, Abbe found that all good lenses had the sine condition. At present, the Abbe sine condition is always considered when calculating any optical systems.

Invariant Abbe

Let two homogeneous transparent media with refractive indices *n* and *n'* be separated by a spherical surface whose radius of curvature is *R*. Draw the optical axis through points *A*, *A'* and the center of curvature of the spherical surface. In paraxial optics, each refractive surface satisfies the Abbe *Q* invariant in the paraxial region, which relates the front focal length d_o of the axial object point to the back focal length d_i of its conjugate point behind the surface:

$$Q = n\left(\frac{1}{d_o} + \frac{1}{R}\right) = n'\left(\frac{1}{d_i} + \frac{1}{R'}\right) = const \quad (32)$$

The Abbe invariant allows calculating the image position of the luminous point located on the optical axis. The Abbe invariant does not include the beam height and the angle between the beam and the optical axis. All rays traveling at a small angle to the axis are directed to one point *A′* (Figure 18).

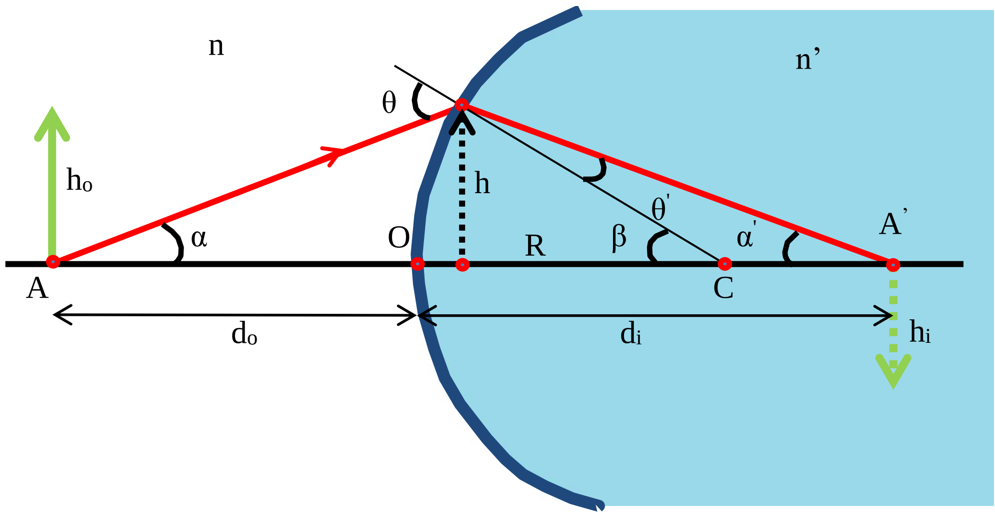


Figure 18 - Illustration for Abbe Invariant

Thus, a homocentric beam refracted by a spherical surface remains homocentric if the paraxiality condition is satisfied. At small angles α, all the rays emanating from the point object *A*, after refraction, intersect at point *A'*. For paraxial rays, the homocentric beam after refraction on the spherical surface remains a homocentric beam and point *A'* is the stigmatic image of point *A*.

The Abbe invariant is applicable for both convex (R> 0) and concave (R <0) surfaces.

LECTURE 7. CHROMATICITY DIAGRAM AND ANALYTICAL CALCULATION

The geometric place of the chromaticity coordinates of pure spectral color is a curve lying in a single plane and referred to the spectral locus or chromaticity diagram (Figure 19).

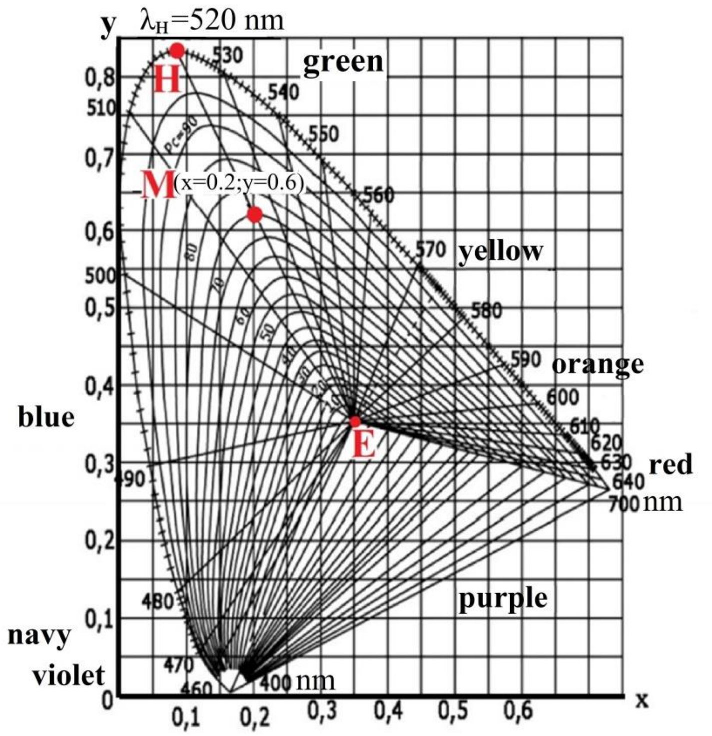


Figure 19 - Chromaticity diagram of the International Commission on Illumination (ICI) [6]

Here, “colorless” or achromatic have a special place, it means that it is white and becomes gray down to black color. Achromatic colors are colors that have no color tone. On the contrary, colors with the most pronounced chromatic component are chromatic colors. Saturated colors refer to that colors. In turn, the less pronounced a color tone we have, the closer a certain color to achromatic, and the less this color is saturated. Saturation, however, is a subjective characteristic, which could be determined by purity. The objective characteristic is color purity and expressed in percent. So, the purity is the degree of color approximation to pure spectral P. In turn, the saturation - a characteristic allow estimating the part of the pure chromatic component in the whole color sensation.

Hue and saturation, or wavelength and purity, called chroma, are considered a quality characteristic of color.

Any ray of light has color coordinates x and y, brightness (or radiation flux) Y and X, color modulus m = Y/y = X/x. Three parameters x, y, Y uniquely determine the brightness and color characteristics of the light beam.

To find the color coordinates of the light beam obtained by summing the color beams with known parameters, it is necessary to sum the brightness parameters Y and X, as well as the color modules m:

$$m_c = m_1 + m_2 + m_3; m_1 = \frac{Y_1}{y_1}; m_2 = \frac{Y_2}{y_2}; m_3 = \frac{Y_3}{y_3};\ Y_c = Y_1 + Y_2 + Y_3;$$

$$y_c = \frac{Y_c}{m_c}; X_1 = m_1 x_1; X_2 = m_2 x_2; X_3 = m_3 x_3; X_c = X_1 + X_2 + X_3; x_c = \frac{X_c}{m_c}$$

The luminous flux, illumination, luminous intensity, brightness - all measured in lighting units: lm, lx, cd, cd/m^2 can act as a brightness parameter. In color studies, they are conventionally called light watts.

Analyzing the ICI chromaticity diagram, one can note:

1. The chromaticity coordinates of all real colors are inside the spectral locus and are determined by the positive values of *x* and *y*.
2. White color *E* is in the center of gravity of the *xoy* triangle. Its chromaticity coordinates will be x = 0.33, y = 0.33.
3. Complementary colors lie on a line segment passing through point *E* with a spectral color curve.
4. The color of the mixture of two colors is displayed by a dot lying on a straight line connecting the mixed colors.

The color tone of any color on the ICI chromaticity diagram is determined (dominant wavelength $\lambda_н$) long monochromatic emission wavelength corresponding to the intersection curve of spectral colors - spectral locus with a line passing through point *E* and the point of displaying the desired color hue. For example, drawing a straight line through a white point, for instance, *E* and a point of a given color $M(x = 0.2; y = 0.6)$ to the intersection with the boundary curve, we get the point *H*. The point *H* determines the radiation color of some monochromatic (i.e. green) radiation with λ_H = 520 nm by the rule of additive color mixing. The mixture with white radiation *E* coincides in color with given radiation, namely the point *M*.

According to the chromaticity coordinates of the point $M(x, y)$, colorimetric purity is calculated through the coordinates "*x*" and "*y*" as follows:

$$P_К = \frac{y_\lambda}{y} \cdot \frac{(x - x_E)}{(x_\lambda - x_E)}; \; P_К = \frac{y_\lambda}{y} \cdot \frac{(y - y_E)}{(y_\lambda - y_E)} \qquad (33)$$

where x_λ and y_λ - are the coordinates of the spectrally pure color «$\lambda_н$» of the same tone as the given color (point *M* for the given color) x_E and y_E- are the coordinates of the white point *E*.

Conditional saturation P_B is introduced by the formulas through the coordinates "x" and "y":

$$P_B = \frac{(x - x_E)}{(x_\lambda - x_E)}; \; P_B = \frac{(y - y_E)}{(y_\lambda - y_E)} \qquad (34)$$

Color purity *P* is determined by concentric curves of equal values of color purity plotted on the color chart, depending on the position of the point characterizing the color of the investigated radiation. From the definition of color purity, it follows that the curve *P* = 100% is the locus of spectral colors and the straight line of magenta that closes it. As a result, the purity *P* (%) can be determined from concentric curves, i.e. estimate the color saturation of the radiation

LECTURE 8. MICHELSON STELLAR INTERFEROMETER. AIRY DISK

Intensity interferometer, i.e. a system for measuring the correlation coefficient of radiation intensity, which comes from two different points in space, is used in radio astronomy to calculate the angular sizes of stars and cosmic radiation sources.

Such interferometers are insensitive to changes in the phase difference, which are caused by mechanical vibrations, turbulence in the atmosphere, etc. However, if external interference is present, the radiation flux sensitivity of the intensity interferometer decreases more than the sensitivity of a conventional phase interferometer. In this regard, the intensity interferometer is used only for bright sources.

The principle of operation is as follows if the angular distance between binary stars is very small, it is necessary to carry out a special shooting to distinguish the distance between stars. For this, you can use the Michelson stellar interferometer, which also allows you to determine the angular distance between the stars. A schematic diagram of a Michelson stellar interferometer is shown in Figure 20 [7].

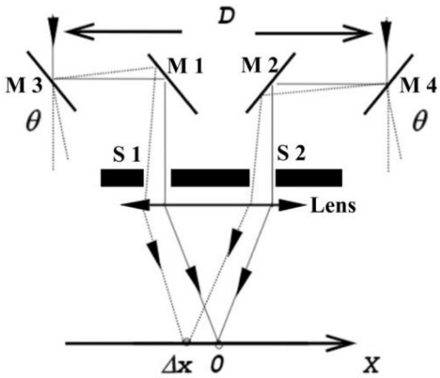


Figure 20 - Scheme of the Michelson interferometer

Parallel light beams from the star fall on two mirrors of the interferometer M_4 and M_3, the distance between which is the base of the interferometer *D* can smoothly change due to the movement of this pair of mirrors. The rays reflected from the mirrors fall on the second, motionless pair of mirrors M_1 and M_2. The rays reflected from these mirrors pass through a pair of slit-like openings S_1 and S_2 located in front of the telescope objective. The rays collected by the lens interfere in the focal plane of the lens and form interference fringes. The distance between the interference fringes (Δx) remains unchanged since the distance between the slits S_1 and S_2 is constant. The contrast of the strips decreases as the base of the interferometer *D*.

The angular size of a star can be calculated with a known value of the coherence radius. By shifting the external mirrors, it is possible to change the travel difference between the beams until the interference pattern disappears. This happens when the relation

$$\theta = \frac{\lambda}{2D} \tag{35}$$

where θ - is the angular distance between the incoming rays from two closely spaced stars, D is the distance between the external mirrors. The interference pattern when determining the angular diameter of a star disappears when the relation:

$$\theta = \frac{1.22\lambda}{2D} \tag{36}$$

where it is assumed that the star is a uniformly luminous disk, where the opposite edges of its disk can be considered as two close sources of radiation. The reciprocal of the minimum angular distance is called the resolution of the optical device. To increase it, it is necessary to increase the light diameters, however, this leads to an increase in distortions (aberrations) of the resulting image (Figure 21).

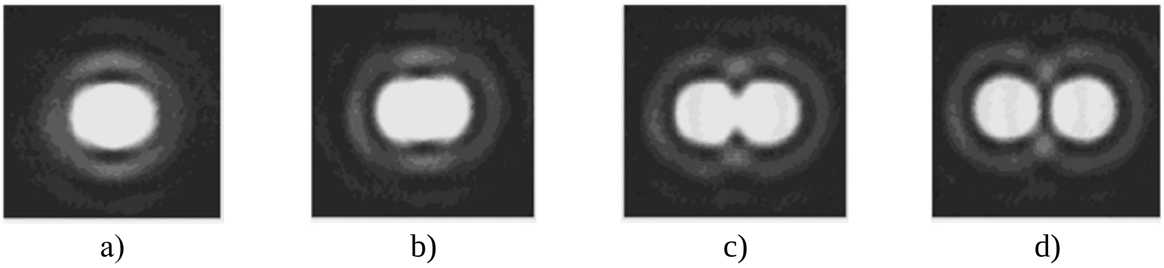


Figure 21 - An example of an increase (a-d) in the degree of resolution of individual stars

In the first two pictures (Figure 21), cases of insufficient resolution are presented, in the next two photographs the desired points are resolved.

Compared with a conventional telescope, a stellar interferometer has a higher resolution, since the base of the interferometer can significantly exceed the diameter of the lens of an astronomical telescope. For a telescope with a diameter of the main mirror D, the resolution is determined by relation (36).

The coherence radius is determined by the magnitude of the base of the interferometer at which the bands are blurred. Limitations in the size of the base of a stellar interferometer were caused by the presence of atmospheric turbulence. The measurement accuracy of a stellar interferometer is greater, the larger the base D. A stellar interferometer was built in which D can reach 18 m. That allows measuring the angular distance with an accuracy of 0.001".

Michelson's stellar interferometer interferes with the Airy disk. Due to the diffraction of light waves, the image of a point light source does not become a point, but a scattered spot called the Airy disk (Figure 22) [8].

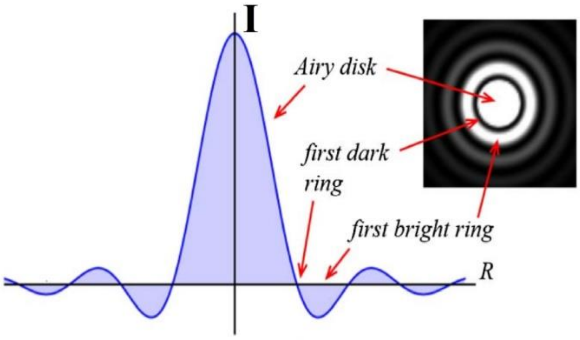


Figure 22 - Intensity distribution within ideal Airy disk

Airy's disc has three very salient features. Rings arise due to the interference of light waves that deviate from the original path with each other, as well as with waves that have kept a straight line. Airy's disk is surrounded by a series of concentric rings. The brightness and intensity of an Airy disk (Figure 22) decrease rapidly with distance from its center. Together with the Airy disk, the rings form a characteristic diffraction pattern known as the Airy model. 85% of the lighting comes from the Airy disk itself and 15% from the surrounding rings. Accordingly, when the relative aperture decreases (i.e., with an increase in the number of apertures), these rings arise due to the interference of light waves that deviate from the original path from each other.

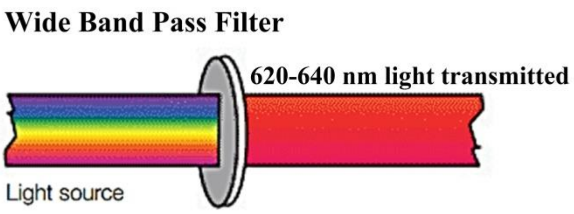
LECTURE 9. LIGHT FILTERS. INTERFERENCE FILTERS

Light filters are devices that change the spectral composition or energy of a light wave incident on them, without changing (or almost not changing) the shape of its front. The main characteristic of the filter is transmission:

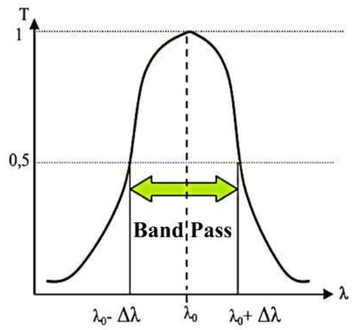
$$T = \frac{I}{I_0} \tag{37}$$

where I and I_0 are the intensities of the light transmitted and incident on the filter.

Filters are called gray, or neutral, if their transmission in the studied spectral range is independent of the wavelength. Neutral Density (ND) filters reduce the amount of light without affecting contrast or image clarity. Filters that do not satisfy this condition are called selective. Selective filters are designed either to separate a wide spectral region or to isolate a narrow spectral region (Figure 23). The filters of the latter type are called narrow-band.



a)



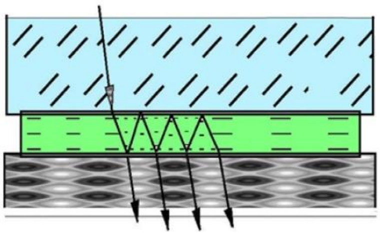
b)

Figure 23 - Illustration for selective filters a) transmission spectrum of a selective filter b)

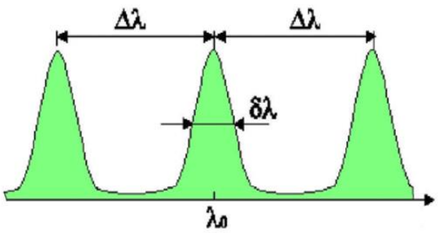
Multipath interference can be used to create narrowband light filters. The simplest interference filter is a Fabry-Perot interferometer with a very small gap between the mirrors (from 1/2 to several wavelengths). When light is incident along a normal with a wide spectral composition in transmitted light, a system of peaks arises, the distance Δλ between them is determined by the optical thickness $n \cdot h$ of the gap between the reflecting layers (Figure 24):

$$\Delta\lambda = \frac{\lambda^2}{2 \cdot n \cdot h} \tag{38}$$

For example, with an optical thickness $n \cdot h = \frac{5}{2}\lambda_0$ we obtain $\Delta\lambda = \frac{\lambda_0}{5}$. If $\Delta\lambda$ =500 nm, then the neighboring maxima lying at $\lambda_0 \pm\Delta\lambda$, correspond to λ_1 =400 nm and λ_2=600 nm. They can be separated from the desired maximum λ_0 using a conventional glass filter (Figure 25).



a)

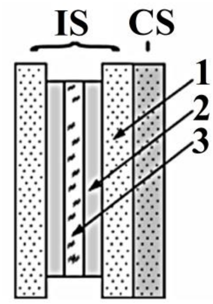


b)

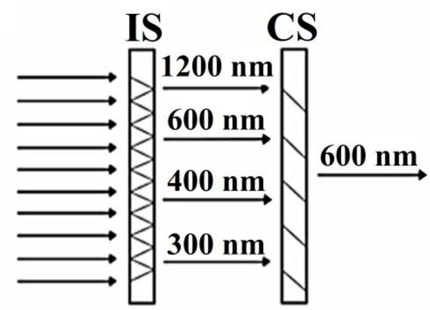
Figure 24 - Interpretation of Interference filter (a) Illustration for bandpass $\delta\lambda$ (b)

Bandpass δλ, i.e. the interference filter width is less than the distance $\Delta\lambda$ between adjacent maxima by a factor of F, where F is the sharpness of the multipath interference bands. At $R \approx 0.9$, the sharpness is $F \approx 30$, and at $\Delta\lambda = 100$ nm we have $\delta\lambda \approx 3$ nm. Such a filter will emit a narrow spectral range with a width of the order of 3 nm near $\lambda_0 = 500$ nm from incident white light normal. So that the filter does not give a noticeable attenuation of light in this band, multilayer dielectric coatings are used as reflective surfaces.

An interference filter consists of a thin plane-parallel dielectric layer with a refractive index n, on both surfaces of which are reflected layers with a reflection coefficient R. At the output of the system, an infinite sequence of rays with decreasing amplitude is formed with an equal path difference between them that interfere with each other.



a)



b)

IS - interference system; CS - cut off system; 1 - glass plane parallel plates; 2 - translucent metal layers; 3 - dielectric layers.

Figure 25 - Interference filter structure (a) and Conversion of white light by an interference filter with a working wavelength of 600 nm (b)

Non-absorbing layers of dielectric materials - oxides Al_2O_3, SiO_2, TiO_2; fluorides MgF_2, CaF_2, LiF; sulfides of ZnS, CdS and other compounds. These layers can be formed on a massive substrate of glass, quartz by vacuum deposition. When white light passes through such a system with numerous interfaces, the light is repeatedly reflected. As a result of interference of reflected rays with transmitted rays (reflected and transmitted rays are coherent), part of the luminous flux is attenuated only slightly, and part - by 10^3 - 10^6 times. The cut off system (CS) is designed to suppress all frequencies except the working one; it can be made of color glass filters or a special interference system. It is technically feasible to produce filters with a passband of up to 1 nm.

Optical filters are commonly used in photography (where filters with special effects are sometimes used, as well as absorption filters), in many optical instruments and for stage lighting. In astronomy, optical filters are used to limit the light transmitted into the spectral region of interest, for example, to study infrared radiation without visible light, which could act on the film or sensors and suppress the desired infrared light.

LECTURE 10. DIELECTRIC MIRRORS. DISTRIBUTED BRAGG REFLECTOR

Using a sequence of alternating dielectric layers with high n_H and low n_L refractive indices, the reflection coefficient of the system can be increased. If the optical thickness of all layers is the same and equal to $\lambda_0/4$ ($n_H l_H = n_L l_L = \lambda_0/4$), then the waves reflected by their boundaries are in the same phase and reinforce each other. Such multilayer coatings are called dielectric mirrors and are one of the types of Bragg reflectors (Figure 26).

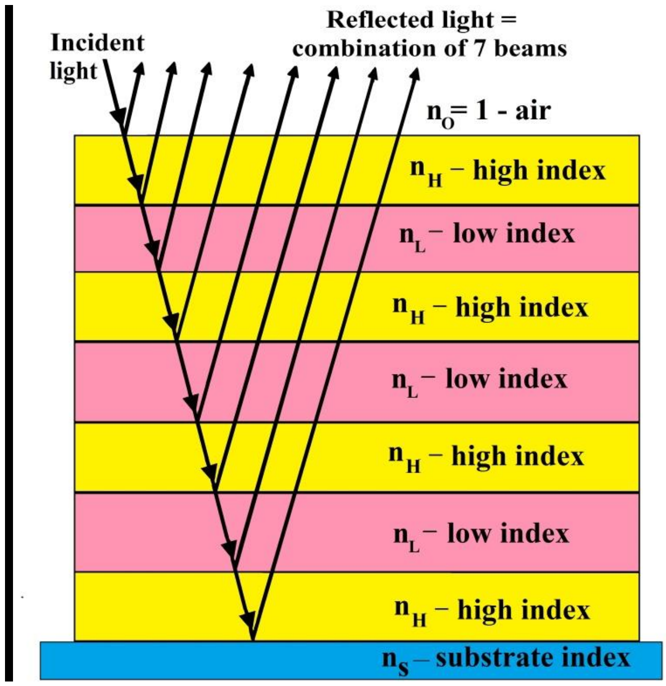


Figure 26 - Structure of a quarter wave thick layers dielectric mirror

They give high reflectivity only in a limited region of wavelengths near the value of λ0, for which the optical thickness of the layers is equal to $\lambda_0/4$. Typically, 5 to 15 layers of zinc sulfide (n_H = 2.3) and cryolite (n_L = 1.35) are applied. With seven layers, it is easy to achieve a reflection coefficient R = 0.9 in the spectral region $\delta\lambda$ with a width of the order of 50 nm. To obtain a reflection coefficient R = 0.99 (such mirrors are used in laser resonators), 11-13 layers must be applied.

Also, this Bragg reflector can be obtained by periodically changing a certain characteristic (for example, height) of a dielectric waveguide, which leads to a periodic change in the effective refractive index n_{eff}:

$$n_{eff} = 2\left(\frac{1}{n_H} + \frac{1}{n_L}\right)^{-1} \tag{39}$$

The range of reflected wavelengths is called the photon stopband. In this wavelength range, light is "forbidden" to propagate in the structure.

Bragg's law defines the conditions under which waves reflected from the medium's boundary of a given dielectric mirror perpendicular to the incident wave are in the same phase.

The reflection coefficient of the Bragg reflector is determined:

$$|R| = \left[\frac{n_o \cdot (n_H)^{2N} - n_S \cdot (n_L)^{2N}}{n_o \cdot (n_H)^{2N} + n_S \cdot (n_L)^{2N}}\right]^2 \tag{40}$$

Where N is the number of pairs of quarter-wave layers ($\boldsymbol{l_H}$+$\boldsymbol{l_L}$), n_H and $n_L$are the refractive indices of the layers, n_o- is the refractive index of the environment, n_S is the refractive index of the substrate.

The maximum reflection coefficient of Bragg reflector in the spectrum falls on the wavelength λ_0, and its spectral width $\Delta\lambda$ (Figure 27) is determined from the expression:

$$\Delta\lambda = \frac{2\lambda_o \Delta n}{\pi n_{eff}} \quad (41)$$

Where λ_o - is the central wavelength, Δn - differences between refractive indices of DBR, n_{eff} - effective index of refraction.

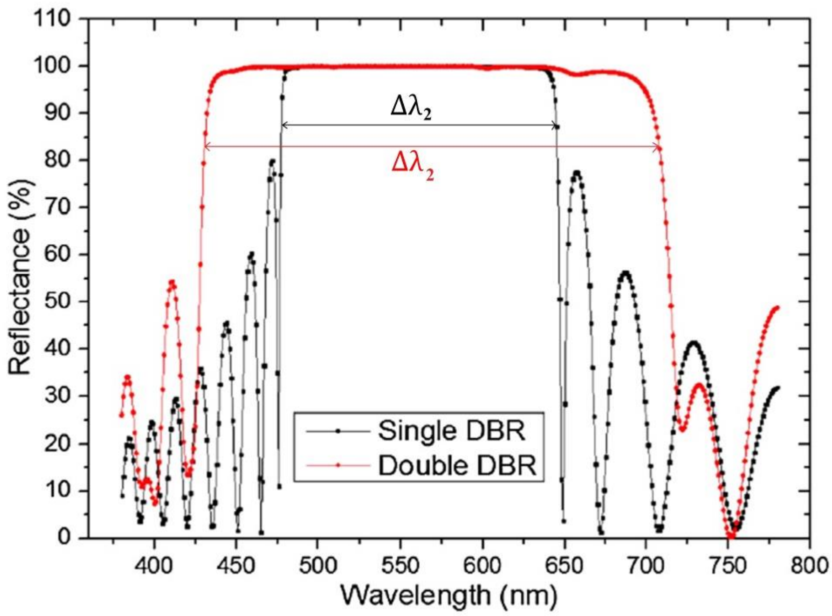
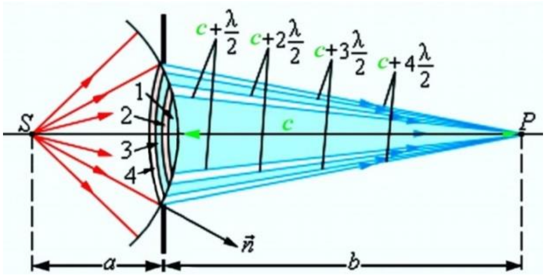


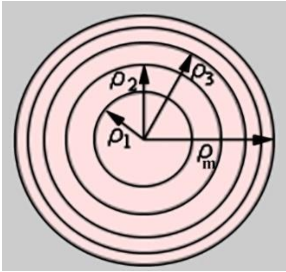
Figure 27 - Comparison between single Bragg reflector stack and double Bragg reflector stacks. The bandwidth of double Bragg reflector stacks is 272 nm while the bandwidth of the single Bragg reflector stack is 170 nm [9]

Bragg reflector allows light waves to be reflected with a much narrower reflection band than between a semiconductor and air. This is what caused the widespread use of such reflectors in optical technologies (filters, reflectors embedded in optical fibers, sensors) and their attractiveness for use as mirrors of semiconductor lasers.

LECTURE 11. ZONE PLATE

Fresnel proposed an original method of dividing the wavefront from the light source *S* into zones (Figure 28). Boundaries of the first (central) zone are the points of the surface S located at a distance *c* from the point *P*. The points of the sphere *S* located at a distance from the point *P*, which are $c + \lambda/2$, $c + 2\lambda/2$, etc. form the second, third, etc. Fresnel zones, respectively.





a) b)

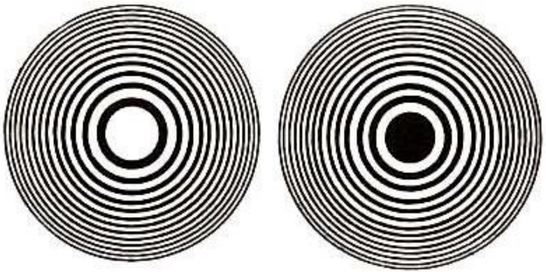
Figure 28 - Splitting the wavefront into Fresnel zones a) a schematic representation of the Fresnel zones radii b)

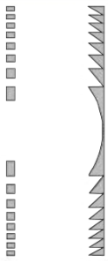
Radii of Fresnel zones based on Figure 25 can be calculated by the formula:

$$\rho_m = \sqrt{m \cdot \lambda \cdot \frac{a \cdot b}{a+b}} \qquad (42)$$

Where *m* is the sequence number of the Fresnel zones, λ is the incident wavelength, a is the distance from the light source to the hole, *b* is the distance from the hole to the observation point on the screen.

If you place a transparent plate with shaded only odd (1st, 3rd, etc.) or even Fresnel (2nd, 4th, etc.) Fresnel zones in the hole area (Figure 29a), then the amplitude vectors from these zones will be co-directed. Such optical elements are called amplitude zone plates. If in the even/odd regions of the Fresnel zones an excess path difference between the rays equal to λ/2 is created, then such zone plates are called phase plates (Wood plates) (Figure 29b).





a) b)

Figure 29 - Even and odd amplitude zone plates a) profile of amplitude and phase zone plates

Fresnel zone plate, like an ordinary lens, focuses a parallel beam of radiation into a point focus. Zone plate has one main focus and an infinite number of side foci. The difference in movement from neighboring open areas to the main focus along the wavelength. Focusing occurs because waves from open zones come into a phase exactly at the focal point where they interfere (Figure 30).

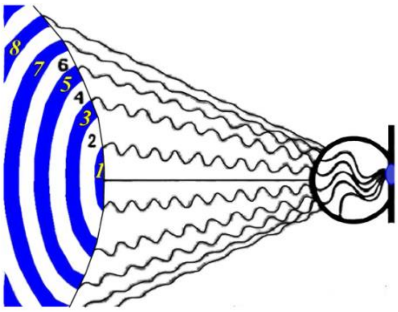


Figure 30 - Beam focusing with a zone plate

Besides the main focus, which is defined:

$$f = \frac{R^2}{\lambda} \tag{43}$$

where R is the radius of the zone plate, zone plate side foci are formed at distances:

$$f_m = \frac{f}{(2m+1)} \tag{44}$$

where m are integers. For example, if we approach the zone plate at a distance $c = f/3$ (Figure 31), then we will fall into one more point of focusing, although weaker one: zones (1,2,3) + (7,8,9 will be open)) + (13,14,15), etc. In each such triad, the waves from odd zones in amplitude almost double the even.

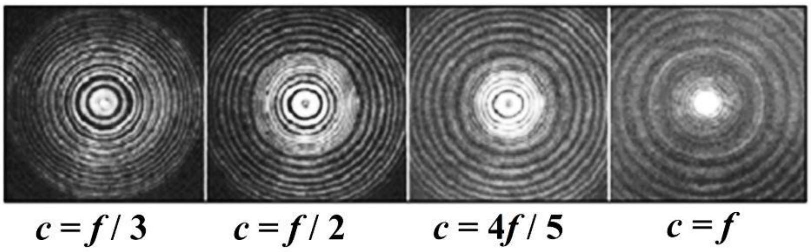


Figure 31 - Focusing on the zone plate depending on the distance to the screen

Now consider phase zone plates, also called Wood plates (Figure 32). If we introduce an additional phase shift $\delta_m = \pi$ with the corresponding path difference $\lambda/2$, then the light intensity at the focus will increase by 4 times.

a) b)

Figure 32 - Wood plate profile options a) path difference between adjacent zones b)

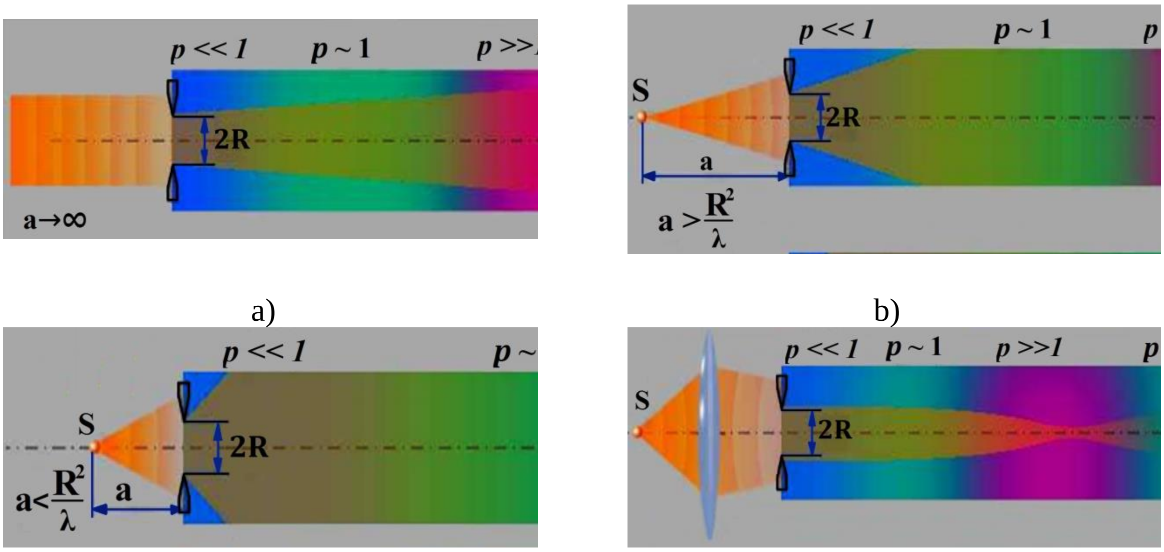
The division into half-wave zones corresponds to a stepwise approximation of the refractive surface of the lens at an equal height of the steps. To do this, the thickness of the glass plate in places corresponding to even Fresnel zones must be reduced or increased by:

$$\Delta d = \frac{\lambda}{2(n-1)} \tag{45}$$

Where n is the refractive index of the phase plate. Wood's two-level plate is capable of concentrating at point P up to 40% of the light energy incident on it.

LECTURE 12. BOUNDARIES OF DIFFRACTION APPROXIMATIONS

Understanding the essence of diffraction allowed us to develop approximate methods for calculating optical instruments, since the edges of the lenses in the frame represent an obstacle to the propagation of light. At the same time, two important special cases stand out. One of them is the location of the obstacle relative to the observation point, at which waves from secondary sources enter it at different angles and overcome paths that differ by several wavelengths. Figure 33 shows four different observation areas in which the obstacle is lit evenly, spherically diverges and converges in the waves.



■ - Fraunhofer diffraction region, ■ - Fresnel diffraction region, ■ - Geometric shadow region

Figure 33 - Diffraction region depending on the location of the light source

Quantitative criterion that diffraction near the axis of a noticeable hole is expressed as follows: the radius ρ_m of the central Fresnel zone must exceed the size of the obstacle R:

$$\rho_m = \sqrt{\frac{ab}{a+b} m\lambda} \geq R \tag{46}$$

Where a - distance from light source to the obstacle, b - distance from the obstacle to the screen where diffraction is observed, m - Fresnel half-wave number:

$$m = \frac{\rho_m^2}{\lambda b} \tag{47}$$

The ratio of the radius of the central Fresnel zone to the linear size of the obstacle R is called the wave parameter**:**

$$p = \frac{\sqrt{\frac{ab}{a+b}\lambda}}{R} \tag{48}$$

Fresnel diffraction is clearly pronounced if $p \geq 1$ or slightly more than 1, and diffraction is practically absent if p << 1 (region of geometric shadow). In the case, we will be p >> 1, we observe the Fraunhofer diffraction (or in parallel beams). Different diffraction variants differ in the number of open Fresnel zones. There is no clear boundary between them, there is a smooth transition.

The result of the diffraction of monochromatic radiation (provided that $a \to \infty$), on any obstacle depends not on its absolute dimensions, but on the number m of half-wave zones covered by it (Figure 34).

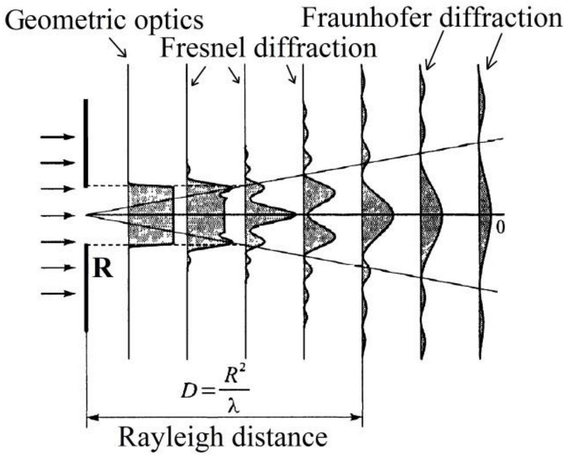


Figure 34 - Diffraction distributions of light intensity at different distances from the hole

If m >> 1, the diffraction effects are insignificant and the intensity distribution is approximately described by the laws of geometric optics. The intermediate condition corresponds to Fresnel diffraction and leads to a complex intensity distribution when both a minimum and a maximum can be observed in the center of the picture. For m <1, a small part of the first zone overlaps and an important case for practice arises - Fraunhofer diffraction. The distance between the hole and the screen, at which the radius of the first Fresnel zone coincides with the radius of the hole, is called the diffraction length of the light beam or the Rayleigh distance:

$$D = \frac{R^2}{\lambda} \tag{49}$$

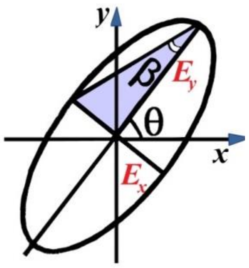
Rayleigh distance corresponding to the distance at which a circular hole of radius R illuminated by a plane monochromatic wave opens one first zone for the central observation point (m = 1).

LECTURE 13 STOKES VECTORS. POINCARE SPHERE

Stokes parameters are used to describe the state and degree of polarization of electromagnetic waves. Up to a constant factor, these quantities have the dimension of light intensity, i.e., they are similar to the Poynting vector. Stokes parameters contain complete information about the intensity, degree and shape of the beam polarization:

$$\begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} I_0 + I_{90} \\ I_0 - I_{90} \\ I_{45} - I_{135} \\ I_{LHC} - I_{RHC} \end{bmatrix} \tag{50}$$

Where I_0 is the intensity of the light reaching the sensor after passing through a linear polarizer at zero degrees (vertical), I_{90} is the intensity with a polarizer at 90 degrees (horizontal), I_{45} and I_{135} are the intensities with a polarizer of 45 and 135 degrees, respectively. These parameters are defined (Figure 35). There are only three independent Stokes parameters. Using them, from the Stokes parameters of a plane wave, it is easy to determine the quantities specifying the direction of oscillations of E. The first Stokes parameter (S_0) is obtained from the addition of horizontal and vertical intensities, while the second parameter (S_1) is the difference between the same two.



$S_0 = E_x^2 + E_y^2 = I_0$	Full intensity
$S_1 = S_0 \cdot \cos 2\beta \cdot \cos 2\theta$	Preferred horizontal polarization parameter
$S_2 = S_0 \cdot \cos 2\beta \cdot \sin 2\theta$	Parameter of predominant diagonal polarization
$S_3 = S_0 \sin 2\beta$	Parameter of predominant right circular polarization

a) b)

Figure 35 - Polarization ellipse a) Stokes parameter equations b)

The third parameter (S_2) is the difference between the two diagonal intensities. The fourth Stokes parameter (S_3) is the difference between two circular polarizations. Stokes parameters are column vectors (Figure 36). A real ray of light is a superposition of a huge number of independent modes of the radiation field that quickly replace each other with random phases and directions of oscillations. The Stokes parameters of the full beam are equal to the sums of Stokes parameters. This property of the Stokes parameters is used in optics.

If the Stokes parameters are used as Cartesian axes, then any polarization state will correspond to a point on the surface of a sphere of radius R, called the Poincare sphere:

$$R = \sqrt{S_1^2 + S_2^2 + S_3^2} \tag{51}$$

Poincare sphere is a graphical tool in real three-dimensional space that allows you to conveniently describe polarized light and polarization transformations caused by propagation through devices. Fully polarized light (linear, circular and elliptical polarized light) is represented by a point on the surface of the Poincare sphere. Partially polarized light, which can be considered as a superposition of polarized and unpolarized light, is represented by a point in the volume of the Poincare sphere (Figure 37).

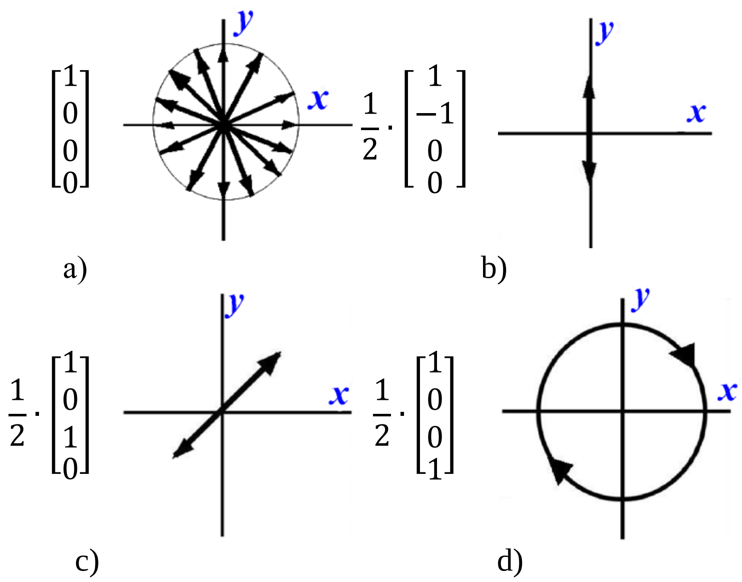


Figure 36 - Example of Stokes parameters for unpolarized light a) for vertically polarized light b) for diagonally polarized light c) for circularly polarized light d)

If the radiation is partially polarized, then the degree of polarization:

$$p = \frac{R}{S_0} \tag{52}$$

Circular states are located at the poles, and intermediate elliptical states are continuously distributed between the equator and the poles. Right elliptical states occupy the northern hemisphere, while left elliptical states occupy the southern hemisphere.

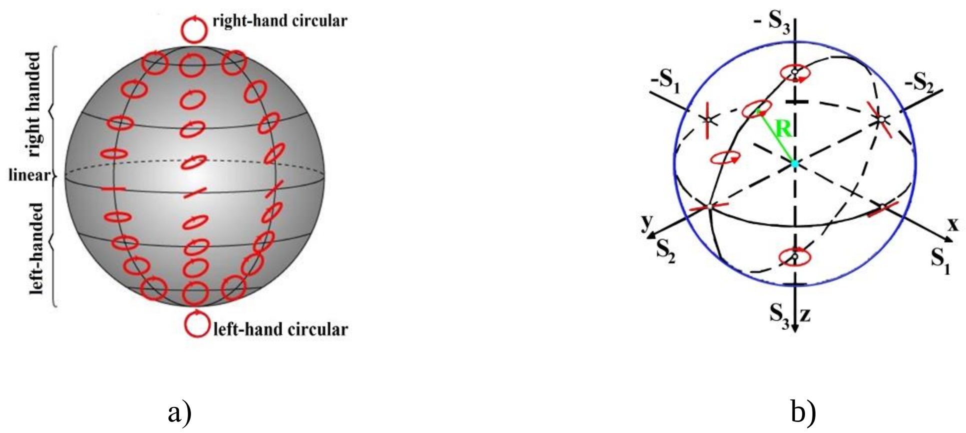


Figure 37 - Poincare sphere illustration

Linear states occupy the equator. The distance from the center of the sphere *O* the the point gives the degree of polarization of light in the range from zero at the origin (non-polarized light) to unity (*1*) on the surface of the sphere (fully polarized light). The points located close to each other on the sphere represent polarizations, which are similar in the sense that the interferometric contrast between the two polarizations is related to the distance between the corresponding two points on the sphere

LECTURE 14. CONOSCOPIC FIGURES

The optical properties of anisotropic materials, which are 90% of all solids, depending on the relative orientation of the incident light and the crystallographic axes. Such substances are characterized by several refractive indices, depending on the direction of light propagation in them and on the coordinates of the oscillation plane. Anisotropy of optical properties is determined primarily by the difference in the electrical properties of the medium in different directions of radiation propagation, since optical radiation is an electromagnetic process. A spectacular demonstration of optical anisotropy is the conoscopic pattern, which in practice is used to identify crystalline structures.

Conoscopic figures (Figure 38) are interference patterns in converging polarized light formed by rays passing through crystalline. A plate with crossed or parallel polarizer and analyzer, and observed in the focal plane of the microscope objective.

a) Uniaxial crystal b) Biaxial crystal

1 - Isogyre, 2 - Melatope, 3 - Isochrome

Figure 38 - Conoscopic figures for uniaxial (a) and biaxial (b) crystals

When anisotropic crystals are illuminated by polarized light, a circular distribution of interference colors is formed. Uniaxial crystals (Figure 38a) show an interference patterns consisting of two intersecting black bands called isogyr. Inner circles of interference pattern are called isochroms. The center of the black cross and isochrome is called the melatope, and denotes the source of light rays passing along the optical axis of the crystal. Biaxial crystals show two melatopes (Figure 38b) and a much more complex picture of interference rings.

You may notice that the isochrumes have certain colors, which are arranged in a special order, which is described by the color scale of Michael-Levy interference colors (Figure 39) [9]

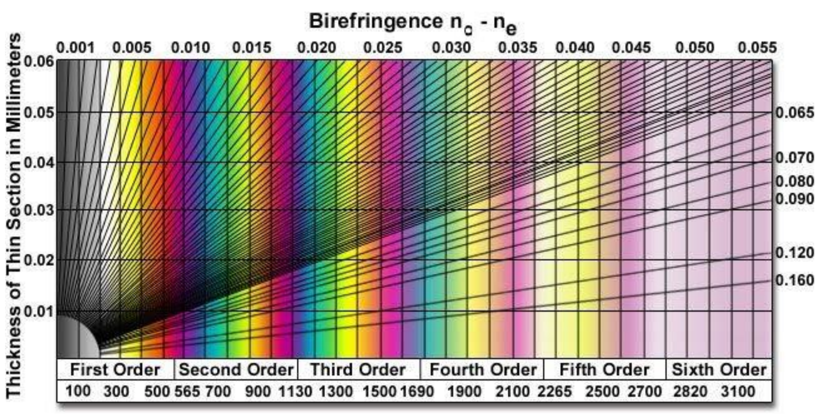


Figure 39 - The color scale of the interference colors of Michel-Levy

The color scale (Figure 39) shows the relationship between interference in polarized light namely retardation by abscissa and thickness of the sample by ordinate. Birefringence in anisotropic crystals is determined by isochromes, each of which has a different measured value of birefringence corresponding to the thickness and interference color. The Michel-Levy diagram is used by comparing interference colors depending on the order displayed by the crystal in a microscope with colors contained in the diagram. Once a matching color is found, the closest vertical line along the interference color follows the nearest horizontal line, representing the thickness and difference in refractive indices between the ordinary ray and the extraordinary one.

Upon entering the crystal, plane-polarized light immediately splits into two mutually perpendicular beams (ordinary and extraordinary), each of which passes through the crystal at different speeds. They have refractive indices n_o and n_e, one moves slowly (direction with a high refractive index), and the other moves fast (direction with a low refractive index). How quickly or slowly depends on the atomic structure. Light passes through crystals in mutually perpendicular directions; only two directions will be visible simultaneously in the microscope; the third will be parallel to the axis of the microscope.

As each of the two rays passes through the crystal at a different speed, a fast beam will exit the crystal sooner than a slow beam. When both rays exit the crystal, there will be a path difference between them, which is called ratardation. In other words, the slow beam is delayed relative to the fast beam. This ratardation distance for microscopic crystals is very small and is measured in nanometers (nm). To be able to observe conoscopic figures, you need to collect the following microscope scheme (Figure 40).

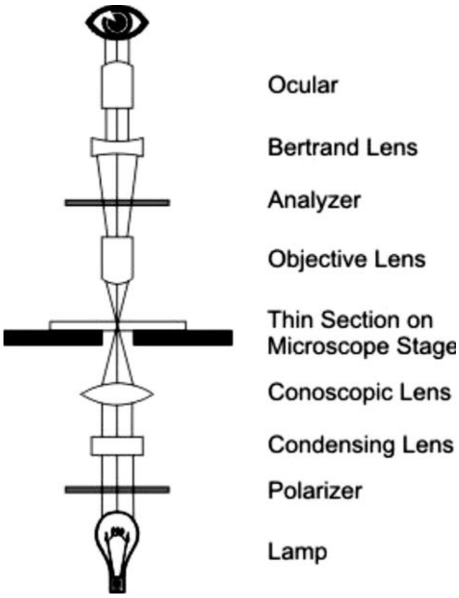


Figure 40 - Light paths in the polarized-light microscope

In this diagram, there are the following additional elements: Benard lens is a special lens placed in the microscope tube above the focus of the lens to adapt the microscope to the study of minerals in convergent polarized light. The condenser lens forms a converging parallel beam of light on one part of the sample. A compensator - anisotropic crystalline plates of constant or variable delay with a known orientation of the directions of wave oscillations. They are used to distinguish between directions of vibration. Most often, compensators are inserted diagonally into the microscope tube under the analyzer. The compensator can be a quartz or gypsum plate, cut parallel to the optical axis, with a thickness of about 62 μm [11].

With the help of conoscopic pictures in anisotropic materials, it is possible to determine whether this crystal is positive, or negative. The optical sign is based on the directions of the refractive index relative to the optical axis. If we consider the definition of a sign in a uniaxial crystal, then, depending on the direction of oscillation of the ordinary and extraordinary rays, two cases should be distinguished (Figure 41):

a)

b)

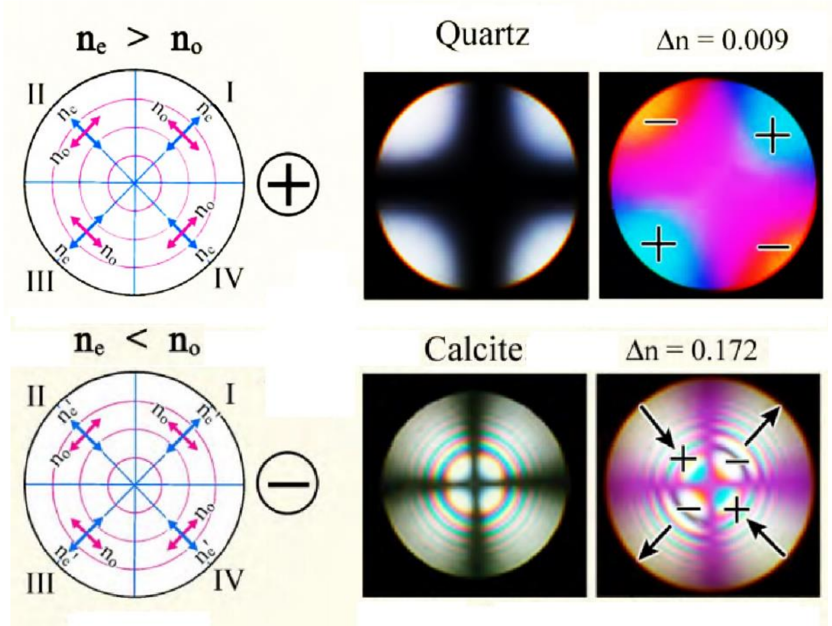


Figure 41 - Determination of the optic sign of a uniaxial crystal

1: For quartz, when a compensating plate is added to the microscope, in quadrants I and III, the interference color increases by one order of magnitude (addition), while in quadrants II and IV, the interference color decreases by one order of magnitude (subtraction). Thus, $n_e > n_o$, and therefore the optical sign of the mineral is positive (Figure 41a).

2: For calcite, when a compensating plate is added to the microscope, the colors in quadrants I and III decrease by one order of magnitude (subtraction), while in quadrants II and IV, the interference colors increase by one order of magnitude (addition). Thus, $n_e < n_o$, and therefore the optical sign of the mineral is negative (Figure 41b)

In other words, we look at what color is in the northeast quadrant directly adjacent to the isogyre. If the first color bar in this position is blue, then the mineral has a positive optical sign, whereas if it is yellow, then the mineral is optically negative. A simple rule to remember this definition - BURP - Blue Upper Right Positive.

LECTURE 15. NORMAL AND ABNORMAL DISPERSION. PHASE AND GROUP VELOCITY

The dependence of refractive index $n = f(v)$ on the frequency of the light wave is called light dispersion. Light dispersion is neither monotonic nor linear. The ranges of frequency *v* where

$$\frac{dn}{dv} > 0 \text{ or } \frac{dn}{d\lambda} < 0 \tag{53}$$

i.e., with increasing frequency *v*, the refractive index *n* increases this is called normal dispersion of light. Normal dispersion is observed in substances transparent to light. For example, ordinary glass is transparent to visible light, and normal dispersion of light in the glass is observed in this frequency range. Based on the phenomenon of normal dispersion, the "decomposition" of light is based on the glass prism of monochromators.

Dispersion is called abnormal if

$$\frac{dn}{dv} < 0 \text{ or } \frac{dn}{d\lambda} > 0 \tag{54}$$

i.e. with increasing frequency *v*, the refractive index *n* decreases. Anomalous dispersion is observed in the frequency regions corresponding to the bands of intense light absorption in a given medium. For example, an abnormal dispersion is observed in ordinary glass in the infrared and ultraviolet parts of the spectrum. Depending on the nature of the dispersion, the group velocity *u* in a substance can be either greater or less than the phase velocity *v*.

Phase velocity *v* of a monochromatic wave is usually called the velocity of propagation of the wavefront. In a medium with a refractive index *n*, the phase velocity *v* is sought from the relation:

$$v = \frac{c}{n} = \frac{dx}{dt} = \frac{\omega}{k} \tag{55}$$

Where x - coordinate, ω is the circular frequency, *k* is the wavenumber, *t* - time. Group wave velocity (Figure 42) is the velocity of a group or train of waves that form at each given point in time a wave packet is localized in space.

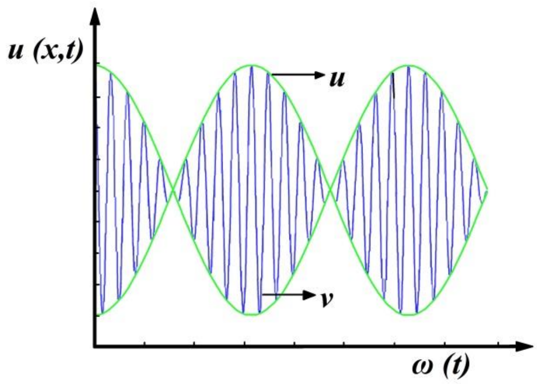


Figure 42 - Group and phase velocity

Based on Figure 42, the green line shows the group velocity *u*. The carrier wave (blue line) moves with phase velocity and is defined as *v*. Any sine wave limited in space and time is an overlap of a large number of sine waves, i.e. an envelope. The wave envelope travels at a group speed. Group velocity is determined by the Rayleigh formula:

$$u = \frac{d\omega}{dk} = v - \lambda \frac{dv}{d\lambda} \tag{56}$$

Wave envelope can be represented as the result of the superposition of a large number of plane waves with different lengths. For normal dispersion $u < \upsilon$, and for anomalous dispersion $u > \upsilon$. The dependence of the refractive index of the circular frequency can be seen in Figure 43.

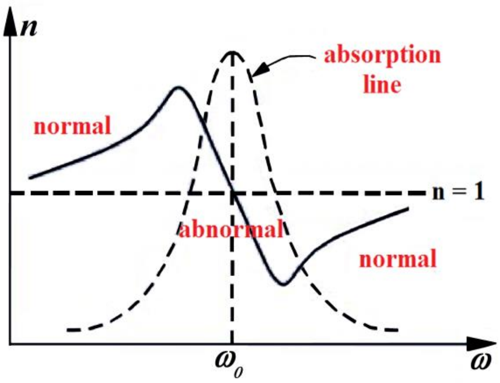


Figure 43 - Dispersion curve for normal and anomalous dispersion

In Figure 43, where $\omega = \omega_0$ is the frequency of the natural vibrations of the electron, it can be seen that the normal dispersion of light is observed far from the natural absorption lines, while the anomalous one is observed within the absorption bands or lines. The dispersion dependence can be experimentally observed using the crossed prism method. The first prism with a vertical edge is glass; it decomposes the white light passing through it into a spectral band. The second prism (with a horizontal rib) is made of the studied material. It shifts each point of the spectrum vertically, and the magnitude of the shift depends on the refractive index at a given frequency. Thus, the shape of the spectral band on the screen reflects the dependence of the refractive index on the wavelength (Figure 44).

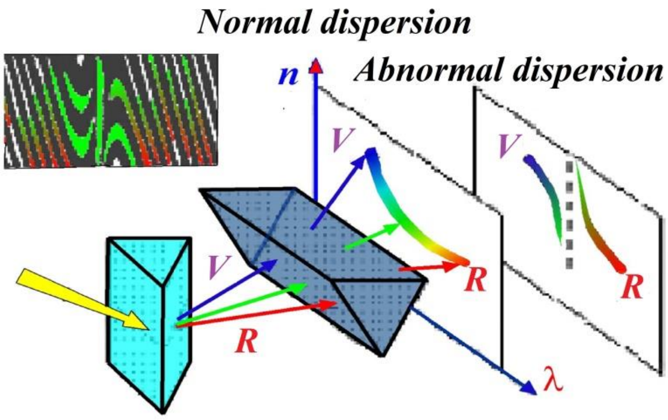


Figure 44 - Experimental observation of normal and abnormal dispersion

The figure shows an approximate view of the spectrum for cases of nominal and anomalous dispersion. To more accurately determine the course of the refractive index in the region of anomalous dispersion where the absorption is high, Rozhdestvensky proposed the spectro-interference method of “hooks” based on introducing an additional difference in the path between the reference and measuring beams in a two-beam interferometer. As a result, the interference bands turn out to be tilted, which makes it possible to quantify the parameters of the anomalous dispersion. The classical method of crossed dispersions is demonstrated.

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Laboratory work № 1

DEFINITION OF GLASS INDEX OF REFRACTION WITH THE MICROSCOPE

Aim of the work: familiarization with the technical details of the microscope; measurement of the refractive index of glass plates.

Instruments and accessories: measuring microscope with a micrometric screw, micrometer, measured glass plates with strokes on both surfaces, illuminator.

Brief theoretical description

It is known that when light passes through a boundary of two transparent media of unequal optical density, the incident ray of light *AO* is divided into two beams - a reflected ray *OB* and a refracted ray *OD* (Figure 45). The directions of these rays are determined by the following laws of reflection and refraction of light:

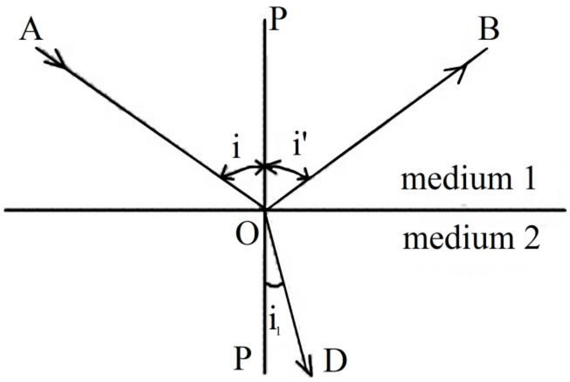


Figure 45 - Illustration of the refraction between two dielectric media

1. The ray *AO* incident on the refractive surface, the normal to the surface at the point of incidence of the *POP*, the reflected ray *OB* and the refracted ray *OD* are in one plane.

2. The *DOM* reflection angle is equal to the *AOP* incidence angle.

3. The sine of the incidence i_1 angle refers to the sine of refraction i_2 angle, as the speed of light in the first medium υ_1 refers to the speed of light in the second medium υ_2:

$$\frac{\sin i_1}{\sin i_2} = \frac{v_1}{v_2} \qquad (57)$$

The last law says that light propagates in different media at different rates.

For two given media and for a beam of a given wavelength, the ratio of the speed of light in the first medium υ_1 to the speed of light in the second medium υ_2 or the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant;

$$\frac{v_1}{v_2} = const = n_{21} \qquad (58)$$

$$n_{21} = \frac{\sin i_1}{\sin i_2} \qquad (59)$$

The quantity n_{21} is called the relative refractive index of the second medium with respect to the first one.

If one of the media, for example, medium 1 is a vacuum or air, then the refractive index *n* of medium 2 (see figure 1.1) with respect to the vacuum is called the absolute refractive index of this medium.

The absolute refractive index of medium 2 is

$$n_2 = \frac{c}{v_2} \quad (60)$$

where *c* is the speed of light in a vacuum; v_2 is the speed of light in a given medium 2.

Thus, the absolute refractive index of a medium is the ratio of the speed of light in a vacuum to the speed of light in a given medium:

$$n = \frac{c}{v} \quad (61)$$

The refractive index depends on the wavelength of light and on the properties of the medium. Absolute refractive indices are greater than 1. This means that the speed of light propagation in a given medium is always less than in a vacuum.

The n_{21} *is* a relative refractive index of the two media. The n_{21} is related to n_1 and n_2 (absolute refractive indices):

$$n_{21} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1} \quad (62)$$

In order to determine the refractive index of the media different methods are typically used. Using a microscope, we can determine the refractive index of a glass.

Description of the working set and method of measurements

The method is based on an apparent decrease in the thickness of a glass due to the refraction of light passing through the glass perpendicular to the surface. The ray's path is shown in Figure 46.

Here, two rays 1 and 2 fall to the point *A* located on the bottom surface of the glass plate. The beam 2 falls on the plate normally to its surface and therefore passes through the plate and leaves the air at point *C*, without refraction. Ray 1 is refracted and leaves the plate at the point *O* in the direction of point *D*.

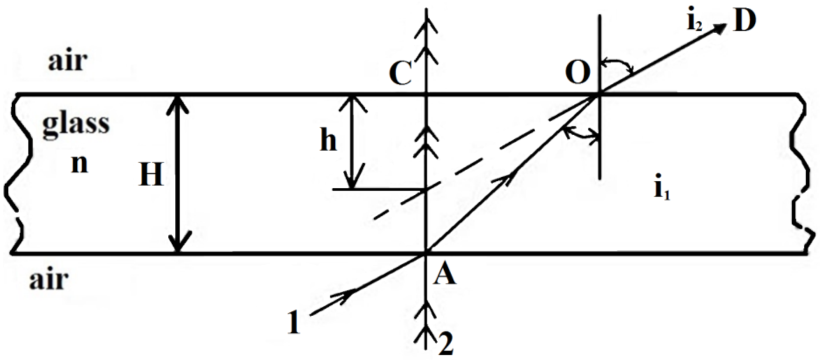


Figure 46 - Illustration of the optical thickness and real thickness of the glass plate

When leaving the plate, the *OD* beam forms an angle of refraction i_2 that is greater than the angle of incidence i_1. If you look from point *D* in the direction of *DO*, the observer will see the point of intersection of the rays *OD* and *AC* not at point *A*, but at point *E*, i.e. the thickness of the plate will appear to be equal to *CE*. The apparent thickness of the plate *CE* = *h* is less than its true thickness *CA* = *H* (see Figure 46).

For rays close to normally incident rays, the angles of incidence and refraction are small. In this case, the sines can be replaced by tangents and, according to the law of refraction of light, write (considering the reverse path of the rays, that is, from *D* to *A*):

$$n_{glass} = \frac{\sin i_2}{\sin i_1} = \frac{\tan i_2}{\tan i_1} \quad (63)$$

Considering that

$$\tan i_2 = \frac{CO}{h} \quad (64)$$

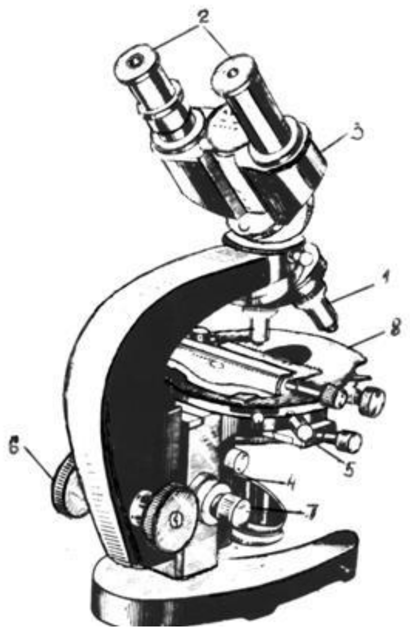
$$\tan i_1 = \frac{CO}{H} \quad (65)$$

(See figure 46), we obtain

$$n = \frac{H}{h} \quad (66)$$

where *H* is the true thickness of the glass plate; *h* is the apparent thickness of the plate.

Consequently, the refractive index of the glass can be found from the ratio of the true thickness of the glass plate to its apparent thickness. The true thickness of the plate is measured by a micrometer and the apparent thickness by a microscope with a micrometer screw. Figure 47 shows the picture of the MBI-3 microscope. The path of the rays in the microscope is shown in Figure 48.



1 - Objective lens, 2 - Eyepiece, 3 - Binocular nozzle, 4 - Lightning mirror, 5 - Table adjustment knob, 6 - Handle for coarse focusing, 7 - Handle for precise focusing (drum), 8 - Stage

Figure 47 - Structure of the MBI-3 microscope

According to figure 48, the subject S_1 is placed between the focal length and the double focal length of the lens (F_1 is the focus of the objective lens, and F_2 is the focus of eyepiece). The image S_2, given by the objective lens 1, is viewed in the eyepiece 2, as in a magnifying glass. The eyepiece is positioned in such a way that the imaginary increase in the image S_3 of the object is at a distance of the best view from the eye (0.25 m).

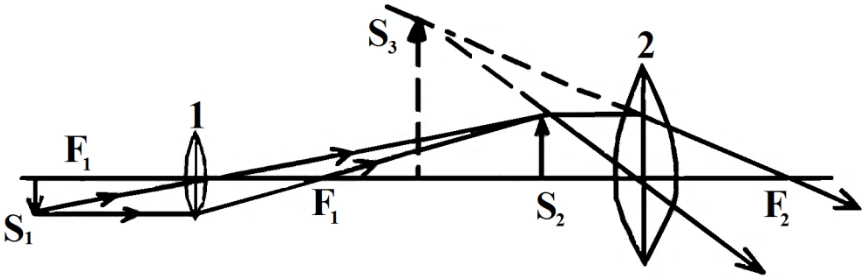


Figure 48 - Optical path of the rays in MBI-3 microscope

Linear lens magnification

$$k_1 = \frac{S_2}{S_1} \tag{67}$$

Eyepiece

$$k_2 = \frac{S_3}{S_2} \tag{68}$$

Microscope

$$k = \frac{S_3}{S_1} \tag{69}$$

Modern microscopes (see figure 47) are complex optical instruments whose lenses and eyepieces consist of several lenses.

Order of performance and processing of the measurements results

1. With a micrometer, measure the true thickness of the glass plate *H* at the point where the strokes are applied, and take its value in millimeters.

2. Determine the apparent thickness of the glass plate h, for which the plate is placed on the microscope subject stage 8 under the objective lens 1 so that both strokes cross the optical axis of the instrument. Rotating the handle for coarse focusing 6 lower the binocular nozzle 3 to its lowest position (see figure 1.3).

3. By rotating drum 7, the mark on the microscope body is aligned with the 0 a scale of the fine-focusing mechanism 7.

4. Watching in the eyepiece 2 and slowly rotating the handle for coarse focusing 6, raise the tube until the appearance in the eyepiece of the sharp image of the risks on the lower surface of the plate.

5. Then, rotating the drum 7 of the precise focusing mechanism and counting the number of revolutions of the micrometer screw, a sharp image of the risks on the upper surface of the plate is obtained. The number of revolutions of the micrometer screw considering the fission rate will give the value h, mm:

$$H = (NZ + 0{,}002\ m) \tag{70}$$

here *N* means full rotations number of the screw drum; *Z* - drum pitch in mm ($Z = 0.002 \times 50 = 0.1$); 50 is the divisions number in one complete drum rotation; 0,002 - the price of one division of the screw drum (mm); *m* - the divisions number in the incomplete rotation of the drum.

6. Using formula (70), calculate the glass refractive index. Then, calculate the error.

7. Measurement of the true and apparent thickness of each plate produces at least three times; determine the average and true value of the refractive index of the glass. The obtained measurement results are entered in the table (the form of Table 1).

Table 1 For the measurement of refraction index

Plate	№ experiment	H, mm	N	m	h, mm	n	<*n*>
1 thick	1						
	2						
	3						
2 thin	1						
	2						
	3						

Questions for the self-test

1. Formulate the basic laws of geometric optics.
2. How are the refractive index of the medium and the speed of light propagated in it?
3. Why does it seem closer to the object through a flat glass plate?
4. Draw a path of rays in a microscope.
5. Under what conditions the formula (69) is valid?
6. Output the formula for calculating the relative error, using a differential method.
7. Critical remarks on the work.

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Laboratory work № 2

DETERMINATION OF THE REFRACTION INDEX OF LIQUID USING REFRACTOMETER

Aim of the work: To study the principle of the refractometer and to determine the dependence of the refractive index of an aqueous solution of glycerol on concentration. The plot of the dependence of the exponent *n* on the concentration solution *C*. Determine the unknown concentration of the solution according to the graph of *n*(C).

Accessories: Refractometer, solutions of various concentrations.

Brief theoretical description

Optical instruments designed to measure the refractive index are called refractometers.

Principle of the refractometer. In this paper, a refractometer is used which operation is based on phenomena occurring when light passes through the interface between two media with different refractive indices (Abbe refractometer). In refractometers of this type, the medium (usually a liquid) is placed in a gap (about 0.1 mm) between the faces of two glass rectangular prisms (see figures 2.1 and 2.2). In the measurements, two methods are used: **the sliding ray method** and the **total reflection method**.

The method of the sliding ray.

The light from the lamp is directed to the face *AB* of the illumination prism P_1, passes through the matte face of the *AC* and, scattered by it, passes a plane-parallel layer (with a refractive index *n*) and falls on the diagonal face *ED* of the measuring prism P_2 at various angles ranging from 0^o to 90^o.

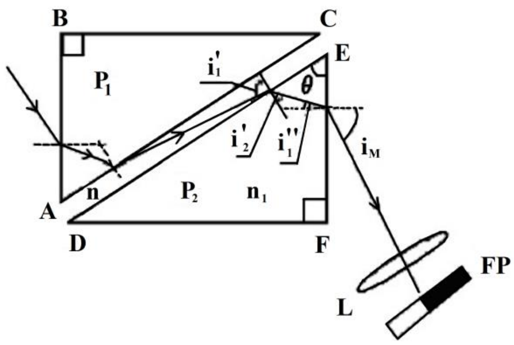


Figure 49 - Optical scheme of the rays for the method of the sliding ray of the refractometer

A beam in which angle of incidence is 90° is called a sliding ray. For the sliding ray *i*'=90^o (it is depicted in figure 49), the refractive law on the *ED* edge has the form

$$n = n_1 \sin i'_2 \qquad (71)$$

where the angle i'_2 is equal to the limiting angle for the boundary "glass - the investigated liquid" *n*' is the refractive index of the prism P_2 (with $n > n_1$).

For the face *EF* the law of refraction has the form

$$n_1 \sin i''_1 = n_0 \sin i_M \qquad (72)$$

where $n_0 = 1$; and i_M is the exit angle of the rays from the prism P_2, and for the sliding rays it has the smallest value (check it on your own).

The rays passing through the face of *EF* will leave the prism P_2 at angles from 90^o to i_M, and if in the path of these rays we put the collecting lens A, then in its focal plane the image is obtained in which the sharp boundary between light and shadow. We obtain a relation between the sought refractive index of the liquid n and the limiting exit angle i_M. From geometric considerations it can be shown that in prism P2 the refracting angle $\theta = i_2' + i_1''$

Then expression (72) can be written in the form $n = n_1 \sin(\theta - i_1'')$.

Using the sine formula for the difference of two angles and taking into account (72), we obtain the relation

$$n = \sin\theta \cdot \sqrt{n_1^2 - \sin^2 i_M} - \cos\theta \cdot \sin i_M \qquad (73)$$

Thus, it can be seen from formula (73) that the position of the boundary of light and shadow depends on the value of the refractive index n (under given prisms). The boundary of light and shadow is considered through the second lens, which together with the lens L forms a telescope, installed at infinity. With this tube, the angle i_M is determined and the refractive index n (that is, the scale of the instrument) is calculated from the known values θ and n_1d. The refractive index scale, visible in the field of view simultaneously with the interface between light and shadow, is graduated in a refractometer for sodium light (λ = 5893 Å) at t = 20 °C.

The method of total reflection

The light from the lamp is introduced into the refractometer through the matte face of the *PD* of the measuring prism P2 and therefore falls on the diagonal face *ED* of these prisms at all possible angles. At angles of incidence $i'_1 > i_1$ there will be a total reflection. The rays passing through the face *EF* and having output angles greater than i_M will give an image with a lower illumination in the focal plane of the photodetector of the lens; and the rays with exit angles less than i_M will give greater illumination of the phase transition. In the visual field of the telescope, in this case, there is a sharp interface between penumbra and light.

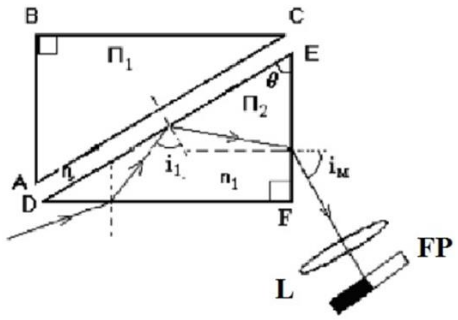


Figure 50 - Optical scheme of the rays for the method of total reflection of the refractometer

The position of the interface between penumbra and light is determined by the angle i_M satisfying (as in the case of a sliding beam) the condition (73). Note that by the method of total reflection it is possible to measure the refractive index and opaque bodies. In conclusion, we emphasize once again: if the rays of light directly or from a mirror installed on the body of the device fall on the entrance window of the illuminating prism, then the measurement of the refractive index is realized by the "sliding-beam method". If the rays get to the input window of the measuring prism, then the "total reflection method" is realized.

The optical system of the refractometer consists of a sighting and counting system and is arranged so that the refractive index scale is visible in the field of view simultaneously with the interface between light and shadow.

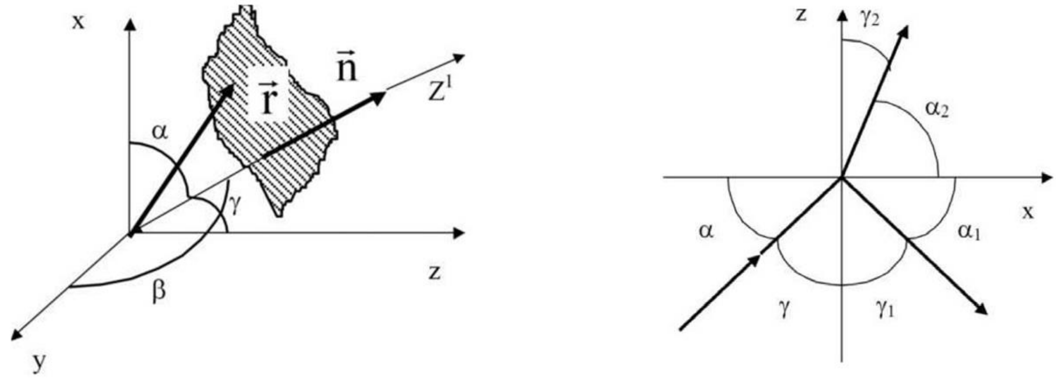
Laws of reflection and refraction of electromagnetic (light) waves

Let us consider the case when an electromagnetic wave falls at an arbitrary angle to the interface between two media.

First of all, it is necessary to form an expression for an electromagnetic wave propagating in an arbitrary direction with velocity v_1(Figure 49).

Let x, y, z be the current coordinates of the point on the plane, the $\vec{n}$ normal to which coincides in direction with z' and the radius vector of this point is $\vec{r}$. If $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are the direction cosines of the normal $\vec{n}$, then for the wave propagating along the chosen direction z', we get the expression:

$$E = ReE_{\infty}\, exp\left[i\omega(t - \frac{\vec{r}\vec{n}}{v_1})\right] = ReE_{\infty} exp[i\omega(t - \frac{x\cos\alpha + y\cos\beta + z\cos\gamma}{v_1})] \qquad (74)$$



a) b)

Figure 51 - a) Illustration to the derivation of the equation of a plane wave propagating along an arbitrary direction b) Illustration to the conclusion of the laws of reflection and refraction of electromagnetic waves

Now it is not difficult to compose expressions for the incident, reflected and refracted waves. As before, the interface between two media is xy, which satisfies the condition $z = 0$. We will also assume that in the incident wave the normal $\vec{n}$ lies in the plane zx ($\cos\beta = 0$, in the plane of Figure 2.3). We will not impose any restrictions on the direction of the normal $\overrightarrow{n_{12}}$ (in the reflected wave) and $\overrightarrow{n_2}$ (in the refracted wave). Let us take the propagation velocity v_2 of the electromagnetic wave in the second medium v_2. Then

$$E = ReE_{00}e^{[i\omega(t - \frac{x\cos\alpha + z\cos\gamma}{v_1})]}; E_1 = ReE_{01}e^{[i\omega_1(t - \frac{x\cos\alpha_1 + y\cos\beta_1 + z\cos\gamma_1}{v_1})]} \qquad (75)$$

$$E_2 = ReE_{02}e^{[i\omega_1(t - \frac{x\cos\alpha_2 + y\cos\beta_2 + z\cos\gamma_2}{v_2})]} \qquad (76)$$

For $z = 0$, the boundary condition - the equality of the tangential components of the electric field strength will have the form

$$E_\tau + E_{\tau_1} = E_{\tau_2} \qquad (77)$$

The latter must be satisfied at any time t and for any x, y coordinates. Otherwise,

$$E_{00_\tau} \cdot e^{\left[i\omega\left(t - \frac{x\cos\alpha}{v_2}\right)\right]} + E_{01_\tau}e^{\left[i\omega_1\left(t - \frac{x\cos\alpha_1 + y\cos\beta_1}{v_1}\right)\right]} = E_{02_\tau}e^{[i\omega_2(t - \frac{x\cos\alpha_2 + y\cos\beta_2}{v_1})]} \qquad (78)$$

This identity will be valid only if several conditions are fulfilled:

1. $\omega = \omega_1 = \omega_2$ (this result is trivial for linear problems, which we consider).

2. $\cos\beta/v_1 = \cos\beta_2/v_2 = 0$. Assuming that the normal $\vec{n}$ to the incident wave $\vec{E}$ lies in

the *zz* plane, we conclude that the normal to the reflected and refracted waves ($\vec{n}$ and $\overrightarrow{n_2}$) also lies in this plane.

3. $\cos\alpha/v_1 = \cos\alpha_2/v_1 = \cos\alpha_2/v_2$ Analysis of these relationships is most convenient in two consecutive phases:
3.1 $\cos\alpha = \cos\alpha_1$, and hence $\alpha = \alpha_1$ i.e. the law of reflection of the electromagnetic (light) waves is obtained - the angle of reflection of the wave is equal to the angle of incidence $\gamma = \gamma_1$.
3.2 $\cos\alpha\,/\cos\alpha_2 = v_1/v_2$ Here, for that $\alpha + \gamma = \pi/2$ we get $\sin\gamma/\sin\gamma_2 = v_1/v_2$ and $\alpha_2 + \gamma_2 = \pi/2$ the law of refraction of electromagnetic waves. The last expression can be represented in $v_1 = c/n_1$ and more familiar form $v_2 = c/n_2$. Then we finally get

$$\frac{\sin\gamma}{\sin\gamma_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1} = n_{21} \tag{79}$$

In optics, this ratio is usually called Snell's law. Here n_1 and n_2 are the absolute refractive indices of the first and second medium, n_{21} the relative refractive index of the second medium to the first medium, and c is the velocity of propagation of electromagnetic (light) waves in a vacuum.

If the light propagates in the opposite direction (i.e., falls at an angle $\gamma_2$from the second medium to the first one), then the incident and refracted rays are exchanged (the property of reversibility of light rays). In this case, the relation holds $n_{21} = 1/n_{21}$

When light propagates from a medium that is optically less dense in the medium, the optically denser value of the reflected light energy increases with increasing angle of incidence. The value of the energy of the refracted beam decreases in this case.

If, however, the light propagates from the optically denser medium to the medium is optically less dense (the smaller absolute refractive index), then the fraction of reflected light energy also increases with increasing angle of incidence. However, starting with a certain angle of incidence γ_{12}. All the light energy of the beam is reflected from the interface and light does not pass from the first medium to the second (the phenomenon of total internal reflection).

The limiting angle of total internal reflection γ_{12} can be determined (considering $\gamma = {}^{\pi}/_{2}$) and $n_1 > n_2$ according to (79) how

$$\sin\gamma_{12} = n_2/n_1 \tag{80}$$

Dispersion of light

By dispersion of light is meant the splitting of white light into a spectrum that occurs during refraction, diffraction, or interference of light. In a narrower sense, the dispersion of light is the dependence of the phase velocity of an electromagnetic (light) wave on its frequency, or the dependence of the refractive index of a substance on the frequency (wavelength) of light

$$n = f(\lambda_0) \tag{81}$$

Where λ_2 is the length of the light wave in a vacuum.

To quantify the dependence of the refractive index of a given substance on the wavelength, the concepts of mean and relative dispersion are introduced. If, for example, two wavelengths λ_1 and λ_2 correspond to the values of the refractive indices n_1 and n_2, then the average dispersion of the substance can be represented by the relation

$$\frac{\Delta n}{\Delta\lambda} = (n_2 - n_1)/(\lambda_1 - \lambda_2) \tag{82}$$

In practice, the measure of variance or take an average dispersion, relative dispersion of either

$$(n_F - n_c)/(n_D - 1) \quad (83)$$

Where (n_F, n_c, n_D) are the refractive indices for Fraunhofer spectral lines F, C, D with wavelengths λ_F = 486,1 nm, λ_D = 589,3 nm. Much less commonly used is the reciprocal of the relative variance, called the dispersion coefficient

$$\nu = (n_D - 1)/(n_F - n_c) \quad (84)$$

For all transparent colorless substances, the value of the function (84) will be $(dn\ /\ d\lambda) < 0$. This type of dispersion is called **normal**. If the substance absorbs, then the dispersion path reveals an **anomaly** in the absorption region and near it $(dn\ /\ d\lambda) > 0$.

An explanation of the phenomenon of light dispersion can be given, for example, from the point of view of the interaction of the electromagnetic field of a light wave with the electric charges of a medium.

Under the influence of the electric field of a light wave, a dielectric (medium) is polarized. The measure of polarization of the medium can be estimated by a vector $\vec{P} = Ne\vec{x}$, where N is the number of dipoles per unit volume, $e\vec{x}$ is the electric moment of the dipole. It is known that the value of the polarization vector is directly proportional to the intensity E of the electric field of the light wave $\vec{P} = \varepsilon_0 \chi \vec{E}$, where χ the electrical susceptibility of the substance is related to the permittivity of the substance by the relation $\varepsilon = 1 + \chi$. According to Maxwell's theory $n = \sqrt{\varepsilon\mu}$, the refractive index (and for most transparent bodies can be taken $\mu = 1$). Then, considering the above, we can obtain the following expression:

$$n^2 = 1 + (Ne/\varepsilon_0 E)x \quad (85)$$

The size of the displacement x can be found by making up the equation of motion of the electron located in the atom of matter under the influence of the electromagnetic field of the light wave. To this end, we consider the forces acting on an individual electron:

1. Forcing force from the electric field of the transmitted lightwave $f_1 = eE_0 \sin \omega t$
2. Quasi-elastic force $f_2 = -m_0\omega_0^2 x$ where ω_0 is the Eigen frequency of an electron, m_0 is its mass;
3. The frictional force, which leads to a damped nature of the oscillations $f_2 = -m_0\beta\dot{x}$, where $\dot{x}$ is the velocity of the electron, β is the attenuation index.

The equation of motion of an electron, considering the forces listed above, takes the form

$$m_0\ddot{x} = eE_0 \sin \omega t - m_0\omega_0^2 - m_0\beta \quad (86)$$

The solution of this differential equation is an expression of the form

$$x = A_0 \sin(\omega t + \varphi) \quad (87)$$

where, $A_0 = eE_0/m_0\sqrt{(\omega_0^2 - \omega^2)} + 4\beta^2\omega^2$, so

$$\tan \varphi = -2\beta\omega/(\omega_0^2 - \omega^2) \quad (88)$$

If the frictional forces are small ($\beta \to 0$), then instead of (11.2) we write

$$x = [eE_0/m_0(\omega_0^2 - \omega^2)] \sin \omega t \quad (89)$$

Substituting the value of *x* from (89) into (85) we finally obtain

$$n^2 = 1 + [Ne^2/\varepsilon_0 m_0(\omega_0^2 - \omega^2)] \tag{90}$$

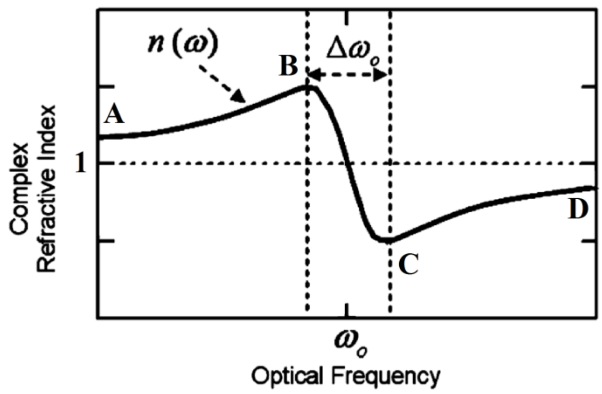


Figure 52 - The form of the dispersion near a single absorption band

The analysis of relation (86) is presented in the form of the graph $n = f(\omega)$ in Figure 52. Here the segments *AB* and *CD* - represent sections of normal variances. When $\omega = \omega_0$ the refractive index does not have a definite value (dotted line) due to the assumption of the absence of frictional force $\beta = 0$. If we consider attenuation, then a segment of the aircraft appears, within which the refractive index decreases with increasing frequency (anomalous dispersion). This manifest itself in the resonance absorption region (for a frequency ω close to ω_0). Equation (90) is incomplete, since the action of neighboring molecules and dipoles is not considered here. If this factor is considered, then, as shown by Lorentz and Lorentz, it will be valid, in the case of no associated liquids with nonpolar molecules, the following dependence for the refractive index

$$(n^2 - 1)/(n^2 + 2) = (4\pi/3)N\alpha \text{ and } [(n^2 - 1)/(n^2 + 2)]M/\rho = (4\pi/3)N_0\alpha \tag{91}$$

where: *N* - number of particles per unit volume, α- polarizability of molecules, *n* - refractive index, N_0 - Avogadro number, ρ - matter density, M - molar mass.

Expression (91) is essentially familiar in the section "Electricity" of the course of general physics by the Clausius-Mosotti formula (it is only necessary to go from n to ε, according to their connection $n\sqrt{\varepsilon}$, which characterizes the electronic polarization of dense dielectrics).

The value $[(n^2 - 1)/(n^2 + 2)]M/\rho = R$ is called the *molecular refraction*. For solutions, in particular, the rule of additivity of molecular refractions takes place (the refraction of the solution consists of refractions of the components $R = \sum_k c_k R_k$, where c_k is the concentration of the component in molar fractions).

If there is, for example, a uniform mixture of two components with the number of molecules per unit volume N_1 and N_2, then formula (90) takes the form

$$(n^2 - 1)/(n^2 + 2) = (4\pi/3)(N_1\alpha_1 + N_2\alpha_2) \tag{92}$$

Here: *n* is the refractive index of the mixture $N_1 = \left(\frac{\rho_1}{M_1}\right)N_0$, $\rho_1 = \frac{M_1}{(V_1+V_2)}$, $N_2 = \left(\frac{\rho_2}{M_2}\right)N_0$, $\rho_2 = \frac{M_2}{(V_1+V_2)}$, where V_1 is the volume of the first component, V_2 is the volume of the second component. We denote the density of the pure components by $\rho_{01} = \frac{M_1}{V_1}$ and $\rho_{02} = \frac{M_2}{V_2}$ and the volume density $= V_1/(V_1 + V_2)$. Then (92) can be represented by the expression

$$\frac{n^2-14\pi}{n^2+12} N_0\alpha_1 \frac{\rho_{01}}{M_1}\delta + \frac{4\pi}{3} N_0\alpha_2 \frac{\rho_{02}}{M_2}(1-\delta) = \frac{n_2^2-1}{n_2^2+2} + \left[\frac{4\pi}{3} N_0\alpha_1 \frac{\rho_{01}}{M_1} - \frac{n_2^2-1}{n_2^2+2}\right]\delta \tag{93}$$

Or final

$$\frac{n^2-1}{n^2+2} = \frac{n_2^2-1}{n_2^2+2} + \left[\frac{n_1^2-1}{n_1^2+2} + \frac{n_2^2-1}{n_2^2+2}\right]\delta \quad (94)$$

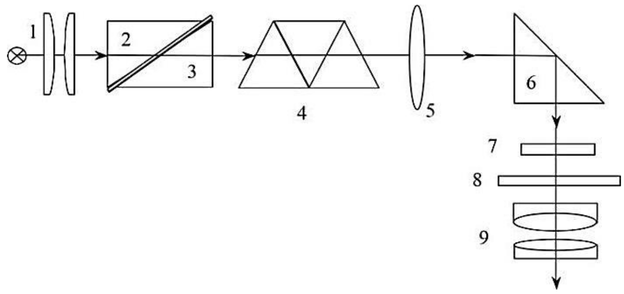
Refractometers RPL-3 AND IRF-22 construction

The optical diagrams of the refractometers RPL-3 and IRF-22 are shown in Figure 53 and Figure 54, correspondingly.

From the illuminator 1 (or mirrors in case the refractometer IRF-22), the light beam is directed to a double prism, between the diagonal planes of which is a thin layer of a liquid test substance. Then the beam passes the dispersion compensator, the objective lens, the prism, the grid with the sight lines (three dashes for the RPL-3 and the cross-line for the IRF-22), the scale and through the eyepiece it enters the eye of the observer.

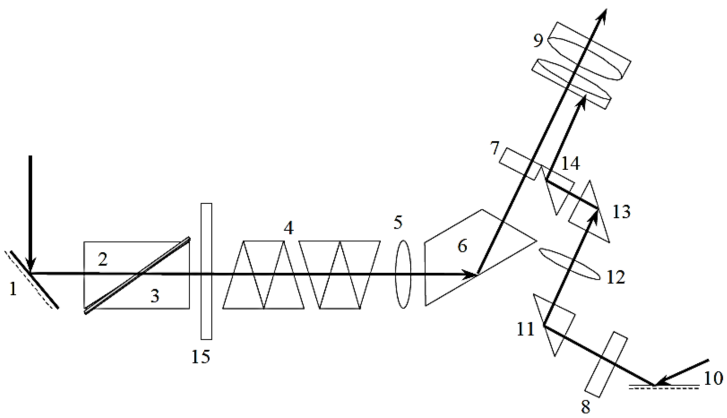
Dispersion compensator is designed to eliminate the spectral color of the boundary of light and shade. The components of the compensator are Amichi prism (direct vision prism, letting yellow rays with λ_D= 589.3 nm without deviation). Two compensating prisms in the IRF-22 refractometer form an optical system with variable dispersion. The correct position of the compensator is selected by rotating the prisms around the beam direction.

The grid line is aligned with the border of light and shade and the index of refraction is measured on the scale.



1 - Illuminator, 2 - Lighting prism, 3 - Measuring prism, 4 - Dispersion compensator, 5 Objective lenses, 6 - Rotary prism, 7 - Grid with sight lines, 8 - Scale of refractive index values, 9 - Eyepiece

Figure 53 - Principal scheme for the ray path in the refractometer RPL-3



1 - Lighting Mirror, 2 - Lighting prism, 3 - Measuring prism, 4 - Dispersion compensator, 5 Objective lenses, 6 - Rotary prism, 7 - Grid with sight lines, 8 - Scale of refractive index values, 9 - Eyepiece, 10 - mirror for the scale lighting, 11,13,14 - System of rotary prisms, 12 - Scale focusing lens, 15 - Protection glass

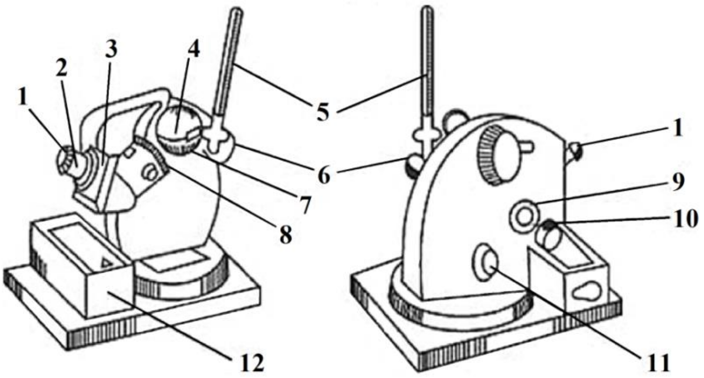
Figure 54 - Principal scheme for the ray path in the refractometer IRF-22

The combination of the visor with the interface between the light and dark fields is carried out in two ways:

1. By turning the objective lens (together with the compensator) around an axis perpendicular to the plane of the drawing at the beam exit from the measuring prism (for RPL-3 and RL refractometers);

2. Rotating the measuring head together with the instrument scale relative to the same axis (for the IRF-22 refractometer).

In the IRF-22 refractometer (Figure 55), the scale 8 is illuminated by a mirror 10 and projected into the focal plane of the eyepiece 9 by means of a micro-lens 12 through a prism system 11, 13 and 14.



1 - Eyepiece, 2 - Tube, 3 - Socket for a key, 4 - Upper hemisphere of the measuring head, 5 - Thermometer, 6 - Lighting mirror, 7 - Lower hemisphere of the measuring head, 8, 11 - Handwheels, 9 - Window, 10 - Mirror for lighting a scale, 12 - Pencil case

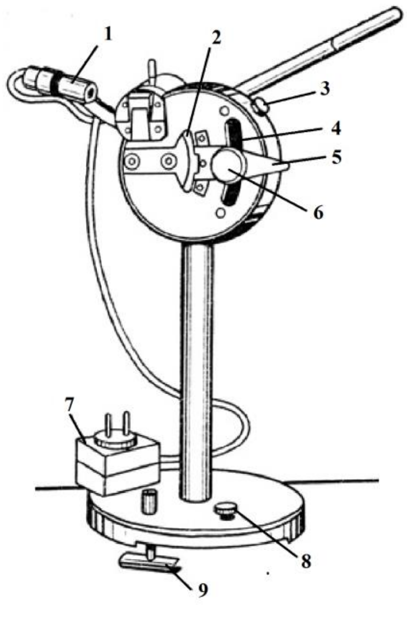
Figure 55 - General view of the IRF-22 refractometer

The measuring head of both refractometers is made in the form of two cameras with illuminating and measuring prisms. The chambers are pivotally interconnected (the upper chamber can be slightly opened by the relative lower one) and have internal channels with external fittings for supply and removal of the thermosetting liquid.

There is also a fitting for fixing the thermometer to estimate the temperature of the test fluid. In addition, both head chambers have windows for directing light beams in them.

In the case of colorless and slightly colored liquids, light is sent to the upper (lighting) prism. When measuring refractive indices of intensely colored liquids that strongly absorb light, the lower (measuring) prism window is used.

In RPL-2 refractometers (Figure 56) to the right of the scale with refractive indices there is a second scale with the values of sugar concentration in percent. *Naturally, this part of the general scale can only be used when working with sugar solutions*.



1 - Illuminator with red filter; 2 - Scale and screw of the compensator; 3 - Cork; 4 - Scale; 5 - Handle, designed to align the sighting line of the grid with the border of light and shade (moving up and down); 6 - Eyepiece; 7 - Illuminator transformer; 8 - Light filter of the illuminator; 9 - Prism screen

Figure 56 - General view of the refractometer RPL-3

Limits of measurement of the refractive index n_D from 1,3000 to 1,5400 (for RPL-3 refractometer) and 1,7000 (for IRF-22 refractometer).

Permissible error in the scale of refractive indices n_D for multiple measurements $\pm 2 \times 10^{-4}$.

Order of performance

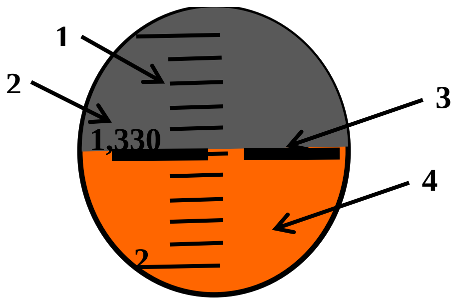
1. Read the RPL-3 (IRF-22) refractometer principle of the action of operation.

2. Check the setting for a zero-point.

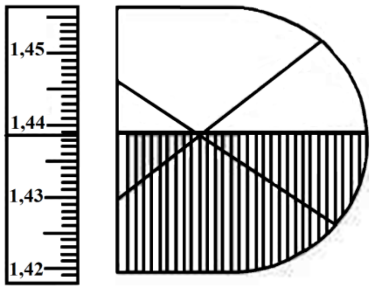
3. For this purpose, the upper chamber of the measuring head is opened. Flush the planes of the upper and lower chambers with distilled water and remove traces of liquid, applying filter paper to the surface of the prisms (do not rub prism surfaces!).

4. Use a dropper (melted glass rod) to set one - two drops of distilled water on the plane of the measuring prism and close the upper chamber of the measuring (prism) head.

5. The RPL-3 handle with the eyepiece is lowered to the lower position and moved until the light and shadow boundary appears in the view field (Figure 57). The same effect is achieved in the case of IRF-22 by turning the corresponding handwheel 8.



1 - Scale, 2 - Index of refraction, 3 - Sighting line at the boundary between dark and illuminated region, 4 - Illuminated region



a) b)

Figure 57 - a) Eyepiece view for the RPL-3 refractometer, b) Eyepiece view for the IRF-22 refractometer (Red color due to illumination with red filter)

6. When observing the view field, the aperture of the dioprintal tip of the eyepiece should be rotated until a sharp image of the scale divisions and the sighting lines of the grid appear (IRF2-2).

7. Moving the illuminator up and down (changing the orientation of the mirror) in front of the input window of the device, achieve the most contrast illumination of the field of view, and by turning the sector with the scale rotate the prisms of the dispersion compensator.

(In case of IRL-3, Combine the sight line of the grid with the border of light and shade and make a countdown on the scale. If the instrument is correctly set to zero, the light and shade boundary at 20 °C should be combined with the division of n_D = 1.33299 of the refractive index scales. If the reading deviates from this value, the difference should be considered for all subsequent measurements).

8. Measure the refractive indices of a series of two-component different concentrations and one-component fluids.

9. Using the refractive index measurements for one-component fluids, calculate the polarizability of *a* and according to formula

10. Draw a graph *n(C)* in order to determine unknown concentration *X* by the method of minimal squares.

Questions for self-test

1. What is called the absolute and relative refractive index?
2. What is the phenomenon of total internal reflection?
3. Explain the principle of the Abbe refractometer (the formation of a certain boundary of light and dark fields).
4. Explain the necessity and the principle of the action of the dispersion compensator in a

refractometer.
5. What is meant by specific, atomic and molecular refraction?

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Laboratory work № 3

DETERMINATION OF THE LIGHT WAVELENGTH USING THE NEWTON RINGS

Aim of the work: Acquaintance with the method of implementation of coherence in optics by the principle of amplitude division (thin films, lines of equal thickness). Mastering the techniques of working with a microscope (evaluation of the magnification of an image obtained using a microscope, determining the price of dividing an ocular micrometer) and measuring linear dimensions with it. To study the interference method of measuring the wavelength of light transmitted by a light filter. To study one of the methods of measuring the radius of curvature of the lenses.

Accessories: Microscope, lens glued with a plate

Brief theoretical description

Newton's rings are a type of interference phenomenon in thin films, called lines of equal thickness. Coherent beams in the installation for observing Newton's rings are obtained by separating the amplitudes of the oscillations. The principle of division can be represented as follows.

There is a small portion of the wavefront falling on a thin plate. The energy of the beam belonging to this section of the front is divided into two parts by reflection on the upper and lower borders of the thin plate. Naturally, these two beams will have smaller amplitudes of oscillations than in the incident beam. Both beams go through different paths and can be reduced to one place, where they will give one or another interference pattern depending on the difference in travel between them.

Let us consider in more detail the method of obtaining Newton rings. Take a plane-convex lens of small curvature and place it on a flat-parallel glass plate, as shown in Figure 58. A kind of air wedge forms between the lens and the plate (gradually thickening from the center to the edges of the air gap).

Optical contact between the lens and the plate, as a rule, is not present, although the smallest thickness of the air gap is small ($OO' << \lambda$).

If a beam of monochromatic light is directed along the normal to the upper boundary of the lens, the light waves will be reflected from the upper and lower faces of the air wedge (respectively, *O*, *O'* and *A*, *A'*).

As a result of the imposition of coherent beams at the upper boundary of the air gap, an interference pattern in the form of alternating dark and light rings will be observed in reflected light. In the center of the picture there will be a dark spot (interference minimum). This is due to the fact that the path difference here is determined only by the loss of the half-wave ($\Delta=\lambda/2$) when reflected from a denser medium at point *O'* (after all, $OO' << \lambda$)

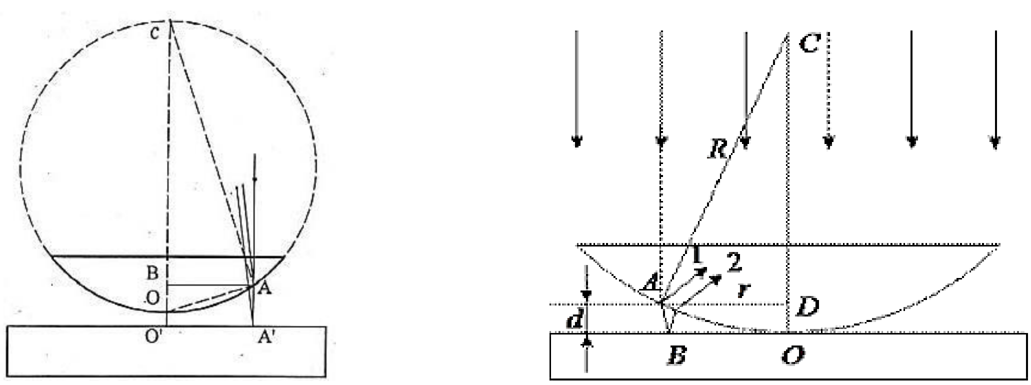


Figure 58 - Illustration for the radii of the Newton rings calculations

Since the geometric position of points having the same path difference as for point *A* will be a circle - The interference pattern looks like rings.

The total optical path difference at point *A* is

$$\Delta = 2\delta_m + \lambda/2 \tag{95}$$

where δ_m is the thickness of the air gap, and $\lambda/2$ is the additional path difference due to a change in the phase of the wave to the opposite when reflected from a denser medium at point *A'*.

The optical path difference (like the optical path length) should generally contain the refractive index of the medium in which the luminous flux propagates. In our case, we can omit the symbol of the index of refraction, since for the air gap, the refractive index is one.

The path difference can be associated with the geometry of the system - the radius of curvature of the lens *R* and the radius *m* of that interference ring. From the *OAC* triangle (Figure 59) it follows: $AB^2 = OB - BC$ or $r^2 = \delta_m(2R - \delta_m)$. Neglecting the value of $-\delta_m$ compared to $2R$, we get

$$\delta_m = r^2/2R \tag{96}$$

For the formation of the m-light ring should be

$$\Delta = 2m\lambda/2 \tag{97}$$

Then from formulas (3.1), (3.2) and (3.3) we obtain the following value of the radius of the bright ring:

$$r_m = [(2m-1)R\,\lambda/2]^{\frac{1}{2}} \tag{98}$$

Accordingly, for the m-dark ring we will have:

$$r_m = (mR\lambda)^{1/2} \tag{99}$$

The greater the number *m*, the smaller the difference between the radii of adjacent rings (the narrower the rings).

To determine the wavelength λ (or *R*), you can use expressions (98) and (99). However, the calculations will be more accurate if calculated by the difference of the radii of the two rings r_m and r_k. In this case, errors introduced by the additional difference of the stroke due to the elastic deformation of the glass at the point of contact of the lens with the plate and contamination (dusting) of the contacting surfaces are excluded. The calculation formulas are as follows:

$$\lambda = \frac{r_m^2 - r_k^2}{(m-k)R} \text{ and } R = \frac{(r_m - r_k)(r_m + r_k)}{(m-k)\lambda} \tag{100}$$

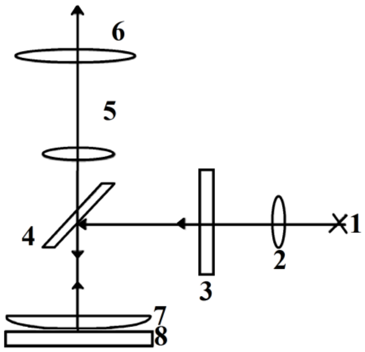
$$\lambda = \frac{(r_m - r_k)(r_m + r_k)}{(m-k)R} \text{ and } R = \frac{r_m^2 - r_k^2}{(m-k)\lambda} \tag{101}$$

The sizes of Newton's rings are small, so they are examined with a microscope.

Newton's rings are inspected with a microscope

The principle path of the rays of the installation (microscope MBU-4) is illustrated in Figure 59.

In the diagram, we can distinguish three main nodes - the illuminator (elements 1, 2, 3), the microscope (elements 5, 6), the object of measurement is the frame with a flat-convex lens on the plate (elements 7, 8). The plane-parallel plate 4 sends a parallel monochromatic beam of light created by the illuminator to the measuring device. Reflecting from the latter, coherent beams interfere. Due to the small radii of Newton's rings, the interference pattern is examined through a microscope

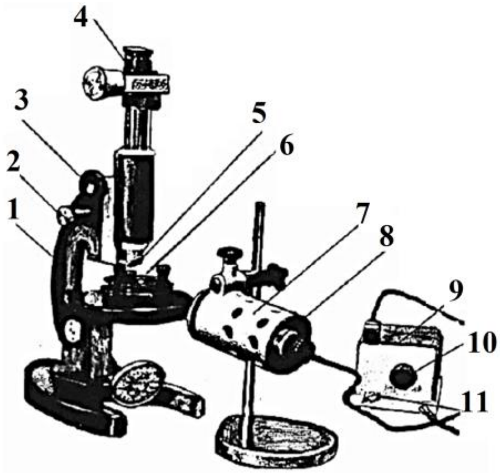


1 - Light source (8 V, 20 W), 2 - Lens, 3 - Monochromatic light filter, 4 - Plane-parallel glass plate at an angle 45°, glued to the lens 5, 5 - Lens, 6 - Ocular micrometer, 7 - Flat-convex lens, 8 - Glass plate

Figure 59 - Schematic diagram of the path of the rays in the installation

The radii of interference rings required for the calculation are determined using an ocular micrometer.

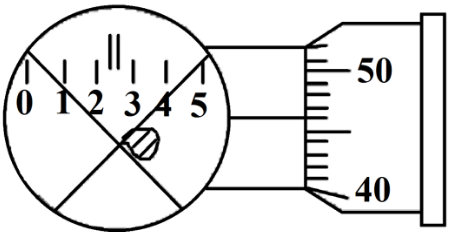
Figure 60 shows a general view of the instruments used in this work.



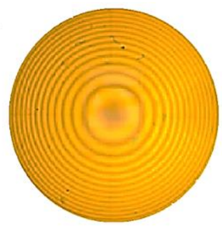
1 - Tube holder, 2,3 - Smooth and Coarse focusing, 4 - Eyepiece micrometer MOV-1-15^x, 5 - Long-focus lens with a plane-parallel plate on its rim, 6 - Measuring device (flat-convex lens and a glass plate in a frame), 7 - Lamp of the illuminator OI-19, 8 - Base of the lamp holder of the illuminator, 9 - Step-down transformer, 10 - Handle of the rheostat, 11 - Toggle-switch

Figure 60 - General view of the installation for observing the rings of Newton

The eyepiece micrometer MOV-1-15^x of the MBU-4 microscope scale with and the Newton Rings is presented in Figure 61.



a)



b)

Figure 62 - a) Eyepiece micrometer MOV-1-15^x, b) Appearance of the Newton rings

Installation preparation for measurements

Begin by getting a beam of light close to parallel at the exit of the illuminator. To do this, move the base of cartridge 8 along with the lamp along the optical axis of the lamp, looking at using a small white paper sheet for changing the configuration of the light beam at the output of the illuminator. If the light spot at almost different distances from the illuminator remains almost constant, the light beam will be sufficiently parallel.

Direct the light beam of the illuminator on the plane-parallel plate of the rim of the microscope objective. Instead of a measurement object, you should put a small piece of white paper with a label in the center (dot, etc.) on the table of the microscope. Changing the height and inclination of the illuminator lamp, they strive to obtain a light spot reflected from the lens plate, symmetrical about the axis of the microscope (index of a piece of paper).

Such a preliminary preparation of devices allows you to go directly to the measurements, during which a more accurate adjustment is made for each type of task. In the process of all the work it is also necessary to remember that to avoid injury to the observer's eye, the glow of the illuminator lamp should not be excessive.

Order of performance

The laboratory work aims to determine the radius of curvature *R* of the lens of the measuring setup and the wavelengths transmitted by the green and red light filters. The content of the work involves the following tasks: It is impossible to determine the magnification of the microscope objective γ and without knowing the magnification when determining the radii of the rings using the scale of the eyepiece micrometer (on the plane of the main scale of the eyepiece micrometer, the magnification of the Newton rings).

To determine the magnification of the microscope objective, an objective micrometer is placed on the table - a transparent plate on which a scale is drawn in the form of small parallel risks with a known division price *l*.

Observing through the eyepiece of the microscope for the object micrometer, put a flat mirror of the microscope in the position of the greatest illumination of the field of view. First, a clear image of the reading scale of the MOB-I-I5$_x$ ocular micrometer is achieved. Then the object micrometer is focused with the help of the focusing (Figure 62). Now it will be clearly - the scale of the ocular micrometer and the image of an objective micrometer.

Using the MOB-I-I5x, the length of *p* divisions of the object micrometer is measured (on the measurement method) and the *q* value is obtained. Then the desired magnitude of the linear increase will be determined from the relation:

$$\gamma = q/p\ l \tag{102}$$

1. Determine the radius of curvature of the lens *R* of the object of measurement. At the output of the illuminator, a red light filter is installed, transmitting a monochromatic beam of light with a wavelength of λ = 650 nm.

2. Put the measuring unit (frame with a lens and a plane-parallel glass plate) on the table of the microscope.

3. Achieve clear visibility of the rings of Newton by focusing the microscope. Sometimes the clarity of the picture increases with a slight change in the angle of inclination of the beam of light coming from the illuminator to the microscope. The rings are located in the center of the field of view. This is confirmed by the situation when both threads of the crosshair of the reading device of the MOB-I-I5$_x$ eyepiece microscope will simultaneously be tangent to any chosen interference ring with two mutually perpendicular micrometer settings.

4. Pay attention to the type of interference pattern in the center. If a maximum is observed here (a bright spot), then this means that there are dust particles between the surface of the lens and the glass, introducing a significant path difference. Dust should be removed with suede or soft cloth.

5. The rings adjacent to the center are usually blurred; therefore, the diameters of five relatively distant from the center of the rings are measured with an eyepiece micrometer. Measurements are carried out in two mutually perpendicular directions. From these measurements, the arithmetic average of the radius of each ring is found.

6. Fulfill table 2 corresponding for the green and red filter.

Table 2 - The Newton rings radii

№	r_1	r_2	$\langle r \rangle$	r, mm	d, mm
1					
2					
3					
4					
5					

6. To improve the accuracy of determining the magnitude of the radius of curvature of the lens R, it is necessary to combine the radii of five measured rings in various combinations in the calculations using formula (102). Of course, the determination of the true value of the radii of the rings should be made considering the increase in the microscope objective (γ times less), or considering the combined dividing price of the eyepiece micrometer $l_o = 1/\gamma$ (l_o times).

7. Determine the wavelengths transmitted by green and blue light filters. The execution of the measuring part is similar to the previous task, with the only difference being that instead of the red light filter, first put green and then blue.

8. Draw a graph dependence d^2 from numbers of rings m.

The calculated part is recommended to carry out in a graphical way. To do this, build a graph, on the ordinate axis of which lay the squares of diameters, and on the abscissa axis - the. The wavelength can be found from the tangent of the angle of inclination of the straight line to the abscissa axis from the relation:

$$\tan\varphi = \frac{d_m^2 - d_k^2}{m-k} = 4\lambda R \qquad (103)$$

It is not difficult to make sure that the latter formula is a certain modification of the formula (103). To assess the accuracy of measuring the radius of curvature of the lens and the wavelengths determined in previous tasks.

9. Calculate the error of the measurements

Questions for self-test

1. What is the phenomenon of interference?
2. What kind of interference pattern will have if the radiation source is white (non-monochromatic)?
3. What the observer will see in the center of the interference pattern
(see the previous question), if the observations are carried out in transmitted light?
4. Why the radius of curvature of the lens of the measurement object should be significant?
5. Explain the formation of light and dark rings, as well as the density of their distribution.
6. Where does the adjustment of the microscope itself begin in the installation?

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Laboratory work № 4

DETERMINATION OF THE LIGHT WAVELENGTH USING THE FRENSEL BIPRISM

Aim of the work. Acquaintance with the method of the coherence principle of wavefront division. Qualitative assessment of the influence of source size (slit) on the clarity of the interference pattern. Mastering basic methods of installation alignment (bringing it into a centered optical system). Using an optical micrometer when measuring linear parameters. Study of the interference method to skip the measurement wavelength filters. Assessment of the coherence measurement accuracy of different values.

Accessories: Optical branch, Illumination source, lens, eyepiece, slit, Fresnel biprism

Introduction to the light interference study

Under the interference represent the phenomenon of redistribution of energy in space, arising from the imposition of wave processes. Let us reveal the meaning of what has been said. Let O_1 and O_2 - sources of any fluctuations are located at distances z_1 and z_2, from the observation point A (Figure 4.1). Then the equations of waves propagating in space from the first and second excitation sources and reaching the observation point A can be written in the following form:

$$x_1=a_1 \sin(\omega t - kz_1); \tag{104}$$

$$x_2=a_2 \sin(\omega t - kz_2), \tag{105}$$

where a_1 и a_2 - oscillation amplitudes, $k = 2\pi/\lambda$ - wave number, λ - wavelength.

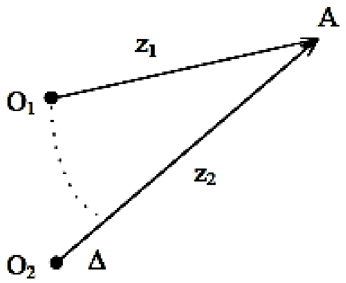


Figure 63 - Illustration for the interference of waves

The resulting oscillation x at point A can be determined by jointly solving the two wave equations (1.4) and (2.4) and A is represented by the following expression in the general form:

$$x = a \sin(\omega t + \varphi). \tag{106}$$

Here a - the resulting amplitude is related to the initial amplitudes by the ratio.

$$a^2 = a_1^2 + a_2^2 + 2 a_1 a_2 \cos \varphi_o , \tag{107}$$

in which the phase difference $\varphi_o = k(z_2 - z_1)$; Resulting phase :

$$\varphi = \tan^{-1} \frac{a_1 \sin kz_1 + a_2 \sin kz_2}{a_1 \cos kz_1 + a_2 \cos kz_2} \tag{108}$$

The amplitude value (107) depends on the phase difference $\varphi_o = k(z_2 - z_1)$ or the path difference $\Delta = z_2 - z_1$ (when considering the interference of light, an optical path difference is usually taken $(z_2n_2 - z_1n_1)$, where n_1 and n_2 are refractions of media in which light waves propagate from their separation points to overlap points).

If during a long observation time the phase difference remains constant, i.e. φ_o = const, such waves are coherent. When this condition is met, two cases are of interest:

1. $\varphi_o = 0; 2\pi; 4\pi; \ldots; 2m\pi$ or $\Delta=2m(\lambda/2)=m\lambda$. So, resulting amplitude is defined as $a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2}$, and the energy of the resulting oscillation (intensity) will be equal to $J = \left(J_1 + J_2 + 2\sqrt{J_1J_2}\right)$, because $J \sim a^2$

In other words, at the observation point there will be an increase in intensity (the resulting energy value is greater than the sum of the energies generated by each of the excitation sources separately).

2. $\varphi_o = \pi; 3\pi; 5\pi; \ldots; (2m+1)\pi$ or $\Delta=(2m+1)(\lambda/2)$. In this case $J = \left(J_1 + J_2 - 2\sqrt{J_1J_2}\right) < (J_1 + J_2)$, i.e. there will be a decrease in intensity at the point of observation.

As can be seen, in the case of the superposition of coherent waves in space, the energy is redistributed. In those places where the path difference from two points of excitation to the point of observation is equal to an even number of half-waves (integer number of waves):

$$\Delta = 2m\frac{\lambda}{2} = m\lambda \qquad (109)$$

We can see a maximum intensity (energy or amplitude). Where the path difference is an odd number of half-waves.

$$\Delta = (2m+1)\frac{\lambda}{2} \qquad (110)$$

- observe a minimum of intensity. This is the phenomenon of wave interference.

In the case of incoherent waves (the phase difference φ_o changes many times during the observation, passing all values from 0 to π), the positions of the minima and maxima will shift in space so that even a short-term pattern of interference cannot be observed, since the phase shift changes rapidly and the whole picture is blurred.

Measuring the amplitude of the resulting wave at each point A will give it mean over time observation value: $\langle a \rangle = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\langle\cos\varphi_0\rangle}$, where $\langle\cos\varphi_0\rangle$ - average value $\cos\varphi_o$ during the observation.

It is easy to verify that in this case $\langle\cos\varphi_0\rangle = 0$, and, therefore, the intensity at the observation point will be equal to the sum of the intensities generated at this point by each of the sources of wave excitation, i.e. $J = J_1 + J_2$. This is a simple or independent imposition of waves (superposition of waves).

To observe the phenomenon of light interference, for example, it is impossible to use two separate natural sources of radiation. The waves coming from such sources will not be coherent. And that's why. As is known, the direct emitters are the atoms of the exciting substance. After one act of radiation (~ 10^{-8} s), an atom may begin to emit light waves again, but the initial phase will be different. Therefore, even one atom in separate acts of emission does not produce coherent waves, not to mention the mass of atoms of two independent radiating bodies (for example, filaments of incandescent bulbs).

However, coherent light waves are not that difficult to get. For this purpose, the radiation coming from one source must first be divided into two streams, and then forced to make these streams again after passing through various paths. Then all elementary acts of radiation occurring in the main source of emission will simultaneously be repeated in both streams. Of course, such wave flows will be coherent. The path difference between the interfering waves must have a value lying within the propagation of a wave of one emission event.

There are two ways to obtain coherent waves in optics: the *splitting of the amplitudes of oscillations* (interference in thin films; for example, Newton's rings); *splitting of the wavefront* (for example, using the Fresnel biprism, in Jung's experience, etc.). More information about how to obtain coherent waves will be discussed when considering the relevant laboratory work.

Brief theoretical description

Biprism is an optical component in the form of two prisms with a small (about half a degree) refractive angle, folded bases. Biprism is usually made from a single piece of glass. We trace the course of the rays through biprism. Take a point monochromatic light source *S* - a narrow infinite silt, located parallel to the refractive edge of the biprism (perpendicular to the plane of the drawing shown in Figure 64). Then the wavefront of the radiation incident on the biprism will be divided into two parts.

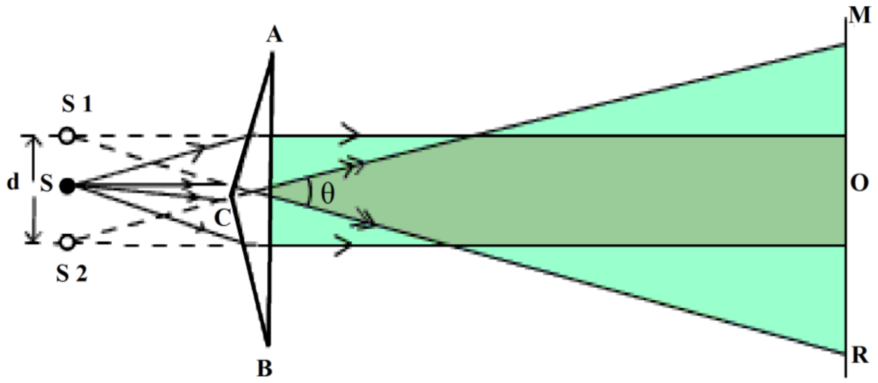


Figure 64 - Schematic diagram of the beam path through the biprism

For biprism each of the two beams, according to the law of geometrical optics, will deviate to the optical axis of the *SO* system. Thus, at the output of the biprism, the beams are superimposed, as if coming from imaginary sources S_1 and S_2.

Both of these bundles are coherent since S_1 and S_2 are images of the same radiation source *S*. At any point where the beams overlap, depending on the path difference, one or another interference effect is observed (in figure 64, the interference observation area is shaded).

To find the relationship between the geometry of the system and the local characteristic of the interference pattern, we use the drawing shown in figure 65

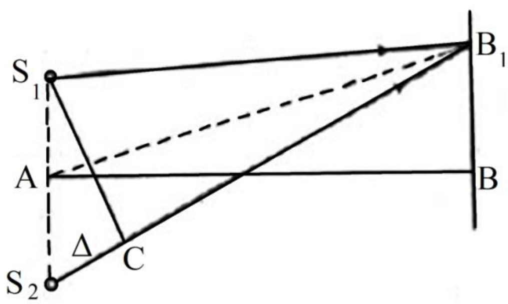


Figure 65 - Illustration for the calculations of interference localization

Here, S_1 and S_2 are imaginary sources, obtained by passing rays through biprism; BB_1 - the screen on which the interference pattern is observed. Denote the distance between imaginary sources S_1S_2 = t, the distance from the sources to the screen $AB = l$, segment $BB_1 = y_m$. We also consider that $l >> t$. At point B_1, a luminance maximum will be observed (a light line perpendicular to the plane of the drawing or parallel to the refractive edge of the biprism) if the path difference

$\Delta = S_2B_1 - S_1B_1 = m\lambda$. Remembering that $l >> t$, from the similarity of triangles S_1S_2C and ABB_1 it will be possible to write $S_2C/S_1C = BB_1/AB$ or $\Delta/t = y_m/l$. From here m maximum must be separated from point B by $y_m = l\Delta/t$.
As for the distance between two adjacent maxima, it can be found from the expression:

$$\delta_y = y_{m+1} - y_m = l\lambda/t \quad (111)$$

This is the main calculation formula for determining the wavelength of the radiant flux using a biprism. For, as we will see a little later, the quantities δ_y, l, t are directly measurable at the appropriate installation and it remains only to calculate the wavelength according to the formula (111). True, the question of measuring the distance between imaginary sources and the distance from imaginary sources to the screen (observation point) is still unspecified, but as will be shown below, there is nothing complicated in this.

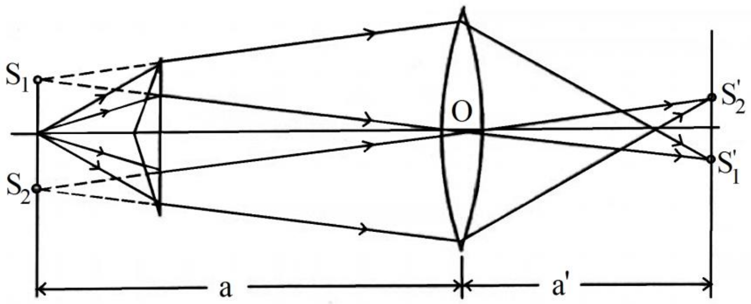


Figure 66 - Illustration for the calculation of the distance between imaginary light sources

To measure the distance $S_1S_2 = t$, proceed as follows (figure 66). An optical power lens of about 5 diopters is placed between biprism and the screen (optical micrometer). Moving the lens to the right and left along the optical axis of the system, it is possible to obtain in the focal plane of the optical micrometer (on the screen) a clear image of two slits in the S_1' and S_2' places. (In general, both enlarged and reduced images of two slits S_1' and S_2' can be obtained. In the present installation, based on its optical-geometrical parameters, as well as the greatest possible measurement accuracy, only a reduced image of slits is obtained).

The distance between these points $S_1'S_2'=t'$ can be measured using an optical micrometer. Then, having measured additionally the sizes of the segments between the slit and the lens a, and also between the lens and the optical micrometer a', from the similarity of triangles ΔS_1OS_2 and $\Delta S_1'OS_2'$ we find the required distance

$$t = t'\frac{a}{a'} \quad (112)$$

Regarding the measurement of the distance l between imaginary sources and the screen, we can say the following.

Actual source (slit) S and imaginary sources S_1 and S_2, strictly speaking, do not lie in the same plane perpendicular to the optical axis. However, this displacement at small refractive angles of the biprism is small compared with the distance between the source and the screen. Therefore, when determining the value of l, one should take the distance between the source (slit) and the screen (with an ocular micrometer in the installation).

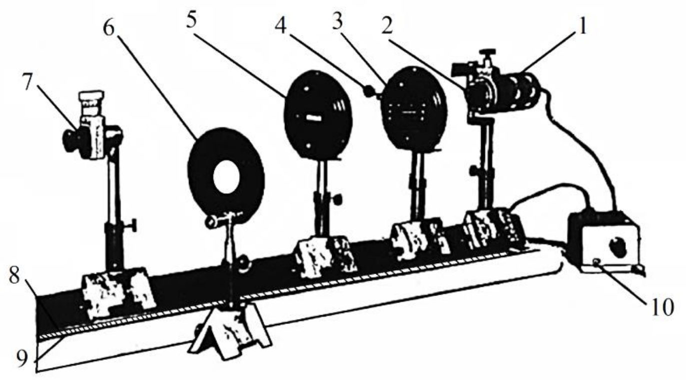
So, from expressions (4.8) and (4.9) it follows that the final calculation formula for finding the radiation wavelength after passing through the light filter will have the following form:

$$\lambda = \frac{at'\delta_y}{a'l} \quad (113)$$

Description of the laboratory installation

All installation units are assembled on an optical bench 9 (Figure 4.5). The following components are installed: Illuminator 1 with a light filter 2 (in the latest models, a revolver head with replaceable light filters is fixed in front of the optical micrometer (along the beam), a slit 3, a biprism 5, a lens 6 and an optical micrometer 7. Note, that lens 6 on the bench is set only for the period of measuring the distance between imaginary sources *t'* of coherent waves. All units are equipped with locking screws. The lower two serve to fix the rater on the optical bench. The upper screw is needed to secure the rack system components (cracks, biprism, etc.). Riders with loose bottom screws can move freely along the optical bench. A scale 9 is set near the base of the optical bench to measure the distances between the various installation units (for example, a slit and an optical micrometer).

The light source is an OI-19 type illuminator, which consists of a flashlight with a light bulb (8 V, 20 W), a two-lens collector with an iris diaphragm. The lamp of the illuminator is powered from the electrical network through a step-down transformer. To adjust the lamp heat in the transformer case there is a rheostat with a handle. The current is turned off by a toggle switch 10. The width of the slit can be changed by turning the handwheel 4. Biprism 5 with the frame can be rotated around the direction perpendicular to the plane of the frame (around the horizontal axis). The same kind of movement is provided at the gap.



1 - OI-19 illuminator, 2 - light filter, 3 - slit, 4 - slit width adjusting screw, 5 - Frensel biprism, 6 - Lens, 7 - ocular micrometer, 8 - Optical bench, 9 - scale, 11 - Toggle switch

Figure 67 - General view of the installation for determining the wavelength using the Fresnel biprism

The interference pattern in the installation is considered in the focal plane of the ocular micrometer, which makes it possible to measure the quantities δ_y and *t'*, which are included in the calculation formula (4.10). We use a micrometer ocular screw type MOB-1-15^x.

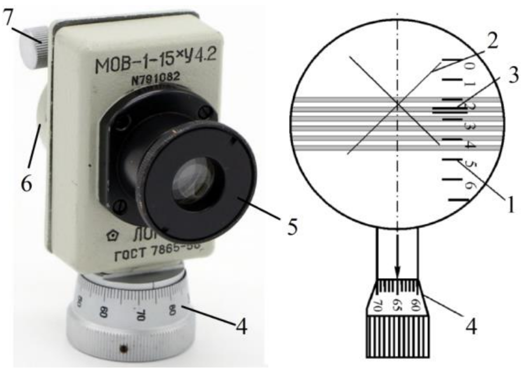
The main elements of the MOB-I-15^x (Figure 67) are a 15-fold compensation eyepiece 5 with a diopter adjustment and a readout device. Installation on a clear image of the reading device is carried out by turning the eyepiece corolla. Limit pickup - 5 diopters. The reading device is located in the focal plane of the eyepiece: a fixed scale with a scale division of 1 mm; a moving grid with a crosshair and an index in the form of two scratches at the level of a fixed scale against a crosshair.

Imaging sources are also located in the focal plane of the eyepiece. Therefore, in the field of view of the adjusted eyepiece is a clearly visible pattern of observation and reading device.

The movable reading device grid is connected with an accurate micrometer screw in such a way that when the corrugated drum screw is rotated, the crosshair and risks move in the field of view of the eyepiece relative to a fixed scale.

When turning the screw drum 4 for one full turn, the risks and crosshairs in the field of view of the eyepiece move one division of the main fixed scale. The screw drum is divided into 100 parts. Therefore, the price of division of the scale on the drum is 0.01 mm (since the price of division of the main scale is 1 mm).

The view field of the micrometer eyepiece consists of a fixed scale and a crosshair. The position of the scratches in the field of view of the eyepiece relative to the zero of the fixed scale indicates the integer number. The fractions of the division are determined by the divisions of the drum, which falls against the index of the fixed screw nozzle. The eyepiece micrometer is fixed on the rack with a clamp 6 by a fixing screw 7.



1 - Fixed scale with divisions, 2 - Crosshair; 3 - Pointer in the form of double risks, coupled with a crosshair; 4 - Drum micrometer screw, 5 - Eyepiece, 6 - Micrometer clamp, 7 - Fixing screw

Figure 68 - General view of the ocular micrometer MOB-I-I5^{x} and corresponding scale in the field of view

Preparation for the measurements

The installation can be considered an alignment if all elements of the optical system have a common optical axis parallel to the optical bench (centered system).

This is achieved as follows. Strengthen on the optical bench with the help of raters illuminator, slit and eyepiece. The gap is oriented in the horizontal direction. The height of the slit and the input window of the eyepiece relative to the optical bench are chosen the same, either with a ruler or by leading the eyepiece close to the slit. Then take the eyepiece from the gap to the maximum possible distance. Turning on the lamp illuminator, check with a sheet of white paper gets a beam of light into the eyepiece. If the beam of light goes above the eyepiece, then the illuminator should be raised relative to the optical bench. If the beam of light goes down, then the illuminator is lowered until a ray of light hits the input window of the eyepiece. Setting a white disc, a target on the input window of the eyepiece, can also be very useful for adjusting.

After that, between the gap and the eyepiece (approximately in the middle) put biprism. Observing the interference fringes through the eyepiece, select the height of the biprism such that they are located in the middle of the field of view of the eyepiece. Adjust the slit width and orientation of the biprism, so that the interference pattern is the clearest. Also pick up the glow of the filament lamp illuminator.

The brightness of the illuminator should provide a clear picture of the phenomenon being studied, at the same time, it should not be excessively high to avoid injury to the observer's eye. Finally, the correctness of the adjustment is checked as follows. Biprism is moved with the rider along the optical bench towards itself, while simultaneously observing through the eyepiece. If the interference pattern remains in the center of the field of view, then the installation is adjusted.

However, most often it happens that the interference pattern smoothly moves up or down. They return biprism to its original position and slightly reduce the height of its setting. Then move it back to itself along the optical bench.

Note, for example, at the same time that the movement of the interference pattern in the field of view has decreased but compared with the first attempt. This will speak in favor of the fact that we are on the right path and that we should lower the biprism a little. And so that the interference pattern does not go beyond the field of view, it will be necessary to lower the eyepiece below. If during retesting, the magnitude of the movement of the interference pattern did not decrease, but increased, then one should not lower the biprism, but set it slightly higher. All these procedures are performed until the interference pattern remains in the center of the eyepiece field when the biprism moves along the optical bench (the central or zero interference maximum always remains within the average risk of the main micrometer scale). That's when the installation will be sufficiently prepared for measurement.

Order of performance

An installation prepared for operation measures the wavelengths transmitted by the red, green, and blue light filters.

The measuring part of the work is performed in the following order.

1. Install in the course of the beam one of the mentioned filters.

2. The biprism from the eyepiece is positioned in such a position that would allow using the lens to obtain in the focal plane of the eyepiece an image of two coherent radiation sources (two slits). Making sure of the latter, measure the distance between the image of imaginary sources t'.

3. To do this, observing through the eyepiece, turn the drum to bring the crosshair of the eyepiece up to coincide with the image of the first selected slit and make reading on the scales of the eyepiece micrometer.

4. Continuing to observe the eyepiece and rotating the drum in the same direction, bring the center of the crosshair to the image of the second slit.

5. Again, read the counting on the micrometer scales. The difference between these two samples is the value of t'.

6. At constant positions of the optical nodes, determine the distances between the lens and the slit - a, between the lens and the focal plane of the eyepiece - a'. Together with them is the distance between the source (slit) and the eyepiece - l.

7. The definition of the mentioned distances is conveniently performed using a triangle. For this purpose, a triangle is tightly applied with one leg to a scale near the base of the optical bench. The other side must coincide with any of the optical nodes (the plane of the slit, the main plane of the thin lens, the focal plane of the ocular micrometer). The difference between two such counts for different optical nodes will give the desired distance.

8. After determining the values of a, a', l by the indicated method, remove the lens from the optical bench. An interference pattern will appear in the field of view of the ocular micrometer.

9. Now the distance between two adjacent interference bands δ_y is measured with an eyepiece micrometer. Note, that in order to increase the accuracy of measuring δ_y, find the coordinate y of dark or light stripes, and then calculate the distance between adjacent stripes.

Table 3 Coordinates and thickness of the interference strips

№	y (coordinates)	δ_y (interference strips thickness)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

10. Note, that you should fulfill the table 1 for chosen light filters.

11. To calculate the interference strips thickness δ_y, use the formula: $\delta_y = y_1 - y_2$, where y_1 and y_2 are coordinates of the neighbor interference strips of the same color.

12. All found values δ_y, t', a, a', are substituted into formula (113) and by doing this the desired wavelength is found.

13. The number of measurements for each light filter should be at least 5. Calculate the error of the measurements.

Questions for self-test

1. What is the phenomenon of interference?
2. Why is the interference pattern observed only with a small distance between coherent sources and a limited path difference?
3. What color will be the zero interference maximum created by the superposition of two coherent waves from white light sources?
4. What view will the interference pattern have when the light filter is retracted?
5. How will the interference pattern change if the refractive angle of the biprism is increased?
6. Why is the horizontal orientation of the gap more efficient in a laboratory setup?
7. Which sign indicates correct installation alignment?

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Laboratory work № 5

DETERMINATION OF THE FOCAL LENGTHS OF THE ZONE PLATE

Aim of the work. Study the working principle of the zone plate and determine the focal lengths of the zone plate.

Accessories: Optical branch, Lenses, zone plate, mate screen, laser

Brief theoretical description

A typical manifestation of the wave properties of light is diffraction. Diffraction is the deviation of light from rectilinear propagation by various inhomogeneities of the medium. Such inhomogeneities can be: a hole in an opaque screen, a small obstacle or a transparent medium, the refractive index of which depends on the coordinates. Diffraction will be observed if the size of the obstacle is commensurate with the wavelength.

Huygens-Fresnel principle

For the first time, the Dutch physicist Huygens considered diffraction from the wave point of view. According to Huygens' principle, every point to which a wave has spread becomes a source of secondary spherical waves. The surface that goes around the secondary waves is the front of the propagating wave. The value of Huygens' principle is that he allowed claiming that in the presence of an obstacle the light wave will necessarily deviate from the straight direction.

Figure 69 shows a slit-shaped obstacle whose dimensions are comparable with the wavelength. A flat monochromatic wave is an incident on the slit. Any point of the wavefront becomes a source of secondary spherical waves. The surface of the secondary waves is a plane ending in areas of the sphere. The last rays, as can be seen from Figure 69, fall into the region of the geometric shadow, i.e. there is a diffraction phenomenon.

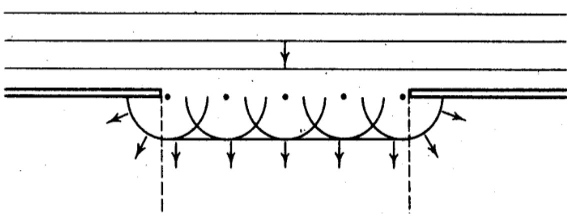


Figure 69 - Illustration for the Huygens principle

The principle of Huygens, explaining the phenomenon of diffraction in a general form, does not touch on the most important question of the intensity of the light waves propagating beyond the barrier.

The French physicist Fresnel supplemented the principle of Huygens with the statement that all secondary sources are coherent, and the waves emitted by them will interfere. He introduced the concept of the amplitude and phase of these waves. Thus, the analysis of the phenomenon of diffraction is carried out based on the principle of Huygens and the laws of interference. In such a combined form, this position of wave optics is called the Huygens-Fresnel principle.

Fresnel zone method

Fresnel used two methods to explain diffraction. One of them, the so-called Fresnel zone method, is a geometric method (Figure 70), suitable for solving diffraction problems on objects with axial symmetry. The second method is analytical, using the Fresnel integrals.

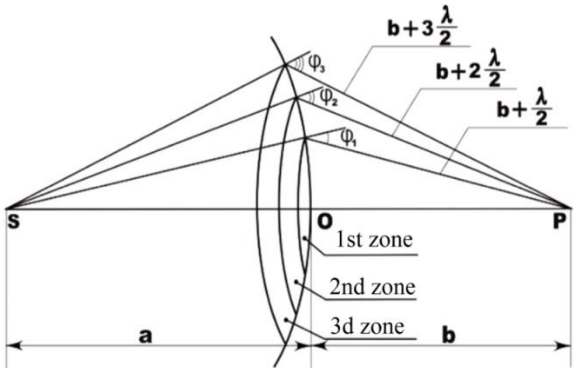


Figure 70 - Demonstration of the Fresnel Zones

When considering diffraction using the Fresnel zone method, the wavefront is divided into small sections - zones with such a condition that the distances from the observation point to the border of each subsequent zone differ by $\lambda/2$ (Figure 70). With this method of selecting zones, for each point of one of the zones, there is a corresponding point in the adjacent zone, which is $\lambda/2$ further from the observation point P than the point of the first zone.

Consequently, the oscillations excited at the point of observation by two adjacent zones are opposite in phase. Therefore, for the amplitude of the resulting oscillation A_0, we write:

$$A_0 = A_1 - A_2 + A_3 - A_4 + \cdots + A_n \tag{114}$$

where A_1, A_2, A_3, A_4, etc. - amplitudes of oscillations excited separately by each zone,

The amplitude A_i, radiated by an arbitrary zone, depends on its area and angle φ, under which this zone is visible from point P. It can be shown that all Fresnel zones are equal in area. At the same time, as the zone number increases, the angle φ increases, and by the Huygens-Fresnel principle, the radiation intensity of the zone decreases in the direction of point P, i.e. amplitude decreases A_n. It decreases with increasing n and due to the distance from the zone to the point P. Thus, $A_1 > A_2 > A_3 > A_4 > \cdots > A_n$. The effect of the open part of the wave surface is determined by the number of zones that fit on this surface relative to the observation point P. If this number turns out to be even (for example, four), the series will be written in this form:

$$A_0 = (A_1 - A_2) + (A_3 - A_4) + \cdots \approx 0 \tag{115}$$

Its sum is close to zero, therefore, the attenuation of light (min) is obtained in the corresponding direction. If the number of zones is odd, then the gain (max) of the light is obtained. In this case, the action of one zone remains uncompensated.

Vector diagrams method

To estimate the contributions from each zone to the total illumination, we use the vector diagram method. To do this, we divide each zone into a series of narrow “subzones” so that each subzone differs from the next only in a small phase shift.

The oscillations of each of the “subzones” will be represented as a vector whose length is determined by the amplitude of the oscillations. Area of the "subzones" choose the same. As can be seen from Figure 71, the vectors of each “subzone” are turned relative to the neighboring ones by a small angle $d\Psi$, and the “subzones” at opposite edges of the zone differ in phase by 180°. The total action of all the “subzones” of the first zone is represented by the vector OO_1.

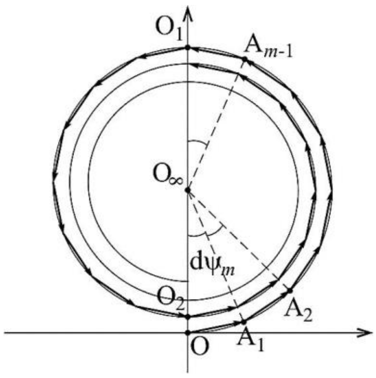


Figure 71 - Illustration of the Fresnel spiral

It is easy to figure out that with the striving of the width of each “subzone” to zero, the resulting broken line turns into a smooth semicircle. The action of the two zones is not zero, since the amplitudes of the zone oscillations are not the same. Their value depends on the cosine of the angle between the normal to the surface of the zone and the direction to the observation point (angle in Figure 70)

The contribution of the first and second zones is characterized by the OO_2 vector. Considering the first, second, third, fourth, etc. zones, we obtain a twisting spiral with the center O_∞, the so-called *Fresnel spiral*. The vector OO_∞ corresponds to the case when the obstacle is absent and all Fresnel zones are open. It turns out to be two times smaller than the vector - the amplitude of the first zone. Using the zone plate, which consists of transparent and opaque Fresnel zones (total zones $2n$), it is possible to exclude the contribution of the “even” (or “odd”) zones. The remaining n open “odd” zones (or n “even”) zones will give the resulting amplitude $n\ OO_1 = 2n\ OO_\infty$, and the intensity $n^2 I_1 = 4n^2 I_\infty$.

Thus, the use of a zone plate allows increasing the intensity at the point of observation P $4n^2$ times. This means that the zone plate acts as a collecting lens, focusing the light energy at point P.

The geometric shape of the Fresnel zones is determined by the shape of the wave surface, which is divided into zones, and the angle at which the observations are made. If the wavefront of a spherical wave from a point source S is divided into annular Fresnel zones, then it can be shown that the radius of the nth zone is determined by the expression:

$$r_n = \sqrt{\frac{n\lambda ab}{a+b}},\ n = 1,2,3\ ... \tag{116}$$

If the source is removed to infinity ($a \to \infty$), then the wave front will be flat (Figure 116). This wave is called flat. In this case, the radius of the n-th zone is determined by the expression:

$$r_n = \sqrt{n\lambda b},\ n = 1,2,3\ ... \tag{117}$$

In this laboratory work, the diffraction of a plane monochromatic wave is studied.

To observe the analogy between the lens and the zone plate most fully, we consider the formula:

$$\frac{n\lambda}{r_n^2} = \frac{1}{f},\ n = 1,2,3\ ... \tag{118}$$

then formula (116) can be represented as $\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$, which is known as the lens formula (a is the distance from the object to the lens, b is the distance from the lens to the image), where f plays the role of the focal length. We get the main focus if $n = 1$, then we can calculate:

$$f_1 = \frac{r_1^2}{\lambda} \tag{119}$$

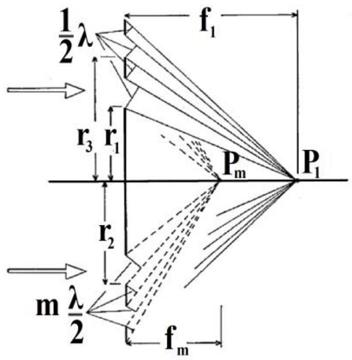


Figure 72 - Geometry of the zone plate

As point P approaches the zone plate, there is a consistent alternation of bright points with darkened gaps between them, which indicates the presence of *several foci* on the zone plate. The focal length of the m-th focus is related to the main focal length by the ratio:

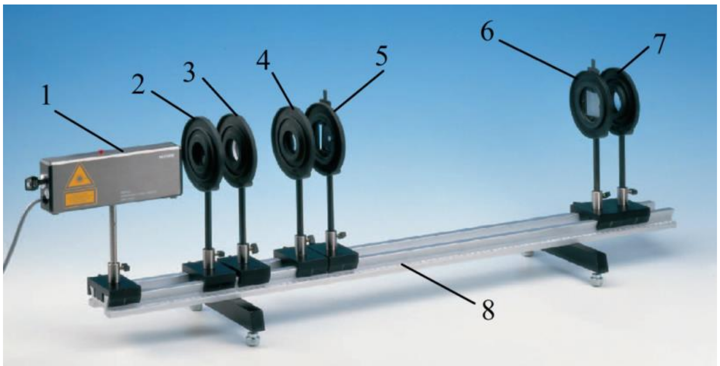
$$f_m = \frac{f_1`}{m}, m = 3,5,7,9, \ldots \tag{120}$$

A helium-neon laser is used to produce coherent radiation with a wavelength of 632.8 nm. The zone plate has 20 zones, the radius of the first light (central) zone is r_1 = 0.6 mm. The radius of the n-th zone is determined by the formula:

$$r_n = r_1\sqrt{n} \tag{121}$$

Description of the installation

In the laboratory, an experimental device is used (Figure 73) to determine the foci of the zone plate.



1 - He-Ne Laser, 2 - Lenz L_1 (f = 20 mm), 3 - Lenz L_2 (f = -50 mm), 4 - Lenz L_3 (f = 100 mm), 5 - Zone Plate (20 zones r_1 = 0,6 mm), 6 - Matt screen with polarizer, 7 - Lenz L_4 (f = 50 mm), 8 - Optical branch

Figure 73 - Experimental device

The lens system L_1 (f = 20 mm), L_2 (f = −50 mm), L_3 (f = 100 mm) forms (if properly tuned) a telescopic system that broadens the parallel laser beam from transverse dimensions d ~ 0.6 mm to dimensions D ~ 6 mm. The scattering lens L_2, located between the eyepiece L_1, and lens 3, allows to reduce the distance between these collecting lenses and, thereby, reduce the length of the optical bench. Correct results for different focal lengths of the zone plate can be obtained only if the incident rays are parallel. The position of; L_1, L_2, L_3 lenses are fixed when the parallelism of the laser beam is ensured. Each time before a new series of measurements, an optical alignment of the parallelism of the laser beam is carried out independently.

The image is observed on a matte screen located on the opposite end of the optical bench. For better observation of focal points, a L_4 lens (f = 50 mm) is used, which serves as a magnifying glass. A polarizing filter is used to reduce the brightness of the image.

Check the position of the optical devices in Figure 74.

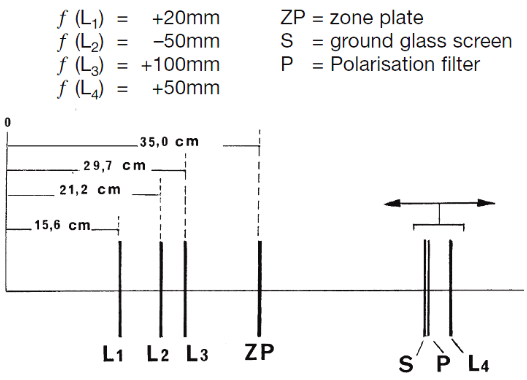


Figure 74 - Position of the optical components on the optical branch

Optical adjustment of the installation

Slowly move the zone plate along the optical bench, while the laser beam, broadened by the optical system, should have in its cross section an almost unchanged round light spot, whose center coincides with the center of the Fresnel zones. The spot diameter should be equal to the outer diameter of the zone plate or slightly exceed it.

Order of performance

1. Put the zone plate on the optical bench at the position indicated in Figure 74.

2. Move the matte screen with the lens L_4 from the edge of the optical bench towards the zone plate.

On a matte screen, there is a system of concentric rings, resembling the rings of a zone plate. Moving the screen, find the positions corresponding to the maxima of the central spot, which correspond to the foci of a certain order. The focus of the first order (the main focus) is the extreme among all the foci: after this point should be observed a monotonous decrease in brightness in the center of the screen. This focus is the brightest one.

3. For each fixed focus (no more than 5), measure the distance from the screen to the zone plate, i.e. focal length f_m (m = 1, 3, 5, 7, 9). Enter the results in Table 5.2 in the line corresponding to №1.

4. Re-measure the focal lengths by moving the screen in the opposite direction, from the zone plate to the edge of the optical bench. The results are listed in Table 4 in the line corresponding to the number 2.

Table 4 - Zone plate foci

	№	f_1, cm	f_3, cm	f_5, cm	f_7, cm	f_9, cm
I	1					
	2					
II	3					
	4					
III	5					
	6					
	f_{avr}					

5. Repeat the focal length measurement 3 times (rows 3 - 6 in Table 4).

6. On graph paper using the experimental data, plot the focal distance versus the reciprocal of the order of the corresponding focal point, i.e formula (119)

7. Calculate the radii of the zone plate using the formula $r_n = \sqrt{n\lambda m f_m}$ The resulting values are listed in Table 4. According to the formula (121), calculate the theoretical values of the radii of Fresnel rings, and record the results in Table 4.

8. Calculate the error using the Appendix

Table 5 - Calculation of the radii of the zone plate

n	r_{theor}, mm	r_{exp}, mm
1	0,60	
2	0,85	
3	1,04	
4	1,20	
5	1,34	

Questions for the self-test

1. The Fresnel zone method. Derive formulas for the radius of the zone and its area.
2. Fresnel diffraction on a round hole and disk.
3. Zone plate. Amplitude and phase zone plates.
4. Why does a zone plate have several foci?

References

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2. Sivukhin D.V. General course of physics. - T. 4. Optics. - Ed. 3rd M.: Fizmatlit, 2005. - § 39, 40.
3. Saveliev I.V. The course of general physics. T. 2. - Ed. 10th - M.: Fizmatlit, 2008. - § 126, 127.

Laboratory work № 6

CHECKING THE LAW OF MALUS

Aim of the work: Estimation of the intensity of polarized rays using a photo-polarimeter. Learning the methods of adjustment and installation work on it. Mastering the method of determining the main reference points in the analysis of measurement results. Using the graphical method of checking the law of Malus. Estimation of the degree of polarization of the light beam at the exit of the polaroid.

Accessories: Polarizer, Analyzer, Illumination source, Multimeter

Brief theoretical description

The polarization of light is one of the main phenomena that characterize light as an electromagnetic wave. Let us remember the basic principles of the theory of polarization of electromagnetic waves.

If the oscillations of the electric intensity vector in an electromagnetic (light) wave occur in one plane, then such wave is called plane polarized. The plane passing through the direction of wave propagation and the direction of oscillations of the electric intensity vector in such plane-polarized electromagnetic (light) wave is called the polarization plane or the plane of oscillation, Figure 75.

It is known that common sources of radiation send non-polarized light waves - natural light. This happens because there is a very large number of elementary emitters (oscillating atoms) in any common source that are not interconnected by the acts of emission.

Some substances (for example, tourmaline) have the ability to pass only a specific component of the natural light wave. In this case, it is possible to isolate a certain polarization state from the initially non-polarized oscillations.

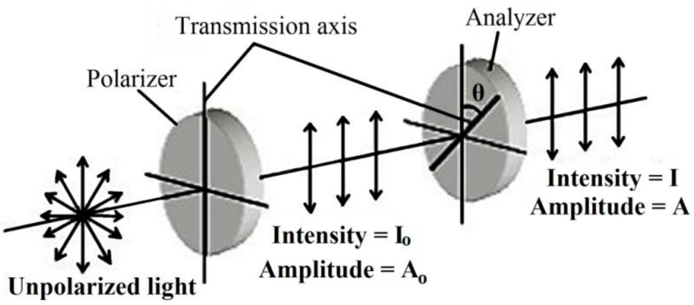

Figure 75 - Schematic diagram of the polarization device

Such substances - systems (polaroid plates, polarizing and birefringent prisms, etc.) are usually called polarizers. The plane of oscillations that they pass is called the polarization plane of the polarizer. If on the path of the beam that emerged from the polarizer, install the second polarizer, usually called an analyzer (Figure 75), then, as experience shows, the amplitude of oscillations of the electric intensity vector of a light wave at the output of such a system depends on the angle between the directions of the polarization planes of both polarizers (polarizer and analyzer). The same applies to the intensity of the transmitted light wave (Malus' law):

$$J = J_o \cos^2 \alpha \quad , \qquad (122)$$

where J_o is the intensity of a polarized light wave that incident on the analyzer, J is the intensity of the light beam at the output of the analyzer. In the case when the directions of the polarization planes of the polarizer and the analyzer are mutually perpendicular (crossed polarizers), the light does not pass through such a system.

Also, one can note that partial polarization of the light beams is observed at the time of reflection and refraction of natural light at the interface with the dielectric medium. At a certain angle of incidence i_p, the reflected beam becomes flat polarized. In this case, the angle between the reflected and refracted ray is 90°, i.e. the condition tan i_p = n is satisfied. (the Brewster law, where $n = n_2/n_1$ is the relative refractive index of the dielectric medium)

In this case (Figure 76), the reflected beam is polarized in a plane perpendicular to the plane of incidence.

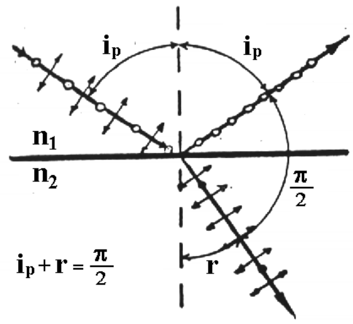

Figure 76 - Illustration of the concept of the law of Brewster

The refracted ray of light is partially polarized in the plane of incidence of the ray. Note also that the degree of polarization can be estimated by the relation:

$$\Delta = (J_{\perp} - J_{II}) / (J_{\perp} + J_{II}) = (J_{max} - J_{min}) / (J_{max} + J_{min}) \qquad (123)$$

If plane-polarized light falls on the interface between two media, then, as Fresnel's research has shown, the reflectivity of a dielectric depends on the angle of incidence of the beam and the nature of polarization of the incident radiant flux:

$$\rho_{\perp} = \frac{sin^2(i-r)}{sin^2(i+r)} \qquad (124)$$

$$\rho_{\|} = \frac{tan^2(i-r)}{tan^2(i+r)} \qquad (125)$$

where $\rho_{\perp}$ is the reflectivity with the incident beam polarized in a plane perpendicular to the plane of incidence; $\rho_{\|}$ - reflectivity with the incident beam polarized in the plane of incidence; and i and r, respectively, are the angle of incidence and the angle of refraction of the beam (Figure 77). Reflectance refers to the ratio of the intensity of the reflected light flux to the intensity of the light flux incident on the dielectric.

When a nonpolarized wave falls on a dielectric, its reflectivity ρ_0 is calculated based on relations (124) and (125) by decomposing the unpolarized wave into two flat polarized $\perp$ and || In this case, receive:

$$\rho_0 = \frac{1}{2}\left[\frac{sin^2(i-r)}{sin^2(i+r)} + \frac{tan^2(i-r)}{tan^2(i+r)}\right] \qquad (126)$$

When considering the graph in Figure 77 it is easy to see that the minimum value of reflectivity ρ|| corresponds to the angle of incidence of polarized beam equal to i_p, i.e. the angle arising from Brewster’s condition.

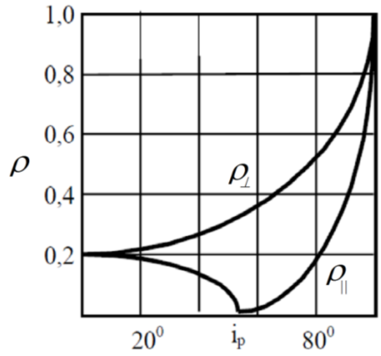

Figure 77 - Light reflectance as a function of angle of incidence i_p with $n_2 > n_1$

Most of the polarization devices used in laboratory installations are based on the phenomenon of double refraction.

Consider the essence of this phenomenon. Pronounced anisotropy crystals (Iceland spar crystals, quartz, and others.) possess the property of double refraction of a substance.

In addition to natural solid crystals, double refraction of rays gives the so-called liquid crystals, several amorphous bodies during deformation, and some liquids in an electric field

The lightwave propagating in such media is divided into two components with different propagation speeds and, therefore, different refractive indices.

Double refraction is observed even when the beam falls along the normal to the face of the crystal. At the same time, there are two directions in crystals in the general case, along which there is no birefringence. These directions were called the optical axes of the crystal. Obviously, the propagation speeds of all the rays along the optical axis are the same, and therefore no separation of the rays is out of the question here.

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Typical representatives of biaxial crystals are: rhombic sulfur, mica. Uniaxial crystals - Iceland spar, quartz. Crystals of the cubic system (for example, rock salt), due to their isotropy, do not have double refraction.

In uniaxial crystals, the passage of one of the rays follows the usual law of refraction. For another ray, the value of the refractive index depends on the direction of its passage in the crystal. The first of them received the name of the ordinary ray, and the second - the extraordinary. In biaxial crystals, both rays do not obey the usual law of refraction. Ordinary and extraordinary rays turn out to be linearly polarized in two mutually polarized planes. If we introduce the concept of the main section (main plane) of a crystal as a plane containing the optical axis and beam, then we can say that the plane of polarization (the plane of oscillation) of the ordinary beam is perpendicular to the main plane of the crystal, and the plane of polarization of the unusual beam lies in the main plane of the crystal.

The brightness of the ordinary and extraordinary rays in uniaxial crystals, as a rule, is the same. However, there are crystals in which one of the rays is absorbed more significantly than the other (tourmaline). This phenomenon is called dichroism. Dichroic films (polaroids) are often used as polarizing devices.

Based on the phenomenon of double refraction, more complex polarization systems are created -

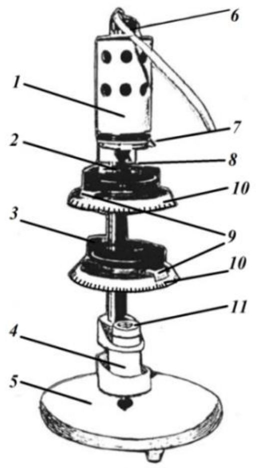
the prism of Nicolas, the prism of Wollaston, etc.

Description of the laboratory installation

The work is performed on a photopolarimeter "Tourmaline", designed at the department of general physics. The main parts of the photopolarimeter (Figure 78) are: Illuminator 1, Polarizer 2, Analyzer 3, Camera of the receiver of radiant energy 4

Each of the listed parts is mounted on a vertical column 8 of the base of device 5 useing fixed couplings. When adjusting the device, the illuminator, polaroids and the camera of the selenium photocell can move relative to each other along the column, and can also be rotated around the latter.

To obtain and study linearly polarized light, polaroids made of very small tourmaline crystals or iodine-quinine sulfate (herapatite) deposited on a celluloid film are used in this setup. All crystals of herapatite are oriented so that their optical axes have the same direction. Incident natural light undergoes birefringence in a polaroid film (herapatite crystals). And, as it was said earlier, the spread of two rays - ordinary and extraordinary - takes place inside the polaroid. The ordinary ray, due to the phenomenon of dichroism, is almost completely absorbed in the herapatite (tourmaline) crystals of the polaroid film. And this means that at the exit from the polaroid a sufficiently flat polarized beam of light is obtained. Nowadays, the dichroism of organic polymer molecules, which are spatially uniformly oriented, is most often used in polaroids. Orientation is carried out using stretching or other special technology (polyvinyl polaroid).



1 - Lamp of the illuminator, 2 - Polarizer, 3 - Analyzer, 4 - Radiant energy receiver chamber, 5 - Instrument base, 6 - Base of the lamp holder lamp holder, 7 - Iris diaphragm lever, 8 - Column, 9 - Circular nonius, 10 - Scales of reference limbs, 11 - Cover - target.

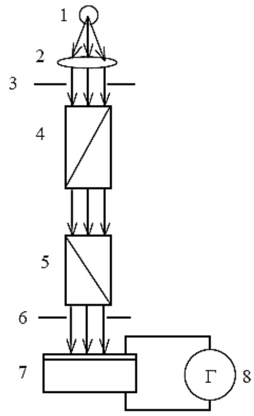
Figure 78 - General view of the photopolarimeter "Tourmaline"

The illuminator OI-19 was used as the light source in the photo-polarimeter. At the output of the illuminator receive a beam of light close to parallel, which is achieved by moving the cap of the lamp illuminator 6 along the optical axis of the device. The cross section of the light beam of the illuminator is regulated by an iris diaphragm. The opening of the diaphragm is changed by lever 7.

Each of the two polaroids of the device is rigidly connected with circular nonius 9 reference limbs 10 polarizer and analyzer. Polaroids, along with nonius, can rotate around the optical axis of the instrument.

By the position of nonius can be found the angle between the polarization planes of the polarizer and the analyzer. It should, however, be borne in mind that the polarization planes of the polaroids are set arbitrarily relative to the readout device.

Figure 79 shows a schematic diagram of the path of the rays in the installation used to verify the law of Malus.



1 - Point- electric incandescent lamp, 2 - Collimator lens necessary to produce a parallel light beam, 3 and 6 - Diaphragm limiting light beam, 4 - Polarizer, 5 - Analyzer, 7 - Selenium photocell, 8 - Galvanometer.

Figure 79 - Schematic diagram of the beam path on the polarimeter

During off-hours, the entrance window of the photocell chamber is covered with a lid. At the end of the lid there is a target (a circle with two mutually perpendicular lines), which makes it easier to align the instrument before the operation.

The device is considered to be adjusted if the beam of light passes through the center of polaroids and falls into the center of the target on the lid of the input window of the photocell, and the beam section is almost invariable at the points where polaroids pass and hit the target. The intensity of the light beam at the output of the analyzer is estimated by the value of the photocurrent of the selenium photocell connected to the galvanometer.

Installation preparation for measurements

After reviewing the installation of "Tourmaline", proceed to its adjustment. First, it is ensured that the light beam at the exit from the illuminator is close in its configuration to the parallel one, and the installation as a whole is centered. After that, select the operating voltage on the lamp illuminator photogoniometer. For this purpose (at first, when the light bulb is low), the analyzer is positioned relative to the polarizer, when the reading of the galvanometer is at its maximum. Then, increasing the glow of the bulb, achieve a significant (full scale) deviation of the galvanometer's light pointer. This ends the preparatory part of the work.

Order of performance

1. The intensities of a light beam passing through a system of two polarizers are measured for various values of the analyzer's dial count from 0 to 180 over an interval of 10° (the polarizer limb position is set by the teacher). The measurement series is reproduced at least three times.

2. Using the averaged measurement data, the intensities of the light beam (photocurrent values) are found, corresponding to a certain angle α between the polarization planes of the polarizer and the analyzer. As starting points take the maximum value of the photocurrent and the minimum value of it. Obviously, the maximum value of the photocurrent will correspond to the angle between the polarization planes of the polarizer and the analyzer $\alpha = 0°$, and the minimum value will be $\alpha = 90°$. Intermediate values of the photocurrent will refer to the angles $\alpha = 10°$; 20; ...; 80°, since the changes in the light beam intensities are carried out every 10° of the analyzer limb.

3. Using the data fulfill table 1. where I_o is the intensity of the light beam at $\alpha = 0°$ (maximum value of the photocurrent); J_α is the intensity of the light beam at a given angle α, and they are analyzed for the implementation of the Malus law.

4. Measurement results are presented as a graph $(I_\alpha/I_o) = f(\cos^2 \alpha)$,

5. Evaluate the degree of polarization of the light beam at the exit of the polaroid according to the data.

6. Calculate the error

Table 6 - Table for calculation for the Malus law checking

№	$\alpha, ^o$	I_1	I_2	I_3	$<I>$	I_α/I_0	$\cos\alpha$	$\cos^2\alpha$
1	0					1		
	10							
	…							
	180							

Questions for self-test

1. What is different from the natural light plane polarized?
2. Are the longitudinal waves to be plane polarized?
3. What is the phenomenon of double refraction?
4. What is the optical axis of the crystal?
5. Why is the intensity of light proportional to the square of the amplitude of the vector of electrical intensity?
6. Formulate the law of Malus.
7. Why the light intensity in the work can be estimated by the value of the photocurrent?

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Laboratory work № 7

STUDY OF THE BEER-LAMBERT LAW AND MEASUREMENT OF THE ABSORPTION INDEX OF THE OPTICAL MEDIUM

Aim of the work: to study the absorption of light in a medium, checking the laws of Beer and Fresnel.

Accessories: Set of plates, laser, multimeter

Brief theoretical description

From experiments, it is known that as the light wave propagates in a substance, its intensity gradually decreases. This phenomenon is called the absorption of light in a substance (adsorption of light). It is associated with the transformation of the energy of the electromagnetic field of a wave into other forms of energy (most often into the energy of the chaotic thermal motion of particles of matter, which are heated by the absorption of light).

Absorption of light can be described in general terms from an energy point of view, without going into the details of the mechanism of interaction of light waves with atoms and molecules of the absorbing substance.

Let a monochromatic wave of intensity I_0 fall on a flat layer of matter with thickness d. We divide this layer into a series of elementary layers with thickness d_x. The intensity of the wave approaching the layer lying at depth x is denoted by I_x. The intensity of the light dI_x absorbed by the layer d_x is proportional to the light I_x incident on this layer and the thickness of the layer dx (Figure 80).

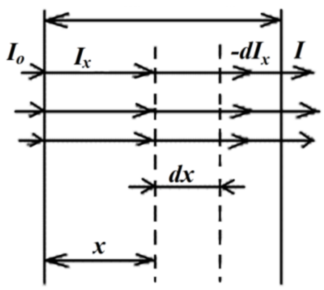


Figure 80 - Illustration for the Beer-Lambert law

Considering that the intensity decreases as we go deeper into the substance, we have

$$dI_x = \alpha I_x dx \qquad (127)$$

where α is the absorption coefficient depending on the frequency of the incident wave (or wavelength).

The "minus" sign means a decrease in the energy of the light wave. Integrating equation (127), we get

$$\int_{I_0}^{I} \frac{dI_x}{I_x} = -\int_0^1 \alpha dx \qquad (128)$$

$$\ln I - \ln I_0 = -\alpha l \qquad (129)$$

$$I = I_0 e^{-\alpha l} \qquad (130)$$

Beer's law (Figure 81) is a law that determines the gradual attenuation of a parallel monochromatic beam of light when propagating in an absorbing substance.

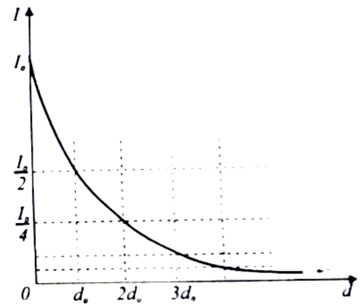


Figure 81 - Dependence of the intensity *I* of the thickness *d*

where *I* is the intensity of the light released from the absorbing substance. Formula (130) is valid only for monochromatic light, since the absorption coefficient depends on the wavelength λ (or frequency). The numerical value of the coefficient α shows the layer thickness *d*, after passing through which the intensity of the plane wave decreases by $e = 2.718$ times. The value of α depends on the nature and state of matter and the frequency of light oscillations in the incident wave. This law shows that the intensity of light decreases exponentially as the light wave propagates in the matter. If the light passes the material with the thickness *d* and its intensity decreases by *N* times, then with the passage of the material with the thickness 2*d*, the transmitted light intensity decreases by N^2 times, see Figure 81.

Usually, absorption is selective, i.e. Light of different wavelengths is absorbed differently. Since the wavelength determines the color of light, therefore, the rays of different colors are absorbed in a given substance in different ways. Transparent unpainted bodies are bodies that give a small absorption of light of all wavelengths related to the interval of visible rays. Thus, in a 1 cm thick layer, glass absorbs only about 1% of visible rays passing through it. The same glass strongly absorbs ultraviolet and far infrared rays.

Colored transparent bodies are bodies that detect absorption selectivity within the limits of visible rays. For example, “red” is glass, slightly absorbing red and orange rays and strongly absorbing green, blue and purple. If white light falls on such glass, which is a mixture of waves of different lengths, only longer waves will pass through it, causing a red sensation, while shorter waves will be absorbed. When the same glass is illuminated with green or blue light, it will appear “black”, since the glass absorbs these rays.

The dependence $\alpha = f(\lambda)$ (Figure 82) is a curve with a series of maxima, which, in turn, are the absorption bands of light substance for a certain range of wavelengths. For various substances, the numerical value of the absorption coefficient α is different and varies widely.

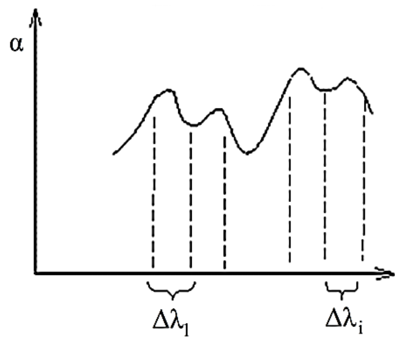


Figure 82 - Dependence of the absorption α on the wavelength λ

As an example, we give the absorption coefficients of visible rays, which are determined by the following values: (0.01-0.03) cm^{-1} - for glass (depending on the grade); 0.001 cm^{-1} - for water; (2-4)10^{-1} cm^{-1} - for air (depending on humidity).

Determination of light transmission and absorption of colorless glass

The light transmission τ of the medium is understood to be the ratio of the luminous flux Φ transmitted through the medium to Φ_0 the incident light. If the luminous flux passes through several media with a transmittance τ_1 τ_2 ... τ_n, then the whole system will have a transmittance

$$\tau = \tau_1 \tau_2 \tau_3 \cdots \tau_M \tag{131}$$

The transmittance τ is equal to the ratio of the luminous flux Φ_τ that passes through the body to the luminous flux Φ that falls on it: $\tau = \Phi/\Phi_o$. The logarithm of the reciprocal of transmission is called the optical density.

$$D = \lg(1/\tau) = -lg\tau \tag{132}$$

The total optical density of the system consisting of M media is $D = D_1 + D_2 + \ldots + D_M$, i.e. there is an additivity law. The formulas defining τ and D are applicable in cases where the incident flow is monochromatic and the media are selective, or when the flow of any spectral composition drops, but the media are not selective.

When measuring α, it is necessary to consider that a part of the light is reflected at the boundary of the substance being studied, and to make appropriate corrections, for example, using Fresnel’s formulas. The transmission spectrum is the dependence of the transmission coefficient on the wavelength or frequency (wavenumber, quantum energy, etc.) of the radiation. Concerning the light, such spectra are also called light transmission spectra.

The light transmission of the plate of colorless glass is calculated by the formula:

$$\tau = (1-\rho)^2 e^{-\alpha l} = R\tau \tag{133}$$

Where ρ is the reflection coefficient from one polished surface, $(1-\rho)^2 = R$ is the correction for reflection, expressed in units of transmission.

The correction ρ can be calculated by the formula:

$$\rho = \left(\frac{n-1}{n+1}\right)^2 \tag{134}$$

Where n is the refractive index of the medium. For M plates (Figure 83) of glass with the same absorption

$$\tau = (1-\rho)^{2M} e^{-\alpha d_t} = R^M \tau_t \tag{135}$$

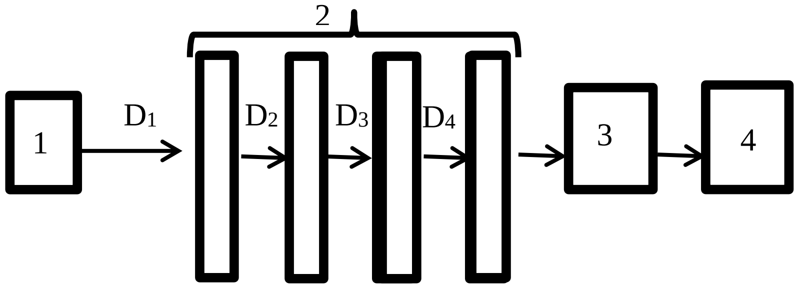
Where d_t is the total thickness of the plates, τ_t is the total transmission coefficient. The absorption coefficient α in its experimental determination is calculated by the formula:

$$\alpha = \frac{\lg R^M - \lg\tau}{d_t \cdot \lg e} \text{ or } \alpha = \frac{D_\rho - D}{0{,}4343 d} \tag{136}$$

where $lg\, e$ = 0,4343, d is in centimeters.

If you measure the intensity of light I_1 and I_2 passed through, respectively, through layers of thickness d_1 and d_2, then you can do without corrections for reflections. In this case, the absorption is determined from the ratio

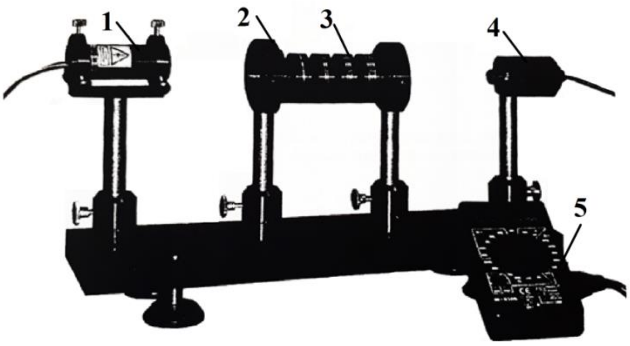
$$\frac{I_1}{I_2} = e^{\alpha(d_2 - d_1)} \tag{137}$$



1-radiation source (semiconductor laser) 2 - object (a set of plates made of neutral glass D_1, D_2 ...) 3 - Radiation receiver (photodiode), 4 - recording device

Figure 83 - Functional installation diagram

Principle of operation:
The PMC-9 installation is made according to the modular principle. All devices entering into it are executed in the form of the replaceable blocks installed on the modular basis and fixed on it by screws. The appearance of the PMC-9 is shown in Figures 84.



1 - Laser source, 2 - Cuvette compartment, 3 - Investigated plates of different thickness in the frames, 4 - Photodetector, 5 - Multimeter

Figure 84 - Appearance of the PMC-9 device

Radiation passing through the set of the studied plates falls on the photosensitive area of the photodetector. The multimeter measurements in the current mode are proportional to the magnitude of the recorded luminous flux. The transmission of the plate is measured as the ratio of the multimeter readings in its absence and installed in the cuvette compartment. During the experiment, the combination of plates according to Table 7 is sequentially established. The data obtained will make it possible to measure the dependence of the absorption in glass on the length of the optical path.

Table 7 - Thickness of the studied plates

№	Marking	Thickness, mm
D1	1 strip	1
D2	2 strips	2
D3	3 strips	4
D4	4 strips	8

The wavelength of the laser used λ - 650 nm, n_D - 1,502, α_λ-0,06

Order of performance:
1. Connect the photodetector to the multimeter.
2. Turn on a multimeter, set current measurement mode

3. Turn on the laser, warm the instruments for 10 minutes.

4. To take the readings of the multimeter I_o corresponding to the intensity of the laser radiation without an object

5. Install the necessary combination of records according to Table 7.1

6. Take multimeter readings

7. Draw a graph *τ(d), τ'(d)*

8. Repeat measurements with all combinations of plates. The results recorded in table 7.2

9. Calculate the errors

Table 8 - Experimental calculations

№ p/p	A set of plates 1 is present , 0 is absent				Total thickness, d_t, mm	Experiment			Calculation	Experiment	α, cm^{-1}
	D1	D2	D3	D4		I_o, mA	I, mA	$\tau=I/I_o$	R^M	$\tau'=\tau/R^M$	
1	1	0	0	0	1						
2	0	1	0	0	2						
3	1	1	0	0	3						
4	0	0	1	0	4						
5	1	0	1	0	5						
6	0	1	1	0	6						
7	1	1	1	0	7						
8	0	0	0	1	8						
9	1	0	0	1	9						
10	0	1	0	1	10						
11	1	1	0	1	11						
12	0	0	1	1	12						
13	1	0	1	1	13						
14	0	1	1	1	14						
15	1	1	1	1	15						
											$<\alpha>=$

Questions for the self-test

1. Formulate the law of the Beer-Lambert law?
2. Explain the difference in the absorption coefficient of dielectrics and metals.
3. Specify the spectral characteristics of the glass.
4. Write down the Fresnel formula for calculating the reflection coefficient.

References

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Laboratory work № 8

STUDYING OF THE UM-2 PRISM MONOCHROMATOR AND ITS GRADUATION

Aim of the work: a study of the device and the principle of operation of the prism monochromator UM-2, familiarization with the method of its graduation with neon and mercury lamps

Accessories: Mercury lamp, neon lamp, prism, collimator, optical branch

Brief theoretical introduction

If the light from the glowing solid body passes through a prism, then we will receive a continuous color bar on the screen behind the prism. Invisible parts as ultraviolet and infrared regions are adjacent to the bar to the right and left. The whole picture together is called the continuous spectrum. The same kind of spectrum is given by hot gases of high density. Very rarefied gases (or vapors) give line spectra (a series of bright lines against a dark background). They arise in the transition of an atom from a higher energy level E_2 to a low E_l one. In this case, the frequency ν of the emitted monochromatic radiation is determined by the condition:

$$E_2 - E_l = h\nu, \qquad (138)$$

Where $h = 6{,}626 \cdot 10^{-34}$ $J\,s$ - Planck's constant.

If the source of radiation is molecules of the substance, then the so-called absorption spectrum is obtained.

By appearance, the emission spectrum is similar to the absorption one, but only in the absorption spectrum there are dark lines on a colored background, and on the emission one there are colored lines on a dark background.

Each chemical element emits a typical spectrum. Therefore, studying the spectrum can be a qualitative analysis of the substance. And by estimating the intensity of the lines, one can determine the quantitative content of a substance, since there is a direct connection between them. The emission spectrum (or absorption) is a set of waves of certain frequencies that the atom emits (or absorbs) of a given substance. Radiation from the infrared, visible and ultraviolet ranges from highly heated substances is usually studied.

Bohr's theory made it possible to explain the existence of line spectra. There are continuous, absorption and emission spectra.

The continuous spectra (Figure 85a) emit all substances in a solid or liquid state. The continuous spectrum contains waves of all frequencies of visible light and therefore looks like a color bar with a smooth transition from one color to another in this order: red, orange, yellow, green, blue and violet.

Emission spectra (Figure 85b) emit all substances in the atomic state. The atoms of all substances emit peculiar only to the sets of waves of quite specific frequencies. Every person has his own personal fingerprints, and the atom of a given substance has its own characteristic spectrum. Emission spectra look like colored lines separated by gaps. The nature of the spectra is explained by the fact that the atoms of a particular substance have only peculiar stationary states with their characteristic energy, and, consequently, their own set of pairs of energy levels that an atom can change, that is, an electron in an atom can transfer only from certain orbits to other well-defined orbits for a given chemical.

Absorption spectra (Figure 85c) are emitted by molecules. Absorption spectra look like line ones, but instead of separate lines there are separate series of lines, perceived as separate bands. It is characteristic that what spectrum is emitted by these atoms is the same and is absorbed, that is, the emission spectra over a set of radiated frequencies coincide with the absorption spectra. Since the atoms of different substances correspond to the spectra peculiar only to them, there is a way to determine the chemical composition of a substance by studying its spectra. This method is called spectral analysis. Spectral analysis is used to determine the chemical composition of fossil ores in the extraction of minerals, to determine the chemical composition of stars, atmospheres, planets; is the main method of controlling the composition of a substance in metallurgy and mechanical engineering.

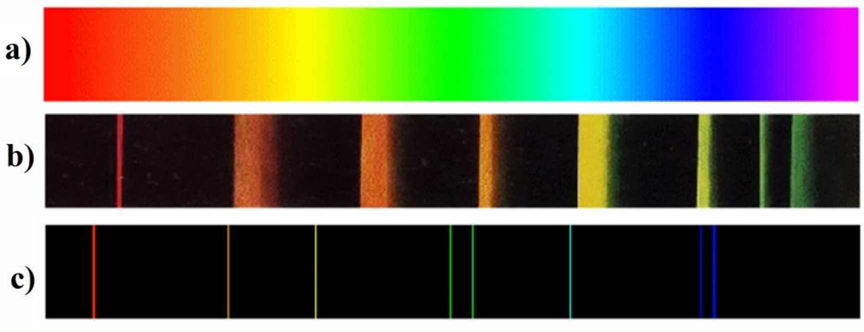

Figure 85 - a) continuous spectrum b) absorption spectrum, c) emission spectrum

Description of the UM-2 monochromator

The schematic diagram of the monochromator is shown in Figure 86. The light from source 1 is focused by lenses 2 and 3 on the entrance slit S_1 of the monochromator, located in the focal plane of its objective 4, and falls on a dispersing element of the monochromator, a prism 5, with a parallel beam. The output lens 6 of the monochromator collects monochromatic beams at various points of the focal plane 7, where the exit slit S_2 is located.

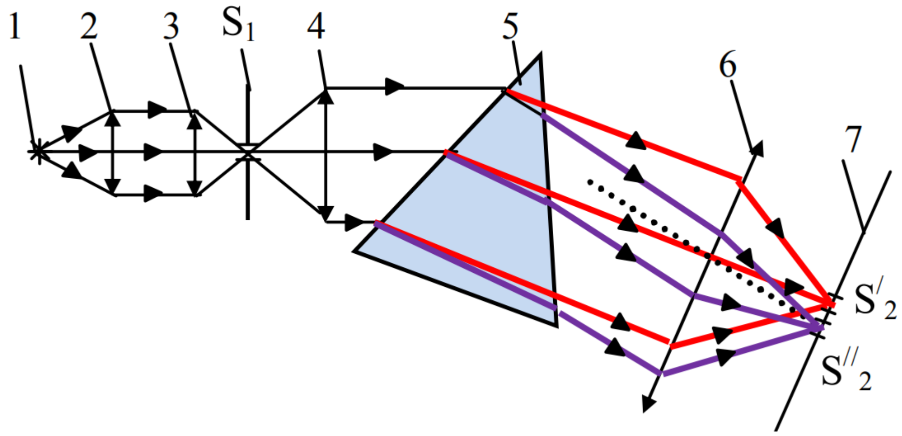

Figure 86 - Schematic diagram of the monochromator

The spectral lines of different colors in the focal plane 7 are monochromatic images of the entrance slit S_1. The combination of these images represents the radiation spectrum of the source. By moving the spectrum relative to the slit S_2 by rotating the prism 5, light beams of different spectral compositions can be obtained in the plane of the exit slit.

The optical scheme of the spectral device (Figure 87) in the general case consists of the following main parts: I - lighting; II - dispersing; III - receiving/recording. The lighting device is designed to create a strong and uniform illumination of the slit diaphragm 3 by the investigated radiation. The lighting part includes a radiation source 1 and a condenser 2, projecting the image of the source on the entrance slit 3 of the collimator. The dispersing part II is used for decomposition into the spectrum of a parallel beam of rays coming from the collimator. A narrow slit input aperture 3 is installed in the focal plane of the lens 4 of the collimator. The collimator directs parallel beams of rays on the dispersing element 5, which is used as a dispersion prism and diffraction gratings. The device receiving/recording part III depends primarily on the destination spectral instrument.

The receiving/recording device with a visual method of observation represents itself like a telescope. It consists of lens 6 and an eyepiece 8. A pointer 7 is located between the lens and the eyepiece. In modern spectral instruments spectral decomposition is carried out using dispersive systems consisting of several prisms. Such an optical system allows to obtain a large dispersion, as well as change the angles of deflection of the rays. In Figure 8 it shows the Abbe prism, which is a block of three glued rectangular prisms

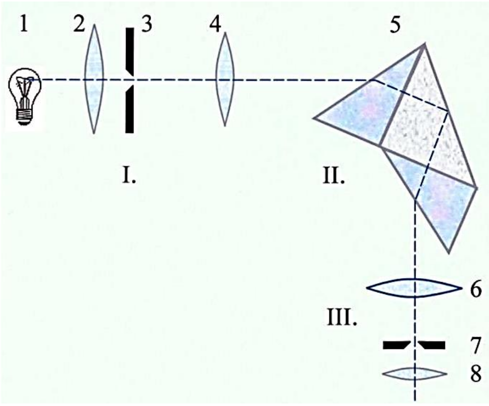

Figure 87 - Optical scheme of UM-2 monochromator

By turning the prism table at different angles relative to the incident beam, light of different wavelengths is obtained at the exit slit, which is passed through the prism at the minimum of deflection. The UM-2 glass-prism monochromator spectrometer is intended for spectral studies in the range from 380 to 1000 nm. The device consists of the following main parts (Figure 88).

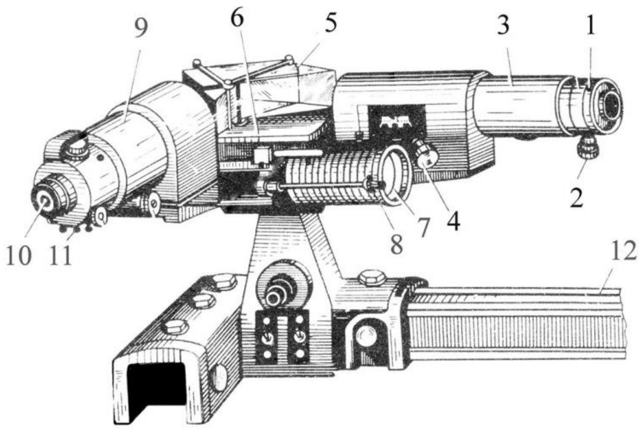

Figure 88 - General view of the monochromator UM-2

The entrance slit 1 is equipped with a micrometric screw 2, which allows opening the slit to the desired width. The collimator lens 3 is used to create a parallel beam of rays. It is equipped with a micrometer screw 4. The screw allows shifting the lens relative to the slit when focusing on spectral lines of various colors. A complex spectral prism 5 is installed on the turntable 6. Prism 5 consists of three glued prisms. The turntable 6 rotates around a vertical axis using a micrometric screw with a reading drum 7. A spiral track with degree divisions is applied to the drum. The turn indicator of drum 8 slides along the track.

As the drum rotates, the prism rotates and different parts of the spectrum appear in the center of the field of view. The telescope consists of a lens 9 and an eyepiece 10. The lens provides an image of the entrance slit 1 in its focal plane. The pointer 11 is located in this plane. The image is viewed through the eyepiece 10. The monochromator is enclosed in a massive case protecting the device from damage and contamination. The device is also equipped with an optical bench 12, on which the reitors with a light source can move. The light source is recommended to be placed at a distance of 45 cm from the slit. Each wavelength of light on the scale of the measuring drum will correspond to its countdown. On the drum there is a spiral track with degree divisions. A drum turn indicator slides along the track. The scale of the drum is applied in degree divisions from 0 to 3500^0 with a division value of 2^0 and digitization in 50^0.

To operate the monochromator, it is necessary to pre-calibrate it and make a passport in the form of a table or a calibration curve $\lambda = f(b)$, where λ is the light wavelength, b is the corresponding degree of the measuring drum. When calibrating, it is necessary to use an eyepiece with a pointer, and light up the entrance slit 6 with a light source having a line spectrum with known wavelengths. Such a source is the DRSh-250 mercury lamp.

For the calibration of the device in the red region of the spectrum, one can use another source - a neon lamp, the spectrum of which is rich in red lines of various shades. The spectra of mercury and neon vapor in the visible region of the spectrum are shown in Figure 89 and Figure 90, respectively.

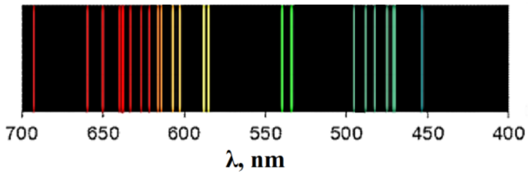

Figure 89 - Neon lamp vapor spectrum

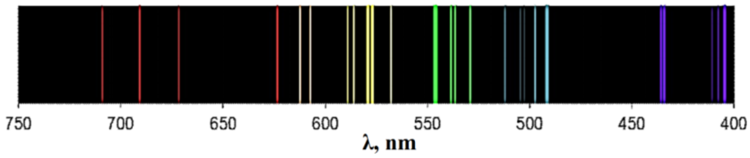

Figure 90 - Mercury lamp vapor spectrum

We can achieve a sharp pointer image when calibrating a monochromator by focusing on the eyepiece. By rotating the measuring drum, one can find the spectral lines of mercury and get their sharp image using a micrometer screw. In this laboratory work, one can use the visual method of observation with an eyepiece, which is installed instead of a slit. In the field of view of the eyepiece, not one line is visible (as behind the exit slit), but several (Figure 91).

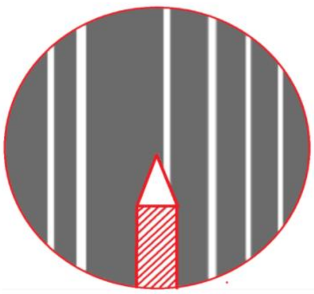

Figure 91 - View of the eyepiece with a set of spectral lines and the needle pointer

To set the position of the spectral line in the plane of the exit slit, there is an index in the form of a sharp needle-pointer. The index is observed through the eyepiece. Entering the spectral line on the index is made using the drum. The eyepiece can be set by the eye of the observer on the sharpness of the image of the index and the spectral line, by rotating it.

It is necessary to observe the visible radiation spectrum of mercury and compare it with the standard one presented in Table 8.1 After that, in some graphics program, for example, Microsoft Excel or OriginPRO, build a calibration curve, putting the wavelengths of light taken from Table 1.8 along the ordinate, and abscissa axis - readings on a drum scale. You need to draw a smooth curve along the points; for this, the experimental dependence can be approximated by a 5 degree polynomial (See figure 91).

For the calibration of a prism monochromator, one can also use the Hartmann formula, which well describes the experimental dependence $\lambda(b)$:

$$\lambda = \lambda_0 - \frac{a}{b - b_0} \tag{139}$$

where b is the division of the drum of the monochromator, λ_0, a and b_0 are constants determined experimentally from the equations

$$\lambda_1 = \lambda_0 - \frac{a}{b_1 - b_0} \tag{140}$$

$$\lambda_2 = \lambda_0 - \frac{a}{b_2 - b_0} \tag{141}$$

$$\lambda_3 = \lambda_0 - \frac{a}{b_3 - b_0} \tag{142}$$

In equations (140 - 142), λ_1, λ_2, and λ_3 are known wavelength values, b_1, b_2, and b_3 are their corresponding readings on the drum. As can be seen from these equations, to determine the constants λ_0, a and b_0, it is necessary to measure on the monochromator three “reference” wavelengths belonging to different parts of the spectrum.

Order of performance:

1. To get acquainted with the device and the principle of operation of the UM-2 monochromator
2. Turn on the eyepiece scale, focusing on the eyepiece to achieve a sharp pointer image. Install the DRSh-250 mercury lamp on the optical bench in front of the entrance slit and turn it on.
3. Rotating the measuring drum 15, find the spectral lines of mercury and obtain a sharp image of them with the help of the micrometric screw 17 and record the corresponding readings of the drum in Table 9.
4. In any graphics program, for example, using the MS Excel spreadsheet processor or OriginPRO, construct an experimental dependence $\lambda = f(b)$, putting the wavelengths of light taken from Table 1.8 along the ordinate axis and the readings on the scale of the drum along the abscissa axis.
5. The obtained points must be linked with a smooth curve, for this it is best to approximate the experimental dependence $\lambda = f(b)$. Figure 8.8 shows an example of such a calibration.
6. Choose three “reference” points - the wavelengths from Table 8.1 and the values of the divisions of the drum for the red, green and violet lines.
7. Determine the constants λ_0, a and b_0 in the Hartman interpolation formula, solving the system of equations (140 - 142). For the spectrum of mercury, reference points can be λ_1 = 690.7 nm, λ_2 = 546.1 nm, λ_3 = 435.8 nm and the corresponding divisions of the monochromator drum.
8. Analyze the results
9. Repeat steps 1-7 for a neon lamp and enter the data in Table 9.

ATTENTION! The mercury lamp emits a powerful stream of ultraviolet rays that are harmful to the eyes, so you should make sure that the window of the housing is covered with a glass filter that *does not transmit ultraviolet radiation. The discharge in the mercury-quartz lamp of ultrahigh pressure DRSh -250 occurs at a pressure of 10-15 atmospheres. The lamp is enclosed in a metal casing to protect the surrounding hot flashes from the lamp bulb in the event of an explosion. After turning off the mercury lamp, it should be re-ignited no earlier than 10 minutes later.*

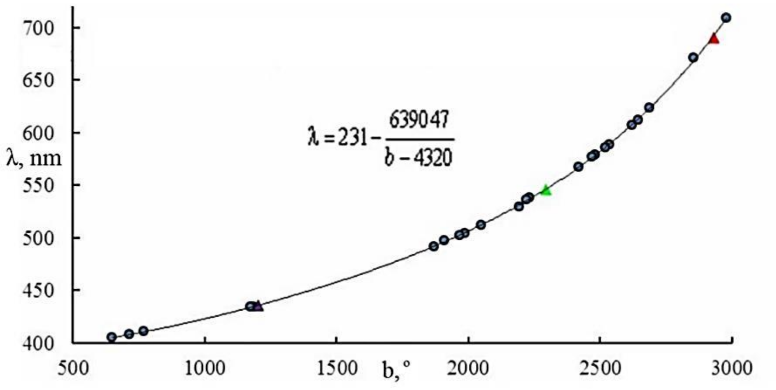

Figure 92 - Calibration dependence $\lambda = f(b)$. Points are experimental data. Triangles denote points that were used to define constants in the Hartmann formula

Table 9 - Wavelengths of the lines of the DRSh-250 mercury lamp

λ, nm	671,7	667,8	659,9	653,3	650,7	640,2	638,3	633,4
$b,^o$								
Color								
λ, nm	630,5	626,6	621,7	616,4	609,6	607,4	603,0	588,2
$b,^o$								
Color								
λ, nm	585,3	576,4	540,1	534,1	533,1	503,1		
$b,^o$								
Color								

Table 10 - Wavelengths of the lines of neon lamp

λ, nm	708,8	690,7	671,6	623,2	612,3	607,3	589	586	579,1
$b,^o$									
Color									
λ, nm	577	567,6	546,1	538,5	536,5	531,7	512,1	504,6	502,5
$b,^o$									
Color									
λ, nm	497,4	491,6	435,8	434,7	433,9	410,8	407,8	404,7	
$b,^o$									
Color									

Questions for the self-test

1. What is a monochromator for?
2. Explain the acquisition of the spectrum. Types of spectra.
3. What is the physical phenomenon based on the obtaining of monochromatic radiation in the UM-2 monochromator?
4. What is the spectral range of the device?
5. What is the purpose of the comparison prism in the monochromator?

6. How can a monochromator be calibrated?
7. Which reference sources can be used for calibration?
8. What is the purpose of the glass filter, which closed the window of the metal casing of the mercury lamp?
9. What reference points should be used to determine the constants in the Hartman formula?

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Laboratory work № 9

STUDYING THE BASIC LAWS OF EXTERNAL PHOTOELECTRIC EFFECT

Aim of the work: To study the external photoelectric effect laws when evaluating the characteristics of a gas-filled photovoltaic cell.

Accessories: Interference filters, FPK-10 device.

Brief theoretical description

Light activated electrons are emitted from some metals. This phenomenon is called the external photoelectric effect, and the obtained light activated electrons are usually called photoelectrons.

The phenomenon of the external photoelectric effect was firstly studied by Stoletov in 1888. The quantum properties of light are clearly manifested in the laws of this phenomenon. To study the photoelectric effect, Stoletov collected the following scheme (Figure 93).

In Figure 93 a metal plate *C* (photocathode) is connected to the negative pole of the battery. The positive pole through galvanometer is connected to the metal grid *A* (anode). Both electrodes stay in an air free glass vessel. When the cathode (plate *C*) is illuminated with light, a current arises in the circuit, which is detected by a galvanometer. This current is called the photoelectric current (or photocurrent), and the electrons pulled out of the cathode by light are photoelectrons. The photocurrent is the motion of electrons emerging from the cathode towards the anode.

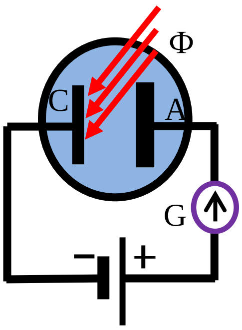


Figure 93 - Structure and wiring diagram photocell, Φ - luminous flux, *A* - anode, *C* - cathode, *G* - galvanometer

Stoletov investigated the dependence of the photocurrent on the magnitude of the applied voltage between the anode and cathode. Figure 94 a, shows the IV curve of the photocurrent (i.e., the dependence of the current value on the potential difference between the anode and the cathode at a constant luminous flux Φ).

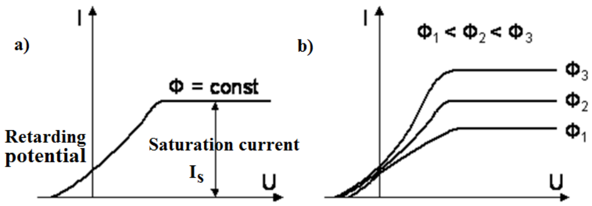


Figure 94 - IV-curves of a photocell

From the graph in Figure 94a, it can be seen that the value of the photocurrent reaches a maximum value at a certain voltage *U*, and then remains constant at any values of voltage. This means that all electrons pulled out of the photocathode by light reach the anode. The maximum current is called the saturation current at a given luminous flux Φ. If we change the magnitude of the luminous flux Φ, then we obtain a family of curves for a given photocathode (Figure 94b).

It can be seen from the IV curve (Figure 94a), that in the absence of voltage between the electrodes, the photocurrent is not zero. This means that electrons ejected from the cathode by the light have a certain initial velocity, and consequently, kinetic energy. So, they can reach the anode without the presence of an external field, forming the initial current.

To weaken or completely stop this current, it is necessary to create a decelerating field ($U < 0$). With a decelerating field increase, the value of the photocurrent weakens gradually, which indicates a large variety of speeds of photoelectrons (electrons are released not only from the surface, but also from the deeper layers of the cathode). If we will choose a potential difference U, at which the photocurrent becomes equal to zero, then we can argue that all electrons are delayed by a decelerating field. The potential difference, at which the photocurrent becomes zero, is called a retarding potential difference.

Stoletov established three laws of the external photoelectric effect.

The first Stoletov law. With a constant spectral composition of the light incident on the photocathode, the saturation photocurrent is proportional to the power of the radiation incident on the cathode. In other words, the number of electrons released by light per second from the cathode (saturation photocurrent I_s) is proportional to the luminous flux Φ:

$$I_s = \gamma \cdot \Phi \tag{143}$$

The coefficient of proportionality γ is called the integral sensitivity of the photocathode if the luminous flux is non-monochromatic (i.e., consisting of a set of different wavelengths, for example, white light). The proportionality coefficient is measured in *A/lm*, *μA/lm* or *A/W*

If the cathode is illuminated with monochromatic light (i.e. light of a certain wavelength), then

$$\gamma = \frac{I_s}{\Phi_\lambda} \tag{144}$$

called the spectral sensitivity of the photocathode. The sensitivity of modern photocathodes reaches 60-100 *μA/lm*.

The second Stoletov law. The maximum initial velocity of photoelectrons depends on the frequency of the light and does not depend on its intensity (on the luminous flux). Therefore, the maximum kinetic energy of photoelectrons is in direct proportion to the frequency of the incident light and does not depend on its intensity.

A theoretical explanation of the second law of Stoletov was proposed by Albert Einstein. He wrote the equation based on the law of conservation of energy and the Planck hypothesis

$$h\nu = A + \frac{mv^2}{2} \tag{145}$$

where $h\nu$ is the quantum energy of light incident on the photocathode of the photocell; h - Planck's constant; ν - the frequency; A - the electron work function of a metal; v - the electron velocity. Work function A depends on the selected material and the purity of the material surface. The maximum speed of the photoelectrons depends only on the frequency of the light causing the photoelectric effect. If the photon energy $h\nu$ is greater than the work function, then the photoelectrons acquire a certain velocity. The greater frequency of the incident light, the grater velocity photoelectrons acquire.

The third Stoletov Law. For each substance, there is a red border of the external photoelectric effect, namely the minimum frequency of light ν_0, at which the photoelectric effect is still possible ($h\nu_0 = A$). The frequency ν_0 depends on the chemical nature of the substance and the quality of the surface. This frequency ν_0 (and the corresponding wavelength $\lambda_0 = \frac{c}{\nu_0}$) is called the "red border" (long-wave border) of the photoelectric effect.

If we consider light as a wave, the kinetic energy of the photoelectrons would have to depend on the intensity of the light, namely with an intensity increase, larger energy would be transmitted to it. But the experiments do not confirm this. It can also be shown that from the wave point of view, the photoelectric effect cannot be inertia-free (which contradicts the experiment).

Thus, neither the presence of a “red border”, nor the dependence of the electron velocity on the luminous flux magnitude, nor the inertia absence of photoelectric effect can be explained by the wave theory of light.

Einstein, using Planck’s theory of quanta, proposed a new explanation of the photoelectric effect in 1905. He suggested that the light is not only emitted, but also absorbed by the substance in individual portions - photons, each of which has an energy equal to

$$E = h\nu \tag{146}$$

When a photon beam falls on the metal surface, the photons collide with electrons. In this case, the photon gives the electron all its energy. An excited electron, having received energy from a photon, can escape from the metal. From the quantum theory point of view, one can explain all the laws of the photoelectric effect. So, from Einstein's formula, it can be seen that the energy of the torn-out particles - electrons depends directly proportionally on the frequency of the incident light, i.e., on the energy of the photons, and does not depend on the intensity of the light.

Stoletov's law: the greater the luminous flux, the greater the number of photons in this flux. Therefore, there will be more torn electrons, namely will be higher saturation current. The absence of inertia of the photoelectric effect is explained by the fact that the energy transfer in the collision of a photon with an electron occurs almost instantaneously.

The phenomenon of the photoelectric effect is widely used in technology (sound film, automatics). All technical applications of the photoelectric effect are based on the use of photocells.

The photocell design is a glass cylinder, often spherical in shape, on one half of the inner surface of which deposited a photosensitive layer of alkali metals or their compounds with a small work function value (Figure 9.3). This photosensitive layer is a photocathode in front of which a small-sized tungsten metal anode is placed. The electrical leads from the cathode and the anode are soldered to the bottom of the cylinder and brought further to the base of the photocell. Either a vacuum (vacuum photocells), or an inert gas (Ar, Ne, He) under a pressure of the order of 10^{-3} mmHg (gas-filled photocells) is created in the photocell balloon.

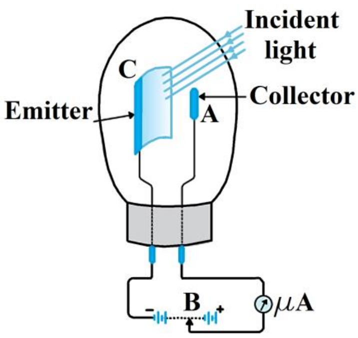
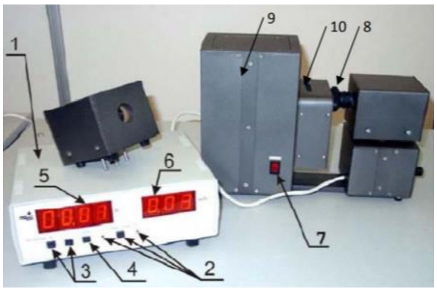


Figure 95 - Photocell device and network connection scheme, C - cathode, A-anode

Photoelectrons, accelerated by an electric field when a voltage is applied between the anode and cathode in a gas-filled photocell, collide with gas molecules and knock electrons out of them. The electrons, accelerated by the same field, increase the ionization of the gas. In a vacuum photocell, current arises only due to photoelectrons. Therefore, the sensitivity of gas-filled photovoltaic cells is higher than that of similar vacuum cells.

Device and principle of operation of the device

Device FPK-10 (Figure 96) is used to carry out the laboratory work. The device allows you to study the current-voltage characteristics in a wide range of illumination.



1 - Object of study and measuring device, 2 - Direct-Reverse button, 3 - Buttons + and -, 4 - Reset button, 5 - Voltage indicator, 6 - Photocurrent indicator, 7 - Power switch, 8 - Light adjustment device, 9 - Power supply, 10 - Interference filters unit

Figure 96 - FPK-10 Device to study the photoelectric effect

The object of study is structurally made in the form of a modular hull. Illuminator (spectral mercury lamp) with a power source 9 an interference filters from 1 to 4 (10) and an illumination control device 8 is installed into the modular hull. The position "0" of the filter unit corresponds to the passage of light without light filters and can be used to remove the integral current-voltage and lux-ampere characteristics. The position "5" overlaps the lamp and is used to set zero. A photocurrent amplifier is attached to the modular hull using a bracket. Replaceable photodetector with an NC-4 photocell is installed on the top cover of the photocurrent amplifier. When a photodetector is installed, its receiving window is combined with the exit window of the illuminator and is closed using a hood.

There is a power switch with a power on indicator 7 on the front panel of the studied object. There is an exit window of the illuminator on the side wall and a device for changing the interference filters and adjusting the illumination. There is a connecting cord for connecting the studied object to the measuring device and the balance control of the amplifier, which can regulate COARSE and ACCURATE on the side surfaces of the photocurrent amplifier. The object of study is connected to a power grid with 220 *V*, 50 *Hz*.

The measuring device is made in the form of a structurally finished product. It uses a single-chip micro-computer with appropriate additional devices used to: 1) measure the current of the photocell, which installed in the object of study, 2) set up and measure the supply voltages on the photocell, 3) provide control functions (setting modes of direct or reverse measurement, etc.). The measuring device also includes power sources.

On the front panel of the measuring device the following controls and displays are placed:

• The DIRECT - REVERSE button with the corresponding indicators 2 is intended to activate the direct or inverse measurement modes.

• Buttons "+", "-" 3 and RESET 4 are used to adjust the voltage on the photocell and reset it to zero.

• Indicators V 5 and μA 6 are intended to indicate the values of voltage and photocurrent on the photocell, correspondingly.

• Button NETWORK, the grounding terminal, the fuse holders (closed with a safety bracket), the power cord with the plug and the connector to the studied object are located on the back panel of the measuring device.

The principle of the device is based on the measurement of the current through the solar cell when the polarity and magnitude of the applied voltage and changing the magnitude and spectral composition of the illumination of the photocell cathode

In the course of laboratory work, the through the photocell and the applied voltage is measures. In this case, the voltage polarity changes (i.e., direct and reverse branches of the IV curve of the photocell are removed separately). Characteristics are taken at different values of illumination and when the wavelength of the photocell illumination changes. Based on the measurement results, curves of IV -curves are constructed.

Order of the performance

1. Turn on the measuring device with the NETWORK button on its back panel. In this case, the REVERSE indicator, the buttons for measuring the voltage and photocurrent of the measuring device should light up. Zeroes should be set on the indicator (indication is allowed up with 2 decimal places).

2. Turn on the button NETWORK on the front panel of the object of study. The indicator NETWORK of the object of study should light up.

3. Let the illuminator lamp warm up for 10 minutes. After warming up with the buttons SETTING ZERO on the object of study, set the zero value on the indicator μA of the measuring device (while the swivel wheel with light filters is in the "5" position, corresponding to the overlap of the light flux from the lamp to the photocell).

4. Install a filter 1 (λ = 407 nm), the data of which are taken from table 9.1

Table 11 - Characteristics of light filters

№ light filter, color	Wavelength(λ), which passes the filter, nm
1. Purple	407
2. Blue	435
3. Green	546
4. Yellow	578

5. Set the minimum photocell illumination. To do this, turn the pivot ring 8 of the lighting device away from you. Write down the photocurrent values from the indicator by changing the voltage values using the "+" and "-" buttons, Measures the dependence of the current through the photocell on the voltage applied to it at a constant illumination and enters the data in Table 11.

6. Press the "Reset" button and set the reverse voltage. Consider reverse polarity as a negative voltage.

7. Reduce the current to zero and determine the blocking voltage.

8. Writ down the current-voltage characteristic with maximum illumination for reverse voltage, data should be entered in Table 11

9. Perform steps 4-8 for maximum and minimum illumination for all remaining light filters (2, 3, 4) Maximum illumination - the extreme position of the ring of the dimming device when it is turned counterclockwise.

10. Build a family of branches of the IV-curves of photocells for optical filters 1-4 for maximum and minimum illumination on a separate sheet of graph paper or in programs such as Microsoft Excel or OriginPRO.

11. Determine the kinetic energy of the photoelectron for chosen light filters using the formula $= eU$, where e - the electron charge, U - the blocking voltage, and write down the data in table 9.3

12. Determine the maximum photoelectron velocity for the chosen light filters by the formula $v_{max} = \sqrt{\frac{2eU}{m}}$, where m - the electron mass, write down the data in Table 13.

13. Determine the work function of the photoelectron from the metal for chosen light filters by the formula, $A = \frac{hc}{\lambda} - eU$, where h - Planck's constant, c - the speed of light in vacuum, λ - the wavelength of the light filter, and write down the data in Table 12

14. Determine the red border of the photoelectric effect by the formula $\lambda_0 = \frac{hc}{A}$, and write down the data in table 12.

Remark. When determining the blocking voltage of the photocell, it is necessary to read out the zero current when the voltage decreases from zero to the value of the blocking voltage, and not *vice versa.*

Table 12 - Indications dependencies *U(I)* for different types of illumination light filters 1 - 4

Light filter 1	Minimum illumination	*U, B*	0	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5
		I,μA											
	Maximum illumination	*U, B*	0	0,05	0,1	0,15	0,2	0,25	0,3	0,35	0,4	0,45	0,5
		I,μA											
Light filter 2	Minimum illumination	*U, B*	0	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5
		I,μA											
	Maximum illumination	*U, B*	0	0,05	0,1	0,15	0,2	0,25	0,3	0,35	0,4	0,45	0,5
		I,μA											
Light filter 3	Minimum illumination	*U, B*	0	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5
		I,μA											
	Maximum illumination	*U, B*	0	0,05	0,1	0,15	0,2	0,25	0,3	0,35	0,4	0,45	0,5
		I,μA											
Light filter 4	Minimum illumination	*U, B*	0	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5
		I,μA											
	Maximum illumination	*U, B*	0	0,05	0,1	0,15	0,2	0,25	0,3	0,35	0,4	0,45	0,5
		I,μA											

It is forbidden to set the voltage value below the blocking voltage. The locking voltage corresponds to a current equal to zero.

Table 13 - Characteristics of the studied object using experimental data

	T, eV	v_{max}, *m/s*	*A, eV*	λ_0, *m*
Light filter 1				
Light filter 2				
Light filter 3				
Light filter 4				

15. At the end of the work, it is necessary to turn off the power supply of the device using the NETWORK switches (on the back panel of the measuring device and the front panel of the studied object) and disconnect the power plugs.

16. Evaluate measurement errors.

Questions for self-test

1. Formulate the laws of the external photoelectric effect.
2. Give an interpretation of the Einstein equation.
3. Tell about Stoletov's laws.
4. Explain the structure and operation of gas-filled and vacuum photocells.

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Laboratory work № 10

DETERMINATION OF THE WAVELENGTH OF RADIATION BY DIFFRACTION ON THE SLIT

Aim of the work: acquaintance with diffraction patterns of various types; determining the width of a rectangular slit when studying the phenomenon of diffraction in monochromatic light; determination of the wavelengths of red and violet light.

Accessories: Laser, tripod; RMS 3 installation, MOL-1

Brief theoretical description

The phenomenon of diffraction is the deviation of light from rectilinear propagation in a media with sharp irregularities in the form of edges of opaque and transparent bodies, narrow holes, protrusions, etc. As a result, light penetrates the region of the geometric shadow and an interference redistribution of the light intensity occurs. Under the diffraction should be understood any deviation from the linear propagation of rays, unless it is a consequence of the usual laws of geometric optics - Reflection and refraction.

The phenomenon of diffraction is explained by the wave properties of light using the principle of Huygens Fresnel.

The main provisions of this principle:

1. Each element of the wave surface, which has been attained by the light wave at the given moment, is a source of secondary waves, whose amplitude is proportional to the area of the element.

2. Secondary waves created by elements of the same surface are coherent and can interfere with themselves.

3. Radiation is maximal in the direction of the outward normal to the surface element. The amplitude of the spherical wave decreases with increasing distance from the source. Only open areas of the wave surface emit.

Huygens Frensel's principle makes it possible to assert deviations from straightforward propagation in the case of any obstacle. Consider the case of a plane wave (a parallel beam of light) falling on an obstacle in the form of an MN hole in an opaque plate (Figure 97).

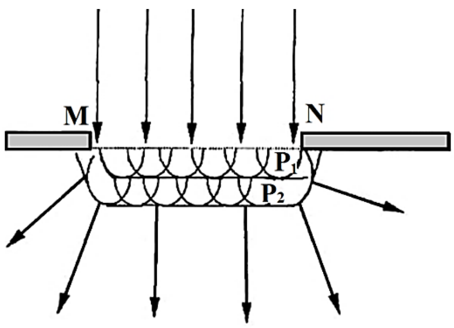


Figure 97 - Illustration of the Huygens-Fresnel principle

Following this principle, each point in the plane of the hole MN can be considered as an independent source of light emitting an elementary spherical wave. Surface P_1, formed by elementary waves, determines the wavefront at time t_1.

This surface P_1 also becomes a source of secondary elementary spherical waves. The curve that envelopes these elementary waves at the instant of time t_2 determines the wavefront with the surface P_2.

From Figure 97 it is seen that the light rays, being perpendicular to the wavefront, deviate from their original direction and fall into the region of the geometric shadow.

To solve the diffraction of a light problem means to investigate questions relating to the intensity of the resultant light wave in different directions. The main issue in this study is to study the interference of light, in which the superimposed waves can not only be amplified, but also weakened. One of the important cases of diffraction is diffraction in parallel rays. It is used when considering the operation of optical devices (diffraction grating, optical instruments, etc.). In the simplest case, the diffraction grating is a glass transparent plate on which strokes of equal width are applied at the same distance from each other. Such a lattice can be used in a spectral setup of the usual type instead of a prism as a dispersing system.

Single-slit diffraction

Consider diffraction in parallel rays on one slit. The type of diffraction in which the diffraction pattern formed by parallel rays is considered is called diffraction in parallel rays or Fraunhofer diffraction. The slot is a rectangular hole in an opaque plate, and one side is much larger than the other. The smaller side is called the **slot width *a***. Such a slit is an obstacle to light waves, and diffraction can be observed on it. Under laboratory conditions, diffraction at the slit is clearly observed if the width of the slit a is comparable in magnitude with the length of the light wave. Let a monochromatic light wave fall normally to a slit plane of width a (distance AB, Figure 98). Behind the slit are mounted a collecting lens and a screen placed in the focal plane of the lens.

According to Huygens' principle, each point of a wavefront that has reached the slit is a new source of oscillations, and the phases of these waves are the same, since with a normal incidence of light, the slit plane coincides with the plane of the wavefront. Let us consider the rays of monochromatic light from the points lying on the front AB, the direction of propagation of which makes an angle with the normal.

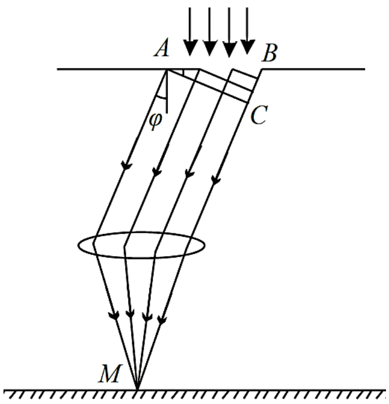


Figure 98 - Illustration for single-slit diffraction considering

Let us drop the perpendicular AC from point A to the direction of the beam propagating from point B. Then, propagating further from the AC, the beams will not change the path difference. The difference in the course of the rays is the segment BC. To calculate the interference of these rays, we apply the Fresnel zone method.

Divide the segment BC into segments of length $\lambda/2$. On the BC, there will be z of such segments:

$$z = \frac{a \sin \varphi}{\frac{\lambda}{2}} \tag{147}$$

Where $a \sin \varphi = BC$. Drawing lines parallel to the AC from the ends of these segments, before meeting with AB, we divide the wavefront into the slit into a series of strips of equal width, the number of which is equal to z. They are the Fresnel zones, since the corresponding points of these strips are sources of waves that have reached the observation point M in this direction with the mutual path difference $\lambda/2$.

The amplitudes of the waves from the strips will be the same, because the front is flat and their areas are equal. According to the theory of Fresnel zones, the rays from two adjacent zones extinguish each other, since their phases are opposite. Then, for an even number of Fresnel zones ($z = 2m$, where m is an integer, $m = 1,2,3$ ...) that fit into the slots, at the point M there will be a minimum of diffraction, and for odd numbers ($z = (2m + 1)$) - maximum. Equation (10.1) is then written as follows:

$$a \sin \varphi = 2m\frac{\lambda}{2} \quad \text{(minimum)}$$
$$a \sin \varphi = (2m + 1)\frac{\lambda}{2} \quad \text{(maximum)} \qquad (148)$$

The intensity distribution in the diffraction pattern from one slit is shown in Figure 10.3. The abscissa axis shows the distance from the zero maximum along with the screen on which the spectral picture is located.

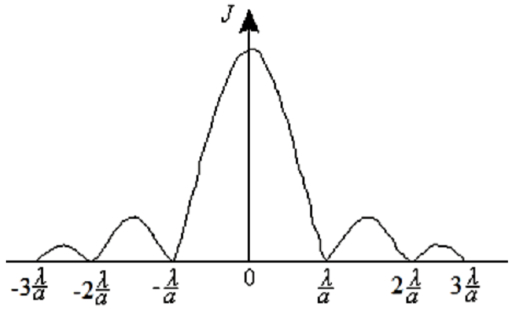
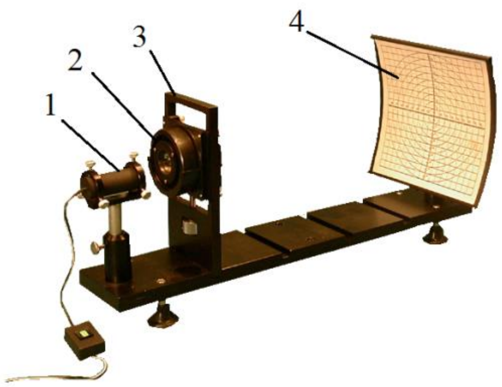


Figure 99 - The intensity distribution in the diffraction pattern from one slit

Description of the laboratory installation

Fraunhofer diffraction is observed at the slit using an RMS 3 setup consisting of an optical bench on which a semiconductor laser is mounted, a vertical alignment module with a photolithographic MOL-1 test object, a screen (Figure 100). The direction of the laser radiation is adjusted by the adjustment screws.



1 - semiconductor laser (GaAs); 2 - photolithographic test object MOL-1; 3 - vertical adjustment module; 4 - screen;

Figure 100 - RMS 3 device structure

The object MOL-1 is a thin glass disc with an opaque coating and transparent structures arranged in three rows: row *A* - double slits, row *B* - round holes, row *C* - single slits (Figure 101). The total number of slots in row *C* is 16. The radiation from the laser is directed to the desired structure on the surface of the MOL-1 object. At the same time, the corresponding diffraction pattern is observed on the screen.

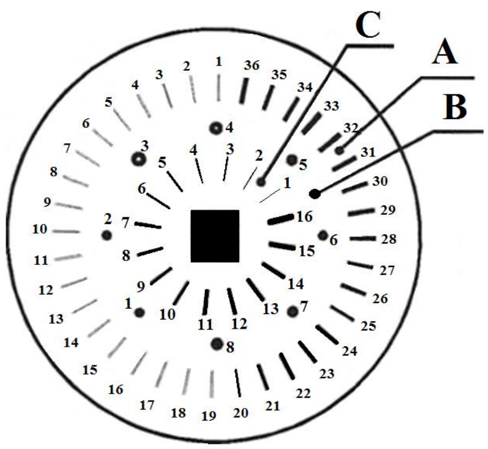


Figure 101 - Layout of the structures of the MOL-1

From the condition of the minimum of the m_{th} order for the slit, the radiation wavelength is expressed, where a - is the slit width; $\sin\varphi$ - is the sine of the angle at which the minimum of the m - order of the minimum is observed

$$\lambda = \frac{a\ \sin\varphi}{m} \tag{149}$$

The angles φ_m under which the minima are observed are small, therefore $\sin\varphi_m \approx \tan\varphi_m$. Using this condition, we get

$$\lambda = \frac{a \tan\varphi}{m} \tag{150}$$

Formula (150) is working to determine the wavelength of laser radiation.

Order of performance:

1. According to Table 14, select a slit to study in a number of slit *C* - at least three (as directed by the laboratory assistant).

Table 14 - The location of the slits on the installation, row C

№ of the slit	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Slit width a, μm	8	10	12	15	20	25	30	35	40	45	50	60	70	80	90	100

2. Turn on the laser. Set the slit at a distance *L* to the screen. By adjusting the adjustment screws, to achieve the desired direction of radiation on the investigated gap in a number *C* on the test - object MOL-1. Get a clear diffraction pattern.

3. Secure a blank sheet of paper on the screen. Mark on it the distances *S* from the middle of the **central maximum** to the **middle of the minima of the first, second and third orders** to the right and the left of the central maximum (i.e., for orders $m = \pm 1, \pm 2, \pm 3$) (Figure 102). Measure distance *L*.

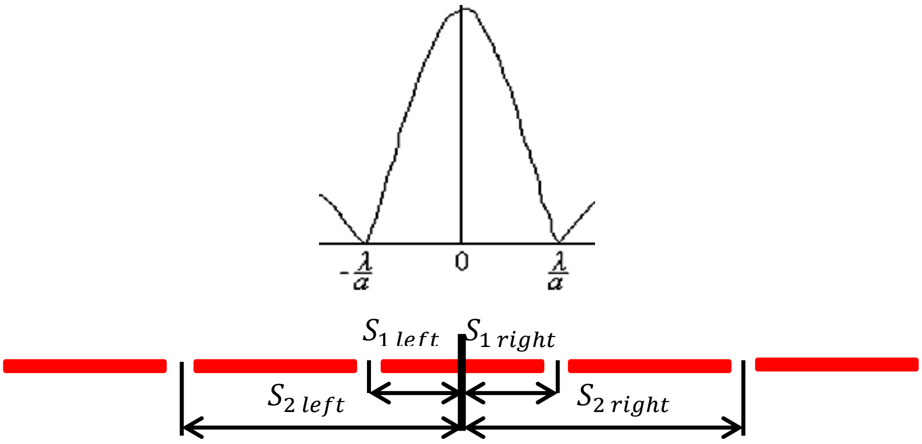


Figure 102 - Illustration of the measurement methods

4. After removing the sheet, carefully measure the marked distances with a ruler. Measurements should be recorded in Table 15. $S_{avr} = \frac{S_{left} - S_{right}}{2}$

Table 15 - Measurement results

№	m	S_{left}	S_{right}	S_{avr}	L, mm	a, µm	λ, nm
1							
2							
…							

5. Calculate $\tan\varphi$ using the formula $\tan\varphi = \frac{S_{avr}}{L}$;
6. According to the formula 150 calculate the wavelength of the laser radiation for the slit;
7. Repeat steps 2-6 for the remaining slits studied;
8. Calculate the measurement error according to the Appendix;

Questions for the self-test:
1. What waves are called coherent?
2. What are the phenomena of interference and diffraction of light?
3. Explain the diffraction patterns obtained from a single slit and the diffraction grating when illuminated with monochromatic and white light.
4. Explain the occurrence of the main maximum, the main minimum and the additional minimum during diffraction on a grating.

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2. Yavorsky B.M., Detlaf A.A. Physics course. - M.: Izdvo "Academy", 2010. - 720 p.
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Laboratory work № 11

DETERMINING THE DISTANCE BETWEEN THE SLITS IN YOUNG'S EXPERIMENT

Aim of the work: familiarization with the phenomena of light interference, understanding of the monochromaticity and spatial coherence of laser radiation on the example of Jung's experience; determination of the distance between the slits by the interference pattern.

Accessories: Laser, tripod; RMS 3 installation, MOL-1

Brief theoretical description

In 1802, Jung received interference from two slits, increasing the spatial coherence of the light incident on the slits (Figure 103). Such an increase was realized by Jung, passing the light through a small hole *S* in the opaque plate. The light passing through this hole illuminated the slots S_1 and S_2 in the opaque plate *2*. The slots S_1 and S_2 are sources of secondary coherent waves. In the region of overlap of light beams, interference phenomena are observed. A system of bright and dark stripes appears on the diffusion-reflecting screen. Thus, for the first time, Jung observed the interference of light waves and determined the lengths of these waves.

Sources of radiation, very close to monochromatic, are quantum generators of light waves - lasers. Laser radiation has a huge temporal and spatial coherence. At the laser outlet, spatial coherence is observed throughout the cross section of the light beam. This makes it possible to carry out Young's experiment with direct illumination of both slits by a complete cross section of a laser light beam. The installation diagram is shown in Figure 103. In this area of the screen, where the light beams emerging from the aperture are superimposed, an interference pattern appears in the form of alternating dark and light stripes along the vertical *y* axis.

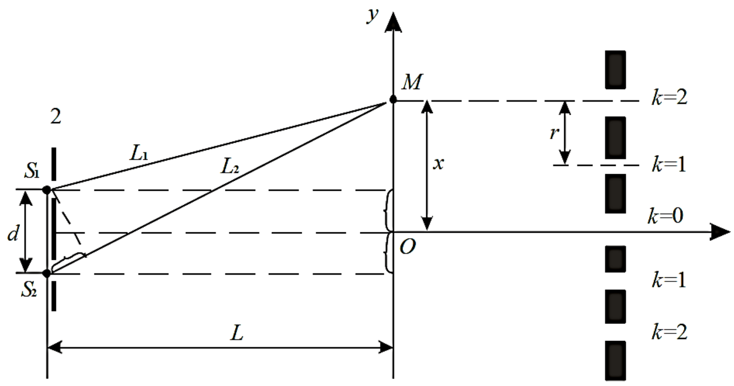


Figure 103 - Installation scheme for observing interference in Jung's experience

We calculate the distance *r* between the centers of the bright bands - the maxima of illumination. The position of the point on the screen will characterize the *x* coordinate along the horizontal axis. We will choose the origin at the point *O*, relative to which the holes S_1 and S_2 are located symmetrically at distances $d/2$ from the horizontal *x* axis. From Figure 103 we have that the optical paths of the rays from the sources S_1 and S_2 to the point *M* are equal:

$$L_1^2 = L^2 + \left(x - \frac{d}{2}\right)^2 \text{ and } L_2^2 = L^2 + \left(x + \frac{d}{2}\right)^2 \qquad (152)$$

Consequently $L_2^2 - L_1^2 = (L_1 + L_2) \cdot (L_2 - L_1) = 2xd$ Because $d \ll L$ and $x \ll L$;

$$\text{So, } L_1 + L_2 \approx 2L \text{ and } L_2 - L_1 = \frac{xd}{L} \qquad (153)$$

Multiplying the optical path difference of the two rays L_2 - L_1 by the absolute refractive index of the medium n, we obtain the optical path difference of the rays:

$$\Delta= n\frac{xd}{L} \qquad (154)$$

For an air n=1, then

$$\Delta= \frac{xd}{L} \qquad (155)$$

At the point M of the screen, the luminance maximum will be observed when an even number of half-waves or an integer number of wavelengths are laid out in the optical path difference, i.e. $\Delta= \frac{2k\lambda}{2} = k\lambda$, where λ - laser wavelength, $k = 0, \pm1, \pm2\ldots$

It means, $\frac{xd}{L} = k\lambda$ (156)

The maximum coordinate is determined by the expression

$$x_k = k\frac{L\lambda}{d}, \text{ where } k = 0, \pm1, \pm2 \qquad (157)$$

Hence we find the distance r between the centers of the peaks:

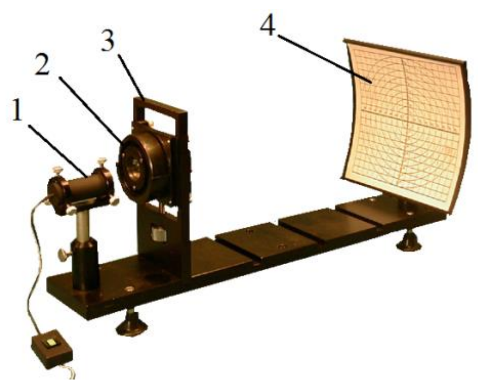
$$r = x_k - x_{k-1} = \frac{L\lambda}{d} \qquad (158)$$

If you measure the distance between the centers of the maxima r, the distance between the plate and the screen L and the distance between the centers of the holes d, then you can determine the laser wavelength:

$$\lambda = \frac{rd}{L} \qquad (159)$$

Description of the laboratory installation

To observe Jung's experience using an RMS 3 setup consisting of an optical bench on which a semiconductor laser is mounted a vertical alignment module with a photolithographic MOL-1 test object, a screen (Figure 104).



1 - semiconductor laser (GaAs); 2 - photolithographic test object MOL-1; 3 - vertical adjustment module; 4 - screen;

Figure 104 - RMS 3 device structure

The direction of the laser radiation is adjusted by the adjustment screws. The object MOL-1 is a thin glass disc with an opaque coating and transparent structures arranged in three rows: row A - double slits, row B - round holes, row C - single slits (Figure 105).

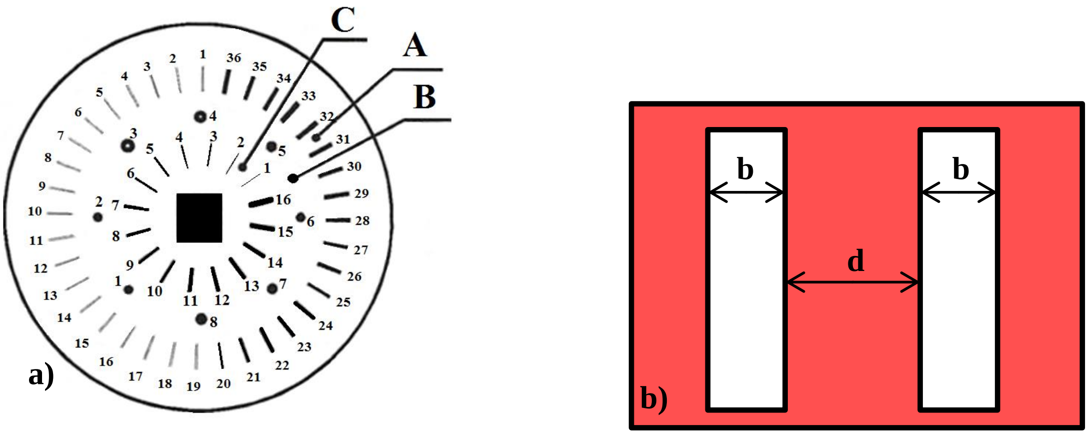


Figure 105 - Layout of the structures of the MOL-1 a), A-type slits construction b)

The number of double strokes in row *A* is 36, the exact values of the distances between the double slits in row A and the width of these slots can be found in Table 11.1. The radiation from the laser is directed to the desired structure on the surface of the MOL-1 object. The light, interfering on a pair of slits, falls on the screen, on which the measurements of the width of the interference fringe Δx are taken.

Table 16 - Number of Double slits of row *A* with the distance between the slits *d* and the width of the slits *b*

№	1	2	3	4	5	6	7	8	9	10	11	12
d, µm	10	15	25	40	25	45	60	75	40	60	80	100
b, µm	5	5	5	5	10	10	10	10	15	15	15	15
№	13	14	15	16	17	18	19	20	21	22	23	24
d, µm	45	70	95	120	60	85	110	135	75	95	120	140
b, µm	20	20	20	20	25	25	25	25	30	30	30	30
№	25	26	27	28	29	30	31	32	33	34	35	36
d, µm	90	125	160	190	75	125	175	225	100	150	200	250
b, µm	40	40	40	40	50	50	50	50	75	75	75	75

Order of performance:

1. Turn on the laser. Adjusting by the screws to achieve the desired direction of radiation to obtain a clear image of interference fringes.
2. Select the double slits to be investigated on the MOL-1 facility in row A (at least three).
3. Set the object MOL-1 at a distance *L* to the screen.
4. Set the screen with a blank sheet of paper. **Mark the midpoints of the observed light bands along the horizontal axis** (Figure 106). For each double slit **take five measurements**.

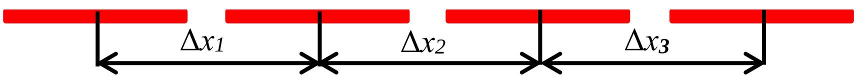


Figure 106 - Illustration for the measuring of the interference strips

5. After removing a sheet of paper, measure with a ruler the **distance between the centers of the light stripes** - the interference bandwidth Δx.
6. Measure the distance *L* from the slits to the screen.

7. Knowing the value of L and the wavelength of the laser radiation (λ = 650 nm), calculate the distance between the slits for each pair using the formula $d = \frac{\lambda L}{\Delta x}$

8. The results obtained are written down in table 17.

Table 17 - Filling the experimental data

№ of a pair of slits							
Δx, mm							
d, μm							

9. Calculate the distance error d between the slits for each of the pairs.

Questions for the self-test:
1. What methods can be used to obtain interference?
2. What is the spatial and temporal coherence of waves? Define of time, length and radius of coherence.
3. What are the properties of laser radiation?
4. How to carry out Young's experience using a conventional incandescent bulb and using a laser?

References

1. Trofimova T.I. The course of general physics: studies. Manual for universities 14th ed., Sr.- M.: Akademiya Publishing Center, 2007, §187. P. 352-354.
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Laboratory work № 12

STUDY OF THE MALUS LAW AND POLARIZED LIGHT PROPAGATION THROUGH THE PHASE PLATE

Aim of the work: To analyze polarized light transmitted through the phase plate, check the of the Malus law implementation

Accessories: Laser, analyzer, phase plate, photodetector

Brief theoretical description

As is known, electromagnetic light waves are transverse: the electric vector $\vec{E}$ (light vector) and the magnetic vector $\vec{H}$ (or $\vec{B}$) are mutually perpendicular and are located in a plane perpendicular to the velocity vector $\vec{v}$. At each point in space, the orientation of a pair of vectors $\vec{E}$ and $\vec{H}$ in the plane perpendicular to the ray (vector $\vec{v}$) in the general case may change with time. Light in which the direction of oscillations of the vector $\vec{E}$ is ordered in some way is called polarized. If the oscillations of the vector $\vec{E}$ are performed only in one plane passing through the beam, then the light is called linearly or flatly polarized. The plane in which the vector $\vec{E}$, oscillates is called the polarization plane (Figure 107).

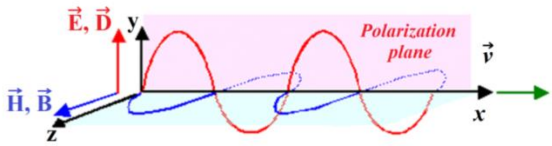


Figure 107 - Illustration of the electromagnetic wave in term of electromagnetic and magnetic vectors oscillations

Common light sources are a collection of a huge number of rapidly illuminating (10^{-10} - 10^{-8} s) elementary sources (atoms and molecules) emitting light independently of each other with different initial phases and with different orientations of the $\vec{E}$ and $\vec{H}$ vectors. Therefore, the orientations of the vectors $\vec{E}$ and $\vec{H}$ in the resulting wave randomly vary in time and a plane perpendicular to the beam, all directions are on average equiprobable. This light is called natural or unpolarized. (Figure 108)

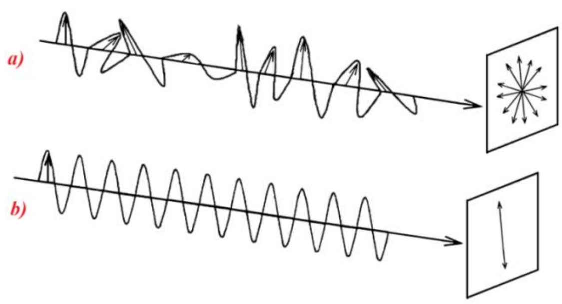


Figure 108 - Directions of oscillations of the vector $\vec{E}$ in the natural (a) and plane polarized (b) waves

Elliptical light polarization.

The most common type of polarization is elliptical polarization. In an elliptically polarized wave, the end of the vector $\vec{E}$ describes some ellipse. Linearly polarized light can be considered as a special case of elliptically polarized light, when the polarization ellipse degenerates into a straight line segment; another particular case is circular polarization (polarization ellipse - circle). A wave with elliptical polarization can always be decomposed into two linearly polarized waves with mutually orthogonal polarization planes (Figure 109).

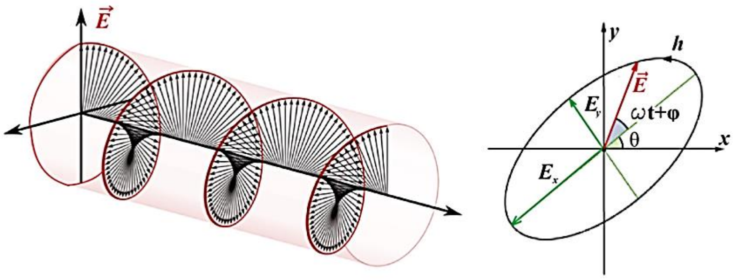


Figure 109 - Electromagnetic wave with elliptical polarization

The phase difference φ of these two waves is kept constant in time, i.e. these waves are coherent. Vectors $\overrightarrow{E_x}$ and $\overrightarrow{E_y}$, describing linearly polarized waves can be written in the form:

$$\overrightarrow{E_x} = \overrightarrow{A_1} \cos \omega t \text{ and } \overrightarrow{E_y} = \overrightarrow{A_2} \cos(\omega t + \varphi) \quad (160)$$

, where: $\overrightarrow{A_1}$, $\overrightarrow{A_2}$, ω and φ don't depend on time.
From natural light, plane-polarized light can be obtained with devices called polarizers. These devices freely transmit vibrations of the light vector $\vec{E}$, the plane, which is called the transmission plane of the polarizer. Oscillations, perpendicular to this plane, completely linger.

Double refraction

Natural light can be polarized in various ways. Polarization of light is observed upon reflection, refraction and the passage of light through anisotropic substances. Many transparent crystalline dielectrics are optically anisotropic, which leads to the appearance of a phenomenon called birefringence. It lies in the fact that the beam of light incident on the crystal is divided inside the crystal into two beams. This phenomenon can be observed even with the normal incidence of the beam on the face of the crystal. (Figure 110).

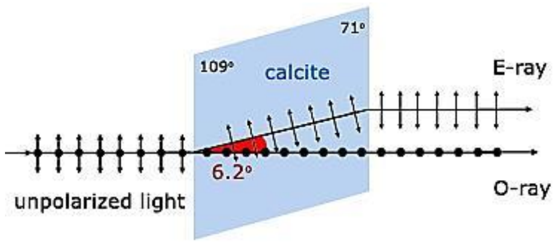


Figure 110 - E-ray and O- ray appearance

One of the rays obeys the usual laws of refraction, its refractive index doesn't depend on the angle of incidence, and the speed of propagation of this beam inside the crystal in all directions is the same. Such a ray is called ordinary (Figure 111, ray o) and denoted by the letter o. The ordinary beam obeys the usual law of refraction.

Another beam, called extraordinary (Figure 111, beam e), doesn't obey the usual law of refraction, its refractive index n_e depends on the direction of the beam in the crystal. Consequently, the speed of propagation of this beam is different in different directions.

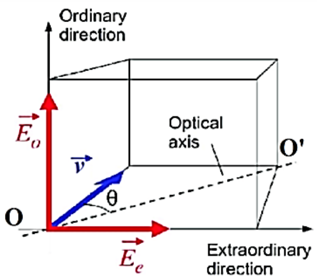


Figure 111 - Ordinary and extraordinary directions in uniaxial birefringent crystals

In the so-called uniaxial crystals, there is a direction - the optical axis OO', along which the speed of propagation of ordinary and extraordinary rays is the same. Along the optical axis, ordinary and extraordinary rays propagate, not separating spatially. The plane containing the optical axis of the crystal and the beam is called the main cross section or the main plane of the crystal to the beam. Both rays, ordinary and extraordinary, are linearly polarized in mutually perpendicular planes. An ordinary ray is polarized in the plane perpendicular to the main section, and the extraordinary one is in the plane of the main section. The directions of oscillations of the light vectors $\vec{E}$ in both beams are shown in Figure 111, where it is assumed that both beams and the optical axis OO' lie in the plane of the figure. In practice, linearly polarized light can be obtained by removing one of the rays produced by double refraction. Some crystals (for example, tourmaline) unequally absorb rays polarized in mutually perpendicular planes. This phenomenon is called dichroism and is used to produce polarized light using so-called polaroids. They are a thin film that linearly polarizes the light passing through it.

Obtaining elliptically polarized light

Elliptically polarized light is obtained from linearly polarized using birefringent plates. Waveplates, also known as retarders, transmit light and modify its polarization state without attenuating, deviating, or displacing the beam. They do this by retarding (or delaying) one component of polarization to its orthogonal component.

As already noted, when light propagates in a uniaxial crystal perpendicular to the optical axis, there will be no spatial separation of the ordinary and extraordinary rays.

For instance, normally linearly polarized light incident into a crystal plate, cut out parallel to the optical axis OO' (Figure 112a) the plane of polarization of which makes an angle φ with the optical axis of the plate. In this case, in a crystal in the same direction (perpendicular to the optical axis), two waves with different velocities ($v_0 = c/n_o$, $v_e = c/n_e$) polarized mutually orthogonally, will propagate. This is shown in Figure 112b, where light propagates perpendicular to the plane of the figure.

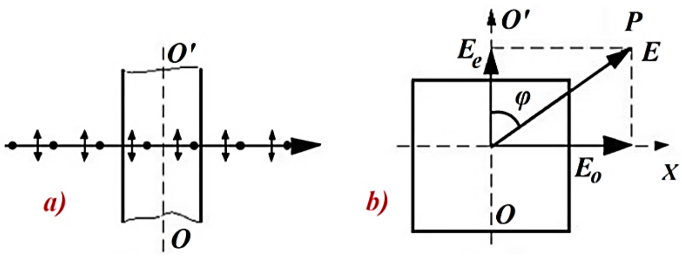


Figure 112 - Passage of a light wave through a crystal with division into ordinary and extraordinary rays

In this figure, P is the plane of polarization of the light incident on the plate, $\vec{E}$ is its amplitude vector, OO' is the optical axis of the crystal, $\vec{E_o}$ and $\vec{E_e}$ are the amplitude vectors of ordinary and extraordinary waves in the crystal plate. We decompose the vector $\vec{E}$ into components $\vec{E_o}$ and $\vec{E_e}$.

At the plate entrance $\overrightarrow{E_o}$ and $\overrightarrow{E_e}$ are in phase. At the exit of the plate with a thickness d between both waves, a phase shift φ appears which depends on the optical path difference Δ of these waves, equal to:

$$\Delta = d(n_o - n_e) \tag{161}$$

Considering that $\varphi = 2\pi/\lambda_0$, where λ_0 is the wavelength for a vacuum, we get:

$$\varphi = 2\pi \frac{d(n_o - n_e)}{\lambda_0} \tag{162}$$

Thus, two mutually orthogonal plane polarized waves emerge from the crystal plate, one polarized perpendicular to the main cross section of the crystal, the other in the plane of this section (Figure 112b). This means that at an arbitrary point behind the plate the corresponding oscillations of the light vector will have the form:

$$E_{0X} = E_0 \cos \omega t \quad \text{and} \quad E_{0Y} = E_0 \cos(\omega t + \varphi) \tag{163}$$

In general, this is the equation of the ellipse in parametric form. Therefore, the light released from the crystal plate is elliptically polarized. It was agreed that if, when observing towards a wave, the rotation of the vector $\vec{E}$ occurs clockwise, then such a wave is called right polarized, if counterclockwise, then left-polarized. Consider the practically important special cases (Figure 113).

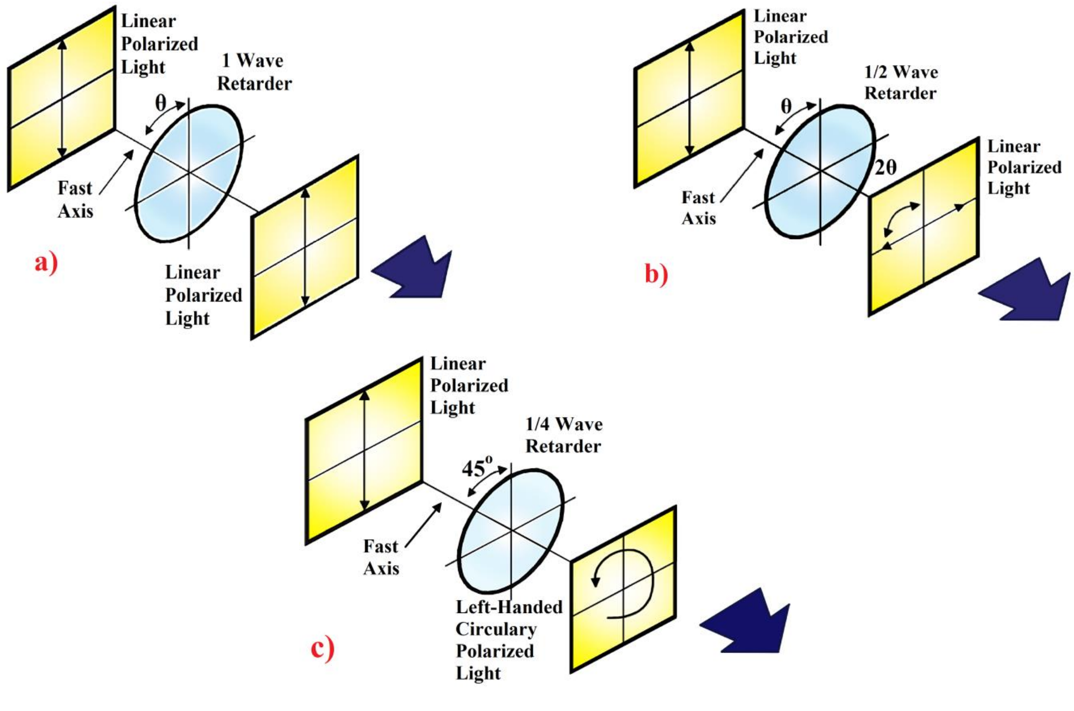


Figure 113 - Particular cases of light passing through a phase plate of different phase shifts a) phase shift *2π*, b) phase shift *π*, c) phase shift *π/2*

a) The plate gives a phase shift of *2π* (plate in wavelength). As a result of the superposition of waves at the plate exit, a linearly polarized wave is formed with the same direction of oscillations of the vector $\vec{E}$ as in the incident wave.

b) The plate gives the phase shift π (plate in half the wavelength). Its thickness satisfies the condition:

$$d|n_o - n_e| = \frac{m\lambda}{2} \quad (164)$$

where: m = 1,3,5..., i.e. for odd values of m. At the plate exit, linearly polarized light is again formed. However, the directions of oscillations of the vector $\vec{E}$ (the plane of polarization) rotate through an angle of 2φ symmetrically to the main section of the plastic (Figure 114). At $\varphi = 45^0$, such a plate "rotates the polarization plane by 90^0, that is, the plane of polarization of the light passing through the plate will be orthogonal to the plane of polarization of the incident light.

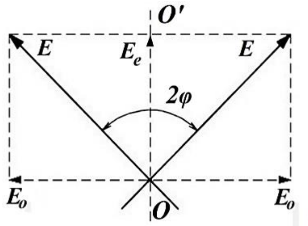


Figure 114 - Vector direction $\vec{E}$ plate section

c) The plate gives the phase shift $\pi/2$ (plastic in a quarter wavelength). Its thickness satisfies the condition:

$$d|n_o - n_e| = \frac{m\lambda}{4} \quad (165)$$

where m = 1,3,5... When adding two mutually perpendicular oscillations with a phase difference, an ellipse is formed. In accordance with formula (165), the ellipse will be reduced to the X and Y axes (Figure 115, where the OO' axis is the optical axis).

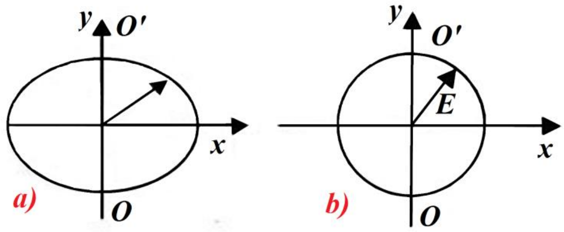
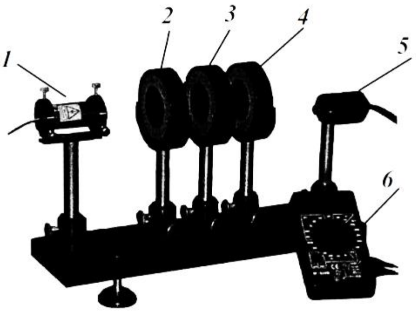


Figure 115 - Transformation of elliptical polarization into circular polarization

If linearly polarized light falls on a quarter-wave plate so that the angle between its plane of polarization P and the optical axis of the plate $\varphi = 45^0$ (Figure 112b), the amplitudes of the ordinary and extraordinary waves are the same and circular light polarized ellipse appears at the exit of the plate - degenerates into a circle (Figure 115). Each birefringent plate has two mutually perpendicular main directions: one parallel to the optical axis of the plate, the other perpendicular to it. Waves linearly polarized along the main directions propagate in a plate with different speeds, without changing the nature of their polarization.

Description of the laboratory installation

Installation diagram is shown in Figure 115. When studying the Malus law, the installation includes a laser 1, a polarizer 2, an analyzer 3, a photodiode 5 and a micro ammeter 6. The light emitted by the semiconductor laser 1 and passed through the polarizer 2 is plane-polarized, its intensity corresponds to the designation I_0 in the formula for Malus' law. The angle φ is changed by the rotation of the analyzer 4. Light passing through the analyzer and having an intensity I falls on the photodetector 5. The current of the photodiode *i*, which is proportional to the intensity of light *I*, is recorded by a micro ammeter.



1- Laser, 2 - Polarizer, 3 - Wave retarder (Phase plate), 4 - Analyzer, 5 - Photodiode, 6 - Micro ammeter

Figure 116 - Installation diagram for the study of the Malus law and the transit of light through the phase plate

When studying the transit of light through the birefringent (phase) plate 3, the latter is placed between polarizer 2 and analyzer 4

Order of performance:

I) Study of the Malus law

1. Assemble the installation consisting of a laser, a polarizer (labeled *P*), an analyzer (labeled *P*), a photodetector and a micro ammeter. Turn on the laser (λ = 650 nm) and wait until it heats up for about 10 minutes.
2. Rotating the analyzer, set it to the position corresponding to $\alpha = 0^o$, relative to the risks. In this case, the intensity of the light falling on the photodiode will be maximum.
3. Then, turning the analyzer through 15^o, record the photocurrent values in table 18. Measurements should be made up to 360^o 3 times.

Table 18 - Table for calculation for the Malus law checking

№	$\alpha, ^o$	I_1	I_2	I_3	$<I>$	$cos^2\alpha$
1	0					
	15					
	…					
	360					

4. Build a graph of the dependence of photocurrent force *i*, proportional to the intensity *I*, released from the light analyzer, on the angle of rotation α and on the square cos α
5. Make the appropriate conclusion.
6. Calculate the error of intensity measurement according to the application.

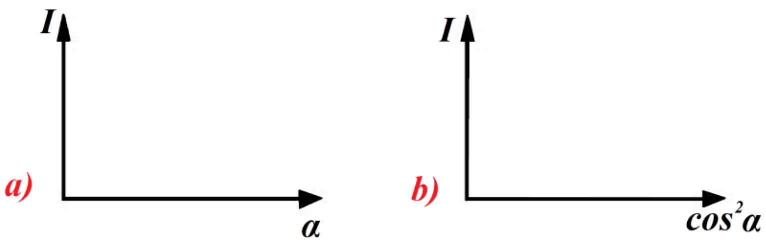


Figure 117 - Plots of intensity *I*, on the angle of rotation α a) and on the square of the cosα b)

II) Working with the phase (birefringent) plate.

1. Rotating the analyzer, set it to the position corresponding to α = 90°. At the same time there should be an almost complete extinguishing of the light falling on the photodetector.

2. Install a phase plate between the polarizer and the analyzer (labeled *C*).

3. Rotating the plate, make sure that there are four of its positions, in which the complete extinction of the light will again be observed. **These positions correspond to the orientation of one of the main directions of the phase plate perpendicular to the transmission plane of the analyzer.**

4. Selecting any of these positions, rotate the plate by 45° and fix it in this position. In this case, the plane of polarization of the incident beam is oriented at an angle of 45° to the main directions of the plate and, thus, the amplitudes of the light vectors of the ordinary and extraordinary rays are the same.

5. Rotating the analyzer, take readings from the microammeter and fill in table 19

Table 19 - Table for calculation for the Malus law checking

№	$\alpha,^o$	I_1	I_2	I_3	$\langle I \rangle$
1	0				
	15				
	…				
	360				

6. Build a graph $I = f(\alpha)$.

7. Find the minimum I_{min} and I_{max} maximum intensity values

8. When the condition of main direction orientation of the plate is fulfilled at an angle of 45° to the plane of polarization of the incident light, determine the phase shift using the formula: $\tan\frac{\varphi}{2} = \frac{a}{b} = \sqrt{\frac{I_{min}}{I_{max}}}$

9. With a known plate thickness d (0.5 mm), calculate the difference between the refractive indices of ordinary and extraordinary rays using the formulas: $\Delta = \frac{\varphi\lambda}{2\pi}$ and $n_o - n_e = \frac{\Delta}{d}$

10. Calculate the error of the measured intensities according to the application.

Questions for the self-test:

1. What light is called natural, linearly polarized and elliptically polarized?
2. What is the phenomenon of birefringence?
3. What properties have ordinary and extraordinary rays?
4. What is the optical axis? Which planes in a uniaxial crystal are called principal?
5. What is dichroism?
6. What are the properties of the birefringent (phase) plate in half the wavelength? A quarter wavelength?

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1. Landsberg G.S. Optics. - Ed. 6th - M.: Fizmatlit, 2006.
2. Sivukhin D.V. General course of physics. - T. 4. Optics. - Ed. 3rd M.: Fizmatlit, 2005. - § 39, 40.
3. Saveliev I.V. The course of general physics. T. 2. - Ed. 10th - M.: Fizmatlit, 2008. - § 126, 127.

Laboratory work № 13

MEASUREMENT OF THE WEDGE ANGLE OF THE INTERFERENCE PATTERN OF THE EQUAL THICKNESS SLOPES

Aim of the work: measure the angle of the air wedge in the gap between the glass plates by the interference equal thickness slopes.

Accessories: basement with vertical with adjustment module, semiconductor laser, a collimator (beam expander), interference object in the frame, screen

Brief theoretical description

Consider the normal incidence of a monochromatic wave on the surface of an air wedge formed by almost parallel glass plates optical path difference of the interfering rays O and O' that takes place when observing the interference of monochromatic light with a wavelength λ passing through a thin air gap between two plane-parallel plates (Figure 118) is:

$$\Delta s = (AD + DC) - n \cdot BC + \lambda \tag{166}$$

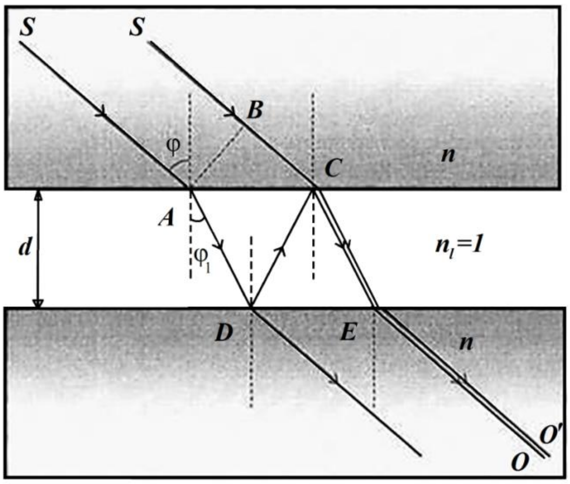


Figure 118 - Illustration of the air gap between the two plates

where *d* is the thickness of the gap, *n* is the refractive index of the plates, φ is an incidence angle of the rays on the glass-air interface, φ_1 is the angle of refraction. The additional path difference is due to reflections from points *C* and *D* which caused due to optically denser medium reflection (where φ_1 is smaller than Brewster angle, at each mentioned reflection, we have shift by λ/2, due to a change in the wave phase by π).

$$AD = DC = \frac{d}{\cos\varphi_l} \tag{167}$$

$$BC = AC \cdot \sin\varphi = 2d\tan\varphi_l \cdot \sin\varphi \tag{168}$$

Considering the Snell's law ($n\sin\varphi = n_l\sin\varphi_l$), we obtain

$$\Delta s = 2d\cos\varphi_l + \lambda \tag{169}$$

Maxima and minima conditions for the interference pattern is formed by coherent waves which are reflected from all surfaces in the gap

$$2d\sqrt{1-\mathrm{n}\sin^2\varphi} = k\frac{\lambda}{2} \quad (170)$$

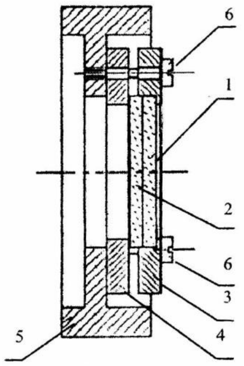
where $k = 2m$, and m is an integer for minima, and for maxima - $k = 2m + 1$.

When, the gap thickness d differs in various places within the monochromatic light beam width, then, dark and light interference strips are seen on the plate surface in the transmitted light and they are called slopes of equal thickness, since each of the lines passes through points with the same thickness. In white light, we can see a system of colored interference strips.

The equal thickness slopes are parallel to the edge of the wedge at interference on a transparent wedge. The interference slopes width B, namely, the distance between two minima or maxima if incidence angles that are close to zero ($\varphi \approx 0$) is in the form:

$$B = \frac{\lambda}{2\alpha} \quad (171)$$

Where α - wedge tip angle (α << 1 rad). The device of the interference object is shown in Figure 119.



1, 2 - glass plates, 3,4 - mandrels, 5 - frame, 6 - screws

Figure 119 - Interference Object Scheme

We can see, that there are two glass plates 1 and 2 on the object. The plates are pressed mutually using mandrels 3 and 4. Reflecting translucent coatings are deposited on the contacting surfaces of the plates, which increases the contrast of the observed interference pattern. The mandrels are pressed by three screws 6 to the frame 5. An air wedge takes place if the mandrels are pressed unevenly with respect to each other, so we should have 2 screws loosened.

Figure 120 presents the RMS 2 optical scheme. The semiconductor laser 1 emits rays, which are expanded using a micro-lens 2 that is fixed on the screen magnetic frame with a hole 3 and therefore the interference object 4 is illuminated. On screen 5 we can see the interference pattern that is remoted from the object to 430 mm. So, the angular divergence of the interfering rays is given 3-4° for the slopes in the screen central zone that is counted 20-30 mm in size. It makes it possible to neglect it and using the model representations.

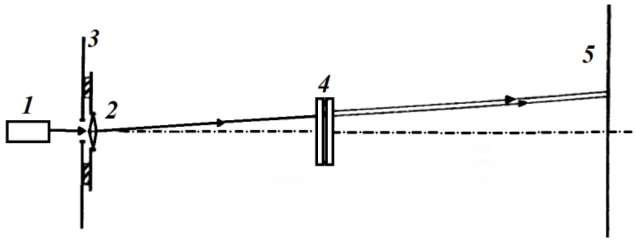
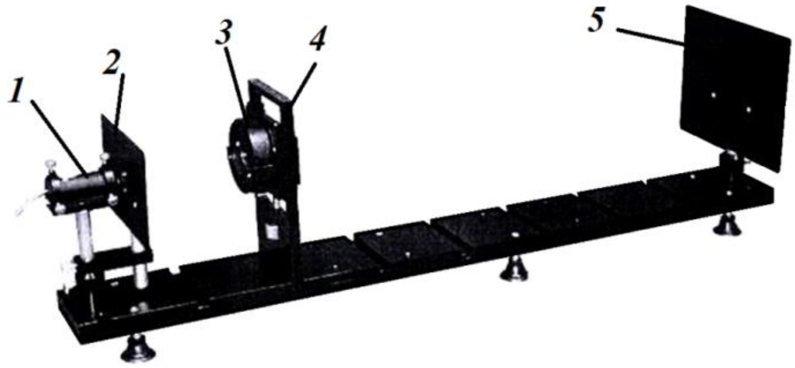


Figure 120 - Optical scheme of ray path to obtain the interference

The interference slopes width B' (mm) is measured in using the scale grid.

Description of the laboratory installation

The appearance of the RMS 2 “Interference” in the assembly is presented in Figure 121. In case of using the installation to observe equal thickness slopes. The radiation from the semiconductor laser 1 falls on the collimator mounted on the screen 2. The advanced beam illuminates the working surface of the interference object 4 mounted on the holder in the vertical module 4. The object is arranged so that there is a wedge-shaped air gap between two plane-parallel plates. Repeatedly reflected from the plates, the rays interfere.



1 - semiconductor laser, 2 collimator, 3 - vertical module holder, 4 - interference object, 5 - screen

Figure 121 - Installation RMS 2

On screen 4, a picture of alternating dark and light stripes will be observed. For measurements, a scaled paper screen is fixed on the screen. By measuring the width and period of the bands, you can calculate the angle of the wedge-shaped gap.

Order of performance

1. Turn on the semiconductor laser, then rotate the adjustment screws to direct the laser beam to the center of the hole.

2. Put a micro objective to the magnetic frame and move it in different directions achieving the best illumination of the chosen object.

3. Turn the screws 6 (see Figure 119) to adjust the gap thickness between the glass plastics.

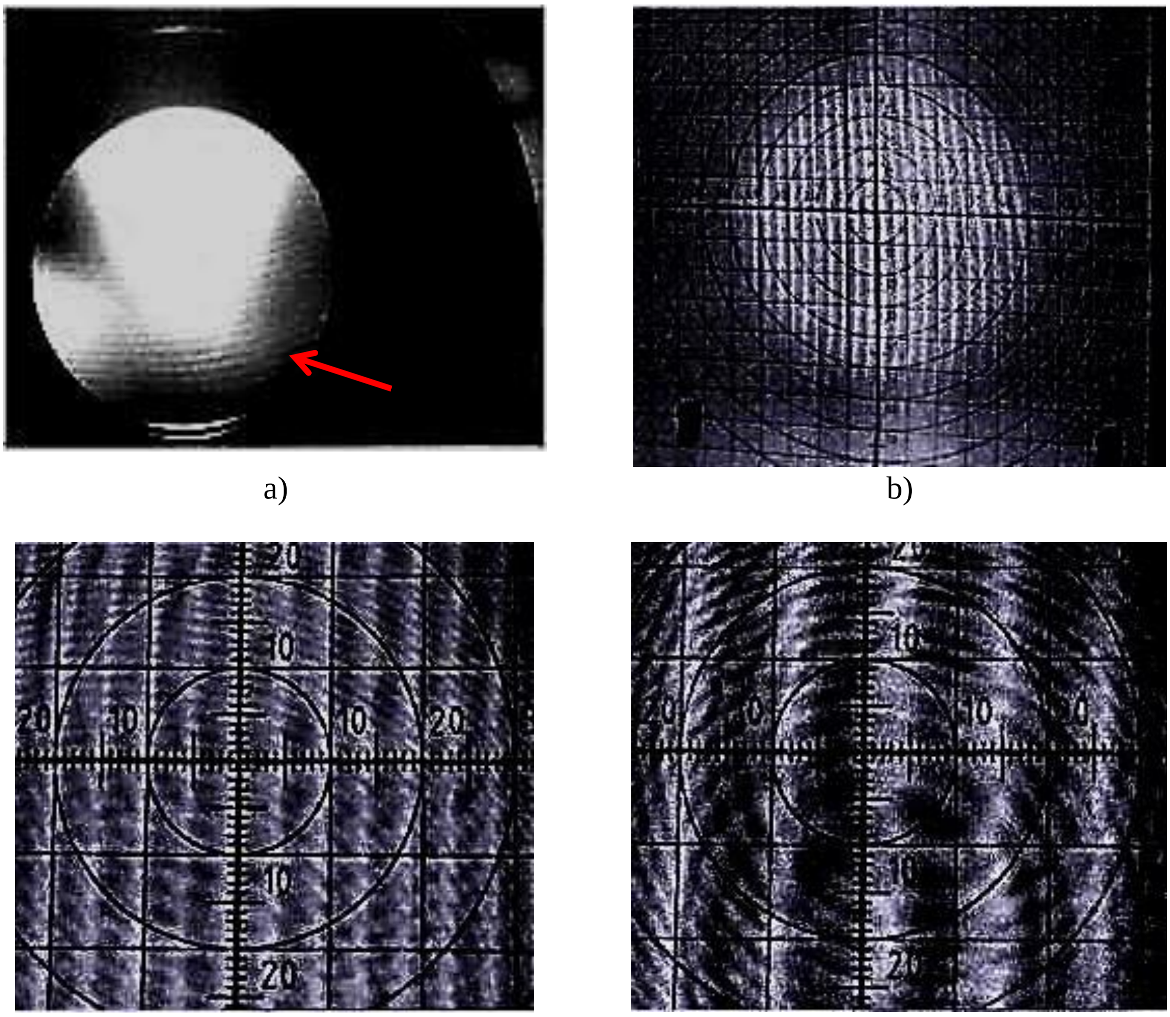
NOTE! It is strictly forbidden to tighten the screws, because it could lead to damaging of the plates. Rotate the screw smoothly, without extra press when there is the final position. To have a wedge-shaped gap you should loose screws 1 or 2.

Figure 122a shows the possibility of observing the interference pattern in the reflected light at 45-60° angles or transmitted light. To obtain straight lines (See Figure 122b-d) the precise adjustment should be done for the laser beam.
The frame with the object should be turned the around the optical axis to set the stripes along the lines of the scale grid.

4. You should measure the interference slopes maxima coordinates at least three times for the adjacent orders. The accuracy must be ± 1 mm. Write down the obtained results in Table 20.

5. For each pair of slopes, calculate the period of interference slopes using the formula according to figure 123 $B'_{ij} = \frac{x_i - x_j}{i-j}$

6. Averaging results. The obtained average value is used to calculate the angle at the top of the air wedge by the formulas (170) and (171): $\alpha = \frac{\lambda}{2B'}$



a) b)

c) d)

Figure 122 - Examples of visually observable interference fringes (a- directly on the object, b, c and d - on the screen for different wedge angles)

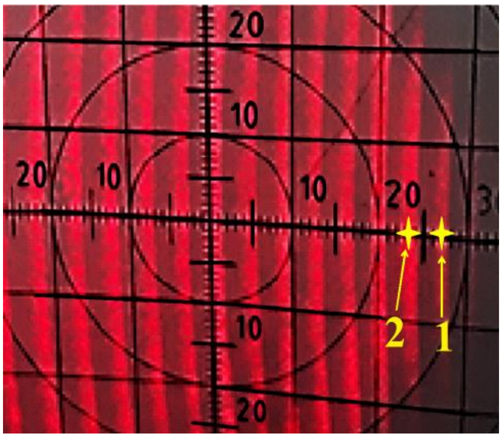


Figure 123 - An example of the measuring of the interference slopes

According to Figure 123, x_i = 27 mm, x_j - 23 mm - coordinates along the x axis of the middle of interference fringes, i, j is the sequence number of the interference fringe.

*The distance from the object to the main screen is L = 430 mm, the distance between adjacent grooves at the base is 100 mm.

Table 20 - Experimental data

№	Linear coordinates of the slopes				
	i	x_i	j	x_j	B'_{ij}
1					
2					
3					
4					
5					
6					

7. Calculate the error of the period of interference fringes.

Questions for the self-test:
1. What is the aim of this laboratory work?
2. What is the phenomenon of interference?
3. Which sources are coherent?
4. What method of obtaining coherent beams used in this work?
5. At what path difference the minimum illumination of the interference pattern is observed? Write down the most common formula.
6. At what path difference the maximum illumination of the interference pattern is observed?
7. What is the method for determining the angle of the air wedge in this laboratory work?

References:
1. Trofimova T. I. The course of physics. - M.: Higher School, -2012.-544 p.
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Laboratory work № 14

STUDY OF THE DISPERSION OF THE OPTICAL GLASS

Aim of the work: Determination of the refractive index of optical glass for different wavelengths

Accessories: Light source, prism, optical branch

Brief theoretical description

Dependence of the refractive index on the wavelength/frequency of electromagnetic light is usually called dispersion. This phenomenon is explained by the different phase velocity of propagation in the substance of different light wavelengths. The refractive index of a substance is the ratio of the phase velocity of light in a vacuum with its speed in a given medium $n = \frac{c}{v}$. If the speed of light in the medium depends on the wavelength, then the refractive index of the medium should depend on the wavelength. The dispersion of a substance is the ratio $\frac{dn}{d\lambda}$, where λ is the wavelength of light in a vacuum. Almost all transparent media have a dispersion, except for vacuum, where the propagation speed of all electromagnetic waves of any length is the same.

Any method that is used to determine the refractive index (refraction in prisms, total internal reflection, interference methods) can be used to detect the phenomenon of dispersion.

In this paper, the measurement of refractive indices is made for optical glass having the shape of a prism. **The decomposition of white light into the spectrum during its passage through a prism is caused by the phenomenon of dispersion.** Light of different wavelengths (different colors) is unequally refracted at the border of two transparent media, since $n = f(\lambda)$.

For an optical prism, there is a relationship between the angle of deflection of the prism from the initial direction δ and the refractive index of prism glass *n*, the refracting angle of the prism A, and the angle of incidence of the rays α. Using this relationship, you can determine the refractive indices of the substance of the prism. This method is used in the work.

At a certain angle of incidence of the rays on the prism, the angle of deflection by the prism δ takes the smallest meaning and is called the angle of the smallest deviation δ_{min}. In this case, the angle of incidence of the rays on the prism α (Figure 124) is equal to the angle of their exit from the prism, that is, the beam in the prism runs parallel to the base. Let us establish for this case the relationship n, A and δ_{min}.

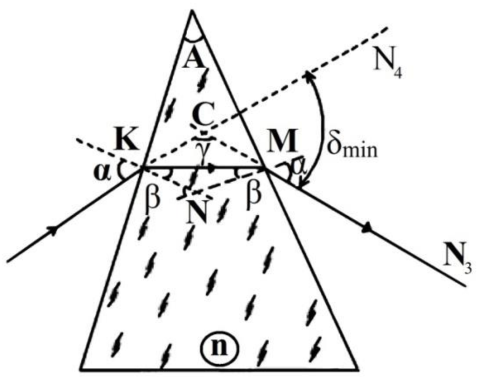


Figure 124 - For the definition of the refractive index

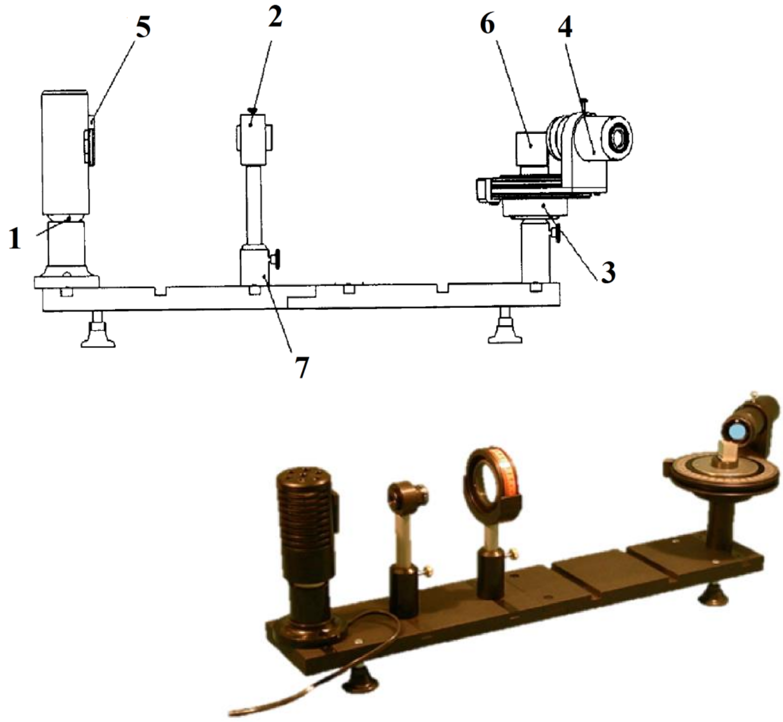
Let us write the law of light refraction for the input face of the prism $n = \frac{\sin\alpha}{\sin\beta}$. From figure 124 it follows that $= A/2$, $\delta_{min} = 180^0 - \gamma$, $\gamma = 360^0 - 2\alpha - (180^0 - A)$ from quadrangle

NKCM, $\delta_{min} = 180^0 - \gamma = 2\alpha - A$. Here $\alpha = \frac{\delta_{min}+A}{2}$ putting values β and α in the law of refraction, we get $n = \frac{\sin\frac{\delta_{min}+A}{2}}{\sin\frac{A}{2}}$.

From the formula it can be seen that in the work the angles A and δ_{min} should be measured for different wavelengths and then the refractive index values are calculated.

Description of the laboratory installation

The installation (Figure 125) is mounted on two component bases on which are fixed: the radiation source - a mercury lamp in housing 1, a collimator 2 of the MGT 2.5*17.5 type on the stand and a goniometric Table with a viewing tube 4 fixed on its alidade. On the lamp case, there is a slot in which the slot is installed.



1- mercury lamp in the housing,
2- MGT type collimator 2.5 * 17.5,
3- goniometric table,
4- telescope
5 output window
6- object under study
7-adjustable screw

Figure 125 - Diagram of the laboratory setup

The investigated object 6 (prism) is fixed in a frame with glued-in magnets and is mounted on the base of the goniometric table. Counting angles of rotation of the table are made on an angular scale with a vernier counting. The radiation from the mercury lamp, filling the slit, is converted by the collimator into a parallel beam, which is directed to a prism mounted on the goniometer table. The deflected radiation is observed visually with the help of a telescope focused on "infinity", which allows you to restore the image of the slit. The angle of radiation deviation is measured on the reading scale of the table. Counting the whole degrees to produce on a scale limb against zero verniers. To these data one should add the number of tenths taken on the vernier scale - the first division of the vernier, coinciding with any division of the limb scale.

The emission spectrum of the lamp contains lines characteristic of mercury vapor. The spectrum consists of the following wavelengths: bright red 631.0 nm; two yellow - 576.9 nm and 579.2 nm; green - 546.0 nm and; blue - 491.6 nm; blue - 435.8 nm; two violet - 407.7 nm and 404.7 nm (not all lines can be visually observed). Not all lines can be visually observed.

Order of performance

1. Turn on the light source, turn the goniometer alidade so that the optical axis of the

telescope coincides with the axis of the collimator. In this case, an image of the entrance slit of the collimator will appear in the field of view of the eyepiece.

2. Check and, if necessary, focus the collimator and telescope in the following sequence:

3. Focus on the optical stand using the autocollimator tube to "infinity". If there is no autocollimator, you can visually focus the pipe on a distant object in the corridor or outside the window.

4. Install the goniometer alidade coaxially with the optical axis of the collimator. Rotate the focusing movement of the collimator to achieve a sharp image of the slit.

5. Place the object to be examined on the sample stage and check for diffracted or deflected radiation.

6. Determine the refracting angle A of the prism (in this work, an AP-90 prism is used, in which two faces are chosen as workers at an angle of 45 °, as shown in Figure 126).

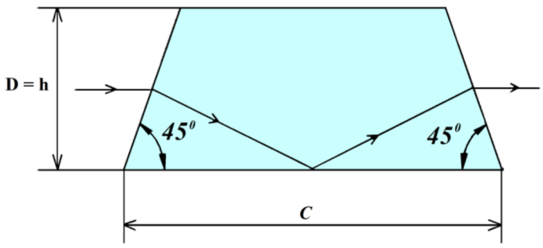


Figure 126 - Diagram of prism AR-90

Place the prism on the object table so that the bisector of the refracting angle of the prism approximately coincides with the axis of the illuminated collimator. In this case, the side faces of the prism work like a mirror.

7. First, with the naked eye, and then with the help of an eyepiece to catch the image of the entrance slit of the illuminated collimator in the direction of the rays reflected from the lateral faces of the prism. Turning the eyepiece, combine its thread with the image of the slit, first to the right of the optical axis of the collimator, and then to the left. At the same time, take readings on the limb (Figure 127) and the goniometer vernier (N_1 and N_2). With this prism position, the required angle A is equal to: $\angle A = \frac{N_1 - N_2}{2}$. If, when moving from the position to the right to the position to the left of the optical axis of the collimator, the eyepiece passes through the limb zero, then $\angle A = \frac{360^0 - (N_1 - N_2)}{2}$.

8. To determine the refractive angle of the prism at least 3 times and find the average value.

9. Measure the angles of least deviation for different wavelengths of the spectrum of the lamp. First you need to see the eyepiece line spectrum of the lamp. To do this, the installation elements must be installed in the following order: place the prism on the stage as shown in Figure 14.2 (the collimator objective and the eyepiece form an angle of approximately 21-25^0). Slightly turning the table with a prism and the eyepiece near this position, you need to achieve a clear image of the lines of the spectrum. Then you should turn the table with the prism in one direction and follow the movement of the spectral lines.

At a certain angle of incidence on the prism, the observed spectral line stops in sight of the eyepiece, and then begins to move in the opposite direction. The position of the spectral line at the moment of stopping corresponds to the angle of the smallest deviation of the beam δ_{min}. Align the reference filament of the eyepiece with the line of the spectrum in the minimum deviation position, remove the N_3 count, over the limb and the Vernier.

10. Next, to measure the angular coordinate of the rays, you need to remove the prism from the table and align the eyepiece with the optical axis of the collimator, combine the reference thread with the image of the entrance slit and remove the N_4 count. Then the smallest angle for any spectral line: $\delta_{min} = N_3 - N_4$ (Figure 127). Take readings at least 3 times for all spectral lines.

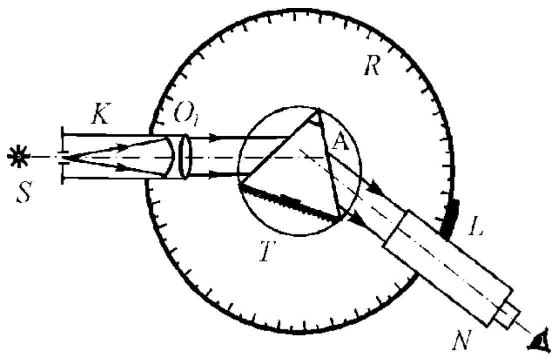


Figure 127 - View of the laboratory setup

11. Using the measured angles A and δ_{min}, calculate the refractive index of the optical glass n of the prism for all the indicated wavelengths.

12. Build a graph depicting the dispersion of light in an optical glass prism $n = n(\lambda)$.

13. To calculate the error for the refractive index of glass.

14. To calculate the dispersion of optical glass in the yellow-green spectral region using the formula $D = \frac{\Delta n}{\Delta \lambda}$.

Questions for the self-test:

1. What is light dispersion?
2. How does normal dispersion differ from abnormal?
3. On what grounds can the spectra obtained with a prism and a diffraction grating be distinguished?
4. What are the main provisions and conclusions of the electronic theory of light dispersion?
5. Why do metals strongly absorb light?

References:

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Laboratory work № 15

DETERMINATION OF THE MAIN CHARACTERISTICS OF THE DIFFRACTION GRATING

Aim of the work: determination of the period, the number of strokes per millimeter, the angular dispersion and resolution of the diffraction grating

Accessories: Light source, diffraction grating, optical bench

Brief theoretical description

The diffraction grating is an optical device, which is a collection of a large number of parallel, equally spaced narrow slits (strokes) of the same shape, applied on any surface. The main property of the diffraction grating is the ability to spread the light falling on it into the spectrum by wavelength. There are reflective and transparent diffraction gratings.

Reflective strokes are applied to the mirror (usually metal) surface, the spectrum is observed in reflected light. For transparent gratings, strokes are applied to the surface of transparent (as a rule, glass) plates or are cut out in the form of narrow slits in an opaque screen and the observation is made in transmitted light. Consider the effect of a transparent diffraction grating. Let a parallel beam of white light fall onto the grating normally to its surface (Figure 128).

On the slits (strokes) of the lattice, commensurate with the wavelength of light, the phenomenon of diffraction occurs, defined as the deviation of the waves from the rectilinear propagation during their interaction with an obstacle. As a result, behind the grating, the rays will go at different angles in all directions from each point of the slit.

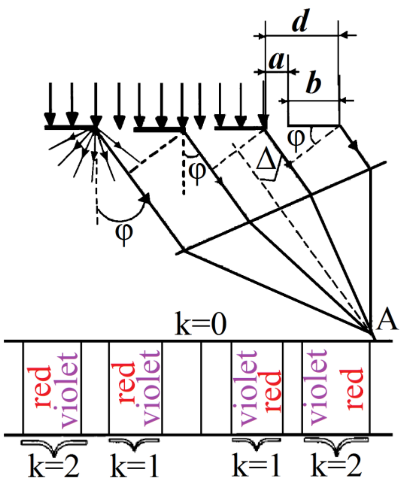


Figure 128 - Principle of operation of the diffraction grating

These rays can be grouped into beams of rays parallel to each other. Install a positive lens behind the grating. Each beam of parallel rays will meet in the back focal plane of the lens at one point (point A for rays diffracted under Figure 128 by the angle φ to the lattice normal). The parallel rays of other diffraction angles are collected by the lens at other points in the focal plane.

At these points, interference of light waves emanating from different grating slots will occur. If an integer number of wavelengths of monochromatic light fit in the path difference between the corresponding rays, then at the point where the rays meet, a maximum of light intensity occurs for a given wavelength, i.e., $\Delta = k\lambda, k = 0, \pm 1, \pm 2 \ldots .$

From figure 15.1 that the path difference Δ between two parallel rays emanating from the corresponding points of the adjacent slits is $\Delta = (a + b)\sin\varphi$, where a is the width of the slit; b - the width of the opaque gap between the slits.

The value $d = a + b$ is called the period, or constant, of the diffraction grating.

Consequently, the condition for the occurrence of the main interference maxima of the lattice has the form.

$$d \sin \varphi = \Delta = k\lambda \tag{171}$$

In the focal plane of the lens for rays that have not experienced diffraction, there is a central white maximum of zero order (φ = 0, k = 0), to the right and left of which are colored maxima (spectral lines) of the first, second and subsequent orders of interference (see Figure 15.1). The intensity of the maxima decreases strongly with an increase in their order, that is, with an increase in the diffraction angle.

Equation (171) allows you to calculate the period of the diffraction grating d, if the measured diffraction angle φ corresponding to the spectral line, for which its wavelength and the order of the spectrum are known. Knowing the lattice period, it is easy to calculate the number of strokes applied to one millimeter of the lattice width:

$$n = \frac{1}{d} \tag{172}$$

One of the main characteristics of the diffraction grating is its angular dispersion. Angular dispersion of the lattice is the value determined by the increment of the diffraction angle when the wavelength is changed by one,

$$D = \frac{d\varphi}{d\lambda} = \frac{\Delta\varphi}{\Delta\lambda} \tag{173}$$

Dispersion determines the angular distance $d\varphi$ between the directions of two spectral lines differing in wavelength by 1 nm ($d\lambda$ = 1 nm), and characterizes the degree of extension of the spectrum near a given wavelength. The formula for calculating the angular dispersion of the grating can be obtained by differentiating the equation that determines the position of the main maxima $d \sin \varphi = k\lambda$, whence

$$D = \frac{d\varphi}{d\lambda} = \frac{k}{d \cos \varphi} \tag{174}$$

From this expression, it follows that the angular dispersion of the grating is the greater, the greater the order of the spectrum. This explains the expansion of the spectrum of the same order of the grating with increasing order.

For gratings with different periods, the width of the spectrum is larger for a grating with a shorter period. Usually, within a single order, $\cos \varphi$ varies insignificantly (especially for gratings with a small number of strokes per millimeter), so the dispersion does not change within one order of magnitude. The spectrum obtained with a constant dispersion is stretched uniformly over the entire wavelength region, which distinguishes the lattice spectrum from the spectrum given by the prism.

In spectroscopy, it is considered that an optical instrument resolved two lines of the spectrum if the images of these lines in the spectrum obtained with this instrument are visible separately. If the images of two lines merge into one, then they say that the device did not allow them. The same lines of the spectrum can be resolved by one device and not allowed by another. This is due to the width of the intensity maxima of these lines.

At the suggestion of Rayleigh, confirmed and verified by experience, the resolution is considered to be complete when the intensity maximum of one of the lines coincides with the minimum of the other (Figure 129).

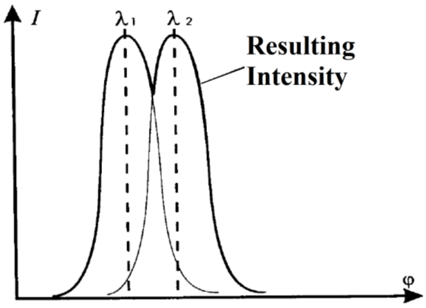


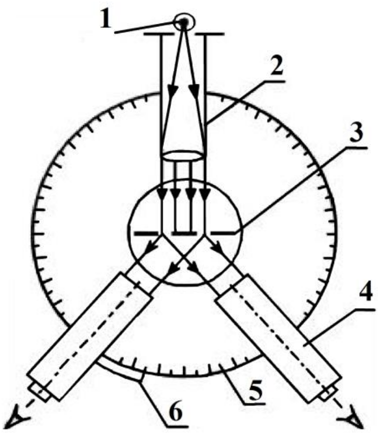
Figure 129 - Illustration of the resolution

If maxima are closer than shown in Figure 129, the images of the lines λ_1 and λ_2 merge into one - the lines are not resolved. When the highs are further apart, the lines are surely resolved. Resolution (or resolving power) is called the ability of the lattice to give to see separately on the screen in the wavelength region λ two wavelengths different from each other by $\Delta\lambda$. The resolution is dimensionless. The larger it is, the closer the wavelength lines are capable of resolving the device. According to the Rayleigh criterion, the resolution of the diffraction grating is determined by the order of the spectrum and the total number of grating lines N.

$$R = kN \tag{175}$$

Description of the laboratory installation

The laboratory setup for determining the basic characteristics of a diffraction grating includes a light source 1 (Figure 130). This is a mercury lamp, giving a line spectrum, which consists of the following spectral lines: two yellow λ_1 = 579.1 nm; λ_2 = 577.0 nm; green λ_3 = 546.1 nm; blue λ_4 = 491.6 nm (weak); blue λ_5 = 435.8 nm; two purple λ_6 = 407.8 nm (weak); λ_7 = 404.7 nm.



1 - Light source
2 - Illumination collimator
3 - Diffraction grating
4 - Viewing tube
5 - Goniometer
6 - Nonius

Figure 130 - Diagram of the laboratory setup

All measurements of diffraction angles are made on a goniometer. It consists of an illumination collimator 2, which gives a beam of parallel rays (the entrance slit of the collimator is located in the focal plane of the objective of the tube). The slot width is adjusted by a screw located on the side. A parallel beam of rays is directed to the diffraction grating 3 mounted on the goniometer table, perpendicular to the axis of the lighting collimator. The spectrum obtained with the help of a diffraction grating is observed into telescope 4, which can rotate around the vertical axis of the goniometer, while remaining always directed along the radius of the circular limbus of the goniometer 5. The limb is divided into 360^0 (the division value of the limb is 30′). Along the limb, along with the telescope, a nonius 6 is fastened to it, the accuracy of which is one angular minute. In the eyepiece of the telescope, there is a vertical filament, which is combined with the examined line of the spectrum.

Order of performance

1. Place the grid (50 lines/mm, 100 lines/mm, 300 lines/mm) on the goniometer table, perpendicular to the axis of the collimator 2. In this case, a series of bright and clear spectral lines of first, second and subsequent orders of mercury vapor will be observed on both sides of the central white maximum (k = 0). To determine the angle φ, it is necessary to combine the vertical filament in the eyepiece of the telescope with the green line in the first-order spectrum, first to the left of the zero maximum. On the goniometer limb and the nonius, take the N_1 countdown (degrees and half-degrees - by the limbo against the nonius, minutes - by the nonius). Then move the viewing tube towards the zero maximum and further until the vertical thread coincides with the green line in the first-order spectrum to the right of the zero maximum. Repeat the countdown N_2. The desired diffraction angle $\varphi = \frac{N_2 - N_1}{2}$. Angle φ measure three times. According to the average value of the measured angle, knowing the wavelength of the green line, calculate the lattice period by the formula $d \sin\varphi = \Delta = k\lambda$.

2. Knowing the grating period to calculate the number of strokes per 1 mm grating width by the formula $n = \frac{1}{d}$.

Table 21 - Recommended values for the measurement

Grating type	Order	Line colors
50 lines/mm	*k=1*	*purple, blue, green, yellow, orange, red*
	k=2	*purple, blue, green, orange, red*
	k=3	*purple, blue, green*
100 lines/mm	*k=1*	*purple, blue, green, yellow, orange, red*
	k=2	*purple, blue, green, yellow, orange, red*
	k=3	*purple, blue, green*
300 lines/mm	*k=1*	*purple, blue, green, yellow, orange, red*
	k=2	*purple, blue, green, yellow, orange*
	k=3	*purple, blue, green, yellow*

3. Measure the diffraction angle for the blue line of the spectrum (λ = 435.8 nm). Knowing the diffraction angles on the green and blue lines and their wavelengths, calculate the angular dispersion using the formula $D = \frac{d\varphi}{d\lambda} = \frac{\Delta\varphi}{\Delta\lambda}$. Also, knowing the grating period, calculate the angular dispersion by the formula $D = \frac{k}{d\ \cos\varphi}$. Compare the results of the two methods for calculating the angular dispersion.

4. Calculate the resolution of the lattice in the spectrum of the considered first order by the formula $R = kN$. Find the total number of grating lines, knowing the number of lines per 1 mm and measuring the width of the cut part of the grid (with an accuracy of 1 mm).

5. Derive the error formula for the lattice period and determine the relative and absolute errors in the calculation of the grating period.

Note. Due to the design features of the gratings made on the plane-parallel glass plastics, additional blurred lines are observed by the holographic method due to multiple reflections in the plate, especially at large diffraction angles. Lines with a clear, bright image are selected for measurements. This effect is most noticeable for two closely spaced yellow lines - the first two lines of the deviation of the central maximum (k = 0), the observed lines with sharp boundaries are the desired ones.

Questions for the self-test

1. How does the diffraction grating decompose white light into a spectrum?
2. What is the method for determining the lattice period in this paper?
3. What is the angular dispersion of the lattice? What does it characterize?
4. What is the resolution criterion?
5. What is the maximum order of the spectrum given by the lattice studied in the work?
6. Why is the central maximum given by the grating not colored (white)?
7. Why are gratings with a small period and a large total number of strokes beneficial?
8. What is the difference between diffraction patterns from a single slit and a grating?
9. At what ratio of the period of the diffraction grating to the width of the slit will not be observed second-order spectrum?
10. How does the work of the diffraction grating and the prism, as the dispersive elements?

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Appendix 1

Types of errors

The result of each measurement of a physical quantity does not coincide with its true value as a result of the action of a multitude of distorting factors. The deviation of the measurement result from the true (valid) value of the measured quantity is called the measurement error (error of measurement).

Absolute measurement error Δ_x is the measurement error, defined as the difference between the measured x_m and the true x_r (real) value of the measured quantity, taken modulo:

$$\Delta_x = |x_m - x_r| \tag{176}$$

Relative measurement error δ_x - measurement error, equal to the ratio of the absolute measurement error to the actual value of the measured value

$$\delta_x = \frac{\Delta_x}{x_r} \cdot 100\% \tag{177}$$

Reduced measurement error γ is the ratio of the absolute error Δ_x to the normalized value x_n, expressed as a percentage:

$$\gamma = \frac{\Delta_x}{x_n} \cdot 100\% \tag{178}$$

For example, the maximum value x_{max} of the measured value $x_{max} = x_n$ can be taken as the normalized value.

There are three types of measurement errors: gross errors, systematic and random errors.

Blunders (gross error) are errors associated with a malfunction of the measuring apparatus, or with the experimenter's error in counting or recording instrument readings, or with a drastic change in conditions. Measurement results in such cases should be discarded and new measurements should be made.

Systematic measurement error is the measurement result error, which remains constant or changes regularly with repeated measurements of the same physical quantity. These errors include methodical and instrumental measurement errors.

Random (or accidental) errors are not directly related to the conditions or circumstances of the observation. For a single measurement or a series of measurements, it is the error remaining after all possible systematic errors and blunders have been eliminated.

Methodical error is due to the lack of measurement method, the imperfection of the theory of physical phenomena and the inaccuracy of the calculation formula used to find the measured value.

Instrumental error is caused by imperfect design and inaccuracy in the manufacture of measuring instruments. The accuracy of the instrument is either specified by the accuracy class (which is usually applied to the instrument), or indicated in the passport attached to the instrument.

Appendix 2

Deviation of the measured physical quantity. Student's distribution.

When a researcher is faced with the task of measuring a specific physical quantity x, then the number of experiments n is limited. If the number of measurements n of the physical quantity x is small, then the true value of the measured quantity $\bar{x}$ differs from the arithmetic mean value $\tilde{x}$. The arithmetic mean value $\tilde{x}$ of measurements, is defined as

$$\tilde{x} = \frac{x_1+x_2+\cdots+x_n}{n} \tag{179}$$

Where x_1, x_2, ..., x_n - values of the measured value corresponding to the serial number of measurements n.

With a small number of measurements $2 \le n \le 10$ to determine the confidence interval α (random error of multiple measurements) Student' coefficients are used to determine the error $\Delta\tilde{x}$.

When performing laboratory work usually $n \le 10$

Let as a result of n measurements of a random variable x, obeying the normal distribution with parameters $\bar{x}$ (the true value of the measured quantity) and $\tilde{\sigma}$ (standard deviation of measurement results from the arithmetic mean $\tilde{x}$ in this series of measurements), different values x_1, x_2, …, x_n. were obtained

The standard deviation of the measurement results σ is calculated by the formula:

$$\tilde{\sigma} = \sqrt{\frac{(\tilde{x}-x_1)^2+(\tilde{x}-x_2)^2+\cdots+(\tilde{x}-x_n)^2}{n(n-1)}} \tag{180}$$

Student reviewed the random variable of the form

$$t = \frac{\bar{x}-\tilde{x}}{\tilde{\sigma}} \tag{181}$$

The values $\tilde{x}$ and $\tilde{\sigma}$ depend on the number of measurements n. Therefore, when the number $n = 1$ measurement, the value of t takes a numerical value of t_1, when $n = 2$, the value of t_2, etc. The student received the distribution law (probability density) $S_n(t)$ of a random variable t (Figure 3) This is some mathematical function of n and t.

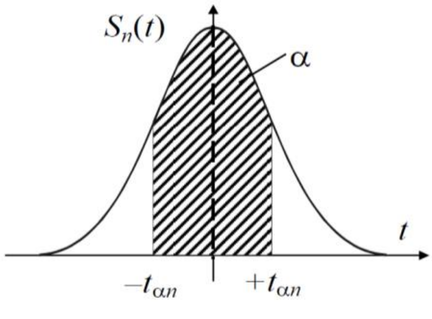


Figure 131 - Student's $S_n(t)$ distribution of magnitude *t*

Student law is the law of the distribution of measurement errors of normal random variables. This function (probability density $S_n(t)$) has a maximum at $t = 0$, when the measurement value is equal to the arithmetic mean value $x = \tilde{x}$.

We denote the probability that the value *t* takes a value from a certain range from $-t_{\alpha n}$ to $+t_{\alpha n}$, through α (shaded area in Figure 131). For a certain number of measurements *n* we set the value of the confidence probability α, then using the function $S_n(t)$, we can calculate the boundaries of the corresponding symmetric interval $t_{\alpha n}$, which depends on α and *n*:

$$\alpha = \int_{-t_{\alpha n}}^{+t_{\alpha n}} S_n(t)dt \qquad (182)$$

The values $t_{\alpha n}$ are called the *Student coefficients*, they are listed in Table 22 for different values of the confidence probability α and the number of measurements *n*.

Table 22 - *Student coefficients*

n \ α	0,9	0,95	0,999
2	6,31	12,7	636,6
3	2,92	4,30	31,6
4	2,35	3,18	12,9
5	2,13	2,78	8,61
6	2,02	2,57	6,37
7	1,94	2,45	5,96
8	1,89	2,36	5,41
9	1,86	2,31	5,04
10	1,83	2,26	4,78
∞		1,96	3

Confidence probability or reliability represents the frequency (i.e. the proportion) of possible confidence intervals that contain the true value of the unknown population parameter. The magnitude of the confidence level is determined by the character of measurements made.

When carrying out educational lab work in a general physics course, the confidence probability α is usually considered to be 95%.

For a known number of experiments *n* and confidence probability *α*, we determine the Student coefficient $t_{\alpha n}$, which corresponds to the maximum deviation of the arithmetic average from the true value. The maximum deviation of $\tilde{x}$ from $\bar{x}$ is equal to the length of the confidence interval $\Delta\tilde{x}$. Then, based on the definition of *t*, we get

$$t_{\alpha n} = \frac{\bar{x}-\tilde{x}}{\tilde{\sigma}} = \frac{\Delta\tilde{x}}{\tilde{\sigma}} \rightarrow \Delta\tilde{x} = t_{\alpha n}\tilde{\sigma} \qquad (183)$$

Thus, the random error (confidence interval) $\Delta\tilde{x}$ with a small number of measurements should be calculated using the Student coefficient $t_{\alpha n}$.

The procedure for calculating the random error:

1. Find the average value $\tilde{x}$
2. Calculate the standard deviation $\tilde{\sigma}$
3. Determine the student coefficient $t_{\alpha n}$ for α=0,95
4. Calculate the error $\Delta\tilde{x}$
5. The result is written as: $x = \tilde{x} \pm \Delta\tilde{x}$

Example of calculation of random error:

Suppose we have received in the process of measuring the following set of values λ_1=701.7 nm; λ_2=625.8 nm; λ_3=590.8 nm; λ_4=663.1 nm, λ_5=578.7 nm; λ_6=708.9 nm; λ_7=605.0 nm; λ_8=580.5 nm.

1. We calculate the arithmetic average value of the wavelength by the formula (179);

$$\tilde{\lambda} = \frac{701.7 + 625.8 + 590.8 + 663.1 + 578.7 + 708.9 + 605 + 580.5}{8} = 632.06$$

We write the value $\tilde{\lambda}$ =632.1 nm (i.e. round up to one decimal place)

2. Calculate the standard deviation of the formula (180).

$$\tilde{\sigma} = \sqrt{\frac{(632.1 - 701.7)^2 + (632.1 - 625.8)^2 + \cdots + (632.1 - 580.5)^2}{8(8-1)}} = 18.79$$

We write the value $\tilde{\sigma}$=18.8

3. Table 22 writes the value of the student $t_{\alpha n}$ for α=0,95. We had 8 dimensions, hence, $t_{\alpha 8}$=2.36

4. Calculate the margin of random error $\Delta\tilde{\lambda} = t_{\alpha 8}\tilde{\sigma}$ = 2.36·18.8=44.37

After the absolute error is calculated, its value is usually rounded to one significant digit. After that, the measurement result is recorded with the number of decimal places no more than they are in absolute error.

5. Write the final version $\lambda = \tilde{\lambda} \pm \Delta\tilde{\lambda}$ or λ = **630** ± **40** nm with reliability α=0,95.

Please note that the degrees of rounding are the same everywhere and cannot be less (more precisely than the data obtained during the experiment)

Appendix 3

Determination of instrument error

Instrumentation always contributes to the measurement uncertainty depending on instrument accuracy.

Instrumental error is caused by imperfect design and inaccuracy in the manufacture of measuring instruments. The accuracy of the instrument is either specified by the accuracy class (which is usually applied to the instrument), or indicated in the passport attached to the instrument.

The main characteristics of measuring instruments that affect the accuracy of measurements performed with their help are the limit of measurement and division value.

The measurement limit (M_l) is the maximum value that can be measured using this instrument scale. If the measurement limit is not specified separately, then it is determined by digitizing the device. So, on the scale of the voltmeter is shown in Figure 132 the measurement limit is 3 V.

Figure 132 - Voltmeter scale

Division value (D_v) - the value of the measured scale corresponds to the smallest division of the scale. If the scale starts from zero, then $D_v = \frac{M_l}{N}$, where N - is the total number of tick marks. For example, from Figure 132 you can see that the price of a voltmeter division $D_v = \frac{3}{30} = 0.1\ B$.

Many electrical measuring instruments have several measuring ranges. When switching from one limit to another, the price of the device changes as well

Accuracy rating (A_r) is the ratio of the absolute instrument error Δ_x to the measurement limit of the scale M_l, expressed as a percentage

$$A_r = \frac{\Delta_x}{M_l} \cdot 100\% \qquad (184)$$

The value of the accuracy rating (without the % symbol) is indicated, as a rule, on electrical measuring instruments. Electrical instruments used in the performance of laboratory work may have an accuracy class of 0.05; 0.1; 0.2; 0.5; 1.0; 1.5; 2.0; 2.5; 4.0. More coarse instruments do not have accuracy rating. The errors that can be determined based on the accuracy rating of the device are listed in Table 23.

Here, $X_k -$ is the largest (in absolute value) of the limits of measurements, c and d - are the numbers given by the accuracy class on the instrument, x_r - is the true (real) value of the measured value.

Table 23 - Designation examples of accuracy rating on measuring instruments

The form of error expressions	Limits of the allowed main error	Limits of the allowed main error, %	Accuracy rating designation	
			In the documentation	On the instrument
Reduced error γ	$\gamma = \frac{\Delta_x}{x_n}$	γ = ±1,5	Accuracy rating 1,5	1,5
Reduced error γ		γ = ±0,5	Accuracy rating 0,5	0,5
Relative error δ_x	$\delta_x = \frac{\Delta_x}{x_r}$	δ = ±0,5	Accuracy rating 0,5	0,5
Relative error δ_x	$\delta_x =$ [c+d($\left\|\frac{X_k}{x_r}\right\| - 1$)]	δ=±[0,02+0,01($\left\|\frac{X_k}{x}\right\| -$ 1)]	Accuracy rating 0,02/0,01	0,02/0,01
Absolute error Δ_x	$\Delta_x = \|x_m - x_r\|$		Accuracy rating M	M
Absolute error Δ_x			Accuracy rating C	C

Error Calculation Example

Let us give an example of calculating the instrumental error. Figure 132 shows a voltmeter with a 1.5 of accuracy rating. The maximum value that it can show is - 500V, $X_k = M_l = x_n = 500V$.

Where $x_n -$ normalized value expressed in a percentage, $X_k -$ greater (in absolute value) of the measurement limits, M_l - maximum value that can be measured using this scale.

Figure 132 - Voltmeter with accuracy class 1,5

1. Calculate the absolute error of the device by the formula (184).

$$\Delta_x = \pm 1{,}5(\%) \cdot \frac{X_k\,(V)}{100(\%)} = \pm 1{,}5(\%) \cdot \frac{500\,(V)}{100(\%)} = \pm 7{,}5\,V$$

Thus, since we r**ound up to the first significant** digit, the absolute error of the voltmeter shown in Figure 132 is equal to $= \pm 8\,V$.

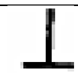
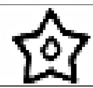
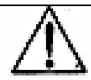
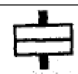
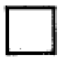
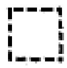
2. Now let's try to calculate the relative error. The value of the arrow of the voltmeter is 200 V, therefore x_r n this particular case becomes equal to 200 V.

$$\delta_x = \pm 1{,}5\ (\%) \cdot \frac{x_{n(V)}}{x_{r(V)}} = \pm 1{,}5 \cdot \frac{500}{200} = \pm 3{,}75\%$$

We write the value $\delta_x = \pm 4\%$, we **round up to the first significant digit** since the relative error of this device cannot be more accurate than the accuracy rating

Appendix 4

Symbols of electrical measuring instruments

Direct current	
Single-phase alternating current	
DC and AC	
Three-phase current (general designation)	
Three-phase current at non-uniform phase load	
The horizontal position of the scale	
The vertical position of the scale	
The inclined position of the scale at a certain angle to the horizon	
The measuring circuit is isolated from the housing and tested for voltage 2kV	
The device is not tested for insulation strength.	
Caution! Insulation strength of the measuring circuit to the body does not meet the standards	
Attention! See additional instructions in the passport of the operating manual.	
Electromagnetic device	
Electrodynamic device	
Protection against external magnetic fields	
Protection against external electric fields	
Magnetoelectric system device	
Rectifier device	

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