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On the optimality one power system

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Abstract. The computational experiments, which show the sufficient efficacy of the proposed procedure for constructing the Bellman-Krotov function and of the synthesing optimal control for the given power system were carried out on the basis of the theoretical results obtained in the work.

Keywords: Electric power system, Nonlinear system, Phase system. Control synthesis. Bellman Krotov function
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INTRODUCTION

Mathematical modeling of a variety of processes and systems is closely linked with the problem of making the best decisions. Therefore, mathematical models of objects for the following use of mathematical methods, including the methods of modern theory of optimal control arc built. As it is known, the basics of this theory arc the principle of maximum by L. S. Pontryagin, the dynamic programming method, sufficient conditions for optimality by V. F. Krotov. In the process of study of practical problems, the last two methods usually require the solution of a nonlinear equation in partial differentials of the first order of Hamilton-Jacobi equations type: it is necessary to find the so-called Bellman-Krotov function which is a well known problem of the theory of differential equations.

The mathematical model of a modern power complex consisting of turbine generators and complex multiply power units is a system of nonlinear ordinary differential equations.

STATEMENT OF THE PROBLEM

Wc will consider a mathematical model that describes the transients in the electrical system, and represents the following system of nonlinear differential equations:

^ - *Si.* *H~ = -DM ~ EfYijsin ctjj* - ftsin($ - a,) - £ Py - *a,)* 4- *u*, / = 17/, r € [0,7], (1)

8ij = 8i~8j, P = EiUY, *P = EjEjYij.*

\

where *8j* is angle of rotation of the rotor of i-th generator with respect to some synchronous rotational axis (the axis of rotation of constant voltage tires, it makes 50 rev / sec), *Sj* is sliding of the i-th generator, H, is constant inertia of i-th machine, *щ* is EMF of i-th machine, Y_{Ij} is mutual conductance of i-th and j-th system branches, U=const is the voltage on constant voltage tires, Y_{un} \ characterizes the contact (conductivity) of i-th oscillator with constant voltage tires, D,-=const>0 is mechanical damping, and $a_{j;}$, a,-, a; are constants, taking into account the influence of active resistances in the generator stator circuits. The complexity of model (1) consists in considering analysis **Of,/,** with the following property «/; = **O/,.** We believe that the conditions 5,7 = *—8ц* are realized. In this case, model (1) is not conservative; so it isn't possible to build for it the Lyapunov function in the form of the first integral. This creates an additional difficulty for its research. System (1) is traditionally called the positional model, and it belongs to non-conservative class.

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Turbine powers are directly accepted as the synthesizing (control) function *u,* for this system. Let the variable states and control in the after-failure mode have the following values:

$$S^i = 0, \quad S^i = Sf^1 \quad Ui = iii. \quad i = Tj-$$

Го obtain the system of equations for the perturbed motion, we will pass to the equations in deviations, assuming

$$U_i = uf.+ Au^h \;\; 8i = 5f + A8i, \;\; 8^1 = AS^l, \;\; i = T7l.$$

Furthermore, denoting for convenience Ди,-, $A8^h$ Д5; again as w,, 8^h 5, and using the formula sin(<5,7 a^{tj}) = cos a_{-j} sin *8jj* - sin c_{Ljj} cos *8jj* , from system (1) we obtain

c

^ = f t , ^ = ^ [- D ' 5< /,($) f M,-(<5) f w/J. r = U Г€[0,Г],

where

/; (50 = /> [sin (5,4- *8f* - a,) - sin (-a,-)],

/ /

/"I.///

/ /

Af,-(*) = £ = I *Г}; [cos(8,, + 8')-cos8f.]*

rjj - />^{7}cosa, ,Г?- = /fysincty .

It should be noted that under the conditions $P^{(J} = P^{fl}$ have Г/- = Г^-, Г?- = Г?..

Control и,-, i - 1. / arc chosen so as to compensate "non-conservative" member *M, (8). i* = **177,** i.e.,

$$iij = Vi-Mi(8)\backslash \;\; / = 1T7,$$

where \>*i e RJ* are unknown synthesizing functions.

Let us consider the following optimal control problem.
Minimize the functional

$$J(v) = 7(v,,\ldots, i^V) = - \hat{} \;\; \frac{1}{} \int^{T\;1} £ \quad + HV,i^{\prime 2}) Л + Л(<\& (7)S(7)), \qquad (2)$$

under conditions

$$lfi\sim^{=S_{,,}} \quad f^{=} jS[bft\hat{}ii») - W*) + i \qquad f€[0,7], \qquad (3)$$

where H;$_{v(i}$w$_{,,}$. arc positive constant weights coefficients, //($) is 2/r-pcriodic continuously diffcrcntiablc function, *N,* **(5)** is 2tf-periodic continuously differentiable function with respect to 8_{u} and the conditions of integrability are realized relatively to *N,-,(8)* **,**

T is the duration of the transition process is assumed to be known and initial conditions are

$$*(0) = \$o, \quad 5,(0) = 5,o, \quad i = tj. \qquad (5)$$

However, the final values of the system state *8(T),* 5(7) are not known in advance, they are to be determined from the solution of the optimal control problem (2) - (5).

To solve this problem, the Krotov theorem on sufficient optimally conditions [1, 21 is used for one power model. As a result, we obtain the following theorem.

THE MAIN THEOREM

Theorem 1. *To ensure the optimality of controls,* $v®\{S[) = —-^-Si,\ i = 1,/,$ *and the corresponding solution of the system* (2) - (5) *it is necessary and sufficient that* Л(5(Г), *S(T)) = K(5(T),* 5(7)) *and* w^{Sl} -= 2Д f > 0, *i* = Г/ *where the Bellman-Krotov function is determined by the formula*

K(S,S) = ^IᵗHⁱSJ + Ig /"*fi(5ⁱ)dSⁱ+ £ /'\(S_b,...,S, ,,4,5_{iM},...,5,)^„
-i-i '⁻i /=I, «,=()(./>;)

«/и/ r/ie *minimum functioned value is equal to*

$$J(v^0)=mnJ(v)=K(So,So). \qquad (6)$$

Proof. For the continuous function *K(8(t),S(t))* of variable /, functional (2.) can be represented as

J(v)=J(8(t),S(t),v{t))= [/o *R(8(t),S(t)Mt))dt* + ***mo(8(0),S(0))*** + *mᵗ(S{t),S(t)),*

where ***K(8,S,*** v) = 1|=1 [tf5.5; + -- ВД + V,) + |(w5,S,^{2} + ny.V^2)].

= Ч = mo(5,5) = AГ(5,5), m/(5,S) - -ЛГ(*,5)+Л(5,5).

To determine the Bellman-Krotov function ***K(S,S),*** let us consider the following Cauchy-Bellman problem

MR(5,S, v) = 0, 0 < r < 7\ *K(8(T),S(T)}* = Л(5(Г),5(T)). (7)

It should be noted that problem (7) with respect to the Bellman-Krotov function is a nonlinear equation in partial differentials of the first order, the solution of which is a well-known problem of the theory of differential equations. This issue was discussed in [3—8J by many authors.

In this paper, using the features of the problem of power system in optimal control, we were able to construct the solution of problem (7). For this purpose, from the necessary condition for the function extremum ***R(S,S.*** v) by v, G we will obtain

Лу. = — Ks, + uv‹v, = 0, *i* = ***TJ.***
Hi

Hence, we find the optimal controls:

yf = --^—KSn **i = M .** (8)
HiwVi

Function ***K(8,S)*** and weight coefficients ***wsⁿwᵛ.*** are found from the condition (7), i.e.,

' * = min*(M,v) = £ ^ . s_r - - ^ A s ^ (9)
i~i *Hi* *2Hfw_{Vi}* *ᵇi* *2*

For this purpose, we assume

K_s,Si='^(fi(8i) + *N_i(8)),,* = *Tj,*

i.e..

K_{si}=-HtS,,Kt,=f,{S,)+N_i{S), / = **17.** (10)

Then, taking into account (10), we obtain (9)

·D·· s 2 _ _ L s 2 1 s s 2 = 0,
w y, ^

$$w_{s_i} = 2Di + \frac{—}{2w_{V_i}} > 0, \quad w_{V_i} > 0, \quad i = 1, /.$$

We obtain from (4) that the synthesizing the optimal controls vf. $i = 1, /$ have the form

$$v?(Si) = -—Si, \quad i = T7. \qquad (ID$$

Let us now consider the following question. How to define the Bellman function - ***K(8,S),*** knowing the partial derivatives Equation (4) takes place indeed, as

$$\frac{dN/(5)}{3g} = \sim ч_K \cos + \%)^p, \quad dN_k(8) = -Vli\cos(8_{kl} + = -\Gamma £\cos(5,* + 5 - J).$$

Therefore, the function ***K(8,S)*** can be represented as

K(S,S)^£tf,-S, + £ I**m)d8,+ £I /%(5,...,5, q,8... 8,)d^ /=1 /=1^0 i=\ ^ -()(/ 1) -()(/ -1)

I > S, + £ [fi(8)d8,+ f £ ^,^,,0....0)</§, 1-1 ,-1 /-Г 70 70 :1

+ Г* I yV2/(5b&.0,....0)^2 ! ...f £ (12)

We note that for the case $1 = 2$:

f^Nn^i^d^+f^Ni] (5ь&)^2 = ^r2 [-cos(5] + 5f?) - 5i sin 5f2 + cos 5£1
.70 70

+Г21 "-cos(5i2 + 5f2) +52sin51^F2-r-cos.(5| + 5^)] = T}2 [-cos(5i2 + 5f2) - 5,2sin5,^F2 + cos5&] .

On the other hand,

I ^l2 N^l2(8)d8 = j ^,2 rl2[sin(jc + 5f2)-sin5f2]^v

= Г}2 [- cos(5i2 + 8^l2) - 8\2sin8^l2 +0085/2] .

Therefore, when / = 2, as well as when any / > 2. the function ***K(8,S)*** of (12) can be written as:

K(S,S) = ^HiSj + £± f /* ^ r m d s , f £ ^I e' Г Ч'(*K*
' /-1

It should be pointed out that in the field of the change of phase variables {5,5} the Bellman-Krotov function #(5,5) is definitely - positive. Furthermore, the Bellman-Krotov function ***K*** (5,5) is at the same time the Lyapunov function for the synthesized optimal system (3), (11). Thus, the synthesized optimal system is stable according to Lyapunov function, i.e. we obtain the solution of the A.M. Letov's problem of the analytical construction design of optimal controllers (ACOC). According to the boundary condition, for Bellman equation (7), the function Л(5,5) will be taken in the form

$$A(8(T),S(T)) = K\{8(T),S\{T)).$$

The direct verification shows that the optimal value of the functional 7(v) is equal to value (6). The theorem is proved. •

CONCLUSION

On the basis of the theoretical results, the computational experiments which show the sufficient the efficacy of the proposed procedure for constructing the Bellman-Krolov function and of the synthesising optimal control for the given power system arc conducted.

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