ON A CONVERGENCE OF THE MODIFICATION OF BROKEN EULER METHOD SOLVING OF THE NONLINEAR BOUNDARY VALUE PROBLEM FOR HYPERBOLIC EQUATION

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We consider in domain $\bar{\Omega} = [0, T] \times [0, \omega]$ boundary value problem for nonlinear hyperbolic equation with two independent variables

$$\frac{\partial^2 u}{\partial x \partial t} = f(x, t, u, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}), \tag{1}$$

$$u(x, 0) = u(x, T), \qquad x \in [0, \omega], \tag{2}$$

$$u(0, t) = \psi(t), \qquad t \in [0, T], \tag{3}$$

Where $f : \overline{\Omega} \times R^3 \to R$, is continuous on $\overline{\Omega}$, $\psi(t)$ is continuously differentiable on $[0, T]$ and satisfying to condition $\psi(0) = \psi(T)$ function.

Modification of Euler broken method [1]is offered for finding solution of problem (1)-(3) . We partition segment $[0, \omega]$ on mN_0 parts with step $h = \frac{\omega}{mN_0} = \frac{h_0}{m}$, $m = = 1, 2, \ldots$ and on each step solve periodical boundary value problem for the ordinary differential equations

$$\frac{dv^{(i+1)}}{dt} = f(ih, t, \psi(t) + h\sum_{j=0}^{i} v^{(j)}(t), \dot{\psi}(t) + h\sum_{j=0}^{i} \dot{v}^{(j)}, v^{(i+1)}), \tag{4}$$

$$v^{(i+1)}(0) = v^{(i+1)}(T), \quad t \in [0, T], \ i = \overline{1, mN_0}. \tag{5}$$

Solvability of boundary value problem (4), (5) were established in [2]. By solutions of problem (4), (5) on $\overline{\Omega}$ we construct the functions $U_h(x, t) = \psi(t) + h\sum_{j=1}^{i-1} v^{(j)}(t) +$ $v^{(i)}(t)(x - (i-1)h)$, $W_h(x, t) = \dot{\psi}(t) + h\sum_{j=1}^{i-1} \dot{v}^{(j)}(t) + \dot{v}^{(i)}(t)(x - (i-1)h)$, $V_h(x, t) =$ $v^{(i+1)}(t)\frac{x - (i-1)h}{h} + v^{(i)}(t)\frac{ih - x}{h}$, $x \in [(i-1)h, ih)$, $i = \overline{1, mN_0}$.

In the paper algorithm of finding of approximate solution to problem (1)-(3) is given and convergence of constructed triple functions $\{U_h(x, t), V_h(x, t),_h (x, t)\}$, $(x, t) \in \overline{\Omega}$, are established under $h \to 0$ to the solution $-u^*(x, t)$ of problem (1)-(3) its partial derivatives in t and x. The necessary and sufficient conditions of existence for "isolated" solution of problem (1)-(3) is obtained.

References

1. *Kabdrakhova S. S.* Modification of Euler's broken line method of solving semiperiodical boundary value problem for nonlinear hyperbolic equation *Mathematical journal* Vol. **8** 2(28) (2008) 55-62,

2. *Kabdrakhova S. S.* On solvability of family periodical boundary value problem o rising in modification Euler broken method *The collection of articles of the III International scientifically- methodical conf.* Vol.**3** (2010) 84-87.