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Computer Algebra Systems in Teaching and Research

Volume VIII

Siedlce University of Natural Sciences and Humanities
Institute of Mathematics and Physics

SIEDLCE 2019

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Secular Perturbations of Translational-Rotational Motion of a Non-stationary Triaxial Body in a Central Gravitational Field

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1 Equations of motion

In this paper the translational-rotational motion of two non-stationary bodies, namely, a spherical body and a triaxial body, is investigated. It is assumed that the initial dynamic shapes of bodies are preserved but their masses and sizes change in time [1]. Besides, the reactive forces and additional torques will be considered negligible and therefore they are equal to zero. An approximate expression for the force function of the Newtonian interaction accurate up to the second zonal harmonics is accepted.

The equations of translational-rotational motion of two bodies in the absolute coordinate system may be written in the form (see [1-5])

$$m_i\ddot{\xi}_i = \frac{\partial U}{\partial \xi_i},\ m_i\ddot{\eta}_i = \frac{\partial U}{\partial \eta_i},\ m_i\ddot{\zeta}_i = \frac{\partial U}{\partial \zeta_i},\ i = 1, 2, \tag{1}$$

$$\frac{d}{dt}(A_1p_1) = 0, \frac{d}{dt}(B_1q_1) = 0, \frac{d}{dt}(C_1r_1) = 0, A_1 = B_1 = C_1, \tag{2}$$

$$\frac{d}{dt}(A_2p_2) - (B_2 - C_2)q_2r_2 = \frac{\sin\varphi_2}{\sin\theta_2}\left(\frac{\partial U}{\partial \psi_2} - \cos\theta_2\frac{\partial U}{\partial \varphi_2}\right) + \cos\varphi_2\frac{\partial U}{\partial \theta_2}, \tag{3}$$

$$\frac{d}{dt}(B_2q_2) - (C_2 - A_2)r_2p_2 = \frac{\cos\varphi_2}{\sin\theta_2}\left(\frac{\partial U}{\partial \psi_2} - \cos\theta_2\frac{\partial U}{\partial \varphi_2}\right) - \sin\varphi_2\frac{\partial U}{\partial \theta_2}, \tag{4}$$

$$\frac{d}{dt}(C_2r_2) - (A_2 - B_2)p_2q_2 = \frac{\partial U}{\partial \varphi_2}, \tag{5}$$

where $m_i = m_i(t)$ are the masses of the bodies, $A_i = A_i(t), B_i = B_i(t), C_i = C_i(t)$ are their principle moments of inertia, and the potential function U is given by

$$U = G\frac{m_1m_2}{R} + Gm_1\frac{A_2 + B_2 + C_2 - 3I}{2R^3}. \tag{6}$$

Here G is a gravity constant, a distance between the centers of mass of the bodies is

$$R = \left((\xi_2 - \xi_1)^2 + (\eta_2 - \eta_1)^2 + (\zeta_2 - \zeta_1)^2\right)^{1/2}, \tag{7}$$

and $I = I(t)$ is the moment of inertia of the non-stationary triaxial body given by

$$I = A_2\alpha^2 + B_2\beta^2 + C_2\gamma^2, \tag{8}$$

where α, β, γ are the cosines of the angles formed by a straight line, connecting the centers of mass of the bodies, with the principal axes of inertia of the non-stationary triaxial body.

Projections of angular velocity of the bodies on the axes of their own coordinate systems are described by the kinematic Euler equations

$$p_i = \dot{\psi}_i \sin\theta_i \sin\varphi_i + \dot{\theta}_i \cos\varphi_i, q_i = \dot{\psi}_i \sin\theta_i \cos\varphi_i - \dot{\theta}_i \sin\varphi_i, r_i = \dot{\psi}_i \cos\theta_i + \dot{\varphi}_i, \tag{9}$$

where $\varphi_i, \psi_i, \theta_i$ are the Euler angles (see, for example, [2], [4]).

Equations (1)–(9) completely characterize the translational-rotational motion of the system but their general solution cannot be found even with the modern computer algebra systems. So the perturbation theory is applied here to their investigation, and the aperiodic motion of the triaxial body m_2 on quasi-canonical section around the body m_1 is considered as the unperturbed one (see [1]).

The main purpose of the present paper is to compute the disturbing functions for the system and to obtain differential equations describing secular perturbations of translational-rotational motion of the non-stationary triaxial body m_2 in the gravitational field of the spherical body m_1. Solving this problem includes several steps which involve quite cumbersome symbolic calculation. At first, we rewrite the equations of motion in the relative coordinate system in which the center of mass of the non-stationary spherical body m_1 is located at the origin, and rotational motion is described in Euler variables.

Then equations of translational-rotational motion in osculating analogues of Delaunay L, l, G, g, H, h and Andoyer L', l', G', g', H', h' elements are derived (see [1-5]). Note that the unperturbed translational motion is described by the aperiodic motion on quasi-conic section [1], while the unperturbed rotational motion is characterized by the Eulerian motion of non-stationary axisymmetric body [1], [2], [4]. The corresponding equations of motion are written in the Hamiltonian form:

$$\dot{L} = \frac{\partial F}{\partial l}, \quad \dot{G} = \frac{\partial F}{\partial g}, \quad \dot{H} = \frac{\partial F}{\partial h}, \quad \dot{l} = \frac{\partial F}{\partial l}, \quad \dot{g} = -\frac{\partial F}{\partial G}, \quad \dot{h} = -\frac{\partial F}{\partial H}, \tag{10}$$

$$\dot{L'} = \frac{\partial F'}{\partial l'}, \quad \dot{G'} = \frac{\partial F'}{\partial g'}, \quad \dot{H'} = \frac{\partial F'}{\partial h'}, \quad \dot{l'} = \frac{\partial F'}{\partial' l}, \quad \dot{g'} = -\frac{\partial F'}{\partial G'}, \quad \dot{h'} = -\frac{\partial F'}{\partial H'}, \tag{11}$$

In these equations the expressions for the disturbing functions F, F' in the analogues of Delaunay-Andoyer elements are very cumbersome, and their computing requires a lot of symbolic computations which are best performed using computer algebra systems. Doing necessary calculations, we obtain analytical expressions for the disturbing functions F,F' in analogues of the Delaunay-Andoyer variables which may be written in the form

$$F = \frac{1}{\nu^2}\frac{\mu_0^2}{2\mu_0 L^2} + \left\{-\frac{1}{2}bR^2 + \frac{(m_1+m_2)}{m_1 m_2}U_2\right\}, \quad (12)$$

$$F' = \frac{1}{2}\left(-\frac{1}{m\chi^2}\left[\frac{G'^2}{A_0} + \frac{A_0 - C_0}{A_0 C_0}L'^2\right]\right) - H^{rot}_{1pert} \quad (13)$$

$$H^{rot}_{1pert} = -\frac{1}{2m\chi^2}\left(\frac{1}{A_0} - \frac{1}{B_0}\right)(G'^2 - L'^2)\cos^2 l' - \left\{U_2 - \frac{1}{2}bR^2\right\} \quad (14)$$

$$U_2 = fm_1\frac{A+B+C-3I}{2R^3}, \quad I = A\alpha^2 + B\beta^2 + C\gamma^2, \quad (15)$$

In the absence of resonances, the equations of secular perturbations are obtained by double averaging the calculated disturbing functions over the mean longitudes l and l'. Solutions of the obtained equations of secular perturbations can be found by Picard's method in the first approximation but a detailed analysis of the equation of secular perturbations is performed by a numerical method. Time-consuming, cumbersome, analytical calculations in the expansion of the perturbation function are performed in the computer algebra system Mathematica [6,7].

Further development of this work involves the study of the obtained equations for secular perturbations of translational-rotational motion of the triaxial body of constant dynamic shape and variable size, and mass, using various analytical and numerical methods.

References

[1] *Minglibayev M.Zh.* Dinamika gravitiruyushchih tel s peremennymi massami i razmerami. Postupatel'noe i postupatel'no-vrashchatel'noe dvizhenie. Lambert Academic Publishing, Germany (2012)

[2] *Arhangel'skij A.Yu.* Analiticheskaya dinamika tverdogo tela. Glavnaya redakciya fiziko-matematicheskoj literatury izdatel'stva "Nauka", Mockva (1977)

[3] *Borisov A.V., Mamaev I.S.* Dinamika tverdogo tela. Gamil'tonovy metody, integriruemost',haos. Moskva-Izhevsk: Institut komp'yuternyh issledovanij (2005)

[4] *Markeev A.P.* Teoreticheskaya mekhanika. Moskva-Izhevsk: NIC Regulyarnaya i haoticheskaya dinamika (2007)

[5] *Lidov M.L.* Kurs lekcij po teoreticheskoj mekhanike.Moskva: Fizmatlit (2010)

[6] *Wolfram S.* An elementary introduction to the Wolfram Language, 2nd ed. Champaign, IL, USA, Wolfram Media (2017).

[7] *Prokopenya A.N.* Reshenie fizicheskih zadach s ispol'zovaniem sistemy Mathematica. Brest:Izdatel'stvo BGTU (2005)