



T. M. Aldibekov

**GENERALIZED
LYAPUNOV
EXPONENTS**

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GENERALIZED
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**A Generalized Lyapunov exponents: Monograph. – Almaty,
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The monograph is devoted to the study of some problems in the qualitative theory of differential equations and is related to the theory of Lyapunov exponents, the linear systems of differential equations under small perturbations and the stability of the asymptotic characteristics of the system are investigated.

In the classical theory of Lyapunov exponents, the linear systems of differential equations with bounded coefficients are considered, in addition, linear systems of differential equations with zero exponents are not considered. In the monograph the generalized Lyapunov exponents, generalized central exponents and generalized singular exponents introduced in the work are effectively used for the study of a system of differential equations with unbounded coefficients, and also for the study of linear systems of differential equations with zero exponents.

The results presented in the monograph have both theoretical and applied significance. It may be applied in the theory of stability, the theory of dynamical systems, control theory and in its applications.

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INTRODUCTION

The monograph is devoted to the study of some problems in the qualitative theory of differential equations and is related to the theory of Lyapunov exponents, the results of scientific research on linear systems of differential equations under small perturbations and the stability of the asymptotic characteristics of the system are presented.

The foundations of the qualitative theory of differential equations were introduced in the late 19th century by A. Poincaré [1] and A.M. Lyapunov [2].

The main problem studied by Lyapunov is the stability of the solution of a differential equation (mainly a solution representing equilibrium or periodic motion) on an infinite interval of the time axis for small changes in the initial conditions that determine the solution [3], see also [4]. The creation of the theory of stability put forward him in the position of the most outstanding mathematicians of the 19th century. Lyapunov proposed two methods for solving this problem. Investigation of stability, which is based on the analytical construction of a family of integral curves, as noted in [5], Lyapunov called by the first method of investigation. The first method is based on the theory of characteristic exponents of linear systems, developed by Lyapunov.

The general theory of Lyapunov exponents is described in detail in the monograph by B.F. Bylov, R.E. Vinograd, D.M. Grobman, V.V. Nemytsky [6]. Also there is information in the monographs of B.P. Demidovich [7], L.Ya. Adrianova [8], N.A. Izobov [9], I.V. Gayshun [10], in the reviews of V.M. Millionshchikov [11-13] and N.A. Izobov [14-18].

In the Lyapunov theory of characteristic exponents of linear differential systems

$$\dot{x} = A(t)x \quad (1)$$

the systems with bounded piecewise-continuous coefficients are considered.

The Lyapunov characteristic exponent [2, p. 27] of the solution $x(t) \neq 0$ of linear system (1) is a number

$$\lambda \equiv \overline{\lim}_{t \rightarrow +\infty} \frac{1}{t} \ln \|x(t)\|.$$

CONTENT

Introduction

1. On Baire classes of the central exponents of homomorphisms of a vector bundle
2. Generalized Lyapunov exponents of the system of differential equations
3. Generalized regular linear systems of differential equations with unbounded coefficients and stability of solutions of a system of differential equations
4. Generalized central and generalized special exponents of a linear system. Uniform estimates of the growth of solutions of a system of differential equations
5. On semicontinuity and continuity points of generalized Lyapunov exponents of the linear systems of differential equations with unbounded coefficients

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References

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