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BEK ISSAKHOV AND FARIZA RAKYMZHANKYZY, *Hyperimmunity and A-*
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$\mathcal{F}$ be a family of total functions which is computable by an oracle A. where A is an
ry set. A numbering $\alpha : \omega \mapsto \mathcal{F}$ is called A-computable if the binary function $\alpha(n)(x)$
mputable, [1].

MA 1. *Let $\mathcal{F}$ be an infinite A-computable family of total functions, where A is an arbitrary*
en $\mathcal{F}$ has an A-computable Friedberg numbering.

gree a is hyperimmune if a contains a hyperimmune set, and a is hyperimmune free
ise. Every nonzero degree comparable with $0'$ is hyperimmune. Dekker showed that
ry nonrecursive c.e. set A there is a hyperimmune set B such that $B \equiv_T A$, which
that every nonrecursive c.e. degree contains a hyperimmune set.

MA 2. *For every hyperimmune set A there exists a nonrecursive A-computable set B.*

known [2], that if A is an arbitrary set. $\mathcal{F}$ is an infinite A-computable family of total
ns and $\mathcal{F}$ has at least two nonequivalent A-computable Friedberg numberings, then
infinitely many pairwise nonequivalent A-computable Friedberg numberings. And
, if $\mathcal{F}$ is an infinite A-computable family of total functions, where $\emptyset' \leq_T A$, then $\mathcal{F}$
nitely many pairwise nonequivalent A-computable Friedberg numberings.
xtend these results:

OREM 3. *Let $\mathcal{F}$ be an infinite A-computable family of total functions, where A is a*
mune set. Then $\mathcal{F}$ has infinitely many pairwise nonequivalent A-computable Friedberg

QUESTION. *Is it true the formulation of previous theorem for infinite family?*
The main talk will be around this question.

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▸ ERIC JOHANNESSON AND ANDERS LUNDSTEDT, *When one must strengthen one's induction hypothesis*.
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Sometimes when trying to prove a fact by induction, one gets "stuck" at the induction step. The solution is often to use a "stronger" induction hypothesis, that is to prove a "stronger" result by induction. But in such cases, can we say that "strengthening the induction hypothesis" is necessary in order to prove the fact?

The general problem of when one must, in order to prove a fact X, first prove another fact Y, seems very hard. Interestingly, the special case of when one must strengthen one's induction hypothesis turns out to be more manageable. We provide the following characterization of when one in fact must strengthen one's induction hypothesis.

Let $\mathsf{Th}(\mathcal{N})$ be the set of sentences of first-order arithmetic that are true in the standard model. Let $T \subseteq \mathsf{Th}(\mathcal{N})$ and let $\varphi(x)$ and $\psi(x)$ be formulas both with at most one free variable x. Say that $\psi(x)$ *witnesses that T proves $\forall x \varphi(x)$ with and only with strengthened induction hypothesis* if and only if

(1) $T \cup \{\varphi(0) \wedge \forall x\,(\varphi(x) \to \varphi(x+1)) \to \forall x\,\varphi(x)\} \nvdash \forall x\,\varphi(x)$,
(2) $T \vdash \varphi(0)$,
(3) $T \vdash \psi(0)$,
(4) $T \vdash \forall x\,(\psi(x) \to \psi(x+1))$,
(5) $T \vdash \forall x\,\psi(x) \to \forall x\,\varphi(x)$.

We show that this definition applies to a number of natural examples. By reflecting on mathematical practice, we argue that this definition does capture the notion of "proof by strengthened induction hypothesis".

▸ DIANA KABYLZHANOVA, *A note on computably enumerable preorders*.
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A preorder is a reflexive and transitive binary relation. We are interested in computably enumerable (c.e.) preorders, in particular, in weakly precomplete c.e. preorders, [2]. Let P and Q be c.e. preorders. We say that P is computably reducible to Q ($P \leq_c Q$) if there is a computable function f such that xPy iff $f(x)Qf(y)$ for every $x, y \in \omega$. A c.e. preorder P is light if there exists a c.e. preorder Q in which all classes are singletones such that $Q \leq_c P$, and c.e. preorder P is called dark if P is not light and has no computable classes [1] A c.e.

in [1]. Looping is the main issue in the system GM^- developed in [1]. Looping may easily be removed by checking if a sequent has already occurred in the branch. Though this is insufficient as it requires much information to be stored. Some looping mechanisms have been considered earlier in ([2,3]). One way to detect loops is adding history to each sequent.

We introduce two systems for first order minimal logic (SwMin and ScMin) which are slightly different. Both systems are based on the idea of adding context to the sequents. In one system, SwMin, the history is kept smaller, but ScMin detects loops more quickly. The heart of the difference between the two systems is that in the SwMin loop checking is done when a formula leaves the goal, whereas in the ScMin it is done when it becomes the goal.

THEOREM.

1. *The systems GM^- and SwMin are equivalent.*
2. *The systems GM^- and ScMin are equivalent.*

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► NIKOLAY BAZHENOV, EKATERINA FOKINA, DINO ROSSEGGER, AND LUCA SAN MAURO, *Computable bi-embeddable categoricity of equivalence structures*.
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We study the algorithmic complexity of embeddings between bi-embeddable equivalence structures. To do this, we use the notions of Δ^0_α *bi-embeddable categoricity* and *relative* Δ^0_α *bi-embeddable categoricity* defined analogously to the standard concepts of Δ^0_α categoricity and relative Δ^0_α categoricity.

We give a characterization of Δ^0_1 bi-embeddably categorical equivalence structures, completely characterize Δ^0_2 bi-embeddably categorical and relatively Δ^0_2 bi-embeddably categorical equivalence structures, and show that all equivalence structures are relatively Δ^0_3 bi-embeddably categorical.

Furthermore, let the *degree of bi-embeddable categoricity* of a computable structure $\mathcal{A}$ be the least Turing degree that, if it exists, computes embeddings between any computable bi-embeddable copies of $\mathcal{A}$. Then every computable equivalence structure has a degree of bi-embeddable categoricity, and the only possible degrees of bi-embeddable categoricity for equivalence structures are $\mathbf{0}, \mathbf{0}'$, and $\mathbf{0}''$.

► NIKOLAY BAZHENOV AND BIRZHAN KALMURZAYEV, *Weakly precomplete dark computably enumerable equivalence relations*.
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