Solution of the inverse problem for n elliptic equations in cylindrical coordinates

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General elliptic equation is a universal mathematical model to describe the set of natural and technological processes, including steady-state diffusion of the temperature field. In actual processes is the problem of determining the temperature at the inner boundary of the cylindrical layer if the researcher is available for observation only the outer boundary. Special case of a mathematical model for this problem is the Cauchy problem for the Laplace equation, which is a classic example of an ill-posed (Hadamard problem). Moreover, in the areas of simple form solution of the problem can be obtained in the form of a series [1], but this solution is exponentially unstable with respect to perturbation of the Cauchy data. This fact can be attributed to this problem greatly flawed, according to the classification of inverse problems [2]. Significant progress has been in the solution of inverse problems at over the past decade, namely, the method of quasisolution [3] and regularization [4]. They permit to solve the task with an acceptable accuracy. Minimization of the residual functional method based on gradient methods is standard method in the numerical solution of the inverse problem. However, analytical methods have an advantage over the numerical methods in accuracy and the analysis of results, depending on the problem parameters. At first time the analytical formulas were obtained in [5] for the regularized solution of the initial-boundary problem of the Laplace equation in a rectangle. The method is based on a system of necessary conditions of a minimum residual functional and constructing the solutions of this system in their final form. In this paper, we implement this approach to solving the inverse problem for three-dimensional steady equation conduction in cylindrical geometry. The inverse problem for the continuation of an elliptic equation with variable coefficient for steady-state diffusion models in the cylindrical layer is considered. We need to restore stationary field at the inner boundary of the cylinder by the Cauchy data on the outer shell of an inhomogeneous cylinder. The problem is reduced to solving three types of Cauchy problems for secondorder ODE. The formulas are derived in the form of series for the regularized problem quasisolution on the basis of the necessary conditions for a minimum residual functional.

References

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