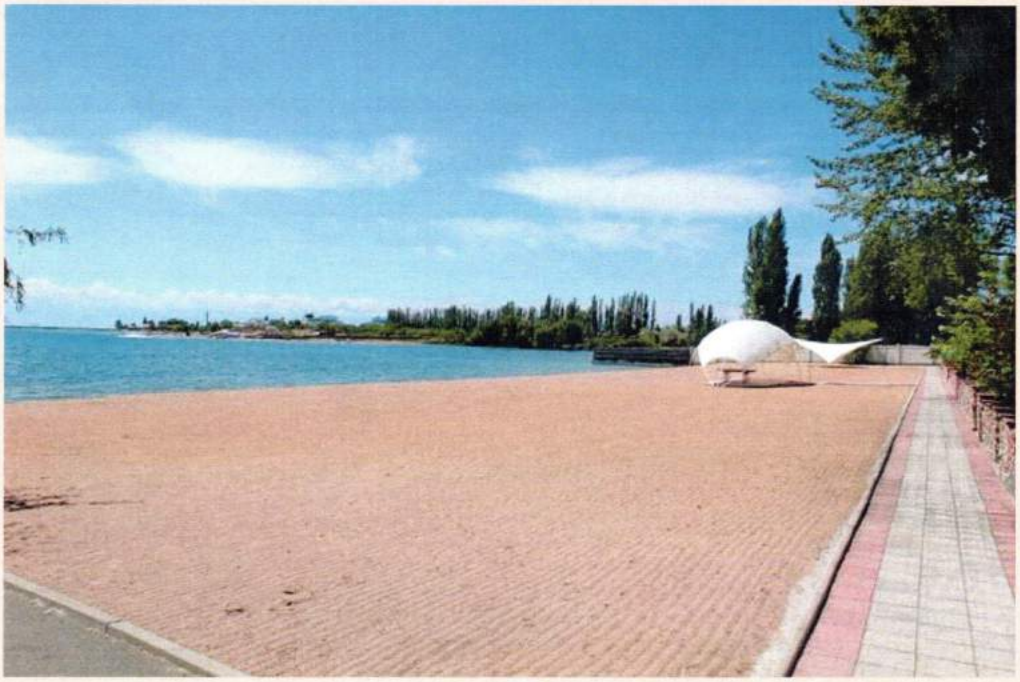

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XIV Международной Азиатской школы-семинара «ПРОБЛЕМЫ ОПТИМИЗАЦИИ СЛОЖНЫХ СИСТЕМ»

20 - 31 июля 2018 года

ЧАСТЬ 1

Кыргызская Республика
оз. Иссык-Куль
пансионат «Отель Евразия»

Алматы 2018

Институт информационных и вычислительных технологий МОН РК (Республика Казахстан, г. Алматы)

Институт вычислительной математики и математической геофизики СО РАН (Россия, г. Новосибирск)

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ON THE THEORY OF INVERSE PROBLEMS IN DEGENERATE DOMAINS

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Abstract. *In the paper we consider a coefficient inverse problem for the heat equation in a degenerating angular domain. It has been shown that the inverse problem for the homogeneous heat equation with homogeneous boundary conditions has a nontrivial solution up to a constant factor consistent with the integral condition. Moreover, the solution of the considered inverse problem is found in explicit form.*

Key words: *Coefficient inverse problem, Heat equation, Degenerating domain.*

Introduction

The inverse problems of this kind were investigated in the papers [1], [2] (see also literature from these works). In that papers it is assumed that the movable boundaries move according to the law obeying Holder class and the domain does not degenerate and the time interval is limited. There uniqueness and existence of the solution of the inverse problem where the required coefficient is a continuous function are established and numerical solutions are obtained.

The peculiarity of our study is that we consider the inverse problem for the heat equation in the degenerating angular domain. For the sake of simplicity and for the purpose of showing the effect of the degeneration of the domain, we consider the problem, where, firstly, the moving part of the boundary changes linearly; secondly, the boundary value problem is completely homogeneous; thirdly, the time interval is semi-bounded. It is known that when a domain degenerates at some points, the methods of separation of variables and integral transformations are generally not applicable to this type of problems. In this paper, to prove the existence of a non-trivial solution for the original problem we use the methods and results of our earlier works [3]-[6] where solutions are found with help of theory of thermal potentials and the Volterra integral equation of the second kind.

We also note works [7] and [8] devoted to the study of the existence of nontrivial solutions for partial differential equations, including for degenerating equations. In the paper [10] a theorem on the unique solvability of the non-homogeneous boundary value problem in weighted Holder spaces was obtained. We also note publications [11]-[15] of other authors that are close by category of this item of work.

The paper is divided as follows. In Section1, we give statement of the problem. In Section 2, we give auxiliary inverse problem in infinite domain. In Section 3, we present equivalent form of auxiliary problem. Section 4 is devoted to existence of the nontrivial solution (up to a constant factor). Nontrivial solution of equivalent form of auxiliary inverse problem is described in Sections 4 and 5. Sections 6 and 7 are devoted to the mathematical justification of the solution of the auxiliary inverse problem obtained in sections 4 and 5. Finally, conclusions are made in Section 8.

1. Statement of the problem

In the domain $G_T=\{(x,t)\,|\,0<x<t,\ 0<t<T,\},T<+\infty$, we consider an inverse problem of finding a coefficient $\lambda(t)$ and the function $u(x,t)$ for following heat equation:

$$u_t(x,t)=u_{xx}(x,t)-\lambda(t)u(x,t), \tag{1}$$

with homogeneous boundary conditions

$$u(x,t)\,|_{x=0}=0,\quad u(x,t)\,|_{x=t}=0,\ 0<t<T, \tag{2}$$

suspect to the overspecification

$$\int_0^t u(x,t)dx=E(t),\quad E(t)\geq\delta>0,\ 0<t<T, \tag{3}$$

where $E(t)\in L_\infty(0,T)$ is the given function.

2. The auxiliary problem

In accordance to the problem (1)-(3) we will set an auxiliary inverse problem in the domain $G_\infty=\{(x,t)\,|\,0<x<t,\ t>0\}$:

$$u_t(x,t)=u_{xx}(x,t)-\lambda(t)u(x,t), \tag{4}$$

with homogeneous boundary conditions

$$u(x,t)\,|_{x=0}=0,\quad u(x,t)\,|_{x=t}=0,\ t>0, \tag{5}$$

suspect to the overspecification

$$\int_0^t u(x,t)dx=\widetilde{E}(t),\ t>0, \tag{6}$$

$$\widetilde{E}(t)=\begin{cases}E(t), & 0<t<T,\\ E_1(t), & T\leq t<\infty,\end{cases} \tag{7}$$

where $E_1(t)\geq\delta>0$ -- an arbitrary bounded function.

Remark 1. Solving in G_∞ the problem (4)-(7) and restricting down its solution to the domain G_T, we can find the solution $\{u(x,t),\lambda(t);(x,t)\in G_T\}$ of the original inverse problem (1)-(3).

3. Equivalent problem

In the problem (4)-(6) we replace the required function by the following transformation

$$w(x,t)=e^{\int_0^t\lambda(s)ds}u(x,t)=\hat{\lambda}(t)u(x,t). \tag{8}$$

Then the inverse problem (4)-(6) reduces to a problem for the homogeneous heat equation:

$$w_t(x,t) = w_{xx}(x,t),\ \{x,t\} \in G_\infty, \tag{9}$$

with homogeneous boundary conditions

$$w(x,t)|_{x=0} = 0,\quad w(x,t)|_{x=t} = 0, t > 0, \tag{10}$$

subject to the over-specification

$$\int_0^t w(x,t)dx = \hat{\lambda}(t)\widetilde{E}(t),\ \ \widetilde{E}(t) \geq \delta > 0, t > 0. \tag{11}$$

4. On a nontrivial solution of the homogeneous boundary value problem (9)-(10)

It follows from our previous results [3]-[6] that a homogeneous boundary value problem (9)-(10) along with a trivial solution has a nontrivial solution up to a constant factor defined by formulas:

$$w(x,t) = \frac{1}{4\sqrt{\pi}}\int_0^t \frac{x}{(t-\tau)^{3/2}}\exp\left\{-\frac{x^2}{4(t-\tau)}\right\}\nu(\tau)d\tau +$$

$$+\frac{1}{4\sqrt{\pi}}\int_0^t \frac{x-\tau}{(t-\tau)^{3/2}}\exp\left\{-\frac{(x-\tau)^2}{4(t-\tau)}\right\}\varphi(\tau)d\tau, \tag{12}$$

$$\nu(t) = \frac{1}{2\sqrt{\pi}}\int_0^t \frac{\tau}{(t-\tau)^{3/2}}\exp\left\{-\frac{\tau^2}{4(t-\tau)}\right\}\varphi(\tau)d\tau, \tag{13}$$

where function $\varphi(t)$ is defined according to the formula:

$$\varphi(t) = C\varphi_0(t),\ \ C = const \neq 0, \tag{14}$$

$$\varphi_0(t) = \frac{1}{\sqrt{t}}\exp\left\{-\frac{t}{4}\right\} + \frac{\sqrt{\pi}}{2}\left[1 + erf\left(\frac{\sqrt{t}}{2}\right)\right], \tag{15}$$

moreover, the function $\varphi(t)$ belongs to the following class:

$$\theta(t)\varphi(t) \in L_\infty(R_+), \tag{16}$$

where

$$\theta(t) = \begin{cases} \sqrt{t}\exp\left\{\dfrac{t}{4}\right\}, & if\quad 0 < t \leq T, \\ 1, & if\quad T < t < +\infty. \end{cases} \tag{17}$$

Substituting $\nu(t)$ (13) in (12), we obtain

$$w(x,t) = w_+(x,t) + w_-(x,t), \tag{18}$$

where

$$w_+(x,t) = \frac{1}{4\sqrt{\pi}} \int_0^t \frac{x+\tau}{(t-\tau)^{3/2}} \exp\left\{-\frac{(x+\tau)^2}{4(t-\tau)}\right\} \varphi(\tau) d\tau, \tag{19}$$

$$w_-(x,t) = \frac{1}{4\sqrt{\pi}} \int_0^t \frac{x-\tau}{(t-\tau)^{3/2}} \exp\left\{-\frac{(x-\tau)^2}{4(t-\tau)}\right\} \varphi(\tau) d\tau. \tag{20}$$

5. The solution of the inverse problem (9)-(11)

From (14) and (18)-(20) we obtain for the solution $w(x,t) = Cw_0(x,t)$ of the homogeneous boundary value problem (9)-(10) the following representation:

$$w_0(x,t) = w_{0+}(x,t) + w_{0-}(x,t), \tag{21}$$

where

$$w_{0+}(x,t) = \frac{1}{4\sqrt{\pi}} \int_0^t \frac{x+\tau}{(t-\tau)^{3/2}} \exp\left\{-\frac{(x+\tau)^2}{4(t-\tau)}\right\} \varphi_0(\tau) d\tau, \tag{22}$$

$$w_{0-}(x,t) = \frac{1}{4\sqrt{\pi}} \int_0^t \frac{x-\tau}{(t-\tau)^{3/2}} \exp\left\{-\frac{(x-\tau)^2}{4(t-\tau)}\right\} \varphi_0(\tau) d\tau. \tag{23}$$

Further using the representation (21)-(22) for the integral condition (11), we get:

$$\int_0^t w_0(x,t)dx = \int_0^t w_{0+}(x,t)dx + \int_0^t w_{0-}(x,t)dx = \hat{\lambda}(t)\widetilde{E}(t). \tag{24}$$

By the commutativity property in the integrals of the formula (24), in the sense of the Dirichlet formula, we have:

$$\int_0^t w_{0\pm}(x,t)dx = \frac{1}{4\sqrt{\pi}} \int_0^t \varphi_0(\tau) d\tau \int_0^t \frac{x \pm \tau}{(t-\tau)^{3/2}} \exp\left\{-\frac{(x \pm \tau)^2}{4(t-\tau)}\right\} dx. \tag{25}$$

Let's calculate the interior integrals from (25). We get

$$\frac{1}{4\sqrt{\pi}} \int_0^t \frac{x \pm \tau}{(t-\tau)^{3/2}} \exp\left\{-\frac{(x \pm \tau)^2}{4(t-\tau)}\right\} dx = \left\| y = \frac{(x \pm \tau)^2}{4(t-\tau)} \right\| =$$

$$= \frac{1}{2\sqrt{\pi(t-\tau)}} \int_{\frac{\tau^2}{4(t-\tau)}}^{\frac{(t \pm \tau)^2}{4(t \mp \tau)}} \exp\{-y\} dy =$$

$$= \frac{1}{2\sqrt{\pi(t-\tau)}}(\exp\left\{-\frac{\tau^2}{4(t-\tau)}\right\} - \exp\left\{-\frac{(t\pm\tau)^2}{4(t-\tau)}\right\}). \quad (26)$$

Then from (11), (24)-(26) we obtain

$$\int_0^t w_0(x,t)dx = \frac{1}{2\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}[2\exp\left\{-\frac{\tau^2}{4(t-\tau)}\right\} -$$

$$-\exp\left\{-\frac{t-\tau}{4}\right\}(\exp\left\{-\frac{t\tau}{t-\tau}\right\}+1)]\varphi_0(\tau)d\tau = \hat{\lambda}_0(t)\widetilde{E}(t). \quad (27)$$

From ratios (8), (11), (27) and $w(x,t) = Cw_0(x,t)$ we find the required coefficient

$$\lambda(t) = \frac{d\ln(\hat{\lambda}(t))}{dt} = \frac{(\hat{\lambda}(t))'}{\hat{\lambda}(t)} = \lambda_0(t), \quad (28)$$

where we have used the equality

$$\left(\frac{\int_0^t w(x,t)dx}{\widetilde{E}(t)}\right)' : \frac{\int_0^t w(x,t)dx}{\widetilde{E}(t)} = \left(\frac{\int_0^t w_0(x,t)dx}{\widetilde{E}(t)}\right)' : \frac{\int_0^t w_0(x,t)dx}{\widetilde{E}(t)}.$$

Thus, the following theorem 1 is proved.

Theorem 1. *The inverse problem (1)-(3) has the following solution* $\{u(x,t),\lambda(t)\}$*: the coefficient* $\lambda(t) = \lambda_0(t)$ *is determined uniquely by the formula (28) by restricting it down to a finite interval* $(0,T)$ *and the solution* $u(x,t)$ *is found by means of the restriction of the function:*

$$u(x,t) = Cu_0(x,t) = C[\hat{\lambda}_0(t)]^{-1}w_0(x,t), \quad (29)$$

on the bounded triangle G_T *where* $w_0(x,t)$ *is defined by formulas (21)-(23).*

Remark 2. Sections 7 and 8 are devoted to the mathematical justification and identification of the features of the solution of the boundary value problem (9)-(10). It will be shown that this solution has a singularity of order $t^{-1/2}$ at small values of t. Since the domain G_T is determined by the relations $0 < x < t, 0 < t < T$, the small value of the variable t provides a small value of the variable x.

According to formulas (21)-(23), (15) the solution $w_0(x,t)$ is a nonnegative function. It should be noted that the function $\widetilde{E}(t)$ from (11) also is a nonnegative function, since the integral (24) is nonnegative and the coefficient $\hat{\lambda}_0(t)$ (8) is nonnegative function.

6. Estimate of the integral (27)

In this section, we will establish the boundedness of the integral (27), considering that the function $\varphi_0(t)$ (15) belongs to the class (16)-(17). The following theorem is true.

Theorem 2. *The integral in (27) is bounded function on semi-axis R_+.*

The proof of theorem 2 will follow from the statements of the following lemmas 1-4.

Lemma 1. *Let $0<t<T$ and $C=\|\theta(t)\varphi_0(t)\|_{L_\infty}(0<t<T)$. Then the following estimate holds*

$$\frac{1}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{\tau^2}{4(t-\tau)}\right\}\varphi_0(\tau)d\tau \le C\sqrt{\pi}. \tag{30}$$

Proof. We obtain

$$\frac{1}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{\tau(t-\tau)}}\exp\left\{-\frac{\tau^2}{4(t-\tau)}-\frac{\tau}{4}\right\}\sqrt{\tau}\exp\left\{\frac{\tau}{4}\right\}\varphi_0(\tau)d\tau \le$$

$$\le C\frac{1}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{\tau(t-\tau)}}\exp\left\{-\frac{\tau^2}{4(t-\tau)}-\frac{\tau}{4}\right\}d\tau \le$$

$$\le C\frac{1}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{\tau(t-\tau)}}d\tau = C\sqrt{\pi}.$$

Lemma 1 is proved.

Lemma 2. *Let $T<t<\infty$ and $C=\|\theta(t)\varphi_0(t)\|_{L_\infty}(T<t<\infty)$. Then the following estimate is true*

$$\frac{1}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{\tau^2}{4(t-\tau)}\right\}\varphi_0(\tau)d\tau \le 2C. \tag{31}$$

Proof. We have

$$\frac{1}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{\tau^2}{4(t-\tau)}\right\}\varphi_0(\tau)d\tau \le$$

$$\le C\frac{1}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{\tau^2}{4(t-\tau)}\right\}d\tau = \left\|z=\sqrt{t-\tau}\right\| =$$

$$=\frac{2C}{\sqrt{\pi}}\exp\left\{\frac{t}{2}\right\}\int_0^{\sqrt{t}}\exp\left\{-\frac{z^2}{4}-\frac{t^2}{4z^2}\right\}dz \le$$

$$\le \frac{2C}{\sqrt{\pi}}\exp\left\{\frac{t}{2}\right\}\int_0^{\infty}\exp\left\{-\frac{z^2}{4}-\frac{t^2}{4z^2}\right\}dz =$$

$$=\frac{2C}{\sqrt{\pi}}\exp\left\{\frac{t}{2}\right\}\frac{1}{2}\cdot\frac{\sqrt{\pi}}{\sqrt{\frac{1}{4}}}\exp\left\{-2\sqrt{\frac{1}{4}\cdot\frac{t^2}{4}}\right\} = 2C.$$

Here we used a well-known equality [9]

$$\int_0^\infty \exp\left\{-\mu x^2 - \frac{\eta}{x^2}\right\}dx = \frac{1}{2}\frac{\sqrt{\pi}}{\sqrt{\mu}}\exp\left\{-2\sqrt{\mu\eta}\right\} \tag{32}$$

Lemma 2 is proved.

Lemma 3. *Let* $0 < t < T$ *and* $C = \|\theta(t)\varphi_0(t)\|_{L_\infty}(0<t<T)$. *Then the following estimate take place*

$$\frac{1}{2\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{t-\tau}{4}\right\}(\exp\left\{-\frac{t\tau}{t-\tau}\right\}+1)\varphi_0(\tau)d\tau \le C\sqrt{\pi}. \tag{33}$$

Proof. We have

$$\frac{1}{2\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{t-\tau}{4}\right\}(\exp\left\{-\frac{t\tau}{t-\tau}\right\}+1)\varphi_0(\tau)d\tau \le$$

$$\le \frac{C}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{\tau(t-\tau)}}d\tau = C\sqrt{\pi}.$$

Lemma 3 is proved.

Lemma 4. *Let* $T < t < \infty$ *and* $C = \|\theta(t)\varphi_0(t)\|_{L_\infty}(T<t<\infty)$. *Then the following estimate is correct*

$$\frac{1}{2\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{t-\tau}{4}\right\}(\exp\left\{-\frac{t\tau}{t-\tau}\right\}+1)\varphi_0(\tau)d\tau \le 2C. \tag{34}$$

Proof. We have

$$\frac{1}{2\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{t-\tau}{4}\right\}(\exp\left\{-\frac{t\tau}{t-\tau}\right\}+1)\varphi_0(\tau)d\tau \le$$

$$\le \frac{C}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{t-\tau}{4}\right\}d\tau = \left\|z = \frac{\sqrt{t-\tau}}{2}\right\| =$$

$$= \frac{4C}{\sqrt{\pi}}\int_0^{\frac{\sqrt{t}}{2}} \exp\left\{-z^2\right\}dz = 2C\cdot erf\left(\frac{\sqrt{t}}{2}\right) \le 2C.$$

Lemma 4 is proved.

From the estimates (30)-(34) established in lemmas 1-4 we obtain the assertion of theorem 2. Theorem 2 is proved.

7. Estimate of the solution (21)-(23)

In this section, we establish the boundedness of the solution (21)-(23) of the boundary value problem (9)-(10), considering that the function $\varphi_0(t)$ (15) belongs to the class (16)-(17). The following theorem is true.

Theorem 3. *The solution (21)-(23) of the problem (9)-(10) is limited, except for the point* $\{x=0, t=0\}$, *where the solution has a singularity of order* $t^{-1/2}$.

The proof of theorem 3 will follow from the statements of the following lemmas 5-6.

Lemma 5. *Let* $0<t<T$ *and* $C=\|\theta(t)\varphi_0(t)\|_{L_\infty}(0<t<T)$. *Then the following estimate holds*

$$w_{0\pm}(x,t)=\frac{1}{4\sqrt{\pi}}\int_0^t\frac{x\pm\tau}{(t-\tau)^{3/2}}\exp\left\{-\frac{(x\pm\tau)^2}{4(t-\tau)}\right\}\varphi_0(\tau)d\tau\le$$

$$\le C(\frac{1}{\sqrt{t}}\exp\left\{-\frac{x^2}{4t}\right\}+\frac{\sqrt{\pi}}{4}). \qquad (35)$$

Proof.

$$w_{0\pm}(x,t)=\frac{1}{4\sqrt{\pi}}\int_0^t\frac{x\pm\tau}{(t-\tau)^{3/2}}\exp\left\{-\frac{(x\pm\tau)^2}{4(t-\tau)}\right\}\varphi_0(\tau)d\tau\le$$

$$\le\frac{C}{4\sqrt{\pi}}\int_0^t\frac{|x\pm\tau|}{\sqrt{\tau}(t-\tau)^{3/2}}\exp\left\{-\frac{(x\pm\tau)^2}{4(t-\tau)}-\frac{\tau}{4}\right\}d\tau=CI_{1\pm}(x,t).$$

We transform the kernel in the last integral. Using the ratios:

$$\frac{|x\pm\tau|}{\sqrt{\tau}(t-\tau)^{3/2}}\le\frac{|x\pm t|+(t-\tau)}{\sqrt{\tau}(t-\tau)^{3/2}}=\frac{|x\pm t|}{\sqrt{\tau}(t-\tau)^{3/2}}+\frac{1}{\sqrt{\tau(t-\tau)}},$$

$$-\frac{(x\pm\tau)^2}{4(t-\tau)}=-\frac{[x\pm t\mp(t-\tau)]^2}{4(t-\tau)}=-\frac{(x\pm t)^2}{4(t-\tau)}+\frac{\pm 2x+t}{4}+\frac{\tau}{4},$$

we have

$$I_{1\pm}(x,t)\le\frac{1}{4\sqrt{\pi}}\exp\left\{\frac{\pm 2x+t}{4}\right\}\int_0^t\frac{|x\pm t|}{\sqrt{\tau}(t-\tau)^{3/2}}\exp\left\{-\frac{(x\pm t)^2}{4(t-\tau)}\right\}d\tau+$$

$$+\frac{1}{4\sqrt{\pi}}\int_0^t\frac{1}{\sqrt{\tau(t-\tau)}}\exp\left\{-\frac{(x\pm\tau)^2}{4(t-\tau)}-\frac{\tau}{4}\right\}d\tau=I_{1\pm}^1(x,t)+I_{1\pm}^2(x,t). \qquad (36)$$

Firstly, we show the boundedness of the first integral (36). For this we introduce the following substitutions $2z_\pm=|x\pm t|(t-\tau)^{-1/2}, z_{1\pm}^2=z_\pm^2-(x\pm t)^2(4t)^{-1}$. Then we obtain

$$I_{1\pm}^1(x,t)=\frac{2\exp\{-\frac{x^2}{4t}\}}{\sqrt{\pi t}}\int_0^\infty\exp\{-z_{1\pm}^2\}dz_{1\pm}=\frac{\exp\{-\frac{x^2}{4t}\}}{\sqrt{t}}. \qquad (37)$$

Remark 3. For the small values of t the following asymptotic is true: $I_{1\pm}^1(x,t)\approx t^{-1/2}$. Indeed, we have

$$\frac{\exp\{-\frac{x^2}{4t}\}}{\sqrt{t}}=\frac{\exp\{-\frac{\varepsilon^2 t}{4}\}}{\sqrt{t}},$$

since $0<x<t$ and $x=\varepsilon t, 0<\varepsilon<1$.

For the second integral $I_{1\pm}^2(x,t)$ in the formula (36) we have:

$$I_{1\pm}^2(x,t)=\frac{1}{4\sqrt{\pi}}\int_0^t\frac{1}{\sqrt{\tau(t-\tau)}}\exp\left\{-\frac{(x\pm\tau)^2}{4(t-\tau)}-\frac{\tau}{4}\right\}d\tau\le$$

$$\le\frac{1}{4\sqrt{\pi}}\int_0^t\frac{1}{\sqrt{\tau(t-\tau)}}d\tau=\frac{\sqrt{\pi}}{4}.$$

Lemma 5 is proved.

Lemma 6. *Let* $T<t<\infty$ *and* $C=\|\theta(t)\varphi_0(t)\|_{L_\infty}(T<t<\infty)$. *Then the following estimate is correct*

$$w_{0\pm}(x,t)=\frac{1}{4\sqrt{\pi}}\int_0^t\frac{x\pm\tau}{(t-\tau)^{3/2}}\exp\left\{-\frac{(x\pm\tau)^2}{4(t-\tau)}\right\}\varphi_0(\tau)d\tau\le C. \tag{38}$$

Proof. As in proof of lemma 5, using similar transformations of independent variables, we obtain

$$w_{0\pm}(x,t)\le\frac{C}{4\sqrt{\pi}}\int_0^t\frac{|x\pm\tau|}{(t-\tau)^{3/2}}\exp\left\{-\frac{(x\pm\tau)^2}{4(t-\tau)}\right\}d\tau\le$$

$$=\frac{C}{4\sqrt{\pi}}\exp\left\{\frac{x\pm t}{2}\right\}\int_0^t\frac{|x\pm t|}{(t-\tau)^{3/2}}\exp\left\{-\frac{(x\pm t)^2}{4(t-\tau)}-\frac{t-\tau}{4}\right\}d\tau+$$

$$+\frac{C}{4\sqrt{\pi}}\exp\left\{\frac{x\pm t}{2}\right\}\int_0^t\frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{(x\pm t)^2}{4(t-\tau)}-\frac{t-\tau}{4}\right\}d\tau=$$

$$=C\left[I_{2\pm}^1(x,t)+I_{2\pm}^2(x,t)\right] \tag{39}$$

Using the substitution $2z_\pm=|x\pm t|(t-\tau)^{-1/2}$, for the first integral we get:

$$I_{2\pm}^1(x,t)=\frac{1}{\sqrt{\pi}}\exp\left\{\frac{x\pm t}{2}\right\}\int_{\frac{|x\pm t|}{\sqrt{t}}}^{\infty}\exp\left\{-z^2-\frac{(x\pm t)^2}{16z^2}\right\}dz\le$$

$$\le\frac{1}{\sqrt{\pi}}\exp\left\{\frac{x\pm t}{2}\right\}\int_0^\infty\exp\left\{-z^2-\frac{(x\pm t)^2}{16z^2}\right\}dz=\frac{1}{2}, \tag{40}$$

here we used a well-known equality (32).

For the second integral $I_{2\pm}^2$ we have

$$I_{2\pm}^2(x,t)=\frac{1}{4\sqrt{\pi}}\exp\left\{\frac{x\pm t}{2}\right\}\int_0^t\frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{(x\pm t)^2}{4(t-\tau)}-\frac{t-\tau}{4}\right\}d\tau=\left\|z_\pm=\frac{2\sqrt{t-\tau}}{|x\pm t|}\right\|=$$

$$= \frac{|x \pm t|}{4\sqrt{\pi}} \exp\left\{\frac{x \pm t}{2}\right\} \int_0^{2\sqrt{t}} |x \pm t| \exp\left\{-\frac{(x \pm t)^2}{16} z_\pm^2 - \frac{1}{z_\pm^2}\right\} dz_\pm \le$$

$$\le \frac{|x \pm t|}{4\sqrt{\pi}} \exp\left\{\frac{x \pm t}{2}\right\} \int_0^{\infty} \exp\left\{-\frac{(x \pm t)^2}{16} z_\pm^2 - \frac{1}{z_\pm^2}\right\} dz_\pm = \frac{1}{2}, \qquad (41)$$

where in (41) the known equality (32) was used.

Lemma 6 is proved.

From the estimates (35) and (38) established in lemmas 5-6 we obtain the assertion of theorem 3. The peculiarity of the solution at the point $\{x = 0, t = 0\}$ follows from the estimate (37) from the proof of lemma 5. Theorem 3 is proved.

8. Conclusion

In the paper we consider an inverse problem for the heat equation in a degenerating angular domain. We have shown that the inverse problem for the homogeneous heat equation with homogeneous boundary conditions has a nontrivial solution $\{u(x,t), \lambda(t)\}$ consistent with the integral condition. It was also proved that the found nontrivial solution is a bounded function for $\forall \{x,t\} \in G_\infty$.

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DEVELOPMENT OF SOFTWARE-HARDWARE FACILITIES FOR CRYPTOSYSTEMS BASED ON THE NONPOSITIONAL NUMBER SYSTEM

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Abstract. *The paper is mainly focused on the design aspects of a cryptographical system based on the nonpositional number system. The structure of the cryptographical primitive of the considered cryptosystem is shown. Some experimental results of the software*

СОДЕРЖАНИЕ

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