**AL-FARABI KAZAKH NATIONAL UNIVERSITY**

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# DATA MINING AND ANALYSIS

Educational manual

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The technology of data mining and analysis is used practically in all spheres of human activity, where retrospective data are accumulated. The methods of data mining and analysis were most widely spread in the following sectors: retail trade; the banking sector; insurance; telecommunications; industrial production; stock and currency markets. In this textbook, the tasks of prediction, classifying, clustering and association rules on specific examples are discussed in more detail.

The textbook is intended for undergraduate and graduate students of natural and technical science specialties.



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CONTENT

# INTRODUCTION

Data mining and analysis is a set of methods for detecting from large quantities of data (big data) previously unknown, practically useful knowledge for decision-making in various spheres of human activity. The peculiarity of the methods of mining and analysis of data, distinguishing them from traditional statistical methods are:

- the discovery of non-obvious regularities: these patterns do not identified by standard methods of information processing or expert by way of;

- the detection of objective laws: the knowledge obtained will be fully correspond to reality;

- finding practically useful regularities: the found knowledge can be found in concrete application in practice;

- reveal regularities without rigid restrictions to initial data and their distribution.

The most common tasks of data development and analysis are:

- classification,

- predicting,

- clustering,

- association.

The basis of methods of data development and analysis is various methods of classification, predicting, clustering, modeling, based on the application of: decision trees; artificial neural networks; genetic algorithms; associative memory; fuzzy logic, etc. Methods of mining and analysis of data also include multidimensional statistical methods: correlation and regression analysis; factor and component analysis; variance analysis; time series analysis and etc.

Data mining and analysis is a multi-stage and very labor-intensive process, which can be divided into three main stages:

- initial research;

- building a model;

- implementation of the model.

The most time-consuming stage of initial data mining involves:

- data cleaning: removal of duplicate observations from the sample, mistakenly

entered data with obvious errors, extreme values (outliers), checking logical rules and conditions;

- analysis and restoration, if necessary, of missing values in data;

- data conversion;

- setting up data properties;

- conducting an exploratory analysis of data using graphic and statistical methods;

- selection of the necessary data for building the model.

At the stage of the model construction, various models of data mining and analysis are considered, and the best of them is selected.

The implementation of the selected model implies its application to new data in order to obtain predictions or estimates of expected results, as well as subsequent monitoring of the quality of the model.

The technology of data mining and analysis is used practically in all spheres of human activity, where retrospective data are accumulated. The methods of development and analysis of data were most widely spread in the following sectors: retail trade; the banking sector; insurance; telecommunications; industrial production; stock and currency markets.

In this educational manual, the tasks of predicting, classifying, clustering and associative rules on specific examples are discussed in more detail. When studying the above problems, Excel is used for predicting tasks and the Rapidminer program for classification, clustering and associative rules tasks. Both programs are easily accessible: Excel is present in almost all local computers, and the Rapidminer program for educational purposes is freely available.

# TOPIC 1

# INTELLIGENT DATA ANALYSIS

## 1.1. Methodology of data mining

Data mining (data mining (DM) - Intelligent data analysis) is a process of discovering meaningful new relationships and dependencies by processing large amounts of data stored in repositories, using pattern recognition technology, statistical and mathematical methods.

There are various methods of data mining, among which is considered the de facto standard methodology CRISP-DM (CRoss-Industry Standard Process for DM) [1]. According to a survey in 2014 methodology CRISP-DM holds a leading position.

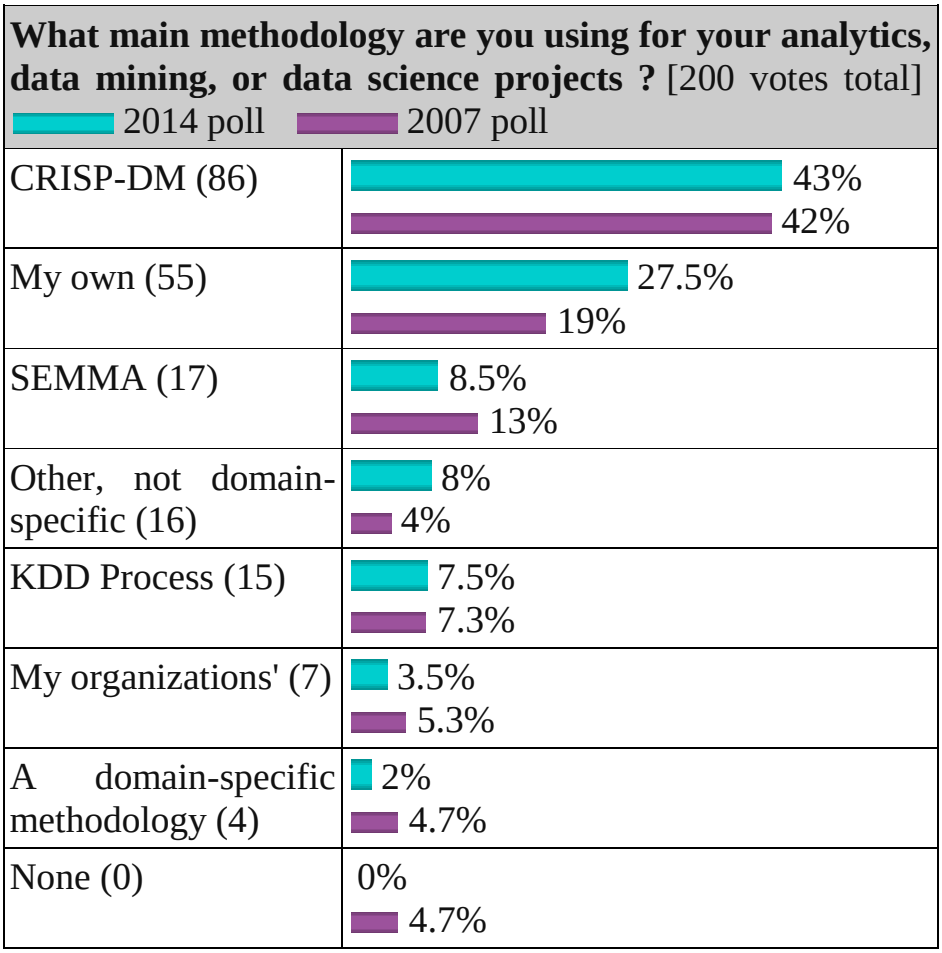



Fig. 1.1. The results of the survey on the use of the methodology of data mining (http://www.kdnuggets.com/)

The methodology CRISP-DM has a life cycle that includes six phases (Figure 1.2.) [2]:

Phase 1. "Business-understanding." At this phase in CRISP-DM standard defines the following tasks:

- The objectives and requirements of the project in terms of business or research units.

- Translating the objectives and constraints in the definition of the problem of data mining.

- Development of the preliminary strategy (plan) to achieve their goals.

Phase 2. "Understanding the data." The second phase begins with data collection, the description thereof. Identify problems with data quality, such as, for errors or omissions. A search of interesting data sets that may contain hidden patterns. Thus, this phase comprising the following tasks:

- The collection of baseline data;

Phase 3. "Preparing data" includes the following tasks:

- Starting from the raw data is preparing the final data set that must be used for all subsequent phases. This phase is very time-consuming;

- Selected cases and variables to be analyzed, and are suitable for the analysis;

- Conversion are performed on certain variables, if necessary;

- Is cleaned original data, so that they were ready for modeling tools.

Phase 4. "Modelling" includes the following tasks:

- Selection of appropriate modeling techniques;

- Calibration of the model parameters to optimize results;

- Can be used several modeling techniques for the same data mining.

- If necessary, loop back to the data preparation phase, to bring the shape data in accordance with the specific requirements of a particular means of data mining.

Phase 5. "assessment" includes the following tasks:

- Evaluation of one or more models are available as modeling phase and the effectiveness of their pre-deployment for use in the field;

- Determine whether the model is actually achieves the objectives set for it in the first phase;

- To establish that some important aspect of the business and research tasks are not taken into account adequately;

- A conclusion regarding the use of the results of data mining.

6. The phase "deployment (implementation)" includes the following tasks:

- Creation of a model does not mean the end of the project;

- An example of a simple deployment: report creation;

- An example of a more complex deployment: the implementation of parallel data mining in another department.

- For enterprises, the customer deploys often based on the model chosen.

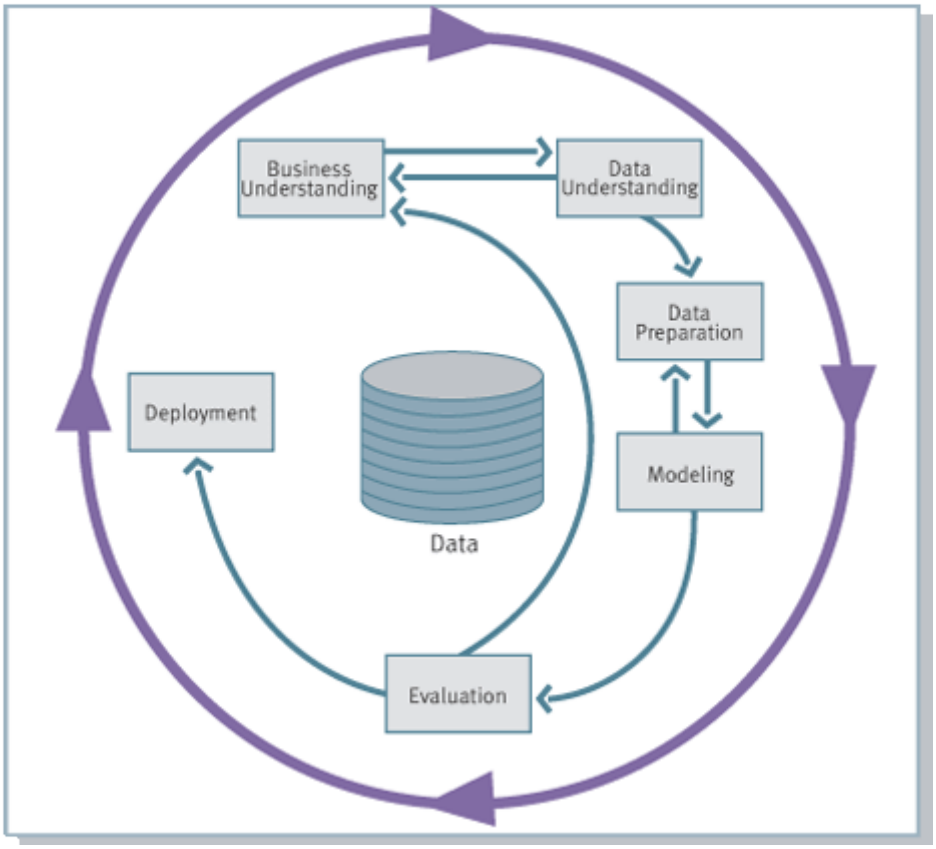



Fig. 1.2. The phases of the life cycle methodology CRISP-DM [1]

## 1.2 Tasks of Data Mining

The main tasks of the most common tasks of data mining include:

- classification

- prediction

- clustering

- Association.

**Classification task.**

Under the classification of objects commonly understood classification (observation events) to one of the previously known classes. For the classification must be present features characterizing the class to which belongs to this or that event or object. The classification relates to the learning strategies with the teacher (supervised learning). The problem of classification is commonly used categorical prediction of the dependent variable (ie, the dependent variable is the category) based on a sample of continuous and / or categorical variables.

Various methods are used for classification. The main ones are:

• Classification using decision trees;

• Bayesian (naive) classification;

• classification using artificial neural networks;

• classification by support vector machines;

• statistical methods such as linear regression;

• classification by using the nearest neighbor method.

**Prediction task.**

Prediction is classification similar, except that for the prediction, the results are in the future. Examples in the field of business studies and predicting tasks include:

- Predict the stock price for three months in the future;

- Prediction of the percentage increase in the incidence of accidents in traffic next year, if the speed limit is increased.

For prediction may be used as a traditional statistical evaluation methods and evaluation points, the confidence interval simple linear regression and correlation,

and multiple regression techniques and Data Mining and Knowledge Discovery, such as neural networks, k-nearest method for neighbor.

**Clustering task**

Clustering refers to the grouping of records, observations or instances into classes of similar objects. A cluster is a set of records that are similar to each other and different from the entries in other clusters. Clustering differs from classification in that there is no target variable for clustering. The clustering problem is not trying to classify, evaluate or predict the value of the target variable. Instead, clustering algorithms searching the initial set of data segmentation in a relatively homogeneous subgroups or clusters, where the cluster records maximized similarity and the similarity of the records of the cluster is minimized.

Clustering is often performed as a preliminary step in the process of data mining, and the resulting clusters are used as additional materials in a variety of technical methods in the following stages, such as neural networks.

**Association**

The task of association for the mining operation is located, which attributes "go hand in hand." Association Tasks are most common in the business world, where it is known as market basket analysis [2]. Association rules have the form of production rules: "if« A », and then« B »" with a measure of support and confidence of the rule. Support call the number or percentage of records containing a particular set of data. The reliability of the rules shows what is the likelihood of that event A implies event B. For example, a particular supermarket can find that out of 1,000 customers in stores Thursday, 200 customers have bought diapers, and of those 200 customers who bought diapers, 50 customers bought beer . Thus, the right of association will be "If you buy diapers, and then buy a beer" with the support of 200/1000 = 20% and reliability rules 50/200 = 25%.

Examples of problems in business and research associations include:

- Study the proportion of cell phone subscribers in the company's plan to respond positively to the proposal to upgrade services;

- The study of the percentage of children whose parents read to them are themselves good readers;

- To find out what items are purchased at the supermarket together and that the details have never purchased together

- Determination of the proportion of cases in which a new drug will exhibit dangerous side effects.

### 1.3 The phases of "Business Understanding" and " Data Understanding"

As it was said above, the following tasks are defined in the "Business-understanding" phase in the CRISP-DM standard:
- presentation of the objectives and requirements of the project in terms of business or research unit.
- translation of goals and limitations into the definition of the problem of data mining.
- development of a preliminary strategy (plan) to achieve the set goals.
In the " Data Understanding " phase, data are collected, described, analyzed and evaluated. This phase includes the following tasks:
- collection and description of source data;
-preliminary, semantic analysis of data on hidden patterns;
- evaluation of data quality.
Let's consider more in detail the tasks of these phases of data mining.
Let's consider on an example of maintenance of quality of cars for manufacturers of cars. Quality assurance is in a top priority for car manufacturers.
The goal of any car company is to reduce the costs associated with warranty claims and improve customer satisfaction. Through interviews with employees of the automotive company, who are technical experts in the production of vehicles, researchers can formulate specific business problems such as:
- Are there interdependencies between warranty claims?
- Are there any previous warranty claims related to similar requirements in the future?
- Is there a link between a certain type of claim and a specific garage?
The data mining plan consists in applying the appropriate data mining techniques to try and identify these and other possible associations.
In the "Understanding Data" phase, the collection and description of the initial data of a particular domain are performed. For the example in question, this is the database of the car manufacturer. How this database is used by consumers, how much this database and its elements are understood by end users. Is it possible to find answers to the questions identified in the previous stage of business understanding on this basis? Preliminary assessment of the quality of the data: the completeness of the data, if there are missing data, if there are data bursts, than what is explained.
In the following sections, the remaining phases of data mining are discussed in detail.

## 1.2 PRACTICAL LESSON 1 AND IWS (INDIVIDUAL WORK OF STUDENT) 1

**Subject.** Methodology and objectives of data mining.

**Lesson plan.**

1) Study the lecture material and methodology problems of data mining, above.

2) Setting the IWS. Make a detailed description of one of the objectives fo intelligent data analysis with examples.

3) A comparative analysis of this problem.

**Literature.**

1. Marbán Ó., Mariscal G. and Segovia J. A Data Mining & Knowledge Discovery Process Model. Data Mining and Knowledge Discovery in Real Life Applications, Book edited by: Julio Ponce and Adem Karahoca, ISBN 978-3-902613-53-0, pp. 438, February 2009, I-Tech, Vienna, Austria

2. Larose D.T. Discovering knowledge in data - an introduction to data mining. Wiley-Interscience, Hoboken, New Jersey, 2005

# TOPIC 2

# MODEL OF PREDICTION TASK

## 2.1. The lecture material

The prediction task is solved in two stages:

1) from the observable data $\boldsymbol{Y, t, X_1, X_2, \dots X_m}$ determine the form of the function F;

2) knowing the form of the prediction function to make a prediction, that is, to find the values of the variable Y for new values of independent variables and the time factor.

Solving the prediction task means making a prediction, which consists in finding the value of the variable Y, for which there are no values in the original data set.

The task of a prediction can be presented in the following form [1]:

$$\boldsymbol{Y = F(t, X_1, X_2, \dots X_m;\ \varepsilon)},$$

where the variable Y is a predictable variable, the variables $\boldsymbol{X_1, X_2, \dots X_m}$ are independent variables, t is the time factor, and ε is a random variable indicating that Y is also a random variable. A random variable determines the inaccuracy in the measurement of the values of the variable Y, as well as the incompleteness of knowledge about the effects of the time factor and factors $\boldsymbol{X_1, X_2, \dots X_m}$ on the variable Y. If the original data is presented in the form of a table (function graph) then $\boldsymbol{y_i}$ are the values of the function F for the specific values of its arguments $\boldsymbol{t_i, x_{1i}, x_{2i}, \dots x_{mi}}$, where i = 1,2, ..., n.

The determination of the form of the function F, which is unknown to the prediction problem, is performed on the basis of the available initial data, as well as some a priori considerations on the possible form of the function. The function is selected: $\boldsymbol{Y = F(b_1, b_2, \dots, b_k; t;\ X_1, X_2, \dots X_m)}$, where the parameters $\boldsymbol{b_1, b_2, \dots, b_k}$ are chosen so that the values of the function $F$ for given The values of the arguments $\boldsymbol{t_i, x_{1i}, x_{2i}, \dots x_{mi}}$ correspond to the values $\boldsymbol{y_i}$ as best as possible. Thus, the selected function $F$ approximates the observed data, that is, the function $F$ can be called an approximating function, since it approximates the observed data.

The constructed function $\boldsymbol{F}$ can be used for the second stage, the step of predicting a data of the dependent predicted variable for future periods of time for which there

are not yet observable data values. In this aspect, the approximating function $\boldsymbol{F}$ is called the prediction function.

The influence of the random variable $\boldsymbol{\varepsilon}$ is usually determined by adding a random effect to the value of the function $F$, i.e. The scheme of the random action $F + \varepsilon$, or the product of the random action and the value of the function $F$, is assumed. A scheme for the random action $\boldsymbol{F} * \varepsilon$ is adopted.

The predicted value of the predicted variable is calculated as the value of the prediction function for the values of the arguments $t_{n+1}, x_{1n+1}, x_{2n+1}, \dots x_{mn+1}$. If the prediction function does not explicitly contain the time argument t, then such data models are called cause-and-effect or casual. If the prediction function does not contain factor arguments $X_1, X_2, \dots X_m$ and depends only on time, then this model is called a time series model.

Models of time series are characterized by a number of concepts, such as a step or projection period, time horizon, the trend in seasonal changes. The prediction period - is the time step, which presents the data in the original data set table $t, X_1, X_2, \dots X_m, Y$. Prediction horizon - is the amount of the prediction period, which will make the prediction. Prediction horizon can be short-term (several periods), medium (about ten periods) or long term (more than ten periods). General trend is a trend in the data table changes the original data set according to the time. Seasonal changes - it is repetitive (periodic) change the values of the factors that influence the behavior of the entire system. Trend and seasonal changes are the components of the prediction function. There are additive model prediction function, when seasonal variations are added to the trend, i.e. $f(t) = T(t) + S(t)$, where T and S represent, respectively, the trend and seasonal components of the function f, and the multiplicative model, where the trend and seasonal variations are multiplied, i.e. $f(t) = T(t) * S(t)$.

## 2.2 PRACTICAL CLASSES 2 AND IWS 2

**Subject.** Building a data model and a trend line for the prediction task.

**Lesson plan.**

1) Formulate the problem of prediction and, by analogy with the table below, make a table of data for its prediction task.

| The period | Time t (Months) | Estimated variable Y (sales) |
|---|---|---|

| | | |
|---|---|---|
| 1 | January | 89390 |
| 2 | February | 95970 |
| 3 | March | 97850 |
| 4 | April | 80940 |
| 5 | May | 114760 |
| 6 | June | 89190 |
| 7 | July | 105235 |
| 8 | August | 108920 |

2) Using Excel tools, build a trend line for this prediction task. It is shown below how to do it. Analyze the results.

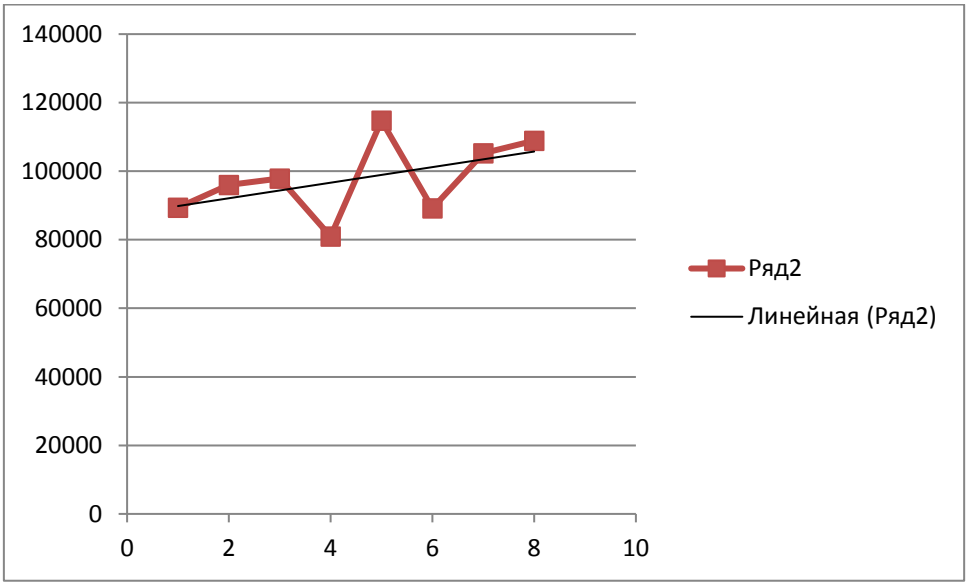
To build in Excel charts or diagrams for a given set of input data must be:

- Select the required set of data;

- Using the tab "Paste" option in the "Chart" to find the "Graphics" section;

- Click on the desired type of graphics.

It should be noted that the trend line cannot be added in bulk, petal, circular, annular and stacked charts. For the original data table above chart will look like:



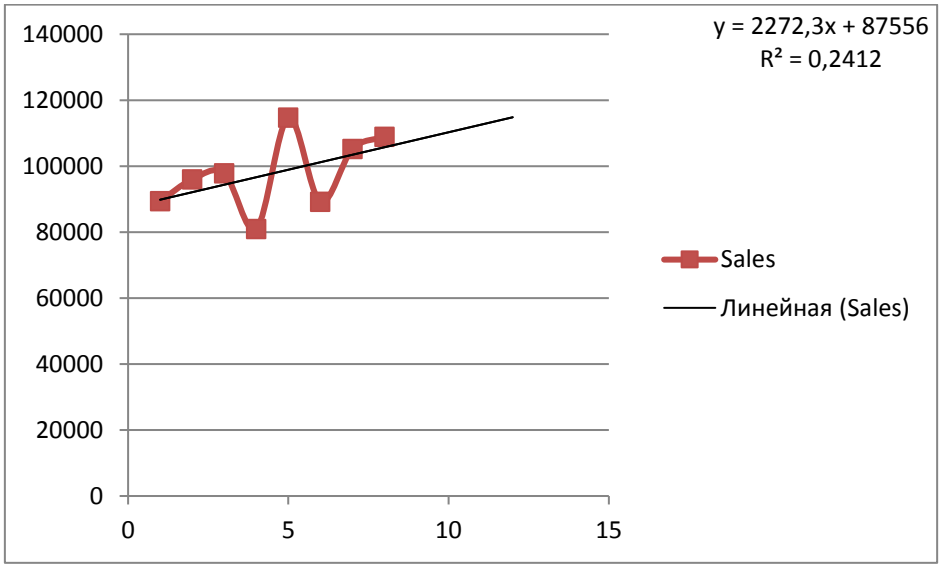

To construct a trend line for this set of data, follow these steps in Excel:

1) Click on the chart to select it;

2) Right-click on the chart, click on the pop-up menu option "format trend line" or "Add trend line";

3) in the Properties window of the trend line, specify parameters such as: the type of trend line forecast (forward or backward) to the desired number of periods, display the equation on the chart, showing squared (coefficient of determination).

The figure below shows a diagram of the forecast values of the predicted variable trend line.




However, the predicted values on the chart gives us a general idea, but not the exact values, i.e., schedule can determine the approximate values of the predicted values predicted variable.

The following topic will be considered standard features predicting and how to obtain specific values of the predicted variable.

Literature.

1. Minko A.A. Prediction in business using Excel. M .: Eksmo, 2007, -208 p.

# TOPIC 3

# PREDICTING USING STANDARD FUNCTIONS

## 3.1. Lecture material

In Excel, there are a number of standard functions that allow to solve the prediction problem using known mathematical functions, such as: linear function, logarithmic function, polynomial function, power function, exponential and other functions. Excel features allow you to select any function from the standard functions available in Excel to approximate the original data set of a specific task. Further, based on the selected function, it is possible predict the value of the dependent variable for a certain number of periods in the future. The use of standard Excel functions for solving the prediction problem is discussed below with a specific example.

The initial data set with the predicted "sales" variable is set:

| Period(month) | Sales |
|---|---|
| 1 | 89390 |
| 2 | 85970 |
| 3 | 97850 |
| 4 | 80940 |
| 5 | 114760 |
| 6 | 89190 |
| 7 | 135235 |
| 8 | 158920 |

When selecting a trend line in Excel, you can specify the following types:

- Linear. The linear trend line is described by the equation $Y = mX + b$, where $X$- is the variable-factor, $m$ and $b$ are the computable parameters of the trend line, where $m$ - determines the slope of the line, $b$ - defines the point of intersection of the line with the vertical coordinate axis (ordinate).

The diagram of the linear trend line is shown below in the figure.

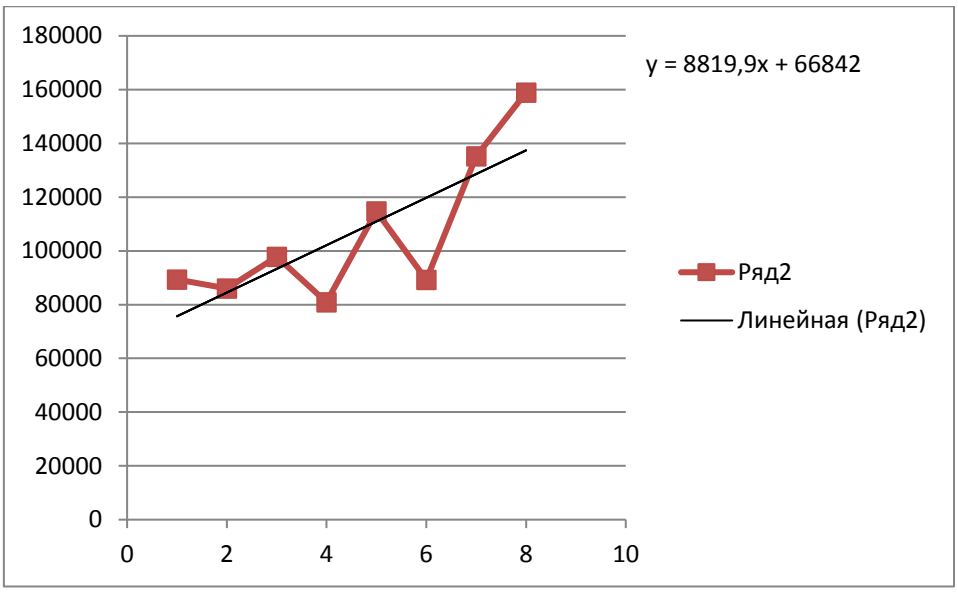



Fig. 3.1. Linear trend line.

- Logarithmic. The logarithmic trend line is described by the equation $Y = c\ln(X) + b$, where $c$ and $b$ are the computable parameters of the logarithmic trend line.

The diagram of the logarithmic trend line is shown below in the figure.

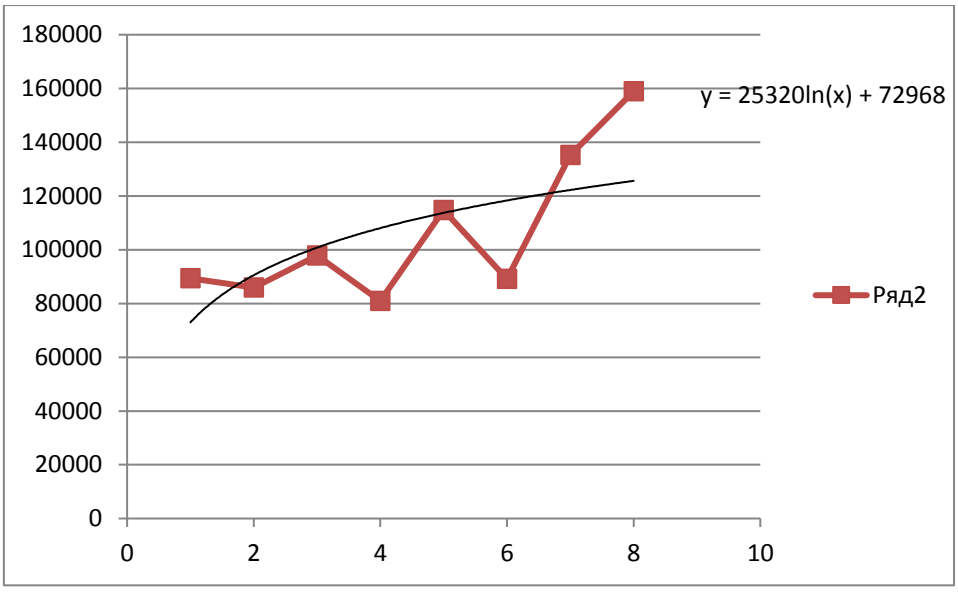



Fig. 3.2. Logarithmic trend line.

- Polynomial. The equation of the polynomial trend line is $Y = c_n\ x^n + c_{n-1}\ x^{n-1} + \cdots c_2\ x^2 + c_1\ x^1$+b, where $c_n, c_{n-1}, \ldots, c_2, c_1$, b are the computable parameters of the polynomial trend line.

The diagram of the polynomial trend line is shown below in the figure.

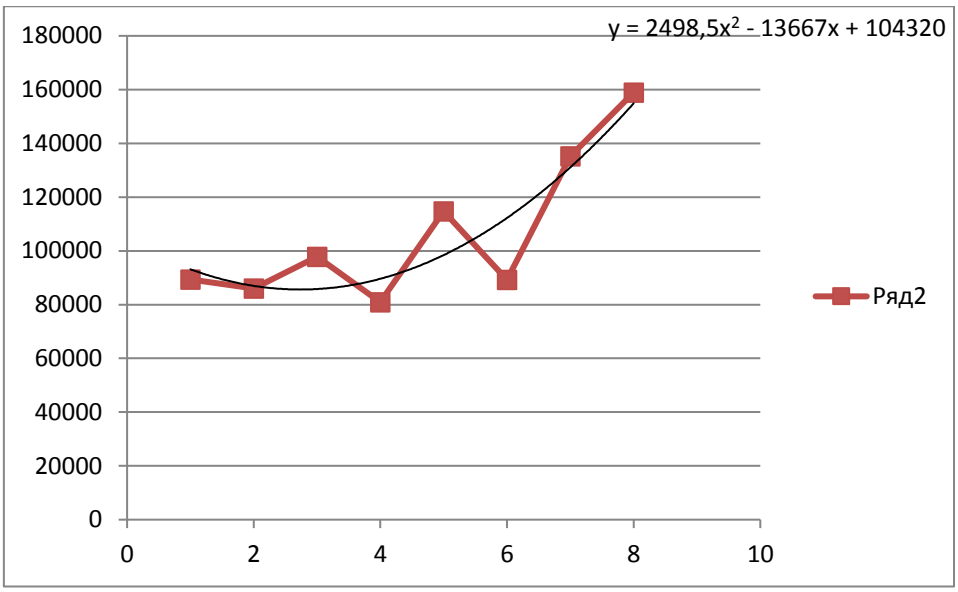



Fig. 3.3. Polynomial trend line.

- Power. The equation of the power trend line has the form $Y = cX^b$ where $c$, b are computable parameters of the power trend line.

The diagram of the power line of the trend is shown below in the figure.

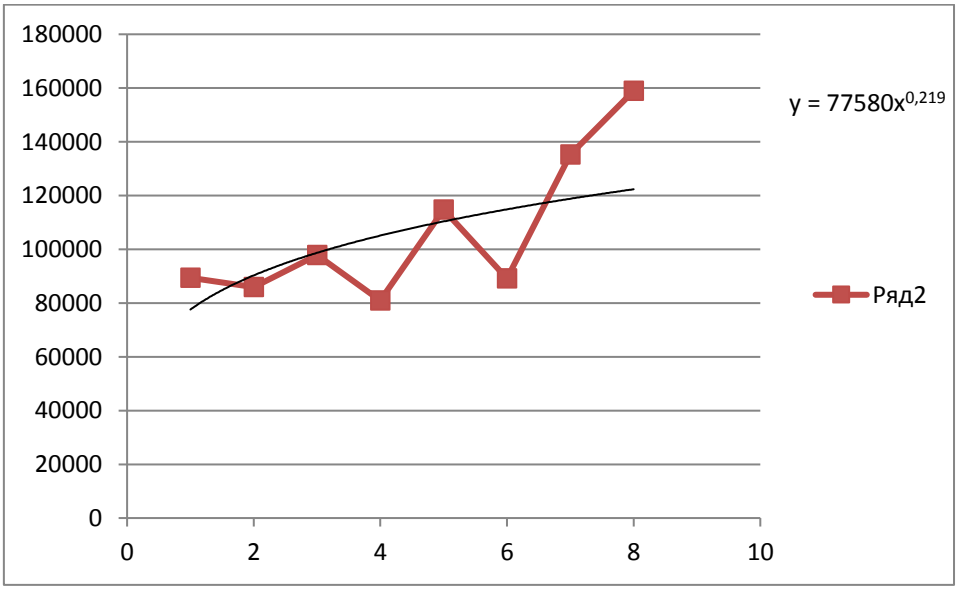



Fig. 3.4. Power line trend.

- The exponential. The equation of the exponential trend line has the form $Y = ce^{bX}$ where $c$, b are computable parameters of the exponential trend line.

The diagram of the exponential trend line is shown below in the figure.

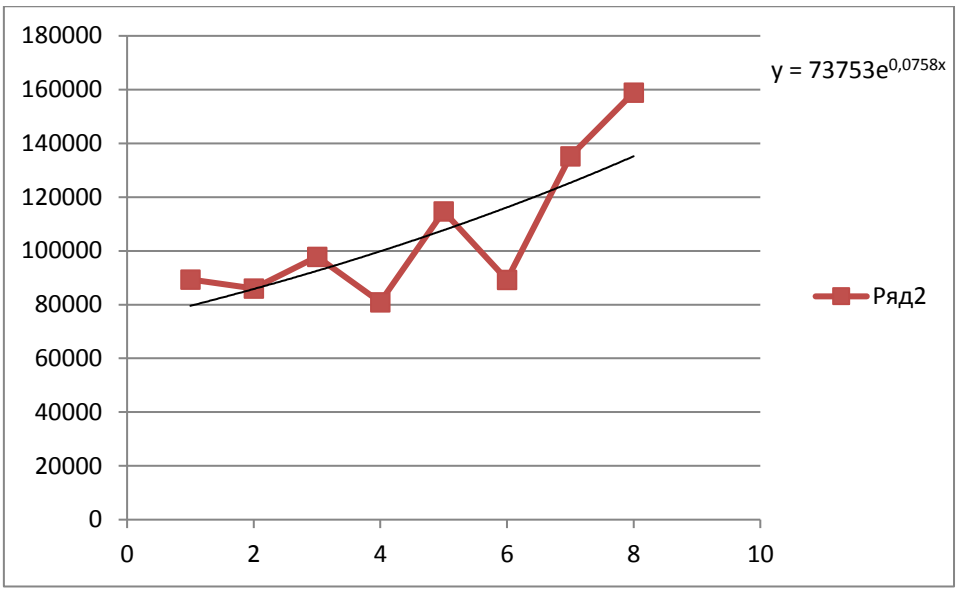



Fig. 3.5. Exponential trend line.

Let's make a prediction for the exponential trend line. To do this, right-click on the trend line you need to open the "trend line format" and specify how many periods to make a prediction. In this example, the prediction is for 4 periods ahead.

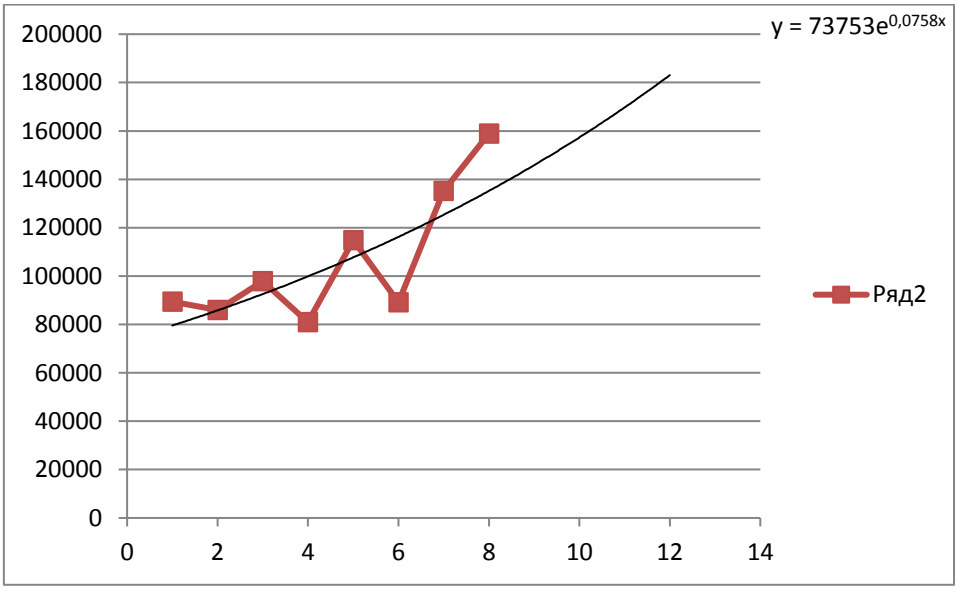



Fig. 3.6. Forecast by exponential trend line.

However, the chart does not allow you to see the exact values of the predicted values of the predicted variable. You can see only approximate values of the forecast. To obtain accurate prediction values, it is necessary to use the obtained trend equation in the diagram. For example, this will be the exponential equation $y = 73753e^{0,0758x}$. Substituting the values of the forecast periods for x, we will obtain the numerical values of the predicted variable for the given four prediction periods.

To do this, you must create columns in other free columns in the Excel workbook for the predicted periods and sales. In the first element of the "sale" column, you must type the trend equation, then copy it and write (extend) it to the other elements of this column. As a result, the following predicted values for the exponential trend line will be obtained.

| Period(month) | Sales |
|---|---|
| 9 | 145900 |
| 10 | 157389,2 |
| 11 | 169783,1 |
| **12** | 183153 |

Below are screenshots of obtaining numerical values of the predicted variable for exponential and power trend lines.

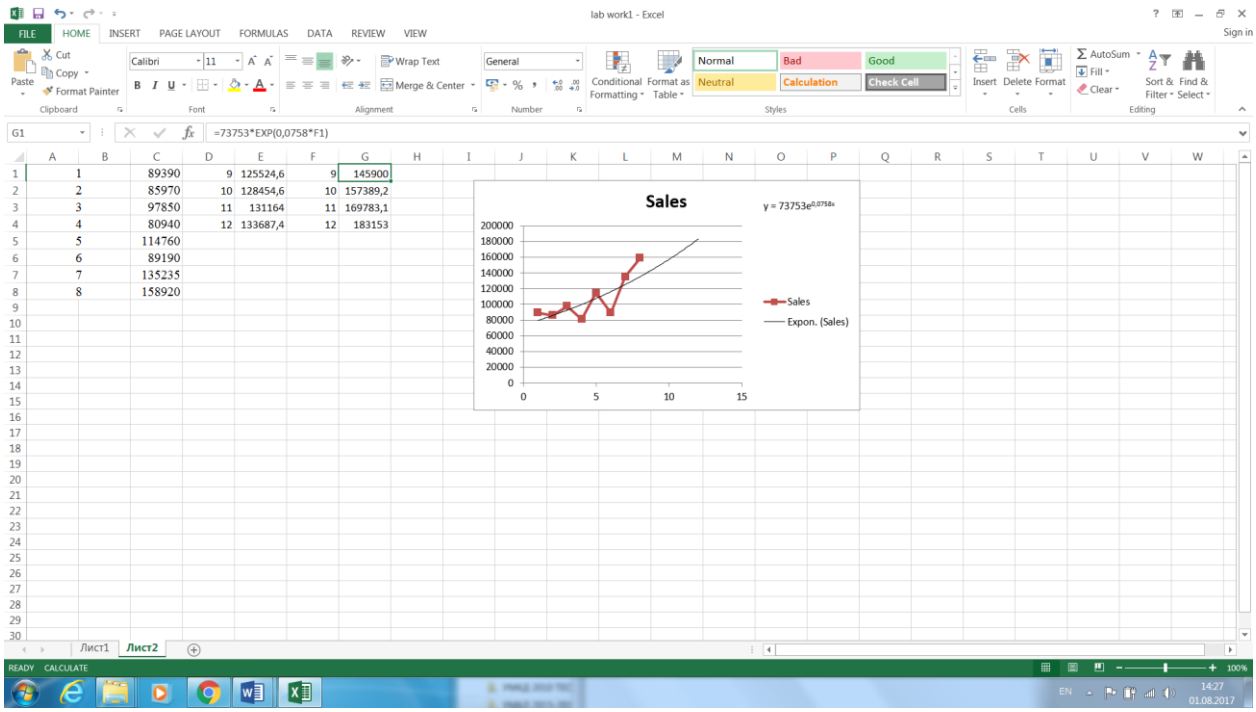



Fig. 3.7. Screenshot of obtaining numerical values of the predicted variable for the exponential trend line

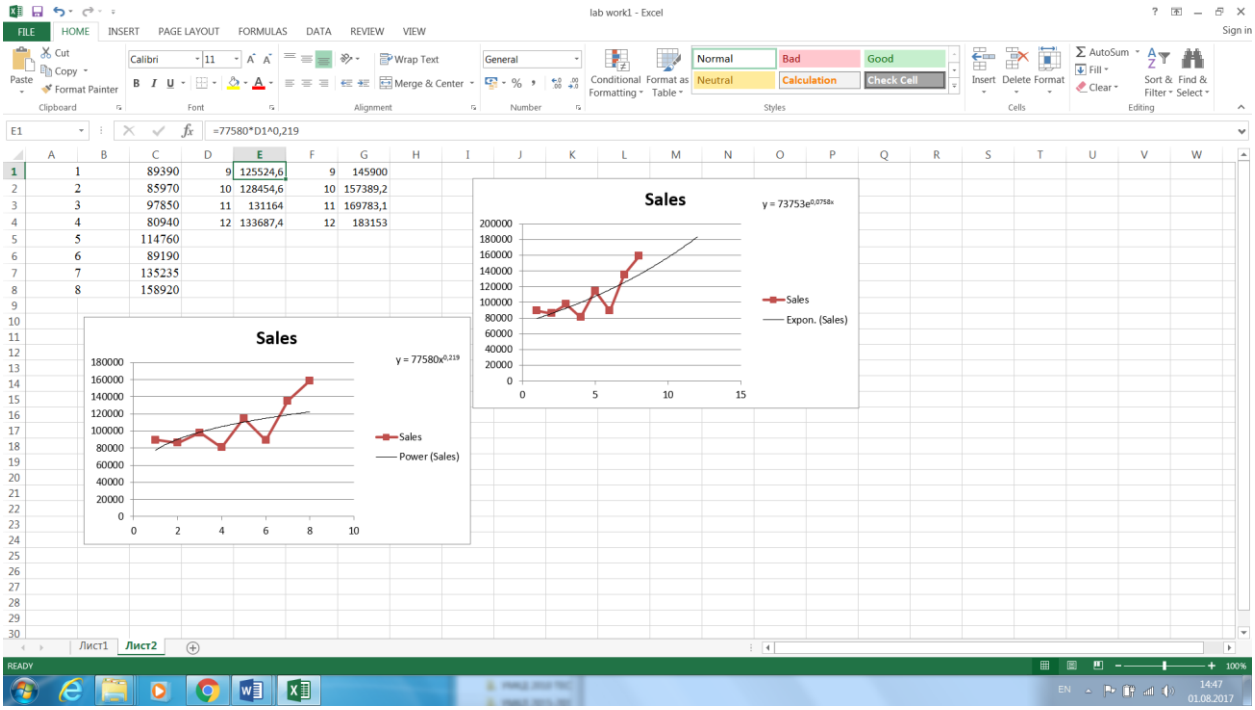



Fig. 3.8. Screenshot of obtaining the numerical values of the predicted variable for the power line trend

## 3.2 PRACTICAL LESSON 3 and IWS 3

**Subject.** Construction of a table of predictive values for the prediction task

**Plan of the lesson.**

1) Construct various trend lines for the prediction task formulated in the previous lesson.

2) The task of the IWS. Construct predictive values for different predicted variables. Analyze the results.

Literature.

1. Minko A.A. Prediction in business using Excel. M.: Eksmo, 2007, -208 p.

**TOPIC 4**

# CALCULATION OF THE DETERMINATION COEFFICIENT FOR PREDICTION FUNCTION

## 4.1. Lecture material

In Section 2, we showed a scheme for solving the prediction task, consisting of two stages: the determination of the approximation function and the determination of the values of the predicted variable from the approximation function by the future values of the independent variables. Naturally, the quality of determining the values of the predicted variable by the future values of independent variables essentially depends on the degree of accuracy of the approximation of the input data by the prediction function. In this section, one of the tools for determining the accuracy of the approximation of the input data by the prediction function is considered, namely, the determination coefficient / 1,2 /. The coefficient of determination shows the degree of accuracy of the approximation of the initial data by the prediction function. The coefficient of determination is denoted as $R^2$ and is calculated by the formula:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} e_i^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}$$

Here $y_i$ - *the observed values* of the predicted variable $Y$;

$\bar{y}$ - *the average value* of $y_i$: $\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$ . This is the average value of the predicted variable for the initial n points of the variable;

$e_i$- *residuals or prediction errors* defined as $e_i = y_i - f(X_i)$and $f(X_i)$ are the values of the prediction function at the i-th point of the original data, i.e. residuals are the difference between the observed values and the values of the prediction function, or in other words are errors of approximation of the observed values by the prediction function;

The sum $\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - f(X_i))^2$ is *the sum of the squares of the residuals*, or the sum of the squares of the prediction (approximation) errors at each point of the original data. Denoted as SSE - the sum of squares of the residuals (sum of squares errors);

The sum in the denominator $\sum_{i=1}^{n}(y_i - \bar{y})^2$, is called *the total sum of squares of deviations* from the mean of the predicted variable (sum of squares total- SST).

It should be noted here that there is another component of the measure of the accuracy of approximation of the observed values by the prediction function. This is the sum of the squares of the regression (sum of squares regression - SSR), this is the sum of the squares of the deviations of the prediction function values from the average observed values of the predicted variable Y.

The sum of squares of the regression SSR is determined by the formula $\sum_{i=1}^{n}(f(X_i) - \bar{y})^2$.

And the sum of SSR and SSE is SST:

SST = SSR + SSE.

Then the coefficient of determination can be represented as:

$$R^2 = 1 - \frac{SSE}{SST}$$

However, the coefficient of determination can also be represented as:

$$R^2 = \frac{SSR}{SST}$$

Then the formula for the determination coefficient will be as follows:

$$R^2 = \frac{\sum_{i=1}^{n}(f(X_i) - \bar{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

From the formula for determining the coefficient of determination, it is clear that the values of $R^2 \leq 1$. The value of the determination coefficient is $R^2 = 1$ if the sum of the squares of the residues is zero, i.e. when all residuals are zero or when the original data is accurately described by the data model.

The coefficient of determination in Excel can be automatically calculated using the standard functions LINEST and LOGEST /1, 3/. The LINEST function is used to calculate the coefficients of multiple linear or polynomial regressions, and the LOGEST function is used to calculate the coefficients of exponential regression. The LINEST function calculates the coefficients $b_0, m_1, \ldots, m_k$ in the equation $Y = b_0 + m_1X_1 + \cdots + m_kX_k$ of the linear multiple regression, or the same coefficients in the

equation $Y = b_0 + m_1X + \cdots + m_kX^k$ of the polynomial regression (from one factor). The LOGEST function calculates the coefficients $b_{0,}, m_1, \ldots, m_k$

In the equation $Y = b_0 * {m_1}^{X_1} * \ldots * {m_k}^{X_k}$ of exponential regression.

The syntax for calling these functions is as follows:

= LINEST (Values _Y; {Values _X}; Constant; Statistics)

= LOGEST(Values _Y; {Values _X}; Constant; Statistics)

The argument "Values _Y" is a one-dimensional array of values for the variable Y (or a reference to a range of cells containing this array). The optional argument {Values _X} is an array of X factor values (or a reference to a range of cells containing this array). If this argument is omitted, then it is assumed that this is an array of natural numbers {1,2,3, ...} of the same size as the array "Values _Y". The "Constant" argument indicates whether the coefficient $b_0$of linear, polynomial or exponential regression should be equal to 0. If this argument is TRUE, 1 or omitted, the coefficient $b_0$ is calculated as usual. If the argument is FALSE or 0, then $b_0$is set to 0, and the values of the coefficients $m_i$are selected with this in mind.

The "Statistics" argument indicates whether additional statistical characteristics of the regression are to be calculated. If this argument is TRUE or 1, then the function calculates and displays additional characteristics. If the Statistics argument is FALSE, O or omitted, then the function returns only the values of the coefficients mi and b0. Calculated additional statistical characteristics:

- $s_1, s_2, \ldots, s_k$- mean-square deviations for the coefficients $m_1, m_2, \ldots, m_k$;

- $s_b$ - mean-square deviations for the coefficient $b_0$;

- $R^2$- coefficient of determination;

- $s_\varepsilon$- residual standard deviation (standard regression error);

- F - criterial statistics for checking the significance of the regression equation;

- *df* is the degree of freedom;

- SSR - the sum of squares of regression (sum of squares regression);

- SSE is the sum of the squares of the residuals (sum of squares error).

Additional statistical characteristics of regression are located in the output array of standard functions LINEST and LGRF as follows:

| | | | | | |
|---|---|---|---|---|---|
| $m_k$ | $m_{k-1}$ | ... | $m_2$ | $m_1$ | $b_0$ |
| $S_k$ | $S_{k-1}$ | ... | $S_2$ | $S_1$ | $S_b$ |
| $R^2$ | $S\varepsilon$ | | | | |
| F | $df$ | | | | |
| SSR | SSE | | | | |

The remaining cells of the output array of additional statistical characteristics of the regression are filled with the values # H / D.

An example of the calculation of the LINEST function of additional statistical characteristics of the regression is shown in Fig.4.1 .

To perform the calculations by the LINEST function it is need:

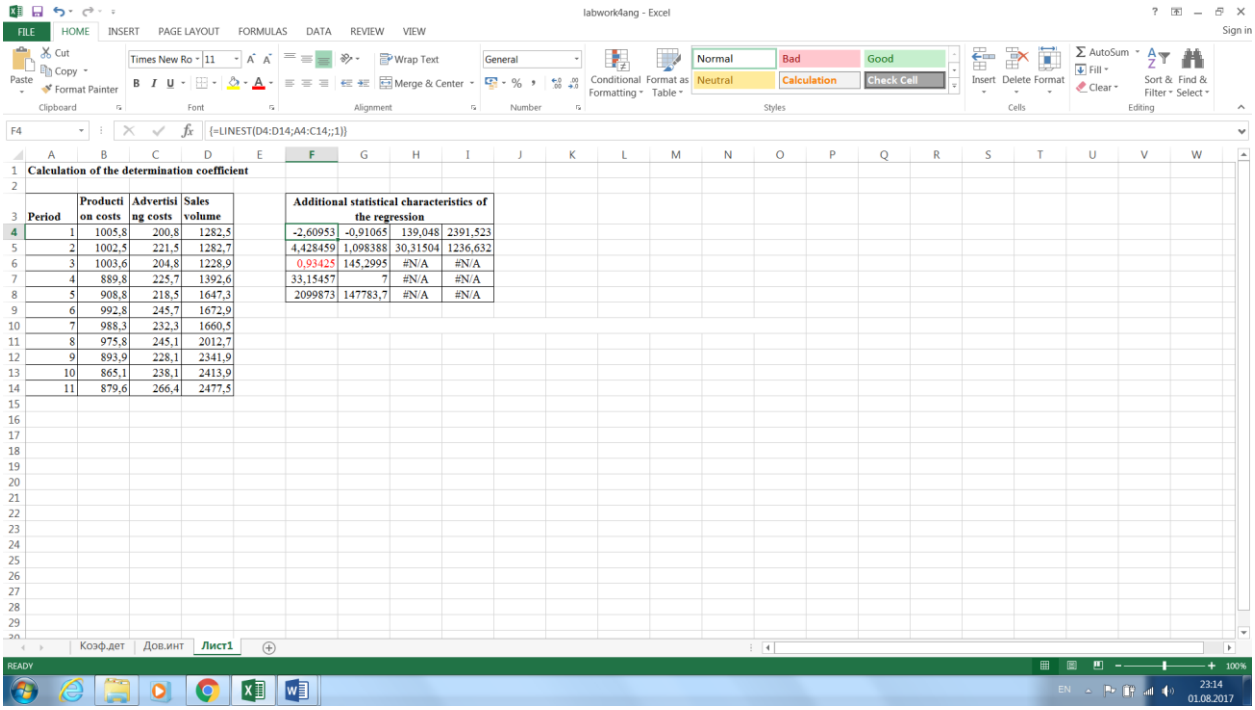
1) select the range F4: I9 (4 columns according to the number of coefficients of the regression equation and 5 lines);

2) without removing selection, enter the formula = LINEST (D4: D14; A4: C14 ;; 1);

3) and then **Press the keys <Ctrl + Shift + Enter>.**

Additional statistical characteristics of regression for the given example are calculated in the field F4: I9.

The value of the determination coefficient is written in cell F6.



Calculation of the determination coefficient

| Period | Production costs | Advertising costs | Sales volume |
|---|---|---|---|
| 1 | 1005,8 | 200,8 | 1282,5 |
| 2 | 1002,5 | 221,5 | 1282,7 |
| 3 | 1003,6 | 204,8 | 1228,9 |
| 4 | 889,8 | 225,7 | 1392,6 |
| 5 | 908,8 | 218,5 | 1647,3 |
| 6 | 992,8 | 245,7 | 1672,9 |
| 7 | 988,3 | 232,3 | 1660,5 |
| 8 | 975,8 | 245,1 | 2012,7 |
| 9 | 893,9 | 228,1 | 2341,9 |
| 10 | 865,1 | 238,1 | 2413,9 |
| 11 | 879,6 | 266,4 | 2477,5 |

Additional statistical characteristics of the regression

| | | | |
|---|---|---|---|
| -2,60953 | -0,91065 | 139,048 | 2391,523 |
| 4,428459 | 1,098388 | 30,31504 | 1236,632 |
| 0,93425 | 145,2995 | #N/A | #N/A |
| 33,15457 | 7 | #N/A | #N/A |
| 2099873 | 147783,7 | #N/A | #N/A |

Fig. 4.1 Calculation of additional statistical characteristics of regression by the LINEST function.

## 4.2 PRACTICAL LESSION 4 and IWS 4

**Subject:** Calculation of the determination coefficient for the prediction problem.

**Plan of the lesson.**

1) Study the calculation scheme for calculating the determination coefficient for the prediction task, presented above.

2) Practice and task of the IWS. Apply the above described calculation scheme of statistical characteristics for its prediction task.

3) Analyze the results.

Literature.

1. Minko A.A. Prediction in business using Excel. M.: Eksmo, 2007, 208 p.

2. Larose D.T. Discovering knowledge in data - an introduction to data mining. Wiley-Interscience, Hoboken, New Jersey, 2005

3. Excel functions. https://support.office.com/ en-us/article/excel-functions.

**TOPIC 5**

**CALCULATION OF CONFIDENCE INTERVALS FOR PREDICTED VALUES**

## 5.1. Lecture material

Confidence interval for the predicted value is a random interval, which with a given probability α contains an unknown exact value of the prediction function F (X). The probability α is called a confidence level.

The confidence interval is calculated using the following formula /1, 2/:

$$y_p \pm t * s_p,$$

where $y_p = f(x_p)$ is the predicted value of the prediction function $f$;
$x_p$ is the value of the factor X for which it is necessary to calculate the value of the predicted variable Y;
t is the quantile of the order (1 + α) / 2 of the Student's distribution with (n-k-1) degree of freedom (k is the degree of the polynomial of the regression function), α is the confidence level;
$s_p$ is the prediction error for the variable Y at the point $y_p$, defined by the formula /1, 2/:

$$s_p = s_\varepsilon \sqrt{1 + \frac{1}{n} + \frac{(x_p - \bar{x})^2}{SS_x}},$$

where

$s_\varepsilon$ - is the standard error of calculating the regression (calculated by the LINEST function;

$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ - is the mean of $x_1, x_2, \dots, x_n$ of the factor X variable;

$SS_x = \sum_{i=1}^{n}(x_i - \bar{x})^2$ - the sum of the squares of the deviations of the factor values from the mean.

The value of the quantile t of the Student's distribution depends on the given confidence level α, usually given as 90, 95 or 99%. Often the confidence level is set at 95%. The use of Student's quantile t imposes a number of conditions on the regression model such as independence and normal distribution of residuals with zero mathematical expectations and the same variances. Often in practice, since the confirmation of these conditions requires enough time resources, an empirical rule of "three sigma" is used.

This rule is not tied to the distribution of residues, and the number 3 is not much larger than the Student's t-test quantile for α = 95%. Then the upper and lower limits of the confidence interval can be calculated by the formula:

$$y_p \pm 3s_p.$$

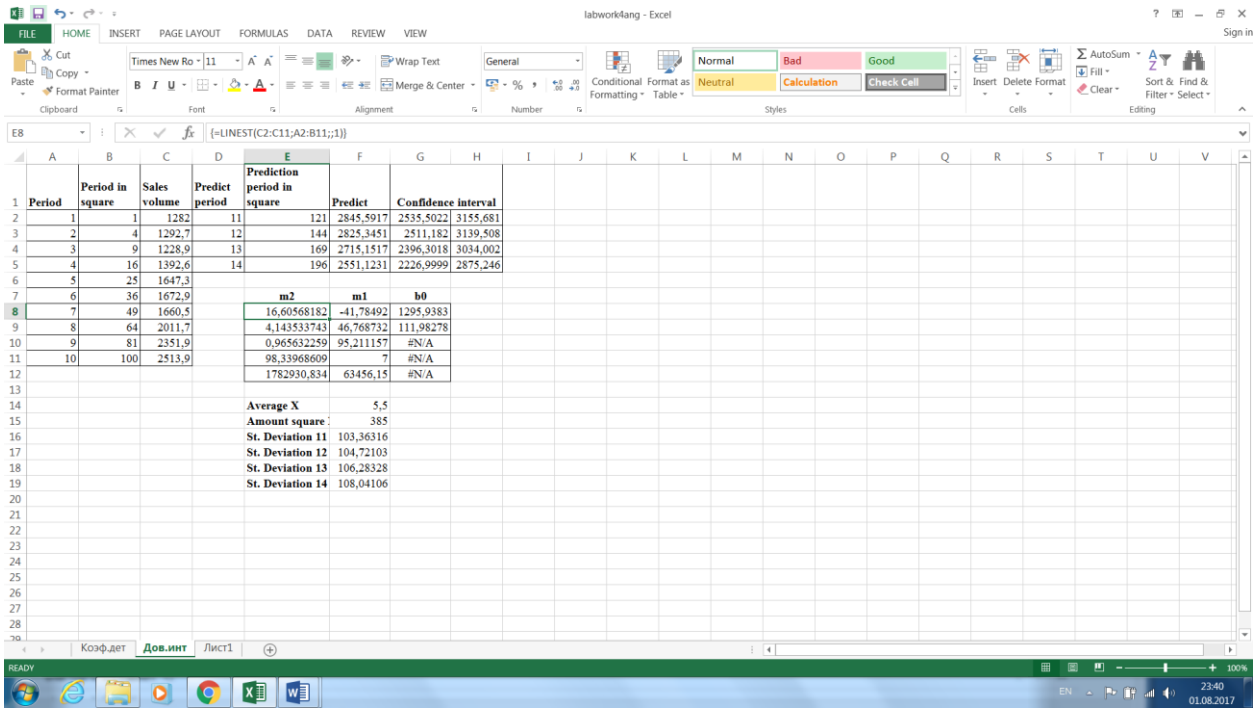
## 5.2 PRACTICAL LESSON 5 and IWS 5

**Subject:** Calculation of confidence intervals for the prediction task.

**Plan of the lesson.**

Study the scheme for calculating confidence intervals for the prediction task, presented above in the lecture material.

Below in the figure is an example for Excel, made in accordance with the described scheme of calculating confidence intervals for the task of forecasting sales volumes as a function of time periods, represented as a polynomial of the second degree $Y = b_0 + m_1 t + m_2 t^2$ [1].




| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | Period | Period in square | Sales volume | Predict period | Prediction period in square | Predict | Confidence interval | |
| 2 | 1 | 1 | 1282 | 11 | 121 | 2845,5917 | 2535,5022 | 3155,681 |
| 3 | 2 | 4 | 1292,7 | 12 | 144 | 2825,3451 | 2511,182 | 3139,508 |
| 4 | 3 | 9 | 1228,9 | 13 | 169 | 2715,1517 | 2396,3018 | 3034,002 |
| 5 | 4 | 16 | 1392,6 | 14 | 196 | 2551,1231 | 2226,9999 | 2875,246 |
| 6 | 5 | 25 | 1647,3 | | | | | |
| 7 | 6 | 36 | 1672,9 | | m2 | m1 | b0 | |
| 8 | 7 | 49 | 1660,5 | | 16,60568182 | -41,78492 | 1295,9383 | |
| 9 | 8 | 64 | 2011,7 | | 4,143533743 | 46,768732 | 111,98278 | |
| 10 | 9 | 81 | 2351,9 | | 0,965632259 | 95,211157 | #N/A | |
| 11 | 10 | 100 | 2513,9 | | 98,33968609 | 7 | #N/A | |
| 12 | | | | | 1782930,834 | 63456,15 | #N/A | |
| 13 | | | | | | | | |
| 14 | | | | | Average X | 5,5 | | |
| 15 | | | | | Amount square | 385 | | |
| 16 | | | | | St. Deviation 11 | 103,36316 | | |
| 17 | | | | | St. Deviation 12 | 104,72103 | | |
| 18 | | | | | St. Deviation 13 | 106,28328 | | |
| 19 | | | | | St. Deviation 14 | 108,04106 | | |

Fig. 5.1 Screenshot of calculating confidence intervals for the task of forecasting sales volumes in relation to time periods.

2) Practice and task of the IWS. Apply the above confidence interval calculation scheme for your prediction task.

3) Analyze the results.

Literature.

1. Minko A.A. Forecasting in business using Excel. M.: Eksmo, 2007, 208 p.

2. Larose D.T. Discovering knowledge in data - an introduction to data mining. Wiley-Interscience, Hoboken, New Jersey, 2005

# TOPIC 6

# RECOVER MISSING DATA FOR THE PREDICTION TASK

## 6.1. Lecture material

Missing or missing data in the intellectual analysis is a problem for the data preparation phase. This lack of data can significantly affect the data analysis process. Lack of information is rarely useful. Other things being equal, more data is almost always better. Therefore, various approaches have been developed in this area to solve this problem or to circumvent it.

There are several approaches to solving this problem of missing data [1, 2]:

- replacement of missing data with a certain constant determined by the analyst;
- replacement of missing data by the average value of the field (for numeric variables) or by the mode (for categorical variables);
- replacement of missing data by a value obtained by using a function that approximates the available data.

One of the simple methods for restoring the missing values of the factor-variable is to replace the missing values with the average arithmetic values of the series of values of this factor. For example, if there is no i-th value $x_i$ of the factor X, then $x_i = (x_{i-1} + x_{i+1})/2$ is assumed. This method of recovery has drawbacks. First, it does not work if there are no successive multiple factor values. Secondly, if the data are ordered in time, then the linear dependence of this factor on time is implicitly implied, which is not always true. Thirdly, if it is assumed the linear dependence of the factor on time, then the simple arithmetic mean of the nearest values is a very inaccurate method of approximation [1].

In the second method of recovering missing values, the regression function of the given (dependent) factor with missing values is first constructed based on one or more other factors (independent factors) also present in the data set. Then the missing values of the dependent factor are replaced by the calculated values of the constructed regression function for this factor. The main problem here is the choice of independent factors by which the regression function for the dependent factor will be built.

Independent factors are subject to certain requirements. First, they must correlate with the dependent factor for which the regression function is built. This means that there must be some dependence between them. Secondly, the values of the independent variables should be as deterministic as possible. Should not depend on random influences, especially on random influences that affect the predicted variable Y. However, the latter condition is difficult to verify in practice, therefore,

the time factor is often taken as independent variables. But, by their nature, changes of the factor are not always determined only by a temporary factor. The way out of this situation is the construction of several data models, where the missing values are restored based on different independent variables [1].

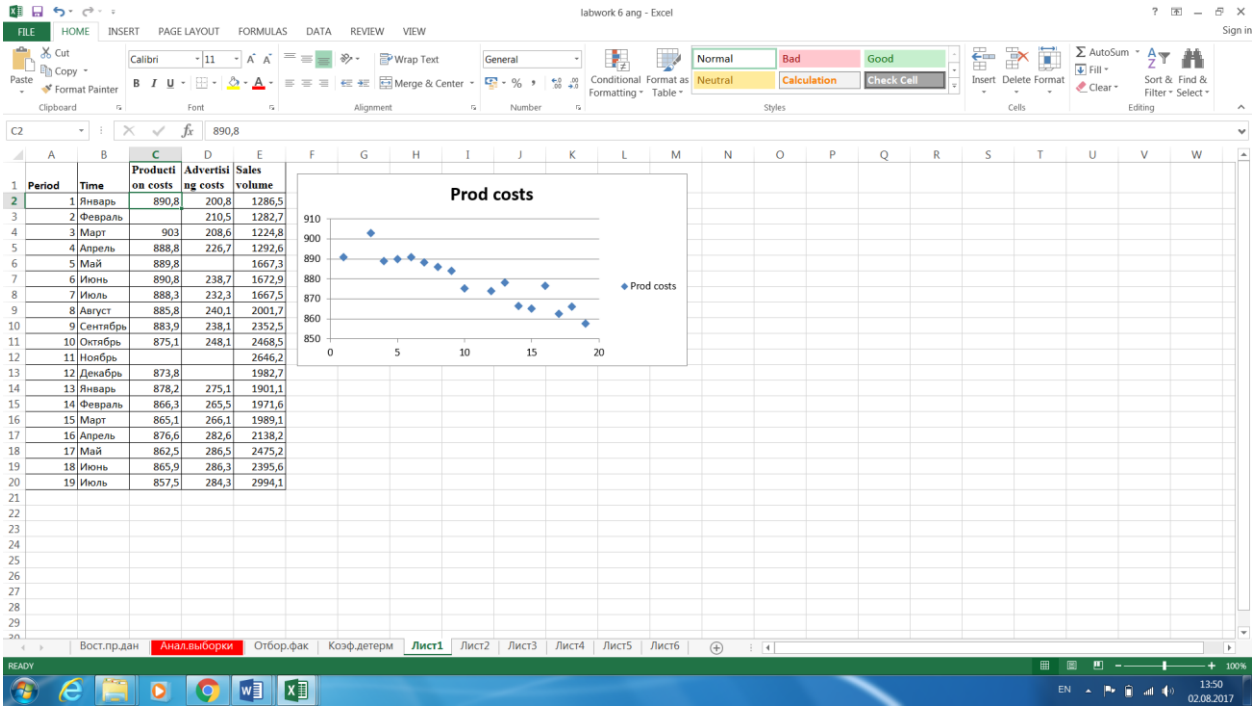
Consider an example of restoring missing values.

In Figure 6.1. The table of initial data of production costs on the time periods is presented. There are missing values from the production cost data. It is necessary to restore the missing values. We can perform the reconstruction of the missing values using the second method described above, restoring the missing values, based on constructing a factor regression line with missing values for other factors in the original data.

In the figure 6.2. Represents the construction of the regression line of production costs from time, which allows you to calculate the missing values of production costs. To do this, you need to add the regression line formula to the missing value cells. This is shown in Figure 6.3.

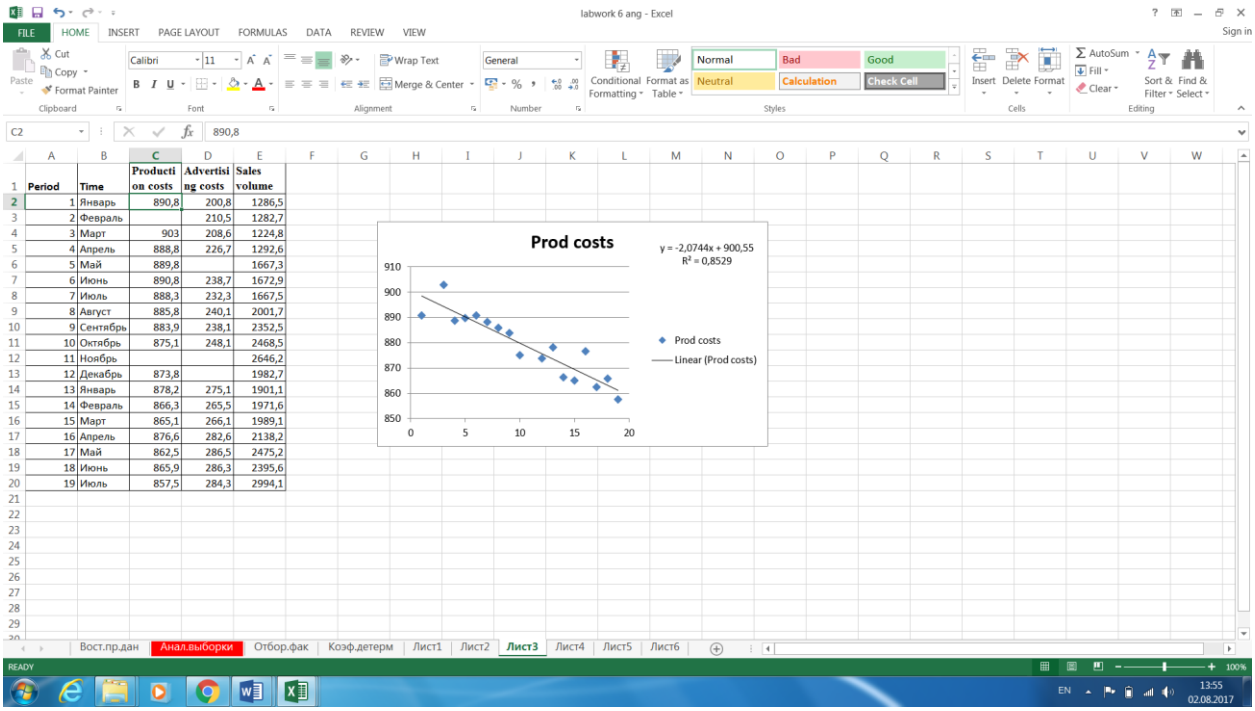
See Figure 6.4. The table of initial data of expenses for advertising on the periods of time is presented. There are missing values from the cost of advertising. It is necessary to restore the missing values of advertising costs. Recover missing costs of advertising costs by using the above method of recovering missed values, based on building a line of regression costs for advertising cost factor production.

In Figure 6.5. The construction of the regression line of the dependence of advertising costs on production costs is presented, which allows calculating the missed costs from time to time. To do this, you need to add the regression line formula to the missing value cells. This is shown in Figure 6.6.



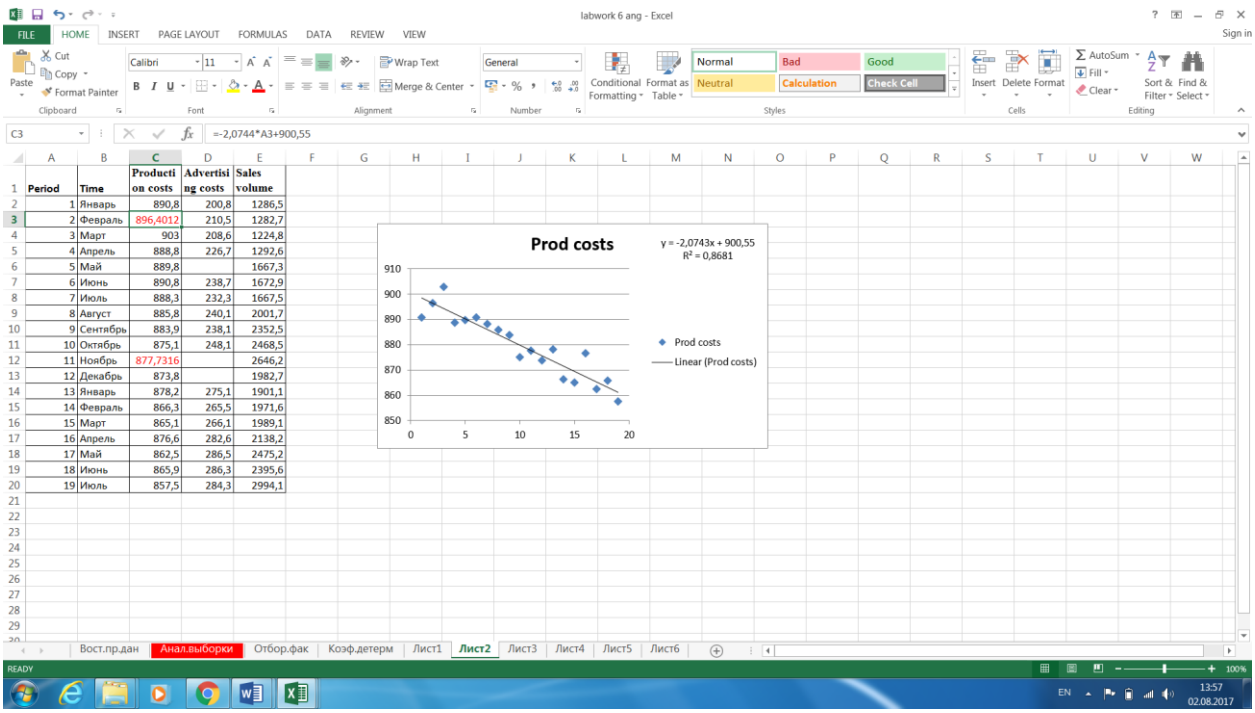

| Period | Time | Production costs | Advertising costs | Sales volume |
|---|---|---|---|---|
| 1 | Январь | 890,8 | 200,8 | 1286,5 |
| 2 | Февраль | | 210,5 | 1282,7 |
| 3 | Март | 903 | 208,6 | 1224,8 |
| 4 | Апрель | 888,8 | 226,7 | 1292,6 |
| 5 | Май | 889,8 | | 1667,3 |
| 6 | Июнь | 890,8 | 238,7 | 1672,9 |
| 7 | Июль | 888,3 | 232,3 | 1667,5 |
| 8 | Август | 885,8 | 240,1 | 2001,7 |
| 9 | Сентябрь | 883,9 | 238,1 | 2352,5 |
| 10 | Октябрь | 875,1 | 248,1 | 2468,5 |
| 11 | Ноябрь | | | 2646,2 |
| 12 | Декабрь | 873,8 | | 1982,7 |
| 13 | Январь | 878,2 | 275,1 | 1901,1 |
| 14 | Февраль | 866,3 | 265,5 | 1971,6 |
| 15 | Март | 865,1 | 266,1 | 1989,1 |
| 16 | Апрель | 876,6 | 282,6 | 2138,2 |
| 17 | Май | 862,5 | 286,5 | 2475,2 |
| 18 | Июнь | 865,9 | 286,3 | 2395,6 |
| 19 | Июль | 857,5 | 284,3 | 2994,1 |

Fig.6.1. Chart of production costs from time.



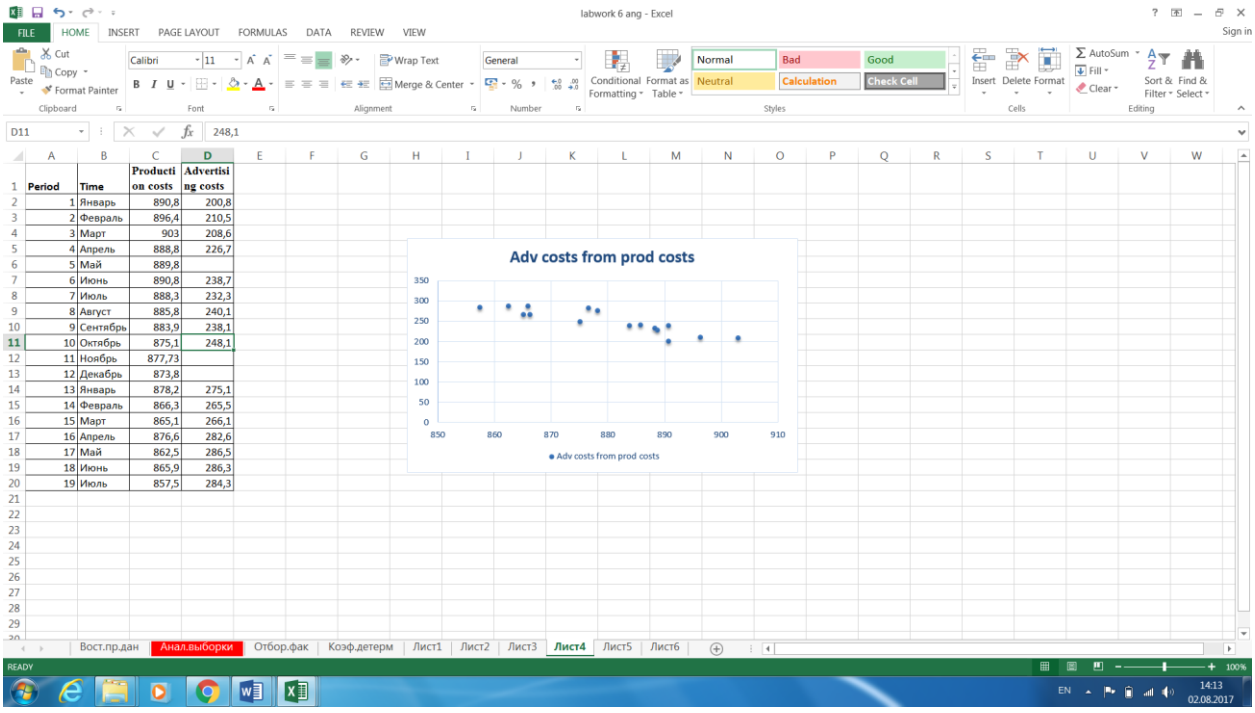

| Period | Time | Production costs | Advertising costs | Sales volume |
|---|---|---|---|---|
| 1 | Январь | 890,8 | 200,8 | 1286,5 |
| 2 | Февраль | | 210,5 | 1282,7 |
| 3 | Март | 903 | 208,6 | 1224,8 |
| 4 | Апрель | 888,8 | 226,7 | 1292,6 |
| 5 | Май | 889,8 | | 1667,3 |
| 6 | Июнь | 890,8 | 238,7 | 1672,9 |
| 7 | Июль | 888,3 | 232,3 | 1667,5 |
| 8 | Август | 885,8 | 240,1 | 2001,7 |
| 9 | Сентябрь | 883,9 | 238,1 | 2352,5 |
| 10 | Октябрь | 875,1 | 248,1 | 2468,5 |
| 11 | Ноябрь | | | 2646,2 |
| 12 | Декабрь | 873,8 | | 1982,7 |
| 13 | Январь | 878,2 | 275,1 | 1901,1 |
| 14 | Февраль | 866,3 | 265,5 | 1971,6 |
| 15 | Март | 865,1 | 266,1 | 1989,1 |
| 16 | Апрель | 876,6 | 282,6 | 2138,2 |
| 17 | Май | 862,5 | 286,5 | 2475,2 |
| 18 | Июнь | 865,9 | 286,3 | 2395,6 |
| 19 | Июль | 857,5 | 284,3 | 2994,1 |

Fig. 6.2. The regression line of production costs from time.




| Period | Time | Production costs | Advertising costs | Sales volume |
|---|---|---|---|---|
| 1 | Январь | 890,8 | 200,8 | 1286,5 |
| 2 | Февраль | 896,4012 | 210,5 | 1282,7 |
| 3 | Март | 903 | 208,6 | 1224,8 |
| 4 | Апрель | 888,8 | 226,7 | 1292,6 |
| 5 | Май | 889,8 | | 1667,3 |
| 6 | Июнь | 890,8 | 238,7 | 1672,9 |
| 7 | Июль | 888,3 | 232,3 | 1667,5 |
| 8 | Август | 885,8 | 240,1 | 2001,7 |
| 9 | Сентябрь | 883,9 | 238,1 | 2352,5 |
| 10 | Октябрь | 875,1 | 248,1 | 2468,5 |
| 11 | Ноябрь | 877,7316 | | 2646,2 |
| 12 | Декабрь | 873,8 | | 1982,7 |
| 13 | Январь | 878,2 | 275,1 | 1901,1 |
| 14 | Февраль | 866,3 | 265,5 | 1971,6 |
| 15 | Март | 865,1 | 266,1 | 1989,1 |
| 16 | Апрель | 876,6 | 282,6 | 2138,2 |
| 17 | Май | 862,5 | 286,5 | 2475,2 |
| 18 | Июнь | 865,9 | 286,3 | 2395,6 |
| 19 | Июль | 857,5 | 284,3 | 2994,1 |

Fig. 6.3. Calculation of the missed values of production costs for the constructed line of regression of production costs from time.




| Period | Time | Production costs | Advertising costs |
|---|---|---|---|
| 1 | Январь | 890,8 | 200,8 |
| 2 | Февраль | 896,4 | 210,5 |
| 3 | Март | 903 | 208,6 |
| 4 | Апрель | 888,8 | 226,7 |
| 5 | Май | 889,8 | |
| 6 | Июнь | 890,8 | 238,7 |
| 7 | Июль | 888,3 | 232,3 |
| 8 | Август | 885,8 | 240,1 |
| 9 | Сентябрь | 883,9 | 238,1 |
| 10 | Октябрь | 875,1 | 248,1 |
| 11 | Ноябрь | 877,73 | |
| 12 | Декабрь | 873,8 | |
| 13 | Январь | 878,2 | 275,1 |
| 14 | Февраль | 866,3 | 265,5 |
| 15 | Март | 865,1 | 266,1 |
| 16 | Апрель | 876,6 | 282,6 |
| 17 | Май | 862,5 | 286,5 |
| 18 | Июнь | 865,9 | 286,3 |
| 19 | Июль | 857,5 | 284,3 |

Fig. 6.4. Spot chart of advertising costs from production costs

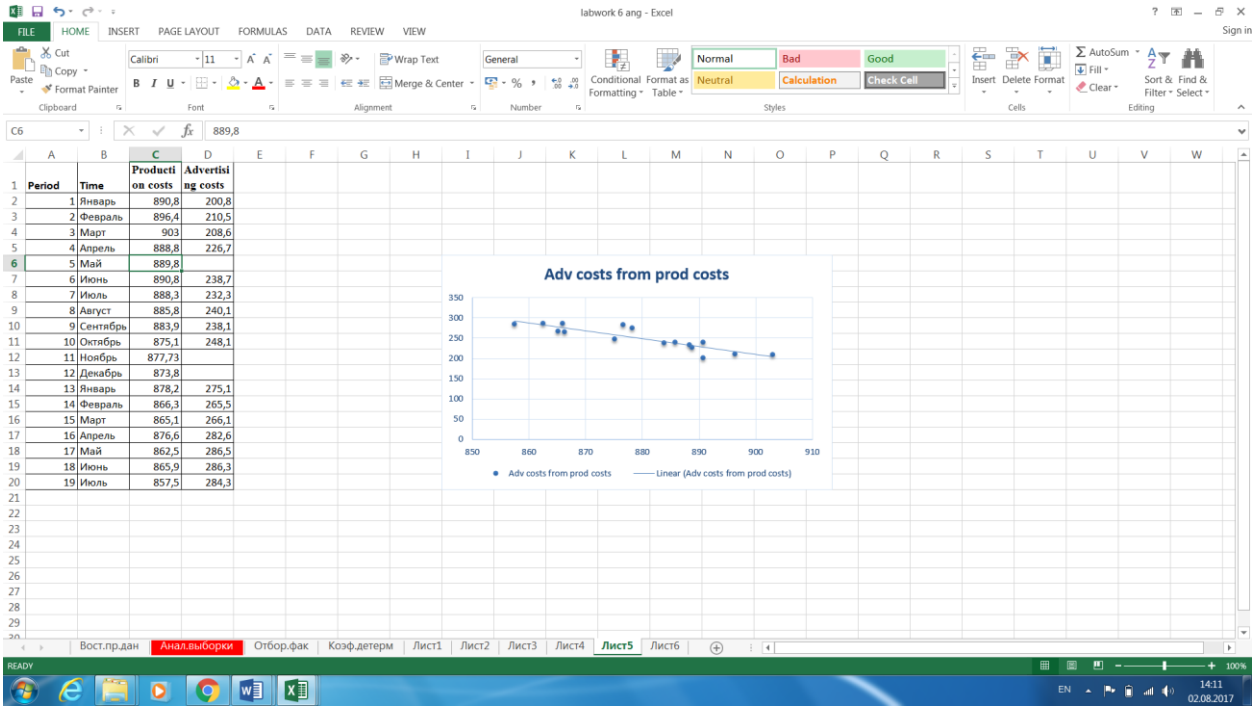



Fig. 6.5. The regression line of the dependence of advertising costs on production costs.

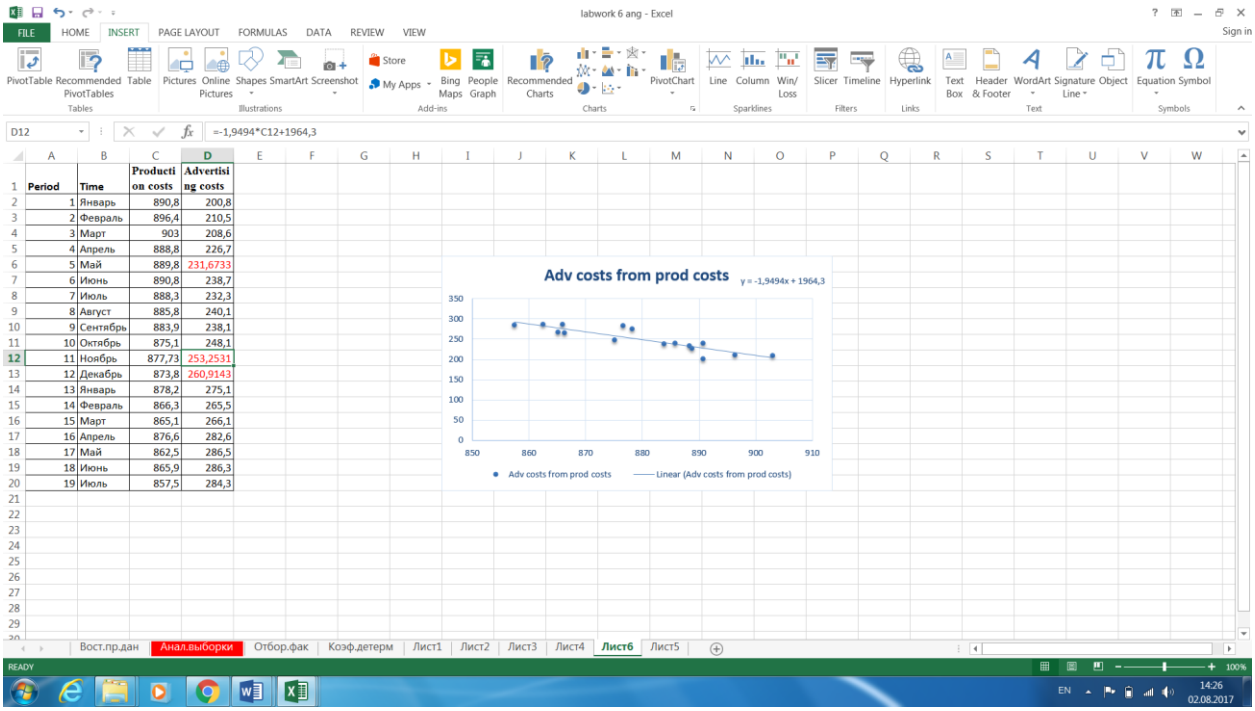



Fig. 6.6. Calculation of the missed costs of advertising costs for the line of regression of advertising costs from production costs.

## 6.2 PRACTICAL LESSON 6 and IWS 6

Subject. Recover missing data for the prediction task.

Plan of the lesson.

1) Study the scheme for recovering missing data for the prediction task, presented above.

2) Practice and task of the IWS. Apply the above-described scheme for recovering missing data for its prediction task.

3) Analyze the results.

Literature.

1. Minko A.A. Forecasting in business using Excel. M.: Eksmo, 2007, 208 p.

2. Larose D.T. Discovering knowledge in data - an introduction to data mining. Wiley-Interscience, Hoboken, New Jersey, 2005

# TOPIC 7

# ANALYSIS OF DATA OUTLIERS FOR THE PREDICTION TASK

## 7.1. Lecture material

Data outliers are extreme values that lie near the limits of the data range or go against the trend of the remaining data. Identifying data outliers is important because they may represent errors in data entry. The task of the data outliers analysis is to find out whether the data outliers are values of "natural" origin, i.e. whether they are caused by the stochastic nature of the initial data or are conditioned by some unaccounted factors. If the data outliers are due to the presence of unknown factors, then it is necessary to deal with this situation, to identify the cause of the unknown factor. This can serve as an incentive for investigating the actual situation described by the initial data, and also for further improving the data model [1, 2].

There are several methods of data outliers detection.

One elementary robust method is to use the interquartile range [2]. The quartiles of a data set divide the data set into four parts, each containing 25% of the data:

-the first quartile (Q1) is the 25th percentile;

-the second quartile (Q2) is the 50th percentile, that is, the median;

-the third quartile (Q3) is the 75th percentile.

The interquartile range (IQR) is a measure of variability that is much more robust than the standard deviation.

The IQR is calculated as IQR = Q3 − Q1 and may be interpreted to represent the spread of the middle 50% of the data. A robust measure of data releases detection is therefore defined as follows. A data value is an outlier if:

1) It is located 1.5(IQR) or more below Q1, or

2) It is located 1.5(IQR) or more above Q3.

For example, suppose that for a set of test scores, the 25th percentile was Q1 = 165 and the 75th percentile was Q3 = 170, so that half of all the test scores fell between 165 and 170. Then the interquartile range, the difference between these quartiles, was IQR = 170 − 165 = 5.

A test score would be robustly identified as an data release if:

a. It is lower than Q1 − 1.5(IQR) = 165 − 1.5*(5) = 157,5, or

b. It is higher than Q3 + 1.5(IQR) = 170 + 1.5(5) = 177,5.

Other method of data outliers detection using regression function is described below [1].

The scheme for determining the data outliers includes the following steps:

1) the data outliers points $x_i$are removed from the initial data set;

2) the regression function f (X) is constructed from the remaining data and its statistical characteristics are calculated using the standard LINEST function (including the standard error $s_\varepsilon$of the constructed regression);

3) find the value of the regression function at the data outliers points f (X).

4) calculate the residuals at data releases points $e_i = y_i - f(X_i)$, where $y_i$- is the value of the i-th outlier.

5) find the normalized residues at the data outliers points $\mathrm{e}_i = \mathrm{e}_i / s_\varepsilon$ ..

6) If the absolute value of the normalized residue exceeds the number 3, it is considered that with a probability of 95% this data outliers is not accidental.

Let us consider an example of determining the randomness of data outliers.

The set of initial data is given, presented in Table 7.1 and in Figure 7.1.

Table 7.1. The set of initial data

| Time | Period | Production costs | Advertising costs | Sales volume |
|---|---|---|---|---|
| January | 1 | 890,8 | 200,8 | 1286,5 |
| February | 2 | 896,4 | 210,5 | 1282,7 |
| March | 3 | 903 | 208,6 | 1224,8 |
| April | 4 | 888,8 | 226,7 | 1292,6 |
| May | 5 | 889,8 | 230,62 | 1667,3 |
| June | 6 | 890,8 | 238,7 | 1672,9 |
| July | 7 | 888,3 | 232,3 | 1667,5 |
| August | 8 | 885,8 | 240,1 | 2001,7 |
| September | 9 | 883,9 | 238,1 | 2352,5 |
| October | 10 | 875,1 | 248,1 | 2468,5 |
| November | 11 | 877,73 | 253,58 | 2946,2 |
| December | 12 | 873,8 | 261,05 | 1982,7 |
| January | 13 | 878,2 | 275,1 | 1901,1 |
| February | 14 | 866,3 | 265,5 | 1971,6 |
| March | 15 | 865,1 | 266,1 | 1989,1 |
| April | 16 | 876,6 | 282,6 | 2138,2 |
| May | 17 | 862,5 | 286,5 | 2475,2 |
| June | 18 | 865,9 | 286,3 | 2395,6 |

| | | | | |
|---|---|---|---|---|
| July | 19 | 857,5 | 284,3 | 2994,1 |

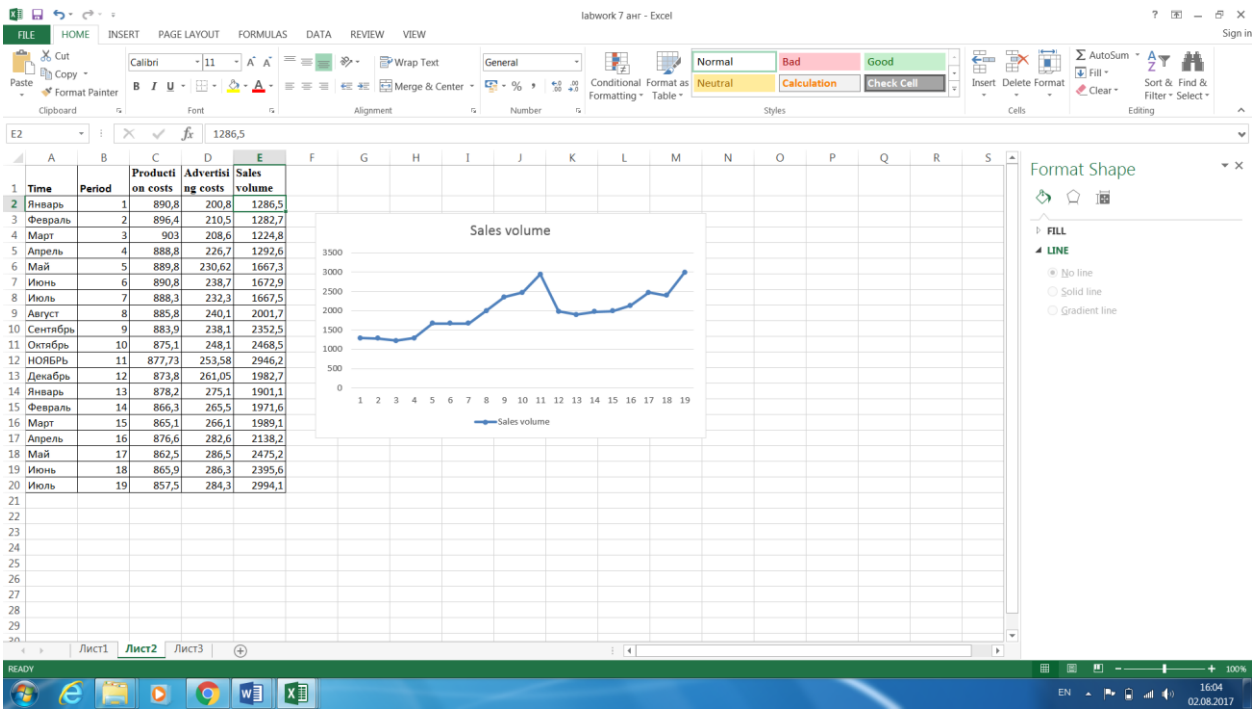
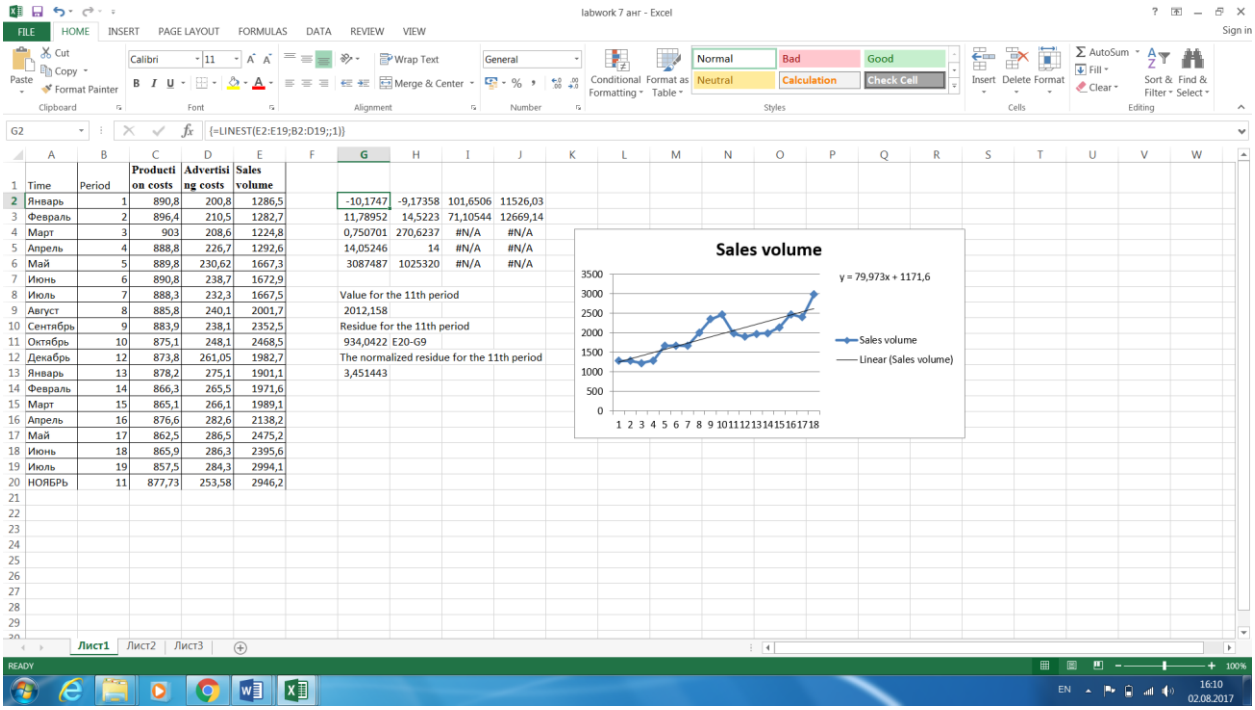



Fig. 7.1. Initial data with data outliers in the 11th period.

In the diagram of Fig. 7.1. One significant data outliers is seen in the 11th period.

1) In accordance with the scheme for determining the accidental data outliers, it is necessary to delete data outliers. For example, the outlier of period 11 is deleted and its data line is stored in free line 20 (see Figure 7.2.).

2) A regression function is constructed for the data of the predicted variable without a deleted data outliers (see Figure 7.2.). The statistical characteristics of the regression are calculated using the standard LINEST function, represented in cells G2: J6 (see Figure 7.2).




| Time | Period | Production costs | Advertising costs | Sales volume |
|---|---|---|---|---|
| Январь | 1 | 890,8 | 200,8 | 1286,5 |
| Февраль | 2 | 896,4 | 210,5 | 1282,7 |
| Март | 3 | 903 | 208,6 | 1224,8 |
| Апрель | 4 | 888,8 | 226,7 | 1292,6 |
| Май | 5 | 889,8 | 230,62 | 1667,3 |
| Июнь | 6 | 890,8 | 238,7 | 1672,9 |
| Июль | 7 | 888,3 | 232,3 | 1667,5 |
| Август | 8 | 885,8 | 240,1 | 2001,7 |
| Сентябрь | 9 | 883,9 | 238,1 | 2352,5 |
| Октябрь | 10 | 875,1 | 248,1 | 2468,5 |
| Декабрь | 12 | 873,8 | 261,05 | 1982,7 |
| Январь | 13 | 878,2 | 275,1 | 1901,1 |
| Февраль | 14 | 866,3 | 265,5 | 1971,6 |
| Март | 15 | 865,1 | 266,1 | 1989,1 |
| Апрель | 16 | 876,6 | 282,6 | 2138,2 |
| Май | 17 | 862,5 | 286,5 | 2475,2 |
| Июнь | 18 | 865,9 | 286,3 | 2395,6 |
| Июль | 19 | 857,5 | 284,3 | 2994,1 |
| НОЯБРЬ | 11 | 877,73 | 253,58 | 2946,2 |

| | | | |
|---|---|---|---|
| -10,1747 | -9,17358 | 101,6506 | 11526,03 |
| 11,78952 | 14,5223 | 71,10544 | 12669,14 |
| 0,750701 | 270,6237 | #N/A | #N/A |
| 14,05246 | 14 | #N/A | #N/A |
| 3087487 | 1025320 | #N/A | #N/A |

Value for the 11th period
2012,158
Residue for the 11th period
934,0422 E20-G9
The normalized residue for the 11th period
3,451443

Fig. 7.2. Calculation of the statistical characteristics of the regression without a deleted data outliers.

3) The values of the predicted variable for the deleted data outliers (see Figure 7.3) are calculated from the parameters of the regression: $Y = b_0 + b_1 t + b_2 X_1 + b_3 X_2$ (formula in Excel: =J2+I2*B20+H2*C20+G2*D20).

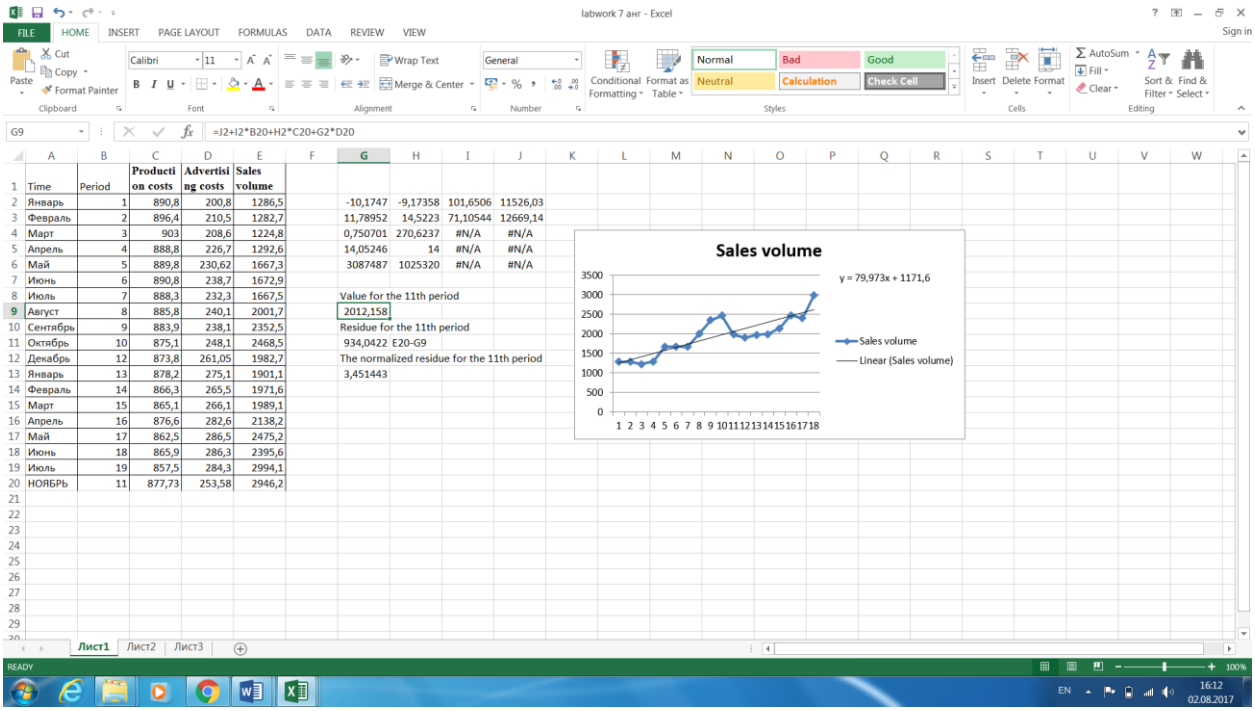


Fig. 7.3. Calculation of the value of the predicted variable for remote emission by regression parameters.

4) The residuals at emission points $e_i = y_i - f(X_i)$, are calculated, where $y_i$- is the value of the i-th outlier (see Figure 7.4).

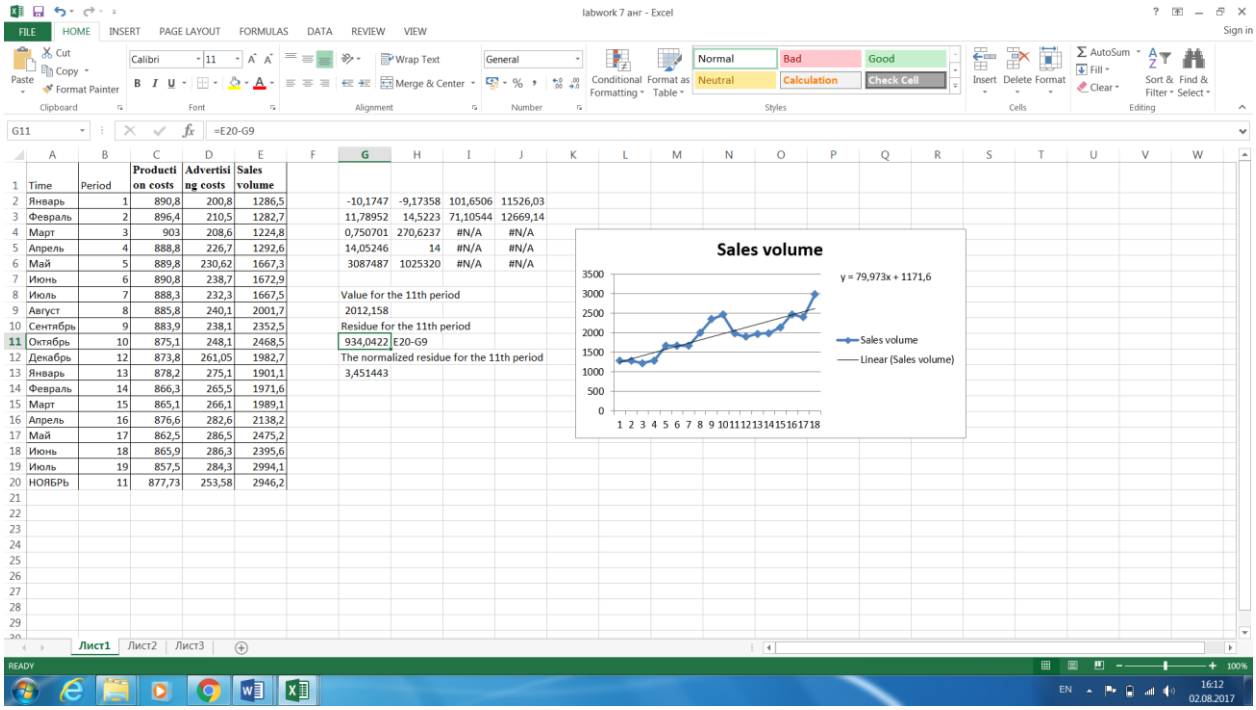



Fig. 7.4. Calculation of residues at data outliers points.

5) Calculation of normalized residues at emission points $e_i = e_i / s_\varepsilon$ for example (see Figure 7.5).

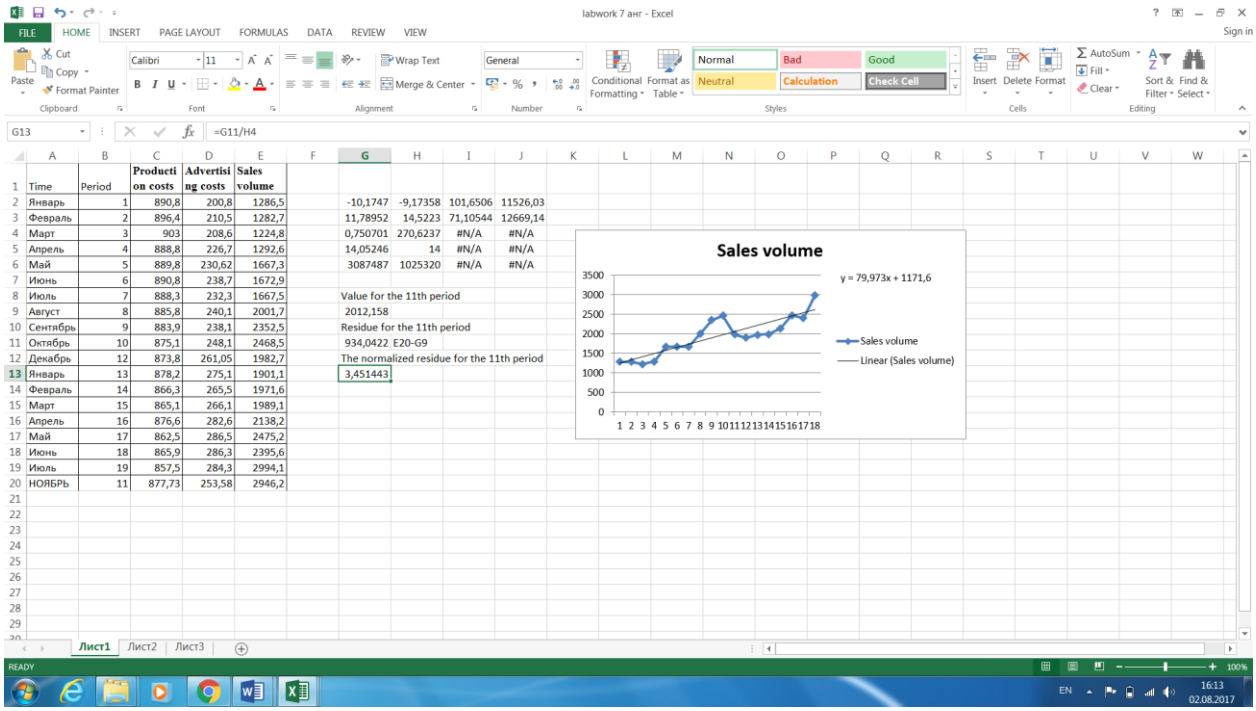


Fig. 7.5. Calculation of normalized residues at data outliers points.

6) If the absolute value of the normalized residue exceeds the number 3, it is considered that with a probability of 95% this data outliers is not accidental.

In this example, the value of the predicted variable of the 11th period is not an accidental outlier, because the value of the normalized remainder is 3.45.

An analysis of the results of a randomized data outliers test includes a sufficiently in-depth analysis of the necessary actions in the event that data outliers are not accidental. Often this is a sign that there is an unaccounted factor.

If, as a result of the analysis, it turns out that these factors will not affect the predicted periods, such data outliers can be removed from the original data set.

If the data outliers are of a recurring nature, then they must be taken into account when allocating the seasonal component of the forecast.

If data outliers can appear in the future, but irregularly (for example, special promotions), these data releases can be taken into account by including additional factors in the data model.

## 7.2 PRACTICAL LESSON 7 and IWS 7

**Subject:** Analysis of data outliers for the predicting problem.

**Plan of the lesson.**

1) Examine the data outlier analysis scheme for the forecasting problem presented above.

2) Practice and task of the IWS. Apply the above described data outliers analysis scheme to your predicting task.

3) Analyze the results.

Literature.

1. Minko A.A. Predicting in business using Excel. Moscow: Eksmo, 2007, -208 p.

2. Larose D.T. Discovering knowledge in data - an introduction to data mining. Wiley-Interscience, Hoboken, New Jersey, 2005

# TOPIC 8

# SELECTION OF FACTORS FOR THE PREDICTION TASK

## 8.1 Lecture material

The task of selecting factors is as follows: **it is necessary to select such factors in the data model that their inclusion increases the accuracy of the approximation of the initial data by the predicting function**.

**One of the most effective methods of selection of factors is the use of the adjusted coefficient of determination**.

Earlier, we considered in detail the coefficient of determination, when the question of the accuracy of the approximation of the initial data by the predicting function was studied. The **coefficient of determination is convenient when estimating the accuracy of approximation by different prediction functions of the same initial data with the same set of factors**.

When it is necessary to **compare the accuracy of approximation with the function of predicting the initial data with a different set of factors**, and the task of selecting factors can be put in this way, then **it is possible to use the adjusted coefficient of determination**.

That is, **you can first calculate the adjusted determination coefficient for a model with a given set of factors, and then remove one of the factors from the model and calculate the adjusted determination coefficient**.

Then compare the obtained values of the adjusted determination coefficient: **if elimination of the factor reduces the quality of the approximation, then this factor is significant for the model**, and vice versa, if **the quality of the approximation does not change or even increases, this means that this factor is not essential for the model**.

The adjusted coefficient of determination $\overline{R_m^2}$ is calculated by the formula:

$$\overline{R_m^2} = \frac{(n-1)(1-R_m^2)}{n-m} \qquad (8.1)$$

Where $R_m^2$- "standard" coefficient of determination; $m$- number of factors; $n$ is the number of data points in the source data set.

The adjusted coefficient of determination has the property that if k new factors are added to the model, then the new adjusted determination coefficient $\overline{R_{m+k}^2}$ can be either larger or smaller than the old adjusted coefficient of determination $\overline{R_m^2}$. If $\overline{R_{m+k}^2}$ is greater than $\overline{R_m^2}$, then it is considered that k new factors significantly affect the predicted variable Y, and these factors need to be added to the model. Otherwise (when $\overline{R_{m+k}^2}$ is less than or equal to $\overline{R_m^2}$, it is assumed that the new factors have little effect on the predicted variable, and therefore they can not be included in the model [1].

Below is an example of an assessment of the inclusion of a factor in the forecasting model. First, we consider a model with three factors:

$$Y = b_0 + b_1 t + b_2 X_1 + b_3 X_2 \qquad (8.2.)$$

Then we add the factor of the time period square to this model, having received the model with four factors:

$$Y = b_0 + b_1 t + b_2 t^2 + b_3 X_1 + b_4 X_2 \qquad (8.3.)$$

Then, from this model, we remove the factor X_2 and get a model with three factors:

$$Y = b_0 + b_1 t + b_2 t^2 + b_3 X_1 \qquad (8.4)$$

For each of these models, we calculate their statistical characteristics and the adjusted determination coefficient. Further, by comparing their values, it is

possible to draw conclusions about the importance of the included or excluded factors for the forecasting function of the problem under consideration.

Below in Tables 8.1, 8.2 and Figure 8.1, the initial data and the results of calculating the estimates for the model (8.2) are presented.

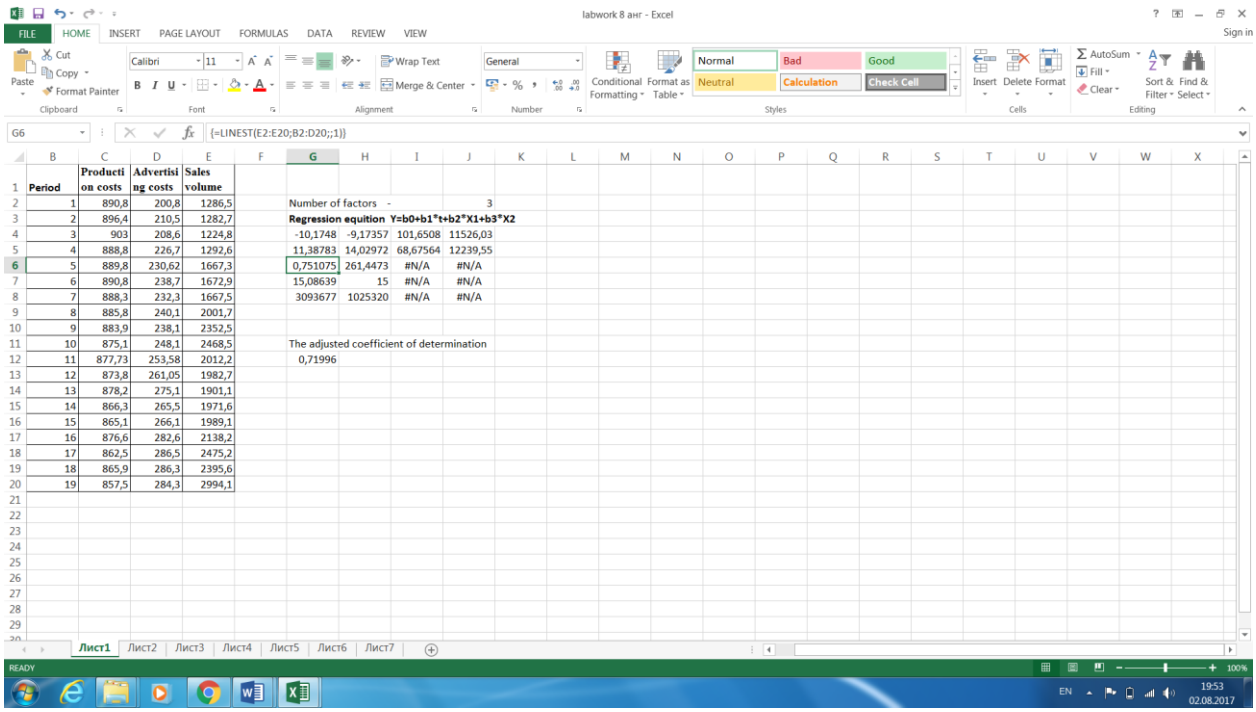
Table 8.1. The initial data for the model (8.2)

| Time | Period | Production costs | Advertising costs | Sales volume |
|---|---|---|---|---|
| January | 1 | 890,8 | 200,8 | 1286,5 |
| February | 2 | 896,4 | 210,5 | 1282,7 |
| March | 3 | 903 | 208,6 | 1224,8 |
| April | 4 | 888,8 | 226,7 | 1292,6 |
| May | 5 | 889,8 | 230,62 | 1667,3 |
| June | 6 | 890,8 | 238,7 | 1672,9 |
| July | 7 | 888,3 | 232,3 | 1667,5 |
| August | 8 | 885,8 | 240,1 | 2001,7 |
| September | 9 | 883,9 | 238,1 | 2352,5 |
| October | 10 | 875,1 | 248,1 | 2468,5 |
| November | 11 | 877,73 | 253,58 | 2012,2 |
| December | 12 | 873,8 | 261,05 | 1982,7 |
| January | 13 | 878,2 | 275,1 | 1901,1 |
| February | 14 | 866,3 | 265,5 | 1971,6 |
| March | 15 | 865,1 | 266,1 | 1989,1 |
| April | 16 | 876,6 | 282,6 | 2138,2 |
| May | 17 | 862,5 | 286,5 | 2475,2 |
| June | 18 | 865,9 | 286,3 | 2395,6 |
| July | 19 | 857,5 | 284,3 | 2994,1 |

Table 8.2. The results of calculating estimates from the model (8.2).

| | | | | | | |
|---|---|---|---|---|---|---|
| | Number of factors - | | | 3 | | |
| | Regression equation Y=b0+b1*t+b2*X1+b3*X2 | | | | | |
| | -10,1748 | -9,17357 | 101,6508 | 11526,03 | | |
| | 11,38783 | 14,02972 | 68,67564 | 12239,55 | | |
| | 0,751075 | 261,4473 | #Н/Д | #Н/Д | | |
| | 15,08639 | 15 | #Н/Д | #Н/Д | | |
| | 3093677 | 1025320 | #Н/Д | #Н/Д | | |
| | | | | | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| | The adjusted coefficient of determination | | | | | |
| | 0,71996 | | | | | |
| | | | | | | |
| | | | | | | |




| Period | Production costs | Advertising costs | Sales volume |
|---|---|---|---|
| 1 | 890,8 | 200,8 | 1286,5 |
| 2 | 896,4 | 210,5 | 1282,7 |
| 3 | 903 | 208,6 | 1224,8 |
| 4 | 888,8 | 226,7 | 1292,6 |
| 5 | 889,8 | 230,62 | 1667,3 |
| 6 | 890,8 | 238,7 | 1672,9 |
| 7 | 888,3 | 232,3 | 1667,5 |
| 8 | 885,8 | 240,1 | 2001,7 |
| 9 | 883,9 | 238,1 | 2352,5 |
| 10 | 875,1 | 248,1 | 2468,5 |
| 11 | 877,73 | 253,58 | 2012,2 |
| 12 | 873,8 | 261,05 | 1982,7 |
| 13 | 878,2 | 275,1 | 1901,1 |
| 14 | 866,3 | 265,5 | 1971,6 |
| 15 | 865,1 | 266,1 | 1989,1 |
| 16 | 876,6 | 282,6 | 2138,2 |
| 17 | 862,5 | 286,5 | 2475,2 |
| 18 | 865,9 | 286,3 | 2395,6 |
| 19 | 857,5 | 284,3 | 2994,1 |

Fig. 8.1. Screenshot of the calculation of the adjusted determination coefficient for the model (8.2).

As can be seen from Table 8.2. And figure 8.1 in regression model 8.2, the determination coefficient is quite high 0.751075, the adjusted determination coefficient is 0.71996.

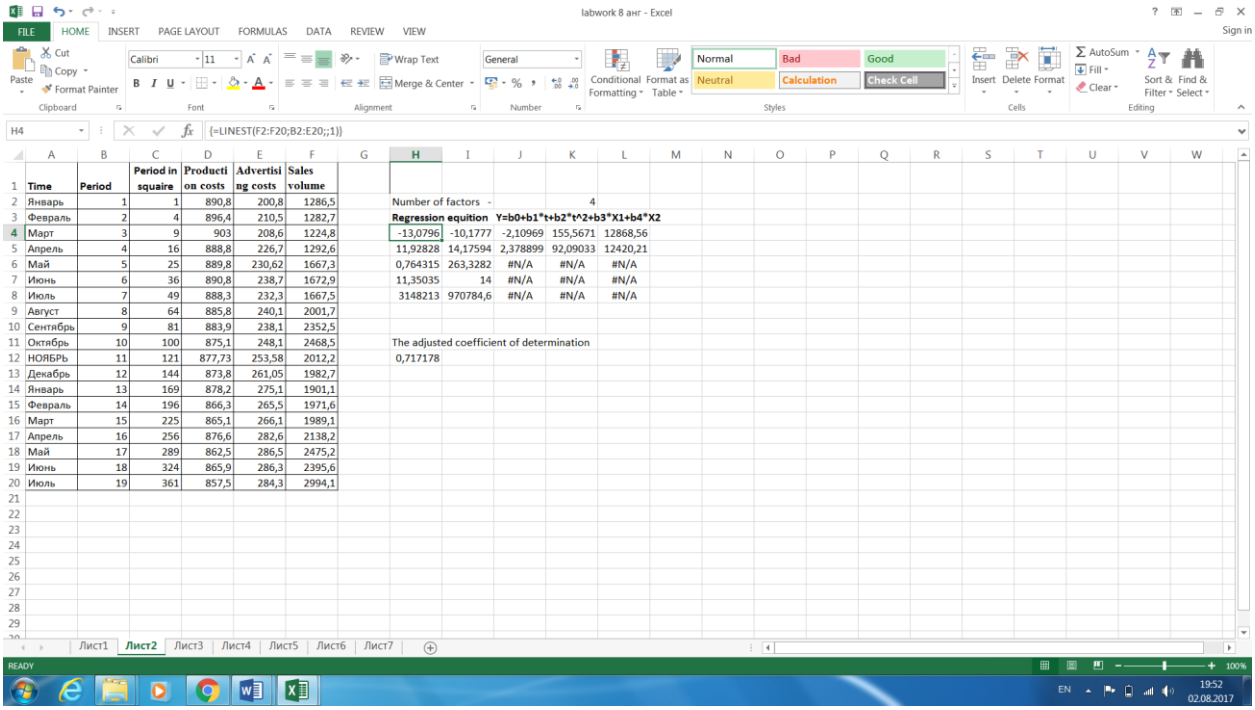
Tables 8.3, 8.4 and figure 8.2 show the initial data and the results of calculating the estimates for the model (8.3), the four-factor model, in which the factor added is the time period squared factor.

Table 8.3. The initial data for the model (8.3)

| Time | Period | Period in square | Production costs | Advertising costs | Sales volume |
|---|---|---|---|---|---|
| January | 1 | 1 | 890,8 | 200,8 | 1286,5 |
| February | 2 | 4 | 896,4 | 210,5 | 1282,7 |
| March | 3 | 9 | 903 | 208,6 | 1224,8 |
| April | 4 | 16 | 888,8 | 226,7 | 1292,6 |
| May | 5 | 25 | 889,8 | 230,62 | 1667,3 |
| June | 6 | 36 | 890,8 | 238,7 | 1672,9 |
| July | 7 | 49 | 888,3 | 232,3 | 1667,5 |
| August | 8 | 64 | 885,8 | 240,1 | 2001,7 |
| September | 9 | 81 | 883,9 | 238,1 | 2352,5 |
| October | 10 | 100 | 875,1 | 248,1 | 2468,5 |
| NOVEMBER | 11 | 121 | 877,73 | 253,58 | 2012,2 |
| December | 12 | 144 | 873,8 | 261,05 | 1982,7 |
| January | 13 | 169 | 878,2 | 275,1 | 1901,1 |
| February | 14 | 196 | 866,3 | 265,5 | 1971,6 |
| March | 15 | 225 | 865,1 | 266,1 | 1989,1 |
| April | 16 | 256 | 876,6 | 282,6 | 2138,2 |
| May | 17 | 289 | 862,5 | 286,5 | 2475,2 |
| June | 18 | 324 | 865,9 | 286,3 | 2395,6 |
| July | 19 | 361 | 857,5 | 284,3 | 2994,1 |

Table 8.4. The results of calculating the estimates from the model (8.3).

| Number of factors - | | | 4 | | |
|---|---|---|---|---|---|
| The regression equation Y=b0+b1*t+b2*t^2+b3*X1+b4*X2 | | | | | |
| -13,0796 | -10,1777 | -2,10969 | 155,5671 | 12868,56 | |
| 11,92828 | 14,17594 | 2,378899 | 92,09033 | 12420,21 | |
| 0,764315 | 263,3282 | #Н/Д | #Н/Д | #Н/Д | |
| 11,35035 | 14 | #Н/Д | #Н/Д | #Н/Д | |
| 3148213 | 970784,6 | #Н/Д | #Н/Д | #Н/Д | |
| | | | | | |
| | | | | | |
| The adjusted coefficient of determination | | | | | |
| 0,717178 | | | | | |




| Time | Period | Period in square | Production costs | Sales volume | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Январь | 1 | 1 | 890,8 | 200,8 | 1286,5 | | | | |
| Февраль | 2 | 4 | 896,4 | 210,5 | 1282,7 | | | | |
| Март | 3 | 9 | 903 | 208,6 | 1224,8 | | | | |
| Апрель | 4 | 16 | 888,8 | 226,7 | 1292,6 | | | | |
| Май | 5 | 25 | 889,8 | 230,62 | 1667,3 | | | | |
| Июнь | 6 | 36 | 890,8 | 238,7 | 1672,9 | | | | |
| Июль | 7 | 49 | 888,3 | 232,3 | 1667,5 | | | | |
| Август | 8 | 64 | 885,8 | 240,1 | 2001,7 | | | | |
| Сентябрь | 9 | 81 | 883,9 | 238,1 | 2352,5 | | | | |
| Октябрь | 10 | 100 | 875,1 | 248,1 | 2468,5 | | | | |
| НОЯБРЬ | 11 | 121 | 877,73 | 253,58 | 2012,2 | | | | |
| Декабрь | 12 | 144 | 873,8 | 261,05 | 1982,7 | | | | |
| Январь | 13 | 169 | 878,2 | 275,1 | 1901,1 | | | | |
| Февраль | 14 | 196 | 866,3 | 265,5 | 1971,6 | | | | |
| Март | 15 | 225 | 865,1 | 266,1 | 1989,1 | | | | |
| Апрель | 16 | 256 | 876,6 | 282,6 | 2138,2 | | | | |
| Май | 17 | 289 | 862,5 | 286,5 | 2475,2 | | | | |
| Июнь | 18 | 324 | 865,9 | 286,3 | 2395,6 | | | | |
| Июль | 19 | 361 | 857,5 | 284,3 | 2994,1 | | | | |
| | | | | | | -13,0796 | -10,1777 | -2,10969 | 155,5671 | 12868,56 |
| | | | | | | 11,92828 | 14,17594 | 2,378899 | 92,09033 | 12420,21 |
| | | | | | | 0,764315 | 263,3282 | #N/A | #N/A | #N/A |
| | | | | | | 11,35035 | 14 | #N/A | #N/A | #N/A |
| | | | | | | 3148213 | 970784,6 | #N/A | #N/A | #N/A |

Fig. 8.2. Screenshot of calculating the corrected determination coefficient for the model (8.3).

As can be seen from Table 8.4. And figure 8.2 in the regression model (8.3), the determination coefficient increased by 0.764315, and the adjusted determination coefficient decreased by 0.717178. This suggests that the added factor is not significant for the forecasting model, i.e. the addition of a "time period square" to the predicting model does not significantly affect the predicted variable.

Below in Tables 8.5, 8.6 and Figure 8.3, the initial data and the results of calculating the estimates for the model (8.4) are presented.

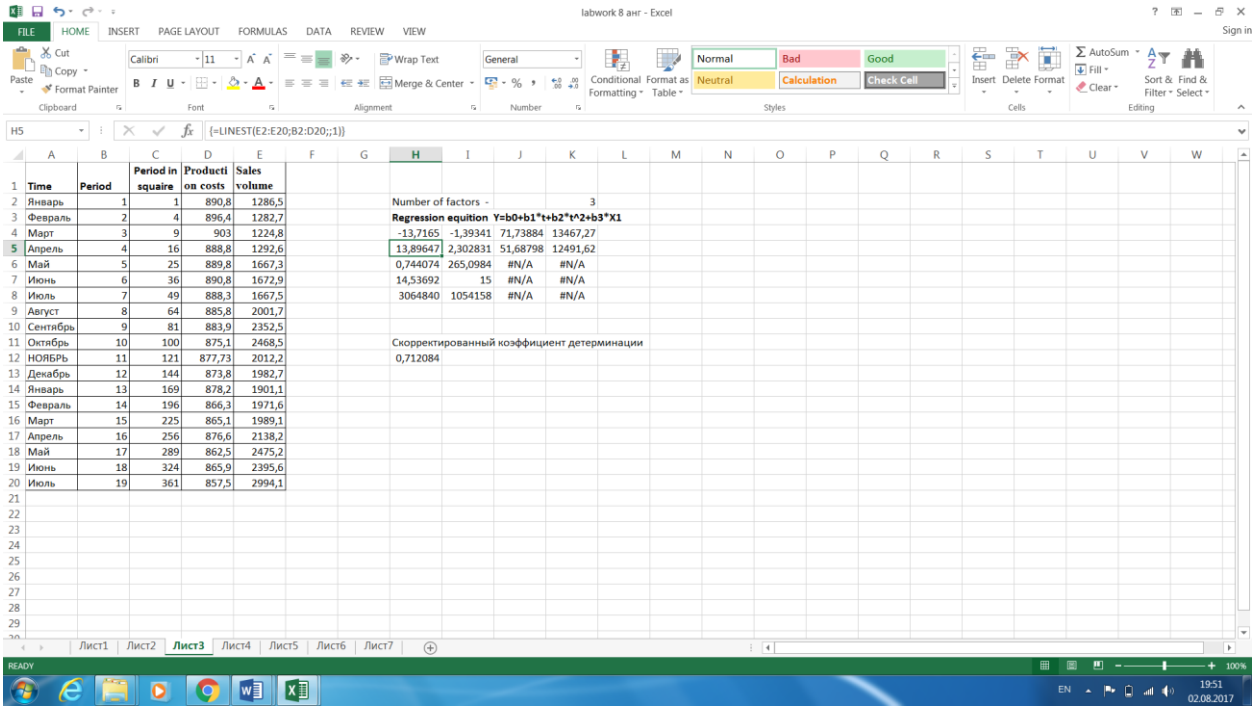
Table 8.5. The initial data for the model (8.4)

| Time | Period | Period in square | Production costs | Sales volume |
|---|---|---|---|---|
| January | 1 | 1 | 890,8 | 1286,5 |
| February | 2 | 4 | 896,4 | 1282,7 |
| March | 3 | 9 | 903 | 1224,8 |
| April | 4 | 16 | 888,8 | 1292,6 |
| May | 5 | 25 | 889,8 | 1667,3 |

| | | | | |
|---|---|---|---|---|
| June | 6 | 36 | 890,8 | 1672,9 |
| July | 7 | 49 | 888,3 | 1667,5 |
| August | 8 | 64 | 885,8 | 2001,7 |
| September | 9 | 81 | 883,9 | 2352,5 |
| October | 10 | 100 | 875,1 | 2468,5 |
| NOVEMBER | 11 | 121 | 877,73 | 2012,2 |
| December | 12 | 144 | 873,8 | 1982,7 |
| January | 13 | 169 | 878,2 | 1901,1 |
| February | 14 | 196 | 866,3 | 1971,6 |
| March | 15 | 225 | 865,1 | 1989,1 |
| April | 16 | 256 | 876,6 | 2138,2 |
| May | 17 | 289 | 862,5 | 2475,2 |
| June | 18 | 324 | 865,9 | 2395,6 |
| July | 19 | 361 | 857,5 | 2994,1 |

Table 8.6. The results of calculating the estimates from the model (8.4).

| | | | | | |
|---|---|---|---|---|---|
| Number of factors - | | | 3 | | |
| The regression equation Y=b0+b1*t+b2*t^2+b3*X1 | | | | | |
| -13,7165 | -1,39341 | 71,73884 | 13467,27 | | |
| 13,89647 | 2,302831 | 51,68798 | 12491,62 | | |
| 0,744074 | 265,0984 | #Н/Д | #Н/Д | | |
| 14,53692 | 15 | #Н/Д | #Н/Д | | |
| 3064840 | 1054158 | #Н/Д | #Н/Д | | |
| | | | | | |
| | | | | | |
| The adjusted coefficient of determination | | | | | |
| 0,712084 | | | | | |
| | | | | | |




| | Time | Period | Period in squaire | Production costs | Sales volume |
|---|---|---|---|---|---|
| 2 | Январь | 1 | 1 | 890,8 | 1286,5 |
| 3 | Февраль | 2 | 4 | 896,4 | 1282,7 |
| 4 | Март | 3 | 9 | 903 | 1224,8 |
| 5 | Апрель | 4 | 16 | 888,8 | 1292,6 |
| 6 | Май | 5 | 25 | 889,8 | 1667,3 |
| 7 | Июнь | 6 | 36 | 890,8 | 1672,9 |
| 8 | Июль | 7 | 49 | 888,3 | 1667,5 |
| 9 | Август | 8 | 64 | 885,8 | 2001,7 |
| 10 | Сентябрь | 9 | 81 | 883,9 | 2352,5 |
| 11 | Октябрь | 10 | 100 | 875,1 | 2468,5 |
| 12 | НОЯБРЬ | 11 | 121 | 877,73 | 2012,2 |
| 13 | Декабрь | 12 | 144 | 873,8 | 1982,7 |
| 14 | Январь | 13 | 169 | 878,2 | 1901,1 |
| 15 | Февраль | 14 | 196 | 866,3 | 1971,6 |
| 16 | Март | 15 | 225 | 865,1 | 1989,1 |
| 17 | Апрель | 16 | 256 | 876,6 | 2138,2 |
| 18 | Май | 17 | 289 | 862,5 | 2475,2 |
| 19 | Июнь | 18 | 324 | 865,9 | 2395,6 |
| 20 | Июль | 19 | 361 | 857,5 | 2994,1 |

Fig. 8.3. Screenshot of calculating the corrected determination coefficient for the model (8.4).

As can be seen from Table 8.6. and figure 8.3 in the regression model (8.4), the determination coefficient decreased by 0.744074, and the adjusted determination coefficient decreased also 0.712084. This suggests that the excluded factor "advertising costs" is essential for the forecasting model, i.e. it is necessary to leave the "advertising costs" factor in the forecasting model.

## 8.2 PRACTICAL LESSON 8 and IWS 8

**Subject.** Selection of factors for the prediction problem.

**Plan of the lesson.**

1) Examine the selection scheme for the factors for the prediction task, presented above.

2) Practical lesson and task of the IWS. Apply the above selection of factors for your prediction task.

3) Analyze the results.

Literature.

Minko A.A. Predicting in business using Excel. M.: Eksmo, 2007, -208 p.

## TOPIC 9

## ESTIMATION OF THE QUALITY OF THE DATA MODEL FOR THE PREDICTION TASK

### 9.1 Lecture material

Evaluation of the quality of the data model means to estimate the accuracy of the approximation of the input data by the prediction function$\boldsymbol{Y} = \boldsymbol{f}(\boldsymbol{t}, \boldsymbol{X_1}, \boldsymbol{X_2}, \dots \boldsymbol{X_m};\ \boldsymbol{\varepsilon})$. There are various methods for estimating the accuracy of the approximation of the input data by the prediction function$\boldsymbol{f} = \boldsymbol{f}(\boldsymbol{b_1}, \boldsymbol{b_2}, \dots, \boldsymbol{b_k}; \boldsymbol{t};\ \boldsymbol{X_1}, \boldsymbol{X_2}, \dots \boldsymbol{X_m})$. The scheme of the effect of the random variable $\boldsymbol{\varepsilon}$ on the prediction function is adopted either additive ($f + \boldsymbol{\varepsilon}$), or multiplicative ($f * \boldsymbol{\varepsilon}$).

The quality of the data model is estimated by various indicators:

- The method of least squares (OLS);

- coefficient of determination;

- adjusted coefficient of determination;

- the average absolute deviation: $\frac{1}{n}\sum_{i=1}^{n} |e_i\ |;;$

- the average absolute error in percent: $(\frac{1}{n}\sum_{i=1}^{n} |\frac{e_i}{y_i}\ |)\ 100\%.$

The coefficient of determination and the adjusted coefficient of determination were previously considered in detail. The average absolute deviation and the mean absolute error in percent as well as the coefficient of determination are used when comparing data models built on the same data sets and containing the same set of factors.

The basic method of constructing the approximating function of the input data is the method of least squares. The main condition for the application of this method is the linear dependence of the approximating function on the unknown (sought) parameters.

The essence of the method of least squares is that the determination of the values of the unknown parameters $b_1, b_2, \dots, b_k$of the approximating function were based on the criterion of the minimum sum of squared deviations of the computed values

of the predicted variable by the approximating function from the observed values of the predicted (dependent) variable:

$$\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - f(x_i))^2 \to min,$$

Where $y_i$are the observed values of the predicted variable at the i-th point of the initial data, $f(x_i)$is the calculated values of the predicted variable from the approximating function at the i-th point.

Finding the "correct" values of the parameters $b_1, b_2, \dots, b_k$by the method of least squares is **due to a number of conditions**. Let $\varepsilon_i$ be the random effects on the i-th point of the original data. Then the **conditions of least squares** have the following form [1]:

1) The mathematical expectation of random variables $\varepsilon_i$is equal to 0 for all data points.

2) The variances of the random variables $\varepsilon_i$must be the same for all data points. This condition is called the condition of data homoscedasticity, and its non-fulfillment is called heteroscedasticity of the data.

3) The random variables $\varepsilon_i$ must be independent of each other. This condition is called the lack of autocorrelation.

4) The random variables $\varepsilon_i$must be independent of the factors.

5) There should be no strong linear relationship between the factors. This condition is called the absence of multicollinearity.

6) The random variables $\varepsilon_i$ have a normal distribution with zero mathematical expectation and the same variance.

It should also be noted that the residuals $e_i$are taken as estimates of the random effects $\varepsilon_i$at the i-th point of the predicted variable. This is due to the fact that random effects can not be observed explicitly.

If these conditions are fulfilled, the values of the parameters $b_1, b_2, \dots, b_k$calculated by the least squares will be "correct": **unbiased, consistent and effective**. The **unbiased** property of the parameter values means that their mathematical expectations will be equal to the true values of these parameters. The property of **consistency** of the values of the parameters means that the variances of these values with an unlimited increase in the number n of data points will tend to zero.

The **effectiveness** property of parameter values means that they have the smallest variance compared to any other estimates of these parameters.

When evaluating the quality of a data model, the feasibility of the method of least squares conditions is important. Thus, if the method of least squares conditions are not fulfilled, it is necessary to consider the correspondence of the chosen type of predicting function to the nature of the initial data. Often in the appearance of the picture of the original data, it is necessary to select a suitable type of prediction function (approximation).

One of the primary conditions of the method of least squares to be checked is the condition of homoscedasticity of data (the uniformity of the variance of residues for all data points) and the lack of autocorrelation of random effects $\varepsilon_i$ (taking into account the use of residues - this will be the verification of the independence of the residues $\varepsilon_i$). Below in Figure 9.1, a graph of the homoscedasticity of the data is shown - the uniformity of the variance of the residues for all data points of factor X, and Figure 9.2 shows the graph of the hetaera.

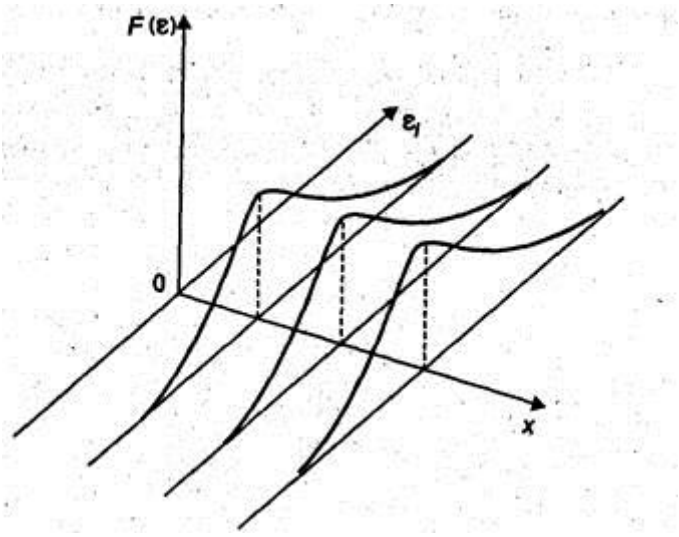



Fig.9.1. Homoscedasticity of residues [2]

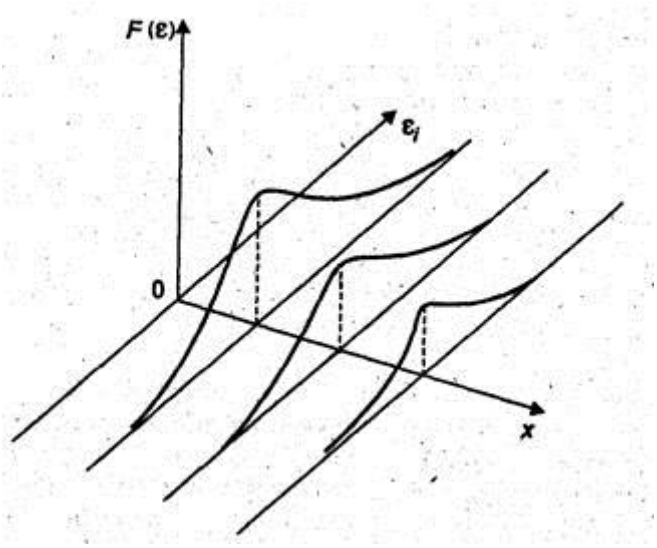



Fig.9.2. Heteroscedasticity of residues [2]

Consider the tests to verify these two conditions for the method of least squares. There are various developed tests to test each of these conditions. We show the tests of these conditions on one of them.

**Verification of the homoscedasticity of data by the Spearman rank correlation test.**

A hypothesis is advanced that there is no heteroscedasticity of the random variable $e_i$. It is assumed that the variance of the random variables $e_i$will either increase or decrease as the factor X increases, and therefore in the method of least squares regression the absolute values of the residuals $e_i$and the values of X will be correlated. Scheme of the test:

1) the data on X and the residuals $e_i$are ranked according to X and their ranks are determined; Ranks are defined as the location numbers of the values of X and $e_i$in the rows ordered in ascending order;

2) the Spearman rank correlation coefficient is determined by the formula

$r = 1 - \frac{6\sum_{i=1}^{n} D_i^2}{n(n^2-1)}$, where $D_i$ is the difference between the ranks of X and $e_i$;

3) The criterial statistics are calculated:

$$t_{x,e} = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

4) The critical value $t_{cr}$ is calculated as a quantile of the order of 1-α / 2 of the Student's distribution with the degree of freedom n-2.

If $t_{x,e} \leq t_{cr}$ , then this will indicate the absence of heteroscedasticity, or, conversely, the presence of homoscedasticity.

The following is an example of how to perform this test. The initial data of the example are presented in Table 9.1. The check on the homoscedasticity of the time factor and the residues in the predict function

$$Y = b_0 + b_1 t + b_2 t^2 + b_3 X_1 + b_4 X_2$$

Where $X_1$- production costs, $X_2$- advertising costs, Y - sales volumes, t - time.

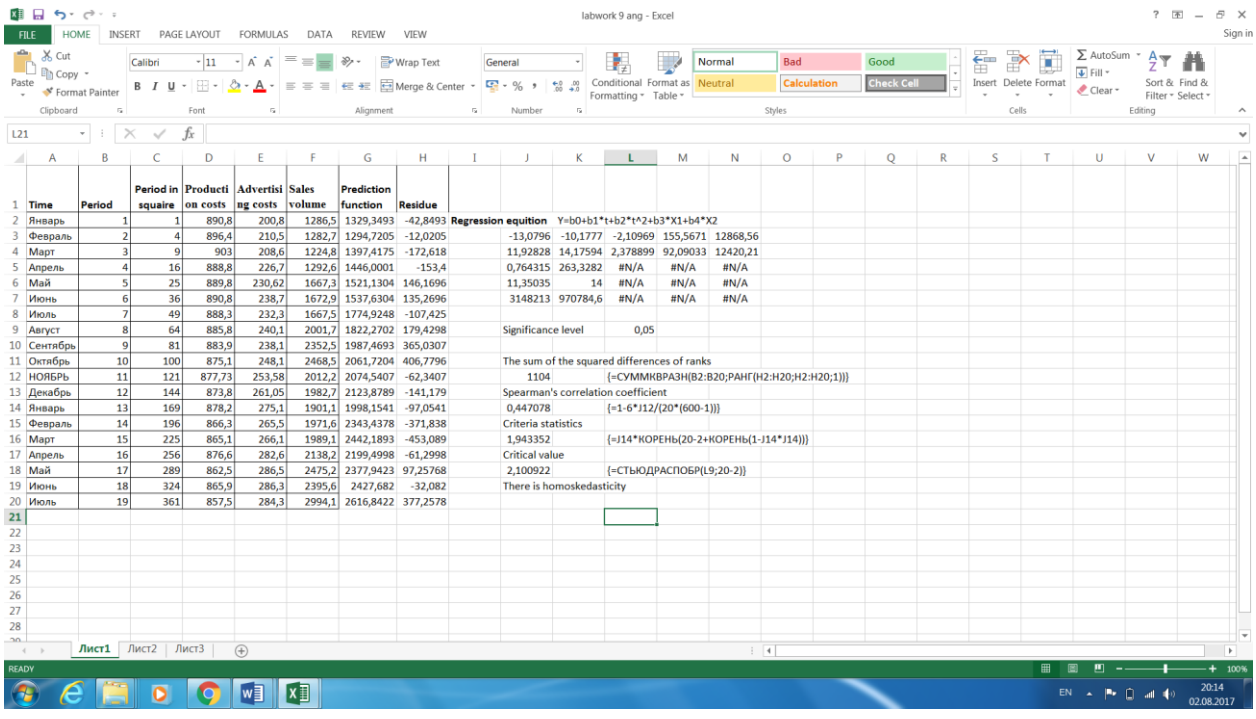
Table 9.1. Initial data for the homoscedasticity test of residues

| Time | Period | Period in square | Production costs | Advertising costs | Sales volume | Function forecast | Residuals |
|---|---|---|---|---|---|---|---|
| January | 1 | 1 | 890,8 | 200,8 | 1286,5 | 1329,349 | -42,8493 |
| February | 2 | 4 | 896,4 | 210,5 | 1282,7 | 1294,721 | -12,0205 |
| March | 3 | 9 | 903 | 208,6 | 1224,8 | 1397,418 | -172,618 |
| April | 4 | 16 | 888,8 | 226,7 | 1292,6 | 1446 | -153,4 |
| May | 5 | 25 | 889,8 | 230,62 | 1667,3 | 1521,13 | 146,1696 |
| June | 6 | 36 | 890,8 | 238,7 | 1672,9 | 1537,63 | 135,2696 |
| July | 7 | 49 | 888,3 | 232,3 | 1667,5 | 1774,925 | -107,425 |
| August | 8 | 64 | 885,8 | 240,1 | 2001,7 | 1822,27 | 179,4298 |
| September | 9 | 81 | 883,9 | 238,1 | 2352,5 | 1987,469 | 365,0307 |
| October | 10 | 100 | 875,1 | 248,1 | 2468,5 | 2061,72 | 406,7796 |
| NOVEMBER | 11 | 121 | 877,73 | 253,58 | 2012,2 | 2074,541 | -62,3407 |
| December | 12 | 144 | 873,8 | 261,05 | 1982,7 | 2123,879 | -141,179 |
| January | 13 | 169 | 878,2 | 275,1 | 1901,1 | 1998,154 | -97,0541 |
| February | 14 | 196 | 866,3 | 265,5 | 1971,6 | 2343,438 | -371,838 |
| March | 15 | 225 | 865,1 | 266,1 | 1989,1 | 2442,189 | -453,089 |
| April | 16 | 256 | 876,6 | 282,6 | 2138,2 | 2199,5 | -61,2998 |
| May | 17 | 289 | 862,5 | 286,5 | 2475,2 | 2377,942 | 97,25768 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| June | 18 | 324 | 865,9 | 286,3 | 2395,6 | 2427,682 | -32,082 |
| July | 19 | 361 | 857,5 | 284,3 | 2994,1 | 2616,842 | 377,2578 |

Table 9.2. Computation of the homoscedasticity test of residues

| | | | | | | |
|---|---|---|---|---|---|---|
| The regression equation Y=b0+b1*t+b2*t^2+b3*X1+b4*X2 | | | | | | |
| -13,0796 | -10,1777 | -2,10969 | 155,5671 | 12868,56 | | |
| 11,92828 | 14,17594 | 2,378899 | 92,09033 | 12420,21 | | |
| 0,764315 | 263,3282 | #Н/Д | #Н/Д | #Н/Д | | |
| 11,35035 | 14 | #Н/Д | #Н/Д | #Н/Д | | |
| 3148213 | 970784,6 | #Н/Д | #Н/Д | #Н/Д | | |
| | | | | | | |
| Significance level | | 0,05 | | | | |
| | | | | | | |
| The sum of the squared differences of ranks | | | | | | |
| 1104 | | {=СУММКВРАЗН(B2:B20;РАНГ(H2:H20;H2:H20;1))} | | | | |
| Spearman's correlation coefficient | | | | | | |
| 0,447078 | | {=1-6*J12/(20*(600-1))} | | | | |
| Criteria statistics | | | | | | |
| 1,943352 | | {=J14*КОРЕНЬ(20-2+КОРЕНЬ(1-J14*J14))} | | | | |
| Critical value | | | | | | |
| 2,100922 | | {=СТЬЮДРАСПОБР(L9;20-2)} | | | | |
| There is homoskedasticity | | | | | | |
| | | | | | | |




| | A | B | C | D | E | F | G | H | I | J | K | L | M | N |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Time | Period | Period in squaire | Production costs | Advertising costs | Sales volume | Prediction function | Residue | | | | | | |
| 2 | Январь | 1 | 1 | 890,8 | 200,8 | 1286,5 | 1329,3493 | -42,8493 | | Regression equition | Y=b0+b1*t+b2*t^2+b3*X1+b4*X2 | | | |
| 3 | Февраль | 2 | 4 | 896,4 | 210,5 | 1282,7 | 1294,7205 | -12,0205 | | -13,0796 | -10,1777 | -2,10969 | 155,5671 | 12868,56 |
| 4 | Март | 3 | 9 | 903 | 208,6 | 1224,8 | 1397,4175 | -172,618 | | 11,92828 | 14,17594 | 2,378899 | 92,09033 | 12420,21 |
| 5 | Апрель | 4 | 16 | 888,8 | 226,7 | 1292,6 | 1446,0001 | -153,4 | | 0,764315 | 263,3282 | #N/A | #N/A | #N/A |
| 6 | Май | 5 | 25 | 889,8 | 230,62 | 1667,3 | 1521,1304 | 146,1696 | | 11,35035 | 14 | #N/A | #N/A | #N/A |
| 7 | Июнь | 6 | 36 | 890,8 | 238,7 | 1672,9 | 1537,6304 | 135,2696 | | 3148213 | 970784,6 | #N/A | #N/A | #N/A |
| 8 | Июль | 7 | 49 | 888,3 | 232,3 | 1667,5 | 1774,9248 | -107,425 | | | | | | |
| 9 | Август | 8 | 64 | 885,8 | 240,1 | 2001,7 | 1822,2702 | 179,4298 | | Significance level | | 0,05 | | |
| 10 | Сентябрь | 9 | 81 | 883,9 | 238,1 | 2352,5 | 1987,4693 | 365,0307 | | | | | | |
| 11 | Октябрь | 10 | 100 | 875,1 | 248,1 | 2468,5 | 2061,7204 | 406,7796 | | The sum of the squared differences of ranks | | | | |
| 12 | НОЯБРЬ | 11 | 121 | 877,73 | 253,58 | 2012,2 | 2074,5407 | -62,3407 | | 1104 | | {=СУММКВРАЗН(B2:B20;РАНГ(H2:H20;H2:H20;1))} | | |
| 13 | Декабрь | 12 | 144 | 873,8 | 261,05 | 1982,7 | 2123,8789 | -141,179 | | Spearman's correlation coefficient | | | | |
| 14 | Январь | 13 | 169 | 878,2 | 275,1 | 1901,1 | 1998,1541 | -97,0541 | | 0,447078 | | {=1-6*J12/(20*(600-1))} | | |
| 15 | Февраль | 14 | 196 | 866,3 | 265,5 | 1971,6 | 2343,4378 | -371,838 | | Criteria statistics | | | | |
| 16 | Март | 15 | 225 | 865,1 | 266,1 | 1989,1 | 2442,1893 | -453,089 | | 1,943352 | | {=J14*КОРЕНЬ(20-2+КОРЕНЬ(1-J14*J14))} | | |
| 17 | Апрель | 16 | 256 | 876,6 | 282,6 | 2138,2 | 2199,4998 | -61,2998 | | Critical value | | | | |
| 18 | Май | 17 | 289 | 862,5 | 286,5 | 2475,2 | 2377,9423 | 97,25768 | | 2,100922 | | {=СТЬЮДРАСПОБР(L9;20-2)} | | |
| 19 | Июнь | 18 | 324 | 865,9 | 286,3 | 2395,6 | 2427,682 | -32,082 | | There is homoskedasticity | | | | |
| 20 | Июль | 19 | 361 | 857,5 | 284,3 | 2994,1 | 2616,8422 | 377,2578 | | | | | | |
| 21 | | | | | | | | | | | | | | |
| 22 | | | | | | | | | | | | | | |
| 23 | | | | | | | | | | | | | | |
| 24 | | | | | | | | | | | | | | |
| 25 | | | | | | | | | | | | | | |
| 26 | | | | | | | | | | | | | | |
| 27 | | | | | | | | | | | | | | |
| 28 | | | | | | | | | | | | | | |

Fig. 9.3. Screenshot of the initial data and calculation of the homoscedasticity test of the time factor and residues

Based on the results of the homoscedasticity test of the time factor and residues, it was found that $t_{x,e}$ = 1,943352, $t_{cr}$ = 2,100922.. Accordingly, $t_{x,e} \leq t_{cr}$ , then there is a homoscedasticity of the time factor and residuals.

**Verification of the condition of independence of the residual data by the Durbin-Watson test.**

The verification of the independence of the residues, that is, the test for the **lack of autocorrelation** of the residues can be performed according to the Darbin-Watson test (criterion). The criterion is named after James Durbin and Jeffrey Watson [3]. The Durbin-Watson criterion is calculated by the following formula:

$$DW = \frac{\sum_{i=2}^{n}(e_i - e_{i-1})^2}{\sum_{i=1}^{n}(e_i)^2}$$

In the absence of autocorrelation DW = 2, with positive autocorrelation, DW tends to zero, and for negative autocorrelation DW approaches 4. And according to the Durbin-Watson criterion, special tables are developed that allow one to determine the critical points of the observable statistics DW for a given number of observations n, the number of factors and a given level of significance. But in practice, the rule is often used:

- the interval from 0 to 4 is divided into several ranges: positive autocorrelation, negative autocorrelation and ranges of uncertainty;

- if DW ≤ 0.5, then there is positive autocorrelation (the range of positive autocorrelation),

- if DW ≥ 3.5, then there is negative autocorrelation (the range of negative autocorrelation),

- if 1.5 ≤ DW ≤ 2.5, then there is no autocorrelation (the range of absence of autocorrelation).

- in other cases, these are uncertainty ranges when there is no confidence in the presence or absence of autocorrelation: 0.5 <DW <1.5 and 2.5 <DW <3.5.

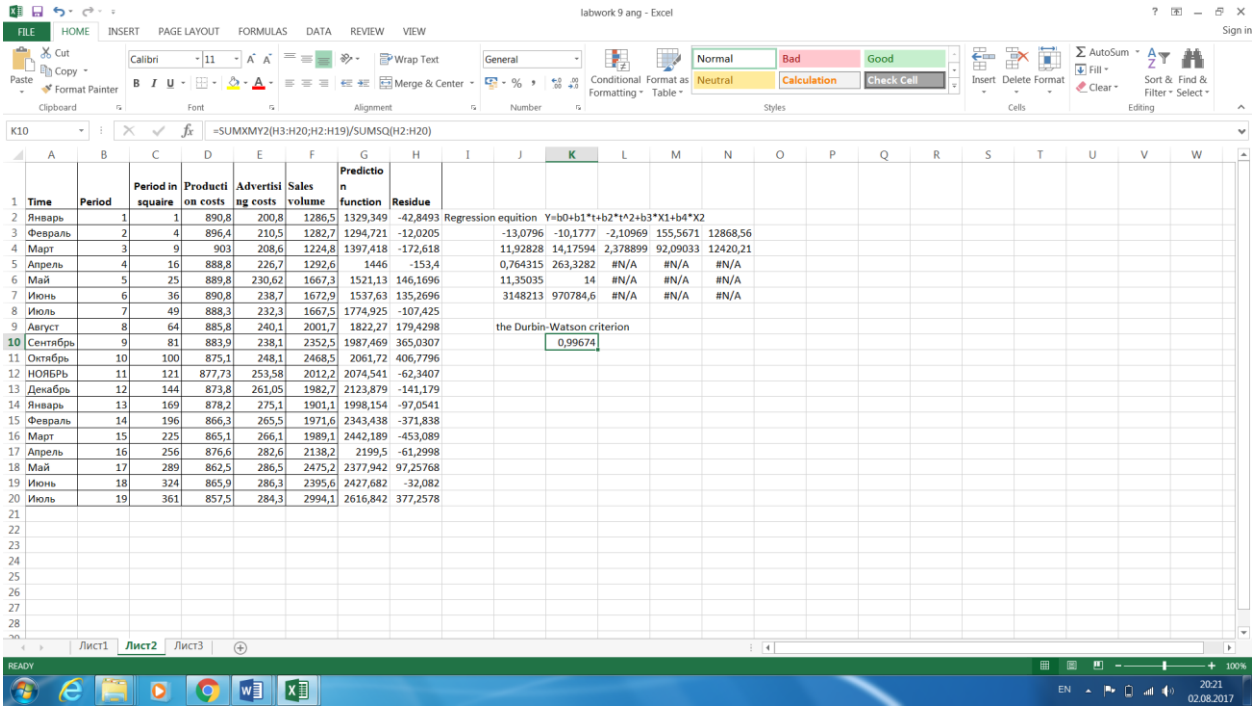
If the value of the Durbin-Watson criterion falls within the autocorrelation ranges or uncertainty ranges when analyzing the given source data model, this should be the reason for further research on the selection of a more suitable prediction function.

Below in Table 9.3 and Figure 9.3, an example of the verification of the independence of residues according to the Durbin-Watson criterion is presented. The initial data is taken from Table 9.1.

Table 9.3 Calculation of the Durbin-Watson criterion for checking the independence of residues

| The regression equation Y=b0+b1*t+b2*t^2+b3*X1+b4*X2 | | | | | | |
|---|---|---|---|---|---|---|
| | -13,0796 | -10,1777 | -2,10969 | 155,5671 | 12868,56 | |
| | 11,92828 | 14,17594 | 2,378899 | 92,09033 | 12420,21 | |
| | 0,764315 | 263,3282 | #Н/Д | #Н/Д | #Н/Д | |
| | 11,35035 | 14 | #Н/Д | #Н/Д | #Н/Д | |
| | 3148213 | 970784,6 | #Н/Д | #Н/Д | #Н/Д | |
| | | | | | | |
| | the Durbin-Watson criterion | | | | | |
| | | 0,99674 | | | | |

In this example, the value of the Durbin-Watson criterion is 0.99674, i.e. It falls within the uncertainty range of 0.5 <DW <1.5. This is the reason for choosing a more advanced forecasting function.




| | Time | Period | Period in squaire | Production costs | Advertising costs | Sales volume | Prediction function | Residue | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Time | Period | Period in squaire | Producti on costs | Advertisi ng costs | Sales volume | Predictio n function | Residue | | | | | | | | | | | | | | | |
| 2 | Январь | 1 | 1 | 890,8 | 200,8 | 1286,5 | 1329,349 | -42,8493 | Regression equition | | Y=b0+b1*t+b2*t^2+b3*X1+b4*X2 | | | | | | | | | | | | |
| 3 | Февраль | 2 | 4 | 896,4 | 210,5 | 1282,7 | 1294,721 | -12,0205 | | | -13,0796 | -10,1777 | -2,10969 | 155,5671 | 12868,56 | | | | | | | | |
| 4 | Март | 3 | 9 | 903 | 208,6 | 1224,8 | 1397,418 | -172,618 | | | 11,92828 | 14,17594 | 2,378899 | 92,09033 | 12420,21 | | | | | | | | |
| 5 | Апрель | 4 | 16 | 888,8 | 226,7 | 1292,6 | 1446 | -153,4 | | | 0,764315 | 263,3282 | #N/A | #N/A | #N/A | | | | | | | | |
| 6 | Май | 5 | 25 | 889,8 | 230,62 | 1667,3 | 1521,13 | 146,1696 | | | 11,35035 | 14 | #N/A | #N/A | #N/A | | | | | | | | |
| 7 | Июнь | 6 | 36 | 890,8 | 238,7 | 1672,9 | 1537,63 | 135,2696 | | | 3148213 | 970784,6 | #N/A | #N/A | #N/A | | | | | | | | |
| 8 | Июль | 7 | 49 | 888,3 | 232,3 | 1667,5 | 1774,925 | -107,425 | | | | | | | | | | | | | | | |
| 9 | Август | 8 | 64 | 885,8 | 240,1 | 2001,7 | 1822,27 | 179,4298 | | | the Durbin-Watson criterion | | | | | | | | | | | | |
| 10 | Сентябрь | 9 | 81 | 883,9 | 238,1 | 2352,5 | 1987,469 | 365,0307 | | | | 0,99674 | | | | | | | | | | | |
| 11 | Октябрь | 10 | 100 | 875,1 | 248,1 | 2468,5 | 2061,72 | 406,7796 | | | | | | | | | | | | | | | |
| 12 | НОЯБРЬ | 11 | 121 | 877,73 | 253,58 | 2012,2 | 2074,541 | -62,3407 | | | | | | | | | | | | | | | |
| 13 | Декабрь | 12 | 144 | 873,8 | 261,05 | 1982,7 | 2123,879 | -141,179 | | | | | | | | | | | | | | | |
| 14 | Январь | 13 | 169 | 878,2 | 275,1 | 1901,1 | 1998,154 | -97,0541 | | | | | | | | | | | | | | | |
| 15 | Февраль | 14 | 196 | 866,3 | 265,5 | 1971,6 | 2343,438 | -371,838 | | | | | | | | | | | | | | | |
| 16 | Март | 15 | 225 | 865,1 | 266,1 | 1989,1 | 2442,189 | -453,089 | | | | | | | | | | | | | | | |
| 17 | Апрель | 16 | 256 | 876,6 | 282,6 | 2138,2 | 2199,5 | -61,2998 | | | | | | | | | | | | | | | |
| 18 | Май | 17 | 289 | 862,5 | 286,5 | 2475,2 | 2377,942 | 97,25768 | | | | | | | | | | | | | | | |
| 19 | Июнь | 18 | 324 | 865,9 | 286,3 | 2395,6 | 2427,682 | -32,082 | | | | | | | | | | | | | | | |
| 20 | Июль | 19 | 361 | 857,5 | 284,3 | 2994,1 | 2616,842 | 377,2578 | | | | | | | | | | | | | | | |

Fig. 9.4. A screenshot of the original data and the calculation of the independence test for residues according to the Durbin-Watson criterion

## 9.2 PRACTICAL LESSON 9 and IWS 9

**Subject.** Estimation of the quality of the data model for the prediction problem.

**Plan of the lesson.**

1) Study the scheme for assessing the quality of the data model for the prediction task, presented above.

2) Practical lesson and task of the IWS. Apply the above-described data quality model for your forecasting task.

3) Analyze the results.

Literature.

1. Minko A.A. Predicting in business using Excel. M.: Eksmo, 2007, -208 p.

2. http://metr-ekon.ru. Lectures on Econometrics with examples of solutions. The essence of heteroscedasticity.

3. https://ru.wikipedia.org/wiki/Robin_Watson_Charter_

**TOPIC 10**

**DATA SMOOTHING FOR THE PREDICTION TASK**

## 10.1. Lecture Material

Data smoothing is a procedure for preliminary processing of the initial data, which allows smoothing (decreasing) the values of random effects in the original data. Consider three methods of smoothing the data: the moving average, exponential smoothing, and the Holt method.

### Moving average method.

Smoothing of data by the moving average method is carried out according to the formula [1]:

$$\widehat{y_i} = \frac{1}{2k+1}(y_{i-k} + y_{i-k+1} + \cdots + y_{i-1} + \boldsymbol{y_i} + y_{i+1} + \cdots + y_{i+k-1} + y_{i+k}), \quad (10.1)$$

where k is a positive integer that is less than n. In practice, the value of k is set to 1, 2 or 3. That is, k values are taken to the left from the right and k values from the averaged value $y_i$, and also $y_i$itself.

The disadvantage of this method is the reduction of the smoothed data on the left and right by k values relative to the original data.

In the case of time series, the moving average method is calculated using the formula:

$$\widehat{y_i} = \frac{1}{k}(y_{i-k+1} + y_{i-k+2} + \cdots + y_{i-1} + y_i). \quad (10.2)$$

In this case, we take on the left k-1 values from the averaged value $y_i$, and also $y_i$ itself.

The disadvantage of this formula is the cut of the smoothed data on the left by k-1 values, i.e. a shift to the right of smoothed data relative to the original data.

Table 10.1. Initial data and results for the moving average method according to the formula (10.1)

| Time | Period | Production costs | The smoothed data (formula 10.1) |
|---|---|---|---|
| February | 2 | 1282,7 | |
| March | 3 | 1224,8 | 1266,7 |
| April | 4 | 1292,6 | 1394,9 |
| May | 5 | 1667,3 | 1544,267 |
| June | 6 | 1672,9 | 1669,233 |
| July | 7 | 1667,5 | 1780,7 |
| August | 8 | 2001,7 | 2007,233 |
| September | 9 | 2352,5 | 2274,233 |
| October | 10 | 2468,5 | 2277,733 |
| NOVEMBER | 11 | 2012,2 | 2154,467 |
| December | 12 | 1982,7 | 1965,333 |
| January | 13 | 1901,1 | 1951,8 |
| February | 14 | 1971,6 | 1953,933 |
| March | 15 | 1989,1 | 2032,967 |
| April | 16 | 2138,2 | 2200,833 |
| May | 17 | 2475,2 | 2336,333 |
| June | 18 | 2395,6 | 2621,633 |
| July | 19 | 2994,1 | 2694,85 |

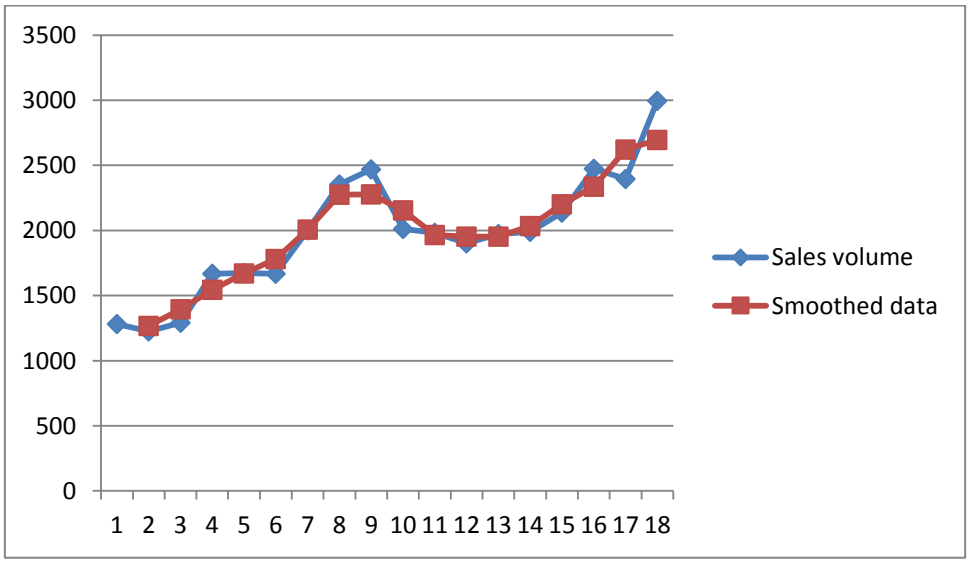


Fig. 10.1 Sales volume charts: the initial (row 1) and smoothed (row 2) by formula (10.1)

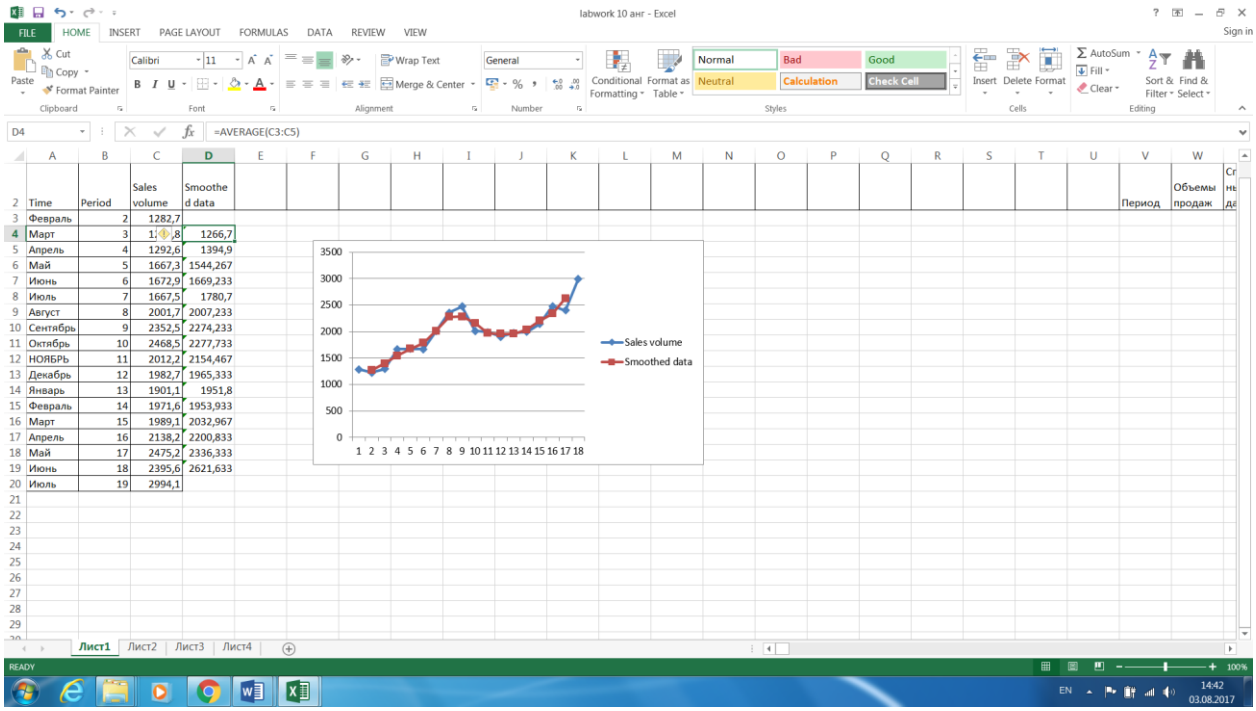



Fig. 10.2. A screenshot of the initial data and smoothing using the moving average method according to the formula (10.1)

Table 10.2 Initial data and results for the moving average method using formula (10.2)

| Time | Period | Production costs | The smoothed data (formula 10.2) |
|---|---|---|---|
| February | 2 | 1282,7 | |
| March | 3 | 1224,8 | 1253,75 |
| April | 4 | 1292,6 | 1258,7 |
| May | 5 | 1667,3 | 1479,95 |
| June | 6 | 1672,9 | 1670,1 |
| July | 7 | 1667,5 | 1670,2 |
| August | 8 | 2001,7 | 1834,6 |
| September | 9 | 2352,5 | 2177,1 |
| October | 10 | 2468,5 | 2410,5 |
| NOVEMBER | 11 | 2012,2 | 2240,35 |
| December | 12 | 1982,7 | 1997,45 |
| January | 13 | 1901,1 | 1941,9 |
| February | 14 | 1971,6 | 1936,35 |
| March | 15 | 1989,1 | 1980,35 |
| April | 16 | 2138,2 | 2063,65 |
| May | 17 | 2475,2 | 2306,7 |

| | | | |
|---|---|---|---|
| June | 18 | 2395,6 | 2435,4 |
| July | 19 | 2994,1 | 2694,85 |

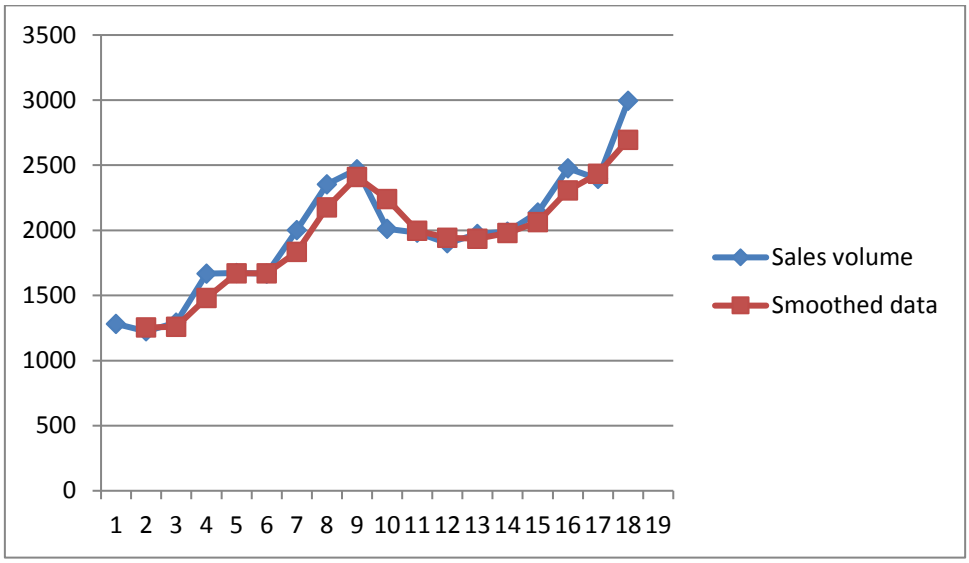



Fig. 10.3 Sales volume charts: the initial (row 1) and smoothed by formula (10.2)

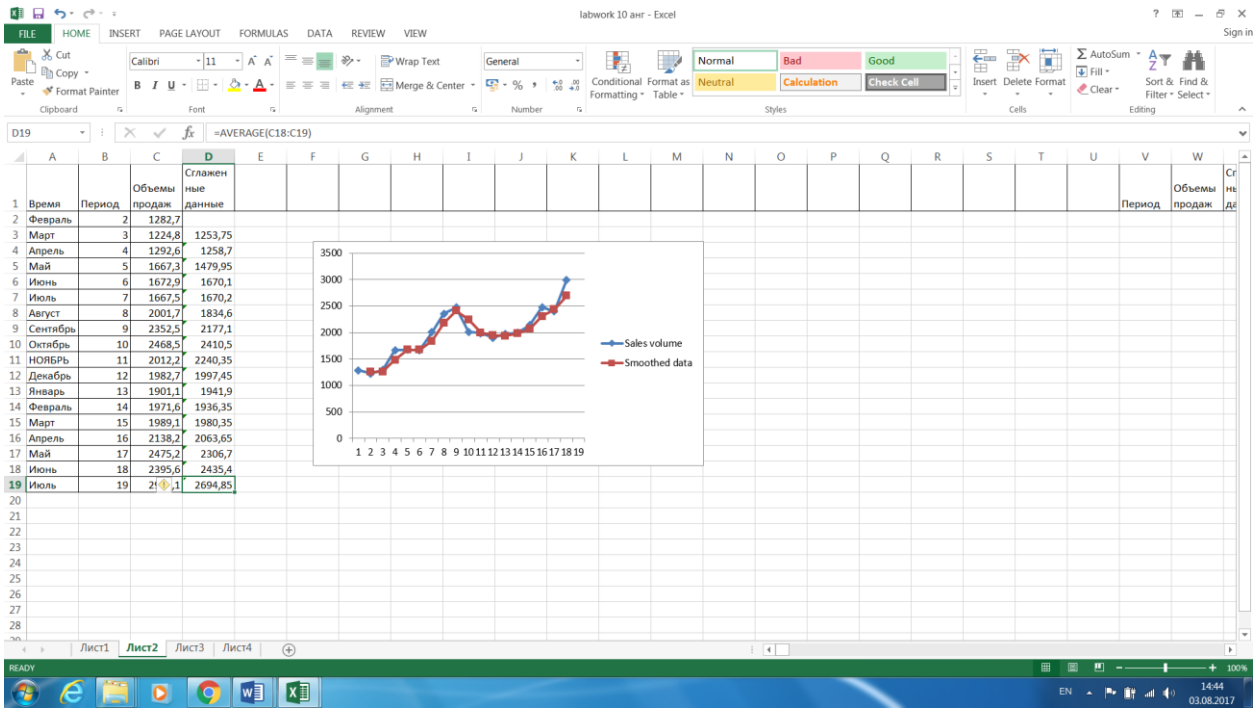



Fig. 10.4. A screenshot of the initial data and smoothing using the moving average method according to the formula (10.2)

**Exponential data smoothing.**

Exponential data smoothing applies to data that depends on time, i.e. to time series. In this method, the smoothed values are calculated using the following formula:

$$\widehat{y_i} = (1 - \alpha)y_i + \alpha\widehat{y_{i-1}}, \quad (10.3)$$

Where α is the smoothing coefficient; $\widehat{y_1}$ is taken as $y_1$. The values of α are taken in the range (0.1), in practice - from 0.2 to 0.8.

The smoothing factor allows some balancing between the previous smoothed value $\widehat{y_{i-1}}$ and the current observed value $y_i$ to get the current smoothed value $\widehat{y_i}$. The closer the smoothing coefficient is to zero, the less is the smoothing of the observed value, i.e. When α = 0, there is no smoothing. Conversely, for α = 1, the current smoothed value will coincide with the previous smoothed value, i.e. the graph of smoothed values will be a horizontal line (see Figure 10.6).

Table 10.3 Values of the α - smoothing coefficient and the smoothing formula with their values

| | | |
|---|---|---|
| 0,2 | coeff smoothing 1 | {=(1-$I$2)*C3+$I$2*D2} |
| 0,4 | coeff smoothing 2 | {=(1-$I$3)*C3+$I$3*E2} |
| 0,6 | coeff smoothing 3 | {=(1-$I$4)*C3+$I$4*F2} |
| 0,8 | coeff smoothing 4 | {=(1-$I$5)*3+$I$5*G2} |
| 1 | coeff smoothing 5 | {=(1-$I$6)*C3+$I$6*H2} |

Table 10.4 Initial data and results for the method of exponential smoothing by the formula (10.3) for the given smoothing coefficients

| Time | Period | Production costs | The smoothed data 1 | The smoothed data 2 | The smoothed data 3 | The smoothed data 4 | The smoothed data 5 |
|---|---|---|---|---|---|---|---|
| February | 2 | 1282,7 | 1282,7 | 1282,7 | 1282,7 | 1282,7 | 1282,7 |
| March | 3 | 1224,8 | 1236,38 | 1247,96 | 1259,54 | 1271,12 | 1282,7 |
| April | 4 | 1292,6 | 1281,356 | 1274,744 | 1272,764 | 1273,167 | 1282,7 |
| May | 5 | 1667,3 | 1590,111 | 1510,278 | 1430,578 | 1336,556 | 1282,7 |
| June | 6 | 1672,9 | 1656,342 | 1607,851 | 1527,507 | 1400,513 | 1282,7 |
| July | 7 | 1667,5 | 1665,268 | 1643,64 | 1583,504 | 1453,464 | 1282,7 |
| August | 8 | 2001,7 | 1934,414 | 1858,476 | 1750,783 | 1549,654 | 1282,7 |
| September | 9 | 2352,5 | 2268,883 | 2154,89 | 1991,47 | 1693,5 | 1282,7 |
| October | 10 | 2468,5 | 2428,577 | 2343,056 | 2182,282 | 1840,515 | 1282,7 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| NOVEMBER | 11 | 2012,2 | 2095,475 | 2144,542 | 2114,249 | 1891,507 | 1282,7 |
| December | 12 | 1982,7 | 2005,255 | 2047,437 | 2061,629 | 1914,257 | 1282,7 |
| January | 13 | 1901,1 | 1921,931 | 1959,635 | 1997,418 | 1915,792 | 1282,7 |
| February | 14 | 1971,6 | 1961,666 | 1966,814 | 1987,091 | 1924,967 | 1282,7 |
| March | 15 | 1989,1 | 1983,613 | 1980,186 | 1987,894 | 1936,696 | 1282,7 |
| April | 16 | 2138,2 | 2107,283 | 2074,994 | 2048,017 | 1970,813 | 1282,7 |
| May | 17 | 2475,2 | 2401,617 | 2315,118 | 2218,89 | 2056,974 | 1282,7 |
| June | 18 | 2395,6 | 2396,803 | 2363,407 | 2289,574 | 2124,94 | 1282,7 |
| July | 19 | 2994,1 | 2874,641 | 2741,823 | 2571,384 | 2274,88 | 1282,7 |

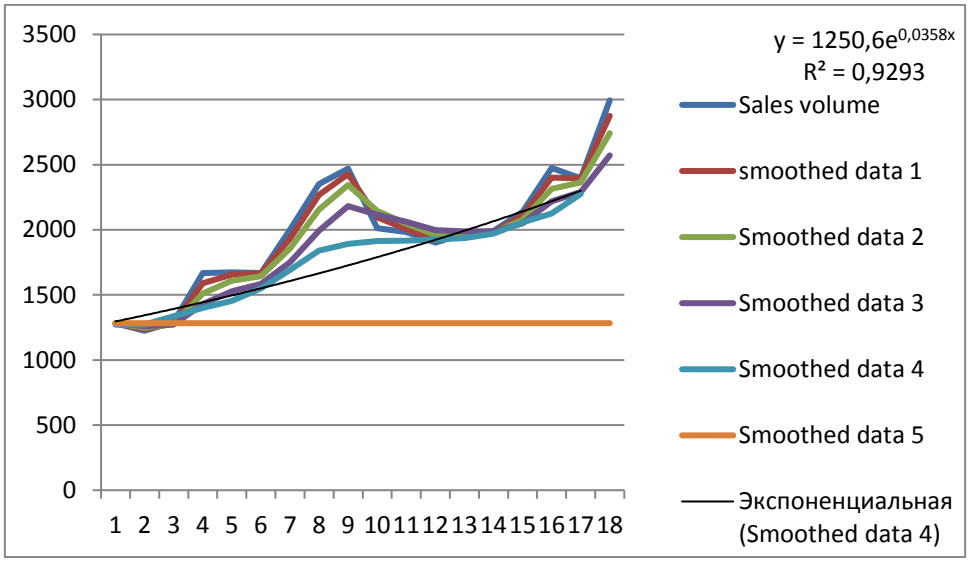



Fig. 10.5 Sales volume charts: the initial (row 1) and smoothed (row 2, row2, row3, row4, row5, row6) by formula (10.3)

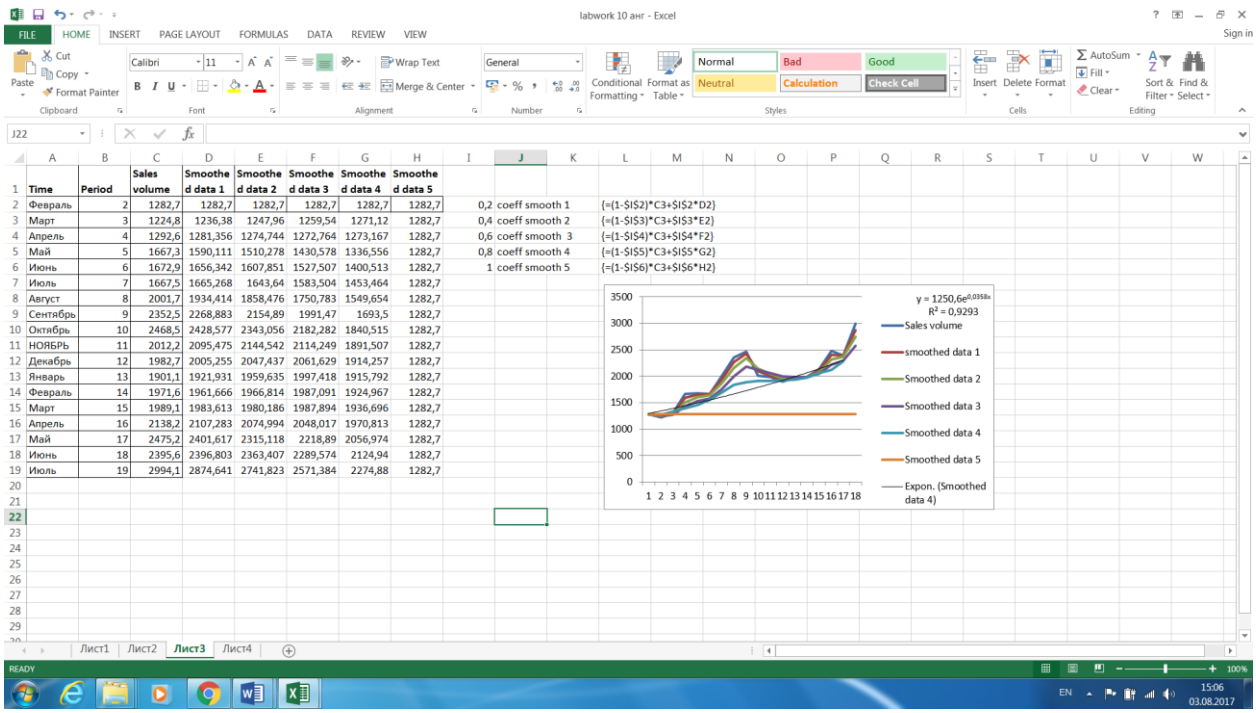


Fig. 10.6 A screenshot of the original data and exponential smoothing using formula (10.3)

**Holt's smoothing method.**

The method of smoothing Holt is based on adding a trend component to the data being smoothed. Calculated by the following formulas:

$$\widehat{y_i} = (1-\alpha)y_i + \alpha(\widehat{y_{i-1}} + T_{i-1}); \quad T_i = (1-\beta)(\widehat{y_i} - \widehat{y_{i-1}}) + \beta T_{i-1}, \qquad (10.4)$$

Where α and β are smoothing coefficients, taking values from 0 to 1. The value $\widehat{y_1}$ is taken as $y_1$, and the value calculated by the formula

$$T_1 = \frac{y_2 - y_1 + y_4 - y_3}{2}$$

is taken as the value of $T_1$.

By Holt's method, it is possible to produce and predict for k periods ahead. Calculations are made by the formula:

$$\widehat{y_{n+k}} = \widehat{y_n} + kT_n \qquad (10.5)$$

Table 10.5 Initial data and results for the Holt method according to the formula (10.4)

| Time | Period | Production costs | The smoothed data | Ti | Alfa = | 0,6 | Beta= | 0,4 |
|---|---|---|---|---|---|---|---|---|
| February | 2 | 1282,7 | 1282,7 | 158,4 | | Ti{=(C3-C2+C5-4)/2} | | |
| March | 3 | 1224,8 | 1164,5 | -7,56 | | $T_i^{sm}${=(1-$I$1)*(D3-2)+$I$1*E2} | | |
| April | 4 | 1292,6 | 1220,276 | 30,4416 | | | | |
| May | 5 | 1667,3 | 1380,821 | 108,5034 | | | | |
| June | 6 | 1672,9 | 1432,55 | 74,43918 | | | | |
| July | 7 | 1667,5 | 1481,867 | 59,36549 | | | | |
| August | 8 | 2001,7 | 1654,181 | 127,1346 | | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| September | 9 | 2352,5 | 1857,228 | 172,682 | | | | |
| October | 10 | 2468,5 | 1998,127 | 153,6126 | | | | |
| NOVEMBER | 11 | 2012,2 | 1911,589 | 9,521932 | | | | |
| December | 12 | 1982,7 | 1934,32 | 17,44755 | | | | |
| January | 13 | 1901,1 | 1910,564 | -7,27493 | | | | |
| February | 14 | 1971,6 | 1939,343 | 14,35775 | | | | |
| March | 15 | 1989,1 | 1950,631 | 12,51597 | | | | |
| April | 16 | 2138,2 | 2018,149 | 45,51715 | | | | |
| May | 17 | 2475,2 | 2173,659 | 111,5129 | | | | |
| June | 18 | 2395,6 | 2195,528 | 57,72631 | | | | |

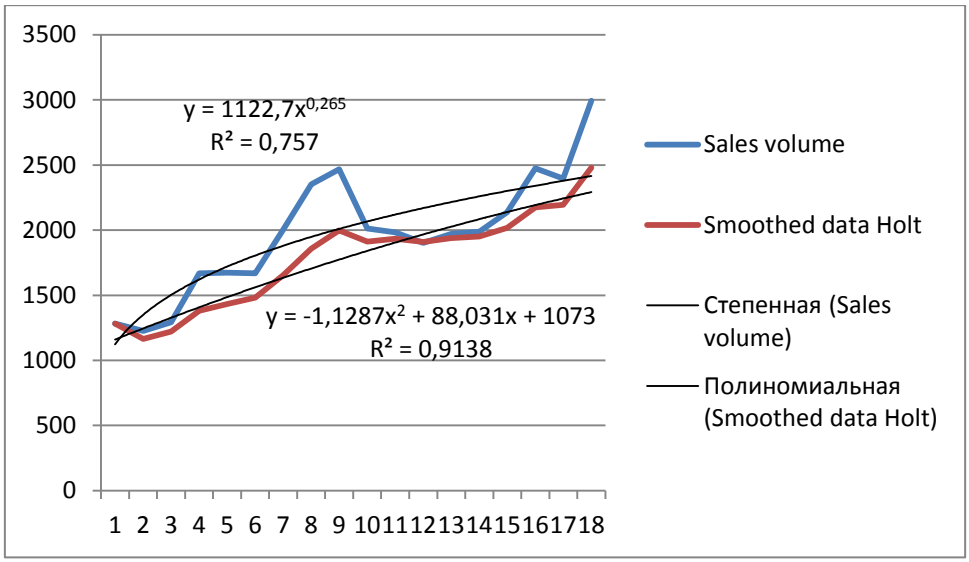
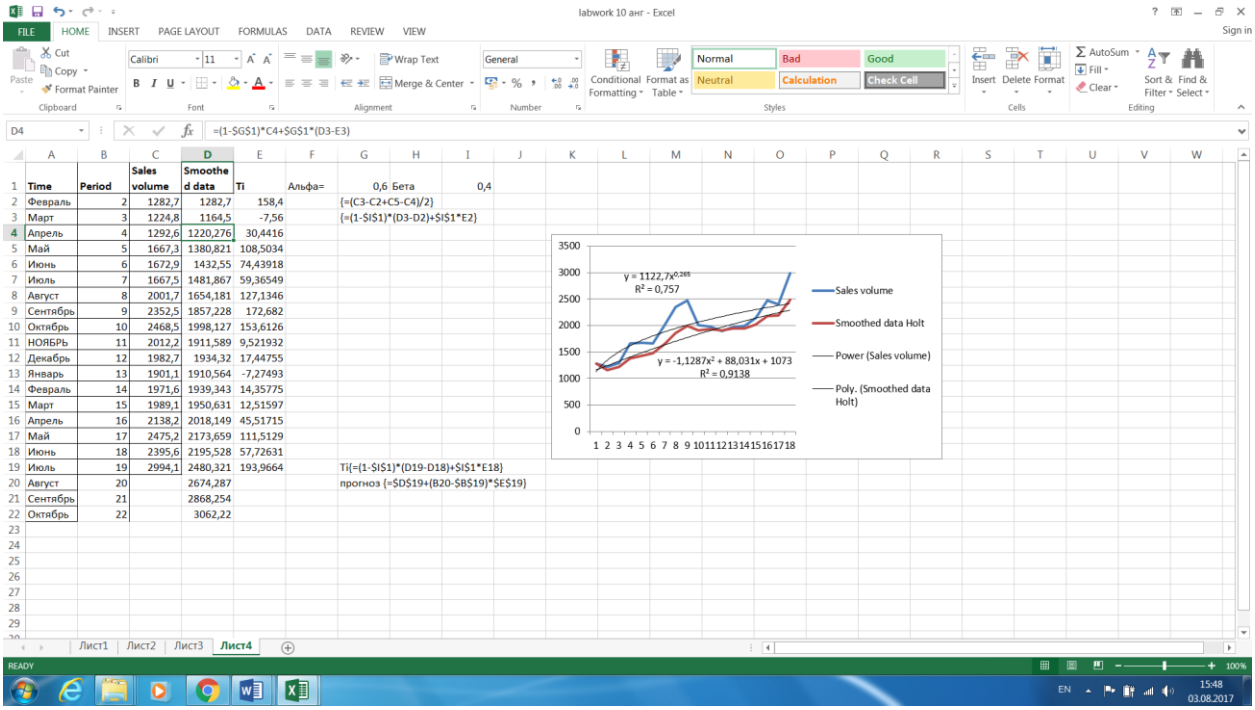



Fig. 10.7 Sales volume graphs: the initial (row 1) and smoothed (row 2) sales volumes for the Holt method using formula (10.4)

In Fig. 10.7 also shows approximations of the original and smoothed sales volume data by predicting functions. The coefficient of determination of the prediction function for smoothed data is much higher.

Table 10.6. The results of calculating the predicting of the volume of sales by the Holt method using formula (10.5)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| July | 19 | 2994,1 | 2480,321 | 193,9664 | Ti{=(1-$I$1)*(D19-D18)+$I$1*E18} | | | |
| August | 20 | | 2674,287 | | прогноз {=$D$19+(B20-$B$19)*$E$19} | | | |
| September | 21 | | 2868,254 | | | | | |
| October | 22 | | 3062,22 | | | | | |




| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Time | Period | Sales volume | Smoothed data | Ti | Альфа= | 0,6 | Бета | 0,4 |
| 2 | Февраль | 2 | 1282,7 | 1282,7 | 158,4 | | {=(C3-C2+C5-C4)/2} | | |
| 3 | Март | 3 | 1224,8 | 1164,5 | -7,56 | | {=(1-$I$1)*(D3-D2)+$I$1*E2} | | |
| 4 | Апрель | 4 | 1292,6 | 1220,276 | 30,4416 | | | | |
| 5 | Май | 5 | 1667,3 | 1380,821 | 108,5034 | | | | |
| 6 | Июнь | 6 | 1672,9 | 1432,55 | 74,43918 | | | | |
| 7 | Июль | 7 | 1667,5 | 1481,867 | 59,36549 | | | | |
| 8 | Август | 8 | 2001,7 | 1654,181 | 127,1346 | | | | |
| 9 | Сентябрь | 9 | 2352,5 | 1857,228 | 172,682 | | | | |
| 10 | Октябрь | 10 | 2468,5 | 1998,127 | 153,6126 | | | | |
| 11 | НОЯБРЬ | 11 | 2012,2 | 1911,589 | 9,521932 | | | | |
| 12 | Декабрь | 12 | 1982,7 | 1934,32 | 17,44755 | | | | |
| 13 | Январь | 13 | 1901,1 | 1910,564 | -7,27493 | | | | |
| 14 | Февраль | 14 | 1971,6 | 1939,343 | 14,35775 | | | | |
| 15 | Март | 15 | 1989,1 | 1950,631 | 12,51597 | | | | |
| 16 | Апрель | 16 | 2138,2 | 2018,149 | 45,51715 | | | | |
| 17 | Май | 17 | 2475,2 | 2173,659 | 111,5129 | | | | |
| 18 | Июнь | 18 | 2395,6 | 2195,528 | 57,72631 | | | | |
| 19 | Июль | 19 | 2994,1 | 2480,321 | 193,9664 | | Ti{=(1-$I$1)*(D19-D18)+$I$1*E18} | | |
| 20 | Август | 20 | | 2674,287 | | | прогноз {=$D$19+(B20-$B$19)*$E$19} | | |
| 21 | Сентябрь | 21 | | 2868,254 | | | | | |
| 22 | Октябрь | 22 | | 3062,22 | | | | | |

Fig. 10.8 The screenshot of the initial data, the results of smoothing and the forecast for the Holt method using formula (10.4) and (10.5)

## 10.2 PRACTICAL LESSON 10 and IWS 10

**Subject.** Smoothing data for the prediction task.

**Plan of the lesson.**

1) Study the data smoothing schemes for the prediction task presented above.

2) Practical lesson and task of the IWS. Apply the above described data smoothing schemes to your prediction task.

3) Analyze the results.

Literature.

1. Minko A.A. Forecasting in business using Excel. M.: Eksmo, 2007, -208 p.

# TOPIC 11

# TREND FUNCTIONS IN PREDICTING TASKS

## 11.1. Lecture material

A trend is the general tendency of changing the initial data. In time-dependent prediction functions, there are usually two components: the trend component, which is the main dependency in the predicting function, and the seasonal component, which may be absent. In this topic, it is consider schemes for building the trend component of the prediction function. The main way to identify the trend is the method of constructing the regression function. As previously shown, the scheme for constructing the regression function is:

- calculation of regression coefficients by the least squares method, which is performed in Excel by standard functions LINEST or LOGEST;

- writing a formula for the chosen regression function using the regression coefficients obtained at the previous stage;

- calculation of the necessary quality indicators of the constructed data model, for example, the adjusted determination coefficient, the mean absolute error or the average absolute error in percent;

- analysis of the quality of the constructed data model.

Below it is consider examples of constructing prediction functions that depend on one factor and prediction functions, depending on several factors.

### 11.1.1 Prediction functions depending on one factor

**Polynomial prediction function.**

The polynomial forecast function from one factor in the general form has the form of an algebraic polynomial [1]:

$$Y = b_0 + b_1X + b_2X^2 + \cdots + b_mX^m, \quad (11.1)$$

Where the coefficients $b_0, b_1, b_2, \dots, b_m$ are determined on the basis of the initial data Y and X.

The following is an example of calculating quality metrics for a parabola and a cubic parabola for given source data:

$$Y = b_0 + b_1 t + b_2 t^2,$$

$$Y = b_0 + b_1 t + b_2 t^2 + b_3 t^3.$$

Table 11.1 The initial data and the results of constructing regression functions for a parabola and a cubic parabola according to formula (11.1)

| Time | Period | Period in square | Period in cube | Production costs | parabola | cubic parabola |
|---|---|---|---|---|---|---|
| January | 1 | 1 | 1 | 1286,5 | 1232,35 | 1014,37 |
| February | 2 | 4 | 8 | 1282,7 | 1322,12 | 1250,66 |
| March | 3 | 9 | 27 | 1224,8 | 1409,84 | 1441,64 |
| April | 4 | 16 | 54 | 1292,6 | 1495,51 | 1583,72 |
| May | 5 | 25 | 125 | 1667,3 | 1579,12 | 1709,36 |
| June | 6 | 36 | 216 | 1672,9 | 1660,68 | 1796,91 |
| July | 7 | 49 | 343 | 1667,5 | 1740,18 | 1860,82 |
| August | 8 | 64 | 512 | 2001,7 | 1817,63 | 1906,47 |
| September | 9 | 81 | 729 | 2352,5 | 1893,03 | 1939,3 |
| October | 10 | 100 | 1000 | 2468,5 | 1966,37 | 1964,72 |
| NOVEMBER | 11 | 121 | 1331 | 2012,2 | 2037,66 | 1988,12 |
| December | 12 | 144 | 1728 | 1982,7 | 2106,9 | 2014,93 |
| January | 13 | 169 | 2197 | 1901,1 | 2174,08 | 2050,56 |
| February | 14 | 195 | 2744 | 1971,6 | 2240,23 | 2128,48 |
| March | 15 | 225 | 3375 | 1989,1 | 2302,28 | 2169,91 |
| April | 16 | 256 | 4096 | 2138,2 | 2363,3 | 2264,46 |
| May | 17 | 289 | 4913 | 2475,2 | 2422,27 | 2389,47 |
| June | 18 | 324 | 5832 | 2395,6 | 2479,18 | 2550,36 |
| July | 19 | 361 | 6859 | 2994,1 | 2534,04 | 2752,54 |

Table 11.2. The calculated characteristics of the regression functions of a parabola and a cubic parabola

| | | | | |
|---|---|---|---|---|
| Coeff of parabola Y=b0-b1*t+b2*t^2 | | | | |
| -1,026883 | 92,853778 | 1140,524 | | |
| 2,2723591 | 46,76701 | 203,22234 | | |
| 0,7273164 | 264,95083 | #Н/Д | | |
| 21,338032 | 16 | #Н/Д | | |
| 2995814,5 | 1123183 | #Н/Д | | |
| | | | | |
| Determination coefficient of parabola | | | | |
| 0,7273164 | | | | |
| Adjusted Determination coefficient of parabola | | | | |
| 0,7112762 | {=1-(19-1)*(1-I4)/(19-2)} | | | |
| Coefficients of cubic parabola | | | | |
| Y=b0+b1*t+b2*t^2+b3*t^3 | | | | |
| 0,9019587 | -28,06199 | 314,15832 | 727,3731 | |
| 0,4451412 | 13,503639 | 117,30559 | 275,9743 | |
| 0,7859135 | 242,46263 | #Н/Д | #Н/Д | |
| 18,355042 | 15 | #Н/Д | #Н/Д | |
| 3237175,6 | 881821,9 | #Н/Д | #Н/Д | |
| | | | | |
| Determination coefficient for cubic parabola | | | | |
| 0,7859135 | | | | |
| Adjusted Determination coefficient for cubic parabola | | | | |
| 0,8897035 | {=1-(19-1)*(1-I14)/(19-3)} | | | |

Comparison of the adjusted coefficients of determination of the regression function of a parabola and a cubic parabola shows that the quality of the approximation of the initial data by a cubic parabola is higher than the parabola. This result can be visually observed by comparing the graphs of the original data and the constructed regression functions in Figure 11.1.

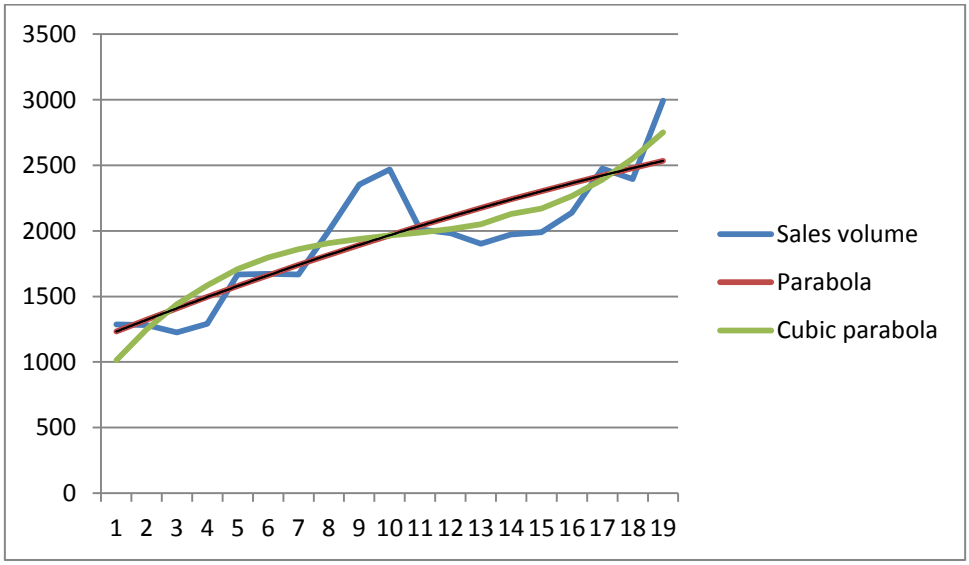


Fig. 11.1. The raw data graphs (row 1), the parabola regression function (row 2), and the cubic parabola regression function (row 3)

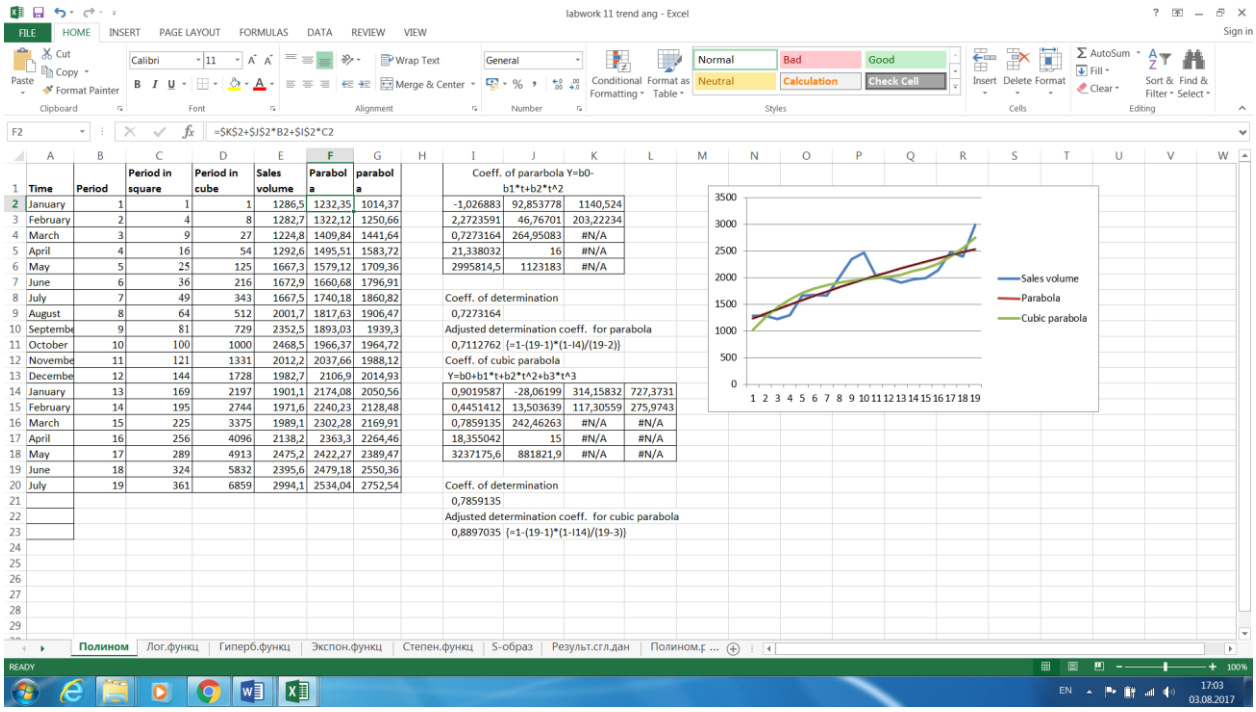



Fig. 11.2. Screenshot of the quality assessment of two models of the polynomial forecasting function: a parabola and a cubic parabola

**Exponential predicting function.**

The exponential function of prediction from one factor is calculated by the formula [1]:

$$Y = b_0 * (b_1)^X \qquad (11.2)$$

To calculate the coefficients of the exponential function, we use the standard LGRF function.

The following is an example of calculating the exponential forecasting performance indicators from a single factor for given source data:

$$Y = b_0 * (b_1)^t.$$

Table 11.3. The initial data and the results of constructing the exponential regression function from one factor using formula (11.2)

| Time | Period | Sales volume | exponential regression | | Coefficients of function | | | |
|---|---|---|---|---|---|---|---|---|
| January | 1 | 1286,5 | 1320,98 | | 1,04 | 1270,29 | | |
| February | 2 | 1282,7 | 1373,69 | | 0,005 | 0,06267 | | |
| March | 3 | 1224,8 | 1428,5 | | 0,749 | 0,13122 | | |
| April | 4 | 1292,6 | 1485,5 | | 50,67 | 17 | | |
| May | 5 | 1667,3 | 1544,78 | | 0,873 | 0,29274 | | |
| June | 6 | 1672,9 | 1606,42 | | {=LOGEST(C2:C20;B2:B20;;1)} | | | |
| July | 7 | 1667,5 | 1670,52 | | Determination coefficient | | | |
| August | 8 | 2001,7 | 1737,17 | | 0,749 | | | |
| September | 9 | 2352,5 | 1806,49 | | | | | |
| October | 10 | 2468,5 | 1878,57 | | | | | |
| NOVEMBER | 11 | 2012,2 | 1953,53 | | | | | |
| December | 12 | 1982,7 | 2031,48 | | | | | |
| January | 13 | 1901,1 | 2112,54 | | {=$G$2*POWER($F$2;B14)} | | | |
| February | 14 | 1971,6 | 2196,83 | | | | | |
| March | 15 | 1989,1 | 2284,49 | | | | | |
| April | 16 | 2138,2 | 2375,64 | | | | | |
| May | 17 | 2475,2 | 2470,44 | | | | | |
| June | 18 | 2395,6 | 2569,01 | | | | | |
| July | 19 | 2994,1 | 2671,52 | | | | | |

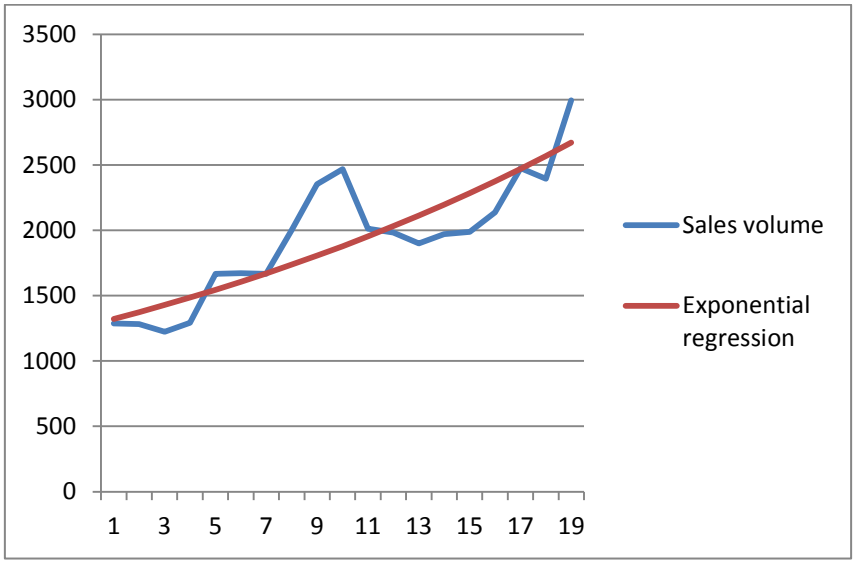



Fig. 11.3. The graphs of the initial data (row 1), the exponential regression function (row 2) of one factor according to the formula (11.2

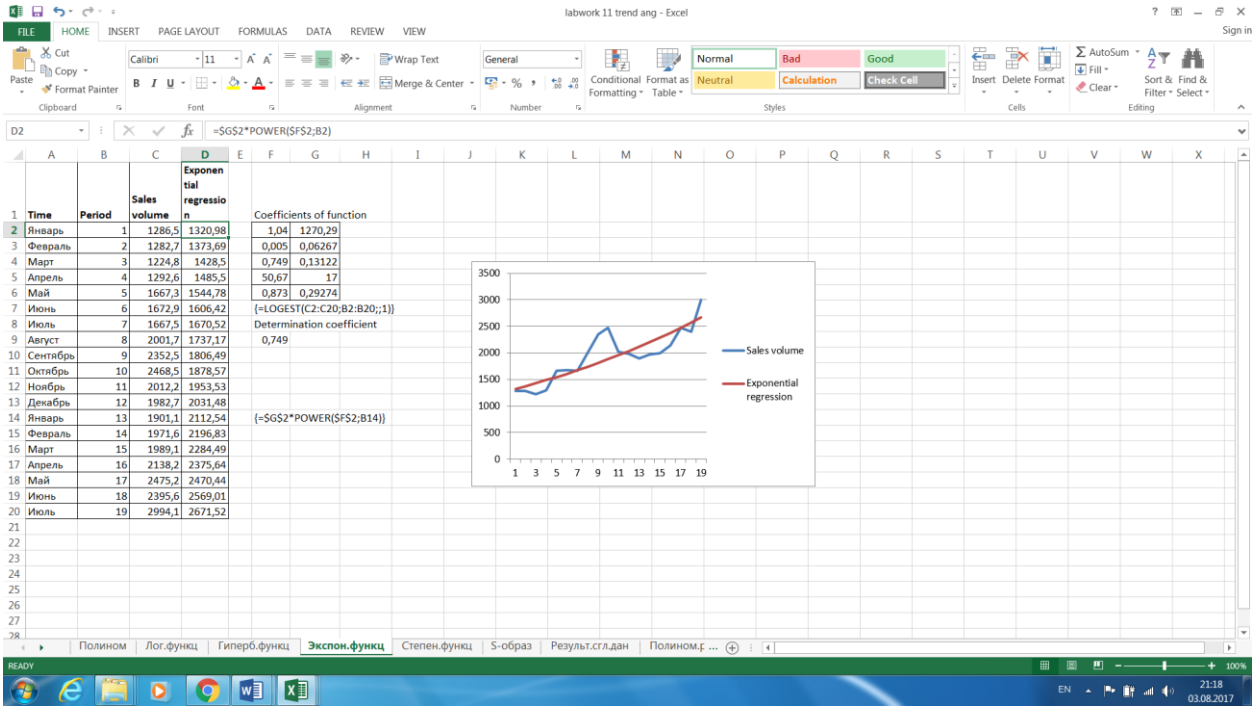



Fig. 11.4. Screenshot of the construction and evaluation of the quality of the exponential regression function from one factor by the formula (11.2)

### 11.1.2. Forecasting functions, depending on several factors.

**Exponential regression.**

The exponential function of the regression from many factors in the general form is calculated by the following formula:

$$Y = b_0 * (b_1)^{X_1} * (b_2)^{X_2} * \dots * (b_m)^{X_m}, \qquad (11.3)$$

Where the coefficients $b_0, b_1, b_2, \dots, b_m$ are determined from the original data $X_1, X_2, \dots, X_m$ and Y.

To calculate the coefficients of the exponential function of regression from many factors, the standard LGRFPRIBL function is used.

The following is an example of calculating the exponential forecasting performance indicators from three factors for given source data:

$$Y = b_0 * (b_1)^{t} * (b_2)^{X_1} * (b_3)^{X_2}, \qquad (11.4)$$

Where t - time period, $X_1$- production costs, $X_2$- advertising costs.

Table 11.4 The initial data and the construction of the trend of the exponential regression function from three factors according to the formula (11.4)

| Time | Period | Production costs | Cost of advertising | Sales volume | Trend |
|---|---|---|---|---|---|
| January | 1 | 905,8 | 199,8 | 1282 | 1303,2 |
| February | 2 | 902,5 | 211,5 | 1292,7 | 1346,58 |
| March | 3 | 903 | 206,6 | 1226,8 | 1375,57 |
| April | 4 | 889,8 | 225,7 | 1392,6 | 1585,13 |
| May | 5 | 889,8 | 219 | 1647,3 | 1636,04 |
| June | 6 | 892,8 | 236,7 | 1672,9 | 1540,28 |
| July | 7 | 888,3 | 231,3 | 1660,5 | 1678,17 |
| August | 8 | 875,8 | 241,1 | 2011,7 | 1956,56 |
| September | 9 | 883,9 | 238,1 | 2351,9 | 1807,64 |
| October | 10 | 875,1 | 248,1 | 2468,5 | 2010,12 |
| NOVEMBER | 11 | 871,6 | 256,9 | 2746,2 | 2095,7 |
| December | 12 | 873,8 | 251,9 | 1942,7 | 2095,67 |
| January | 13 | 868,2 | 273,1 | 1901,1 | 2183,13 |
| February | 14 | 866,3 | 264,5 | 1971,6 | 2317,85 |
| March | 15 | 862,1 | 267,1 | 1989,1 | 2471,69 |
| April | 16 | 866,6 | 282,9 | 2139,2 | 2292,83 |
| May | 17 | 862,5 | 287,5 | 2474,2 | 2431,14 |
| June | 18 | 863,9 | 286,3 | 2393,6 | 2435,33 |
| July | 19 | 858,5 | 285,3 | 2990,1 | 2657,8 |

The calculation of the trend of the exponential forecasting function from three factors: Y = b0 * (b1) ^ t * (b2) ^ X1 * (b3) ^ X2 is performed in Excel using the formula:

= $ K $ 3 * $ J $ 3 ^ B2 * $ I $ 3 ^ C2 * $ H $ 3 ^ D2.

Table 11.5. The results of calculating the characteristics of the exponential regression function from three factors using formula (11.4)

Regression Y=b0*(b1)^t*(b2)^X1*(b3)^X2

| | | | |
|---|---|---|---|
| 0,997792 | 0,987413 | 1,016948 | 191595799 |
| 0,00664 | 0,009145 | 0,035555 | 8,7102376 |
| 0,725445 | 0,150918 | #Н/Д | #Н/Д |
| 13,21128 | 15 | #Н/Д | #Н/Д |
| 0,902716 | 0,341646 | #Н/Д | #Н/Д |

Determination coefficient
0,725445

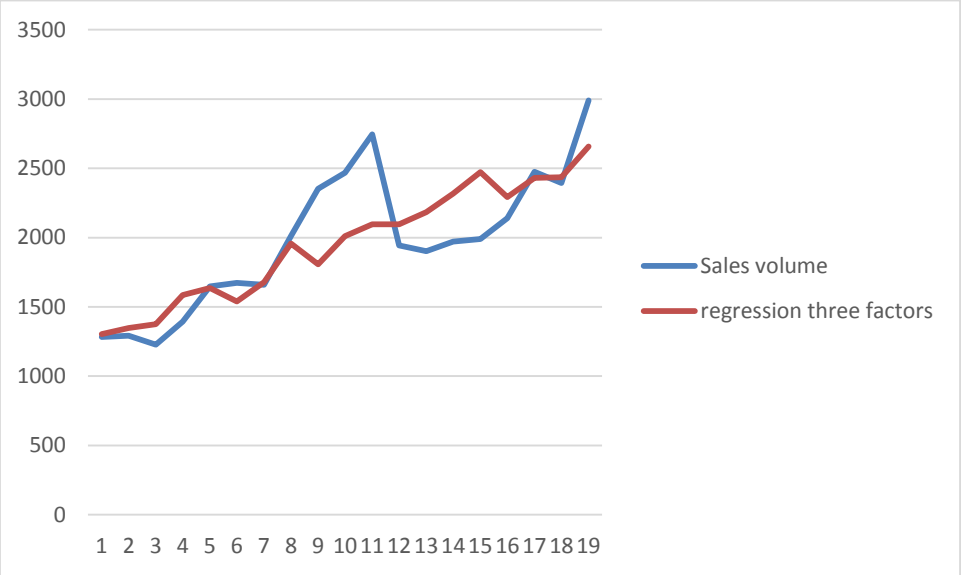



Fig. 11.5. The graphs of the initial data (row 1), the exponential regression function from the three factors (row 2) by the formula (11.4)

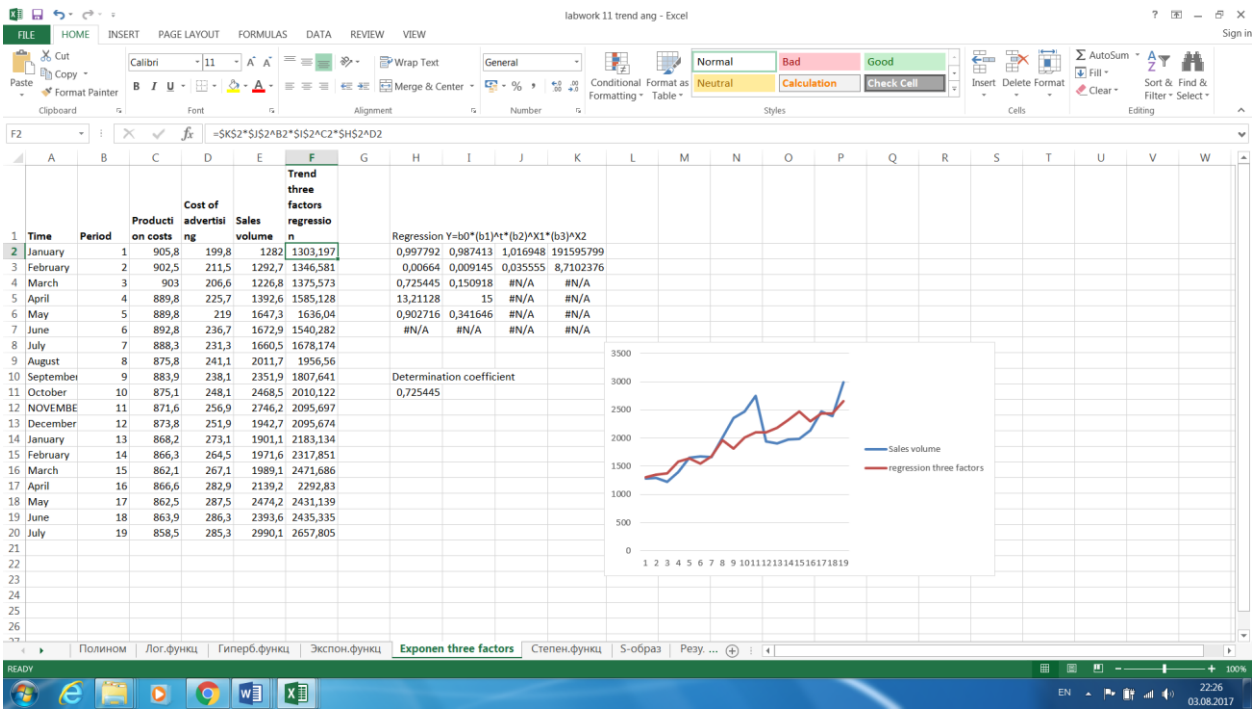



Fig. 11.6 Screenshot of the construction and evaluation of the quality of the exponential function of the regression from three factors by the formula (11.4)

In Excel, there are a number of implemented standard functions that allow you to approximate the original data. And a number of functions, such as: linear,

exponential, logarithmic, polynomial, power functions allow you to build trend lines, determine the coefficient of determination, make a prediction for a specified number of periods forward (backward). Often it is necessary to build the forecasting function using the standard LINEST and LOGEST functions, calculating the coefficients and statistical characteristics of the approximating regressions. This scheme is used in the examples discussed above.

## 11.2 PRACTICAL LESSON 11 and IWS 11

**Subject.** Highlighting the trend function for the prediction task.

**Plan of the lesson.**

1) Study the trend allocation scheme for the prediction task presented above.

2) Practical lesson and task of the IWS. Apply the above described schemes to the selection of the trend function for its prediction task.

3) Analyze the results.

Literature.

1. Minko A.A. Predicting in business using Excel. M.: Eksmo, 2007, -208 p.

**TOPIC 12**

**SEASONAL COMPONENT IN PREDICTION FUNCTIONS**

## 12.1. Lecture material.

The seasonal component in the prediction functions determines periodic changes in the values of the prediction function. The type (model) of interaction between the trend component and the seasonal component of the prediction function can be additive or multiplicative. In the additive model of the prediction function, seasonal changes are added to the trend, i.e. F (t) = T (t) + S (t), where T and S denote, respectively, the trend and seasonal components of the function f, and in the multiplicative model, the trend and seasonal variations are multiplied; F (t) = T (t) * S (t).

The scheme for allocating the seasonal component from the initial data and including it in the prediction function has the following stages:

1) Based on the input data, the trend of the predicted variable (trend of the prediction function) is constructed T. The previous chapter shows how this is done. This is usually done by constructing a regression function.

2) Seasonal components $s_i$are calculated for each regression data point using the formulas:

- for the additive model $s_i = y_i - T_i$, where i = 1, ..., n;

- for the multiplicative model $s_i = y_i / T_i$, where i = 1, ..., n,

Where $y_i$are the observed values of the predicted variable at the i-th data point; $T_i$- the value of the trend of the predicted variable in the i-th data point; Total points of input data n.

3) The seasonal components $s_1, s_2, \dots, s_n$ are divided into groups according to k seasons, for example, 12 months of the year; For each season there is an average $\overline{S_i}$

4) The seasonal coefficients $S_1$, $S_2$ , … , $S_k$are calculated:

- for the additive model $S_i = \overline{S_i} - m, i = \overline{1 \dots k}$, where m is the correction factor for computing each seasonal coefficient: $m = \frac{1}{k}(\overline{S_1} + \overline{S_2} + \dots + \overline{S_k})$

- for the multiplicative model $S_i = \overline{S_\iota} * m, i = \overline{1 \dots k},$, where m is the correction factor for computing each seasonal coefficient: $m = \frac{k}{\overline{S_1}+\overline{S_2}+\cdots+\overline{S_k}}$.

5) The prediction function with seasonal coefficients is calculated:

- for the additive model $f_i = T_i + S_p;;$

- for the multiplicative model $f_i = T_i * S_p$.

Where $S_p$is the seasonal coefficient for the p-th season, which includes $T_i$-the trend function at the i-th data point,

6) The residuals (prediction errors) are calculated $e_i = y_i - f_i$.

7) The remainder of the constructed data model is analyzed and the model quality indicators are computed.

If the residue analysis and the quality indicators of the constructed model are satisfactory, i.e. the constructed model is adequate to the initial data, then a prediction is made for the constructed model. If analysis of the residuals shows unsatisfactory results, then a change in the trend function approximating the initial data is necessary.

8) Calculation of the prediction. If it is necessary to predict the depended variable at time$t_{n+j}$, then the value of the prediction function $f_{n+j}$is calculated using the formulas:

- for the additive model $f_{n+j} = T_{n+j} + S_p,$

- for the multiplicative model $f_{n+j} = T_{n+j} * S_p$.

As can be seen from the formulas, in order to make a prediction, it is necessary to calculate the value of the trend $T_{n+j}$at the time of the forecast. The trend value of the prediction function $T_{n+j}$at time $t_{n+j}$ in the case when the predicted variable depends only on time is not difficult: it is calculated by the regression formula of this prediction function for the required time moment.

The situation is more complicated if the prediction function depends on several factors. In this case, to calculate the predicted values of the trend prediction function, it is necessary to know (calculate) the predicted values of all factors. In this case, it is need first calculate the predicted values of the factors, and then calculate the predicted values of the function. That is, to predict the values of

factors, it is need follow the same steps as for the prediction function. For this it is necessary for each factor to construct a function of regression by another factor, usually in this capacity the time factor is taken. And the regression function for this factor is to calculate the predicted values of this factor at the necessary instants of time.

Below is an example of constructing a prediction function, taking into account the allocation and use of the seasonal component in the prediction function.

1) Based on the input data, the trend of the predicted variable (trend of the prediction function) is constructed T. In the example, this is done by constructing the regression function. In the example, the regression equation is taken

$Y = b_0 + b_1 t + b_2 t^2 + b_3 X_1 + b_4 X_2$, where t - time, $X_1$- production costs, $X_2$- advertising costs.

Table 12.1 Initial data and constructed regression function.

| Period | Period in square | Production costs | Advertising costs | Sales volume | Regression function |
|---|---|---|---|---|---|
| 1 | 1 | 890,8 | 200,8 | 1286,5 | 1329,349 |
| 2 | 4 | 896,4 | 210,5 | 1282,7 | 1294,721 |
| 3 | 9 | 903 | 208,6 | 1224,8 | 1397,418 |
| 4 | 16 | 888,8 | 226,7 | 1292,6 | 1446 |
| 5 | 25 | 889,8 | 230,62 | 1667,3 | 1521,13 |
| 6 | 36 | 890,8 | 238,7 | 1672,9 | 1537,63 |
| 7 | 49 | 888,3 | 232,3 | 1667,5 | 1774,925 |
| 8 | 64 | 885,8 | 240,1 | 2001,7 | 1822,27 |
| 9 | 81 | 883,9 | 238,1 | 2352,5 | 1987,469 |
| 10 | 100 | 875,1 | 248,1 | 2468,5 | 2061,72 |
| 11 | 121 | 877,73 | 253,58 | 2012,2 | 2074,541 |
| 12 | 144 | 873,8 | 261,05 | 1982,7 | 2123,879 |
| 13 | 169 | 878,2 | 275,1 | 1901,1 | 1998,154 |
| 14 | 196 | 866,3 | 265,5 | 1971,6 | 2343,438 |
| 15 | 225 | 865,1 | 266,1 | 1989,1 | 2442,189 |
| 16 | 256 | 876,6 | 282,6 | 2138,2 | 2199,5 |

| | | | | | |
|---|---|---|---|---|---|
| 17 | 289 | 862,5 | 286,5 | 2475,2 | 2377,942 |
| 18 | 324 | 865,9 | 286,3 | 2395,6 | 2427,682 |
| 19 | 361 | 857,5 | 284,3 | 2994,1 | 2616,842 |

Calculation of regression coefficients by the standard LINEST function.

The regression equation is Y = b0 + b1 * t + b2 * t ^ 2 + b3 * X1 + b4 * X2

| | | | | |
|---|---|---|---|---|
| -13,0796 | -10,1777 | -2,10969 | 155,5671 | 12868,56 |

2) Seasonal components $s_i$are calculated for each regression data point using the formulas:

- for the additive model $s_i = y_i - T_i$, where i = 1, ..., n;

Where $y_i$are the observed values of the predicted variable in the i-th data point, $T_i$is the trend value of the predicted variable in the i-th data point.

3) The seasonal components $s_1, s_2, \dots, s_n$are divided into groups according to k seasons; for each season there is an average $\overline{S_i}$.

4) The seasonal coefficients $S_1,\ S_2, \dots, S_k$are calculated:

- for the additive model $S_i = \overline{S_i} - m, i = \overline{1 \dots k}$,, where m is the correction factor for computing each seasonal coefficient: $m = \frac{1}{k}(\overline{S_1} + \overline{S_2} + \dots + \overline{S_k})$

For the example in question, Table 12.2 shows the values of the calculated seasonal components, the mean seasonal component, seasonal coefficients.

Table 12.2 Calculation of seasonal components, mean seasonal component, seasonal coefficients

| Sales volume | Regression function | Residuals | Average season | Seasonal coefficient |
|---|---|---|---|---|
| 1286,5 | 1329,349 | -42,8493 | -69,9517 | -101,107 |
| 1282,7 | 1294,721 | -12,0205 | -191,929 | -223,084 |
| 1224,8 | 1397,418 | -172,618 | -312,853 | -344,008 |
| 1292,6 | 1446 | -153,4 | -107,35 | -138,505 |
| 1667,3 | 1521,13 | 146,1696 | 121,7136 | 90,55862 |
| 1672,9 | 1537,63 | 135,2696 | 51,59384 | 20,43882 |
| 1667,5 | 1774,925 | -107,425 | 134,9165 | 103,7615 |

| | | | | |
|---|---|---|---|---|
| 2001,7 | 1822,27 | 179,4298 | 179,4298 | 148,2747 |
| 2352,5 | 1987,469 | 365,0307 | 365,0307 | 333,8757 |
| 2468,5 | 2061,72 | 406,7796 | 406,7796 | 375,6246 |
| 2012,2 | 2074,541 | -62,3407 | -62,3407 | -93,4957 |
| 1982,7 | 2123,879 | -141,179 | -141,179 | -172,334 |
| 1901,1 | 1998,154 | -97,0541 | -97,0541 | |
| 1971,6 | 2343,438 | -371,838 | -371,838 | |
| 1989,1 | 2442,189 | -453,089 | -453,089 | |
| 2138,2 | 2199,5 | -61,2998 | -61,2998 | |
| 2475,2 | 2377,942 | 97,25768 | 97,25768 | |
| 2395,6 | 2427,682 | -32,082 | -32,082 | |
| 2994,1 | 2616,842 | 377,2578 | 377,2578 | |

Excel formulas for computed columns (on the first line, column names and line numbering, see Figure 12.1):

The regression function is = $ R $ 2 + $ Q $ 2 * A2 + $ P $ 2 * B2 + $ O $ 2 * C2 + $ N $ 2 * D2;

Residuals of regression function = E2-F2;

Mean $\overline{S_i}$ - = AVERAGE (G2; OFFSET (G2; 12; 0));

The seasonal coefficients $S_i$ - = H2-AVERAGE ($ H $ 2: $ H $ 13).

5) The forecasting function with seasonal coefficients is calculated:

- for the additive model $f_i = T_i + S_p$,

Where $S_p$ is the seasonal coefficient for the p-th season, which includes $T_i$ - the trend function at the i-th data point.

For the example in question, Table 12.3 shows the values of the prediction function.

Table 12.3 Construction of the prediction function as the sum of the trend and the calculated seasonal coefficients

| Sales volume | Regression function | Remaining regression function | Average season | Seasonal coefficient | Prediction function |
|---|---|---|---|---|---|
| 1286,5 | 1329,349 | -42,8493 | -69,9517 | -101,107 | 1228,243 |
| 1282,7 | 1294,721 | -12,0205 | -191,929 | -223,084 | 1071,636 |

| | | | | | |
|---|---|---|---|---|---|
| 1224,8 | 1397,418 | -172,618 | -312,853 | -344,008 | 1053,409 |
| 1292,6 | 1446 | -153,4 | -107,35 | -138,505 | 1307,495 |
| 1667,3 | 1521,13 | 146,1696 | 121,7136 | 90,55862 | 1611,689 |
| 1672,9 | 1537,63 | 135,2696 | 51,59384 | 20,43882 | 1558,069 |
| 1667,5 | 1774,925 | -107,425 | 134,9165 | 103,7615 | 1878,686 |
| 2001,7 | 1822,27 | 179,4298 | 179,4298 | 148,2747 | 1970,545 |
| 2352,5 | 1987,469 | 365,0307 | 365,0307 | 333,8757 | 2321,345 |
| 2468,5 | 2061,72 | 406,7796 | 406,7796 | 375,6246 | 2437,345 |
| 2012,2 | 2074,541 | -62,3407 | -62,3407 | -93,4957 | 1981,045 |
| 1982,7 | 2123,879 | -141,179 | -141,179 | -172,334 | 1951,545 |
| 1901,1 | 1998,154 | -97,0541 | -97,0541 | | 1897,047 |
| 1971,6 | 2343,438 | -371,838 | -371,838 | | 2120,354 |
| 1989,1 | 2442,189 | -453,089 | -453,089 | | 2098,181 |
| 2138,2 | 2199,5 | -61,2998 | -61,2998 | | 2060,995 |
| 2475,2 | 2377,942 | 97,25768 | 97,25768 | | 2468,501 |
| 2395,6 | 2427,682 | -32,082 | -32,082 | | 2448,121 |
| 2994,1 | 2616,842 | 377,2578 | 377,2578 | | 2720,604 |

The Excel formula for the prediction function (on the first line, column names and line numbering, see Figure 12.1): = F2 + I2.

6) Calculation of residuals (prediction errors) $e_i = y_i - f_i$.

For the example in question, Table 12.4 shows the values of the prediction residuals (errors).

Table 12.4 Calculations of residuals (prediction errors).

| Sales volume | Regression function | Remaining regression function | Average season | Seasonal coefficient | Prediction function | The prediction residuals (errors) |
|---|---|---|---|---|---|---|
| 1286,5 | 1329,349 | -42,8493 | -69,9517 | -101,107 | 1228,243 | 58,25742 |
| 1282,7 | 1294,721 | -12,0205 | -191,929 | -223,084 | 1071,636 | 211,0636 |
| 1224,8 | 1397,418 | -172,618 | -312,853 | -344,008 | 1053,409 | 171,3909 |
| 1292,6 | 1446 | -153,4 | -107,35 | -138,505 | 1307,495 | -14,8951 |
| 1667,3 | 1521,13 | 146,1696 | 121,7136 | 90,55862 | 1611,689 | 55,61099 |
| 1672,9 | 1537,63 | 135,2696 | 51,59384 | 20,43882 | 1558,069 | 114,8308 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 1667,5 | 1774,925 | -107,425 | 134,9165 | 103,7615 | 1878,686 | -211,186 |
| 2001,7 | 1822,27 | 179,4298 | 179,4298 | 148,2747 | 1970,545 | 31,15502 |
| 2352,5 | 1987,469 | 365,0307 | 365,0307 | 333,8757 | 2321,345 | 31,15502 |
| 2468,5 | 2061,72 | 406,7796 | 406,7796 | 375,6246 | 2437,345 | 31,15502 |
| 2012,2 | 2074,541 | -62,3407 | -62,3407 | -93,4957 | 1981,045 | 31,15502 |
| 1982,7 | 2123,879 | -141,179 | -141,179 | -172,334 | 1951,545 | 31,15502 |
| 1901,1 | 1998,154 | -97,0541 | -97,0541 | | 1897,047 | 4,052622 |
| 1971,6 | 2343,438 | -371,838 | -371,838 | | 2120,354 | -148,754 |
| 1989,1 | 2442,189 | -453,089 | -453,089 | | 2098,181 | -109,081 |
| 2138,2 | 2199,5 | -61,2998 | -61,2998 | | 2060,995 | 77,20515 |
| 2475,2 | 2377,942 | 97,25768 | 97,25768 | | 2468,501 | 6,699056 |
| 2395,6 | 2427,682 | -32,082 | -32,082 | | 2448,121 | -52,5208 |
| 2994,1 | 2616,842 | 377,2578 | 377,2578 | | 2720,604 | 273,4963 |

The Excel formula for calculating the remainders of the prediction function (for the first line, column names and line numbers, see Figure 12.1): = E2-J2.

7) Analysis of the constructed data model for quality indicators.

Table 12.5. Calculation of the quality indicators of the constructed data model

| Seasonal coefficient | Prediction function | the prediction residuals (errors) | | b4 | b3 | b2 | b1 | b0 |
|---|---|---|---|---|---|---|---|---|
| -101,107 | 1228,243 | 58,25742 | | -13,0796 | -10,1777 | -2,10969 | 155,5671 | 12868,56 |
| -223,084 | 1071,636 | 211,0636 | Regression equation Y=b0+b1*t+b2*t^2+b3*X1+b4*X2 | | | | | |
| -344,008 | 1053,409 | 171,3909 | | | | | | |
| -138,505 | 1307,495 | -14,8951 | | | | | | |
| 90,55862 | 1611,689 | 55,61099 | Determination coeff regression equation | | | | | |
| 20,43882 | 1558,069 | 114,8308 | | 0,764315 | | | | |
| 103,7615 | 1878,686 | -211,186 | | | | | | |
| 148,2747 | 1970,545 | 31,15502 | Determination coeff of prediction function | | | | | |
| 333,8757 | 2321,345 | 31,15502 | | 0,936665 | | | | |
| 375,6246 | 2437,345 | 31,15502 | | | | | | |
| -93,4957 | 1981,045 | 31,15502 | | | | | | |
| -172,334 | 1951,545 | 31,15502 | | | | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1897,047 | 4,052622 | | | | | | |
| | 2120,354 | -148,754 | | | | | | |
| | 2098,181 | -109,081 | | | | | | |
| | 2060,995 | 77,20515 | | | | | | |
| | 2468,501 | 6,699056 | | | | | | |
| | 2448,121 | -52,5208 | | | | | | |
| | 2720,604 | 273,4963 | | | | | | |

Excel formula for calculating the coefficient of determination of the regression function (see column 12.1 for column names and line numbering):

= 1- (SUMSQ (G2: G20) / SUMSQ (E2: E20-AVERAGE (E2: E20))).

The Excel formula for calculating the determination coefficient of the prediction function (column names and line numbering, see Figure 12.1): = 1- (SUMSQ (K2: K20) / SUMSQ (E2: E20-AVERAGE (E2: E20))).

The value of the coefficient of determination of the prediction function is 0.936665, which is much better than the value of the coefficient of determination of the regression function 0, 764315, which indicates that the quality of the model with seasonal component is more qualitative than the model without taking it into account.

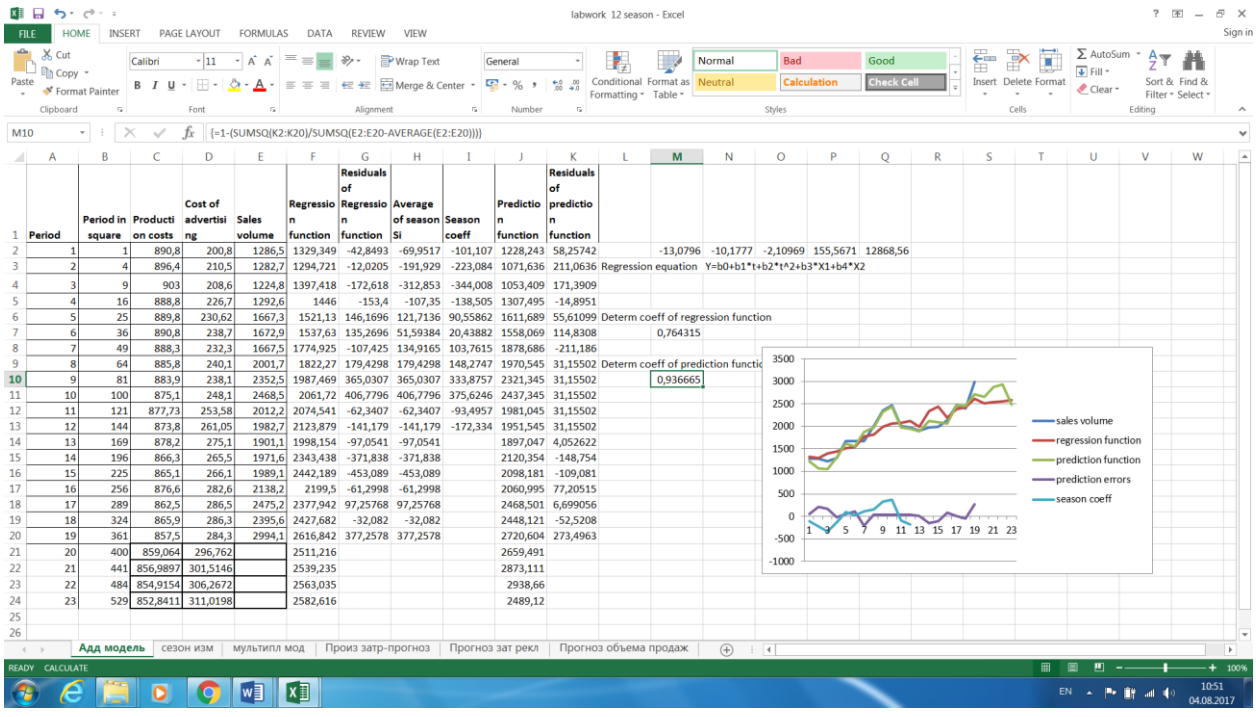



Fig. 12.1 A screenshot of the construction of the prediction function taking into account the seasonal component.

8) Calculation of the prediction.

The example uses the regression equation

$Y = b_0 + b_1 t + b_2 t^2 + b_3 X_1 + b_4 X_2$,, where t - time, $X_1$- production costs, $X_2$- advertising costs.

Since the prediction function depends on several factors, to calculate the predict from the predicted variable, it is necessary to make a prediction for all factors of the regression equation. The prediction by time and square of time is not difficult. It is necessary to construct regression functions for $X_1$- production costs, $X_2$ - advertising costs.

The prediction of values of the factor "production costs" is presented below.

Table 12.6. Input and prediction for the "production cost" factor

| Period | Production costs | Prediction period | Prediction of production costs | |
|---|---|---|---|---|
| 1 | 890,8 | 20 | 859,064 | |
| 2 | 896,4 | 21 | 856,9897 | |
| 3 | 903 | 22 | 854,9154 | |
| 4 | 888,8 | 23 | 852,8411 | |
| 5 | 889,8 | | | |
| 6 | 890,8 | | | |
| 7 | 888,3 | | | |
| 8 | 885,8 | | | |
| 9 | 883,9 | | | |
| 10 | 875,1 | | | |
| 11 | 877,73 | | | |
| 12 | 873,8 | | | |
| 13 | 878,2 | | | |
| 14 | 866,3 | | | |
| 15 | 865,1 | | | |
| 16 | 876,6 | | | |
| 17 | 862,5 | | | |
| 18 | 865,9 | | | |

| | | | | |
|---|---|---|---|---|
| 19 | 857,5 | | | |

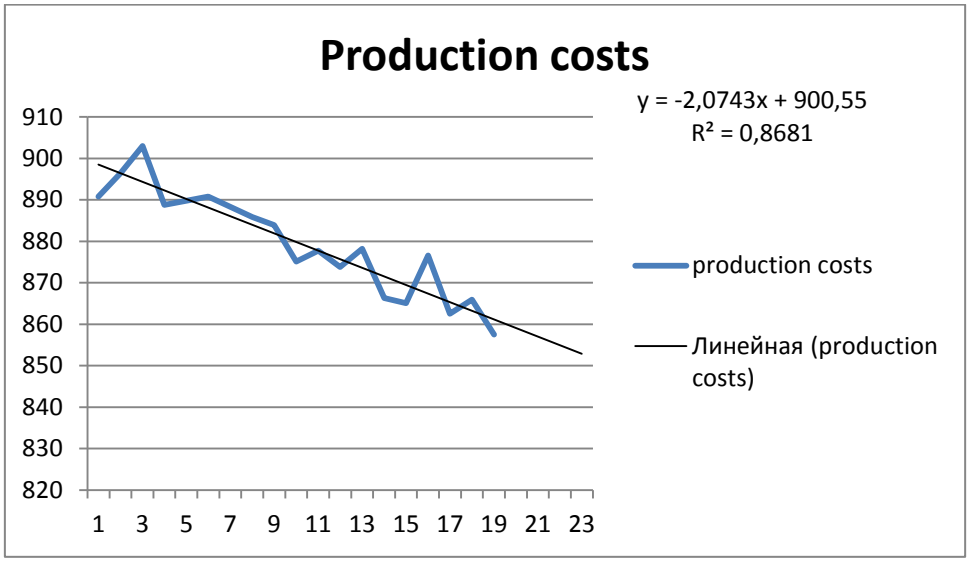
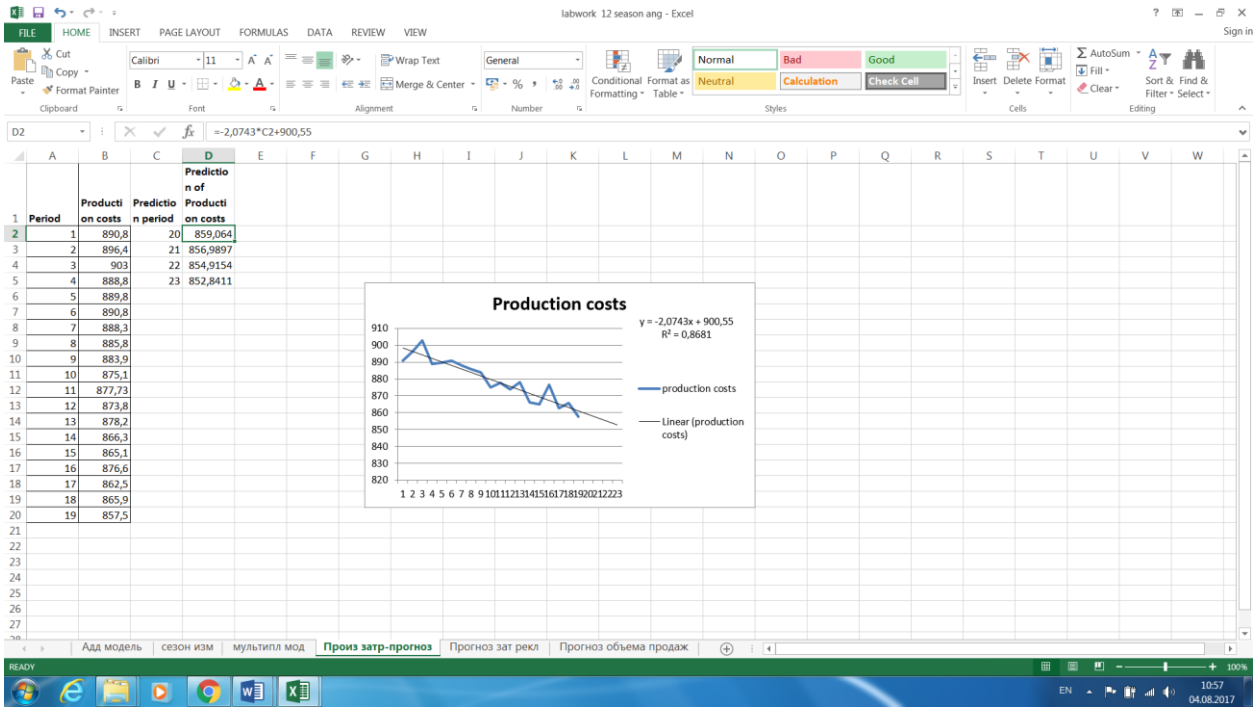



Fig. 12.2. Graph of the regression function with the prediction for the "production cost" factor




| Period | Production costs | Prediction period | Prediction of Production costs |
|---|---|---|---|
| 1 | 890,8 | 20 | 859,064 |
| 2 | 896,4 | 21 | 856,9897 |
| 3 | 903 | 22 | 854,9154 |
| 4 | 888,8 | 23 | 852,8411 |
| 5 | 889,8 | | |
| 6 | 890,8 | | |
| 7 | 888,3 | | |
| 8 | 885,8 | | |
| 9 | 883,9 | | |
| 10 | 875,1 | | |
| 11 | 877,73 | | |
| 12 | 873,8 | | |
| 13 | 878,2 | | |
| 14 | 866,3 | | |
| 15 | 865,1 | | |
| 16 | 876,6 | | |
| 17 | 862,5 | | |
| 18 | 865,9 | | |
| 19 | 857,5 | | |

Fig. 12.3. Screenshot of the prediction for the factor "production costs"

Below is a prediction of the values of the factor "advertising costs"

Table 12.6. Baseline data and prediction for the "advertising costs" factor

| Period | Advertising costs | Prediction period | Prediction of advertising costs | |
|---|---|---|---|---|
| 1 | 200,8 | 20 | 296,762 | |
| 2 | 210,5 | 21 | 301,5146 | |
| 3 | 208,6 | 22 | 306,2672 | |
| 4 | 226,7 | 23 | 311,0198 | |
| 5 | 230,62 | | | |
| 6 | 238,7 | | | |
| 7 | 232,3 | | | |
| 8 | 240,1 | | | |
| 9 | 238,1 | | | |
| 10 | 248,1 | | | |
| 11 | 253,58 | | | |
| 12 | 261,05 | | | |
| 13 | 275,1 | | | |
| 14 | 265,5 | | | |
| 15 | 266,1 | | | |
| 16 | 282,6 | | | |
| 17 | 286,5 | | | |
| 18 | 286,3 | | | |
| 19 | 284,3 | | | |

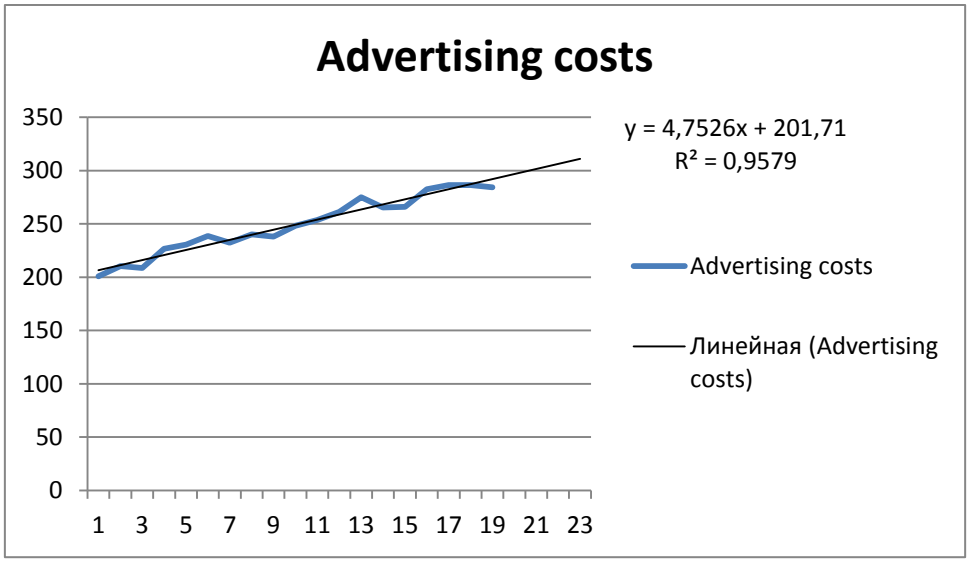



Fig. 12.4. Graph of the regression function with the prediction for the "advertising costs" factor

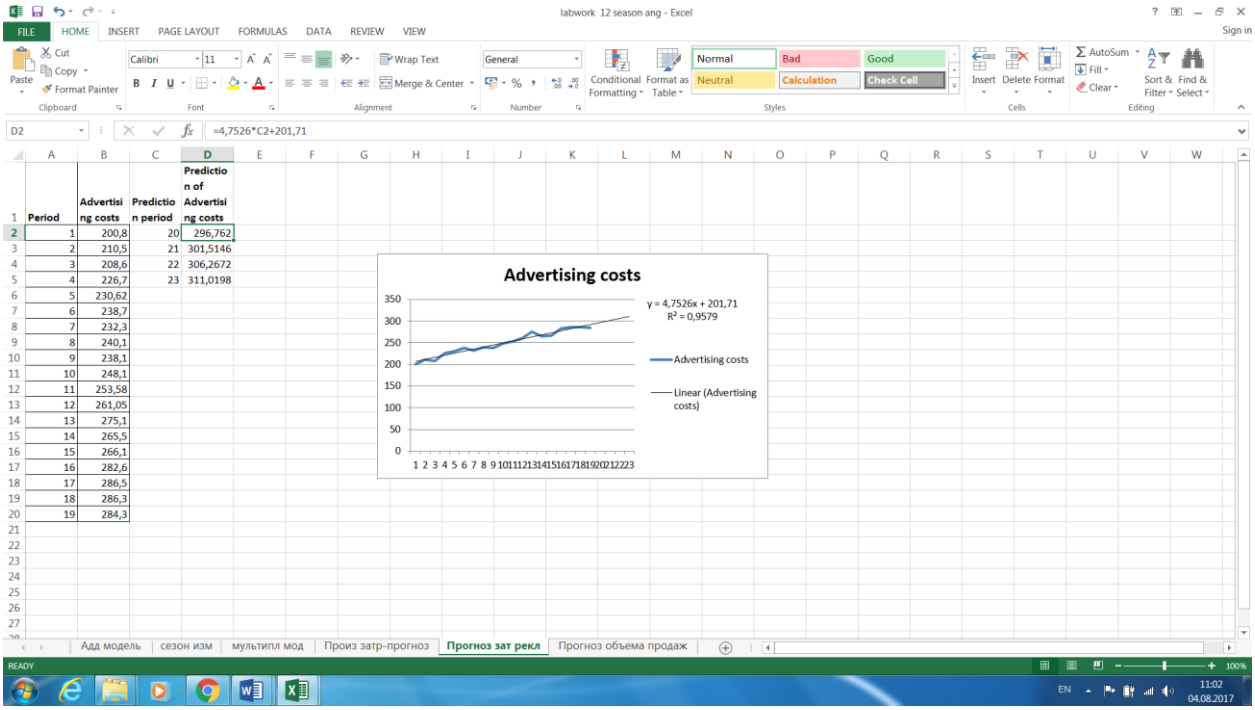



Fig. 12.5. Screenshot of the prediction for the factor "advertising costs"

The prediction for the prediction variable (sales volume) is presented below.

Table 12.6. Initial data and prediction for the prediction function for time periods 20-23

| Период | Производственные затраты | Затраты на рекламу | Объем продаж | Функция регрессии | Остаток функции регрессии | Среднее сезона Si | Сезонные коэф | Функция прогноз |
|---|---|---|---|---|---|---|---|---|
| 1 | 890,8 | 200,8 | 1286,5 | 1329,349 | -42,8493 | -69,9517 | -101,107 | 1228,243 |
| 2 | 896,4 | 210,5 | 1282,7 | 1294,721 | -12,0205 | -191,929 | -223,084 | 1071,636 |
| 3 | 903 | 208,6 | 1224,8 | 1397,418 | -172,618 | -312,853 | -344,008 | 1053,409 |
| 4 | 888,8 | 226,7 | 1292,6 | 1446 | -153,4 | -107,35 | -138,505 | 1307,495 |
| 5 | 889,8 | 230,62 | 1667,3 | 1521,13 | 146,1696 | 121,7136 | 90,55862 | 1611,689 |
| 6 | 890,8 | 238,7 | 1672,9 | 1537,63 | 135,2696 | 51,59384 | 20,43882 | 1558,069 |
| 7 | 888,3 | 232,3 | 1667,5 | 1774,925 | -107,425 | 134,9165 | 103,7615 | 1878,686 |
| 8 | 885,8 | 240,1 | 2001,7 | 1822,27 | 179,4298 | 179,4298 | 148,2747 | 1970,545 |
| 9 | 883,9 | 238,1 | 2352,5 | 1987,469 | 365,0307 | 365,0307 | 333,8757 | 2321,345 |
| 10 | 875,1 | 248,1 | 2468,5 | 2061,72 | 406,7796 | 406,7796 | 375,6246 | 2437,345 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 11 | 877,73 | 253,58 | 2012,2 | 2074,541 | -62,3407 | -62,3407 | -93,4957 | 1981,045 |
| 12 | 873,8 | 261,05 | 1982,7 | 2123,879 | -141,179 | -141,179 | -172,334 | 1951,545 |
| 13 | 878,2 | 275,1 | 1901,1 | 1998,154 | -97,0541 | -97,0541 | | 1897,047 |
| 14 | 866,3 | 265,5 | 1971,6 | 2343,438 | -371,838 | -371,838 | | 2120,354 |
| 15 | 865,1 | 266,1 | 1989,1 | 2442,189 | -453,089 | -453,089 | | 2098,181 |
| 16 | 876,6 | 282,6 | 2138,2 | 2199,5 | -61,2998 | -61,2998 | | 2060,995 |
| 17 | 862,5 | 286,5 | 2475,2 | 2377,942 | 97,25768 | 97,25768 | | 2468,501 |
| 18 | 865,9 | 286,3 | 2395,6 | 2427,682 | -32,082 | -32,082 | | 2448,121 |
| 19 | 857,5 | 284,3 | 2994,1 | 2616,842 | 377,2578 | 377,2578 | | 2720,604 |
| **20** | **859,064** | **296,762** | | **2511,216** | | | | **2659,491** |
| **21** | **856,989** | **301,514** | | **2539,235** | | | | **2873,111** |
| **22** | **854,915** | **306,267** | | **2563,035** | | | | **2938,66** |
| **23** | **852,841** | **311,019** | | **2582,616** | | | | **2489,12** |

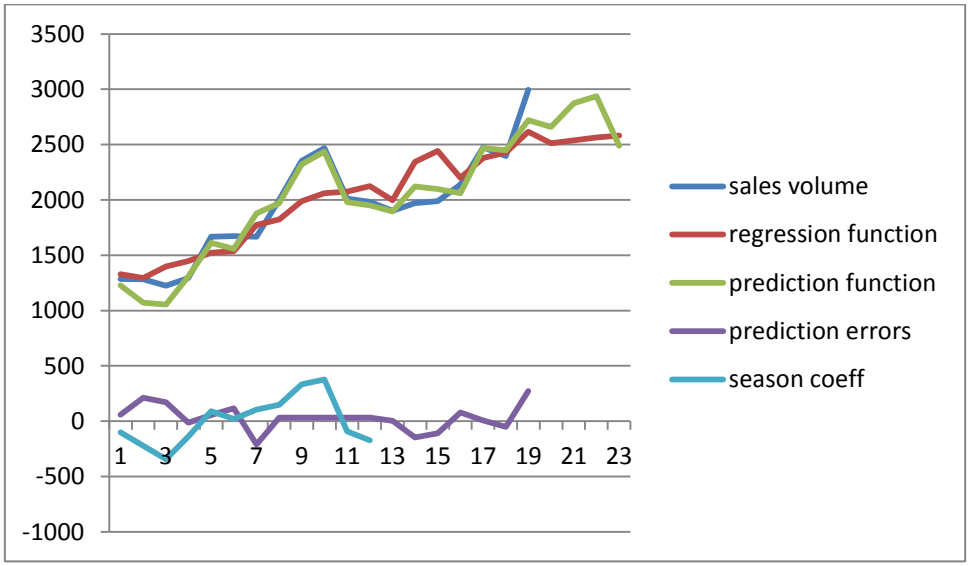



Fig. 12.6. The prediction graphic for the prediction function, the seasonal coefficients graphic, the prediction function error graphic.

## 12.2 PRACTICAL LESSON 12 and IWS 12

**Subject.** Segregation of the seasonal component of the data model for the prediction task.

**Plan of the lesson.**

5) Study the scheme for allocating the seasonal component of the data model for the prediction problem presented above.

6) Task of the IWS. Apply the above chart of the seasonal component of the data model for its prediction task.

7) Analyze the results.

Literature.

Minko A.A. Predicting in business using Excel. M.: Eksmo, 2007, -208 p.

# TOPIC 13

# DECISION TREES

## 13.1. Lecture material

The work of decision trees is based **on the process of recursive partitioning of the initial set of observations or objects into subsets** associated with **classes defined by the selected attributes of objects** on each cycle of the recursive partition.

**The partition is performed using the decision rules, in which the values of the attributes are checked according to the specified condition**. Let's consider the main idea of algorithms for constructing decision trees using the example.

The scheme for using decision trees is as follows. Let's consider the scheme of construction and use of decision trees on **the standard example of issuing a loan to the client** [1].

The database on which to base the forecasting, for example, contains the following initial data on the bank's customers that are attributes of this database:
- age,
- availability of real estate,
- education,
- the average monthly income,
- return a loan by the client on time.

The task is to predict, based on the data listed above (except of the last attribute), it is worth whether giving out a loan to a new client.

The task is solved in two stages:
**- construction of a classification model (training stage);**
**- use of the constructed model for making decisions on new clients.**
At the stage of building the classification model, a classification tree is constructed or a set of classification rules is created.
At the stage of using the model, the built decision tree, namely, the path from its root to one of the vertices, which is a set of rules for a particular client, is used to answer the question «Whether to issue the loan? "

**The rule is a logical construction, represented in the form**

**"If: <condition> then: <operator>".**

Let's assume for this example there are statistics presented in Table 13.1.

Table 13.1. Customer input data

| | Age | property availability | education | average monthly income | loan repayment |
|---|---|---|---|---|---|
| 1 | 40(age>30) | yes | Higher education | high | yes |
| 2 | 50(age >30) | yes | Higher education | high | yes |
| 3 | 65(age >60) | no | specialized secondary | average | no |
| 4 | 32(age >30) | yes | Higher education | average | yes |
| 5 | 25(age ≤30) | no | secondary education | low | no |
| 6 | 20(age ≤30) | no | secondary education | low | no |
| 7 | 28(age ≤30) | yes | secondary education | average | yes |

In Fig. 13.1. an example of a classification tree, which solves the problem of "Whether to issue the loan?".

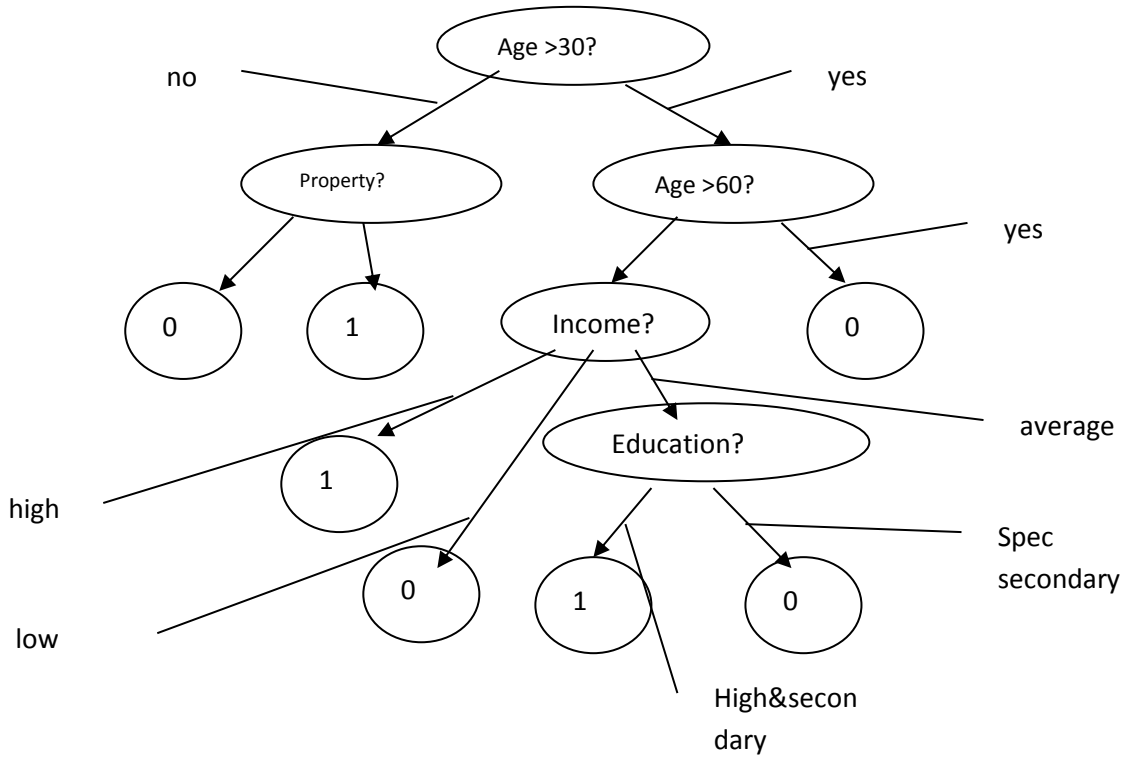



Fig.13.1. A variant of the decision tree for the task of "issuing a loan to a client" (1-issuing a loan, 0-not issuing).

As can be seen from the decision tree of this example, the internal nodes of the tree (age, availability of real estate, income and education) are attributes of the database described above. These attributes are called *splitting attributes*. The final nodes of a tree, or sheets, are called *class labels*, which are the values of the dependent categorical variable "to give out" or "not to issue" a loan.

At each internal node, it is necessary **to specify a validation condition (*splitting predicate*) that would split the set associated with this node into subsets**. For this check, one of the database attributes must be selected, except for the dependent variable. **This question of selecting the attribute of splitting is the main point in the automated construction of the decision tree.**

The general rule for selecting an attribute can be formulated as follows: **the selected attribute must split the set so that the resulting subsets consist of objects belonging to the same class** or are as close to it as possible, i. e. the number of objects from other classes ("impurities") in each of these sets was as small as possible.

A number of methods for constructing decision trees have been developed, including: Classification and regression trees (CART) method, C4.5 method [1].

Algorithm C4.5 uses the concept of **information gain or entropy reduction** to select the optimal splitting (split) in each node of the decision tree. **Because the decrease in entropy leads to an increase in information and vice versa.**

Let us first consider **the concept of entropy**. Suppose that there is a variable $X$ whose $k$ *values* have probabilities $p_1, p_2, \ldots, p_k$. **How much information is needed to transmit a stream of symbols representing the values of the observed $X$**? The answer to this question is the entropy X and is defined as

$$H(X) = -\sum_j p_j log_2(p_j).$$

Algorithm C4.5 uses the concept of entropy as follows.

Suppose that there is a splitting attribute s that divides the learning set of data $T$ into several subsets, $T_1, T_2, \ldots, T_k$. **The average amount of information** on these subsets

can be calculated as the weighted sum of the entropies of the individual subsets, as follows:

$$H_s(T, D) = \sum_{i=1}^{s} P_i \, H_s(T_i, d),$$

Where $P_i$ represents the fraction of records in the subset i, $H_s$(T,D)is the entropy of the splitting of the data set T by the splitting attribute S with the dependent variable D, $H_s(T_i, D)$- is the entropy of the splitting subset $T_i$from the splitting attribute S with the dependent variable D, $s$ is the number of values of the splitting attribute *S* (determines the number of subsets (classes) of splitting).

Then, information gain or entropy reduction can be determined by splitting the initial set T into subsets of the splitting attribute s as:

$$Gain(T,S) = H(T,D) - H_s(T,D),$$

Where H (T, D) is the entropy of the initial data set T by the attribute of the D-dependent variable.

$$H(T, D) = \sum_{i=1}^{d} P_i \, H(T_i, D),$$

Where d is the number of values of the attribute D-dependent variable.

Thus, the information is incremented by dividing the initial training data set T in accordance with the splitting attribute S. At each node, C4.5 selects the optimal splitting, which has the highest gain of information Gain (T, S).

We calculate the entropy and increment of information by the definition of the optimal attribute for an example.

We calculate the entropy of the initial data set H (T).

H (T, Credit) = $-\frac{4}{7} log_2 \frac{4}{7} - \frac{3}{7} log_2 \frac{3}{7} \approx 0{,}9852$. Here, Credit is a dependent variable.

We calculate the increments of information for various attributes.

The increase in information on the splitting attribute "Income":

$$Gain\ (T, \text{Income}) = H(T, \text{Credit}) - \frac{2}{7}H(T_{income=high}, \text{Credit}) - \frac{3}{7}H(T_{income=average}, \text{Credit}) - \frac{2}{7}H(T_{income=low}, \text{Credit})$$
$$\approx 0{,}9852 - \frac{2}{7}\left(-\frac{2}{2}log_2\frac{2}{2} - \frac{0}{2}log_2\frac{0}{2}\right) - \frac{3}{7}\left(-\frac{2}{3}log_2\frac{2}{3} - \frac{1}{3}log_2\frac{1}{3}\right) - \frac{2}{7}\left(-\frac{0}{2}log_2\frac{0}{2} - \frac{2}{2}log_2\frac{2}{2}\right) \approx 0{,}9181$$

The increase in information on the splitting attribute "Age":

$$Gain\ (T, \text{age}) = H(T, \text{Credit}) - \frac{3}{7}H(T_{age>30}, \text{Credit}) - \frac{1}{7}H(T_{age>60}, \text{Credit}) - \frac{3}{7}H(T_{age\leq 30}, \text{Credit})$$
$$\approx 0{,}9852 - \frac{3}{7}\left(-\frac{3}{3}log_2\frac{3}{3} - \frac{0}{3}log_2\frac{0}{3}\right) - \frac{1}{7}\left(-\frac{0}{1}log_2\frac{0}{1} - \frac{1}{1}log_2\frac{1}{1}\right) - \frac{3}{7}\left(-\frac{1}{3}log_2\frac{1}{3} - \frac{2}{3}log_2\frac{2}{3}\right) \approx 0{,}9181$$

Increase in information on the attribute of splitting " Property ":

$$Gain\ (T, \text{property}) = H(T, \text{Кредит}) - \frac{4}{7}H(T_{property=yes}, \text{Кредит}) - \frac{3}{7}H(T_{property=no}, \text{Кредит}) \approx 0{,}9852 - \frac{4}{7}\left(-\frac{4}{4}log_2\frac{4}{4} - \frac{0}{4}log_2\frac{0}{4}\right) - \frac{3}{7}\left(-\frac{0}{3}log_2\frac{0}{3} - \frac{3}{3}log_2\frac{3}{3}\right) \approx 0{,}9852$$

The increase in information on the splitting attribute "Education":

$$Gain\ (T, \text{education}) = H(T, \text{Credit}) - \frac{3}{7}H(T_{education=high}, \text{Credit}) - \frac{1}{7}H(T_{education=spec\ secondary}, \text{Credit}) - \frac{3}{7}H(T_{education=secondary}, \text{Credit})$$
$$\approx 0{,}9852 - \frac{3}{7}\left(-\frac{3}{3}log_2\frac{3}{3} - \frac{0}{3}log_2\frac{0}{3}\right) - \frac{1}{7}\left(-\frac{0}{1}log_2\frac{0}{1} - \frac{1}{1}log_2\frac{1}{1}\right) - \frac{3}{7}\left(-\frac{1}{3}log_2\frac{1}{3} - \frac{2}{3}log_2\frac{2}{3}\right) \approx 0{,}9181$$

Thus, calculations of the information gain show that the optimal at the first level is the verification of the attribute "Property".

Thus, the calculation of the information gain shows that the optimal at the first level is the verification of the attribute "Property", in which the gain of information Gain (T, property) = 0.9852. With this in mind, the decision tree of this example will look like Figure 13.2.

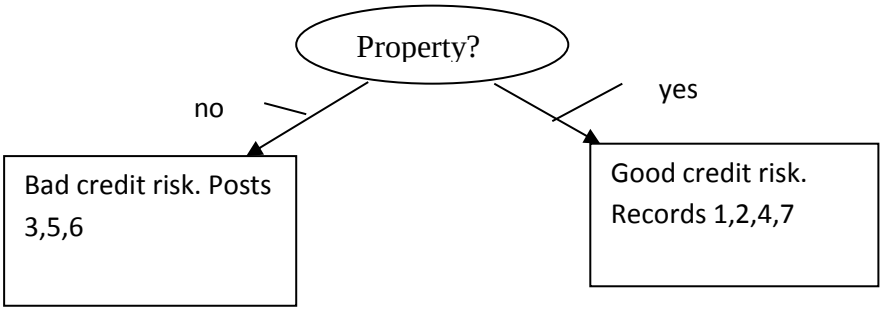



Fig. 13.2 Optimal solution tree for example

Select the data set for the right branch from the source data set (Table 13.2.). This is necessary for the recursive application of the algorithm for constructing a decision tree to determine the splitting attribute for the next level. To do this, the set of possible attributes includes: age, education, income. The attribute "property" is used for splitting on the first level.

Table 13.2 Data set for selecting the second splitting attribute.

| | Age | property availability | education | average monthly income | loan repayment |
|---|---|---|---|---|---|
| 1 | 40(age>30) | yes | Higher education | high | yes |
| 2 | 50(age >30) | yes | Higher education | high | yes |
| 4 | 32(age >30) | yes | Higher education | average | yes |
| 7 | 28(age ≤30) | yes | secondary education | average | yes |

We calculate the entropy of the initial data set H (T) for the next level of splitting.

$$H(T,Credit)=-\frac{4}{4}log_2\frac{4}{4}-\frac{0}{4}log_2\frac{0}{4}\approx 0$$

Since the entropy of a given set for the next level is zero, this means that the process of building the decision tree for this example is complete. The resulting decision tree for the given source data set of the statistics of the example is optimal (Figure 13.2). However, one should bear in mind that the original sample may not cover all possible cases in reality, so the final decision on the structure of the decision tree remains with the user.

Below is represented the solution of the above example using data mining tool Rapidminer.

In the Fig.13.3 screenshot of source data for the standard example of issuing a loan to the client is presented.

In the Fig.13.4 screenshot of the developing process structure for the standard example of issuing a loan to the client is presented. Input and output points of process operators (mod = Model, unl = Unlabelled data, lab = Labelled data, exa = Example set, ori = Original data).

In the Fig.13.5-13.8. screenshot of set role operator parameters for the standard example of issuing a loan to the client are presented.

Parameters of the operator "Set role" [2]:

1) The parameter 'attribute_name', which role should be changed. The name can be selected from the dropdown menu or manual typed.

Range of the parameter 'attribute_name':

- target_role;
- set of additional role.

The target role of the selected Attribute is the new role assigned to it. As possible target roles are used:

- regular: attributes without a special role. Regular Attributes are used as input variables for learning tasks;

- label: this is a special role; an attribute with the label role acts as a target Attribute for learning Operators; the label is also often called 'target variable' or 'class';

- prediction: this is a special role; an Attribute with the prediction role is the result of an application of a learning model.

More detail description of the parameters is in the Rapidminer Manual [2].

And in the Fig.13.9. screenshot of results for the standard example of issuing a loan to the client is presented.

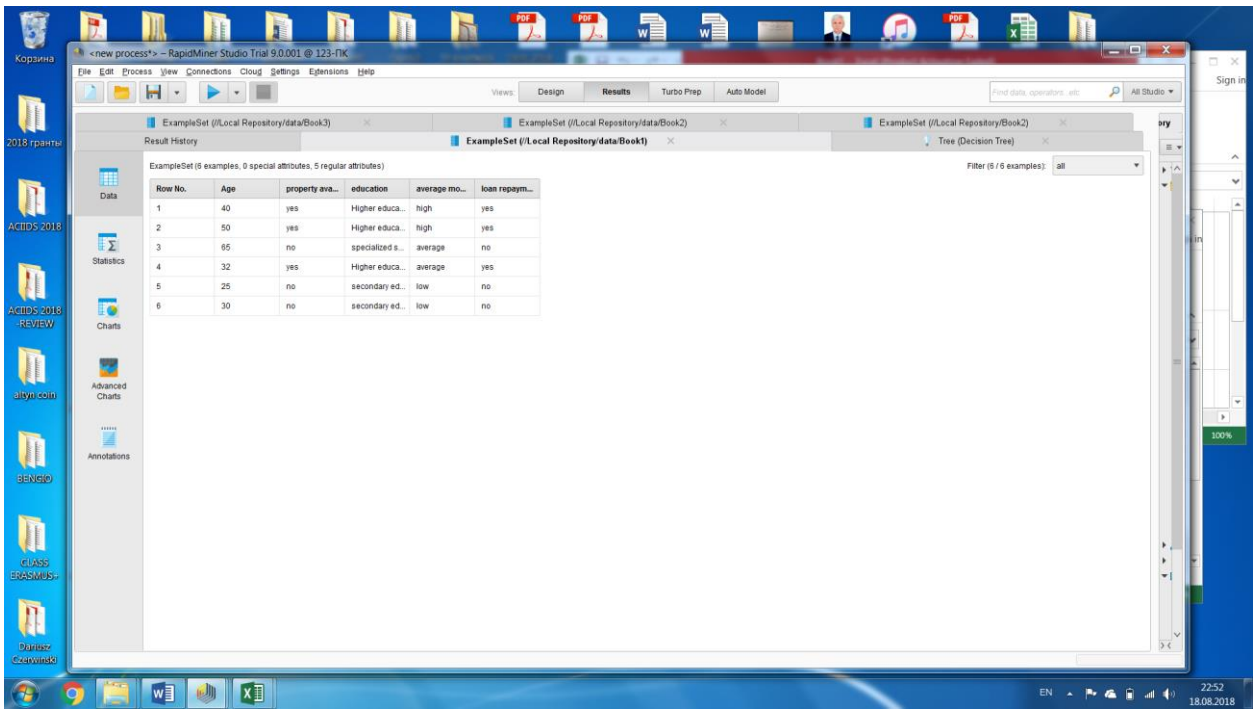



Fig.13.3. Screenshot of source data for the standard example of issuing a loan to the client

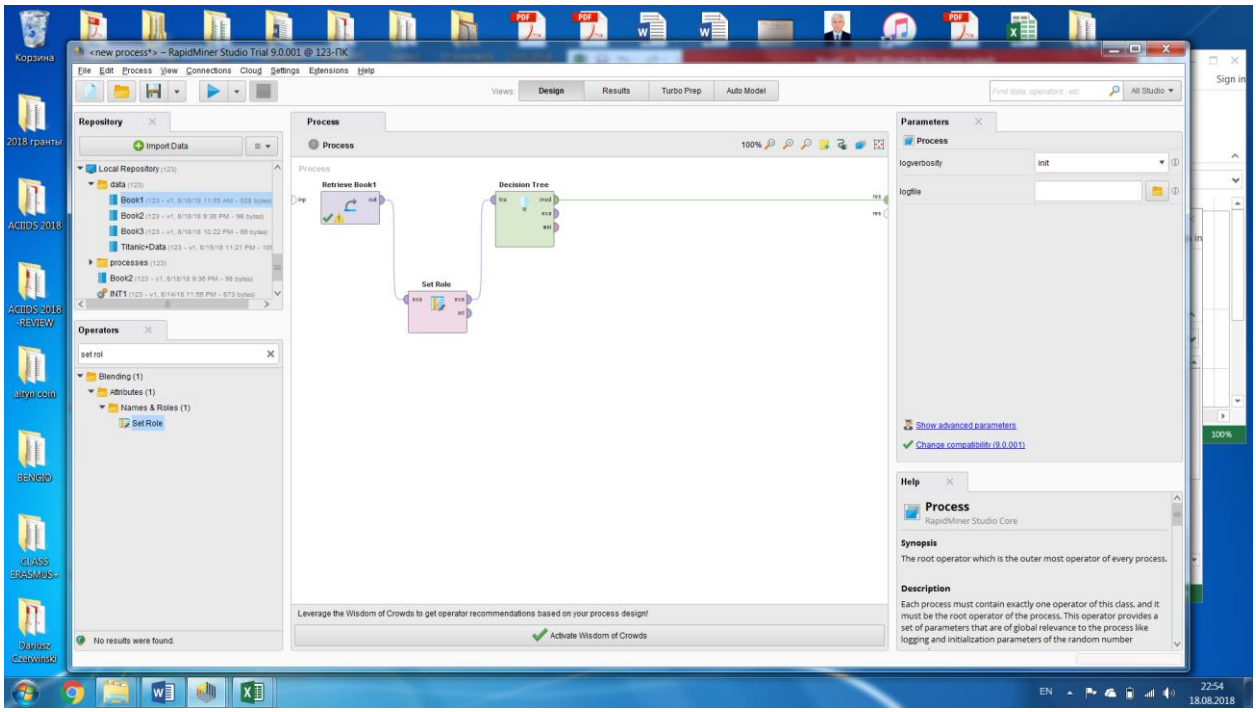



Fig.13.4. Screenshot of the developing process structure for the standard example of issuing a loan to the client.

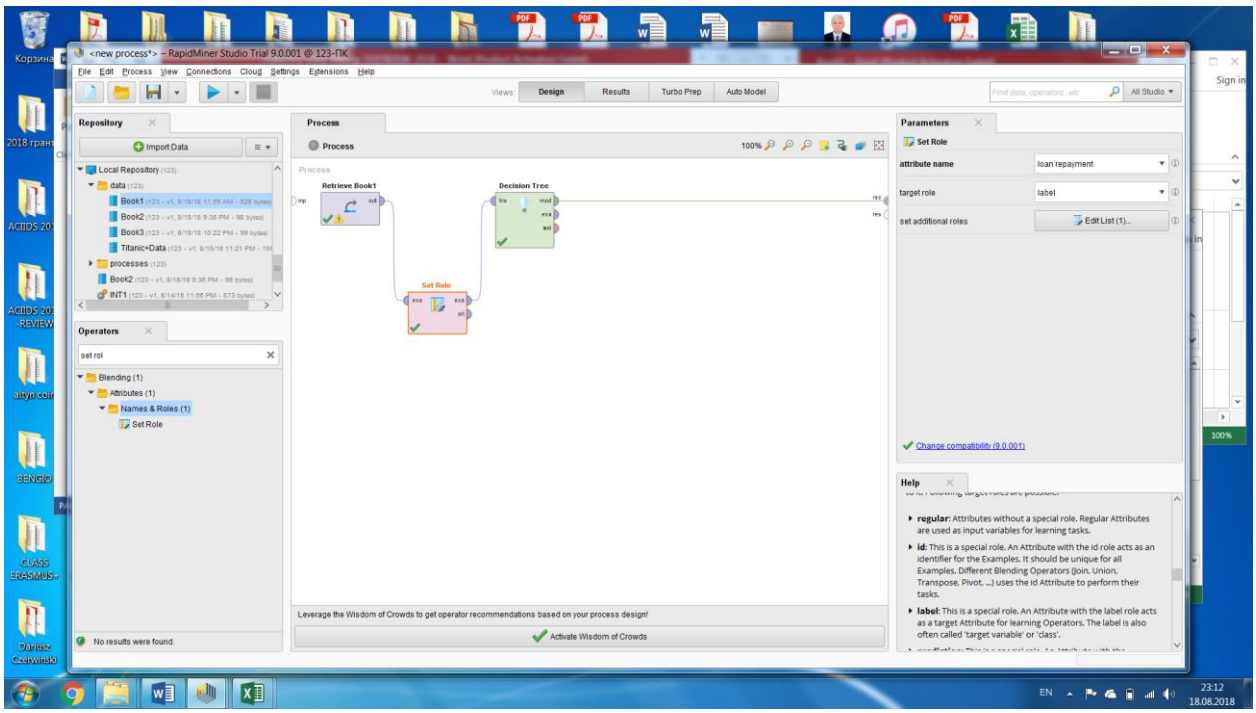



Fig.13.5. Screenshot of set role operator parameters (target attribute “loan repayment”) for the standard example of issuing a loan to the client

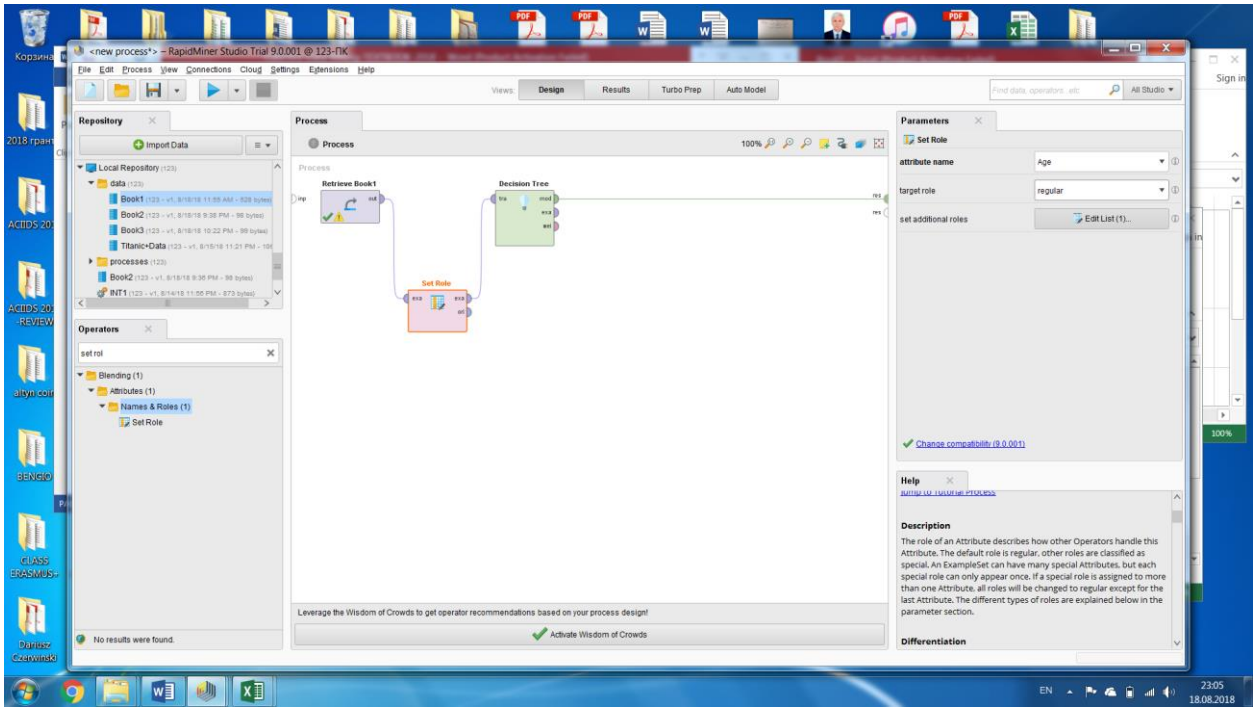



Fig.13.6. Screenshot of set role operator parameters (regular attribute “age”) for the standard example of issuing a loan to the client

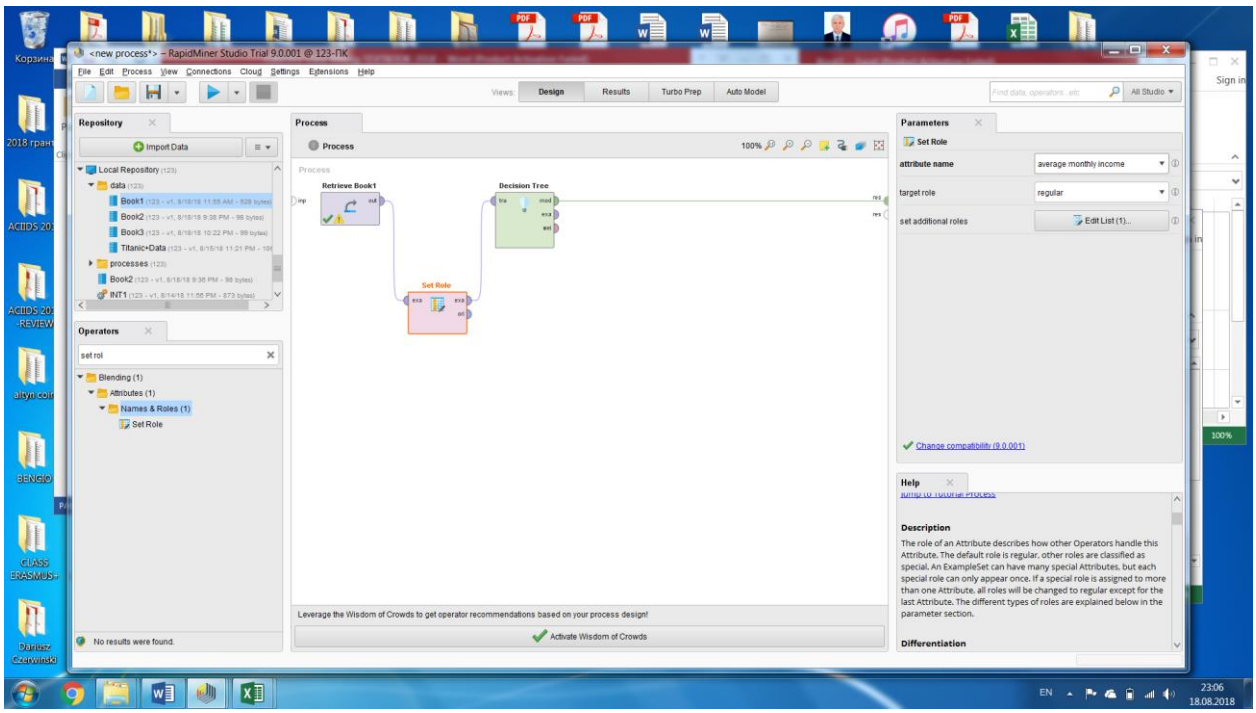



Fig.13.7. Screenshot of set role operator parameters (regular attribute “average monthly income”) for the standard example of issuing a loan to the client

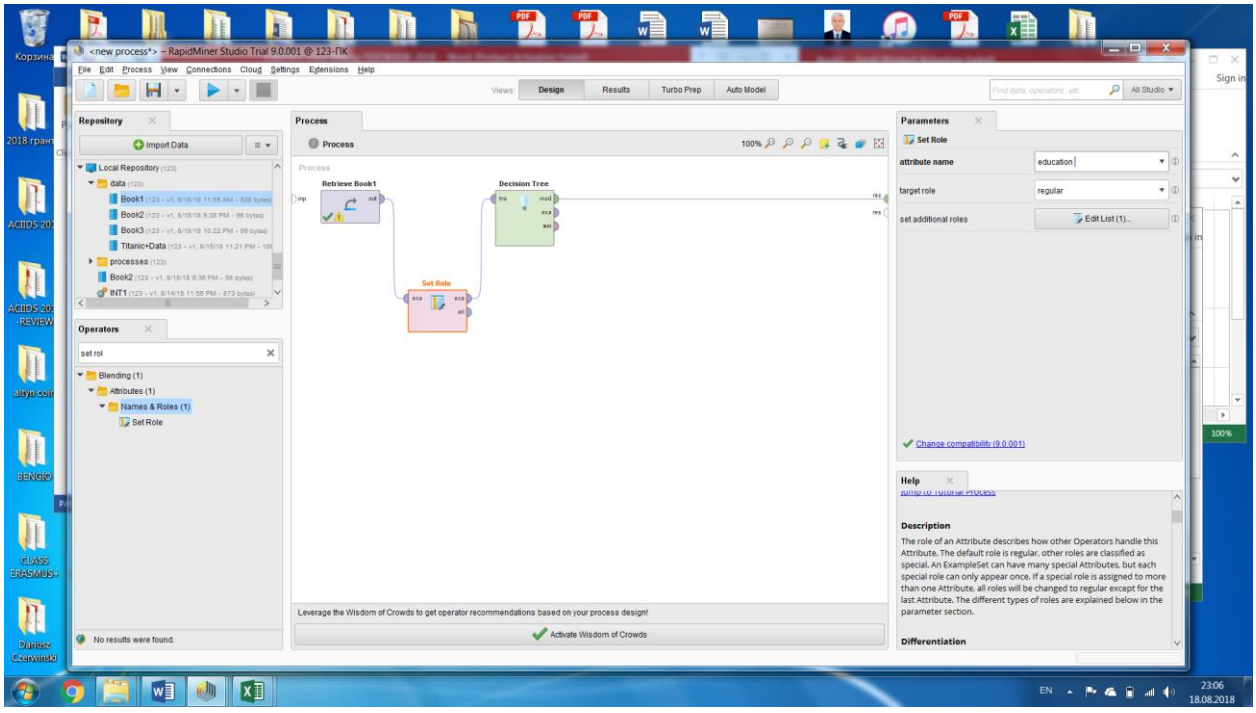



Fig.13.7. Screenshot of set role operator parameters (regular attribute “education”) for the standard example of issuing a loan to the client

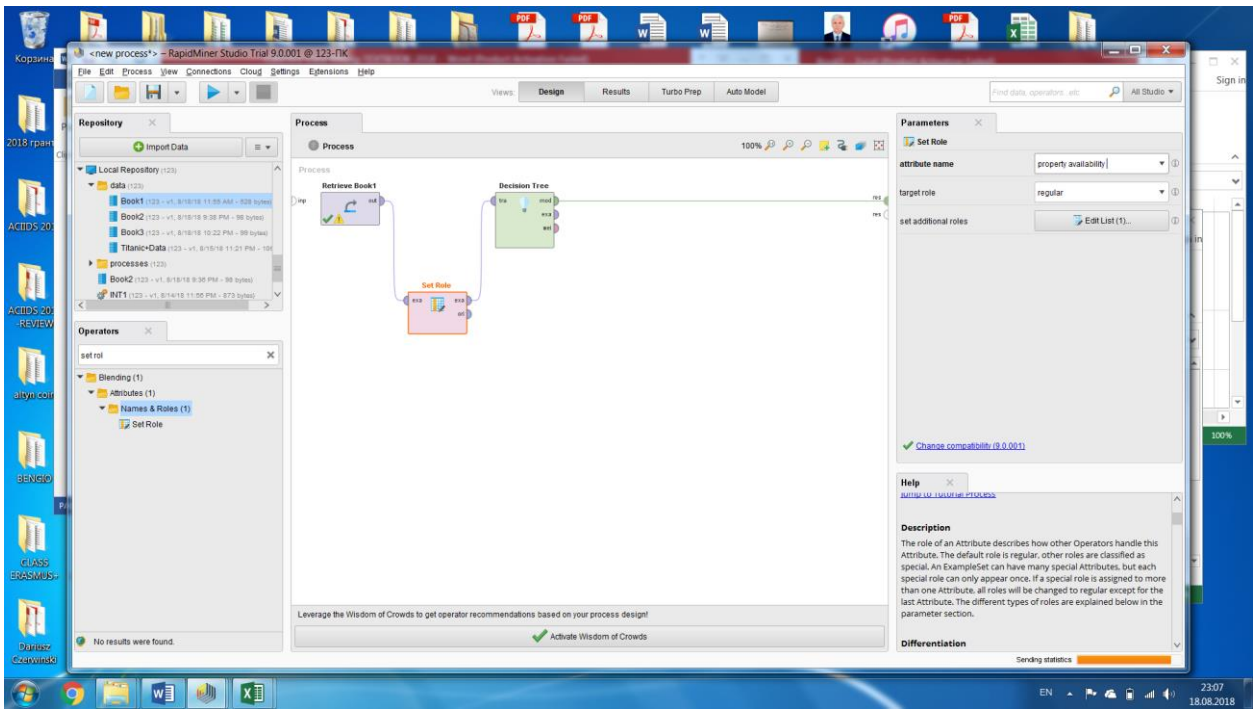



Fig.13.8. Screenshot of set role operator parameters (regular attribute "property availability") for the standard example of issuing a loan to the client

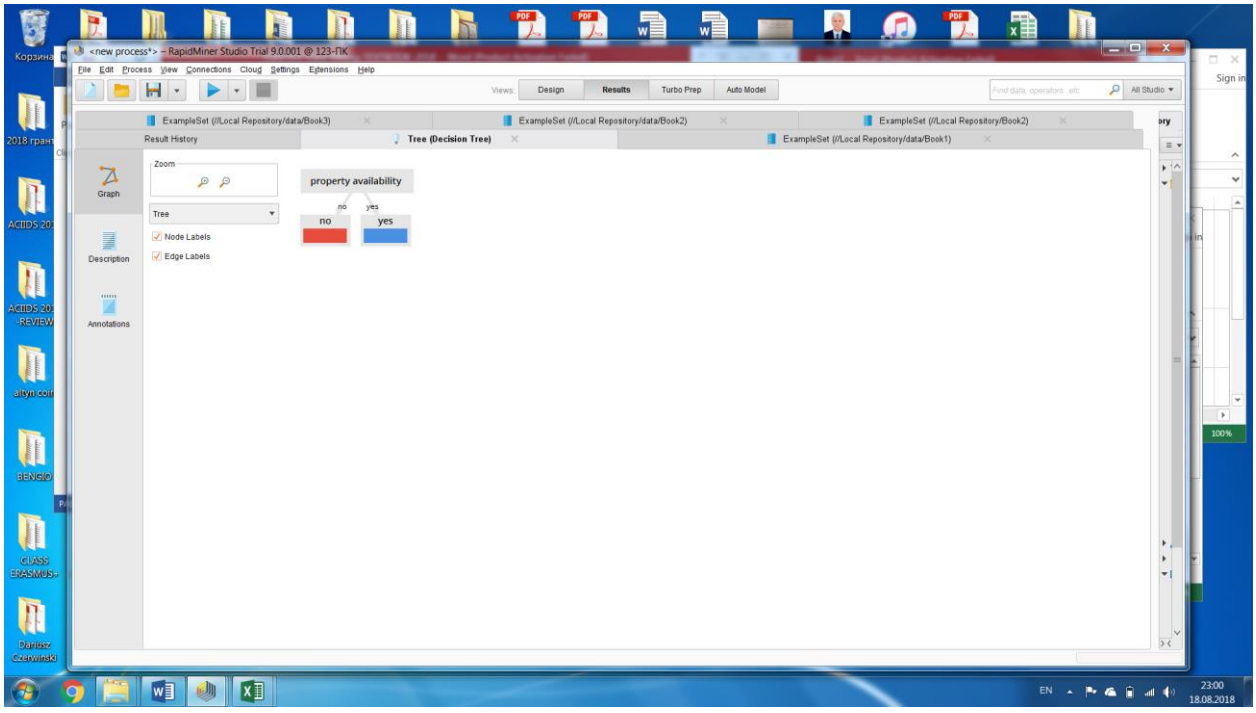



Fig.13.9. Screenshot of results for the standard example of issuing a loan to the client

## 13.2 PRACTICAL LESSON 13 and IWS13

**Subject.** The construction and use of decision trees for the classification problem.

**Plan of the lesson.**

1) Study the scheme for constructing and using decision trees for the classification problem, presented above.

2) The task of the IWS. Apply the above scheme of constructing and using the decision tree for your classification problem.

3) Analyze the results.

Literature

1. Larose D.T. Discovering knowledge in data - an introduction to data mining. Wiley-Interscience, Hoboken, New Jersey, 2005

2. Rapidminer Studio, manual. https://docs.rapidminer.com/latest/studio/

# TOPIC 14

# CLUSTERING

## 14.1. Lecture material

**Clustering refers to the grouping of observation records into classes of similar objects.**

**A cluster is a collection of records that are similar to each other, and are not similar to records in other clusters.**

Clustering differs from **classification** in that **there is no target variable for clustering**. The clustering task does not attempt to classify, evaluate, or predict the value of the target variable. **Instead, clustering algorithms tend to segment the entire data set into relatively homogeneous subgroups or clusters**, where **the similarity of records in a cluster is maximized**, and **the similarity of records outside this cluster is minimized** [1].

Examples of clustering tasks in business and research include the following:

• Targeted marketing of niche products for small businesses that does not have a large marketing budget.

• For the purpose of accounting audit, for the segmentation of financial behavior in benign and suspicious categories.

• As a measurement tool to reduce, when a data set has hundreds of attributes.

Clustering is often performed as a preliminary step in the data mining process, with clusters being used as additional inputs to other downstream techniques, such as neural networks.

### 14.1.1 Similarity of measures.

Cluster analysis is confronted with many of the same issues that are dealt with in the classification problem. For example, the following issues should be addressed:

- How to measure the similarity?

- How to standardize or normalize numeric variables?

One measure of the similarity of records is the Euclidean distance between records:

$$d_{Euclidean}(x,y) = \sqrt{\sum_i (x_i - y_i)^2}$$

Where $x = x_1, x_2, ..., x_m$, and $y = y_1, y_2, ..., y_m$ represent the m attribute values of the two records.

Of course, there are many other indicators, such as the distance of Manhattan:

$$d_{Manhatten}(x,y) = \sum_i |x_i - y_i|$$

The Minkowski distance is the general case of the above metrics for the total exponent r:

$$d_{Minkowski}(x,y) = \sqrt[r]{\sum_i |x_i - y_i|^r}$$

A particular case of the Minkowski distance is the Manhattan distance corresponding to r = 1. For r = 2, the usual Euclidean distance is obtained.

### 14.1.2. Transformation of data.

Variables, as a rule, have ranges that differ significantly from each other. Therefore, for data analysis programs, it is necessary to normalize numerical variables, standardize the scale of the effect of each variable on the results. There are several methods for normalization, two of the most common methods: min-max normalization and Z-score standardization. Let X refer to the initial value of the field and X * refers to the normalized value of the field.

The Min-Max normalization method scales by the difference in range:

$$X^* = \frac{X - \min(X)}{range(X)} = \frac{X - \min(X)}{\max(X) - \min(x)},$$

Where X is the value of the normalized field, min (X) is the minimum value of the normalized field, max (X) is the maximum value of the normalized field.

For example, consider the variable "time to 60" from the car data set, which determines the time (in seconds) how much each car spends to reach 60 km per hour. Let's calculate the min-max normalization for three cars (Car1, Car2, Car3), having "time to 60" 8, 15 and 25 seconds respectively.

Then for Car1 the normalized value of the field "time to 60" will be equal to:

$$X^* = \frac{X - \min(X)}{\max(X) - \min(x)} = \frac{8 - 8}{25 - 8} = 0$$

For Car2, the normalized field value "time to 60" will be:

$$X^* = \frac{X - \min(X)}{\max(X) - \min(x)} = \frac{15 - 8}{25 - 8} = 0{,}41$$

For Car3, the normalized field value "time to 60" will be:

$X^* = \frac{X - \min(X)}{\max(X) - \min(x)} = \frac{25 - 8}{25 - 8} = 1.$

Thus, min-max values of normalization will fluctuate from zero to one if new data values do not occur that lie outside the original range.

Z-score standardization is very widespread in the world of statistical analysis, it works as a difference between the field value and the mean value of the field and the scaling of this difference by the standard deviation (SD) values of the field. I.e,

$X^* = \frac{X - \text{mean}(X)}{SD(X)}$.

Then for Car1 the normalized value of the field "time to 60" will be equal to:

$$X^* = \frac{X - \text{mean}(X)}{SD(X)} = \frac{8 - 15}{2{,}9} = -2{,}4$$

For Car2, the normalized field value "time to 60" will be:

$$X^* = \frac{X - \text{mean}(X)}{SD(X)} = \frac{15 - 15}{2{,}9} = 0$$

For Car3, the normalized field value "time to 60" will be:

$X^* = \frac{X - \text{mean}(X)}{SD(X)} = \frac{25 - 15}{2{,}9} = 3{,}4.$

Summarizing, the standardization values of the Z-count are generally in the range of -4 to 4, with an average value having Z-account of zero standardization.

### 14.1.3 Clustering algorithms. Hierarchical cluster method.

**In this method, a hierarchical agglomerate algorithm is realized, which consists in uniting smaller clusters into larger clusters.**

Before starting clustering, **all objects are considered separate clusters**.

In the course of the algorithm, clusters are combined**. First, select a pair of nearby clusters, which are combined into one cluster.** As a result, the number of clusters becomes N-1.

**The procedure is repeated until all the classes are combined**. **At any stage, the union can be interrupted by obtaining the required number of clusters.**

**The closeness of clusters is determined by the distance between them.**

You can use several criteria to determine the distance between clusters A and B:

- **The "nearest neighbor" method**, based on **the minimum distance between any record of cluster A and any record of cluster B**.

- **The method of " distant neighbor",** based on **the maximum distance between any record of cluster A and any record of cluster B**. Moreover, cluster clusters that are smaller in size are the priority for the association.

- The method of "**the least mean connection between the records of cluster A and records of cluster B**".

The "nearest neighbor" method.

For example, take a one-dimensional data set [1]:

1 4 9 15 16 18 25 32 32 44

It is assumed that each element is a separate cluster. The process of combining clusters and constructing a hierarchy of clusters for an example is shown in Fig. 14.1. Consider the sequence of algorithm steps for this example.

Step 1. Since singleton clusters are considered, the minimum distance between any record of cluster A and any record of cluster B will be searched between the

elements. **These are clusters with 32,32 elements, the distance between them is 0.**

Step 2. The search of variants of clusters shows that clusters with **elements 15,16 with a distance equal to 1 should unite.**

Step 3. At this step it is necessary to **combine the cluster (15,16) with the cluster (18) with the distance equal to 2.**

Step 4. At this step, **clusters (1) and (4) are combined with a distance of 3**.

Step 5. Clusters (1,4) and (9) are united with a minimum distance of 5 between elements 4 and 9.

Step 6. Clusters (1,4,9) and (15,16,18) are united with a minimum distance equal to 6 between cluster element 9 (1,4,9) and cluster element 15 (15,16,18).

Step 7. Clusters (1,4,9, 15,16,18) and (25) are combined with a minimum distance of 7 between the cluster element 18 (1,4,9,15,16,18) and the cluster element 25 (25 ).

Step 8. Clusters (1,4,9, 15,16,18, 25) and (32,32) are united with a minimum distance of 7 between cluster element 25 (1,4,9,15,16,18,25) And cluster element 32 (32,32).

Step 9. Clusters (1,4,9, 15,16,18,25,32,32) and (44) are combined with a minimum distance of 12 between the cluster element 32 (1,4,9,15,16,18, 25,32,32) and the cluster element 44 (44).

At each step, the variants are sorted and there is a variant of combining clusters with the minimum distance.

| steps | 1 | 4 | 9 | 15 | 16 | 18 | 25 | 32 | 32 | 44 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | 32,32 d=0 | | |
| 2 | | | | 15,16 d=1 | | | | | | |
| 3 | | | | 15,16,18 d=2(16,18) | | | | | | |
| 4 | 1,4 d=3 | | | | | | | | | |
| 5 | 1,4,9 d=5(4,9) | | | | | | | | | |
| 6 | 1,4,9, 15,16,18 d=6(9,15) | | | | | | | | | |
| 7 | 1,4,9, 15,16,18,25 d=7(18,25) | | | | | | | | | |
| 8 | 1,4,9, 15,16,18,25,32,32 d=7(25,32) | | | | | | | | | |
| 9 | 1,4,9, 15,16,18,25,32,32,44 d=12(32,44) | | | | | | | | | |

Fig. 14.1. Steps for clustering the data set using the "nearest neighbor" method.

**The method of "distant neighbor".**

For example, the "distant neighbor" method is shown in Fig. 14.2.

**The difference in the distant neighbor algorithm for this example begins with step 4**, where the distance is taken as the maximum between the elements. The situation is similar in step 5.

**The structural difference in the hierarchy begins with step 6, where, unlike the previous method, other clusters are combined.**

| шаги | 1 | 4 | 9 | 15 | 16 | 18 | 25 | 32 | 32 | 44 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | 32,32 d=0 | | |
| 2 | | | | 15,16 d=1 | | | | | | |
| 3 | 1,4 d=3 | | | | | | | | | |
| 4 | | | | 15,16,18 d=3(15,18) | | | | | | |
| 5 | 1,4,9 d=8(1,9) | | | | | | | | | |
| 6 | | | | | | | 25,32,32 d=7(25,32) | | | |
| 7 | 1,4,9, 15,16,18 d=17(1,18) | | | | | | | | | |
| 8 | | | | | | | 25,32,32,44 d=19(25,44) | | | |
| 9 | 1,4,9, 15,16,18,25,32,32,44 d=43(1,44) | | | | | | | | | |

Fig. 14.2. Steps for clustering a data set using the far-neighbor method.

**The method of the least mean connection between the records of cluster A and records of cluster B.**

"The method of the least mean connection between records of cluster A and records of cluster B" is shown for example in Fig. 14.3. As you can see from the comparison of the clustering patterns in three methods, the result of the clustering of the "least mean communication method between the records of cluster A and records of cluster B" coincides with the result of the "distant neighbor" method.

| steps | 1 | 4 | 9 | 15 | 16 | 18 | 25 | 32 | 32 | 44 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | 32,32 d=0 | | |
| 2 | | | | 15,16 d=1 | | | | | | |
| 3 | 1,4 d=3 | | | | | | | | | |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 4 | | | 15,16,18<br>d=2,5(15,18=3;16,18=2) | | | | | | |
| 5 | 1,4,9<br>d=6,5(1,9=8;4,9=5) | | | | | | | | |
| 6 | | | | | | 25,32,32 d=7 | | | |
| 7 | 1,4,9, 15,16,18 d=11,66 | | | | | | | | |
| 8 | | | | | | 25,32,32,44 d=14,33 | | | |
| 9 | 1,4,9, 15,16,18,25,32,32,44 d=23.91 | | | | | | | | |

Fig. 14.3. Steps for clustering a data set using the method of least average communication between cluster records.

### 14.1.4 Algorithms for clustering. Method K-means (k-means).

The k-means method is a simple and effective algorithm for building data clusters. The algorithm includes the following steps [1]:

**1) The user sets the number of clusters to which the original data set should be divided.**

2) Randomly assign records as cluster centers.

3) For each record is the closest cluster to it.

4) For each cluster there is a centroid of the cluster and the center of each cluster changes according to the centroid value.

5) Steps 3-5 are repeated **until there is a minimum of the sums of the quadratic deviation of cluster points from the center of the cluster or centroid** of the clusters will cease to change.

The criterion of **"closeness" in step 3 usually uses the Euclidean distance**

$d_{Euclidean}(x, y) = \sqrt{\sum_i (x_i - y_i)^2}$.

The cluster centroid in step 4 is as follows. Suppose that there are n data points ($a_1$,$b_1$,$c_1$), ($a_2$,$b_2$,$c_2$),…, ($a_n$,$b_n$,$c_n$). The centroid of these points is at the point ($\sum a_i/n$, $\sum b_i/n$, $\sum c_i/n$,). If the centroids of the clusters cease to change, the algorithm terminates. The completion of the algorithm can also be determined by minimizing the sums of quadratic deviation of cluster points (sum of squared errors-SSE) from the center of the cluster:

$SSE = \sum_{i=1}^{k} \sum_{p \in C_i} d(p, m_i)^2$,

Where $p \in C_i$is the point $C_i$of the cluster, $m_i$is the centroid of the cluster, $d(p, m_i)$is the deviation (error) of the points from the center of the cluster.

Also, to assess the rationality of clustering, the ratio of the variations between clusters and the sums of the quadratic deviation of cluster points is used:

$\frac{d(m_1,m_2)}{SSE}$.

This indicator is growing for a more rational version of clustering.

Example. Let there be given 5 points of a two-dimensional space:

| a | b | c | d | e |
|---|---|---|---|---|
| (1,3) | (3,3) | (4,3) | (5,3) | (1,2) |

The number of clusters is set to -2.

The centroid for cluster C1 will be (1,2), and the centroid for cluster C2 will be (5,3). This is usually determined randomly.

Table 14.1. Calculation of the distance of Euclid for example.

| | $m_1$ | $m_2$ | Cluster |
|---|---|---|---|
| A | 1 | 4 | C1 |
| B | 2.24 | 2 | C2 |
| C | 3.16 | 1 | C2 |
| D | 4.12 | 0 | C2 |
| E | 0 | 4.12 | C1 |

Cluster C1 includes (a, e), and cluster C2 includes (b, c, d).

Calculating the sums of the quadratic deviation of cluster points (sum of squared errors-SSE) from the center of the cluster for an example:

$SSE = \sum_{i=1}^{k} \sum_{p \in C_i} d(p, m_i)^2 = 1^2 + 2^2 + 1^2 + 0^2 + 0^2 = 6.$

Calculating the ratio:

$\frac{d(m_1,m_2)}{SSE} = \frac{4,12}{6} = 0,68.$

In the next iteration, new values of cluster centroids are determined.

For cluster C1, the centroid value is: $\left(\frac{1+1}{2}, \frac{3+2}{2}\right) = (1; 2.5)$

For cluster C2, the centroid value is: $\left(\frac{3+4+5}{3}, \frac{3+3+3}{3}\right) = (4; 3).$

Now, for the new values of cluster centroids, the distances from the center of the clusters for each point are calculated.

Table 14.2. The calculation of the Euclidean distance for an example is the second iteration

| | $m_1$ | $m_2$ | Claster |
|---|---|---|---|
| a | 0.5 | 3 | C1 |
| b | 2.06 | 2 | C2 |
| c | 3.04 | 1 | C2 |
| d | 4.03 | 1 | C2 |
| e | 0.5 | 3.16 | C1 |

Calculating the sums of the squared deviation of cluster points (sum of squared errors-SSE) from the cluster center for the second iteration:

$SSE = \sum_{i=1}^{k} \sum_{p \in C_i} d(p, m_i)^2 = 0.5^2 + 2^2 + 1^2 + 1^2 + 0.5^2 = 6.5$

Calculating the ratio:

$\frac{d(m_1, m_2)}{SSE} = \frac{3,04}{6.5} = 0,46.$

As SSE increases, and the evaluation of the rationality of clustering decreases, the previous version is considered the most rational.

Below is presented an example of solving the above problem using the Rapidminer software tool [2].

In the Fig. 14.1 the screenshot of source data for the a one-dimensional data set is presented.

In the Fig. 14.2 the screenshot of process structure of clustering task by k-means method is presented.

In the Fig. 14.3 the screenshot of clustering task results by k-means method is presented.

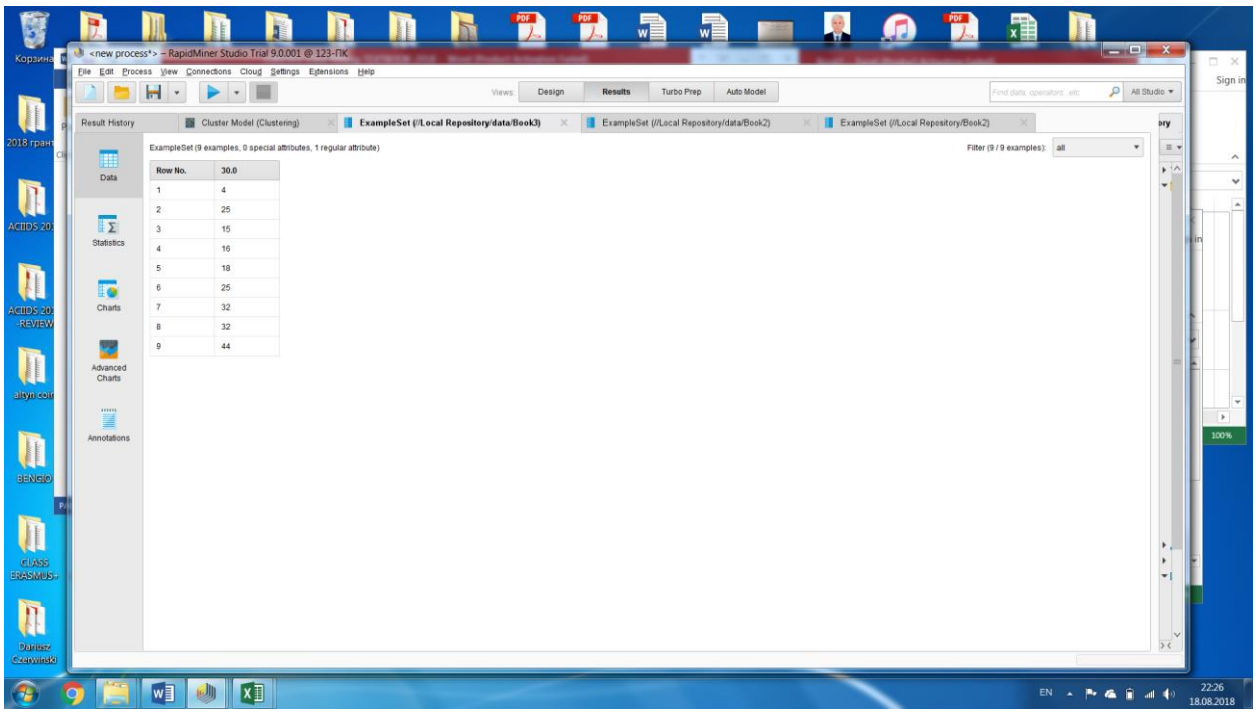



Fig. 14.1. Screenshot of source data for the a one-dimensional data set

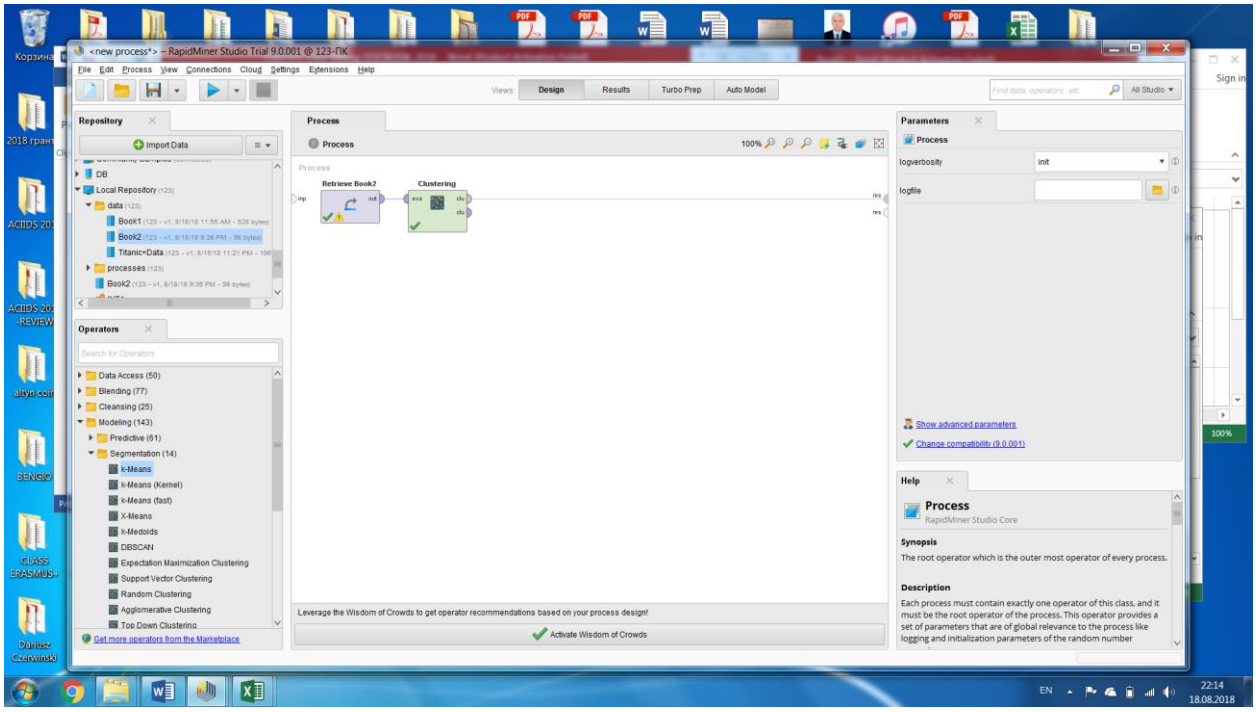



Fig. 14.2. Screenshot of process structure of clustering task by k-means method

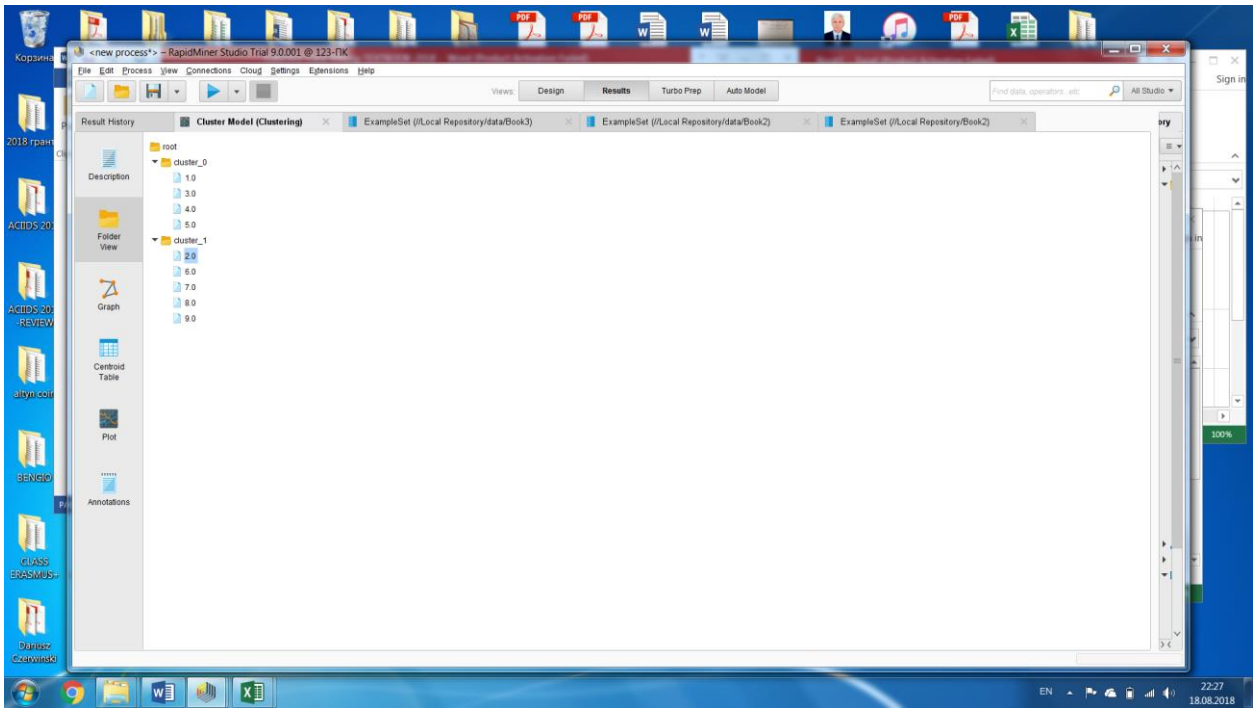



Fig. 14.3. Screenshot of clustering task results by k-means method

## 14.2 PRACTICAL LESSON 14 and IWS 14

**Subject.** Building and using clustering methods.

**Plan of the lesson.**

1) Study the scheme for constructing and using the clustering methods presented above.

2) The task of the IWS. Apply the above scheme of building and using clustering methods for your clustering task.

3) Analyze the results.

Literature.

1. Larose D.T. Discovering knowledge in data - an introduction to data mining. Wiley-Interscience, Hoboken, New Jersey, 2005

2. Rapidminer Studio, manual. https://docs.rapidminer.com/latest/studio

# TOPIC 15

# ASSOCIATION RULES

## 15.1. The lecture material

The purpose of the search for association rules is to find patterns between related events in databases. For the first time, the association rule mining problem was proposed to find typical shopping patterns made in supermarkets, so sometimes it is also called market basket analysis. Very often buyers buy more than one product. In most cases, there is a relationship between these goods. A market basket is a set of goods purchased by a buyer within the framework of a single transaction. Transactions are quite typical operations, for example, they can describe the results of visits to the store buyer. A transaction is a set of events that occurred simultaneously. Each such transaction is a set of goods purchased by the buyer for one visit. By registering all business transactions during the whole period of their activity, trading companies accumulate transaction collections, formed as transaction database.

Transaction database is a two-dimensional table, which consists of the transaction number (TID) and the list of purchases purchased by the buyer during this transaction. Each transaction corresponds to the purchase of an individual buyer. An example of such a table is shown in Table 15.1.

Table 15.1. Transaction database example

| TID | The list of purchases |
|---|---|
| 001 | Bread, milk, tea, sugar |
| 002 | Milk, cheese, sour cream |
| 003 | Milk, bread, cheese, tea |
| 004 | Sausage, cheese, bread, sugar |
| 005 | Bread, tea, milk, sour cream |
| 006 | Sugar, tea, butter |

Let D be the set of transactions presented in Table 15.1, where each transaction T in D represents a set of elements contained in a variety of possible goods, the set denoted as I (bread, milk, tea, sugar, cheese, sour cream, sausage, butter). Suppose that there are a certain set of elements A (for example, bread and milk) and another set of objects B (for example, tea). Then the association rule takes the form, if A, then B (i.e,), where *antecedent* A and *consequent* B are the proper subsets of I. For example, trivial rules such as: *"bread and milk*, and then *tea"*.

As a result of this type of analysis, one can establish a regularity of the following kind: "If a set of goods (or a set of elements) A has been encountered in the transaction, then one can do the conclusion that in the same transaction a set of elements B should appear. Establishment such regularities enable us to find very simple and understandable rules, called *associative*.

The method of associative rules uses two parameters that characterize this method: *support* and *confidence* of the rule. *Support* for a specific association rule is the proportion of transactions in D which contain both A and B. That is,

$$Support = \frac{Number\ of\ transactions\ containing\ both\ A\ and\ B}{Total\ number\ of\ transactions}.$$

*The confidence* of the association rule is a measure of the accuracy of the rule,

the percentage of transactions containing A that also contain B.

$$Confidence = \frac{Number\ of\ transactions\ containingboth\ A\ and\ B}{Number\ of\ transactions\ containing\ A}.$$

A set of elements is a set of elements contained in I, and k-itemset is a set of elements containing k elements. For example, {bread, milk} is a set of 2 items, and {sugar, tea, butter} is a 3-set of items, each of the set I. The frequency of the elements of a set is simply the number of transactions that contain a particular set of elements. A frequent set of items is a set of elements that arises at least a certain minimum number of times, with a set frequency $\geq\varphi$. For example, suppose that we set $\varphi = 4$. Then the sets that occur more than four times are called often. We denote the set of frequent sets of k-objects as $F_k$.

The mining of association rules from large databases is a two-step process [1]:
1. Find all frequent itemsets; that is, find all itemsets with frequency $\geq \phi$.
2. From the frequent itemsets, generate association rules satisfying the minimum support and confidence conditions.
The *a priori algorithm* takes advantage of the a priori property to shrink the search space. The *a priori property* states that if an itemset *Z* is not frequent, then adding another item *A* to the itemset *Z* will not make *Z* more frequent. That is, if *Z* is not frequent, will not be frequent. In fact, no *superset* of *Z* (itemset containing *Z*) will be frequent. This helpful property reduces significantly the search space for the a priori algorithm.

Below is an example of mining associative rules using the Rapidminer program for the data presented in Table 15.1 (Fig. 15.1-15.6) [2].

In the Fig. 15.1 is presented screenshot of source data with items in separate columns. In the Fig. 15.2-15.4 is presented screenshot of process structure with define of characteristics of process operators.

In Rapidminer there are possibility to write a list of items in one column. For above example we prepare source data as list of items in one column, rename items by short name (Table 15.2).

Table 15.2. Transaction database example with a list of items in one column

| TID | The list of purchases | The list of purchases in one column |
|---|---|---|
| 001 | Bread, milk, tea, sugar | a\|b\|c\|g |
| 002 | Milk, cheese, sour cream | b\|d\|f |
| 003 | Milk, bread, cheese, tea | b\|a\|d\|c |
| 004 | Sausage, cheese, bread, sugar | e\|d\|a\|g |
| 005 | Bread, tea, milk, sour cream | a\|c\|b\|f |
| 006 | Sugar, tea, butter | g\|c\|h |

In the Fig. 15.5 is presented screenshot of source data with list items in one column.

In the Fig. 15.6 is presented screenshot of process structure with define of characteristics of process operators FP- growth with list items in one column.

In the Fig. 15.7 is presented screenshot of frequent items of association rules with items on separate columns.

In the Fig. 15.8 is presented screenshot of association rules with items on separate columns.

In the Fig. 15.9 is presented screenshot of frequent items of association rules with list items in one column.

In the Fig. 15.10 is presented screenshot of association rules with list items in one column.

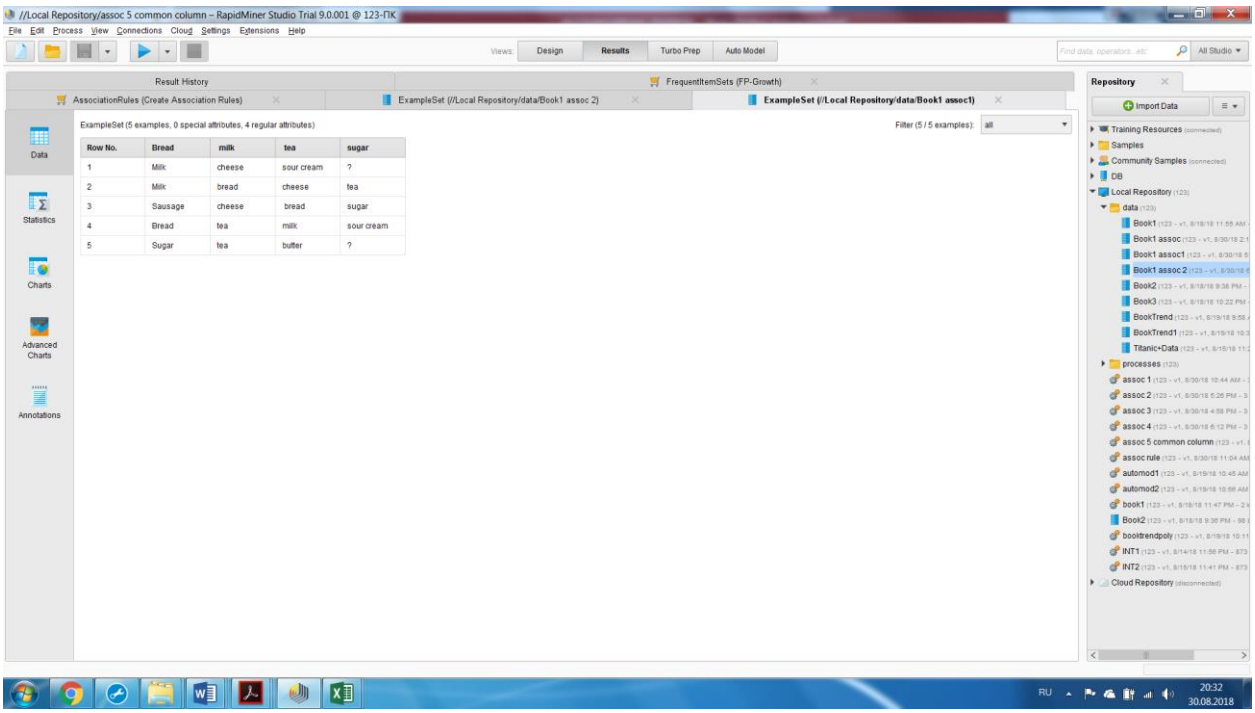



Fig. 15.1. Screenshot of source data with items in separate columns

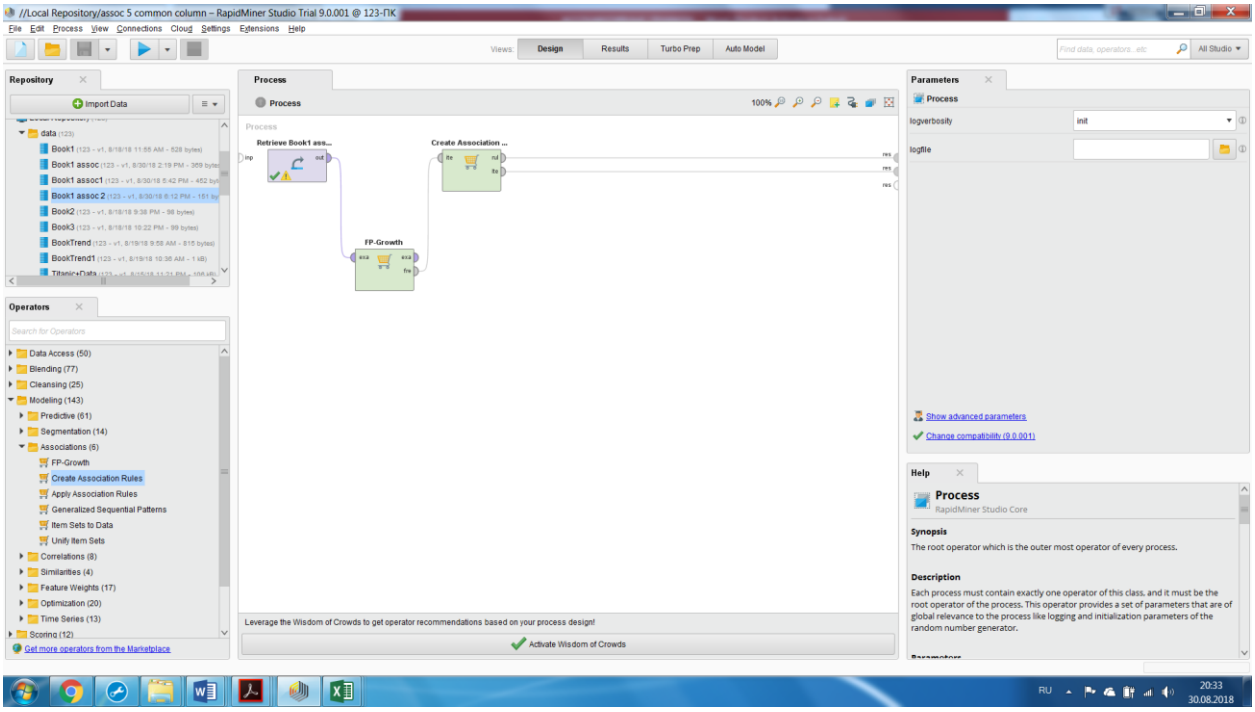



Fig. 15.2. Screenshot of process structure of association rules defining

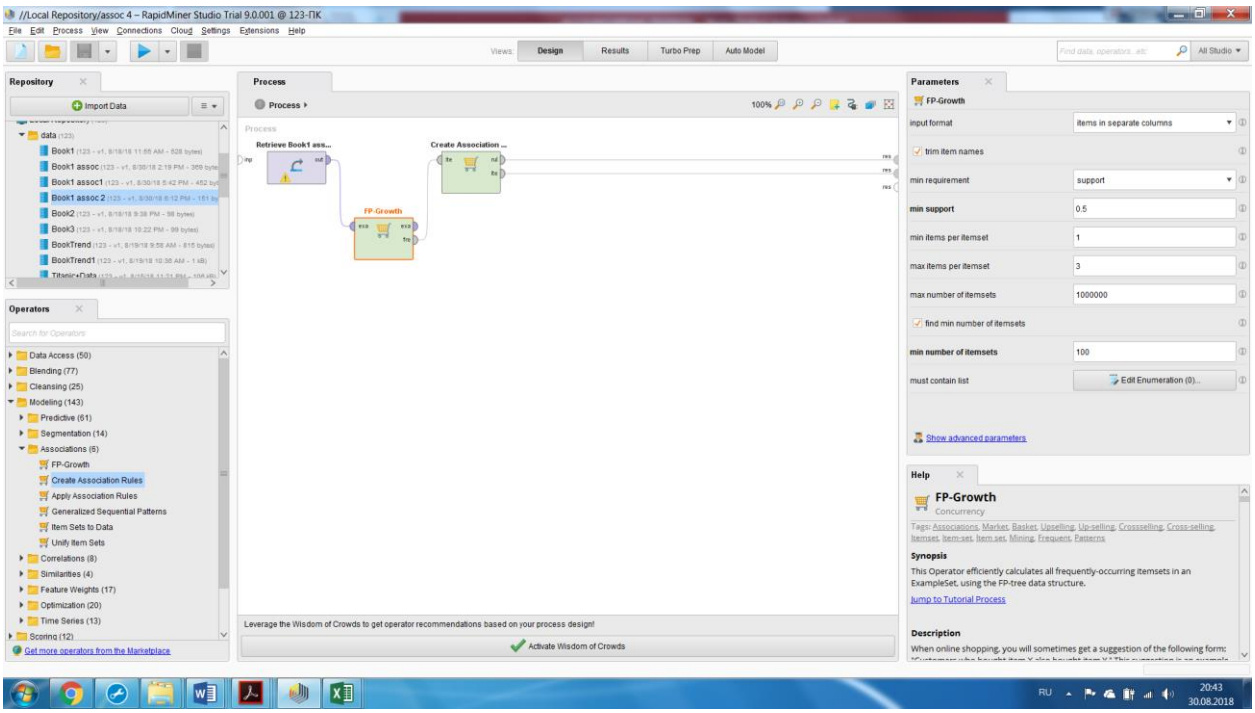



Fig. 15.3. Screenshot of process structure with define of characteristics of process operators FP- growth with items on separate columns

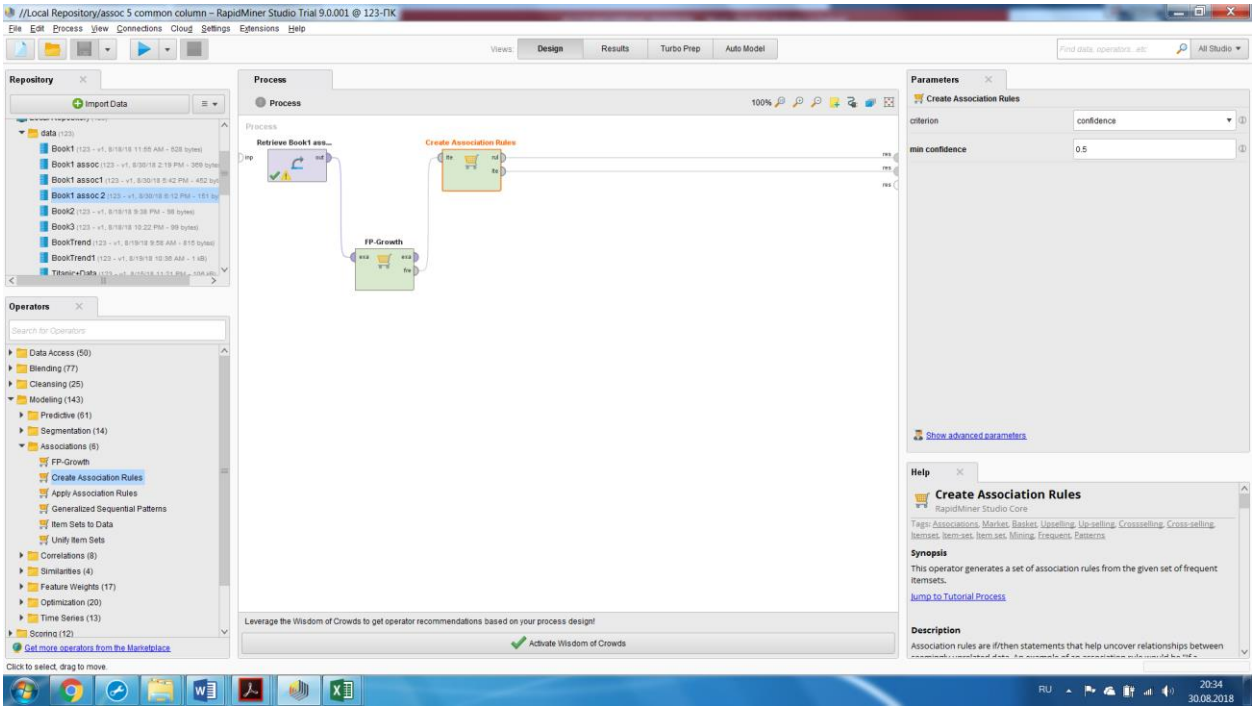



Fig. 15.4. Screenshot of process structure with define of characteristics of process operator “Create associative rules” with items on separate columns

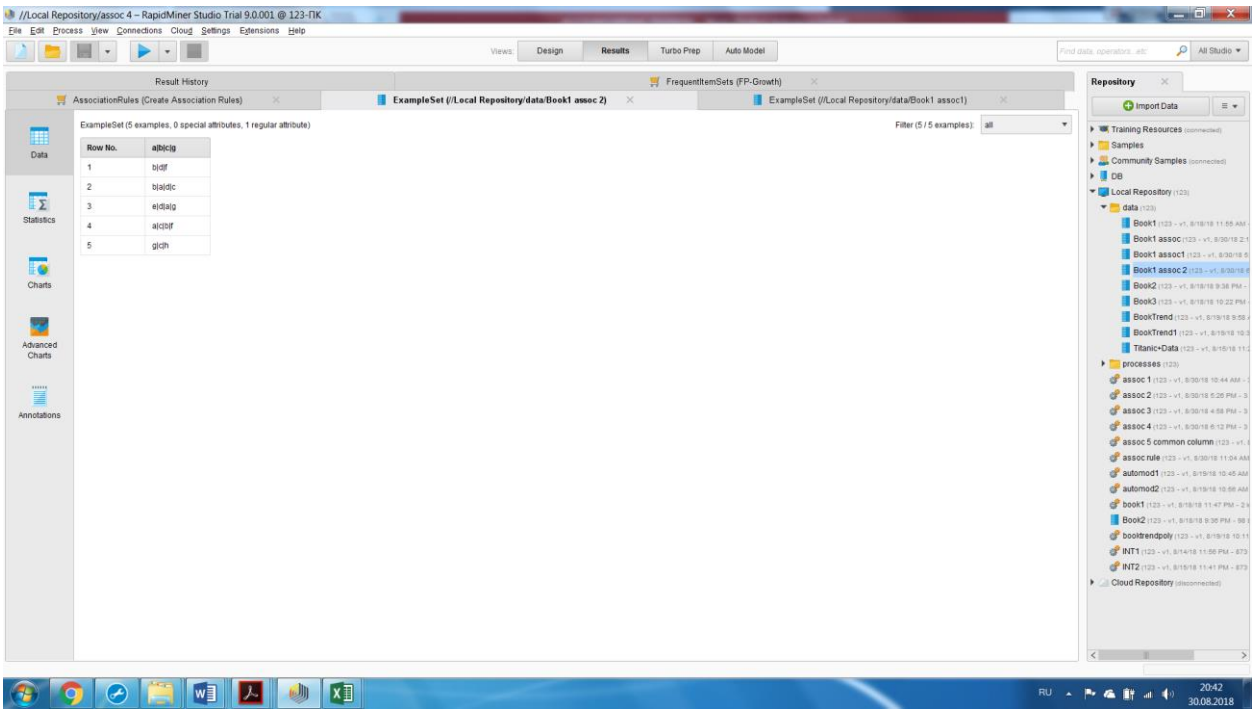



Fig. 15.5. Screenshot of source data with list items in one column

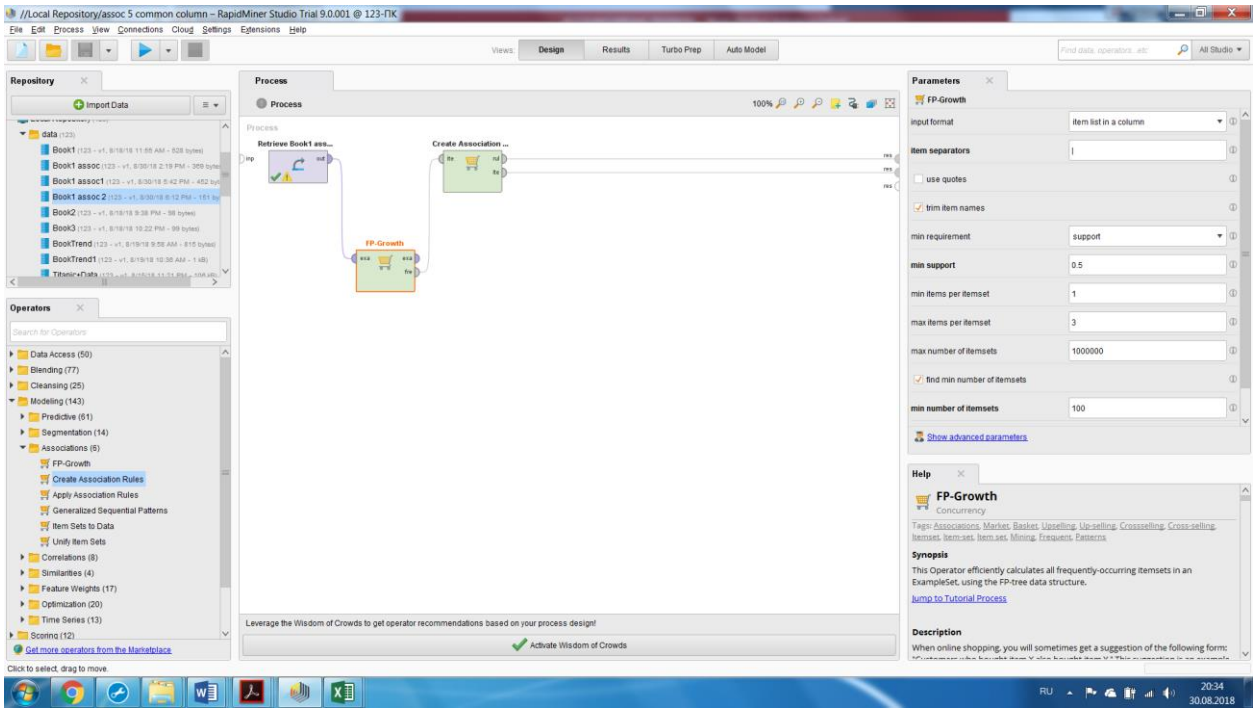



Fig. 15.6. Screenshot of process structure with define of characteristics of process operators FP- growth with list items in one column

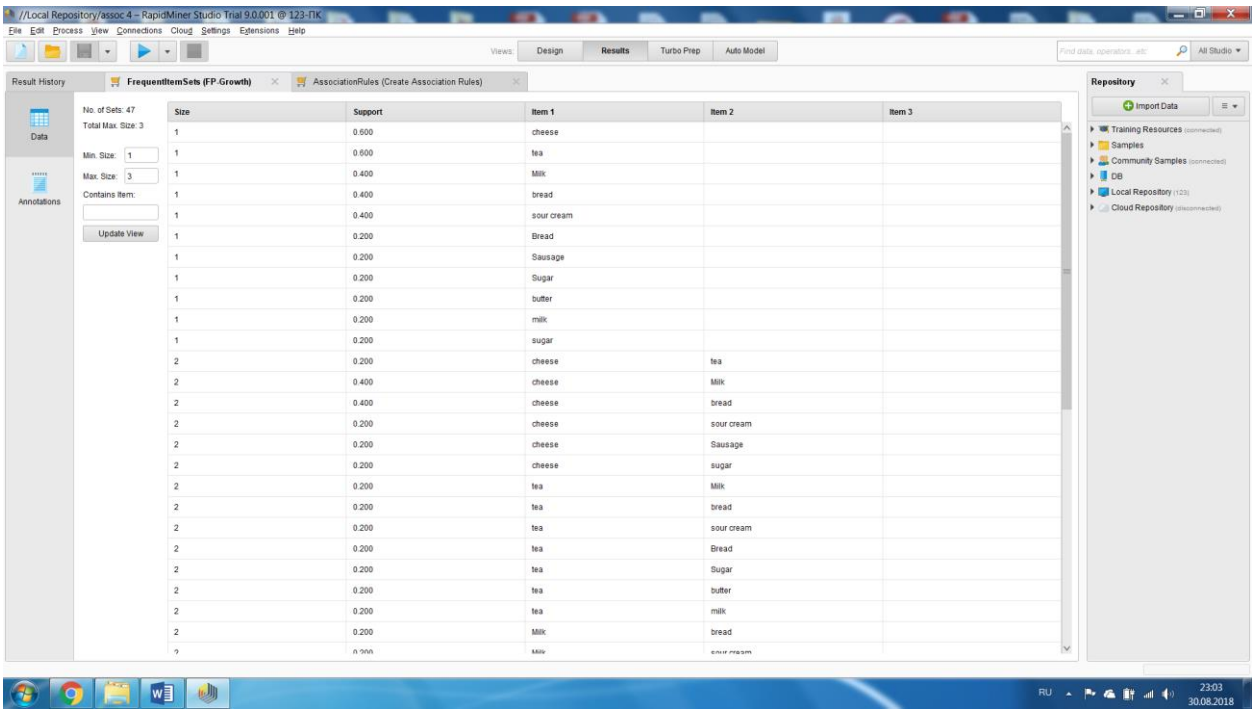



Fig. 15.7. Screenshot of frequent items of association rules with items on separate columns

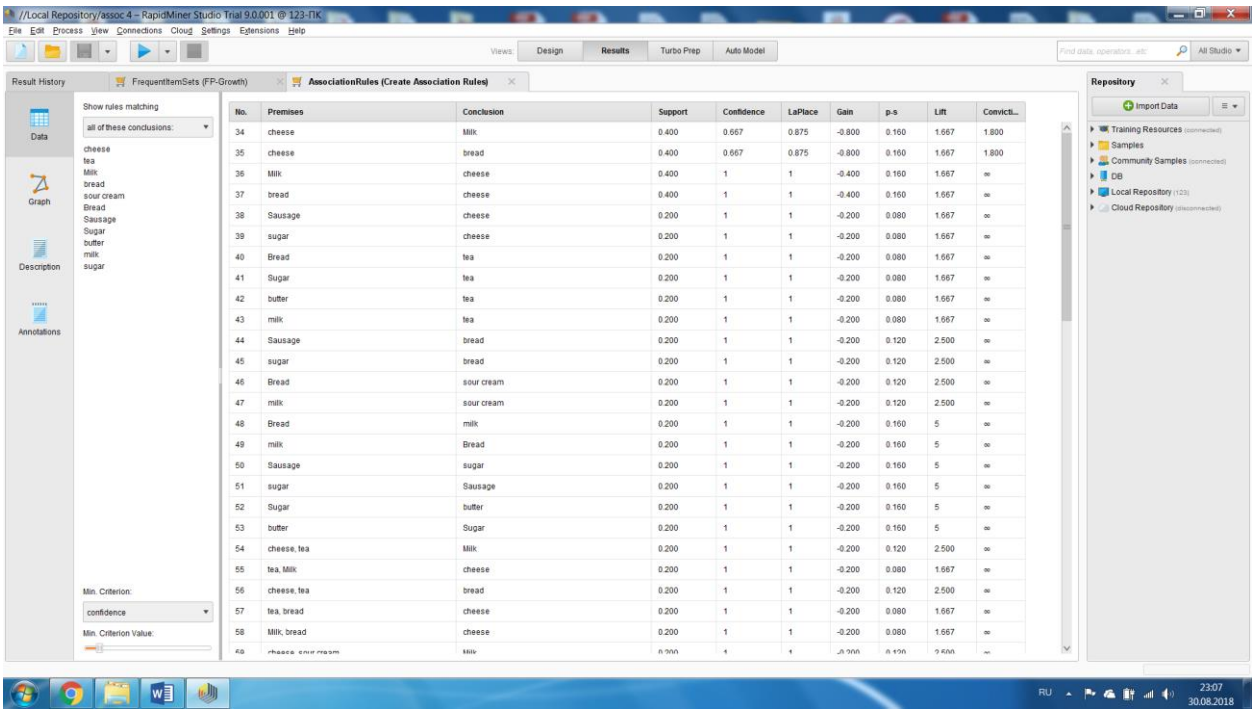



Fig. 15.8. Screenshot of association rules with items on separate columns

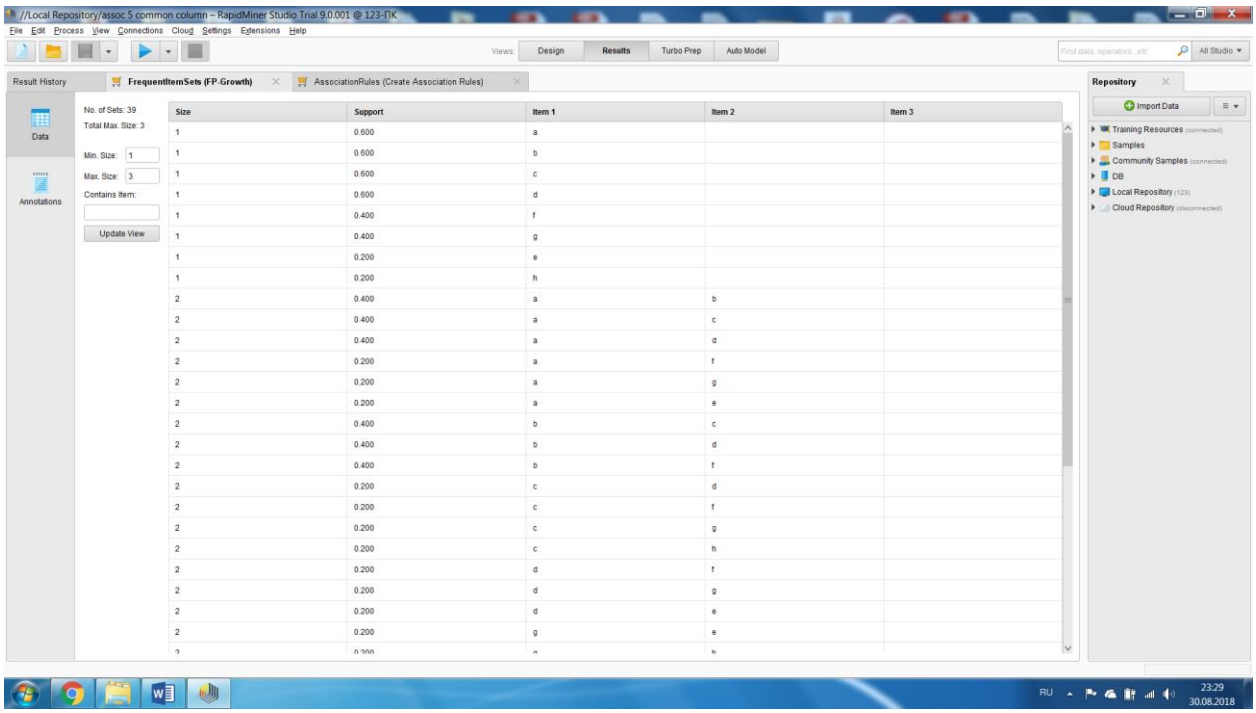



Fig. 15.9. Screenshot of frequent items of association rules with list items in one column

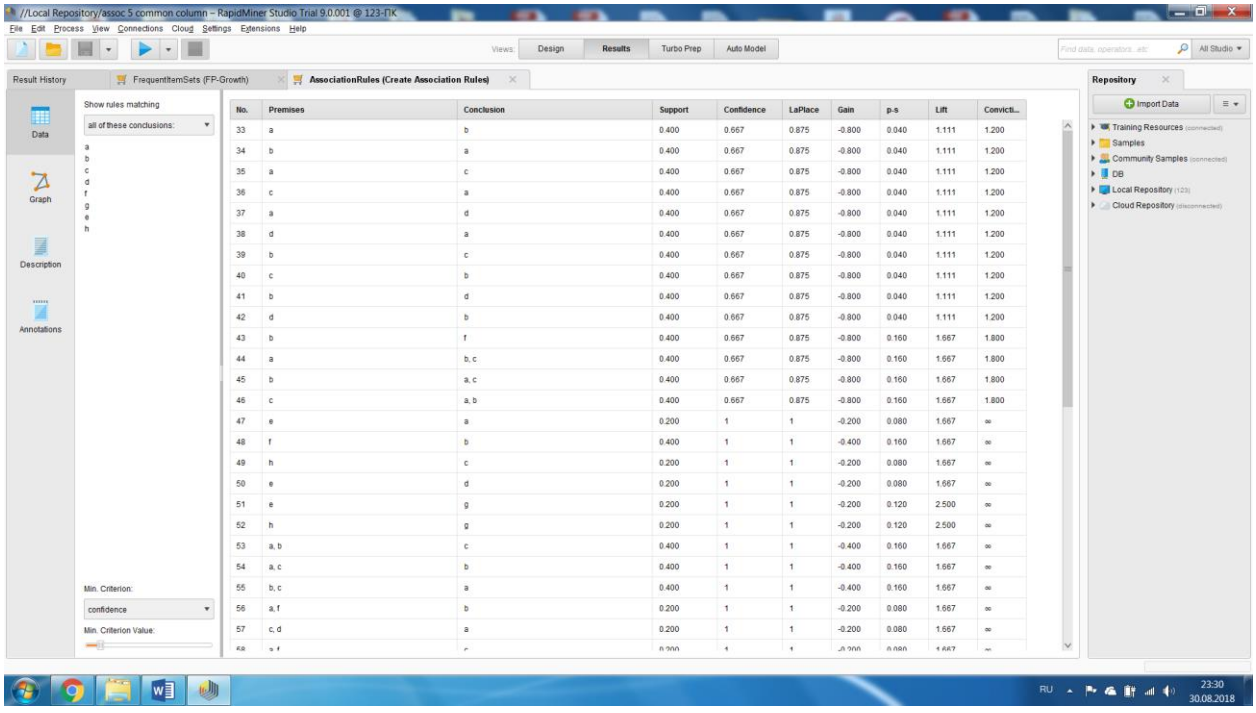



Fig. 15.10. Screenshot of association rules with list items in one column

## 15.2 PRACTICAL LESSON 15 and IWS 15

**Subject.** Building and using association rules methods.

**Plan of the lesson.**

1) Study the scheme for constructing and using the association rules methods presented above.

2) The task of the IWS. Apply the above scheme of building and using association rules methods for your clustering task.

3) Analyze the results.

Literature.

1. Larose D.T. Discovering knowledge in data - an introduction to data mining. Wiley-Interscience, Hoboken, New Jersey, 2005

2. Rapidminer Studio, manual. https://docs.rapidminer.com/latest/studio