Resonance Vibrations and Stability of Drill Strings Complicated by Frictional Forces

Lelya Khajiyeva, Aliya Umbetkulova
Department of Mathematical and Computer Modeling
Al-Farabi Kazakh National University
Almaty, Kazakhstan
e-mail: khadle@mail.ru, aliya1988kz@mail.ru

Abstract—In this work the nonlinear analysis of a drill string movement taking into account a complicating factor was studied. As a complicating factor is taken a frictional force arising between the string and the borehole. The resonance modes and its stability are investigated here. For that reason the amplitude-frequency characteristics of the first and the third harmonic resonance are constructed. The consideration of friction forces leads to a decrease in the amplitudes of transverse vibrations and to a weakening of resonance oscillations with a narrowing of the zone of dangerous frequencies. Qualitative and quantitative agreement of the stability zones of the resonance on the basic frequency with bifurcation zones on the resonant amplitude-frequency characteristics was also established. Obtained here responses determinate the instability zone of the basic resonance allows to avoid non-working resonance frequencies at the early stage of drilling wells. Modeling of resonance regimes of the drill string dynamics along with the analysis of its stability has a great importance for development of drilling equipment and improving its dynamic characteristics.

Keywords – amplitude-frequency characteristic; finite deformation; frictional force; nonlinearity; resonance vibrations; stability zones;.

I. Introduction

A drill string is a part of the rotary drill rig used for mining oil and gas wells. During drilling, there are strong vibrations that damage drill pipes and strings and drilling equipment. Drill string vibration is one of the major causes for a deteriorated drilling performance, and if left untreated may result in a complete failure of the drilling process. It is well known that drill string vibrations may lead to fatigue failures and abrasive wear of tubulars, damage to the drill bit and the borehole wall. As a consequence, oil well drilling becomes inefficient and costly. On the other hand, measurements of these vibrations may provide valuable information about the drilling assembly and formation characteristics. Therefore, vibrations must be fully understood and their effects should be minimized in any approach to drilling optimization. Most studies of the drill string dynamics are directed to modeling and analysis of columns vibration, which leads to a loss of movement stability of drilling equipment and violation of their strength properties. Loss of stability of drill string rectilinear form leads to a curvature of wells, which is cause of their unfitness. Therefore, research on modeling of drill strings dynamics considering their deformation properties, taking into account the complicating factors and the influence of external disturbing forces have a scientific and practical interest. As a complicating factor is taken an inertia and damping forces, which are caused by the frictional forces arising between the string and the borehole, between the drilling tool and the borehole at longitudinal feed during drilling. Previously, the influence of external friction on the dynamic characteristics of the drill strings oscillations was studied [1]. It is established that friction qualitatively changes the shape of the transverse vibrations of the pipes. It was also found that the dynamic characteristics of the lateral oscillations of the column depend on the nature of the axial load. In [2], [3] linear problems of bending vibrations of the bottom structure of a rotating drill string under the action of friction forces are considered. The analysis is performed and the effect of the resistance forces on the oscillatory process is revealed.

II. Mathematical Model

Thus, during operation, the drill string, depending on its flexibility, is bent on several half-waves. At the same time, due to the elastic deformation force, it is pressed against the walls of the borehole, creating an additional frictional force against the movement of the cutting tool supply to the bottom. That is, there takes place an additional longitudinal force ΔN in the rod. This force is proportional to the speed of movement of the drill string cross-section in planes xOz and yOz:

$$\Delta N_u = -\varepsilon \frac{\partial e_u(z,t)}{\partial t}, \qquad \Delta N_v = -\varepsilon \frac{\partial e_v(z,t)}{\partial t}. \tag{1}$$

Here ε is coefficient of friction, $\frac{\partial e_u(z,t)}{\partial t}$, $\frac{\partial e_v(z,t)}{\partial t}$ are feed speeds of the drill in planes xOz and yOz respectively.

The movements e_u, e_v are found as the difference between the initial length of the drill string l and the projections of their curved axes in the corresponding planes, and can be written as follows:

$$e_u(z,t) = \frac{1}{2}\int_0^l \left(\frac{\partial u}{\partial z}\right)^2 dz + \frac{3}{8}\int_0^l \left(\frac{\partial u}{\partial z}\right)^4 dz + ...,$$

$$e_v(z,t) = \frac{1}{2}\int_0^l \left(\frac{\partial v}{\partial z}\right)^2 dz + \frac{3}{8}\int_0^l \left(\frac{\partial v}{\partial x}\right)^4 dz + ... \tag{2}$$

Giving an increment ΔN of the external compressive longitudinal force $N(z,t)$, the following nonlinear dynamic model (3) considering the friction force was obtained. It describes the interaction of flexural vibrations of a drill string, which rotates with angular velocity ω, compressed by the longitudinal force $N(z,t)$ and twisted by the moment $M(t)$.

$$\rho A\frac{\partial^2 u}{\partial t^2} - \rho I\frac{\partial^4 u}{\partial z^2 \partial t^2} + EI\frac{\partial^4 u}{\partial z^4} + \frac{\partial}{\partial z}\left[\left(N(z,t) - \varepsilon\frac{\partial e_u(z,t)}{\partial t}\right)\frac{\partial(u+u_0)}{\partial z}\right] - M(t)\frac{\partial^3 v}{\partial z^3} -$$
$$-\frac{EA}{(1-\nu)}\frac{\partial}{\partial z}\left[\left(\frac{\partial u}{\partial z}\right)^3\right] - \frac{EA(5-6\nu)}{2(1-\nu)}\frac{\partial}{\partial z}\left[\frac{\partial u}{\partial z}\left(\frac{\partial v}{\partial z}\right)^2\right] - \rho A\omega^2 u + 2\rho A\omega\frac{\partial v}{\partial t} = 0,$$
$$\rho A\frac{\partial^2 v}{\partial t^2} - \rho I\frac{\partial^4 v}{\partial z^2 \partial t^2} + EI\frac{\partial^4 v}{\partial z^4} + \frac{\partial}{\partial z}\left[\left(N(z,t) - \varepsilon\frac{\partial e_v(z,t)}{\partial t}\right)\frac{\partial(v+v_0)}{\partial z}\right] + M(t)\frac{\partial^3 u}{\partial z^3} -$$
$$-\frac{EA}{(1-\nu)}\frac{\partial}{\partial z}\left[\left(\frac{\partial v}{\partial z}\right)^3\right] - \frac{EA(5-6\nu)}{2(1-\nu)}\frac{\partial}{\partial z}\left[\frac{\partial v}{\partial z}\left(\frac{\partial u}{\partial z}\right)^2\right] - \rho A\omega^2 v - 2\rho A\omega\frac{\partial u}{\partial t} = 0, \tag{3}$$

where ρ is density, A is cross-sectional area, $u(z,t)$ and $v(z,t)$ are displacements of the flexural center of the cross-section along the planes *Oxz* and *Oyz* owing to bending; E is Young's modulus, I is axial moment of inertia, ν is Poisson's ratio, u_0, v_0 are initial curvatures of the drill string.

The boundary conditions for the drill string with hinged ends are set as equality to zero of deformations and bending moments at the ends:

$$u(z,t)\big|_{\substack{z=0\\z=l}} = v(z,t)\big|_{\substack{z=0\\z=l}} = 0,$$

$$EI\frac{\partial^2 u(z,t)}{\partial z^2}\bigg|_{\substack{z=0\\z=l}} = EI\frac{\partial^2 v(z,t)}{\partial z^2}\bigg|_{\substack{z=0\\z=l}} = 0. \tag{4}$$

Initial conditions are presented in the form:

$$u(z,t)\big|_{t=0} = v(z,t)\big|_{t=0} = 0,$$

$$\frac{\partial u(z,t)}{\partial t}\bigg|_{t=0} = C_1,\ \frac{\partial v(z,t)}{\partial t}\bigg|_{t=0} = C_2, \tag{5}$$

where C_1, C_2 are constant values.

III. Numerical Analysis

Because of the complexity of direct integration of the nonlinear model (3)-(5), it is reduced to a form convenient for numerical integration. For this, the known method of separation of variables - the Bubnov-Galerkin method is used, where the shape of the bend of the drill string axis is given by the spectrum of harmonic forms. In [4] has been shown that this method allows to successfully analyze the behavior of drill strings used for oil production in the vertical and deviated wells. Importance of considering inertial forces to investigate stability of the drill string when drilling vertical holes is also shown there. The convergence of the Bubnov-Galerkin method for the system is proved in the papers [5].

In contrast to [6], we consider the multimode approximation of the solution here. The initial curvatures of the drill string have a smooth form [7], it can be presented in the form of a periodic trigonometric function. Hence, the components of transverse displacements $u(z,t)$, $v(z,t)$ and initial curvatures u_0, v_0 are presented in the form of series:

$$u(z,t) = \sum_{i=1}^{n}\overline{u_i}(t)\sin\left(\frac{i\pi z}{l}\right),$$
$$v(z,t) = \sum_{i=1}^{n}\overline{v_i}(t)\sin\left(\frac{i\pi z}{l}\right),$$
$$u_0(z) = \overline{u_0}\sin\left(\frac{\pi z}{l}\right),$$
$$v_0(z) = \overline{v_0}\sin\left(\frac{\pi z}{l}\right), \tag{6}$$

where basis function $\sin\left(\frac{\pi z}{l}\right)$ are chosen so that they satisfy the boundary conditions (4). Applying this method to a first approximation $n=1$, the dynamic model taking into account the friction force is reduced to the following form:

$$
\begin{aligned}
& a_1\overline{u_1}''(t)+a_2\overline{v_1}'(t)+c\overline{u_1}(t)^3+b\overline{u_1}(t)+k\overline{u_1}(t)\cos(\overline{\omega}t)+\\
& +d\overline{v_1}(t)^2\overline{u_1}(t)+\left[f_{31}\overline{u_1}(t)\overline{u_1}'(t)+f_{32}\left(\overline{u_1}(t)\right)^3\overline{u_1}'(t)\right]\cdot\\
& \cdot\left(\overline{u_0}+\overline{u_1}(t)\right)=h_1+h_2\cos(\overline{\omega}t),\\
& a_1\overline{v_1}''(t)-a_2\overline{u_1}'(t)+c\overline{v_1}(t)^3+b\overline{v_1}(t)+k\overline{u_1}(t)\cos(\overline{\omega}t)+\\
& +d\overline{u_1}(t)^2\overline{v_1}(t)+\left[f_{31}\overline{v_1}(t)\overline{v_1}'(t)+f_{32}\left(\overline{v_1}(t)\right)^3\overline{v_1}'(t)\right]\cdot\\
& \cdot\left(\overline{v_0}+\overline{v_1}(t)\right)=h_1+h_2\cos(\overline{\omega}t),
\end{aligned}
\tag{7}
$$

where $a_1=\rho A+\rho I\frac{\pi^2}{l^2},\ a_2=2\rho A\omega,$

$$b=EI\frac{\pi^4}{l^4}+\rho A\omega^2-\frac{\pi^2}{l^2}N_0,\quad c=\frac{6EA\pi^4}{8(1-\nu)l^4},$$

$$k=-\frac{\pi^2}{l^2}N_t,\quad d=\frac{3EA(5-6\nu)}{8(1-\nu)}\frac{\pi^4}{l^4},$$

$$h_1=\frac{\pi^2}{l^2}N_0\overline{u_0},\quad h_2=\frac{\pi^2}{l^2}N_t\overline{u_0},$$

$$f_{31}=\frac{\pi^4}{4l^3}\varepsilon,\quad f_{32}=\frac{9\pi^6}{16l^5}\varepsilon.$$

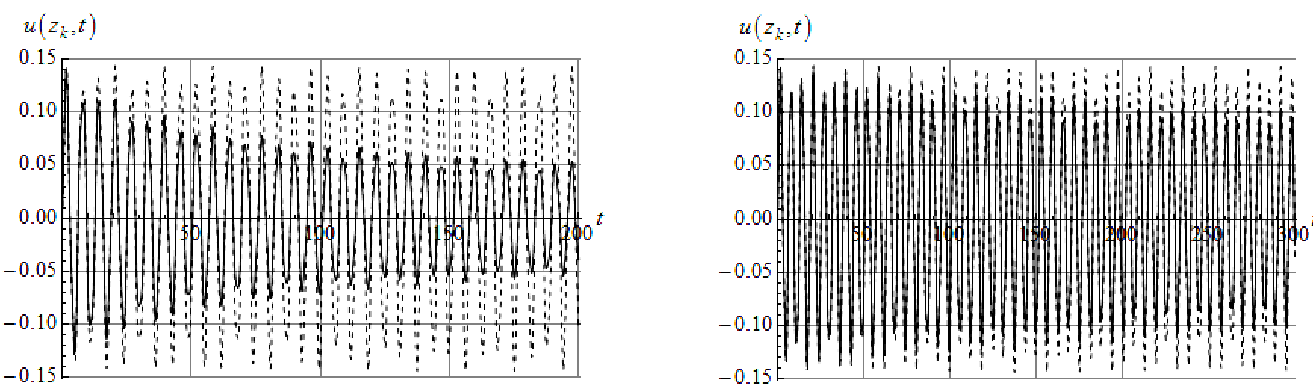
However, it was previously established that taking into account only the first form of approximation is insufficient for a complete description of the oscillatory process, so numerical analysis is carried out for the third approximation of the solution of the oscillation model, $n=3$ in (6).

Numerical calculations were carried out at the following values of parameters of the duralumin drill string: Young's modulus $E=0.7\times10^5\,\text{MPa}$, material density $\rho=2700\ \text{kg/m}^3$, Poisson's ratio $\nu=0.34$, outer diameter of the string $D=0.2\text{m}$, inner diameter $d=0.12\text{m}$, length of string $l=100\text{m}$, angular speed $\omega=1\text{rad/s}$, longitudinal compressive load $N(t)=5.23\times10^3\,\text{N}$ and twisting moment $M(t)=10^4\cos(\overline{\omega}t)\,\text{N}\times\text{m}$. The data for the value ε is taken from [8]-[10].

Fig. 1-2 show the influence of frictional force on the oscillating process of a duralumin drill string. A comparison of models of drill strings vibrations with and without consideration of frictional forces is made (Fig. 1) and there was obtained the influence of frictional forces on oscillation process. It is established that taking friction into account leads to a decrease in the amplitude of vibrations of the drill strings, and over time the difference between the amplitudes of the vibrations increases. However, for smaller coefficients of friction this difference is small. An analysis of the influence of the friction coefficient on the oscillation process of drill strings is also carried out and it is established that for a larger value of the friction coefficient the amplitude of the oscillations decreases (Fig. 2).

Fig. 3 analyzes the effect of an external compressive longitudinal force on the amplitudes of its vibrations, and it is established that the influence of the friction force is not so great as the compressive load increases. This is due to the fact that as the external load increases, the drilling speed of the well also increases. But it is important that the load should not go beyond the critical value.

Figures 4 show the drill string spatial bending forms, taking into account the nonlinear frictional force at various time moments. The friction coefficient was taken as $\varepsilon=0.3$, other parameters remained unchanged. The shape of the bending depends on the chosen number of modes according to Bubnov-Galerkin's expansion (6). Three modes of bending can be observed and are presented below.



(a) $\varepsilon=0.3$ (b) $\varepsilon=0.03$

Fig. 1. Influence of the friction force on the vibrations of the drill string
(– – – – without friction, ——— with friction)

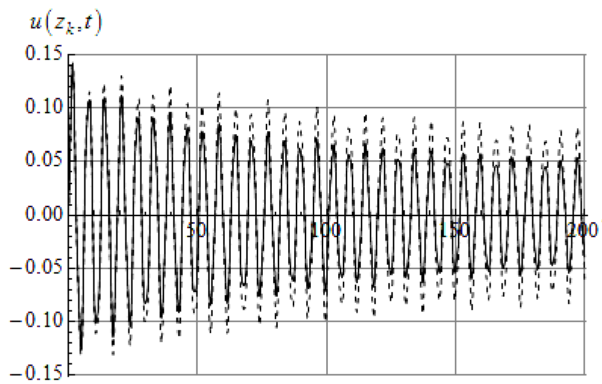


Fig. 2. Influence of the value of the friction coefficient on the vibrations of the drill string
(– – – – $\varepsilon = 0.3$, —— $\varepsilon = 0.6$)

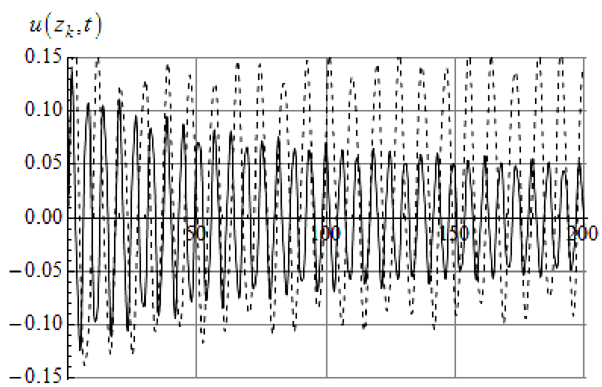
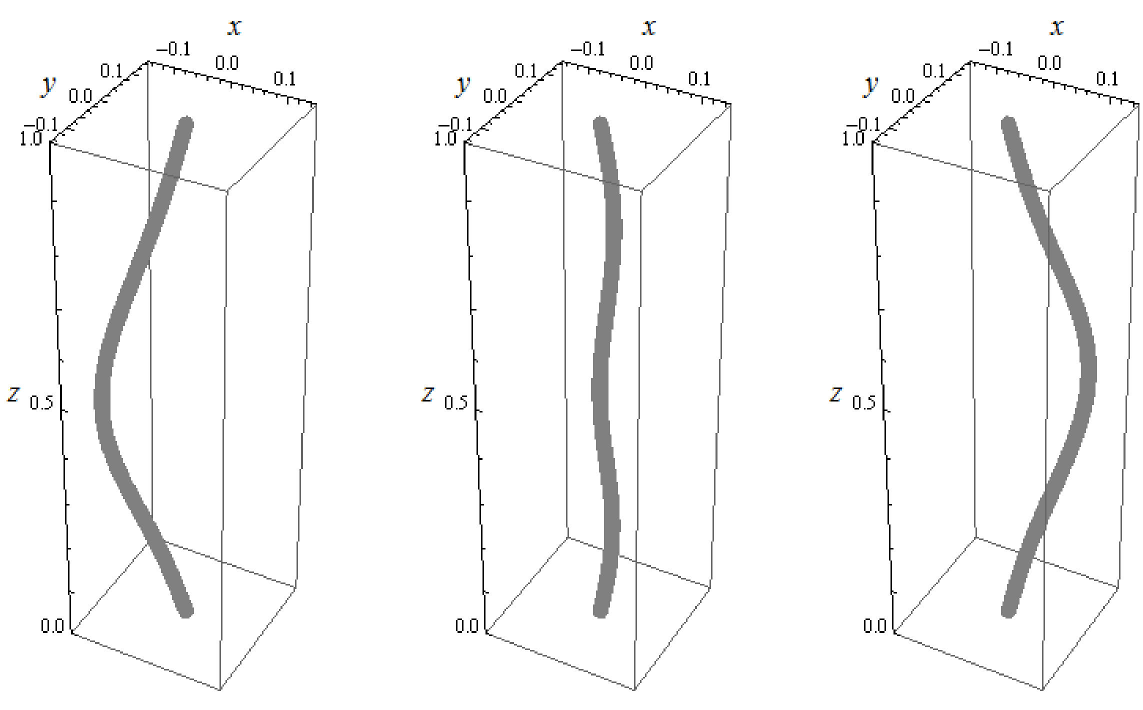


Fig. 3. Influence of the compressive load on the vibrations of the drill string considering the friction force
(– – – – $N_t = 5.23\times10^4$ N , —— $N_t = 5.23\times10^3$ N)



a) $t_m = 8.58s$ b) $t_m = 16.86s$ c) $t_m = 22s$

Fig. 4. Spatial bending forms of the drill string considering the friction force

IV. Resonance

Deformation finiteness in (3) leads to a nonlinearity of dynamic model and can make essential impact on character of movement of drill string such as the imposition of vibration process on the nominal condition of the drill string, change of amplitude-frequency response, and, as a consequence, the loss of the dynamic stability of the rod, as well as the appearance of dangerous resonant modes of vibration. Resonances are accompanied by large vibration amplitudes, which significantly weakens the strength characteristics of the considering system. Therefore, it becomes necessary to study resonant modes of drill string vibrations at multiple frequencies in order to exclude them from the operating frequency area.

In contrast to [11], [12] in the present work resonant vibrations of drill string considering the friction forces are investigated.

Examination of resonant regimes of movement of the drill string and its stability can be reduced to analysis of the amplitude-frequency characteristics of their transverse vibrations. In nonlinear system along with vibrations, which frequency coincides with frequency of the external force, higher and subharmonic oscillations can arise [13]. In the resonance case difference of phases between natural vibrations and external effects may have a great impact on the magnitude of amplitudes and the frequency of vibrations.

Introducing a dimensionless time parameter τ, the nonlinear model (3) reduces to the form:

$$\begin{aligned}&\overline{u}_1''(\tau)+\overline{A}\overline{v}_1'(\tau)+A\overline{u}_1(\tau)+K\overline{u}_1(\tau)\cos(\Omega\tau)+B\overline{u}_1(\tau)^3+\\&+D\overline{v}_1(\tau)^2\overline{u}_1(\tau)+\left[F_1\overline{u}_1(\tau)\overline{u}_1'(\tau)+F_2\left(\overline{u}_1(\tau)\right)^3\overline{u}_1'(\tau)\right]\cdot\\&\cdot\left(\overline{u}_0+\overline{u}_1(\tau)\right)=C_1+C_2\cos(\Omega\tau),\\&\overline{v}_1''(\tau)-\overline{A}\overline{u}_1'(\tau)+A\overline{v}_1(\tau)+K\overline{v}_1(\tau)\cos(\Omega\tau)+B\overline{v}_1(\tau)^3+\\&+D\overline{u}_1(\tau)^2\overline{v}_1(\tau)+\left[F_1\overline{v}_1(\tau)\overline{v}_1'(\tau)+F_2\left(\overline{v}_1(\tau)\right)^3\overline{v}_1'(\tau)\right]\cdot\\&\cdot\left(\overline{v}_0+\overline{v}_1(\tau)\right)=C_1+C_2\cos(\Omega\tau),\end{aligned}\tag{8}$$

where $\overline{A}=\dfrac{a_2}{a_1\omega_0}$, $A=\dfrac{b}{a_1\omega_0{}^2}$,

$$B=\frac{c}{a_1\omega_0{}^2},\ D=\frac{d}{a_1\omega_0{}^2},$$

$$F_1=\frac{f_{31}}{a_1\omega_0},\ F_2=\frac{f_{32}}{a_1\omega_0},$$

$$K=\frac{k}{a_1\omega_0{}^2},\ C_1=\frac{h_1}{a_1\omega_0{}^2},$$

$$C_2=\frac{h_2}{a_1\omega_0{}^2},\ \Omega=\frac{\overline{\omega}}{\omega_0},$$

ω_0 is the frequency of the drill string natural vibrations.

The general method to solve such system is expansion of the functions $\overline{u_1}(\tau), \overline{v_1}(\tau)$ into the Fourier series with undefined coefficients [14]:

$$\begin{aligned}\overline{u_1}(\tau)&=r_{u_0}+r_{u_1}\cos(\Omega\tau-\varphi_{11})+r_{u_2}\cos(2\Omega\tau-\varphi_{12})+...,\\\overline{v_1}(\tau)&=r_{v_0}+r_{v_1}\cos(\Omega\tau-\varphi_{21})+r_{v_2}\cos(2\Omega\tau-\varphi_{22})+...,\end{aligned}\tag{9}$$

which can be determined by the method of harmonic balance when taking into account the finite and usually a small number of members.

Considering the resonance on the basic frequency a solution of (9) can be approximated by a simple harmonic with frequency Ω:

$$\begin{aligned}\overline{u_1}(\tau)&=r_{u_1}\cos(\Omega\tau-\varphi_1),\\\overline{v_1}(\tau)&=r_{v_1}\cos(\Omega\tau-\varphi_2).\end{aligned}\tag{10}$$

On substituting (10) into (8) and applying the method of harmonic balance [15], the following system of equations defining the dependence between the amplitudes r_{u_1}, r_{v_1} and the frequency Ω:

$$\begin{aligned}&\left[Ar_{u_1}-r_{u_1}\Omega^2+\frac{3}{4}\left(Br_{u_1}{}^3+Dr_{u_1}r_{v_1}{}^2\right)\right]^2+\\&+\left[\overline{A}r_{v_1}\Omega+F_{31}r_{u_1}{}^3\Omega\right]^2=C_2{}^2,\\&\left[Ar_{v_1}-r_{v_1}\Omega^2+\frac{3}{4}\left(Br_{v_1}{}^3+Dr_{v_1}r_{u_1}{}^2\right)\right]^2+\\&+\left[\overline{A}r_{u_1}\Omega+F_{32}r_{v_1}{}^3\Omega\right]^2=C_2{}^2,\end{aligned}\tag{11}$$

where $F_{31}=-\dfrac{1}{4}\dfrac{f_{31}}{a_1\omega_0}-\dfrac{1}{8}\dfrac{f_{32}}{a_1\omega_0}r_{u_1}{}^2$,

$$F_{32}=-\frac{1}{4}\frac{f_{31}}{a_1\omega_0}-\frac{1}{8}\frac{f_{32}}{a_1\omega_0}r_{v_1}{}^2.$$

The amplitude-frequency characteristics (11) depend on geometrical and physical parameters of the dynamic system. It allows examining the effects of these parameters on the resonance regimes of the drill string external vibrations to separate the resonant frequencies from drilling operating frequencies or to control them.

Fig. 5 shows the influence of the friction force and the effect of the friction coefficient (Fig. 6) on the resonance modes of the duralumin drill strings vibrations at the basic frequency. It is established that taking into account the friction force leads to the extinction of the resonance curve, and an increase in the coefficient of friction reduces the amplitude of the oscillations and narrows the band of non-operating frequencies. All resonant curves stretch out to the right because of existence of geometrical nonlinearity in the system; meanwhile, shifting of the resonances curves towards the growth of external vibration frequency Ω takes place due to the initial curvature of the drill string axis.

According the bifurcation of amplitude-frequency response, pronounced on the presented figures, the instability zone of the basic resonance corresponding the frequencies of the external load can be determined.

Numerical calculations were carried out at the following values of parameters of the duralumin drill string: outer diameter of the string $D = 0.168\text{m}$, inner diameter $d = 0.12\text{m}$, length of string $l = 100\text{m}$, angular speed $\omega = 5\,\text{rad/min}$, longitudinal compressive load $N_0 = 0.7 \times 10^3\,\text{N}$, $N_t = 2.3 \times 10^4\,\text{N}$, and the values of initial curvatures $u_0 = 0.05$, $v_0 = 0.05$. Coefficients of friction was varying.

To verify the reliability of the obtained results and the developed amplitude-frequency characteristics, it is necessary to linearize the nonlinear model (3). The linearization of the model with respect to the finite deformation led to the known curves of the amplitude-frequency characteristics of linear oscillations, which confirms the reliability of the studies (Fig.7).

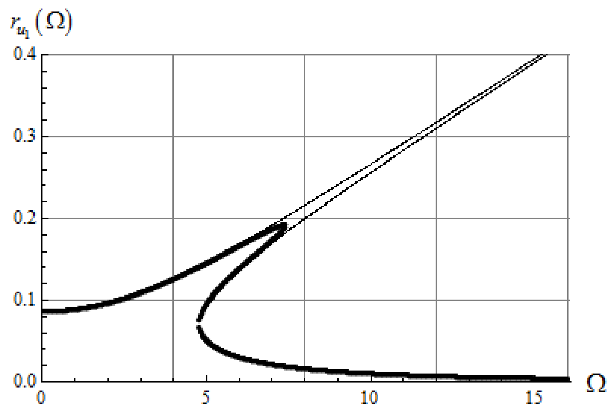
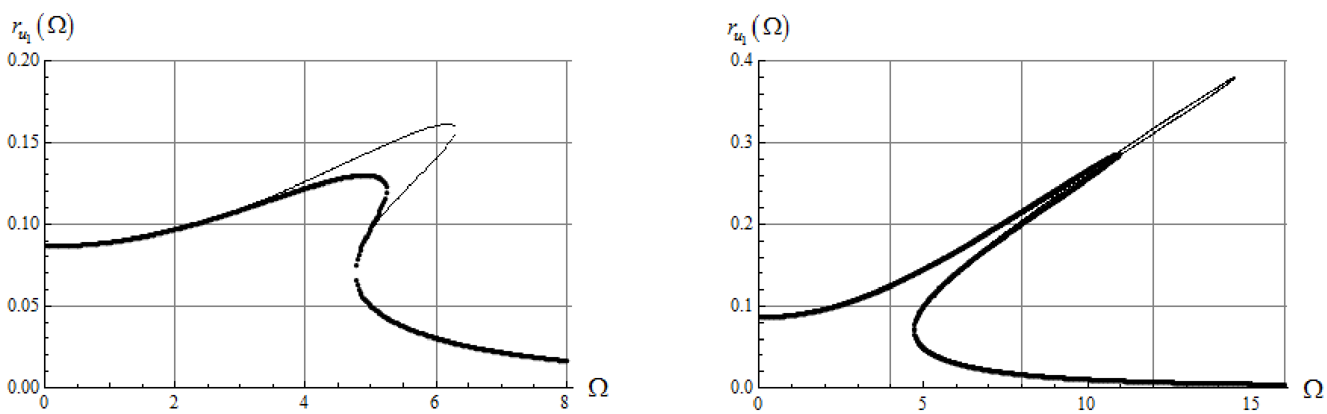


Fig. 5. Influence of the friction force on the basic resonance
(——— without friction, ▬▬ with friction $\varepsilon = 0.08$)



a) (——— $\varepsilon = 0.3$, ▬▬ $\varepsilon = 0.6$) b) (——— $\varepsilon = 0.03$, ▬▬ $\varepsilon = 0.06$)

Fig. 6. Influence of the coefficient of friction on the basic resonance

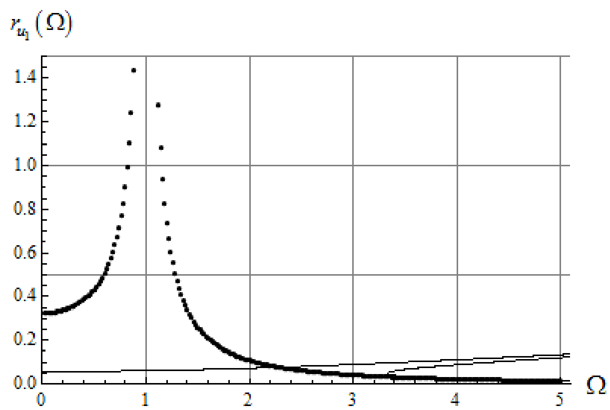


Fig. 7. Linearization of the model of the drill strings movement considering the friction force

It is known that in nonlinear systems, in addition to vibrations with a frequency equal to the frequency of the external force, higher harmonic vibrations may arise. For a more complete analysis of the resonant phenomena, a third harmonic term is added to the basic harmonic. It was previously established that the second harmonic is not captured for systems with geometric nonlinearity, so the term with the second harmonic is omitted. Then an approximate solution of (9) can be represented in the form:

$$\overline{u_1}(\tau) = r_{u_1}\cos(\Omega\tau - \varphi_{11}) + r_{u_3}\cos(3\Omega\tau - \varphi_{13}),$$
$$\overline{v_1}(\tau) = r_{v_1}\cos(\Omega\tau - \varphi_{21}) + r_{v_3}\cos(3\Omega\tau - \varphi_{23}), \qquad (12)$$

Applying the method of harmonic balance, the equations defining the dependence between the amplitudes $r_{u_1}, r_{u_3}, r_{v_1}, r_{v_3}$ and the frequency Ω was obtained:

$$A2_u^2 + B2_u^2 + 9\left(\bar{A} r_{v3}\Omega + F_{31} r_{u3}^3\Omega\right)^2 + \frac{A2_u B2_u}{A1_u B1_u}\cdot \left[C_2^2 - A1_u^2 - B1_u^2 - \left(\bar{A} r_{v1}\Omega + F_{31} r_{u1}^3\Omega\right)^2\right] + \frac{A2_u r_{v1} + A1_u r_{v3}}{3A1_u r_{v3}}\left[A2_u^2 - B2_u^2 - 9\left(\bar{A} r_{v3}\Omega + F_{31} r_{u3}^3\Omega\right)^2\right] = 0,$$

$$B1_u^2 + \left(\bar{A} r_{v1}\Omega + F_{31} r_{u1}^3\Omega\right)^2 - A1_u^2 + \frac{B1_u r_{v1}}{3B2_u r_{v3}}\cdot \left[A2_u^2 - B2_u^2 - 9\left(\bar{A} r_{v3}\Omega + F_{31} r_{u3}^3\Omega\right)^2\right] + \frac{4C_1C_2A1_u}{K r_{u1}} = C_2^2, \qquad (13)$$

where

$$A1_u = -r_{u1}\Omega^2 + A r_{u1} + \frac{3}{4}B r_{u1}^3 + \frac{3}{2}B r_{u1} r_{u3}^2 + \frac{1}{2}D r_{u1} r_{v3}^2 + \frac{3}{4}D r_{u1} r_{v1}^2 + D r_{u3} r_{v1} r_{v3},$$

$$B1_u = \frac{3}{4}B r_{u1}^2 r_{u3} + \frac{1}{4}D r_{u3} r_{v1}^2 + \frac{1}{2}D r_{u1} r_{v1} r_{v3},$$

$$A2_u = -9r_{u3}\Omega^2 + A r_{u3} + \frac{3}{4}B r_{u3}^3 + \frac{3}{2}B r_{u3} r_{u1}^2 + \frac{1}{2}D r_{u3} r_{v3}^2 + \frac{3}{4}D r_{u3} r_{v3}^2 + D r_{u1} r_{v1} r_{v3},$$

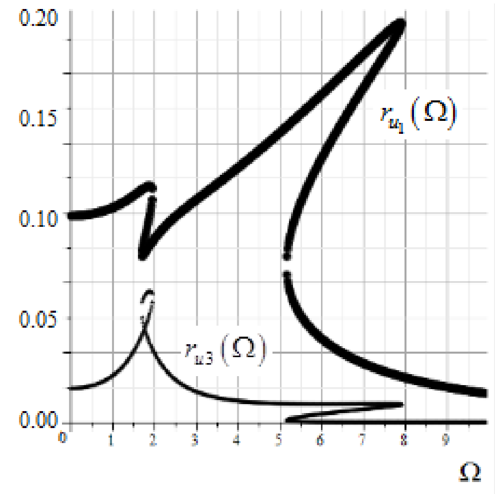
$$B2_u = \frac{1}{4}B r_{u1}^3 + \frac{1}{4}D r_{u1} r_{v1}^2.$$

Similar equations were obtained for the second component of transverse vibrations, where the amplitudes of the oscillations are expressed by r_{v1}, r_{v3}. Thus, the system of 4 equations with unknowns $r_{u1}, r_{u3}, r_{v1}, r_{v3}$ and the frequency Ω is obtained. By solving this system with respect to unknowns, amplitude-frequency characteristics can be found for the resonance at higher frequencies. Numerical analysis is presented in the following Fig. 8-11.

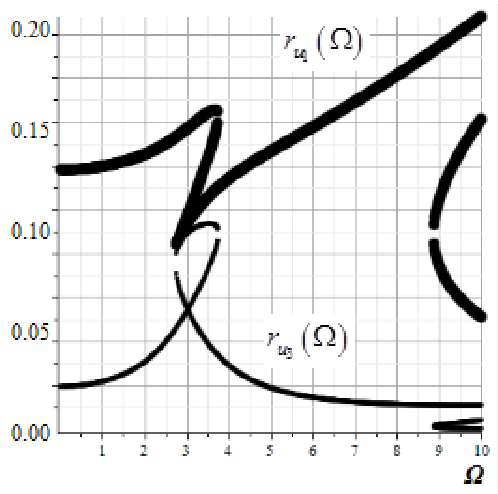
The results of the investigations for steel and duralumin drill strings are shown in Fig. 8. It is established that in the case of resonance at higher frequencies of the duralumin drill string, its amplitude-frequency characteristics has larger values than for the steel string. This suggests that the duralumin drill string is subject to smaller deviations from the rectilinear form than the steel one under the same drilling conditions, which significantly improves its dynamic and strength characteristics. There is a "dragging" of the amplitude-frequency characteristic to the region of high frequencies. That is, in the case of duralumin rods, resonance should be expected at high frequencies. In addition, along with the basic resonance, a third harmonic resonance arises. For the case of duralumin rods, the bifurcation of the amplitude-frequency characteristics is more pronounced.

As a result of the research, the energy of the basic resonance is transferred to the third harmonic resonance. Its amplitude reaches critical values, which is characteristic of strongly nonlinear systems.

In Fig. 9-10, the influence of the length of the steel drill rod on the resonance modes of their vibrations is investigated. It is established that as the length of the strings increases, their frequency response is shifted to a zone of lower frequencies, and resonance should be expected at earlier frequencies than for drill strings of shorter lengths.

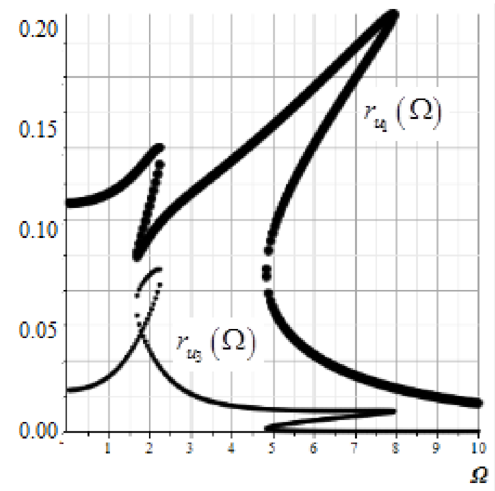


(a) steel drill string

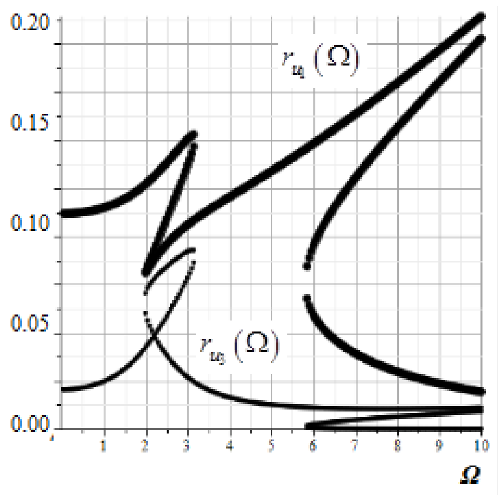


(b) duralumin drill string

Fig. 8. Resonance curves of the first (———) and third (———) harmonic vibrations of drill strings of different materials



(a) $l = 200m$



(b) $l = 150m$

Fig. 9. Resonance curves of the first (———) and third (———) harmonic vibrations of drill strings of different materials

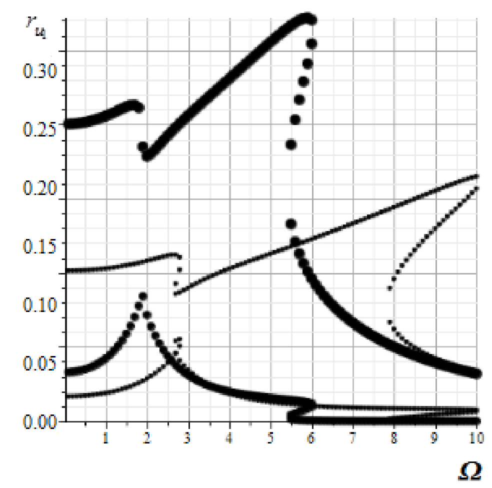


Fig. 10. Resonance curves of the first and third harmonic vibrations of drill strings of different lengths
(——— $l = 500\text{m}$, ——— $l = 300\text{m}$)

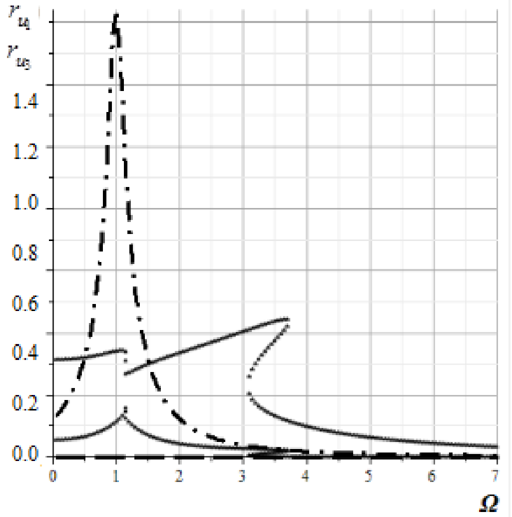


Fig. 11. Comparison of linear (– · – · – ·) and nonlinear (———) models of drill strings movement

As before, to confirm the validity of solutions of nonlinear problems, they were tested by their linear cases obtained as a result of linearization of the obtained models. For this purpose, a comparative analysis of the amplitude-frequency responses of the nonlinear model (3) and its particular linear case was done (Fig. 11). In the study of resonance modes of vibration, the linearization of the nonlinear model with respect to the finite deformation leads to the absence of a third harmonic in the oscillatory process. And according to the first harmonic, the known amplitude-frequency characteristic of linear oscillations is obtained. All this confirms the reliability of the studies.

By the bifurcations of the amplitude-frequency characteristics, which are clearly expressed in the presented graphs, it is possible to determine the zone of resonance instability at the corresponding external impact frequencies. A more complete analytical representation of the instability zones of resonance phenomena is carried out below.

V. Stability Zones

Stability of the given modes of the drill string movement has a fundamental importance for ensuring its troubleproof operation. The steady movement of a drill string is understood as its movement in the absence of the dangerous resonance modes of fluctuations. For this, the stability of the basic resonance is investigated.

If, with an unlimited increase in time, all the solutions of the system remain in the neighborhood of the solution corresponding to the equilibrium state, then the system and the solution are stable. Otherwise, when the values of the variables go away from their initial values with increasing τ, the system is considered unstable.

The stability problem of a periodic solution of the model (8) is investigated here. Considering the periodic solution as $u_0(\tau), v_0(\tau)$ and let the small variation $\delta u, \delta v$, the solution of (8) $\bar{u}_1(\tau), \bar{v}_1(\tau)$ will be presented in the next form:

$$u_1(\tau) = u_0(\tau) + \delta u,$$
$$v_1(\tau) = v_0(\tau) + \delta v. \tag{14}$$

The stability of the solutions $u_0(\tau), v_0(\tau)$ depend on the behavior of the small deviations $\delta u, \delta v$ in time as follows [16]: the solutions $u_0(\tau), v_0(\tau)$ is considered unstable if the magnitudes $\delta u, \delta v$ unrestrictedly increase at $\tau \to \infty$; and the solutions $u_0(\tau), v_0(\tau)$ is considered stable if $\delta u, \delta v$ are restricted at $\tau \to \infty$.

The case of the basic resonance (10) is examined there. The equations of the perturbed state for the considered case of Hill type is received:

$$\frac{d^2\delta u}{d\tau^2} + \bar{A}\frac{d\delta v}{d\tau} + \left(F_1 u_0^{\,2} + F_2 u_0^{\,4}\right)\frac{d\delta u}{d\tau} +$$
$$+\left(A + K\cos\bar{\Omega}\tau + 3Bu_0^{\,2} + 2Dv_0\delta v\right)\delta u = 0,$$
$$\frac{d^2\delta v}{d\tau^2} - \bar{A}\frac{d\delta u}{d\tau} + \left(F_1 v_0^{\,2} + F_2 v_0^{\,4}\right)\frac{d\delta v}{d\tau} +$$
$$+\left(A + K\cos\bar{\Omega}\tau + 3Bv_0^{\,2} + 2Du_0\delta u\right)\delta v = 0. \tag{15}$$

Behavior character of the solutions (15) indicates stability or instability of the basic resonance according to Lyapunov.

Setting the variations $\delta u, \delta v$ in the form of a spectrum of oscillations [16] according to Floquet theory:

$$\delta u = e^{\eta\tau} b_1 \cos(\Omega\tau - \psi_1),$$
$$\delta v = e^{\mu\tau} b_1 \cos(\Omega\tau - \psi_1), \tag{16}$$

where η, μ are the characteristic indexes.

Applying the method of harmonic balance, the characteristic determinants are defined, which sets the borders of the first instability zone of the basic resonance:

$$\Delta(\eta=0)=1+\Omega^4-2\Omega^2+\frac{27}{16}(B+D)^2 r_{u_1}{}^4+$$
$$+3(B+D)(1-\Omega^2)r_{u_1}{}^2+\left[\bar{A}\Omega+F_{31}r_{u_1}{}^2\Omega\right]^2=0,$$
$$\Delta(\mu=0)=1+\Omega^4-2\Omega^2+\frac{27}{16}(B+D)^2 r_{v_1}{}^4+$$
$$+3(B+D)(1-\Omega^2)r_{v_1}{}^2+\left[\bar{A}\Omega+F_{32}r_{v_1}{}^2\Omega\right]^2=0. \quad (17)$$

Numerical analysis of the basic resonance instability zones of the nonlinear dynamic system (8), based on the amplitude-frequency relations (17) is obtained above at Fig. 12-16. Calculations were carried out using the same parameters of the drill string as at analysis of resonance, angular speed $\omega=0.5\text{rad/s}$, a friction coefficient $\varepsilon=0.8$, longitudinal compressive load at Fig. 13 $N_0=0.7\times10^3\,\text{N},\ N_t=2.3\times10^4\,\text{N},$ at Fig. 14-15 $N_0=0.7\times10^3\,\text{N},\ N_t=2.3\times10^5\,\text{N}.$.

Figure 12 examines the effect of taking into account the Coriolis force and the friction force in the model of the drill string vibrations on the instability zones at the basic frequency. The comparative analysis of two models with and without the Coriolis force and friction force is carried out. It is established that in the second case, the instability zone of the basic resonance begins with a zero amplitude and originates at the point $\Omega=1$. In the model considering frictional forces, the instability zone does not reach the zero amplitude, and is rounded off before reaching the axis. At that case resonance should be expected at higher frequencies.

Further, the effect of the friction force on the stability zones of the basic resonance for different cases of external loading is investigated (Fig. 13-14). It is established that consideration the friction force shifts the instability zone of the basic resonance to the region of high frequencies. However, this shifting occurs at large friction coefficients ($\varepsilon>0.6$). At small coefficients, which were considered earlier, the influence of the nonlinear friction force is not so significant. It is also confirmed in Fig. 15 where influence of value of the friction coefficient on the instability zone of the basic resonance is investigated. Only an insignificant shift in the resonance instability zone is noticeable with an increase in the friction coefficient.

In Fig. 13-14, the studies were carried out at the same parameters, and only the magnitude of the external load was changed. It is noted that an increase in the external load leads to a shift in the instability zone of the basic resonance to the region of lower frequencies. Those, resonance should be expected at earlier frequencies. This shifting is typical both for the model taking into account the friction force, and for the model without taking it into account.

The influence of the length of the drill string on the stability of the basic resonance is investigated (Fig. 16). It is established that as the length increases, the instability zone of the basic resonance shifts to the region of high frequencies. An increase in the amplitude-frequency characteristics of the drill string is also observed. There is also an expansion of the instability zone, which significantly reduces the zone of operating frequencies.

Just as before, in order to confirm the correctness of the obtained results, the zones of instability of the basic resonance and resonance curves at the basic frequency are superimposed at the same parameters and technical characteristics of the duralumin drill strings and the same external loads acting on it, taking into account friction forces (Fig. 17). The zones of instability of the basic resonance completely cover the undesirable working region, obtained as a result of numerical analysis of the resonance modes of vibration. All this also validates the received results.

Thus, obtained here amplitude-frequency responses determinate the instability zone of the basic resonance allows to avoid non-working resonance frequencies at the early stage of drilling wells.

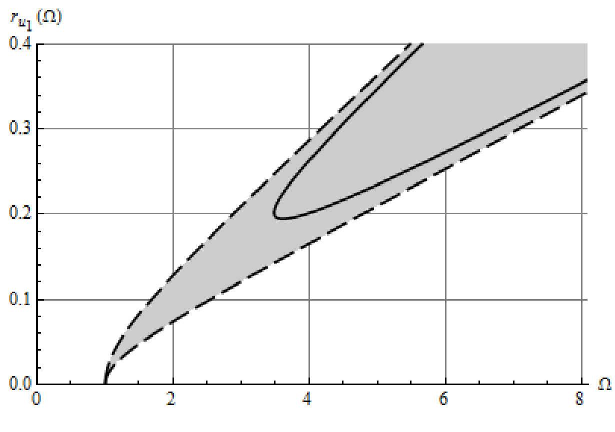


Fig. 12. Influence of Coriolis force and friction force on the instability zones (– – – – without, ——— with their considering)

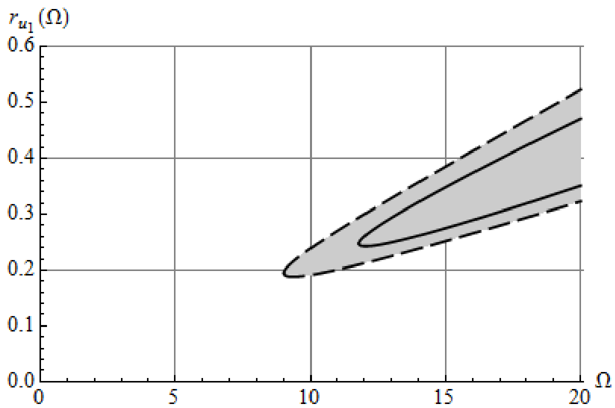


Fig. 13. Influence of the friction force on the instability zones at $N_t = 2.3\times10^4$ N (– – – without, ——— with friction)

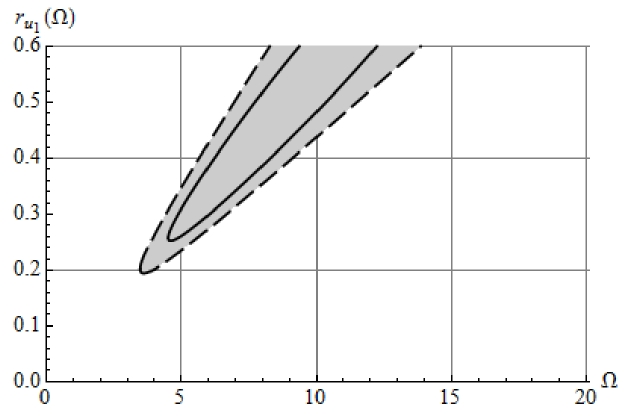


Fig. 14. Influence of the friction force on the instability zones at $N_t = 2.3\times10^5$ N (– – – without, ——— with friction)

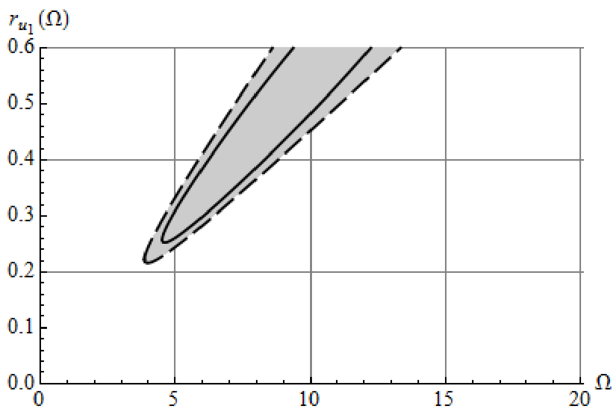


Fig. 15. Influence of the friction coefficient on the instability zones (– – – $\varepsilon = 0.7$, ——— $\varepsilon = 0.8$)

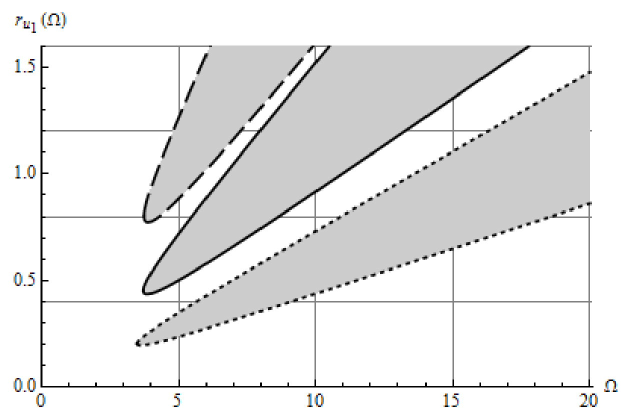


Fig. 16. Influence of the drill strings lengths on the instability zones (– – – – $l = 200$m, ——— $l = 150$m, - - - - $l = 100$m)

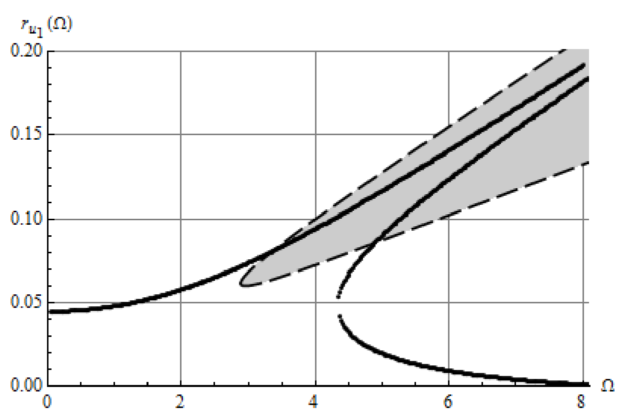


Fig. 17. Curve of the resonance on the basic frequency and its zone of instability

VI. Conclusion

According to the results of the research, it is possible to see the influence of the frictional force between the drill rod and the borehole, between the drilling tool and the borehole on the deviation the drill string from its rectilinear form during drilling. An increase in the coefficient of friction leads to a decrease in the amplitude of the drill strings vibrations, but the frequency of the oscillations does not change and, on the whole, the nature of the oscillations remains unchanged. That is, with an increase in the friction coefficient, the oscillatory process is more damped. The consideration of friction forces leads to a decrease in the amplitudes of transverse vibrations and to a weakening of resonance oscillations with a narrowing of the zone of dangerous frequencies. By increasing the magnitude of the external compressive load, it is possible to overcome the friction forces, while increasing the drilling speed of the well.

Revealed here nonlinear effects are out of the linear theory of vibrations. Therefore, the development and research of nonlinear models of shallow drilling strings for the finite deformation has a fundamental nature.

As a result of studies of the nonlinear model of drill string vibrations, the appearance of a resonance at the third frequency is established, which introduces qualitative and quantitative changes in the oscillation process. This phenomenon occurs at certain frequencies due to the "dragging" of the energy of the oscillatory process at the basic frequency into the oscillatory process at higher frequencies. There is a loss of stability of the basic resonance. In places of its bifurcation the amplitude-frequency characteristics of the third harmonic resonance are observed. This fact confirms the appearance of resonance at higher frequencies in nonlinear systems with a stiff characteristic.

As a result of modeling and analysis of the drill strings dynamic stability, it was established that taking into account the nonlinear frictional force leads to a narrowing of the instability zones of the basic resonance, and an increase in the coefficient of friction leads to a further narrowing of these zones. Qualitative and quantitative agreement of the stability zones of the resonance on the basic frequency with obtained in this work bifurcation zones on the resonant amplitude-frequency characteristics, was also established.

Modeling of resonance regimes of the drill string dynamics along with the analysis of its stability has a great importance for development of drilling equipment and improving its dynamic characteristics. In doing so, it is essential to take into account the geometrical nonlinearity of the system and the frictional force between the drill rod and the borehole.

Despite the fact that the proposed methods were used to study the stability of the basic resonance of elastic dynamical systems, they can also be successfully applied to analyze the stability of resonances at higher frequencies.

ACKNOWLEDGMENT

This research work was performed within the framework of the SF 54/53 (2015-2017) scientific project funded by the Ministry of Science and Education of the Republic of Kazakhstan.

REFERENCES

[1] B.M. Mardonov, A.S. Karimova, “Established transverse vibrations of drill columns with areas of external friction under the action of compressive loads,” Oil and Gas, vol. 1, 2008, pp. 74-79. (in Russian)

[2] V.I. Gulyaev, S.N. Khudolii, E.I. Borshch, “Wirl vibrations of the drillstring bottom hole assembly,” Strength of Materials, vol. 42(6), 2010, pp. 637-646.

[3] V.I. Gulyayev, S.N. Hudoly, L.V. Glovach, “The computer simulation of drill column dragging in inclined bore-holes with geometrical imperfections,” Int. J. Solids and Struct., vol. 48(1), 2010, pp. 110-118.

[4] M.A. Vaz, M.N. Patel, “Analysis of drill strings in vertical and deviated holes using the Galerkin technique,” Engineering structures, vol. 17 (6), 1995, pp. 437-442.

[5] E.I. Kucherenko, “On a modification of the Galerkin method for the boundary-value problem of an ordinary nonlinear differential equation,” Ukr. Math. J., vol. 33(5), 1981, pp. 597-603. (in Russian)

[6] L. Khajiyeva, A. Kudaibergenov, A. Kudaibergenov, A. Kydyrbekuly, “Analysis of drill string motion in a gas stream,” Proc. 8th European Nonlinear Dynamics Conf. (ENOC 2014), Vienna, Austria, 2014, pp. 593-596.

[7] A.S. Sergaliyev, L.A. Khajiyeva, “Flat flexural vibration of drill-string with an initial curvature,” Advances in Mechanism Design II, vol. 44, 2017, pp. 231-237.

[8] A.P. Christoforou, A.S. Yigit, “Dynamic modeling of rotating drillstrings with borehole interactions,” Journal of Sound and Vibration, vol. 206 (2), 1997, pp. 243-260.

[9] W. Zhu, D. Qinfeng, “Effect of Prebent Deflection on Lateral Vibration of Stabilized Drill Collars,” Society of Petroleum Engineers, vol. 16(1), 2011, pp. 200-216.

[10] D.B. McIver, “Hamilton’s principle for systems of changing mass,” Journal of Engineering Mathematics, vol. 7, 1973, pp. 249−261.

[11] L.A. Khajiyeva, A. Sergaliyev, A. Umbetkulova, “Dynamic Analysis of Steel and Dural Drill Rods,” Advanced Materials Research, vol. 705, 2013, pp. 91-96.

[12] A. Umbetkulova, L.A. Khajiyeva, Y. Sato, “Analysis of vibrations of a chisel column taking nonlinear factors into account,” Proc. of the 3rd Japan - Korea Joint Symposium of Dynamics&Control, Fukuoka, Japan, 2013, pp. 187-190.

[13] K.V. Avramov, “A qualitative analysis of the subharmonic oscillations of a parametrically excited flexible rod,” J. Appl. Math. Mech., vol. 70, 2006, pp. 284-291.

[14] W. Szemplinska-Stupnicka, “Non-linear normal modes and the generalized Ritz method in the problems of vibrations of non-linear elastic continuous systems,” International Journal of Non-Linear Mechanics, vol. 18, 1983, pp. 149-165.

[15] V.I. Gulyayev, V.V. Gaidaichuk, I.L. Solovjov, I.V. Gorbunovich, “The buckling of elongated rotating drill strings,” J. Petroleum Science and Engineering, vol. 67, 2009, pp. 140-148.

[16] Ch. Hayashi, *Nonlinear oscillations in Physical Systems*. New Jersey: Princeton University Press, 1986.