THE CONSTRUCTION OF CONSTRAINED CONTROL FOR LINEAR SYSTEMS

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The method of control construction for the control system described by

$$\dot{x} = A(t)x + B(t)u + \mu(t), \quad t \in I = [t_0, t_1], \quad x(t_0) = x^0, \tag{1}$$

$$x(t_1) = x^1, \tag{2}$$

$$u(t) \in U, \quad U = \Big\{u(\cdot) \in L_2(I, R^r) \ / \ \gamma(t) \le u(t) \le \delta(t), \quad t \in I\Big\}, \tag{3}$$

is developed. Here $A = A(t), B = B(t), \mu = \mu(t)$ are given $n \times n, n \times m, n \times 1$ matrices, respectively, with piecewise continuous elements, $u = u(t)$ is control-function, t_0, t_1 are fixed time moments, x^0, x^1 are given vectors.

The problem is to construct the control $u(t) \in U$ such that transfers trajectory of system (1) to the given state x^1.

Let W to be the set of all the controls $u(\cdot) \in L_2(I, R^r)$ such that transfer trajectory of system (1) to the given state x^1. The set W is determined by theorem 3 in [1]. Then the solution to the formulated problem is contained in W.

Let us introduce some notations:

$\theta(t)$ is a fundamental matrix of solutions for the linear homogeneous system $\dot{\eta} = A(t)\eta$,

$$\Phi(t,\tau) = \theta(t)\theta^{-1}(\tau), \quad T(t_0,t_1) = \int_{t_0}^{t_1} \Phi(t_0,t)B(t)(t)B^*\Phi^*(t_0,t)dt,$$

$$C(t) = B^*\Phi^*(t_0,t)T^{-1}(t_0,t_1), \quad a = \Phi(t_0,t_1)x^1 - x^0 - \int_{t_0}^{t_1} \Phi(t_0,t)\mu(t)dt.$$

$$\phi(t,\alpha) = C(t)\Big[a - \int_{t_0}^{t_1} \Phi(t_0,t)B(t)[\alpha\gamma(t) + (1-\alpha)\delta(t)]dt\Big].$$

The following result which yields control construction method for the considered problem is obtained:

Theorem 1. *Let the matrix $T(t_0,t_1)$ be positively definite. If the following inequalities*

$$(1-\alpha_*)[\gamma(t) - \delta(t)] \le \phi(t,\alpha_*) \le 0$$

hold for some $\alpha_* \in [0,1]$, then the control $u(t) = \alpha_*\gamma(t) + (1-\alpha_*)\delta(t) + \phi(t,\alpha_*)$, $t \in I$, is the solution of problem (1)-(3).

References

[1] Aisagaliev S.A. (2005) Obschee reshenie odnogo klassa integralnykh uravneniy. *Matematicheskiy zhurnal*, no. 4, pp. 3–10.