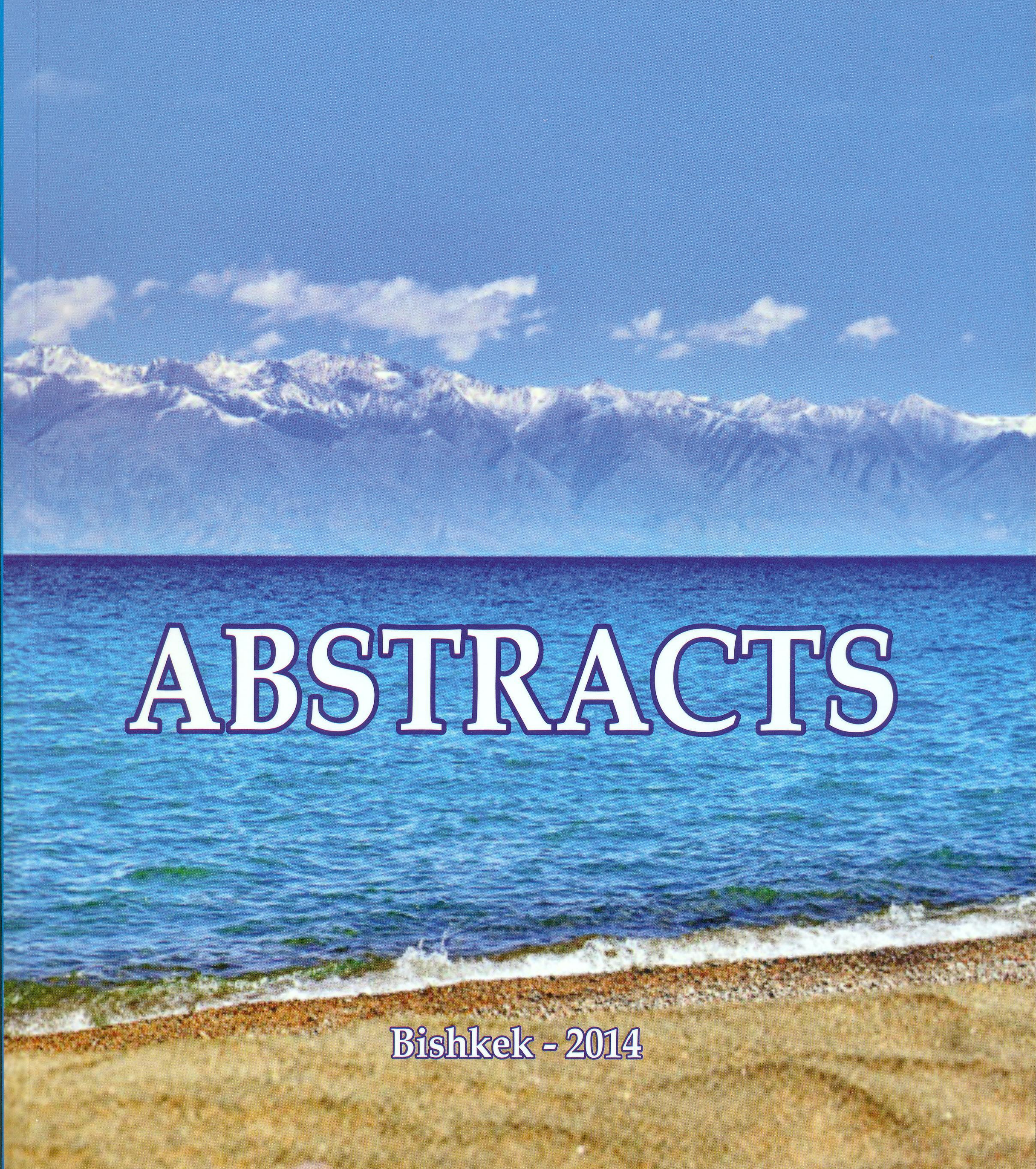
V Congress of the

TURKIC WORLD MATHEMATICIANS

Kyrgyzstan, «Issyk-Kul Aurora», 5-7 June, 2014

TWMS



ABSTRACTS

Bishkek - 2014

ASYMPTOTICAL CONVERGENCE OF SOLUTIONS BOUNDARY VALUE PROBLEMS FOR SINGULARLY PERTURBED INTEGRO-DIFFERENTIAL EQUATIONS

Dauylbayev Muratkhan, Atakhan Nilupar.
Almaty (Kazakhstan)
dmk57@mail.ru

Consider the following linear singularly perturbed integro-differential equation on the interval [0, 1]:

$$L_\varepsilon y \equiv \varepsilon y^{(n)} + A_1(t)y^{(n-1)} + \ldots + A_n(t)y = F(t) + \int_0^1 \sum_{i=0}^{m_1+1} H_i(t,x)y^{(i)}(x,\varepsilon)dx \qquad (1)$$

with boundary conditions:

$$h_i y \equiv \sum_{j=0}^{m_i} \alpha_{ij} y^{(j)}(0,\varepsilon) + \sum_{j=0}^{l_i} \beta_{ij} y^{(j)}(1,\varepsilon), \;\; i = \overline{1,n}, \qquad (2)$$

where $\varepsilon > 0$ is a small parameter, $A_1(t) > 0$, α_{ij}, β_{ij}, a_i are some known constants independent on ε, $\alpha_{1,m_1} \neq 0$, $n-1 > m_1 > m_2 \geq \ldots \geq m_n$, $n-1 > l_1 > l_2 \geq \ldots \geq l_n$.

Solution of boundary value problem (1), (2) has an initial jump of m_1–th order at the point $t = 0$, i.e. $y^{(i)}(0,\varepsilon) = O(1), i = \overline{0,m_1}$, $y^{(m_1+1)}(0,\varepsilon) = O(1/\varepsilon)$, $\varepsilon \to 0$.

Analogical problem for differential equations was investigated in [1].

We prove that as $\varepsilon \to 0$ the solution of the boundary value problem (1), (2) tends to the solution of the following unperturbed equation:

$$L_0\overline{y} \equiv A_1(t)\overline{y}^{(n-1)}(t) + \sum_{i=2}^{n} A_i(t)\overline{y}^{(n-i)}(t) = F(t) + \int_0^1 \sum_{i=0}^{m_1+1} H_i(t,x)\overline{y}^{(i)}(x)dx + \Delta(t)$$

with boundary conditions:

$$h_1\overline{y}(t) \equiv \sum_{j=0}^{m_1} \alpha_{1j}\overline{y}^{(j)}(0) + \sum_{j=0}^{l_1} \beta_{1j}\overline{y}^{(j)}(1) = a_1 + \alpha_{1,m_1}\Delta_0,$$

$$h_i\overline{y}(t) \equiv \sum_{j=0}^{m_i} \alpha_{ij}\overline{y}^{(j)}(0) + \sum_{j=0}^{l_i} \beta_{ij}\overline{y}^{(j)}(1) = a_i, \;\; i = \overline{2,n}$$

where $\Delta(t) = \Delta_0 H_{m_1+1}(t,0)$, Δ_0 is the initial jump of solution.

References

[1] Kasymov K.A., Nurgabyl D.N. (2003) Asymptotic Behavior of Solutions of Linear Singularly Perturbed General Separated Boundary-Value Problems with Initial Jump. *Ukrainian Mathematical Journal* . Vol. 55, No. 11, pp. 1777-1792.