Modeling of a non-perturbative spinor vacuum and the investigation of gravitational waves interacting with the non-perturbative spinor vacuum

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The propagation of strong gravitational waves interacting with a nonperturbative vacuum of spinor fields is considered. We suggest an approximate model of a nonperturbative spinor vacuum. The gravitational-wave solution of Einstein equations is analogous to the solution of the Schrödinger equation for an electron moving in a periodic potential. The analog of the Kronig-Penney model for gravitational waves is considered. It is shown that the suggested gravitational-wave model permits the existence of weak electric charge and current densities concomitant with the gravitational wave. Based on this observation, a possible experimental verification of the model is suggested.

Keywords: strong gravitational waves, nonperturbative vacuum, spinor field, Schrödinger equation, Kronig-Penney model.

1. Introduction

This talk is based on Ref. [1].

An experimental search for gravitational waves (GW) is one of the most intriguing problems in modern physics. One of the effects accompanying the propagation of GWs could be its interaction with a nonperturbative spinor vacuum. The reason for such an interaction to occur is that the energy-momentum tensor of a spinor field contains the spin connection, which in turn contains first derivatives of tetrad components with respect to the coordinates. As a result, the Einstein equations give the wave equation for a GW which contains second derivatives of the tetrad components on the left-hand side and their first derivatives on the right-hand side.

In Ref. [2] we have considered the propagation of a weak GW interacting with the nonperturbative spinor vacuum. Here we extend those results to the case of a strong GW. In doing so, as in Ref. [2], to model the nonperturbative vacuum of a spinor field, we will use a phenomenological approach. Within the framework of this approach, we make some physically reasonable assumptions about expectation values of the spinor field and its dispersion. This will permit us to reduce the infinite system of differential equations for all Green's functions of the nonperturbative quantum spinor field to the finite set of equations (for more details, see Refs. [2,3]).

Following this approach, here we will discuss the solution of the Einstein equations for a strong GW propagating on the background of the nonperturbative vacuum of spinor fields, which is a generalization of the weak, plane gravitational wave of Ref. [2].

2. Strong GW in a nonperturbative spinor vacuum

The metric for a strong GW propagating in one direction is sought in the form

$$ds^2 = 2d\xi d\eta - \chi^2(\eta)\gamma_{ab}(\eta)dx^a dx^b, \tag{1}$$

where $x^0 = \xi, x^1 = \eta$ are lightlike coordinates; the indices a, b run over the values $2, 3$, and $\det(\gamma_{ab}) = 1$. Our goal here is to consider the propagation of GWs in a nonperturbative spinor vacuum. We expect that such a physical system has to be considered in nonperturbative language when both a metric and a spinor field are regarded as quantum quantities and are quantized in a nonperturbative manner.

Our nonperturbative approach for quantizing nonlinear fields is described in Ref. [2]. In such an approximation the Einstein-Dirac operator equations can be written in the following manner [2]:

$$G_{\bar{a}\mu} = \varkappa \left\langle Q \left| \hat{T}_{\bar{a}\mu} \right| Q \right\rangle, \tag{2}$$

$$\left\langle Q \left| \nabla_\mu \hat{T}_{\bar{a}}{}^{\mu} \right| Q \right\rangle = 0, \tag{3}$$

Also, to check the consistency of the ansätze for a 2-point Green's function, instead of solving the Dirac equation, we will use the Bianchi identities (3), as we did in Ref. [2].

3. Approximate model of the nonperturbative spinor vacuum

Our strategy in the formulation of the model of the nonperturbative spinor vacuum is as follows: (i) we write some classical ansätze for a spinor; (ii) we derive the corresponding energy-momentum tensor; and (iii) we then write hats over the corresponding spinor components.

We suggest the following approximate model of the nonperturbative vacuum of the spinor field:

(a) The nonperturbative vacuum is described by the following operator of the spinor field:

$$\hat{\psi}^T = e^{i\omega\eta} \left(\hat{A}, \hat{A}, \hat{V}, \hat{V} \right). \tag{4}$$

The constant operators $\hat{A}, \hat{V}$ appearing here are independent of η.

(b) The corresponding energy-momentum tensor of the spinor field is

$$\left\langle Q \left| \hat{T}_{\bar{1}\eta} \right| Q \right\rangle = -2 \left\langle \hat{V}\hat{V}^\dagger \right\rangle \left(\beta' \cosh\beta + \alpha' \sinh\beta + 4\omega \right). \tag{5}$$

(c) To check this model, we calculate the divergence of the energy-momentum tensor and show that it vanishes.

4. Einstein equations for a strong GW interacting with the nonperturbative spinor vacuum

For the metric (1), we have foloowing Einstein equation

$$-\chi'' + \left[\beta' \cosh\beta + \alpha' \sinh\beta - \frac{1}{4}\left({\alpha'}^2 + {\beta'}^2\right)\cosh^2\beta - \frac{1}{4}\alpha'\beta' \sinh 2\beta\right]\chi = -4\tilde{\omega}\chi, \tag{6}$$

One sees immediately that Eq. (6) is a Schrödinger-like equation with an effective potential.

In what follows we will seek periodic solutions to Eq. (6). It is clear that for periodic functions α, β we obtain an equation similar to that describing the movement of a single electron in a crystal. Such an equation has been well studied in the literature (see, for example, the textbook[4]), and we can apply all mathematical methods used in solving the Schrödinger equation to our case of a strong GW.

5. General solution

In order to solve Eq. (6) for a periodic potential, in this section we apply the methods developed in solid state theory. The only difference is that the function $\chi(\eta)$ is a real function, unlike the usual quantum mechanics where a wave function is complex.

First of all, let us rewrite Eq. (6) in the form

$$-\chi''(\eta) + V_{eff}(\eta)\chi(\eta) = -4\omega\chi(\eta). \tag{7}$$

In order to find a solution of the type of GW, we have to investigate the case of a periodic potential $V_{eff}(\eta + \eta_0) = V_{eff}(\eta)$.

We seek a solution to Eq. (7) in the form

$$\chi(\eta) = \chi_0 + \sum_{k=1}^{\infty}\left[a_k \cos\left(\frac{2\pi k}{\eta_0}\eta\right) + b_k \sin\left(\frac{2\pi k}{\eta_0}\eta\right)\right]. \tag{8}$$

In principle, by using well-developed methods of solid state theory (see, for example, the textbook[4]), one can find a GW solution for any periodic metric functions $\alpha(\eta)$ and $\beta(\eta)$.

6. The Kronig-Penney model for gravitational waves

Let us consider the simplest case, $\beta = 0$. In order to apply the Kronig-Penney approximation, we will use the metric function $\alpha(\eta)$ and corresponding effective potential as in Fig. 1.

For the periodic potential, Eq. (7) takes the form

$$\begin{cases} \chi'' = K^2\chi & \text{if} \quad -a < \eta < 0, \\ \chi'' = -Q^2\chi & \text{if} \quad 0 < \eta < b, \end{cases} \tag{9}$$

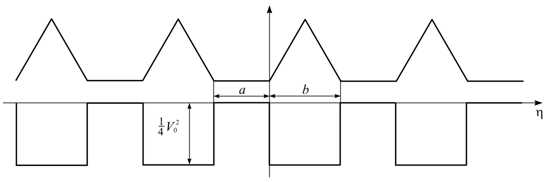

Fig. 1. The profiles of $\alpha(\eta)$ (the top graph) and $V_{eff} = -\frac{\alpha'^2(\eta)}{4}$ (the bottom graph).

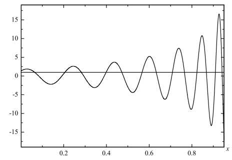

Fig. 2. The profile of the left-hand side of the constraint equation (10) for $\tilde{a} = 3$ and $\tilde{b} = 50$. The straight line corresponds to 1.

where $K^2 = 4\omega$, $Q^2 = V_0^2/4 - K^2$. The dimensionless constraint equation is

$$\frac{2x-1}{2\sqrt{x-x^2}} \sinh\left(\tilde{a}\sqrt{x}\right) \sin\left(\tilde{b}\sqrt{1-x}\right) + \cosh\left(\tilde{a}\sqrt{x}\right) \cos\left(\tilde{b}\sqrt{1-x}\right) = 1. \tag{10}$$

The typical profile of the left-hand side (lhs) of the constraint equation (10) is shown in Fig. 2.

7. Experimental verification

Let us discuss now possible experimental consequences coming from a consideration of the propagation of a strong GW through the nonperturbative spinor vacuum. Direct calculations show that for such a case there exists the electric current with following non-zero components

$$j^t = -j^x = 2\sqrt{2}\left\langle Q \left| \hat{V}\hat{V}^\dagger \right| Q \right\rangle. \tag{11}$$

This means that we have the electric charge and current densities concomitant with the GW. The electric current is directed along the direction of propagation of the GW. This observation allows us to suggest the following experimental verification of the GW and nonperturbative spinor vacuum models considered here: *It is possible to try to measure the weak electric charge and current densities together with the standard measurements of GWs (LIGO, LISA, and so on).*

References

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