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Cosmological constant, supersymmetry, nonassociativity, and big numbers

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Abstract The nonassociative generalization of supersymmetry is considered. It is shown that the associator of four supersymmetry generators has the coefficient $\sim \hbar / l_0^2$ where l_0 is some characteristic length. Two cases are considered: (a) l_0^{-2} coincides with the cosmological constant; (b) l_0 is the classical radius of the electron. It is also shown that the scaled constant is of the order of 10^{-120} for the first case and 10^{-30} for the second case. The possible manifestation and smallness of nonassociativity is discussed.

1 Introduction

The observed value of the cosmological constant is smaller than that predicted by an effective local quantum field theory by a factor of 10^{120}. This discrepancy has been called "the worst theoretical prediction in the history of physics" [1,2]. Great interest would be aroused by the appearance of the cosmological constant in any formula, even if obtained by some qualitative reasonings.

It is shown in Ref. [3] that the idea of supersymmetry can be extended with the inclusion of nonassociativity into supersymmetry. The main idea presented there is that an associator of four supersymmetrical quantities $Q_a, Q_{\dot{a}}, Q_b, Q_{\dot{b}}$ can be connected with the angular momentum operator.

We want to consider the proportionality coefficient in this relation and, using some qualitative reasonings, to show that it may contain the factor which can be identified either with the cosmological constant or with the classical radius of the electron.

2 Nonassociative decomposition of the angular momentum operator

In Ref. [3], a nonassociative generalization of a supersymmetry algebra with the supersymmetry generators $Q_a, Q_{\dot{a}}$ (here $a = 1, 2, \dot{a} = \dot{1}, \dot{2}$) is considered. The simplest supersymmetry algebra considered there is (in this section we follow Ref. [3])

$$\{Q_a, Q_{\dot{a}}\} = Q_a Q_{\dot{a}} + Q_{\dot{a}} Q_a = 2\sigma^{\mu}_{a\dot{a}} P_{\mu}, \tag{1}$$
$$\{Q_a, Q_b\} = \{Q_{\dot{a}}, Q_{\dot{b}}\} = 0, \tag{2}$$
$$[Q_a, P_{\mu}] = [Q_{\dot{a}}, P_{\mu}] = 0, \tag{3}$$
$$[P_{\mu}, P_{\nu}] = 0. \tag{4}$$

The anticommutator (1) connects the momentum operator $P_{\mu} = -i\partial_{\mu}$ (here $\mu = 0, 1, 2, 3$) and the generators $Q_{a,\dot{a}}$. The Pauli matrices $\sigma^{\mu}_{a\dot{a}}, \sigma^{a\dot{a}}_{\mu}$ are

$$\sigma^{\mu}_{a\dot{a}} = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\}, \tag{5}$$
$$\sigma^{a\dot{a}}_{\mu} = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\}. \tag{6}$$

Let us define an associator as follows:

$$[x, y, z] = (xy)\, z - x\, (yz). \tag{7}$$

It is assumed that the associator $[Q_a, Q_{\dot{a}}, (Q_b Q_{\dot{b}})]$ is

$$[Q_a, Q_{\dot{a}}, (Q_b Q_{\dot{b}})] = 2\zeta \sigma^{\mu}_{a\dot{a}} \sigma^{\nu}_{b\dot{b}} M_{\mu\nu}, \tag{8}$$

where the operator

$$M_{\mu\nu} = x_{\mu} P_{\nu} - x_{\nu} P_{\mu} \tag{9}$$

is the angular momentum operator, and ζ is the still undefined numerical factor that equalizes the dimensions of the right- and left-hand sides of Eq. (8). Our main goal here is to derive this factor using some plausible physical arguments.

3 Definition of ζ

We see from Eq. (1) that the dimension of Q is

$$[Q] = \left(\frac{\text{g cm}}{\text{s}} \right)^{1/2}. \tag{10}$$

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From Eq. (8), one can find that

$$[\zeta] = \frac{\text{g}}{\text{s}}. \tag{11}$$

We think that ζ should be constructed from fundamental constants. In this case one can consider the following possibilities.

The first one is that

$$\zeta \propto \frac{c^3}{G}, \tag{12}$$

where c is the speed of light and G is the gravitational constant. But we think that the relation (8) is a quantum relation, and in some sense it should be similar to the commutation relation

$$\left[x, \hat{p}_x\right] = i\hbar. \tag{13}$$

This means that the Planck constant should be included in ζ. For example, it can be done as follows:

$$\zeta \propto \frac{\hbar}{l_0^2}, \tag{14}$$

where l_0 is some characteristic length and should be constructed from physical constants. One can find the following possibilities:

$$l_0 = \begin{cases} \frac{1}{\sqrt{\Lambda}}, & \Lambda \text{ is the cosmological constant,} \\ r_0 = \frac{e^2}{m_e c^2} & \text{is the classical radius of electron.} \end{cases} \tag{15}$$

Choosing ζ in the form (15) we can rewrite (8) in the dimensionless form

$$\left[\tilde{Q}_a, \tilde{Q}_{\dot{a}}, \left(\tilde{Q}_b \tilde{Q}_{\dot{b}}\right)\right] = 2\tilde{\zeta}\sigma^{\mu}_{a\dot{a}}\sigma^{\nu}_{b\dot{b}}\tilde{M}_{\mu\nu}, \tag{16}$$

where the quantities with tildes are dimensionless, and

$$\tilde{\zeta} = \begin{cases} l_{\text{Pl}}^2 \Lambda \approx 10^{-120}; \\ \frac{G m_e e^2}{c^3 \hbar^2} \approx 10^{-30} \end{cases} \tag{17}$$

where $l_{\text{Pl}} = \sqrt{\frac{\hbar G}{c^3}}$ is the Planck length; e, m_e are the charge and mass of electron. Finally, write (8) in the form

$$\left[Q_a, Q_{\dot{a}}, \left(Q_b Q_{\dot{b}}\right)\right] = \begin{cases} 2\zeta_0 \hbar \Lambda \sigma^{\mu}_{a\dot{a}}\sigma^{\nu}_{b\dot{b}} M_{\mu\nu}; \\ 2\zeta_0 \frac{\hbar}{r_0^2} \sigma^{\mu}_{a\dot{a}}\sigma^{\nu}_{b\dot{b}} M_{\mu\nu} \end{cases} \tag{18}$$

where the coefficient ζ from Eq. (8) is defined as $\zeta = \zeta_0 \tilde{\zeta}$ and $\zeta_0 = \pm 1, \pm i$ is now a dimensionless number. The inverse relation is

$$M_{\mu\nu} = \begin{cases} \frac{1}{8\zeta_0}\frac{1}{\hbar\Lambda}\sigma^{a\dot{a}}_{\mu}\sigma^{b\dot{b}}_{\nu}\left[Q_a, Q_{\dot{a}}, \left(Q_b Q_{\dot{b}}\right)\right]; \\ \frac{1}{8\zeta_0}\frac{r_0^2}{\hbar}\sigma^{a\dot{a}}_{\mu}\sigma^{b\dot{b}}_{\nu}\left[Q_a, Q_{\dot{a}}, \left(Q_b Q_{\dot{b}}\right)\right]. \end{cases} \tag{19}$$

We see that there are different possibilities for choosing ζ. Probably it is due to the fact that the coefficient ζ in the relation (8) can be different for different physical situations. For example, on the large scales (∼Universe) $l_0 \approx 1/\sqrt{\Lambda}$ but on the micro scales ($\sim r_0$) $l_0 \approx r_0$.

4 Discussion and conclusions

We have shown that the nonassociative generalization of supersymmetry has some coefficient that can be associated with some characteristic length. After scaling this dimensionless factor shows how small can be the manifestation of possible nonassociativity in physics. For the first case $\zeta \sim c^3/G$ and the dimensionless $\tilde{\zeta} \sim 1$, which is too large. For the second case the scaled value of this constant is the product of the squared Planck length and the cosmological constant $\zeta \sim \hbar\Lambda$, and consequently is $\approx 10^{-120}$. The possible manifestation of nonassociativity in this case would become apparent in quantum gravity on large scales since $\tilde{\zeta}$ contains $\hbar$, G, and Λ. For the third case the scaled value of this constant is $\zeta \sim G m_e e^2/(c^3\hbar^2)$, and consequently is $\approx 10^{-30}$. The possible manifestation of nonassociativity in this case would become apparent in quantum gravity + electrodynamics on micro scales since $\tilde{\zeta}$ contains $\hbar$, G, e, and m_e.

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