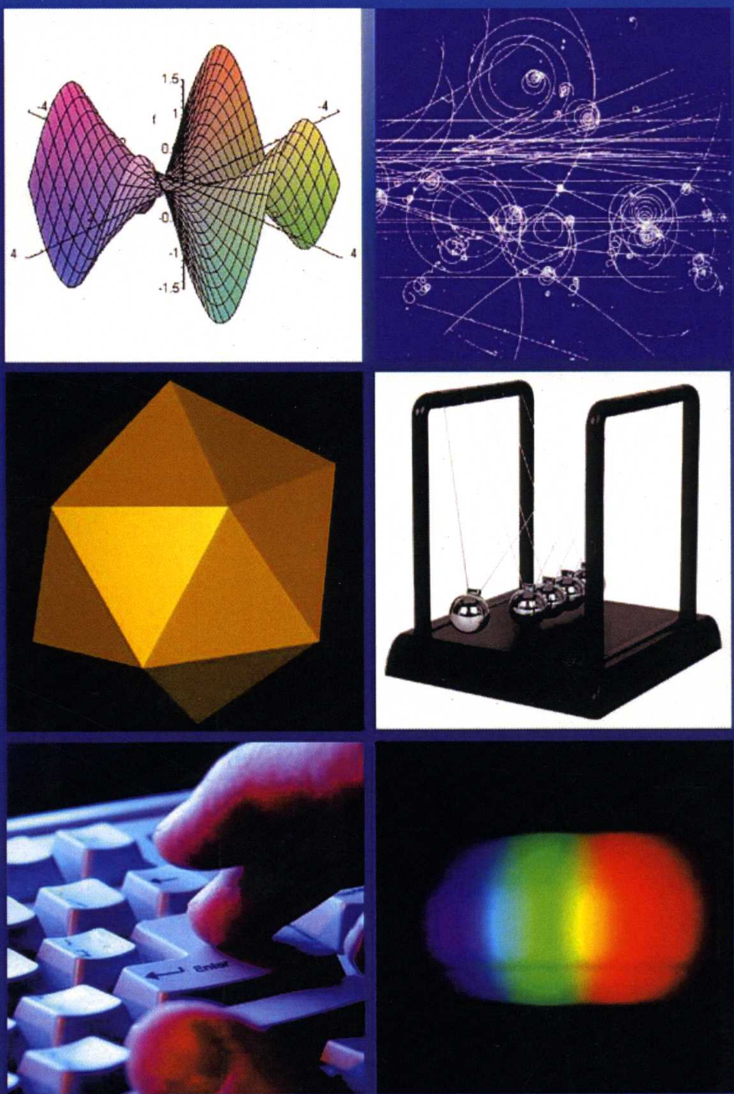
VOLUME 7 NUMBER 1 2016 ISSN 2218-7987

International Journal of Mathematics and Physics



Al-Farabi Kazakh National University

UDC 621.548

[1]Yershin Sh.A., [2]Yershina A.K., [3*]Manatbayev R.K., [3]Tulepbergenov A.K.

[1] Faculty of Mechanic-Mathematical, al-Farabi Kazakh National University, Almaty, Kazakhstan
[2]Kazakh State Women Pedogogical University, Almaty, Kazakhstan
[3] Physico-Technical Faculty, al-Farabi Kazakh National University, Almaty, Kazakhstan
*e-mail: rustem1977@mail.ru

Definition of aerodynamic characteristics wind turbine to Darrieus of system troposcino

Abstract: Recently the majority show interest in connection with a number of merits to vertically-axial wind turbine type to Darrieus. It is possible to tell that age such wind energy installations of 25-30 years whereas others types wind turbine (sailing, propeller) have seriously started to study almost 1,5 centuries. The theory of these wind turbine with sufficient completeness are resulted in known E. M.Fateyev's monograph.
In the present to article results of a theoretical substantiation for one of design kinds wind turbine are stated Darrieus - systems troposcino. This constructive form of the device to Darrieus becomes more and more popular. The general theory is constructed, almost all constructive and aerodynamic characteristics (linear speed, carrying power, front resistance are defined; wind power operating ratio; the moment of rotation of the turbine) this device. Thus, attempt to put basic bases of the theory of devices to Darrieus of system troposcino is made. This report shows main theoretical principles of troposkino wind-turbine. Theoretically were determined dynamical characteristics of the wind-turbine, such as: rotation moment, power, useful wind energy coefficient and physical model of the plant. The theoretical results were compared with well-known experimental data.
Key words: aerodynamic characteristics, wind turbine, wind turbine to Darrieus, system troposcino.

Introduction

The form troposcino (see Fig. 1) is rather close to a parabola described by the equation

$$x = r = r_m - \frac{9z^2}{16r_m} = \frac{3}{8}H - \frac{3}{2}\frac{z^2}{H}, \qquad (1)$$

where $r_m = \frac{3}{8}H$ – the maximum radius of rotation of the turbine (in considered by us of a design), - height. Therefore (1) it is possible to present the formula in a kind

$$r = r_m\left[1-\left(\frac{z}{H/2}\right)^2\right]. \qquad (2)$$

In case of the turbine with direct blades "r" does not depend on "z" and $r_m = r_0$.

The area of the window formed in midlives metrically located pair of blades, is the area m sections of a surface of rotation of the turbine and is equal

$$F = \frac{4}{3}r_m H = \frac{H^2}{2}. \qquad (3)$$

Let's find total length of 2 blades. As it is known, the length $L_{1,2}$ of a curve on $[z_1, z_2]$ a piece is defined by the formula $L_{1,2} = \int_{Z_1}^{Z_2}\sqrt{(dx)^2+(dz)^2}$. From here length of both blades is equal:

$$2L = 4\int_0^{H/2}\sqrt{1+\left(\frac{dx}{dz}\right)^2}dz = 4\int_0^{H/2}\sqrt{1+9\frac{z^2}{H^2}}dz = 2\left[\frac{H}{2}\sqrt{1+\frac{9}{4}}+\frac{H}{3}\ln\left(\frac{3}{2}+\sqrt{1+\frac{9}{4}}\right)\right] \approx 2,6H. \qquad (4)$$

Let's find the moment of force developed two blade by the turbine to Darrieus, executed in a kind troposcino which at us is replaced by a parabola (2). With that end in view we will define in the beginning components of a vector of speed of attack in the M point on the bent element of a wing of infinitesimal length System of co-ordinates it is located so that the direction of speed of a wind coincides with a direction of co-ordinate at, and a vertical axis of rotation wind turbine – with an axis z.

Method of research

Direction rotation wind turbine with some angular speed we will choose so that if to take a detached view of the turbine of a positive direction of an axis z, rotation will be counter-clockwise. We will consider instant position of blades developed concerning an axis x on a corner θ (Fig. 1 see). For simplification of the analysis (Fig. 2 see) we will choose three mutually perpendicular straight lines. Two of them are tangents to a blade surface in M. This is AA' point - a tangent to a parabola line, defines a blade bend, that is a corner γ between a straight line AA' and a vertical NN', and BB' - a tangent in the M point To a circle described in radius at rotation of the turbine. And, at last, the third straight line represents CC' a normal to the blade surface, directed to the M.

The point vector $\vec{V}$ of speed of attack represents a resultant from addition of two vectors. One of them is normal to a surface of the blade a component of a vector of speed of a wind $\vec{U}$ and is equal $\vec{U}\sin\theta\cos\gamma$ (Fig. 2), the second see - is directed on a tangent BB' and represents the sum: linear speed of rotation of a point $(\vec{r}\times\vec{\omega})$ of M plus a component of speed of the wind, designed on a direction $BB'(\vec{U}\cos\theta)$. As a line on BB' which total speed $\vec{r}\times\vec{\omega}+\vec{U}\cos\theta$, and CC' on which the normal component of speed of a wind $\vec{U}\sin\theta\cos\gamma$ operates, are mutually perpendicular, their equally effective it is equal

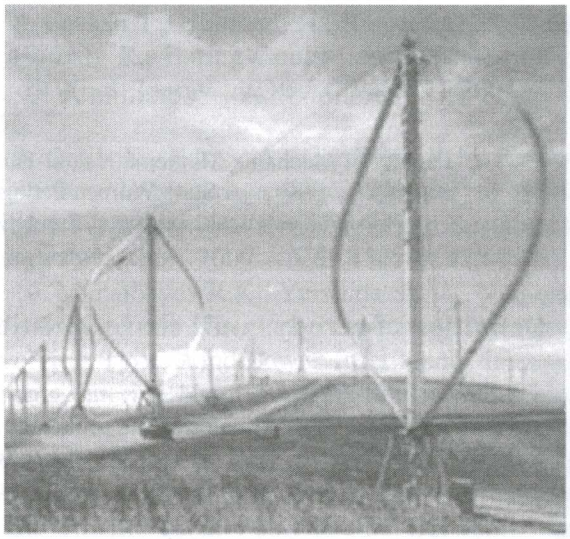


Figure 1 – Wind turbine with the turbine to Darrieus of system troposcino

$$\left|\vec{V}\right| = \sqrt{U^2\sin^2\theta\cos^2\gamma + (r\omega + U\cos\theta)^2} \quad (5)$$

Also gives value of speed of attack of an air stream in point M the answer an angle of attack it will be defined by expression

$$tg\alpha = \frac{U\sin\theta\cos\gamma}{r\omega + U\cos\theta}. \quad (6)$$

From (5) and (6) it is easy to establish connection between an angle of attack and a blade angle of rotation:

$$\begin{aligned} \vec{V}\cos\alpha &= \vec{U}(\chi + \cos\theta) \\ \vec{V}\sin\alpha &= \vec{U}\sin\theta\cos\gamma. \end{aligned} \quad (7)$$

The moment created tangential making carrying power $\vec{R}_l$, is equal $M = rR_\tau = r\left|\vec{R}_l\right|\sin\alpha$. Carrying power $\vec{R}_l$ is directed perpendicularly to a vector of speed of attack and connected with last means of factor of carrying power. Counteracting force is resistance of air to blade movement $\left|\vec{R}_D\right|$.

From here the moment created by an element dS of a wing, registers as follows:

$$dM_l = r\rho \frac{V^2}{2} в \left[C_y \sin\alpha - C_x \cos\alpha \right] dS, \quad (8)$$

Substituting values C_y and C_x for profile NASA-0021 in (8), we will receive

$$M_l = \rho \frac{V^2}{2} вr \left[\sqrt{2}\pi \sin^2\alpha - (0{,}014 + \sin^2\alpha)\cos\alpha \right] dS \quad (9)$$

The decision (9) we will break into 2 separate problems

$$dM_l = dM_{l1} + dM_{l2},$$

where

$$dM_{l1} = \sqrt{2} вr\pi \frac{\rho V^2}{2} \sin^2\alpha dS, \quad (10)$$

$$dM_{l2} = -вr \frac{\rho V^2}{2} (0{,}014 + \sin^2\alpha)\cos\alpha dS. \quad (11)$$

The first problem (10) dares simply enough. Addressing to (7), we will receive

$$dM_{l1} = \sqrt{2}\pi вr(z) \frac{\rho U^2}{2} \sin^2\theta \cos\gamma dz, \quad (12)$$

$$\text{As } \frac{dz}{dS} = \cos\gamma.$$

Using the formula (2), we will receive the following:

$$dM_{l1} = \sqrt{2}\pi в \frac{r_m\left(1-\bar{z}^2\right)H}{2} \rho \frac{U^2}{2} \sin^2\theta \cos\gamma d\bar{z}, \quad (13)$$

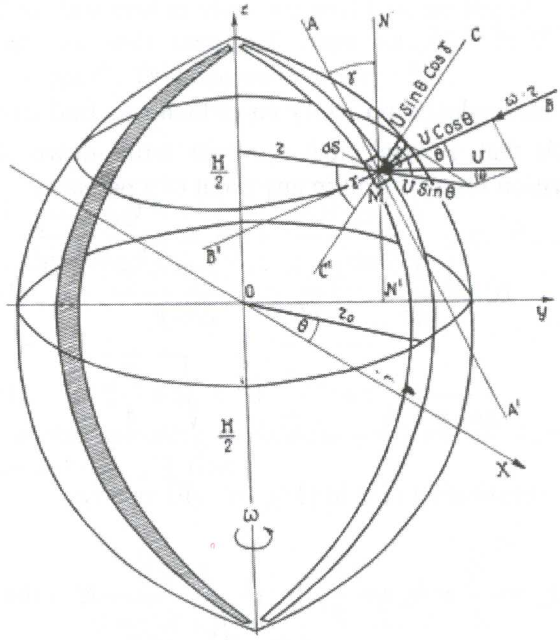


Figure 2 – The scheme of interaction of the blade troposcino with a wind stream

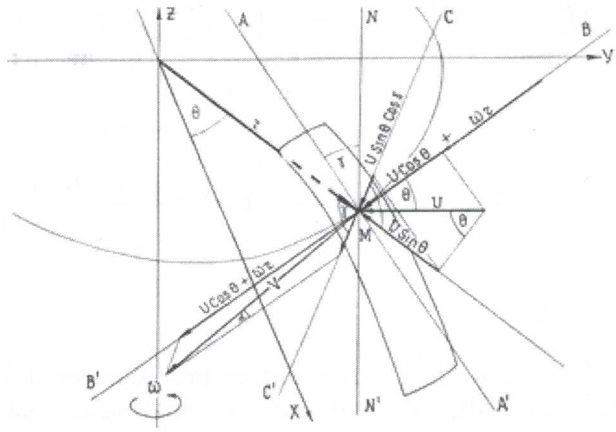


Figure 3 – A blade element troposcino for definition of speed and an angle of attack

where $\bar{z} = \dfrac{z}{H/2}$.

Dependence on $\cos\gamma$ co-ordinate to find easy. With that end in view we will write down the equation for a tangent in any point of a parabola:

$$tg\gamma = \frac{dx}{dz} = -3\frac{z}{H} = \frac{\sqrt{1-\cos^2\gamma}}{\cos\gamma}, \qquad (14)$$

$$\frac{1}{\cos\gamma} = \sqrt{1+9\left(\frac{z}{H}\right)^2} = \sqrt{1+\frac{9}{4}\bar{z}^2}. \qquad (15)$$

Substituting (15) in (13), we will receive

$$dM_{l1} = \frac{\sqrt{2}}{2}\pi H r_m в\rho\frac{U^2}{2}\sin^2\theta\frac{1-\bar{z}^2}{\sqrt{1+9/4\,\bar{z}^2}}d\bar{z} \qquad (16)$$

And, integrating last expression on $\bar{z}$, we will receive size of the rotary moment of the blade

$$M_{l1} = \frac{\sqrt{2}}{4}\pi H r_m в\rho U^2\sin^2\theta\int_{-1}^{1}\frac{1-\bar{z}^2}{\sqrt{1+9/4\,\bar{z}^2}}d\bar{z}\,. \qquad (17)$$

Tabular integral

$$\int_{-1}^{1}\frac{1-\bar{z}^2}{\sqrt{1+9/4\,\bar{z}^2}}d\bar{z} \approx 1{,}37\,.$$

Thus, for two blade turbines we will have:

$$2M_{l1} = 0{,}685\sqrt{2}\pi в r_m H\rho U^2\sin^2\theta. \qquad (18)$$

Average value of the rotary moment at one turn of the turbine we will receive, having integrated $2M_{l1}$ from zero to 2π and having divided on 2π:

$$M_{l1} = \frac{1}{\pi}\int_0^{2\pi}M_{l1}d\theta = 0{,}685\sqrt{2}в r_m H\rho U^2\int_0^{2\pi}\sin^2\theta d\theta$$

$$M_{l1} = 1{,}37\sqrt{2}\pi в r_m H\rho\frac{U^2}{2} \qquad (19)$$

Let's address now to the second problem (11). If to take out from a bracket and to take into consideration (5) and (7) expression (11) will become:

$$dM_{l2} = -в r(z)\frac{\rho U^2}{2}\times\left(1+0{,}014\frac{\sin^2\theta\cos^2\gamma+(\chi+\cos\theta)^2}{\sin^2\theta\cos^2\gamma}\right)\times\frac{\sin^2\theta\cos^2\gamma(\chi+\cos\theta)dS}{\sqrt{\sin^2\theta\cos^2\gamma+(\chi+\cos\theta)^2}}. \qquad (20)$$

Before to pass to the further procedure of the decision (20), we will receive some additional parities and communications. First of all we will notice that in a case troposcino degree of rapidity of the turbine changes on height

$$\chi(z) = \frac{\omega r(z)}{U} \qquad (21)$$

And it brings serious difficulties in the decision of a problem (10). They can be overcome if to take into consideration that the basic contribution to resistance of the air environment to blade movement is brought by the central part troposcino. And it occupies about 85 % of length of the blade. Really, from (2) follows

$$\bar{z} = \frac{z}{H/2} = \sqrt{1-\bar{r}} \qquad (22)$$

where $\bar{r} = \dfrac{r(z)}{r_m}$.

From here it is possible to estimate $\bar{z}$, for example, at reduction $\bar{r}$ in 3 times

$$\bar{z} = \sqrt{0{,}7} \approx 0{,}85. \qquad (23)$$

In view of (21), (22) it is possible to express dependence through size of rapidity of elements of the turbine $\chi(z)$

$$\bar{z} = \sqrt{1-\bar{\chi}}, \qquad (24)$$

where $\bar{\chi} = \dfrac{\chi(z)}{\chi_m}$, $\quad \chi_m = \dfrac{\omega r_m}{U}$.

That circumstance that the basic contribution to turbine work is brought by the central part of blades (85 %) allows to soften nonlinearity of a parity (20).

With that end in view we will remove the brackets and we will take out from under a root size $\chi + \cos\theta$. Then we will receive

$$dM_{l2} = -вr(z)\frac{\rho U^2}{2}\times\left[\frac{1}{\sqrt{1+\left(\frac{\sin\theta\cos\gamma}{\chi+\cos\theta}\right)^2}} + 0,014\left(\frac{\chi+\cos\theta}{\sin\theta\cos\gamma}\right)^2\times\sqrt{1+\left(\frac{\sin\theta\cos\gamma}{\chi+\cos\theta}\right)^2}\right]\sin^2\theta\cos^2\gamma dS$$

The size estimation $\beta = \left(\frac{\sin\theta\cos\gamma}{\chi+\cos\theta}\right)^2$ shows that with reference to the central part troposcino, occupying 85 % of length of blades, it is possible to consider that $\beta << 1$. Really, in view of (21) and (15), we will β present size in a following kind

$$\beta = \left[\frac{\sin\theta}{\left(1+\frac{9}{4}\bar{z}\right)(\cos\theta + \chi_m\bar{\chi})}\right]^2$$

Owing to the accepted assumption $\bar{z} = 0,85$ at $\bar{\chi} = \frac{r}{r_m} = 0,3$ taking into account that the operating conditions wind turbine demand we will $\chi_m \geq 3$, find

$$\beta = \left[\frac{\sin\theta}{\left(1+0,85\frac{9}{4}\right)(\cos\theta + 3\cdot 0,3)}\right]^2$$

Whence follows, what even at the maximum value sinq=1, cosq = 0, It allows to spread out and in numbers $\frac{1}{\sqrt{1-\beta}}$ and $\sqrt{1-\beta}$ to be limited to first two members of these numbers

$$dM_{l2} = -вr(z)\frac{\rho U^2}{2}\times\left[1-\frac{1}{2}\left(\frac{\sin\theta\cos\gamma}{\chi_m\bar{\chi}+\cos\theta}\right)^2 + 0,014\left(\frac{\chi_m\bar{\chi}+\cos\theta}{\sin\theta\cos\gamma}\right)^2 - 0,007\right]\times\sin^2\theta\cos^2\gamma dS = -вr(z)\frac{\rho U^2}{2}\times$$

$$\times\left[1,007\sin^2\theta\cos^2\gamma - \frac{1}{2}\frac{(\sin\theta\cos\gamma)^4}{(\chi_m\bar{\chi}+\cos\theta)^2} + 0,014(\chi_m\bar{\chi}+\cos\theta)^2\right]dS.$$

In square brackets, in comparison with two others, it is possible to neglect obviously, second member. As a result the decision breaks up again to 2 problems

$$dM_{l2} = dM'_{l2} + dM''_{l2}, \qquad (25)$$

where

$$dM'_{l2} = -1,007вr(z)\frac{\rho U^2}{2}\sin^2\theta\cos\gamma dz, \qquad (26)$$

$\frac{dz}{dS} = \cos\gamma$, and

$$dM''_{l2} = -0{,}014 в r(z)\frac{\rho U^2}{2}(\cos\theta + \chi_m \bar{\chi})^2 dS. \quad (27)$$

The decision of the first problem (26) to be reduced to already known decision (18) problems (12) and looks like

$$2M'_{l2} = -0{,}577 r_m в H \frac{\rho U^2}{2}(1 - \cos 2\theta). \quad (28)$$

The problem (27) also easily gives in to the decision. For this purpose on the basis of (2) and (24) we will write down

$$\bar{z} = \frac{2z}{H} = \sqrt{1 - \bar{r}} = \sqrt{1 - \bar{\chi}} \quad (29)$$

Then (27) it is led to a kind

$$dM''_{l2} = -0{,}014 в r_m \bar{r} \rho \frac{U^2}{2}(\cos\theta + \chi_m \bar{r})^2 \frac{dz}{\cos\gamma}$$

$$dM''_{l2} = -0{,}014 \frac{вH}{4} r_m \rho U^2 (1 - \bar{z}^2)\left[\cos\theta + \chi_m (1 - \bar{z}^2)\right]^2 \sqrt{1 + 9/4\, \bar{z}^2}\, d\bar{z} \quad (30)$$

Let's open square brackets in the second degree

$$dM''_{l2} = -0{,}007 в r_m H \rho \frac{U^2}{2}\left[(\cos\theta + \chi_m)^2 (1 - \bar{z}^2) - 2\chi_m \times (\cos\theta + \chi_m)\bar{z}^2 (1 - \bar{z}^2) + \chi_m^2 \bar{z}^4 (1 - \bar{z}^2)\right]\sqrt{1 + \frac{9}{4}\bar{z}^2}\, d\bar{z} \quad (31)$$

For integration we will result (31) in a kind

$$dM''_{l2} = A_0 (A_1 + A_2 y^2 + A_3 y^4 + A_4 y^6)\sqrt{1 + y^2}\, dy,$$

where $y = \frac{3}{2}\bar{z}$, $A_0 = -\frac{0{,}007}{3} в r_m H \rho U^2$, $A_1 = (\cos\theta + \chi_m)^2$,

$$A_2 = -\frac{4}{9}(\cos^2\theta + 4\chi_m \cos\theta + 3\chi_m^2),$$

$$A_3 = \frac{16}{81}(2\chi_m \cos\theta + 3\chi_m^2), \quad A_4 = -\frac{64}{729}\chi_m^2.$$

Integration leads to tabular integrals

$$I_1 = 2\int_0^{3/2\sqrt{0{,}7}} \sqrt{1 + y^2}\, dy = 3{,}12;$$

$$I_2 = 2\int_0^{3/2\sqrt{0{,}7}} y^2\sqrt{1 + y^2}\, dy = 1{,}43;$$

$$I_3 = 2\int_0^{3/2\sqrt{0{,}7}} y^4\sqrt{1 + y^2}\, dy = 2{,}36;$$

$$I_4 = 2\int_0^{3/2\sqrt{0{,}7}} y^6\sqrt{1 + y^2}\, dy = 3{,}34.$$

Integration limits are limited by values $\overline{z}=-\sqrt{0,7}$ and $\overline{z}=\sqrt{0,7}$ (see (23)). As a result for two blade turbines it is had

$$2M''_{l2}=-\frac{0,014}{3}r_m вH\rho U^2\Big[3,12\times(\chi_m+\cos\theta)^2-1,43\frac{4}{9}\times$$
$$\times(\cos^2\theta+4\chi_m\cos\theta+3\chi_m^2)+2,37\frac{16}{81}(2\chi_m\cos\theta+3\chi_m^2)-3,34\frac{64}{729}\chi_m^2\Big]$$

Let's open simple brackets and we will collect similar members. Then we will receive

$$2M''_{l2}=-\frac{0,014}{3}r_m вH\rho U^2\times\left(2,48\cos^2\theta+4,64\chi_m\cos\theta+2,32\chi_m^2\right)$$

Let's unite with the decision (28)

$$2M_{l2}=2(M'_{l2}+M''_{l2})=-\frac{0,014}{3}r_m вH\rho U^2\times\begin{bmatrix}0,577\frac{3}{0,014}(1-\cos 2\theta)+2,48\cos^2\theta+\\+4,63\chi_m\cos\theta+2,32\chi_m^2\end{bmatrix} \quad (32)$$

The received dependence expresses the counteracting moment of resistance of air to turbine rotation. The general rotary moment of the turbine consists of the algebraic sum positive (18) and the negative (32) moments of forces

$$2M_l=2(M_{l1}+M_{l2})=\frac{0,014}{3}r_m вH\rho U^2\times\left\{\left[\left(\frac{0,685\sqrt{2}\pi}{2}-0,577\right)\times\frac{3}{0,014}(1-\cos 2\theta)\right]-1,24(1+\cos 2\theta)-4,63\chi_m\cos\theta-2,32\chi_m^2\right\} \quad (33)$$

Average value of the rotary moment operating on the turbine we will find, having integrated (33) from zero to 2π and 2π having divided on:

$$M_{turb}=\frac{1}{\pi}\int_0^{2\pi}M_l d\theta=\frac{0,028}{3}r_m вH\rho\frac{U^2}{2}\times(177-2,72\chi_m^2) \quad (34)$$

Capacity of the turbine is defined by product of angular speed of rotation of the turbine at the moment of forces

$$N_{turb}=\omega M_{turb}. \quad (35)$$

Substituting here dependence (34) and entering $\chi_m=\frac{\omega r_m}{U}$, we will receive

$$N_{turb}=\frac{0,028}{3}вH\rho\frac{U^3}{2}\chi_m(177-2,72\chi_m^2) \quad (36)$$

From here it is easy to define wind power operating ξ ratio if (35) to divide into capacity of the wind stream which is passing through by the turbine the area F

$$N_в = F\rho\frac{U^3}{2}. \quad (37)$$

Thus

$$\xi = \frac{N_{turb}}{N_в} = \frac{0,028}{3F} вH\chi_m(177 - 2,72\chi_m^2). \quad (38)$$

From last formula we will define value χ_m, at which the maximum value coefficient wind power uses is reached. $\xi_{\max}$ With that end in view we will equate the first derivative ξ on χ_m zero

$$\frac{d\xi}{d\chi_m} = 177 - 8,16\chi_m^2 = 0 \quad (39)$$

From here $\chi_m = 4,66$.

The area surfaces F we will find under the known formula for rotation bodies

$$F = 2\pi\int_{-1}^{1}\bar{r}(\bar{z})\sqrt{1+\left(\frac{d\bar{r}}{d\bar{z}}\right)^2}d\bar{z}.$$

Having substituted expression for $\bar{r}(\bar{z})$ under the formula (2), we will define

$$F = \frac{3\pi}{8}H^2\int_0^1(1-\bar{z}^2)\sqrt{1+\frac{9}{4}\bar{z}^2}d\bar{z}$$

Let's enter a new variable $y = \frac{3}{2}\bar{z}$. Then

$$F = \frac{\pi H^2}{4}\int_0^{3/2}\sqrt{1+y^2}dy - \frac{\pi H^2}{9}\int_0^{3/2}y^2\sqrt{1+y^2}dy.$$

Again we come to tabular integrals I_1 and I_2 (see 28). Calculating these certain integrals, we will receive

$$F = 0,337\pi H^2. \quad (40)$$

Substituting expressions (39) and (40) in the formula (38), we will find

$$\xi_{\max} = 0,363. \quad (41)$$

Value $\xi_{\max}$ and size χ_m at which it is reached $\xi_{\max}$, lie close enough to the known data resulted on Fig. 4. On Fig. 4 skilled values of factor for ξ various types and designs wind turbine depending on degree their rapidity's taken $\chi = \frac{|\vec{\omega}|r_0}{|\vec{U}|}$ from [1,2] are resulted.

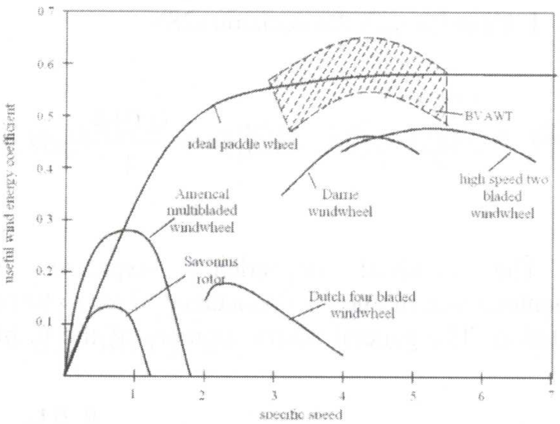


Figure 4 – Dependences of operating ratio of wind power x for various types and designs wind turbine from degree of their rapidity c.

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