

Star-plus-wormhole systems with two interacting scalar fields

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We study static, spherically symmetric mixed configurations with a nontrivial (wormhole) spacetime topology provided by the presence of two interacting ghost scalar fields. Wormhole is assumed to be filled by a perfect relativistic neutron fluid modeled by a polytropic equation of state. For such mixed configurations, we find regular, asymptotically flat general relativistic solutions. It is shown that the maximum of the fluid density is always shifted from the center and the resulting configurations represent, in general, double-throat systems.

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1. Introduction

In recent years interest in obtaining wormhole-type solutions has increased appreciably, primarily because of the discovery of the accelerated expansion of the present universe. At the present time it is widely believed that such an acceleration is due to the presence of some special form of matter, called dark energy (DE).¹ One of the key features of the latter is its ability to violate various energy conditions. In the most extreme case, DE is modeled by matter violating the null energy condition,

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when the effective pressure p of matter filling the Universe is negative and larger (modulus) than its energy density ε , i.e. $p < -\varepsilon$. Such matter is called exotic.

In modeling the accelerated expansion of the Universe, it is usually assumed that DE is distributed homogeneously and isotropically on the largest scales. This, however, does not exclude a possibility that DE might cluster on relatively small scales comparable to sizes of galaxies or even of separate stars. In the latter case, the literature in the field offers compact objects composed of DE (the so-called DE stars), which are modeled by some form of matter possessing the properties of DE.² Such objects may have both a trivial and a nontrivial topology of spacetime. In the latter case, to construct localized solutions, one needs to invoke some form of exotic matter which permits compact configurations to have a nontrivial (wormhole-like) topology. In the simplest case, one can employ the so-called ghost (or phantom) scalar fields, which may be massless³ or possess a potential energy⁴ (other examples of phantom field wormholes can be found, e.g. in Ref. 5, and for a general overview on the subject of Lorentzian wormholes, see Ref. 6).

Another possibility is to consider the case where exotic matter is clustered in the presence of ordinary (baryonic) matter. Such mixed compact configurations, with a nontrivial topology provided by using ghost scalar fields, have been studied in Refs. 7–10. In that case, a wormhole is assumed to be filled by ordinary (neutron) matter and the resulting systems possess properties both of wormholes and of ordinary stars: On one hand, the topology of spacetime is nontrivial and on the other hand, there is a visible surface created by a neutron fluid. Due to the presence of the nontrivial topology, the physical properties (masses, sizes and distributions of matter) of such configurations may differ considerably from those of systems with a trivial topology (for example, ordinary neutron stars).

The possibility to construct objects which acquire new properties in the presence of a nontrivial topology motivates one to study mixed systems supported by other sources of exotic matter. In the simplest case, one can modify the systems of Refs. 7–10 “one scalar field plus ordinary matter” by adding to them one more scalar field interacting with the first one. Systems with two scalar fields are themselves well known in quantum field theory, where they are used to obtain solitary wave solutions.¹¹ When a gravitational field is present, such systems have also been repeatedly considered in the cosmological and astrophysical contexts (see, for example, in Ref. 12, as well as at the beginning of Sec. 2). The presence of a second scalar field allows one to get new interesting solutions, not shared by one-field systems (for example, the cosmological quintom models containing usual and ghost scalar fields and possessing the properties both of quintessence and of phantom models¹). We will show below that the inclusion in our mixed system of the second field will allow us to obtain configurations that possess new physical properties and differ substantially from one-field systems.

Consistent with this, in this paper, we consider mixed astrophysical systems consisting of a wormhole supported by two interacting ghost scalar fields and filled by ordinary matter. Our goal is to clarify the question of how the presence of

such a wormhole changes the distribution of the ordinary matter and influences the physical characteristics of the resulting objects.

The paper is organized as follows. In Sec. 2, we present the statement of the problem and derive the corresponding general-relativistic equations for the mixed systems under consideration. In Sec. 3, we solve these equations numerically for different amount of ordinary matter and compare the systems under consideration with configurations studied earlier. Finally, in Sec. 4, we summarize the results obtained.

2. Statement of the Problem

As pointed out in Introduction, here, we study a mixed gravitating system consisting of two interacting ghost scalar fields φ, χ and a perfect fluid. The Lagrangian for this system can be presented in the form

$$L = -\frac{c^4}{16\pi G}R - \left[\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V(\varphi, \chi) \right] + L_m, \quad (1)$$

with the curvature scalar R and Newton's constant G . Here L_m is the Lagrangian of the perfect isotropic fluid (where isotropic means that the radial and the tangential pressure of the fluid agree), which has the form $L_m = p$.

We choose the scalar field potential energy $V(\varphi, \chi)$ as

$$V(\varphi, \chi) = \frac{\lambda_1}{4}(\varphi^2 - m_1^2)^2 + \frac{\lambda_2}{4}(\chi^2 - m_2^2)^2 + \varphi^2\chi^2 - V_0. \quad (2)$$

Here, m_1 and m_2 are the masses of the scalar fields, λ_1, λ_2 are the coupling constants and V_0 is some constant whose value is chosen from the statement of the problem. (Note that one of these free parameters can always be eliminated by the corresponding rescaling.) Using this potential, we obtained earlier a number of solutions which can be employed both in describing astrophysical objects and when considering cosmological problems. Namely, we have shown that: (i) For the four-dimensional case, there exists regular spherically and cylindrically symmetric solutions,^{13,14} and also cosmological solutions both for usual fields (i.e. for the fields having the usual sign in front of the kinetic energy term) and for ghost scalar fields^{15,16}; (ii) for the higher-dimensional cases, there exist thick brane solutions supported by usual and ghost scalar fields.¹⁷

An important feature of this potential is the presence of two local minima at $\chi = 0$ and $\varphi = \pm m_1$ to which the solutions tend asymptotically at spatial infinity. These local minima corresponds to two vacua. For wormhole-type systems, the solution starts in one of these vacua as $r \rightarrow -\infty$ and returns to it back as $r \rightarrow +\infty$ ¹⁴ (r is the spatial coordinate). Such solutions are called nontopological,¹¹ in contrast to soliton-like solutions with one scalar field when the solution starts in one of the vacua and goes to the other. The latter solutions occurring only in the presence of two or more vacua are called topological. In the present paper, we will discuss only the nontopological solutions.

2.1. Field equations

To describe the mixed equilibrium system under consideration, let us employ the polar Gaussian coordinates, in which the metric has the form

$$ds^2 = e^\nu(dx^0)^2 - dr^2 - R^2d\Omega^2, \tag{3}$$

where ν and R are functions of r only and $x^0 = ct$ is the time coordinate.

The total energy–momentum tensor can be obtained from the Lagrangian (1) in the following form:

$$T_i^k = (\varepsilon + p)u_i u^k - \delta_i^k p - \partial_i \varphi \partial^k \varphi - \partial_i \chi \partial^k \chi - \delta_i^k \left[-\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - V(\varphi, \chi) \right], \tag{4}$$

where ε and p are the energy density and the pressure of the fluid, and u^i is the four-velocity. By using (4), the $\binom{0}{0}$, $\binom{1}{1}$ and $\binom{2}{2}$ components of the Einstein equations with metric (3) take the form

$$- \left[2 \frac{R''}{R} + \left(\frac{R'}{R} \right)^2 \right] + \frac{1}{R^2} = \frac{8\pi G}{c^4} T_0^0 = \frac{8\pi G}{c^4} \left[\varepsilon - \frac{1}{2} (\varphi'^2 + \chi'^2) - V(\varphi, \chi) \right], \tag{5}$$

$$- \frac{R'}{R} \left(\frac{R'}{R} + \nu' \right) + \frac{1}{R^2} = \frac{8\pi G}{c^4} T_1^1 = \frac{8\pi G}{c^4} \left[-p + \frac{1}{2} (\varphi'^2 + \chi'^2) - V(\varphi, \chi) \right], \tag{6}$$

$$\frac{R''}{R} + \frac{1}{2} \frac{R'}{R} \nu' + \frac{1}{2} \nu'' + \frac{1}{4} \nu'^2 = - \frac{8\pi G}{c^4} T_2^2 = \frac{8\pi G}{c^4} \left[p + \frac{1}{2} (\varphi'^2 + \chi'^2) + V(\varphi, \chi) \right], \tag{7}$$

where the prime denotes differentiation with respect to r .

The equations for the scalar fields φ and χ result from the Lagrangian (1) in the general form as

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left[\sqrt{-g} g^{\mu\nu} \frac{\partial(\varphi, \chi)}{\partial x^\nu} \right] = - \frac{\partial V}{\partial(\varphi, \chi)}. \tag{8}$$

In the metric (3), these equations yield

$$\varphi'' + \left(\frac{1}{2} \nu' + 2 \frac{R'}{R} \right) \varphi' = \frac{dV}{d\varphi}, \tag{9}$$

$$\chi'' + \left(\frac{1}{2} \nu' + 2 \frac{R'}{R} \right) \chi' = \frac{dV}{d\chi}. \tag{10}$$

Finally, the hydrostatic equation for the fluid can be obtained from the law of the conservation of energy and momentum, $T_{i;k}^k = 0$. The $i = 1$ component of this equation gives

$$\frac{dT_1^1}{dr} + \frac{1}{2} (T_1^1 - T_0^0) \nu' + 2 \frac{R'}{R} \left[T_1^1 - \frac{1}{2} (T_2^2 + T_3^3) \right] = 0. \tag{11}$$

Taking the components of the energy–momentum tensor from Eqs. (5)–(7) and also taking into account that $T_3^3 = T_2^2$, from (11) we have

$$\frac{dp}{dr} = -\frac{1}{2}(\varepsilon + p)\frac{d\nu}{dr}. \quad (12)$$

The mixed system under investigation can be regarded as follows. Initially, we have a wormhole geometry provided by two interacting ghost scalar fields. In the absence of ordinary matter (fluid), such systems have been considered in Ref. 14. By adding a fluid to the system, we consider the influence that a wormhole has on the spatial distribution of the fluid and on the physical characteristics (masses, sizes) of the resulting mixed systems.

As the fluid one can choose any type of matter used in modeling astrophysical objects (for instance, stars). One of the simplest ways to describe star’s matter is to employ a polytropic fluid. The latter is applied both in describing nonrelativistic objects (Newtonian stars, see, e.g. Ref. 18) and when considering relativistic systems, including those with a relativistic neutron fluid in a strong gravitational field. In doing so, a polytropic equation of state (EOS) can adequately represent more realistic EOSs used in modeling neutron stars (see, e.g. Ref. 19).

For our purpose, we choose the following relativistic EOS:

$$p = K\rho_b^{1+1/n}, \quad \varepsilon = \rho_b c^2 + np, \quad (13)$$

with the constant $K = kc^2(n_b^{(ch)}m_b)^{1-\gamma}$, the polytropic index $n = 1/(\gamma - 1)$ and $\rho_b = n_b m_b$ denotes the rest-mass density of the neutron fluid. Here n_b is the baryon number density, $n_b^{(ch)}$ is a characteristic value of n_b , m_b is the baryon mass and k and γ are parameters whose values depend on the properties of the neutron matter. In particular, we choose $m_b = 1.66 \times 10^{-24}$ g, $n_b^{(ch)} = 0.1 \text{ fm}^{-3}$, $k = 0.1$ and $\gamma = 2$.²⁰

With the EOS in the form of (13), one can integrate Eq. (12). To do this, it is convenient to introduce the new variable θ ,¹⁸

$$\rho_b = \rho_{bc}\theta^n, \quad (14)$$

where ρ_{bc} is some characteristic density of the neutron fluid. For the mixed configurations with an isotropic fluid considered by us earlier in Refs. 7 and 8 ρ_{bc} corresponds to the central (maximum) density at the wormhole throat. However, for systems with an anisotropic fluid a situation is possible where the maximum density of the neutron fluid is shifted away from the center.¹⁰ Then ρ_{bc} no longer plays the role of maximum density, but it is just some characteristic value for the configuration under consideration. As we will see below, here we deal with such a situation even in the case of an isotropic fluid.

Making use of expression (14) we have from Eq. (12)

$$2\sigma(n+1)\frac{d\theta}{dr} = -[1 + \sigma(n+1)\theta]\frac{d\nu}{dr}, \quad (15)$$

where $\sigma = K\rho_{bc}^{1/n}/c^2$ is the relativistic parameter.²¹ Integrating this equation, one can find

$$\theta = \frac{1}{\sigma(n+1)} \{ [1 + \sigma(n+1)\theta_c] e^{(\nu_c - \nu)/2} - 1 \}, \quad (16)$$

where e^{ν_c} is the value of e^ν at the center. The integration constant ν_c is fixed by requiring that the spacetime is asymptotically flat, i.e. $e^\nu = 1$ at infinity. In turn, the arbitrary constant θ_c , contained in the boundary conditions (24), corresponds to the central value of the function θ .

For the numerical calculations, it is convenient to rewrite the obtained equations in terms of dimensionless variables. Since below we will seek solutions with zero central values of the scalar field derivatives, $\varphi'(0), \chi'(0) = 0$ and a nonzero central value of the potential energy, $V(\varphi(0), \chi(0)) \neq 0$, we normalize the characteristic size of the system L with respect to this value. Namely, let us introduce dimensionless variables

$$\xi = \frac{r}{L}, \quad \Sigma = \frac{R}{L}, \quad \tilde{\varphi}(\xi), \tilde{\chi}(\xi), \mu_{1,2} = \frac{\sqrt{8\pi G}}{c^2} \varphi(r), \chi(r), m_{1,2}, \quad \text{where} \\ L = \frac{c^2}{\sqrt{8\pi G |V(\varphi(0), \chi(0))|}}. \quad (17)$$

In such variables Eqs. (5)–(7), (9) and (10) take the form

$$- \left[2 \frac{\Sigma''}{\Sigma} + \left(\frac{\Sigma'}{\Sigma} \right)^2 \right] + \frac{1}{\Sigma^2} = B(1 + \sigma n \theta) \theta^n - \frac{1}{2} (\tilde{\varphi}'^2 + \tilde{\chi}'^2) - \tilde{V}, \quad (18)$$

$$- \frac{\Sigma'}{\Sigma} \left(\frac{\Sigma'}{\Sigma} + \nu' \right) + \frac{1}{\Sigma^2} = -B\sigma\theta^{n+1} + \frac{1}{2} (\tilde{\varphi}'^2 + \tilde{\chi}'^2) - \tilde{V}, \quad (19)$$

$$\frac{\Sigma''}{\Sigma} + \frac{1}{2} \frac{\Sigma'}{\Sigma} \nu' + \frac{1}{2} \nu'' + \frac{1}{4} \nu'^2 = B\sigma\theta^{n+1} + \frac{1}{2} (\tilde{\varphi}'^2 + \tilde{\chi}'^2) + \tilde{V}, \quad (20)$$

$$\tilde{\varphi}'' + \left(\frac{1}{2} \nu' + 2 \frac{\Sigma'}{\Sigma} \right) \tilde{\varphi}' = \tilde{\varphi} [2\tilde{\chi}^2 + \Lambda_1(\tilde{\varphi}^2 - \mu_1^2)], \quad (21)$$

$$\tilde{\chi}'' + \left(\frac{1}{2} \nu' + 2 \frac{\Sigma'}{\Sigma} \right) \tilde{\chi}' = \tilde{\chi} [2\tilde{\varphi}^2 + \Lambda_2(\tilde{\chi}^2 - \mu_2^2)], \quad (22)$$

where the dimensionless potential energy is

$$\tilde{V} = \frac{\Lambda_1}{4} (\tilde{\varphi}^2 - \mu_1^2)^2 + \frac{\Lambda_2}{4} (\tilde{\chi}^2 - \mu_2^2)^2 + \tilde{\varphi}^2 \tilde{\chi}^2 - \tilde{V}_0. \quad (23)$$

Here, $B = (\rho_{bc} c^2)/|V(\varphi(0), \chi(0))|$ is the dimensionless ratio of the characteristic fluid energy density to that of the scalar field at the center; the dimensionless coupling constants $\Lambda_{1,2} = (c^4/8\pi G \sqrt{|V(\varphi(0), \chi(0))|})^2 \lambda_{1,2}$ and $\tilde{V}_0 = V_0/|V(\varphi(0), \chi(0))|$.

Note that, using L from (17), the expression for B can be recast in the following form: $B = 8\pi G\rho_{bc}(L/c)^2$. This expression will be used below in performing numerical calculations. In the case of $B = 0$, we deal with a system consisting of a pure scalar field configuration with no ordinary matter.¹⁴

2.2. Boundary conditions

Here, we consider neutron-star-plus-wormhole configurations that are asymptotically flat and symmetric under $\xi \rightarrow -\xi$. The metric function $\Sigma(\xi)$ may be considered as a circumferential radial coordinate. Asymptotic flatness requires that $\Sigma(\xi) \rightarrow |\xi|$ for large $|\xi|$. Because of the assumed symmetry, the center of the configurations at $\xi = 0$ should correspond to an extremum of $\Sigma(\xi)$, i.e. $\Sigma'(0) = 0$. If $\Sigma(\xi)$ has a minimum at $\xi = 0$, then $\xi = 0$ corresponds to the throat of the wormhole. If, on the other hand, $\Sigma(\xi)$ has a local maximum at $\xi = 0$, then $\xi = 0$ corresponds to an equator. In that case, the wormhole will have a double-throat surrounding a belly (for examples of double-throat systems, see, e.g. Refs. 9 and 22).

Consistent with this, Eqs. (18)–(22), together with the expression (16), will be solved for given parameters of the fluid σ , n and B , subjected to the boundary conditions at the center of the configuration $\xi = 0$,

$$\begin{aligned} \theta(0) &= \theta_c, & \Sigma(0) &= \Sigma_c, & \Sigma'(0) &= 0, & \nu(0) &= \nu_c, & \nu'(0) &= 0, \\ \tilde{\varphi}(0) &= \tilde{\varphi}_c, & \tilde{\varphi}'(0) &= 0, & \tilde{\chi}(0) &= \tilde{\chi}_c, & \tilde{\chi}'(0) &= 0, \end{aligned} \quad (24)$$

where $\theta_c, \Sigma_c, \tilde{\varphi}_c$ and $\tilde{\chi}_c$ are some constants.

To find the location of the throat, expand the metric function Σ in the neighborhood of the center as

$$\Sigma \approx \Sigma_c + \frac{1}{2}\Sigma_2\xi^2.$$

Then, using Eqs. (18) and (19), we find the relations

$$\Sigma_c = \frac{1}{\sqrt{-B\sigma\theta_c^{n+1} - \tilde{V}(\tilde{\varphi}_c, \tilde{\chi}_c)}}, \quad \Sigma_2 = -\frac{\Sigma_c}{2}B\theta_c^n[1 + (n+1)\sigma\theta_c]. \quad (25)$$

It is seen from these expressions that: (i) to obtain physically reasonable solutions, one needs to choose the free parameters of the system in such a way that the radicand in the first formula be positive, and (ii) at $\theta_c > 0$ (as it should be for ordinary matter) Σ_2 is negative, corresponding to a solution with an equator surrounded by a double-throat.

3. Numerical Results

In Ref. 14, we studied wormhole systems supported by ghost scalar fields with the potential (2). Here, we modify that system by adding to it ordinary matter in the form of a polytropic fluid. Our aim is to study such mixed configurations, in particular, with respect to the effect that the presence of the nontrivial topology has on the distribution of the neutron matter along the radius of the system.

We will solve the system of Eqs. (18)–(22) together with (16) numerically using the boundary conditions (24) and (25). Since a characteristic property of a gravitating polytropic fluid is the presence of an edge where its pressure (density) goes to zero, the configurations under consideration can be subdivided into two regions: (i) The internal one, where both the scalar fields and the fluid are present; (ii) the external one, where only the scalar fields are present. Correspondingly, the solutions in the external region are obtained by using Eqs. (18)–(22), in which θ is set to zero.

The internal solutions must be matched with the external ones at the boundary of the fluid, $\xi = \xi_b$, by equating the corresponding values of the functions $\tilde{\varphi}$, $\tilde{\chi}$, Σ , ν and their derivatives. The boundary of the fluid ξ_b is defined by $p(\xi_b) = 0$. Knowledge of the asymptotic solutions in turn allows one to determine the value of the integration constant ν_c at the center, proceeding from the requirement of asymptotic flatness of the external solutions.

As in the case without ordinary matter studied in Ref. 14, the system of Eqs. (18)–(22) has regular solutions (satisfying the necessary boundary conditions) only for certain values of the coupling constants Λ_1, Λ_2 and of the masses of the scalar fields μ_1, μ_2 . As a result, the problem reduces to a search for *eigenvalues* of the parameters μ_1, μ_2 and for the corresponding *eigenfunctions* $\nu, \Sigma, \tilde{\varphi}$, and $\tilde{\chi}$ of the nonlinear system of differential equations (18)–(22). We will seek the specified eigenvalues by using the shooting method. A step-by-step description of the procedure for finding solutions can be found in Refs. 14 and 15.

Proceeding in this way, we have obtained the results shown in Figs. 1–3. The parameters of the system have been chosen so that the masses of the mixed configurations under consideration be of the order of 1–2 solar masses and their sizes ~ 20 km. These characteristics are typical for neutron stars.²³

The results were obtained by choosing the following fixed values of the parameters: The characteristic fluid density $\rho_{bc} \simeq 10^{14} \text{ g cm}^{-3}$ and its size $L \simeq 10$ km, the coupling constants $\Lambda_1 = 0.1$ and $\Lambda_2 = 1$ and the central values of the scalar fields $\tilde{\varphi}_c = 5$ and $\tilde{\chi}_c = 1.1$. This is allowed to get configurations with the required characteristics, as shown in Table 1.

As seen in Fig. 3, $\tilde{\varphi} \rightarrow \mu_1$ and $\tilde{\chi} \rightarrow 0$ at large distances, i.e. the solutions approach asymptotically the local minimum of the potential (23). The arbitrary constant \tilde{V}_0 is chosen so that the value of the potential (23) be equal to zero at the local minimum, i.e. we take $\tilde{V}_0 = \Lambda_2 \mu_2^4 / 4$. Such a choice ensures a zero value for the scalar field energy density as $\xi \rightarrow \pm\infty$, as one can see from Fig. 1.

The asymptotic behavior of the scalar fields can be found in the following form:

$$\tilde{\varphi} = \mu_1 - \delta\tilde{\varphi}, \quad \tilde{\chi} = \delta\tilde{\chi}, \quad (26)$$

where $\delta\tilde{\varphi}, \delta\tilde{\chi} \ll 1$ behave as

$$\delta\tilde{\varphi} \simeq C_\varphi \frac{\exp\left(-\sqrt{2\Lambda_1\mu_1^2}\xi\right)}{\xi}, \quad \delta\tilde{\chi} \simeq C_\chi \frac{\exp\left(-\sqrt{(2\mu_1^2 - \Lambda_2\mu_2^2)}\xi\right)}{\xi}, \quad (27)$$

where C_φ and C_χ are integration constants.

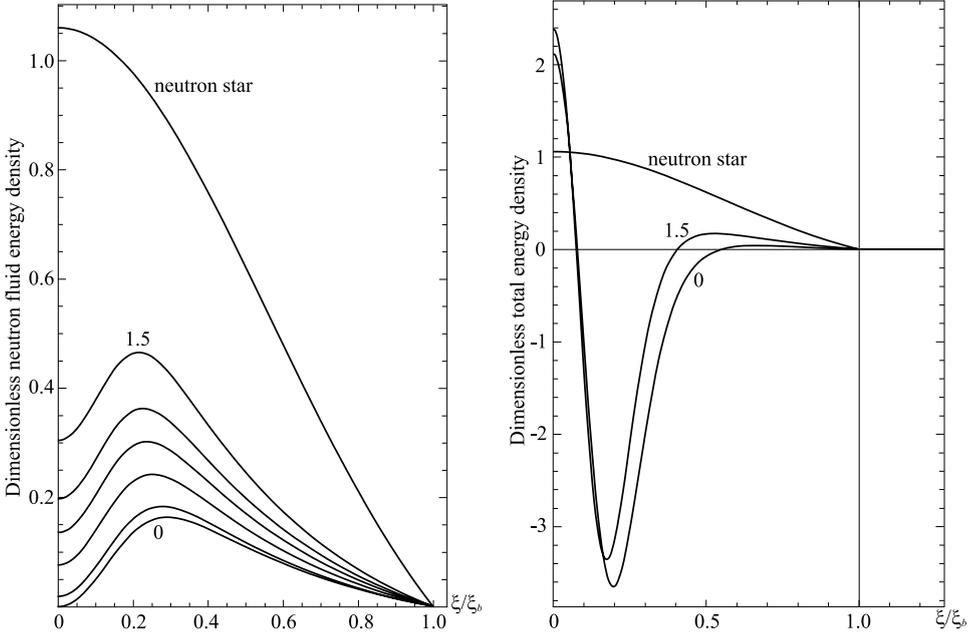


Fig. 1. The fluid energy density $B(1 + \sigma n \theta) \theta^n$ (left panel) and the total energy density T_0^0 (right panel) from the right-hand side of Eq. (18) (in units of $|V(\varphi(0), \chi(0))|$) are shown as functions of the relative radius ξ/ξ_b . Since the solutions with the wormhole are symmetric with respect to $\xi = 0$, the graphs are shown only for $\xi > 0$. In the left panel, the parameter θ_c runs the values $0, 0.1, 0.4, 0.7, 1.0$ and 1.5 , from bottom to top. In the right panel, all the remaining curves with $0 < \theta_c < 1.5$ lie between the presented curves. The thin vertical line corresponds to the boundary of the fluid.

Table 1. Characteristics of the mixed configurations at fixed $\rho_{bc}, L, \Lambda_1, \Lambda_2, \tilde{\varphi}_c$ and $\tilde{\chi}_c$ (see in the text). Here, the central $\rho_c = \rho_{bc} \theta_c^n$ and the maximum ρ_{\max} densities of the neutron fluid (both in units of $10^{14} \text{ g cm}^{-3}$), the total mass M_{tot} , the proper mass M_{prop} of the neutron matter (both in solar mass units M_\odot), the radius of the neutron fluid R (in kilometers) and the masses of the scalar fields μ_1 and μ_2 are shown. For the neutron star with the same values of the polytropic parameters and ρ_{bc} , we have $M_{\text{tot}} = 1.876 M_\odot$ and $R = 32.812 \text{ km}$.

ρ_c	ρ_{\max}	M_{tot}	M_{prop}	R	μ_1	μ_2
0.0	0.838	1.087	0.526	18.555	5.97803	7.67266
0.1	0.932	1.173	0.623	19.201	5.97824	7.67252
0.4	1.213	1.419	0.902	20.437	5.97884	7.67213
0.7	1.489	1.631	1.147	20.980	5.97940	7.67175
1.0	1.761	1.805	1.351	21.132	5.97994	7.67140
1.5	2.206	2.022	1.613	20.930	5.98078	7.67084

The total masses of the configurations shown in Table 1 were calculated as follows. Consider a volume enclosed by a sphere with circumferential radius R_c , corresponding to the center of the configuration and another sphere with circumferential radius $R > R_c$. Then, using the spherically symmetric metric (3), the mass

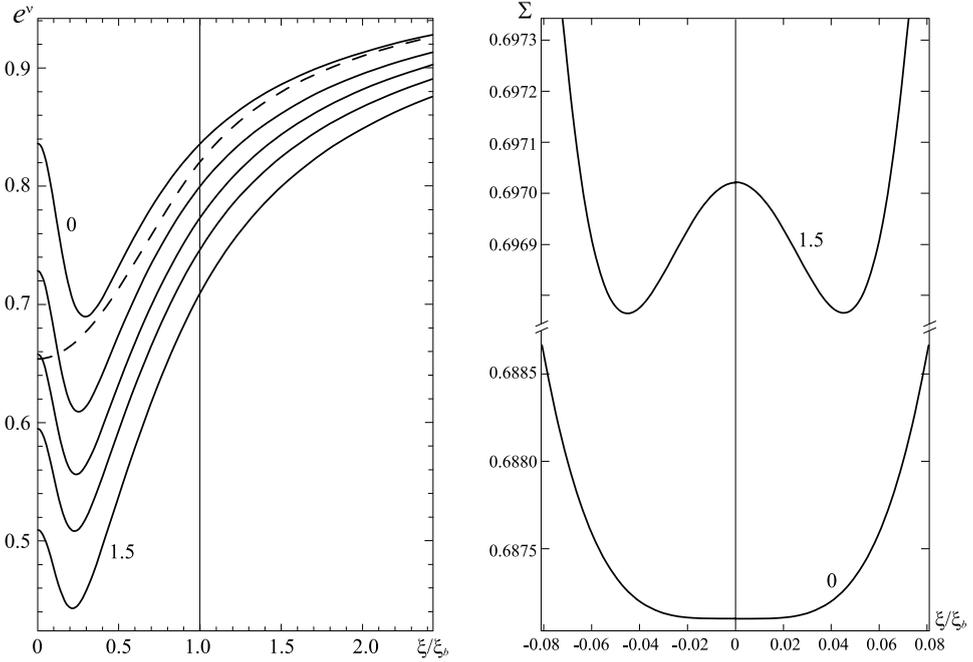


Fig. 2. The metric functions $g_{tt} = e^\nu$ (left panel) and Σ (right panel) are shown as functions of the relative radius ξ/ξ_b . In the left panel, the thin vertical line corresponds to the boundary of the fluid; the dashed line refers to the neutron star; for the mixed configurations, the parameter θ_c runs the values 0, 0.4, 0.7, 1.0 and 1.5, from top to bottom. Asymptotically, as $\xi \rightarrow \pm\infty$, the spacetime is flat with $\Sigma \rightarrow |\xi|$ and $e^\nu \rightarrow 1$ from below.

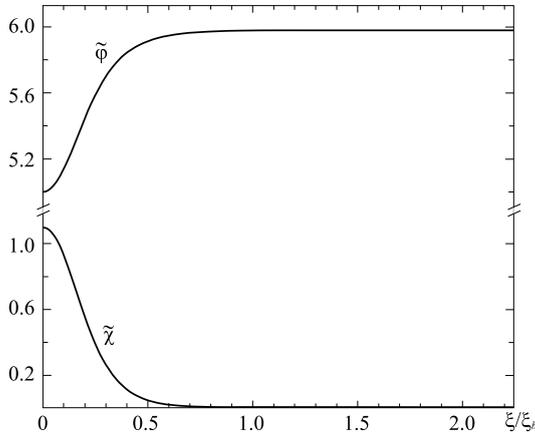


Fig. 3. The typical behavior of the scalar fields. Asymptotically, as $\xi \rightarrow \pm\infty$, the field $\tilde{\varphi}$ tends to μ_1 and $\tilde{\chi}$ goes to 0 [see Table 1 and Eqs. (26) and (27)].

$m(r)$ associated with this volume can be defined as follows:

$$m(r) = \frac{c^2}{2G}R_c + \frac{4\pi}{c^2} \int_{R_c}^r T_0^0 R^2 dR. \quad (28)$$

For single-throat configurations, the circumferential radius R_c corresponds to the radius of the wormhole throat defined by $R_c = \min\{R(r)\}$. However, for the systems considered here, a single-throat configuration will exist only when $\theta_c = 0$. When $\theta_c \neq 0$, as mentioned above, R_c corresponds to an equator, while the two throats are located symmetrically away from the center (see in the right panel of Fig. 2).

In the dimensionless variables of Eqs. (17) and (14) the expression (28) takes the form

$$m(\xi) = M^* \left\{ \Sigma_c + \int_0^\xi \left[B(1 + \sigma n \theta) \theta^n - \frac{1}{2} (\tilde{\varphi}'^2 + \tilde{\chi}'^2) - \tilde{V} \right] \Sigma^2 \frac{d\Sigma}{d\xi'} d\xi' \right\}, \quad (29)$$

where the coefficient M^* in front of the curly brackets has the dimension of mass

$$M^* = \frac{c^3}{2} \sqrt{\frac{B}{8\pi G^3 \rho_{bc}}}.$$

Note that, formally, the total mass M_{tot} is then obtained by taking the upper limit of the integral to infinity, since the energy density of the scalar fields becomes equal to zero only asymptotically, as $\Sigma \rightarrow \infty$. However, since the scalar fields decay exponentially, see Eq. (27), the integral converges rapidly to its asymptotic value even at small $\xi \sim \mathcal{O}(1)$. Note also that in evaluating the above integral it is necessary to perform the calculations in the internal and external regions separately.

One more interesting characteristic of the configurations under consideration is the proper mass M_{prop} of the neutron matter

$$M_{\text{prop}} \equiv m_b N = 4\pi \int_0^{r_b} \rho_b R^2 dr = 4\pi \rho_{bc} L^3 \int_0^{\xi_b} \theta^n \Sigma^2 d\xi. \quad (30)$$

M_{prop} is equal to the mass which the baryons of the system would possess altogether, if they were dispersed throughout a volume so large that all types of interactions between them could be neglected. Evaluating the expression (30) for the proper mass, we can find the number of neutrons N in the system.

From the results obtained, it is possible to delineate the following features of the systems under consideration:

- (i) For the values of the parameters used here, it is seen from Table 1 that while the central value of the neutron fluid density increases, the masses of the scalar fields remain practically unchanged. This corresponds to the fact that the spatial distribution of the scalar fields changes also very slightly for different amount of the neutron matter. In this sense, the scalar fields can be regarded as the background ones. But, the total mass of the configuration varies substantially that is obviously caused by a change in the number of neutrons N in the system (see Table 1).

- (ii) The presence of the ordinary matter results in the following changes in the geometry of the system: (a) As the amount of the neutron matter increases, the redshift function $g_{tt} = e^\nu$ increasingly differs from 1 (see Fig. 2), i.e. the system becomes more and more relativistic; (b) at $\theta_c \neq 0$ two weakly marked throats are necessarily present in the system (see in the right panel of Fig. 2), in contrast to systems supported by scalar fields only where just one throat is present.
- (iii) One more interesting feature associated with the presence of the two scalar fields is a unique shape of the neutron matter distribution along the radius. The literature in the field offers different systems with an isotropic fluid having a maximum density at the center, like that illustrated by the curve labeled by “neutron star” in Fig. 1. This applies both to systems with a trivial topology of spacetime (ordinary stars) and to the mixed systems of the “wormhole plus ordinary matter” type of Refs. 7 and 8. When a system contains an anisotropic fluid, its maximum density may already be located somewhere between the center and the edge of the system (see Ref. 24 concerning ordinary stars and Ref. 10 where a mixed system is discussed). For the mixed configurations considered here such a shift of the maximum fluid density takes place even for isotropic matter, and the central density of the neutron matter can vary from 0 to the values of the order of nuclear density (see Table 1 and Fig. 1).

Of course, the obtained quantitative characteristics (masses, sizes and shapes of the neutron matter distribution) depends essentially on the specific values of scalar field parameters (coupling constants and boundary conditions) and of ordinary matter characteristic densities and sizes. But one might expect that the qualitative behavior of the solutions will remain the same even with a different, physically reasonable choice of the parameters of the system under consideration.

4. Conclusion

We have studied equilibrium mixed configurations consisting of a wormhole supported by two interacting ghost scalar fields and threaded by a relativistic polytropic neutron fluid. The scalar fields play the role of exotic matter providing a nontrivial topology of spacetime in the system.

Our goal was to investigate the influence that such a nontrivial topology has on the distribution of the neutron matter and on physical characteristics of resulting objects. In doing so, we compared the obtained mixed configurations with a neutron star modeled by matter with the same EOS. Having fixed the polytropic parameters and coupling constants of the scalar fields, we kept track of changes in the distribution of the neutron matter depending on the central value of the neutron matter rest-mass density.

The main results can be summarized as follows:

- (i) There exist static regular asymptotically flat solutions describing mixed neutron-star-plus-wormhole systems in which the neutron matter is

concentrated in a finite-size region. Such configurations may be regarded as consisting of a neutron star having either a single-throat at the center (at zero central density of the neutron matter) or an equator surrounded by a double-throat (see Fig. 2).

- (ii) Due to the presence of two scalar fields, the neutron fluid always has a maximum density somewhere between the center of the configuration and the edge of the fluid. In this case, the central density of the neutron matter can vary from 0 to the values of the order of nuclear density (see Table 1 and Fig. 1). Such a behavior, which is usually characteristic of anisotropic fluids, is attained here even in the case of isotropic fluid. This is because of the characteristic behavior of the redshift function $g_{tt} = e^\nu$ which, in systems with two scalar fields, has a minimum shifted away from the center (see Fig. 2, and also the pure scalar field systems of Ref. 14).

Altogether, as regards the form of the neutron matter distribution, the mixed systems “two scalar fields plus isotropic matter” discussed here are similar to the mixed systems “one scalar field plus anisotropic matter” of Ref. 10, with the main difference related to the behavior of the redshift function. On the other hand, the mixed two-field systems considered here differ in principle from the mixed one-field systems of Refs. 7 and 8, for which the maximum of the neutron fluid density is *always* located at the center of a configuration.

In conclusion, we may say a little regarding the question of stability of the considered mixed systems. The results available in the literature indicate that for pure field configurations (with no ordinary matter) wormhole solutions are linearly^{25–27} and nonlinearly^{28,29} unstable (see, however, Ref. 30 where a method of obtaining general-relativistic stable wormhole solutions is presented). As shown in our previous studies of mixed “wormhole plus star” systems with one scalar field,^{8–10} the presence of ordinary matter did not result in stabilization of the configuration. One might expect that the inclusion in the system of one more ghost scalar field will not change the stability situation. Nevertheless, this issue requires special study and we plan to perform such a stability analysis later on.

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