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Methodology of Projection of Wave Functions of Light Nuclei on Cluster Channels on the Example of Quantum $^{11}B\{\alpha\text{-}^{7}Li\}$-System

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Abstract

In the present paper the authors investigate the cluster structure of ^{11}B nucleus by a method of projection of its three-body wave function on the cluster channel ^{7}Li + α. An estimation of the wave function of ^{11}B nucleus in the three-body $\alpha\alpha t$-model on the cluster channel ^{7}Li $\{\alpha t\}$ + α has been obtained. It is shown that the account of only one configuration in the wave function of ^{11}B nucleus does not describe completely the cluster structure of this nucleus.

Keywords: light nuclei, cluster structure, many-particle shell model, wave function, ^{11}B, ^{7}Li, projection

1 Introduction

The projection includes several steps of transformations, knowledge and ability of which are necessary for investigations of the light nuclei. For estimation of the wave function of ^{11}B nucleus let's project the wave function of this nucleus in the three-body {ααt}-model on the cluster channel ^{7}Li + α. The nuclei under consideration have the following quantum numbers of spin, parity and isospin in the ground state (fig. 1) [1]:

$$^{11}\mathrm{B}_{g.s.}\left(\frac{3}{2}^-,\frac{1}{2}\right);\quad ^{7}\mathrm{Li}_{g.s.}\left(\frac{3}{2}^-,\frac{1}{2}\right);\quad \begin{matrix}\lambda=0,\\ \ell=1;\end{matrix}\quad \vec{\lambda}+\vec{\ell}=\vec{L};\quad \begin{matrix}L=1,\\ \Lambda=0,2.\end{matrix}$$

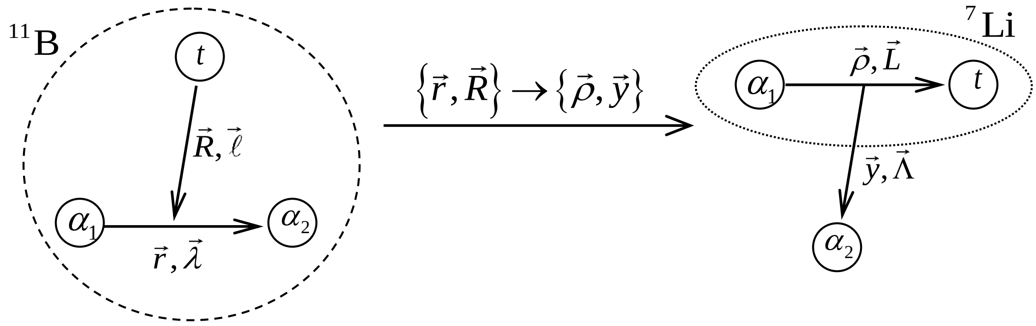

Figure 1 – Relative Jacobi coordinates for the channel ^{11}B → ^{7}Li + α

According to the many-particle shell model [2] the wave function of the ground state of the ^{11}B nucleus has the configuration $(1s)^4(1p)^7$, that is it contains N = 7 quanta of excitation when decaying by the channel: ^{11}B → ^{7}Li ($(1s)^4(1p)^3$) + α ($(1s)^4$). There are N = 4 quanta for the relative motion of nuclei in the final state and the wave function of the relative motion has the shell $R_{4\Lambda}$ form. The radial wave function of ^{11}B nucleus has two components: $R_{40}\equiv|S\rangle$ and $R_{42}\equiv|D\rangle$.

For the realization of the procedure of projection it is necessary to know the formulas of transition from the set of $\{\vec{\rho},\vec{y}\}$-coordinates, when ^{7}Li nucleus and α-particle are given, to the set of $\{\vec{R},\vec{r}\}$-coordinates, when the ^{11}B nucleus coordinates are given (fig. 1) and vice-versa:

$$\begin{cases}\vec{r}=\vec{r}_{\alpha_1}-\vec{r}_{\alpha_2},\\ \vec{R}=\frac{1}{8}\left(4\vec{r}_{\alpha_1}+4\vec{r}_{\alpha_2}\right)-\vec{r}_t,\end{cases}\qquad \begin{cases}\vec{\rho}=\vec{r}_{\alpha_1}-\vec{r}_t,\\ \vec{y}=\left(\frac{4}{7}\vec{r}_{\alpha_1}+\frac{3}{7}\vec{r}_t\right)-\vec{r}_{\alpha_2}.\end{cases}\qquad (1)$$

Then the transition from one Jacobi coordinates to another is realized by the formulas:

$$\begin{cases}\vec{\rho}=\frac{1}{2}\vec{r}+\vec{R},\\ \vec{y}=\frac{11}{14}\vec{r}-\frac{3}{7}\vec{R},\end{cases}\qquad \begin{cases}\vec{r}=\vec{y}+\frac{3}{7}\vec{\rho},\\ \vec{R}=\frac{11}{14}\vec{\rho}-\frac{1}{2}\vec{y}.\end{cases}\qquad (2)$$

2 Wave functions

Let's write down the relative radial wave function of the $^{11}\mathrm{B}\{\alpha\alpha t\}$ nucleus [3]:

$$\Phi^{^{11}B}_{\lambda,\ell}\left(\vec{r},\vec{R}\right)=N\cdot\vec{r}^{\,\lambda}\cdot\vec{R}^{l}\sum_{j}C_j\exp\left(-\alpha_j\vec{r}^{\,2}-\beta_j\vec{R}^2\right). \qquad (3)$$

Then the total wave function of $^{11}B\{\alpha_1\alpha_2 t\}$ nucleus has the form:

$$\left|^{11}B\{\alpha_1\alpha_2 t\}\right\rangle=\sum_{\substack{00,1m_\ell\\1M_{\tilde{L}},\tilde{m}_t}}\left(001m_\ell\middle|1M_{\tilde{L}}\right)\left(1/2\,\tilde{m}_t1M_{\tilde{L}}\middle|3/2M_j\right)\cdot Y_{00}\left(\vec{r}\right)\cdot Y_{1m_\ell}\left(\vec{R}\right)\times$$
$$\times\chi^{(\sigma)}_{1/2\tilde{m}_t}\ \chi^{(\tau)}_{1/2,-1/2}\cdot\chi^{(\sigma,\tau)}_{00,00}\left(\alpha_1\right)\ \chi^{(\sigma,\tau)}_{00,00}\left(\alpha_2\right)\cdot\Phi_{000}\left(\alpha_1\right)\ \Phi_{000}\left(\alpha_2\right)\ \Phi_{000}\left(t\right)\times$$
$$\times N\sum_{j}C_j\exp\left(-\alpha_j\vec{r}^{\,2}-\beta_j\vec{R}^2\right). \qquad (4)$$

For the projection of the wave function of $^{11}\mathrm{B}\{\alpha\alpha t\}$ nucleus on the cluster channel $^{7}\mathrm{Li}\{\alpha t\}+\alpha$ one needs to calculate the overlapping integral:

$$\Psi\left(\vec{y}\right)=\left\langle\Psi_{^7\mathrm{Li}},\Psi_{\alpha}\,|\,\Psi_{^{11}\mathrm{B}}\right\rangle=\int\Psi^{*}_{^7\mathrm{Li}}\left(\vec{\rho}\right)\cdot\Psi^{*}_{\alpha_2}\cdot\Psi_{^{11}\mathrm{B}}\ \left(\vec{r},\vec{R}\right)\ d\vec{\rho}. \qquad (5)$$

Let's write down the total wave function of the ^{7}Li nucleus in the two-body model:

$$\Psi_{^7Li}\left(\vec{\rho}\right)=\sum_{M_L,m_t}\left(1/2\,m_t1M_L\middle|3/2\,m_j\right)\Phi_{000}\left(\alpha_1\right)\cdot\Phi_{000}\left(t\right)\cdot\chi^{(\sigma)}_{1/2m_t}\left(t\right)\cdot\chi^{(\tau)}_{1/2,-1/2}\left(t\right)\times$$
$$\times\sum_i A_i e^{-a_i\vec{\rho}^2}\cdot Y_{1M_L}\left(\vec{\rho}\right), \qquad (6)$$

where the coefficients of expansion of the relative function are taken from [4]. Now let's substitute the expressions (4) and (6) into the expression (5):

$$\Psi\left(\vec{y}\right)=\sum_{M_L,m_t}\left(1/2\,m_t1M_L\middle|3/2\,m_j\right)\sum_{\substack{00,1m_\ell\\1M_{\tilde{L}},\tilde{m}_t}}\left(001m_\ell\middle|1M_{\tilde{L}}\right)\left(1/2\,\tilde{m}_t1M_{\tilde{L}}\middle|3/2M_j\right)\times$$
$$\times\sum_{i\ j}A_iC_j\cdot\int e^{-a_i\vec{\rho}^2-c_j\vec{r}^2-d_j\vec{R}^2}\cdot Y^{*}_{1M_L}\left(\vec{\rho}\right)\cdot Y_{00}\left(\vec{r}\right)\cdot Y_{1m_l}\left(\vec{R}\right)\,d\vec{\rho}; \qquad (7)$$

3 Method of diagonalization of the squared form

Let's diagonalize the squared form on the exponent in the expression (7). Firstly let's transform the form with account of the transformations (2):

$$h=\left(a_i+\frac{9}{49}c_j+\frac{121}{196}d_j\right)\vec{\rho}^2+\left(c_j+\frac{1}{4}d_j\right)\vec{y}^2+\left(\frac{6}{7}c_j-\frac{11}{14}d_j\right)\vec{\rho}\vec{y}. \qquad (8)$$

Change of variables and new denotations:

$$\begin{cases}\vec{\rho}=\vec{x}_1+\alpha\vec{y},\\ \vec{y}=\vec{y}.\end{cases}\qquad \begin{cases}f_1=a_i+\frac{9}{49}c_j+\frac{121}{196}d_j,\\ f_2=c_j+\frac{1}{4}d_j,\\ f_3=\frac{6}{7}c_j-\frac{11}{14}d_j.\end{cases}\tag{9}$$

Then with account of the expression (9) the expression (8) will take the form:

$$h=f_1\vec{x}_1^2+\left(2f_1\alpha+f_3\right)\vec{y}\vec{x}_1+\left(f_3\alpha+f_1\alpha^2+f_2\right)\vec{y}^2.\tag{10}$$

For the diagonalization it is necessary to put the coefficient at the crossing term to be equal to zero:

$$2f_1\alpha+f_3=0\ \Rightarrow\ \alpha=-\frac{f_3}{2f_1}.\tag{11}$$

With account of the expression (11) let's find the coefficient at the third term:

$$f_3\alpha+f_1\alpha^2+f_2=\left(f_2-\frac{f_3^2}{4f_1}\right).\tag{12}$$

Now let's substitute the expression (12) into the expression (10):

$$q=f_2-\frac{f_3^2}{4f_1},\quad h=f_1\vec{x}_1^2+q\vec{y}^2.\tag{13}$$

Let's express $\vec{R}$ through $\vec{x}_1$ and $\vec{y}$. For this let's use the expression $\vec{R}=\frac{11}{14}\vec{\rho}-\frac{1}{2}\vec{y}$ from (2) and substitute $\vec{\rho}=\vec{x}_1+\alpha\vec{y}$ from (9) in it:

$$\vec{R}=\frac{11}{14}\vec{x}_1+\omega\vec{y},\ \ \omega=\frac{11}{14}\alpha-\frac{1}{2}.\tag{14}$$

4 Transformation of the spherical functions

For the transformation of the expressions for $Y_{1M_l}\left(\vec{R}\right)$ and $Y^*_{1M_L}\left(\vec{\rho}\right)$ let's use the table formulas from [5], then one obtains:

$$Y_{1m_l}\left(\vec{R}\right)=\sqrt{4\pi}\sum_{m_1,\,m_2}\left[\left(1m_1 00|1m_\ell\right)Y_{1m_1}\left(\frac{11}{14}\vec{x}_1\right)Y_{00}\left(\omega\vec{y}\right)+\left(001m_2|1m_\ell\right)Y_{00}\left(\frac{11}{14}\vec{x}_1\right)Y_{1m_1}\left(\omega\vec{y}\right)\right]\tag{15}$$

Further let's use the formulas for the Clebsch-Gordan coefficients [5] and the expressions of the spherical function will take the form:

$$Y_{1m_\ell}\left(\vec{R}\right)=\frac{11}{14}Y_{1m_\ell}\left(\vec{x}_1\right)+\omega Y_{1m_\ell}\left(\vec{y}\right),\ \ Y^*_{1M_L}\left(\vec{\rho}\right)=Y^*_{1M_L}\left(\vec{x}_1\right)+\alpha Y^*_{1M_L}\left(\vec{y}\right),\ \text{where}\ \alpha=-\frac{f_3}{2f_1}\tag{16}$$

5 Calculation of the integral with respect to $\vec{\rho}$ variable

Let's write down without account of Clebsch-Gordan coefficients algebra the separate integral from the expression (7) with account of formulas (16):

$$\psi(\vec{y}) \ = \ \int e^{-\left(f_1\vec{x}_1^2+q\vec{y}^2\right)} \cdot \left(Y^*_{1M_L}(\vec{x}_1)+\alpha Y^*_{1M_L}(\vec{y})\right)\cdot\left(\frac{11}{14}Y_{1m_\ell}(\vec{x}_1)+\omega Y_{1m_\ell}(\vec{y})\right) d\vec{\rho}. \qquad (17)$$

Let's use the change of variables (9) $\vec{\rho}=\vec{x}_1+\alpha\vec{y}$ and, taking into account that $d\vec{\rho}=d\vec{x}_1=x_1^2dx_1d\Omega_{x_1}$, substitute it into the expression (17):

$$\psi(\vec{y})=e^{-q\vec{y}^2}\int e^{-f_1\vec{x}_1^2}x_1^2dx_1\cdot\int d\Omega_{x_1}\Big[Y^*_{1M_L}(\vec{x}_1)\cdot\frac{11}{14}Y_{1m_\ell}(\vec{x}_1)+\alpha Y^*_{1M_L}(\vec{y})\cdot\frac{11}{14}Y_{1M_l}(\vec{x}_1)+$$
$$+\omega Y^*_{1M_L}(\vec{x}_1)Y_{1m_\ell}(\vec{y})+\alpha\omega Y^*_{1M_L}(\vec{y})Y_{1m_\ell}(\vec{y})\Big]. \qquad (18)$$

Let's consider separately the integral of the spherical functions from the expression (18):

$$I_\Omega = \ \frac{11}{14}x_1^2\int d\Omega_{x_1}Y^*_{1M_L}\left(\Omega_{x_1}\right)\cdot Y_{1m_\ell}\left(\Omega_{x_1}\right) \ + \ \frac{11}{14}\alpha\cdot Y^*_{1M_L}(\vec{y})\cdot x_1\int d\Omega_{x_1}Y_{1m_\ell}\left(\Omega_{x_1}\right) \ +$$
$$+ \ \omega\cdot\vec{y}Y_{1m_\ell}\left(\Omega_y\right)\cdot x_1\int d\Omega_{x_1}Y^*_{1M_L}\left(\Omega_{x_1}\right) \ + \ \alpha\omega\cdot y^2Y^*_{1M_L}\left(\Omega_y\right)Y_{1m_\ell}\left(\Omega_y\right)\int d\Omega_{x_1}. \qquad (19)$$

Further let's use the table formulas [5] and the expression (19) will take the form:

$$I_\Omega=\frac{11}{14}x_1^2\delta_{11} \ \delta_{m_\ell M_L}+\alpha\omega\cdot4\pi\cdot y^2(-1)^{M_L} \ Y_{1,-M_L}\left(\Omega_y\right)Y_{1m_\ell}\left(\Omega_y\right). \qquad (20)$$

Now let's use the table formula [5] for the product of two spherical vector functions and obtain for the expression (20):

$$I_\Omega=\frac{11}{14}x_1^2\delta_{m_\ell M_L}+4\pi\alpha\omega\cdot y^2(-1)^{M_L} \ \times$$
$$\times\sum_{\Lambda M_\Lambda}\frac{3}{\sqrt{4\pi}}\frac{1}{\sqrt{2\Lambda+1}}(1010|\Lambda 0)(1-M_L1m_\ell|\Lambda M_\Lambda) \ Y_{\Lambda M_\Lambda}\left(\Omega_y\right). \qquad (21)$$

Having transformed it one obtains:

$$I_\Omega=\frac{11}{14}x_1^2\delta_{m_\ell M_L}+4\pi\alpha\omega\cdot y^2(-1)^{M_L+m_\ell+1}\sum_{\Lambda M_\Lambda}\sqrt{\frac{3}{4\pi}}(1010|\Lambda 0)(\Lambda\text{-}M_\Lambda 1m_\ell|1M_L) \ Y_{\Lambda M_\Lambda}\left(\Omega_y\right). \ (22)$$

Now let's return to the expression (18) with account of the formula (22):

$$\psi(\vec{y})= \ e^{-q\vec{y}^2}\cdot\left\{\frac{11}{14}\delta_{m_\ell M_L}\int e^{-f_1\vec{x}_1^2}x_1^4dx_1+4\pi\alpha\omega\cdot y^2\times\right.$$
$$\times(-1)^{M_L+m_\ell+1}\sum_{\Lambda M_\Lambda}\sqrt{\frac{3}{4\pi}}(1010|\Lambda 0)(\Lambda\text{-}M_\Lambda 1m_\ell|1M_L)Y_{\Lambda M_\Lambda}\left(\Omega_y\right)\int e^{-f_1\vec{x}_1^2}x_1^2dx_1. \qquad (23)$$

Then the expression (23) with account of the table integrals will take the form:

$$\psi(\vec{y}) = e^{-q\vec{y}^2}\left[\frac{33\sqrt{\pi}}{112 f_1^{5/2}}\delta_{m_\ell M_L} + \right.$$

$$\left. + \frac{\pi\sqrt{3}}{2}\frac{\alpha\omega\cdot y^2}{f_1^{3/2}}(-1)^{M_L+m_\ell+1}\sum_{\Lambda M_\Lambda}(1010|\Lambda 0)(\Lambda\text{-}M_\Lambda 1 m_\ell|1M_L)Y_{\Lambda M_\Lambda}(\Omega_y)\right]. \quad (24)$$

Now let's write the expression (7) with account of the formula (24):

$$\Psi(\vec{y}) = \frac{1}{\sqrt{4\pi}}\sum_{i\,j}A_i C_j \sum_{M_L, m_t}(1/2\, m_t 1 M_L|3/2\, m_j) \times$$

$$\times \sum_{\substack{00,1m_\ell\\ 1M_{\tilde{L}},\tilde{m}_t}}(001m_\ell|1M_{\tilde{L}})(1/2\,\tilde{m}_t 1 M_{\tilde{L}}|3/2\, M_j)\cdot\psi(\vec{y}). \quad (25)$$

6 Transformation of the algebra of the Clebsch-Gordan coefficients

Let's substitute the expression (24) into the formula (25) and represent the obtained expression in the following form:

$$\Psi(\vec{y}) = \frac{1}{\sqrt{4\pi}}\sum_{i\,j}A_i C_j e^{-q\vec{y}^2}\cdot[I_2 + I_1], \quad (26)$$

where

$$I_2 = \frac{33\sqrt{\pi}}{112 f_1^{5/2}}\delta_{m_\ell M_L}\sum_{\substack{M_L,\\ m_t}}(1/2\, m_t 1M_L|3/2\, m_j)\sum_{\substack{00,1m_\ell\\ 1M_{\tilde{L}},\tilde{m}_t}}(001m_\ell|1M_{\tilde{L}})(1/2\,\tilde{m}_t 1M_{\tilde{L}}|3/2\, M_j), \quad (27)$$

$$I_1 = \frac{\pi\sqrt{3}}{2}\frac{\alpha\omega\cdot y^2}{f_1^{3/2}}\sum_{M_L, m_t}(1/2\, m_t 1M_L|3/2\, m_j)\sum_{\substack{00,1m_\ell\\ 1M_{\tilde{L}},\tilde{m}_t}}(001m_\ell|1M_{\tilde{L}})(1/2\,\tilde{m}_t 1M_{\tilde{L}}|3/2\, M_j) \times$$

$$\times(-1)^{M_L+m_\ell+1}\sum_{\Lambda M_\Lambda}(1010|\Lambda 0)(\Lambda\text{-}M_\Lambda 1m_\ell|1M_L)Y_{\Lambda M_\Lambda}(\Omega_y), \quad (28)$$

Having transformed the product of the Clebsch-Gordan coefficients in the expressions (27) and (28), let's write down the final form of the wave function (26) with account of the last two expressions:

$$\Psi(\vec{y}) = \frac{1}{\sqrt{4\pi}}\sum_{i\,j}A_i C_j e^{-q\vec{y}^2}\cdot[I_2 + I_1],$$

where

$$I_2 = \frac{33\sqrt{\pi}}{112 f_1^{5/2}}\,\delta_{m_t \tilde{m}_t}\,\delta_{m_j M_j}, \quad (29)$$

$$I_1 = \frac{3\pi\alpha\omega\cdot y^2}{f_1^{3/2}}(-1)^{M_L+M_{\tilde{L}}+4\Lambda-M_\Lambda}\sum_{\Lambda M_\Lambda}(1010|\Lambda 0)(3/2\, m_j \Lambda M_\Lambda|3/2\, M_j)\times$$

$$\times\begin{Bmatrix}1 & 1/2 & 3/2\\ 3/2 & \Lambda & 1\end{Bmatrix}\cdot Y_{\Lambda M_\Lambda}(\Omega_y). \quad (30)$$

The quantum numbers ΛM_Λ can take the following values: $\Lambda M_\Lambda = 00$, $\Lambda M_\Lambda = 2M_\Lambda$. The expression (29) and the case 1 for the expression (30) give us the contribution into the S-component of the wave function. The case 2 for the expression (30) – is the D-component of the wave function. Let's consider them separately.

1. $\Lambda M_\Lambda = 00$.

$$I_1 = \frac{\pi\alpha\omega \cdot y^2}{2 f_1^{3/2}} \; \delta_{m_j M_j} Y_{00}\left(\Omega_y\right). \tag{31}$$

Then the radial S-component of the wave function will take the form:

$$\left|\Psi_S\right\rangle = \frac{1}{\sqrt{4\pi}} \sum_{i\,j} A_i C_j e^{-q\vec{y}^2} \cdot \left[\frac{33\sqrt{\pi}}{112 f_1^{5/2}} \; \delta_{m_t \tilde{m}_t} \; \delta_{m_j M_j} \; + \; \frac{\pi\alpha\omega \cdot y^2}{2 f_1^{3/2}} \delta_{m_j M_j} \right]. \tag{32}$$

2. $\Lambda M_\Lambda = 2M_\Lambda$.

$$I_1 = \frac{\pi\alpha\omega \cdot y^2}{2 f_1^{3/2}} (-1)^{M_L + M_{\tilde{L}} + 4\Lambda - M_\Lambda + 1} \left(3/2\, m_j\, 2M_\Lambda \middle| 3/2\, M_j\right) \cdot Y_{2M_\Lambda}\left(\Omega_y\right), \tag{33}$$

Then the radial D-component of the wave function will take the form:

$$\left|\Psi_D\right\rangle = \frac{1}{\sqrt{4\pi}} \sum_{i\,j} A_i C_j e^{-q\vec{y}^2} \cdot \left[\frac{\pi\alpha\omega \cdot y^2}{2 f_1^{3/2}} (-1) \left(3/2\, m_j\, 2M_\Lambda \middle| 3/2\, M_j\right) \cdot Y_{2M_\Lambda}\left(\Omega_y\right) \right]. \tag{34}$$

Then the components of the radial part of the wave function are obtained from the expressions (32) and (34) without angular functions $Y_{\Lambda M_\Lambda}\left(\Omega_y\right)$, since the wave function of ^{11}B nucleus has the form:

$$^{11}B\left\{{}^7Li\alpha\right\} = \sum_{m_j, M_\Lambda} \left(3/2\, m_j \Lambda M_\Lambda \middle| 3/2\, M_j\right) \, {}^7Li \left| jm_j \right\rangle \cdot \Phi_{000}(\alpha) \cdot Y_{\Lambda M_\Lambda}\left(\Omega_y\right) \cdot R_{4\Lambda}(\vec{y}).$$

$$\left|S\{40\}\right\rangle = \sum_{i\,j} A_i C_j \left(\frac{33}{224} \frac{1}{f_1^{5/2}} \; + y^2 \frac{\sqrt{\pi}}{4} \frac{\alpha\omega}{f_1^{3/2}} \right) \cdot e^{-q\vec{y}^2}, \tag{35}$$

$$\left|D\{42\}\right\rangle = -\frac{\sqrt{\pi}}{4} \sum_{i\,j} A_i C_j \cdot \frac{\alpha\omega}{f_1^{3/2}} \cdot y^2 e^{-q\vec{y}^2}, \tag{36}$$

where

$$\omega = \left(\frac{11}{14}\alpha - \frac{1}{2}\right), \; \alpha = -\frac{f_3}{2 f_1}, \; q = f_2 - \frac{f_3^2}{4 f_1}, \; \begin{cases} f_1 = a_i + \dfrac{9}{49} c_j + \dfrac{121}{196} d_j, \\ f_2 = c_j + \dfrac{1}{4} d_j, \\ f_3 = \dfrac{6}{7} c_j - \dfrac{11}{14} d_j. \end{cases}$$

7 Numerical calculations

In the figure 2 there are represented the *S*- and *D*-components of the radial wave functions of the ^{11}B nucleus, calculated by the formulas (35) and (36) in the range of 0 to 5 fermi. The radial wave function of the ^{11}B nucleus in the three-body ααt-model is projected on the cluster channel ^{7}Li {αt} + α.

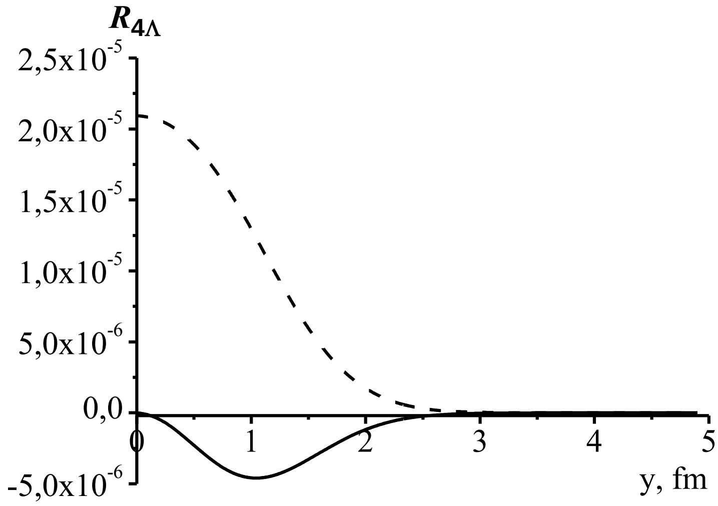

Figure 2 – The radial part of the wave function of ^{11}B nucleus: dashed line – *S*-component, solid line – *D*-component

The amplitudes of the *S*- and *D*-components of the radial wave function of the ^{11}B nucleus are turned out to be small by values, this means that in this case the account of only one configuration in the wave function of the ^{11}B nucleus, exactly the {ααt} configuration, having the Young scheme [443], is not able to describe well the wave function of this nucleus. Since the weight of this component in the wave function of the ground state of the ^{11}B nucleus in the many-particle shell model is not more than 40 % [2] the account and contribution of the components with Young schemes [4421], [4331] is turned out to be important. A calculation with account of such configurations presents an interest and is the subject of the future investigations for the authors.

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