

Extended objects in nonperturbative quantum-field theory and the cosmological constant

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We consider a gravitating extended object constructed from vacuum fluctuations of nonperturbatively quantized non-Abelian gauge fields. An approximate description of such an object is given by two gravitating scalar fields. The object has a core filled with a constant energy density of the vacuum fluctuations of the quantum-fields. The core is located inside a cosmological event horizon. An exact analytical solution of the Einstein equations for such a core is presented. The value of the energy density of the vacuum fluctuations is connected with the cosmological constant.

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1. Introduction

Probably, the first explanation for the present acceleration of the universe came from quantum-field theory: here the energy of the vacuum quantum fluctuations gives rise to a cosmological constant and thus a cosmological term in the Einstein equations. However, this model yields a disagreement of more than 100 orders of magnitude between the measured value of the cosmological constant and the

theoretical zero-point energy as obtained from perturbative quantum-field theory within the Standard Model, leading to its description as being “the worst theoretical prediction in the history of physics.”¹ In the theoretical procedure employed, we would like to emphasize the word “perturbative,” which means that the calculation is done by perturbative techniques. The problem of these perturbative calculations is that without a cutoff the zero-point fluctuations yield an infinite gravitational energy, whereas a cutoff at the Planck scale leads to the Planck energy density which is $\sim 10^{120}$ bigger than the measured cosmological constant.

Currently, there are many models for explaining the accelerated expansion of the universe: a cosmological constant; quintessence; modified gravity models; ideas from string theory like brane cosmology; etc. (For details on this problem see, for example, the reviews.²⁻⁴) Many of these models are based on using some fields which effectively lead to the acceleration. The essential difference of such models from the model based on the assumption that the cosmological constant corresponds to the energy of quantum fluctuations is that in the first case we must have some field equation(s) describing such (an) additional field(s), whereas in the second case some field (gauge field, spinor field) is in the ground state and quantum fluctuations around this state lead to the appearing of the cosmological constant. While this is an advantage for such a point of view, the calculations in perturbative quantum-field theory lead to a huge discrepancy between this theoretical value and the observed value of the cosmological constant.

The goal of this paper is to show that the nonperturbative vacuum in quantum-field theory has a good chance to yield the cosmological constant. The idea presented here is as follows. Let us imagine that we can: (a) construct a gravitating nonperturbative model of an extended object in quantum-field theory; (b) calculate the gravitational field created by this object. The object itself consists of vacuum fluctuations of non-Abelian gauge fields, that are present in the Standard Model. Then, the structure of such an object can be as follows: it possesses a core inside the cosmological event horizon and a tail outside the horizon. The core and the tail are filled with vacuum fluctuations. The boundary between the core and the tail is given by the cosmological event horizon indicating that we have obtained a cosmological constant originating from the nonperturbatively quantized fields.

2. Extended Objects from an Approximate Nonperturbative Quantization

Here, we follow Refs. 5 and 6 where an approximate nonperturbative quantization procedure was applied to modeling a glueball. To begin with, let us consider the $SU(3)$ Lagrangian in quantum chromodynamics

$$\begin{aligned} \mathcal{L} = & \frac{1}{4} F_{\mu\nu}^B F^{B\mu\nu} = f_{\mu\nu}^a f^{a\mu\nu} + f_{\mu\nu}^m f^{m\mu\nu} \\ & + g^2 C^{ab_1c_1} C^{ab_2c_2} A_{\mu}^{b_1} A_{\nu}^{c_1} A^{b_2\mu} A^{c_2\nu} + g^2 C^{ap_1q_1} C^{ap_2q_2} A_{\mu}^{p_1} A_{\nu}^{q_1} A^{p_2\mu} A^{q_2\nu} \\ & + g^2 C^{ab_1c_1} C^{ap_1q_1} A_{\mu}^{b_1} A_{\nu}^{c_1} A^{p_1\mu} A^{q_1\nu} + \dots, \end{aligned} \quad (1)$$

where $B = 1, 2, \dots, 8$ is the $SU(3)$ index; $F_{\mu\nu}^B = \partial_\mu A_\nu^B - \partial_\nu A_\mu^B + gC^{BCD}A_\mu^C A_\nu^D$ is the field strength operator; $f_{\mu\nu}^B = \partial_\mu A_\nu^B - \partial_\nu A_\mu^B$; g is the coupling constant; $a, b_{1,2}, c_{1,2} = 1, 2, 3$ are the $SU(2)$ indices; $m, p_{1,2}, q_{1,2}$ are the coset indices, and C^{BCD} are the $SU(3)$ structure constants.

In order to obtain an approximation yielding a nonperturbative description of a glueball, we assume that

- expectation values of non-Abelian gauge fields inside a glueball are zero;
- 2-point Green functions of gauge fields can be approximately represented through two scalar fields ϕ, χ ;
- the behavior of the $SU(2)$ and coset $SU(3)/SU(2)$ components is different; they are described by different scalar fields – ϕ and χ ;
- 4-point Green functions can be approximately decomposed as the product of 2-point Green functions;
- the expectation value of the product of an odd number of gauge potentials is zero.

Thus, we assume that it is possible to approximately describe these quantum-fields in the form of two scalar fields, where one of them (namely ϕ) describes $SU(2) \in SU(3)$ gauge fields, and the other one (namely χ) — the coset $SU(3)/SU(2)$ gauge fields.

2.1. The effective Lagrangian

The next step is to obtain some effective Lagrangian for these two scalar fields. This is done by performing the quantum averaging of the initial $SU(3)$ Lagrangian. Here, we assume that the 2- and 4-point Green functions are described in terms of these scalar fields ϕ and χ by using the following relations:

$$\langle A_\mu^a A^{a\mu} \rangle \approx m_1^2 - \phi^2, \quad (2)$$

$$\langle \partial_\mu A_\alpha^a \partial^\mu A^{a\alpha} \rangle \approx \frac{1}{2} \partial_\mu \phi \partial^\mu \phi, \quad (3)$$

$$C_{ab_1c_1} C_{ab_2c_2} \langle A_\mu^{b_1} A_\nu^{c_1} A^{b_2\mu} A^{c_2\nu} \rangle \approx \frac{\lambda_1}{4} (m_1^2 - \phi^2)^2 \quad (4)$$

$$\langle A_\mu^m A^{m\mu} \rangle \approx m_2^2 - \chi^2, \quad (5)$$

$$\langle \partial_\mu A_\alpha^m \partial^\mu A^{m\alpha} \rangle \approx \frac{1}{2} \partial_\mu \chi \partial^\mu \chi, \quad (6)$$

$$C_{ap_1q_1} C_{ap_2q_2} \langle A_\mu^{p_1} A_\nu^{q_1} A^{p_2\mu} A^{q_2\nu} \rangle \approx \frac{\lambda_2}{4} (m_2^2 - \chi^2)^2, \quad (7)$$

$$C_{abc} C_{amn} \langle A_\mu^b A_\nu^c A^{m\mu} A^{n\nu} \rangle \approx \frac{1}{2} \phi^2 \chi^2, \quad (8)$$

$$\langle A_\mu^B \rangle = 0, \quad (9)$$

$$\langle \partial_\mu A_\nu^B A_\rho^C A_\sigma^D \rangle = 0, \quad (10)$$

where $a, b, c, d = 1, 2, 3$ are the $SU(2)$ indices, $m, n, p, q = 4, 5, 6, 7, 8$ are the coset indices, and $\lambda_{1,2}$ and $m_{1,2}$ are closure constants. Equations (9) and (10) imply that the expectation value of odd products is zero. Note here that because of the positivity of the relations (2), (5) and (3), (6) the squares of the scalar fields ϕ, χ should be less than $m_{1,2}^2$, respectively. The effective Lagrangian then becomes

$$\mathcal{L}_{\text{eff}} = \langle \mathcal{L}_{SU(3)} \rangle \approx \frac{1}{2} (\nabla_\mu \phi)^2 + \frac{1}{2} (\nabla_\mu \chi)^2 - \frac{\lambda_1}{4} (\phi^2 - m_1^2)^2 - \frac{\lambda_2}{4} (\chi^2 - m_2^2)^2 - \frac{1}{2} \phi^2 \chi^2. \quad (11)$$

The procedure of obtaining the effective Lagrangian (11) from the initial Lagrangian (1) is similar to what happens in turbulence modeling when one gets the Reynolds equation starting from the Navier–Stokes equation.⁷ (A detailed discussion of the similarity between the nonperturbative quantization and turbulence modeling is given in Ref. 6.)

The field equations derived from the Lagrangian (11) are as follows:

$$\partial_\mu \partial^\mu \phi = -\phi [\chi^2 + \lambda_1 (\phi^2 - m_1^2)], \quad (12)$$

$$\partial_\mu \partial^\mu \chi = -\chi [\phi^2 + \lambda_2 (\chi^2 - m_2^2)]. \quad (13)$$

2.2. Extended objects

Before turning to gravitating systems of quantum fluctuations, let us consider the simpler case of Minkowski spacetime. In the case of spherical symmetry and no time-dependence, Eqs. (12) and (13) take the following dimensionless form

$$\phi'' + \frac{2}{x} \phi' = \phi [\chi^2 + \lambda_1 (\phi^2 - m_1^2)], \quad (14)$$

$$\chi'' + \frac{2}{x} \chi' = \chi [\phi^2 + \lambda_2 (\chi^2 - m_2^2)], \quad (15)$$

where the dimensionless radial coordinate $x = r/l_0$ has been introduced and the following redefinitions have been used: $\phi \rightarrow l_0 \phi$, $\chi \rightarrow l_0 \chi$, $m_{1,2} \rightarrow l_0 m_{1,2}$.

We are looking for regular solutions in Minkowski spacetime whose energy density takes asymptotically a constant value. An asymptotic expansion of the fields leads to

$$\phi \approx m_1 - \phi_\infty \frac{e^{-x\sqrt{2\lambda_1 m_1^2}}}{x}, \quad (16)$$

$$\chi \approx \chi_\infty \frac{e^{-x\sqrt{m_2^2 - \lambda_2 m_1^2}}}{x}, \quad (17)$$

where ϕ goes to a finite constant, whereas χ vanishes asymptotically.

We solve the above set of equations as a nonlinear eigenvalue problem, where $m_{1,2}$ are eigenvalues and ϕ, χ are eigenfunctions. This yields regular solutions with

a finite asymptotic energy density. One can consider this energy density as the nonperturbative energy density of the vacuum.

Typical profiles of the functions $\phi(x), \chi(x)$ are presented in Fig. 1. In turn, Fig. 2 shows the energy density profiles for the system as a whole, $\epsilon(x)$, and for the fields ϕ and χ , separately:

$$\epsilon = \frac{1}{2}\phi'^2(x) + \frac{1}{2}\chi'^2(x) + \frac{\lambda_1}{4}(\phi^2 - m_1^2)^2 + \frac{\lambda_2}{4}(\chi^2 - m_2^2)^2 + \frac{1}{2}\phi^2\chi^2, \quad (18)$$

$$\epsilon_\phi = \frac{1}{2}\phi'^2(x) + \frac{\lambda_1}{4}(\phi^2 - m_1^2)^2, \quad (19)$$

$$\epsilon_\chi = \frac{1}{2}\chi'^2(x) + \frac{\lambda_2}{4}(\chi^2 - m_2^2)^2. \quad (20)$$

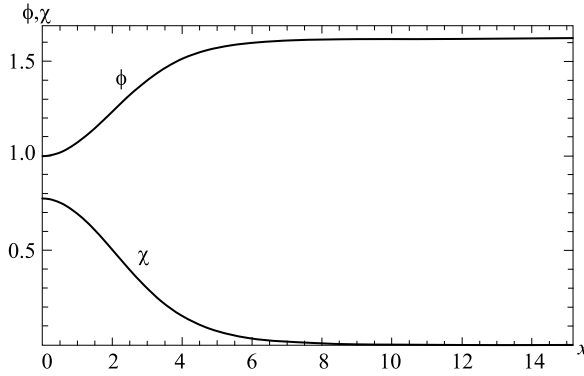


Fig. 1. The profiles of $\phi(x)$ and $\chi(x)$ in Minkowski spacetime. $\lambda_1 = 0.1; \lambda_2 = 1.0; \phi_0 = 1.0; \chi_0 = \sqrt{0.6}; m_1 = 1.617168; m_2 = 1.492735$.

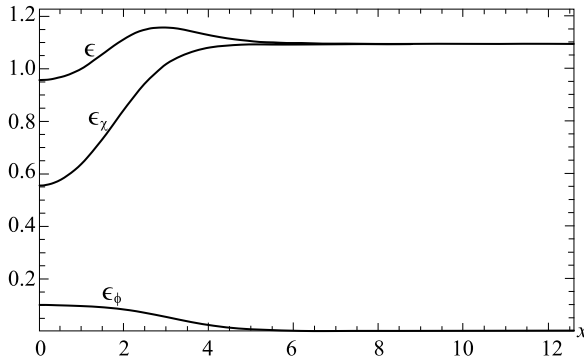


Fig. 2. The energy density profiles: for the system as a whole $\epsilon(x)$ (solid line) and for the scalar fields $\epsilon_\phi(x)$ and $\epsilon_\chi(x)$ (dashed lines) in Minkowski spacetime. $\lambda_1 = 0.1; \lambda_2 = 1.0; \phi_0 = 1.0; \chi_0 = \sqrt{0.6}; m_1 = 1.617168; m_2 = 1.492735$.

The asymptotic behavior of the dimensionless energy densities is as follows:

$$\begin{aligned} \epsilon \approx & 2\lambda_1 m_1^2 \frac{e^{-2x\sqrt{2\lambda_1 m_1^2}}}{x^2} + \frac{1}{2}\chi_\infty^2 (m_2^2 - \lambda_2 m_1^2) \frac{e^{-2x\sqrt{m_2^2 - \lambda_2 m_1^2}}}{x^2} \\ & + \frac{1}{2}m_1^2 \chi_\infty^2 \frac{e^{-2x\sqrt{m_2^2 - \lambda_2 m_1^2}}}{x^2} + \frac{\lambda_2}{4}m_2^4, \end{aligned} \quad (21)$$

$$\epsilon_\phi \approx 2\lambda_1 m_1^2 \frac{e^{-2x\sqrt{2\lambda_1 m_1^2}}}{x^2}, \quad (22)$$

$$\epsilon_\chi \approx \frac{1}{2}\chi_\infty^2 (m_2^2 - \lambda_2 m_1^2) \frac{e^{-2x\sqrt{m_2^2 - \lambda_2 m_1^2}}}{x^2} + \frac{\lambda_2}{4}m_2^4. \quad (23)$$

Thus, this extended object is constructed in such a way that the quantum fluctuations of the non-Abelian gauge field A_μ^m (described here by the scalar field χ) displace the quantum fluctuations of the non-Abelian gauge field A_μ^a (described here by the scalar field ϕ). This effect is similar to the Meissner effect in superconductivity when a magnetic field is expelled from the superconductor during its transition to the superconducting state.

According to (21), the vacuum energy density (with the dimension cm^{-4}) is approximately equal to $(\lambda_2/4)m_2^4 \text{cm}^{-4}$, where we went back to the dimensional quantity $m_2 \text{cm}^{-1}$.

3. Gravitating Extended Objects

Now we wish to consider gravitating scalar fields describing gravitating quantum fluctuations of non-Abelian gauge fields. Here, one may expect that the presence of the density of vacuum fluctuations gives rise to the appearance of a cosmological event horizon.

To obtain the Lagrangian, we couple the effective fields ϕ and χ to Einstein gravity. Thus, the Lagrangian consists of the Einstein-Lagrangian and the effective Lagrangian from Eq. (11)

$$L = -\frac{R}{16\pi G} + \left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V(\phi, \chi) \right] \quad (24)$$

with

$$V(\phi, \chi) = \frac{\lambda_1}{4}(\phi^2 - m_1^2)^2 + \frac{\lambda_2}{4}(\chi^2 - m_2^2)^2 + \frac{1}{2}\phi^2\chi^2. \quad (25)$$

The corresponding field equations are

$$R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R = \varkappa T_\mu^\nu, \quad (26)$$

$$\square\phi = -\frac{\partial V(\phi, \chi)}{\partial\phi}, \quad (27)$$

$$\square\chi = -\frac{\partial V(\phi, \chi)}{\partial\chi}, \quad (28)$$

where $\varkappa = 8\pi G$, and T_μ^ν is the energy–momentum tensor

$$T_\mu^\nu = \partial_\mu \phi \partial^\nu \phi + \partial_\mu \chi \partial^\nu \chi - \delta_\mu^\nu \left[\frac{1}{2} \partial_\alpha \phi \partial^\alpha \phi + \frac{1}{2} \partial_\alpha \chi \partial^\alpha \chi - V(\phi, \chi) \right]. \quad (29)$$

3.1. Spherically symmetric Ansatz

Again we are looking for static spherically symmetric solutions. Therefore, we adopt for the metric the Ansatz

$$ds^2 = A(r) e^{2\alpha(r)} dt^2 - \frac{dr^2}{A(r)} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2). \quad (30)$$

Also, the fields ϕ and χ again depend only on the radial coordinate.

Substituting this Ansatz into Eqs. (26)–(29), we find

$$-A\alpha'' - \frac{A''}{2} - A\alpha'^2 - \frac{3}{2}A'\alpha' - \frac{A\alpha'}{x} - \frac{A'}{x} = \tilde{\varkappa} \left[\frac{A}{2} (\tilde{\phi}'^2 + \tilde{\chi}'^2) + V(\tilde{\phi}, \tilde{\chi}) \right], \quad (31)$$

$$\alpha' = \tilde{\varkappa} \frac{x}{2} (\tilde{\phi}'^2 + \tilde{\chi}'^2), \quad (32)$$

$$A' - \frac{1-A}{x} = -\tilde{\varkappa} x \left[\frac{A}{2} (\tilde{\phi}'^2 + \tilde{\chi}'^2) + V(\tilde{\phi}, \tilde{\chi}) \right], \quad (33)$$

$$\tilde{\phi}'' + \left(\frac{2}{x} + \alpha' + \frac{A'}{A} \right) \tilde{\phi}' = \frac{\tilde{\phi}}{A} \left[\tilde{\chi}^2 + \lambda_1 (\tilde{\phi}^2 - \tilde{m}_1^2) \right], \quad (34)$$

$$\tilde{\chi}'' + \left(\frac{2}{x} + \alpha' + \frac{A'}{A} \right) \tilde{\chi}' = \frac{\tilde{\chi}}{A} \left[\tilde{\phi}^2 + \lambda_2 (\tilde{\chi}^2 - \tilde{m}_2^2) \right]. \quad (35)$$

Here, we have introduced the following dimensionless quantities: $\tilde{\phi} = l_0 \phi$, $\tilde{\chi} = l_0 \chi$, $\tilde{\varkappa} = \varkappa/l_0^2$, $\tilde{m}_{1,2} = l_0 m_{1,2}$, $x = r/l_0$, and l_0 is some characteristic length. (We will see below that this length should be identified with the radius of the cosmological event horizon.)

3.2. Taylor expansion of the solutions at the points $x = 0, x_H, \infty$

We wish to find solutions which possess a cosmological event horizon. The presence of the latter implies that there will be a point $x_H = r_H/l_0$ at which

$$A(x_H) = 0. \quad (36)$$

Again we aim at solutions where the scalar fields tend to limiting values for $x \rightarrow \infty$, with $\tilde{\phi} \rightarrow \tilde{m}_1$ and $\tilde{\chi} \rightarrow 0$. The asymptotic geometry would, however, now be of de Sitter type.

In order to find such solutions, let us first consider their Taylor expansions at the origin, $x = 0$, at the cosmological horizon, $x = x_H$, and at infinity, $x \rightarrow \infty$.

3.2.1. Taylor expansion at the origin

We assume that at the origin the solutions behave as

$$A(x) = 1 + A_2 \frac{x^2}{2} + \dots, \quad (37)$$

$$\alpha(x) = \alpha_0 + \alpha_2 \frac{x^2}{2} + \dots, \quad (38)$$

$$\tilde{\phi}(x) = \tilde{\phi}_0 + \tilde{\phi}_2 \frac{x^2}{2} + \dots, \quad (39)$$

$$\tilde{\chi}(x) = \tilde{\chi}_0 + \tilde{\chi}_2 \frac{x^2}{2} + \dots. \quad (40)$$

This implies for the center the boundary conditions

$$A(0) = 1, \quad \tilde{\phi}'(0) = \tilde{\chi}'(0) = 0. \quad (41)$$

3.2.2. Taylor expansion at the cosmological event horizon

At the cosmological event horizon, the solutions should satisfy

$$A(x) = A'_H (x - x_H) + \dots, \quad (42)$$

$$\alpha(x) = \alpha_H + \dots, \quad (43)$$

$$\tilde{\phi}(x) = \tilde{\phi}_H + \tilde{\phi}'_H (x - x_H) + \dots, \quad (44)$$

$$\tilde{\chi}(x) = \tilde{\chi}_H + \tilde{\chi}'_H (x - x_H) + \dots. \quad (45)$$

From Eqs. (34) and (35), one can find

$$\tilde{\phi}'(x_H) = \tilde{\phi}'_H = \frac{\tilde{\phi}_H}{A'_H} \left[\tilde{\chi}_H^2 + \lambda_1 (\tilde{\phi}_H^2 - \tilde{m}_1^2) \right], \quad (46)$$

$$\tilde{\chi}'(x_H) = \tilde{\chi}'_H = \frac{\tilde{\chi}_H}{A'_H} \left[\tilde{\phi}_H^2 + \lambda_2 (\tilde{\chi}_H^2 - \tilde{m}_2^2) \right], \quad (47)$$

where A'_H is obtained from (33) as

$$A'_H = \frac{1}{x_H} - \tilde{\varkappa} x_H V(\tilde{\phi}_H, \tilde{\chi}_H). \quad (48)$$

3.2.3. Taylor expansion at infinity

At infinity, the solutions possess the asymptotic behavior

$$A(x) \approx -A_\infty x^2, \quad (49)$$

$$\tilde{\phi}(x) \approx \tilde{m}_1 + \tilde{\phi}_\infty (x^{\beta_1} + x^{\beta_2}), \quad \beta_{1,2} = -\frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{2m_1^2 \lambda_1}{A_\infty}}, \quad (50)$$

$$\tilde{\chi}(x) \approx \tilde{\chi}_\infty (x^{\gamma_1} + x^{\gamma_2}), \quad \gamma_{1,2} = -\frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{m_1^2 - \lambda_2 m_2^2}{A_\infty}}, \quad (51)$$

where $\tilde{\phi}_\infty$ and $\tilde{\chi}_\infty$ are integration constants. Using these expressions, the asymptotic form for $\alpha(x)$ can be found from Eq. (32).

Depending on the sign of the expressions under the square roots in $\beta_{1,2}$ and $\gamma_{1,2}$, the asymptotic behavior of the scalar fields changes drastically. For positive values of the expressions under the square roots, a power damping will take place. For negative values of the expressions under the square roots, however, Eqs. (50) and (51) take the following form

$$\tilde{\phi}(x) \approx \tilde{m}_1 + 2\tilde{\phi}_\infty x^{-3/2} \cos\left(\sqrt{\left|\frac{9}{4} - \frac{2m_1^2\lambda_1}{A_\infty}\right|} \ln x\right), \quad (52)$$

$$\tilde{\chi}(x) \approx 2\tilde{\chi}_\infty x^{-3/2} \cos\left(\sqrt{\left|\frac{9}{4} - \frac{m_1^2 - \lambda_2 m_2^2}{A_\infty}\right|} \ln x\right). \quad (53)$$

3.3. Numerical solutions

We solve the system of Eqs. (32)–(35) numerically. As discussed above, we seek solutions possessing a cosmological event horizon.

Our strategy for obtaining such solutions is to employ a two-step procedure. We first solve the equations in the inner region $x < x_H$. Subsequently, we integrate the equations in the outer region $x > x_H$. We solve the set of Eqs. (32)–(35) as a nonlinear eigenvalue problem with eigenvalues $m_{1,2}$ and $\tilde{\kappa}$ and eigenfunctions $\tilde{\phi}, \tilde{\chi}$ and A, α . For the numerical computations, we employ the shooting method. In the first step, we start the solution at the point $x_H - \delta$ with $\delta = 10^{-3}$, and integrate towards the center of the system, i.e. in the direction $x < x_H$. As boundary conditions, we choose a set of values $A_H = 0$, α_H , $\tilde{\phi}_H$ and $\tilde{\chi}_H$ at the cosmological horizon, and then determine the unknown values $m_{1,2}$ and $\tilde{\kappa}$, by requiring the boundary conditions (41).

Once a numerical solution inside the cosmological event horizon is found, we seek the solution outside the event horizon, $x > x_H$. To do this, we solve Eqs. (32)–(35) with the values of the parameters $m_{1,2}$ and $\tilde{\kappa}$ found in the region with $x < x_H$.

Figures 3 and 4 show an example of such solutions for the following parameters and horizon values of the fields:

$$\lambda_1 = 0.1, \quad \lambda_2 = 1.0, \quad x_H = 1.0, \quad \alpha_H = 1.0, \quad \tilde{\phi}_H = 3.0, \quad \tilde{\chi}_H = 3.0, \quad (54)$$

and the associated eigenvalues

$$\tilde{m}_1 = 10.07462, \quad \tilde{m}_2 = 4.2241352, \quad \tilde{\kappa} = 0.01095. \quad (55)$$

This solution exhibits the oscillating behavior according to the asymptotic expansions (52) and (53).

As one can see in Fig. 3, inside the cosmological event horizon the scalar fields change very slowly. Therefore, the energy density of the quantum fluctuations is

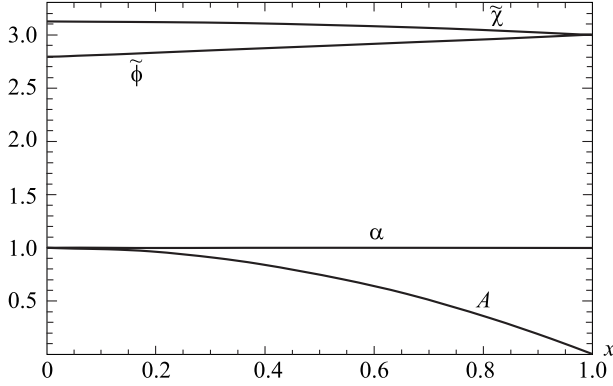


Fig. 3. Solution inside the cosmological event horizon.

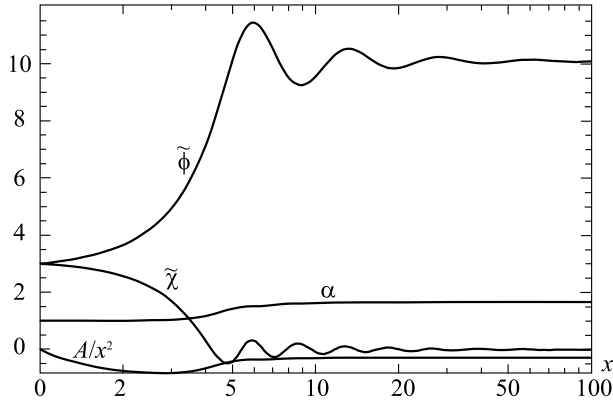


Fig. 4. Solution outside the cosmological event horizon, where asymptotically $\tilde{\phi} \rightarrow \tilde{m}_1$, $\tilde{\chi} \rightarrow 0$.

also almost constant with the main contribution coming from the potential part. Since the solutions start at the horizon with the boundary conditions (44) and (45), the energy density approximately corresponds to the cosmological constant:

$$\Lambda = \frac{1}{l_0^2} \tilde{T}_0^0 = \frac{1}{l_0^2} \tilde{\varkappa} \left[\frac{\lambda_1}{4} (\tilde{\phi}_H^2 - \tilde{m}_1^2)^2 + \frac{\lambda_2}{4} (\tilde{\chi}_H^2 - \tilde{m}_2^2)^2 + \frac{1}{2} \tilde{\phi}_H^2 \tilde{\chi}_H^2 \right] = \frac{3}{l_0^2}. \quad (56)$$

If we choose $l_0 \approx 10^{28}$ cm (i.e. the radius of the observable universe), we obtain the expected value of the cosmological constant $\Lambda \approx 10^{-56}$ cm⁻².

The presented numerical solution is given for $l_0 \approx l_{P1}$, and it would be very important to show that such a solution does exist for $l_0 \gg l_{P1}$. It will be done in the next section.

3.4. Analytical solution inside the cosmological event horizon

The system of equations (31)–(35) has a trivial de Sitter solution of the form

$$\tilde{\phi}(x) = \tilde{\phi}_H, \quad (57)$$

$$\tilde{\chi}(x) = \tilde{\chi}_H, \quad (58)$$

$$A(x) = 1 - x^2, \quad (59)$$

$$\alpha(x) = \alpha_H \quad (60)$$

with the following eigenvalues

$$\tilde{m}_1 = \sqrt{\frac{\tilde{\chi}_H^2}{\lambda_1} + \tilde{\phi}_H^2}, \quad (61)$$

$$\tilde{m}_2 = \sqrt{\frac{\tilde{\phi}_H^2}{\lambda_2} + \tilde{\chi}_H^2}, \quad (62)$$

$$\tilde{\varkappa} = \frac{3}{\frac{\tilde{\chi}_H^4}{4\lambda_1} + \frac{\tilde{\phi}_H^4}{4\lambda_2} + \frac{\tilde{\phi}_H^2 \tilde{\chi}_H^2}{2}}. \quad (63)$$

In principle, this solution is valid over all space.

Let us consider the case where $\tilde{\varkappa}$ in (63) is $\tilde{\varkappa} \approx 8\pi l_{\text{Pl}}^2/l_0^2$. In this case

$$\frac{\chi_H^4}{4\lambda_1} + \frac{\phi_H^4}{4\lambda_2} + \frac{\phi_H^2 \chi_H^2}{2} \approx \frac{3}{8\pi} \left(\frac{1}{l_0 l_{\text{Pl}}} \right)^2 = \frac{3}{8\pi} \frac{\Lambda}{l_{\text{Pl}}^2} \approx 10^9 \text{ cm}^{-4}. \quad (64)$$

For these values of the fields ϕ, χ , we have the observed value of the cosmological constant $\Lambda \approx 10^{-56} \text{ cm}^{-2}$. These values of the fields give us the following vacuum energy for the nonperturbatively quantized fields:

$$\epsilon_{\text{vac}} \approx \hbar c V(\phi_H, \chi_H) = \hbar c \left(\frac{\chi_H^4}{4\lambda_1} + \frac{\phi_H^4}{4\lambda_2} + \frac{\phi_H^2 \chi_H^2}{2} \right) \approx 10^{-8} \text{ erg/cm}^3, \quad (65)$$

i.e. the energy density of the present universe. The solution for the fields ϕ and χ from (64) is a very good approximation for the real universe since it has a constant distribution of the nonperturbative vacuum energy density for the gauge fields A_μ^B . It must be remembered here that ϕ describes the dispersion of nonperturbative quantum fluctuations of the gauge fields $A_\mu^a \in SU(2) \subset SU(3)$ and χ describes the dispersion of nonperturbative quantum fluctuations of the coset fields $A_\mu^m \in SU(3)/SU(2)$.

4. Discussion and Conclusion

Here, we have shown that a nonperturbatively quantized non-Abelian gauge field permits the existence of a regular extended object, which in the presence of gravity may possess a cosmological event horizon. Inside the horizon, the energy density of

the scalar fields can be made practically constant, and thus may be (approximately) considered as a cosmological constant.

The size of such an extended object corresponds to the size of the universe as a whole, and its core, the observable universe, corresponds to the region located inside the cosmological event horizon. Let us note that in Sec. 3.3 we have obtained a solution with

$$\bar{\kappa} = \frac{\kappa}{l_0^2} = \frac{8\pi l_{\text{Pl}}^2}{l_0^2} \approx 10^{-2}.$$

This means that $l_0 \approx 50l_{\text{Pl}}$. In Sec. 3.4, we have shown that on the event horizon there exist such values of the fields ϕ_H, χ_H which give the observable value of the cosmological constant:

$$l_0 \approx \Lambda^{-1/2}, \quad \bar{\kappa} \approx l_{\text{Pl}}^2 \Lambda \approx 10^{-123}.$$

In this case, an extended object created by two scalar fields ϕ, χ (describing the nonperturbatively quantized non-Abelian $SU(3)$ gauge field) will be immersed into the universe with the observed value of the cosmological constant.

The main goal of the paper was to show that (in contrast to perturbative quantization) nonperturbative quantization leads to a finite vacuum energy density, which can be regarded as the cosmological constant. The main difference compared to perturbative quantization is thus the finiteness of the vacuum energy density.

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