

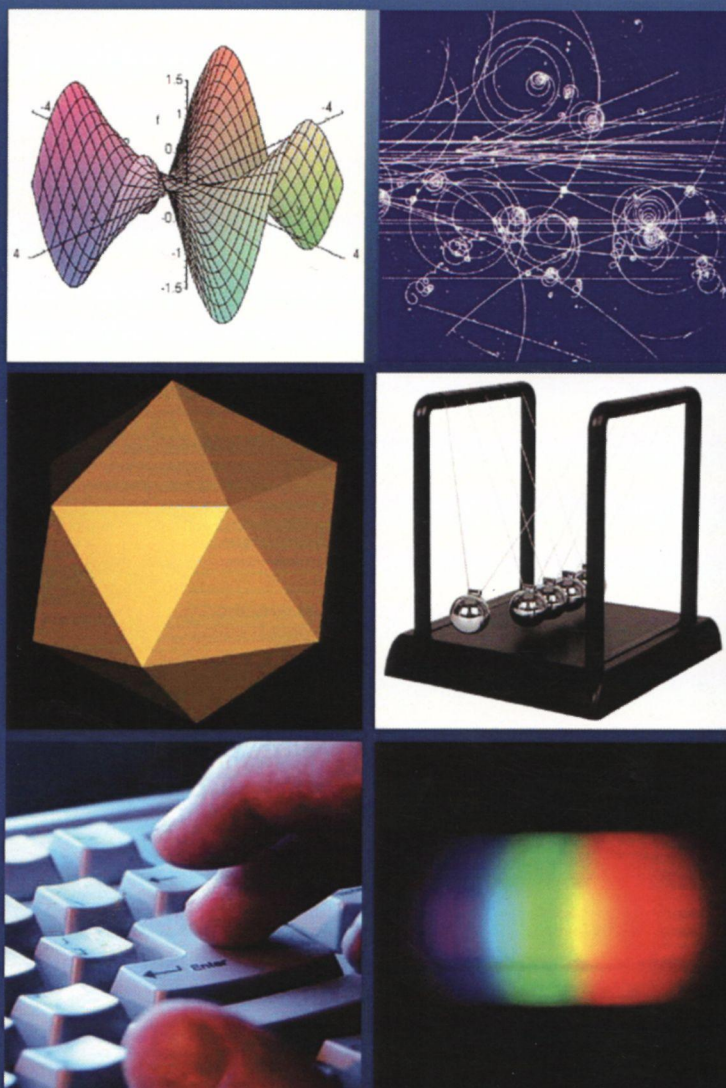
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## Editorial

The most significant scientific achievements are attained through joint efforts of different sciences, mathematics and physics are among them. Therefore publication of the Journal, which shows results of current investigations in the field of mathematics and physics, will allow wider exhibition of scientific problems, tasks and discoveries.

One of the basic goals of the Journal is to promote extensive exchange of information between scientists from all over the world. We propose publishing service for original papers and materials of Mathematical and Physical Conferences (by selection) held in different countries and in the Republic of Kazakhstan.

Creation of the special International Journal of Mathematics and Physics is of great importance because a vast amount of scientists are willing to publish their articles and it will help to widen the geography of future dissemination. We will also be glad to publish papers of scientists from all the continents.

The Journal will publish experimental and theoretical investigations on Mathematics, Physical Technology and Physics. Among the subject emphasized are modern problems of Calculus Mathematics, Algebra and Mathematical Analysis, Differential Equations and Mechanics, Informatics and Mathematical Modeling, Calculus of Approximations and Program Systems, Astronomy and Space Research, Theoretical Physics and Plasma Physics, Chemical Physics and Radio Physics, Thermophysics, Nuclear Physics and Nanotechnology.

The Journal is issued on the base of al-Farabi Kazakh National University. Leading scientists from different countries of the world agreed to join the Editorial Board of the Journal.

The Journal will be published two times a year by al-Farabi Kazakh National University. We hope to receive papers from many laboratories which are interested in applications of the scientific principles of mathematics and physics and are carrying out researches on such subjects as production of new materials or technological problems.

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### Study of anisotropy of extremely high energy cosmic rays

**Abstract.** The article shows investigations of anisotropy of high energy cosmic rays. We calculated the number of cosmic rays (events) of extremely high energies with different threshold energy, the direction of arrival of which lies on the angular distances from the active galactic nuclei, not large than given. Similar calculations were also carried with maximum of one hundred numerical experiments of concentrating around active galactic nuclei isotropic distribution of particles. For interval to  $\sim 3^\circ$  in all cases, except for a local minimum at  $\sim 2^\circ$  for distance  $< 100$  Mpc, these values were less than the observed in real distribution. Thus, our results indicate a clear correlation between the arrival directions of cosmic rays of extremely high energy and direction to close (within 100 Mpc) to the solar system active galactic nuclei, therefore, these objects are one of the real sources of such particles.

**Key words:** anisotropy, cosmic rays, galactic nuclei, isotropic distribution, galaxy

#### Introduction

Cosmic ray particles with energies more than 40 EeV are called particles of extremely high energy (or ultra high energy). The complexity of such studies is that their intensity is very low, for example, the flux of cosmic rays with an energy of about 100 EeV is only 1 particle/(100 km<sup>2</sup> year). It is the reason why less than 100 of such particles have been registered, until 2000 in all over the world.

Since 2004, the installations of cosmic rays registration of Pierre Auger Observatory were started to operate. They registered particles of ultra-high energies, the square of each area were about 3,000 km<sup>2</sup>, with an angular resolution of about 1%, and an energy resolution of  $\sim 10\%$ . These parameters give the possibility to register for 10 years, more than two hundred of extensive air showers (EAS) generated by ultra-high energy particles of cosmic rays that allowed us to obtain a lot of new valuable information about their origin. Despite this, the issue of sources of ultra-high energy particles of cosmic rays still remains open. The most likely candidates for this role are being considered are active galactic nuclei.

In connection with the above, in the present work new observational data obtained in Pierre Auger Observatory were used. The correlation between the arrival directions of ultra-high-energy

particles and directions to the closest solar system, galaxies with active nuclei were investigated.

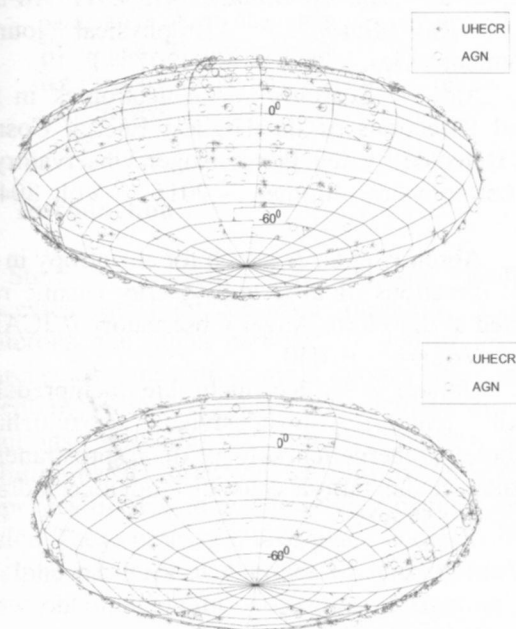
In addition of Pierre Auger Observatory data of energies and coordinate axes of extensive air showers from cosmic rays particles of extremely high energies, we used data on the equatorial coordinates and redshifts of active galactic nuclei presented in Swift-BAT directory, which includes sources that are registered for 70 months of observations by BAT-detector of hard X-ray radiation on Swift satellite board [1].

#### Instruments and methods

Figure 1 shows the distribution at the celestial sphere of coordinates on the arrival of cosmic ray particles with extremely high energies by measuring of Pierre Auger Observatory and galaxies with active nuclei of Swift-BAT 70-month X-ray catalog in equatorial coordinates.

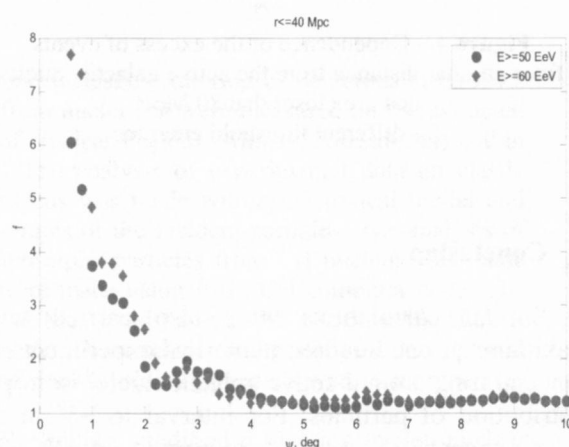
Using these data, we calculated the number of cosmic rays (events) of extremely high energies with different threshold energy, the direction of arrival of which lies on the angular distances from the active galactic nuclei, not large than given. The angular distance varied from  $0.6^\circ$  to  $10^\circ$  in increments of  $0.2^\circ$ , the threshold energy were taken to be 50 and 60 EeV, for these values the amount of rainfall is still ensures a sufficient statistic. Calculations were made for active galactic nuclei

with different maximum heliocentric distance to them: from 40 to 100 Mpc. Galaxies, which were lying at great distances, were not considered as due to the interaction of cosmic rays of extremely high energy with microwave background radiation of the Universe, particles with such distances should not reach the observer (blackbody cutoff of cosmic rays spectrum, Greisen – Zatsepin – Kuzmin effect (GZK effect)). To estimate the excess of particles at small angular distances from active galactic nuclei (i.e. directions concentration of their arrival around active galactic nuclei) the ratio particles number were calculated in the observed distribution with their numbers in isotropic distribution at the same other conditions (the same total number of particles, the same angular distance, the maximum distance from active galactic nuclei, etc.). In calculating the number of particles concentrating around active galactic nuclei in isotropic distribution, we took the average value of that number of hundreds numerical experiments results on the generation of random uniform distribution. The results of this study are shown in Figures 1-5.

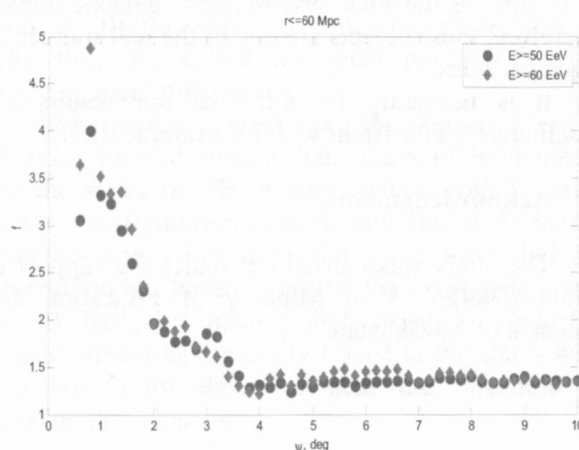


**Figure 1** – Distribution of arrival directions of cosmic ray particles with energies  $E > 5 \cdot 10^{19}$  eV on celestial sphere on measurements of Pierre Auger Observatory and galaxies with active nuclei of Swift-BAT 70-month X-ray catalog in equatorial coordinates (hemisphere of spring equinoxes points (a) and autumn equinoxes points (b))

From figures it is clear that the number of observed events within  $\sim 30^\circ$  of active galactic nuclei exceeds the number of those for isotropic distribution, it is particularly clearly seen in the case of smaller distances to active galactic nuclei. The fact that this excess is greater the closer you are to the considered active galactic nuclei, can be explained by the influence of GZK effect. At an angular distance of  $\sim 20^\circ$  in considered dependence in nearly all cases there is a local minimum, and we do not have assumptions about the reason for its occurrence.

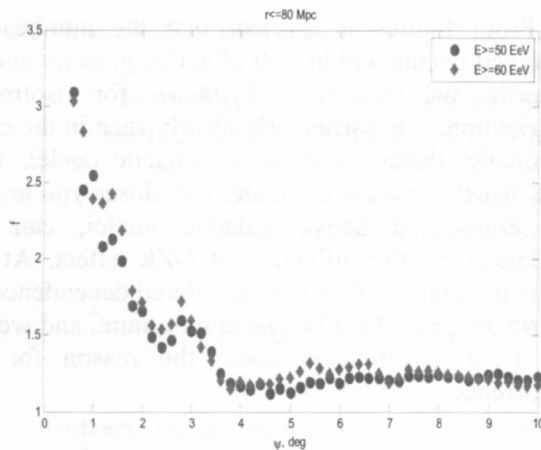


**Figure 2** – Dependence of excess of events from the angular distance to active galactic nuclei, located closer than 40 Mpc, for different threshold energies

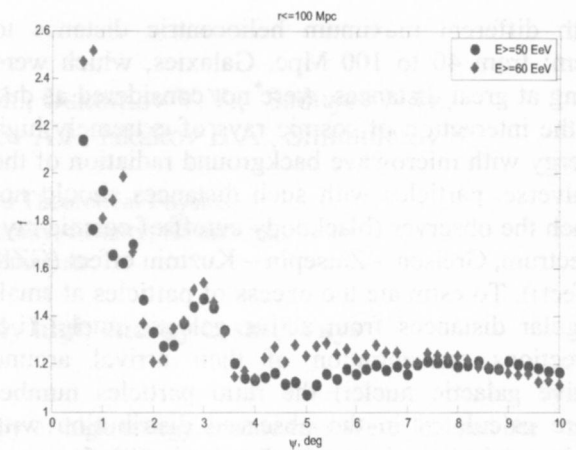


**Figure 3** – Dependence of the excess of events from angular distance from the active galactic nuclei that are closer than 60 Mpc, for different threshold energies





**Figure 4** – Dependence of the excess of events from angular distance from the active galactic nuclei that are closer than 80 Mpc, for different threshold energies



**Figure 5** – Dependence of the excess of events from angular distance from the active galactic nuclei that are closer than 100 Mpc, for different threshold energies

## Conclusion

Similar calculations were also carried with maximum of one hundred numerical experiments of concentrating around active galactic nuclei isotropic distribution of particles. For interval to  $\sim 3^\circ$  in all cases, except for a local minimum at  $\sim 2^\circ$  for distance  $< 100$  Mpc, these values were less than the observed in real distribution.

Thus, our results indicate a clear correlation between the arrival directions of cosmic rays of extremely high energy and direction to close (within 100 Mpc) to the solar system active galactic nuclei, therefore, these objects are one of the real sources of such particles.

It is necessary to note that our results are qualitatively consistent with for example, [2-4].

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