SCATTERING CROSS SECTIONS OF THE PARTICLES IN THE PARTIALLY IONIZED DENSE NONIDEAL PLASMAS

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Abstract. The electron-atom interaction with taking into account of dynamic screening is considered in the dense partially ionized plasmas. The phase-functions method is used. It is shown that the phase shifts and differential cross sections of the electron scattering on the helium and argon atoms, calculated with taking into account of dynamic screening, are bigger than those, obtained on the basis of the static interaction potential. This can influence on macroscopic properties such as transport coefficients.

Key words: phase shift, scattering cross section, dynamic interaction potential, dense nonideal plasma.

I. Introduction

Interaction of the electron with atom is a fundamental problem in investigation of the physical properties of a partially ionized plasma. Reaction cross sections are required in calculating macroscopic properties such as composition and transport coefficients. It is known, that in a dense plasma the electron-atom interaction potentials have to take into account the polarization effect as well as many-particles effects, which lead to screening of the interaction potential. In dense semiclassical plasma the quantum-mechanical effects such as diffraction effect also have to be taken into consideration. In this paper, the electron-atom scattering in a dense complex plasma is considered on the basis of the dynamic polarization potential. The effective polarization potential of the electron-atom interaction, which was presented in works [1-2], takes into account the effects of static screening and diffraction:

$$\Phi_{ea}(r) = -\frac{e^2\alpha}{2r^4(1-4\lambda\!\!\!^{-}_{ea}{}^2/r_D{}^2)}\left(e^{-Br}(1+Br)-e^{-Ar}(1+Ar)\right)^2, \tag{1}$$

here $A^2 = \left(1+\sqrt{1-4\lambda\!\!\!^{-}_{ea}{}^2/r_D^2}\right)\Big/2\lambda\!\!\!^{-}_{ea}{}^2$, $B^2 = \left(1-\sqrt{1-4\lambda\!\!\!^{-}_{ea}{}^2/r_D^2}\right)\Big/2\lambda\!\!\!^{-}_{ea}{}^2$. $\lambda\!\!\!^{-}_{ea} = \hbar\Big/\sqrt{2\pi\mu_{ea}k_BT} \approx \lambda\!\!\!^{-}_e$ is the de Broglie thermal wavelength; $\mu_{ea} = m_e m_a/(m_e+m_a)$ is the reduced mass of the atom and the electron; $r_D = \left(k_BT\Big/\left(4\pi e^2 n_e\right)\right)^{1/2}$ is the Debye length; n_e is the numerical density of electrons; T is the plasma temperature; k_B is the Boltzmann constant, α is the atomic polarizability. Potential (1) is screened and has finite values at the distances close to zero.

Accounting of the influence of the different dynamic effects in the interaction potential, in particular, of the dynamic screening, has reasonable grounds [3-10]. In Refs. [5,6], the elastic scattering at the electron-charge collisions in the dense semiclassical plasma was investigated on the basis of the interaction potential, which takes into account the effects of diffraction and the effect of dynamic screening. In this potential the Debye length in the interaction potential was replaced by the screening radius, which takes into account the relative velocity of the colliding particles:

$$r_0 = r_D (1 + \upsilon^2 / \upsilon_{Th}^2)^{1/2}, \tag{2}$$

here υ is the relative velocity of the colliding particles, υ_{Th} is the thermal velocity. If one apply this replacement to the potential (1), the energy of the electron-atom interaction, which takes into account dynamic screening, can be rewritten as:

$$\Phi_{ea}^{dyn}(r) = -\frac{e^2\alpha}{2r^4(1-4\lambda_{ea}^2/r_0^2)}\left(e^{-Br}(1+B_0 r) - e^{-Ar}(1+A_0 r)\right)^2, \tag{3}$$

where $A_0^2 = \left(1+\sqrt{1-4\lambda_{ea}^2/r_0^2}\right)\Big/2\lambda_{ea}^2$, $B_0^2 = \left(1-\sqrt{1-4\lambda_{ea}^2/r_0^2}\right)\Big/2\lambda_{ea}^2$.

Cross sections of electron scattering on the atoms in partially ionized plasma within the static model (1) were calculated in Refs. [2,3,11]. In this paper, we investigated electron elastic scattering on the atoms of helium and argon on the basis of the dynamic electron-atom interaction model (3).

For calculation of the scattering cross section of particles many theoretical approaches were developed, for example, the Born and eikonal approximations, also, the phase-functions method. All they are important and have their own application areas. The Born approximation is the most simple one. It gives the qualitative description of the processes due to the limitation in the energy of the colliding particles. Other two methods have not such limitation and have main advantage that the wave equation reduce to a differential equation in a single variable. At the same time there is a reason to believe that the eikonal approximation can reliably describe the collision properties at short wave lengths or large values of the orbital quantum number, as the wave equation is approximated using the WKB theory. For these reasons, we have chosen the phase-functions method [12,13], which works in a wide range of parameters of the colliding particles.

By this method we have already studied scattering of the electron on the charges within the static and the dynamic models of the electron-charge interactions, which are described in Refs. [14,15] and [6], correspondingly. The feature of the present paper is to study the scattering of the electrons by an atom for the first time on the basis of the dynamic model (3).

II. Method and parameters

The phase-functions method is widely used for the investigation of the scattering processes. The monographs [13,16] provides a detailed description of this method. Thus, within this approach we have solved a first-order differential equation for the scattering phase, i.e. the Calogero equation:

$$\frac{d}{dr}\delta_l(k,r) = -\frac{1}{k}\frac{2m}{\hbar^2}\Phi_{\alpha\beta}(r)\left[\cos\delta_l(k,r)\,j_l(kr) - \sin\delta_l(k,r)\,n_l(kr)\right]^2,\ \delta_l(k,0) = 0, \quad (4)$$

where l is orbital quantum number; $\delta_l(k,0)$ is the phase function; $\Phi_{\alpha\beta}(r)$ is the interaction potential; k is the wave number of the particle; $j_l(kr)$ and $n_l(kr)$ are regular and irregular solutions of the Schrödinger equation. The phase shift is the asymptotical value of the phase function at the large distances:

$$\delta_l(k) = \lim_{r\to\infty}\delta_l(k,r)\,. \quad (5)$$

In the Calogero equation (4), the relation between the interaction potential and the scattering phase shift is clearly seen. Besides, it is quite easy to solve this equation numerically.

If the scattering phase shifts are known one can calculate the total cross section for the elastic scattering of the plasma particles according to [11]:

$$Q^{\alpha\beta}(k) = \frac{4\pi}{k^2}\sum_{l=0}(2l+1)\sin^2\delta_l^{\alpha\beta} \quad (6)$$

The wave number is rebated to the energy of the incident particle, $k^2 = \mu_{ea}E/\hbar^2$.

Following dimensionless parameters were used:

$$\Gamma = \frac{e^2}{a\,k_B T} \quad (7)$$

is the coupling parameter and

$$r_s = \frac{a}{a_B} \quad (8)$$

is the density parameter. Here $a = (3/4\pi n)^{1/3}$ is the average distance between particles, n is the numerical density of the plasma particles and $a_B = \hbar^2/m_e e^2$ is the Bohr radius.

III. Results

We calculated the phase shifts of the electron scattering on the helium atom and on the argon atom and presented them in Fig. 1 and Fig.2, correspondingly, as the dependences on the electron impact energy. As can be seen in Figs 1 and 2, phase shifts calculated within the dynamic potential (3) lay higher than those obtained on the basis of the static model (1), since the dynamic screening of the electric field is weaker than the static one at any values of plasma density. From Fig. 3 one can see that for relatively more dense plasmas (cases with $r_s = 4$ and $r_s = 8$) the scattering phase shifts $\delta_{l=0}(0)$ tend to the value of π. According to the Levinson theorem [17-19], it corresponds to one bound state ($\delta_l(0) = n\pi$, where n is the number of bound states). In this case, it is the negative ion of argon atom (electron capture by the polarized atom). For helium atom we do not observe the electron capture at given parameters because the atomic polarizability of the helium atom ($\alpha_{He} = 1.38 a_B^3$) [20] is smaller than that of the argon atom ($\alpha_{Ar} = 11.084 a_B^3$) [21].

Figs. 3 and 4 shows the dependence of the differential cross sections of the electron-atom scattering on the scattering angle. As one can see, the differential cross sections calculated within the dynamic potential (3) are bigger than those obtained in framework of the static model (1). This result follows from the behavior of the phase shifts (Eq.(6)). With decrease in the plasma density the difference between static and dynamic screening diminishes due to weakening of the many-particles effects in general. On Fig.5 the total cross sections for electron scattering on the helium and argon atoms as the functions of the wave vector are presented. At $r_s = 2$ in the region of the wave vector near $ka_B = 1$ one can see the appearance of the difference between data obtained on the basis of the static and dynamic potentials. They coincide at less and bigger values of ka_B. With an increase in r_s this difference diminishes.

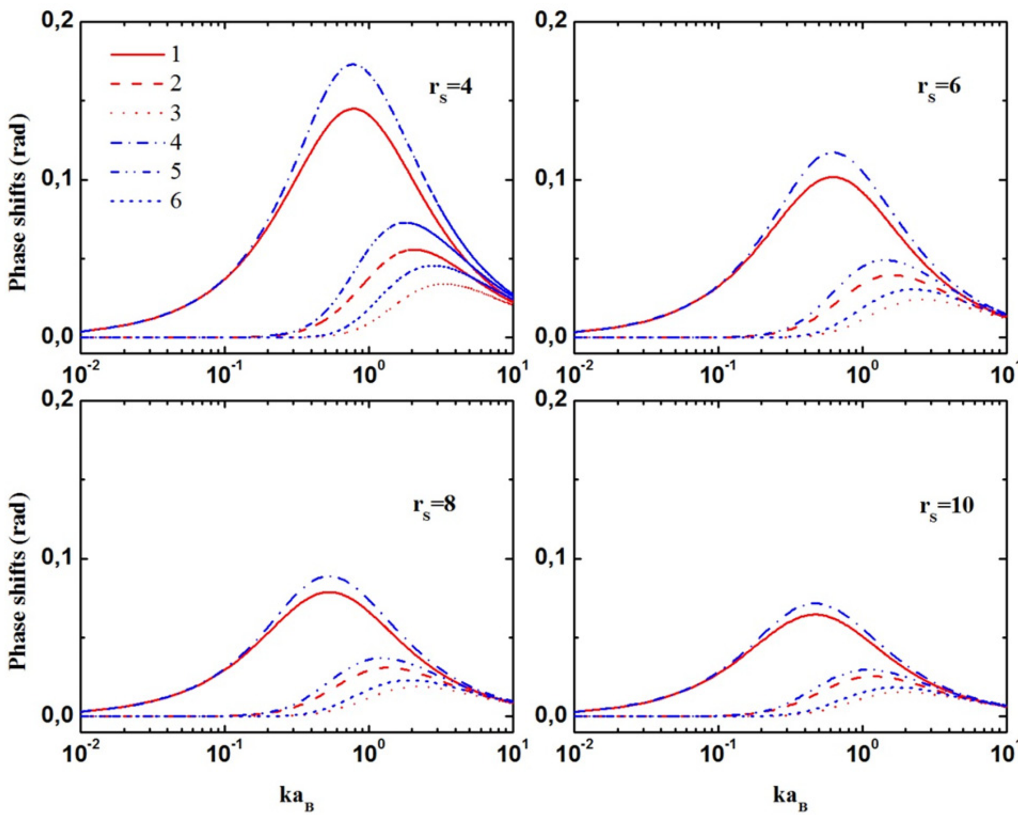

Figure 1 – Phase shifts for electron scattering on the helium atom, $\Gamma = 0.5$. On the basis of the static potential (red lines in electronic version): 1 - $l = 0$; 2 - $l = 1$; 3 - $l = 2$. On the basis of the dynamic potential (blue lines in electronic version): 4 - $l = 0$; 5 - $l = 1$; 6 - $l = 2$.

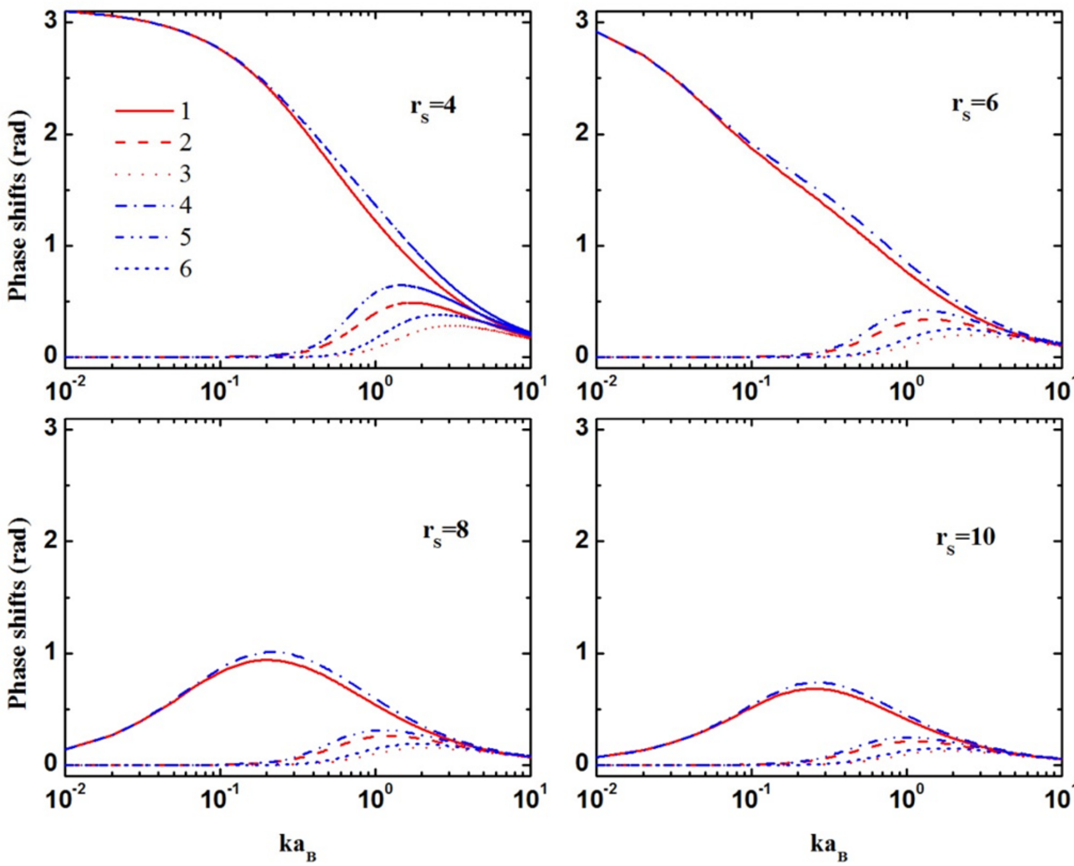

Figure 2 – Phase shifts for electron scattering on the argon atom. $\Gamma = 0.5$. On the basis of the static potential (red lines in electronic version): 1 - $l = 0$; 2 - $l = 1$; 3 - $l = 2$. On the basis of the dynamic potential (blue lines in electronic version): 4 - $l = 0$; 5 - $l = 1$; 6 - $l = 2$.

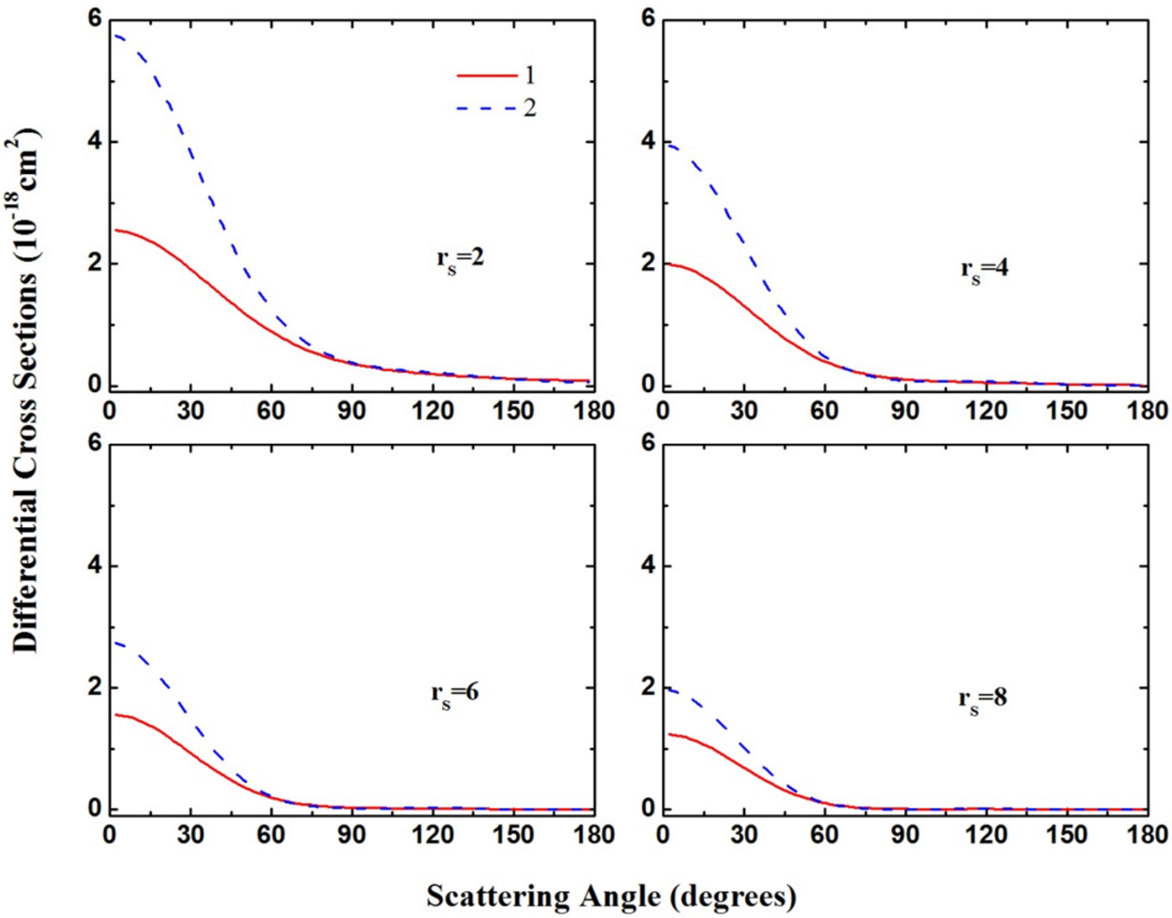

Figure 3 – Differential cross sections for electron scattering on the helium atom, $\Gamma = 0.5$, $ka_B = 1$. 1 - On the basis of the static potential (red lines in electronic version); 2 - On the basis of the dynamic potential (blue lines in electronic version).

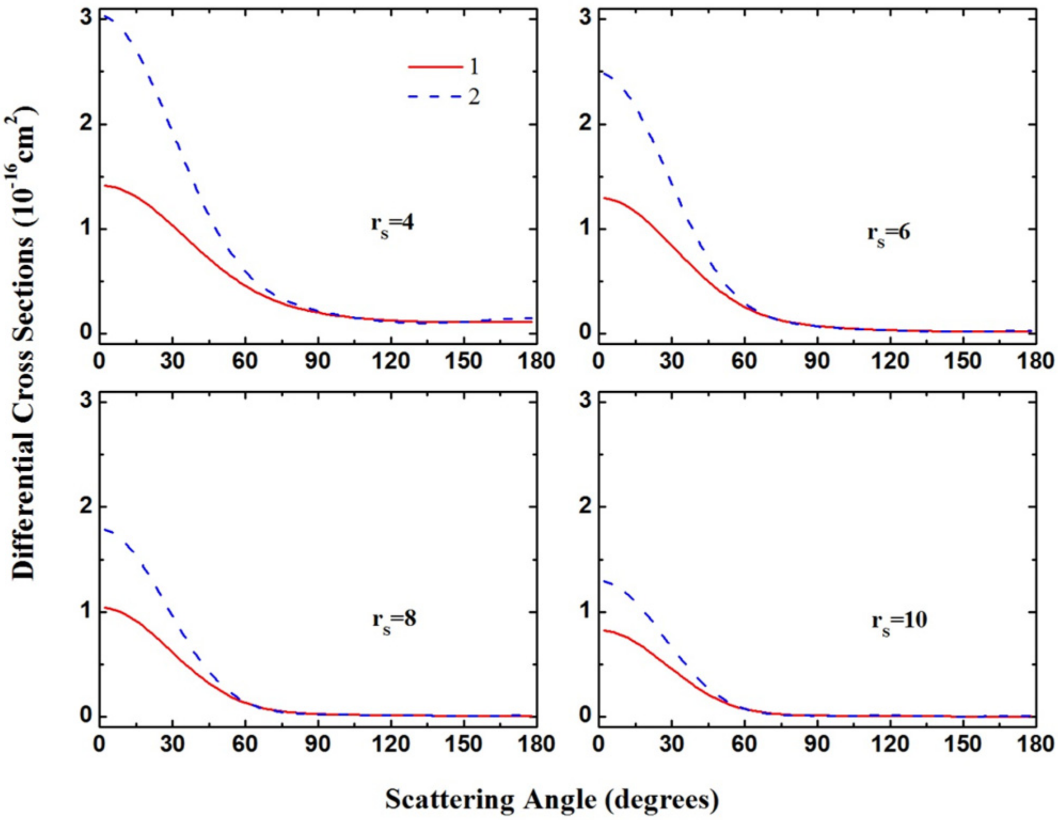

Figure 4 – Differential cross sections for electron scattering on the argon atom, $\Gamma = 0.5$, $ka_B = 1$. 1 - On the basis of the static potential (red lines in electronic version); 2 - On the basis of the dynamic potential (blue lines in electronic version).

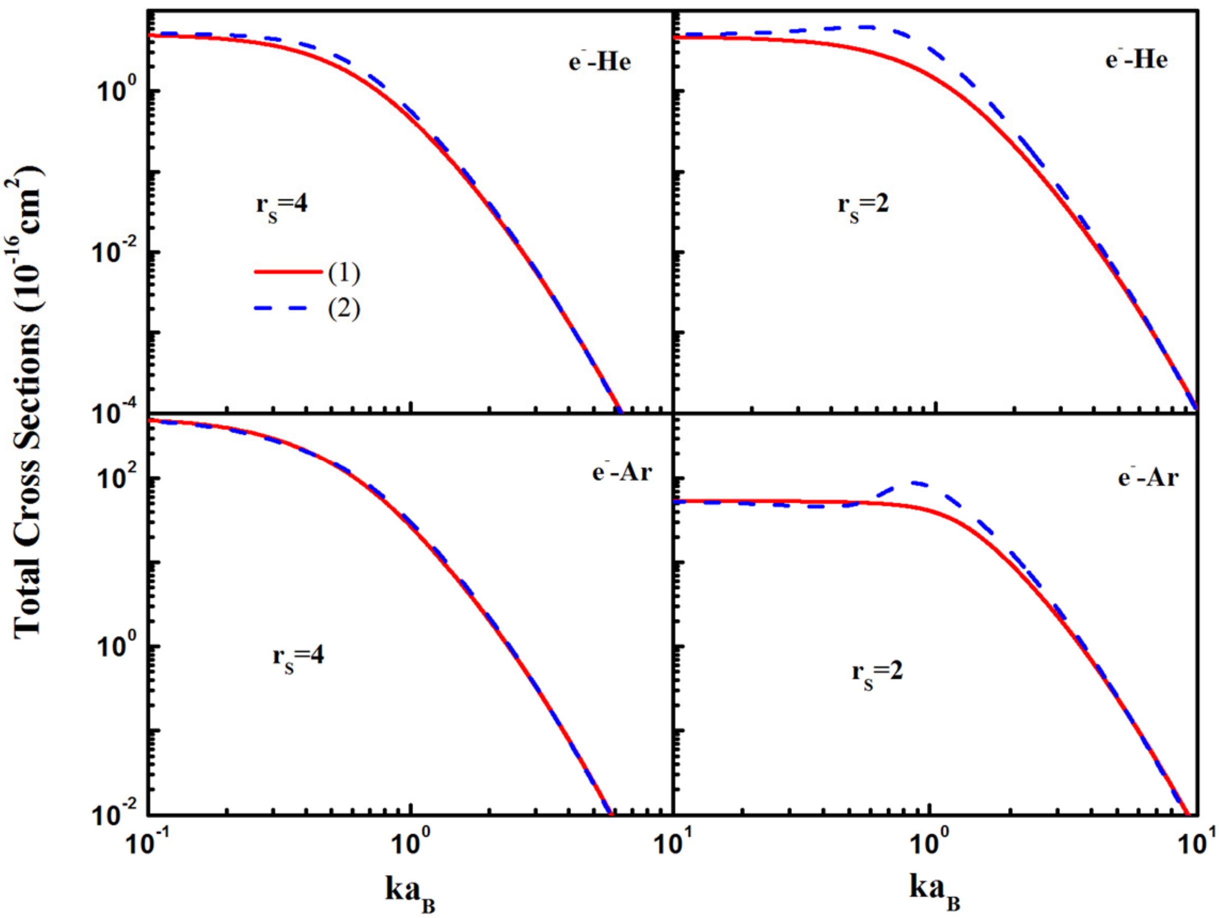

Figure 5 – Total cross sections for electron scattering on the helium and argon atoms, $\Gamma = 0.5$. 1 - On the basis of the static potential (red lines in electronic version; 2 - On the basis of the dynamic potential (blue lines in electronic version).

IV. Conclusion

In contrast to the work [6], where the characteristics of the electron scattering on the charges were investigated, in present work, the scattering phase shifts and cross sections of the electron scattering on the helium and argon atoms were considered. In both of works, the studying was carried out on the basis of the dynamic model of the electron-charge and the electron-atom interaction, correspondingly. Analysis of the results obtained in this work has shown that taking into account dynamic screening leads to increase in the phase shifts, the differential and total cross sections of the electron scattering on the atom in comparison with those, obtained by taking into consideration static screening, that is in agreement with results obtained for electron-charge scattering [6]. This increase can be considerable for the dense plasmas. The appearance of the bound state Ar^{-} at some values of the density parameter is also seen in the density-dependent electron-atom phase-shifts. In particular, it can influence on the recent results for the thermodynamic and transport properties of the dense semiclassical plasmas.

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