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About Dynamics of Muffs from Physically Nonlinear Material

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Abstract. The paper is devoted to simulation of nonlinear dynamic processes in mechanical systems with nonlinear characteristics of the physical nature. As an example torsional oscillation of rubber-cord muff, which connects rotating shafts of a drive of machines, is investigated. Influence of nonlinear properties of a muff on amplitude-frequency characteristics of a kinematic circuit is established. The research results are in accord with experimental data of other works.

Introduction

From the analysis of dynamics of drives of machines it is known, that the reason for the breakage of rotating elements is the vibrations, which are not taken into consideration when designing. Internal and external activators of vibrations are: non-uniform distribution of capacity on branches of transverse transmission, misalignment of axles in the zone of a suspension bracket of working devices, and also in the junctions with chain muffs; their insufficient compensatory ability, etc. However, the basic source of excitation of the vibrations is the variable torsional moment of the engine.

Effective means of struggle against dangerous development of torsional vibrations is change of frequencies of proper vibration of multimass system and removal of resonant frequencies from the zone of operational modes. This occurs due to the change of shaft rigidity, where there is a node of vibrations, or the moments of inertia of weight with the greatest amplitude of vibrations. In a kinematic circuit of machines replacement of chain muffs with rubber-cord muffs takes place. These muffs, possessing good damping properties, are remarkable for raising durability due to element reinforcement.

The purpose of the work is research of influence of caoutchouc properties on amplitude-frequency characteristics of the rubber-cord muff.

Simulation of torsion of rubber-cord muff

Rubber-cord muffs represent threw-reinforced rubber-elastic muffs, which work as zero-clearance connections. The diameter of the reinforcing wire reaches 0,2 mm. The muff due to the elasticity of the rubber element has six limited mobilities. The reinforced rubber element possesses the greatest rigidity at its torsion relative to a shaft axis, and significantly smaller rigidity - in radial and axial directions. These muffs possess good damping properties. They are characterized by raising durability for the account of reinforcement of the element, and dimensions, which are smaller than threw-rubber muffs with vulcanized metal plugs.

Here, considering the rubber-cord muffas part of the shafts which it connects, torsional oscillations of the rubber-cord shaft are modeled in view of physical nonlinearity of the material. Elastic potentials most profoundly reflect nonlinear properties of rubber-cord material. They are received experimentally for various types of rubber and characterize their stress - deformable state. As an example in the paper we consider Mooney elastic potential, which describes the properties of an isotropic incompressible body [1] and is received at uniaxial stretching of a rubber sample:

$$W = C_1(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3) + C_2\left[\frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} + \frac{1}{\lambda_3^2} - 3\right], \tag{1}$$

$\lambda_1, \lambda_2, \lambda_3$ - are coefficient of extensionin three main directions, C_1 and C_2 - modules of shear, which values are known. In case of turgid caoutchouk they are connected as follows: $C_2 / C_1 = 0{,}1$ [1].

The authors of works [2, 3], excluding shaft rotation as the common to all its sections, have developed a nonlinear dynamic model of torsional vibrations of rubber-cord muff with regard to its nominal rotation:

$$G_0 J_p \frac{\partial^2 \varphi}{\partial z^2} + \frac{1}{8} G_0 J_p \varphi \frac{\partial^2 \varphi}{\partial z^2} + \frac{1}{4} G_0 J_p \left(\frac{\partial \varphi}{\partial z} \right)^2 - \rho J_p \frac{\partial^2 \varphi}{\partial t^2} + \xi_1 \frac{\partial^3 \varphi}{\partial z^2 \partial t} - \xi_2 \frac{\partial \varphi}{\partial t} = F(z,t), \tag{2}$$

$\varphi(z,t)$ - is a corner of twisting of shaft cross-section;

$G_0 J_p$ - rigidity of a shaft on torsion;

ρJ_p - moment of inertia of shaft length unit;

ξ_1 , ξ_2 - factors, which characterize internal and external friction, accordingly;

$F(z,t)$ - intensity of the external torsional moment.

Thus the condition of isotropy and incompressibility of the material are used:

$$\lambda_1 \lambda_2 \lambda_3 = 1 \tag{3}$$

and the transition from lengthening factors $\lambda_1, \lambda_2, \lambda_3$ in three main directions to the main components of deformation tensor $\varepsilon_1, \varepsilon_2, \varepsilon_3$:

$$\lambda_1 = \sqrt{1 + 2\,\varepsilon_1}, \quad \lambda_2 = \sqrt{1 + 2\,\varepsilon_2}, \quad \lambda_3 = \sqrt{1 + 2\,\varepsilon_3}, \tag{4}$$

Applying Green's approach, with a view of the utility of the dynamic analysis of mechanisms and machines, the dynamic model is under construction in traversing. Using nonlinear dependence of stress tensor and deformations [4]:

$$\sigma_{ij} = \frac{1}{2} \left(\frac{\partial}{\partial \varepsilon_{ij}} + \frac{\partial}{\partial \varepsilon_{ji}} \right) W(\varepsilon_{11}, \ldots, \varepsilon_{33}), \tag{5}$$

the value of a vector of shearing stress in a shaft section is determined:

$$|\sigma| = \frac{r \frac{\partial \varphi}{\partial z} \sqrt{\left(1 + \frac{1}{4}\varphi^2\right)}}{a} \sqrt{C_1^2 - 4C_2^2 + 4a(C_1 C_2 - C_2^2) + a^2 (C_2^2 - 2C_1 C_2)}, \tag{6}$$

where $a = \left[1 - r^2 \left(\frac{\partial \varphi}{\partial z} \right)^2 \left(1 + \frac{1}{4}\varphi^2 \right) \right]^2$.

The model, described by the Eq. 2, takes into account both physical nonlinearity of muff material, and its finite strain. Thus the hypothesis of flat sections and the second Novozhilov V.V. system of the simplifications is accepted.

For the research of dynamic characteristics of physically nonlinear systems with the equations of movement of the type Eq. 2 the authors of the work have developed the technique for research of resonant vibrations of systems and definition of their stability zones. It allows to define resonant frequencies of vibrations and to exclude them from operational modes. Thus a nonlinear model with distributed parameters (Eq. 2) with the method of division of variables leads to the following:

$$\ddot{f} + K\dot{f} + \alpha_1 f + \alpha_2 f^2 = \Phi(t)\,, \tag{7}$$

where K - coefficient of linear - viscous resistance; α_1 and α_2 - coefficients of linear and nonlinear components of the elastic characteristic, accordingly; Φ(t) - periodic disturbing force.

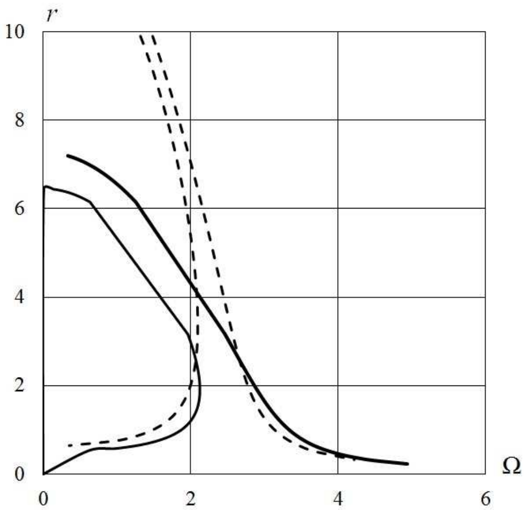
Force Φ(t) can be presented as a spectrum of harmonic components with frequencies, which are divisible by frequency of the disturbing force Ω.

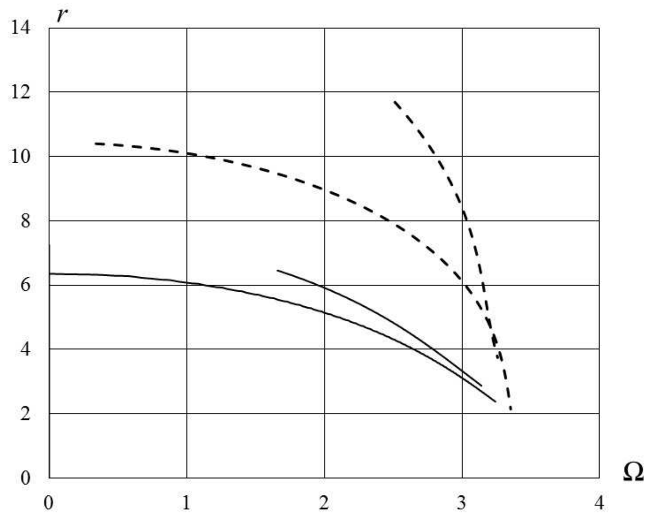
With the method of harmonic balance the authors of the work have investigated resonance in a physically nonlinear system on the basic frequency.

It is known that research of the resonance on the basic frequency in nonlinear systems gives the first approximation to the solution of quite a difficult problem of analysis of the behavior of the present system on frequencies, which are divisible by frequency of the disturbing force (sub- and ultraharmonious oscillations). Besides, specification of instability zones of the basic resonance (the first, the second, etc.) allows the researcher to judge at the stage of studying of the basic resonance about an opportunity of occurrence of resonances on the maximum frequencies with their frequencies definition.

Their displacement to the zone of the lowest frequencies in comparison with linear cases is established. In Fig. 1 and Fig.2 curve amplitude-frequency characteristics of the basic resonance and the zones of its stability, accordingly, are submitted.

In Fig. 1 we can observe that in the case of physical nonlinearity of the system (aheavy full line) there is a reduction in size of amplitude of the basic resonance and displacement of resonant frequency into the area of smaller frequencies. The dashed line submits a case of the basic resonance for a linear case.





_ _ $\alpha_2 = 0{,}5$; ___ $\alpha_2 = 1$ at $K_1 = 0{,}05$; $\alpha_1 = 5$

Fig.1. - Amplitude-frequency characteristic for systems with nonlinear-viscous resistance ($F_0 = 5$; $F_1 = 15$)

Fig. 2. - Influence of physical nonlinearity of the system on stability zone

The analysis of instability zones of the basic resonance also confirms its displacement into the area of smaller frequencies (Fig. 2). This illustrates good vibroprotective properties of the considered rubber-cord material and efficiency of its application for removal of resonant frequencies of a drive of machines from an area of operating conditions.

Potentials of elastic deformation of physically nonlinear mediums are received experimentally for particular cases of deformation [4]. Absence of a universal form of elastic potential, which describes the behavior of bodies at various forms of their deformation, brings attention to the question on choice and applicability of various potentials for the considered case.

Quasianalytic estimation of models of rubber-cord muffs, which are set by Treloar and Mooney potentials

In practice there are enough elastic potentials of physically nonlinear mediums [1,4]. It is known that the potential of trinomial theory gives the greatest discrepancy in research results. The potentials of Hart - Smith, Mooney and Treloar theory give a picture of a body mode of deformation in small quantitative discrepancies. Thus Treloar theory gives certain "averaging" of the first two theories (Fig.3) [4]. Being guided by the aforesaid, with a view of the comparative analysis of rubber-cord muff models, properties of the rubber-cord muff material are set by Treloar potential.

With a view of comparison with the previous example, the properties of material of rubber-cord muff are set by Treloar potential. Mooney potential is an expansion of Treloar potential. The latter is a particular case of the general Mooney theory, when one of the constants C_2 is equal to zero, that is [1]:

$$W = \frac{1}{2} G(\lambda_1^2 + \lambda_2^2 + \lambda_3^3 - 3)\,. \tag{8}$$

Treloar potential describes "neo-Hooke" material and is applied, as well as in the first case, only to incompressible bodies. Following the above specified technique, the value of shearing stress in a section of a shaft is determined:

$$|\tau| = \frac{Gr\dfrac{\partial\varphi}{\partial z}\sqrt{\left(1+\dfrac{1}{4}\varphi^2\right)}}{\left[1 - r^2\left(\dfrac{\partial\varphi}{\partial z}\right)^2\left(1+\dfrac{1}{4}\varphi^2\right)\right]^2}\,. \tag{9}$$

Thus the dynamic model of shaft torsional vibrations looks as follows:

$$GJ_p\frac{\partial^2\varphi}{\partial z^2} + \frac{1}{8}GJ_p\varphi\frac{\partial^2\varphi}{\partial z^2} + \frac{1}{4}GJ_p\left(\frac{\partial\varphi}{\partial z}\right)^2 - \rho J_p\frac{\partial^2\varphi}{\partial t^2} + \xi_1\frac{\partial^3\varphi}{\partial z^2\partial t} - \xi_2\frac{\partial\varphi}{\partial t} = F(z,t)\,, \tag{10}$$

G - module of material shear. Model (Eq. 10) is identical to model (Eq. 2), received at the same assumptions. The distinction is only in the "total" module of shear G_0.The difference of the model has quantitative character, which basically is justified by the concept of construction of the most elastic Mooney potential.

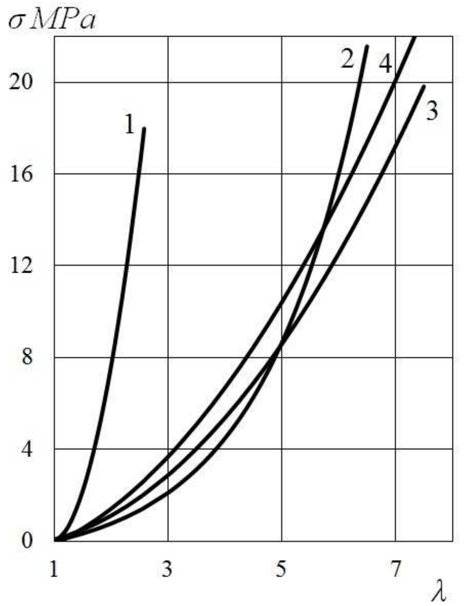
Setting edge and starting conditions for models, described by the Eq. 2 and Eq. 10:

$$\varphi(x,t)\Big|_{x=0} = 0, \qquad \varphi(x=L, t=0) = \frac{\pi}{3},$$

$$G_0 J_p\frac{\partial\varphi}{\partial x}\Big|_{x=L} = 0, \qquad \frac{\partial\varphi}{\partial x}(x=L, t=0) = 0, \tag{11}$$

the comparative analysis of models is carried out. Proper vibrations of systems are investigated.

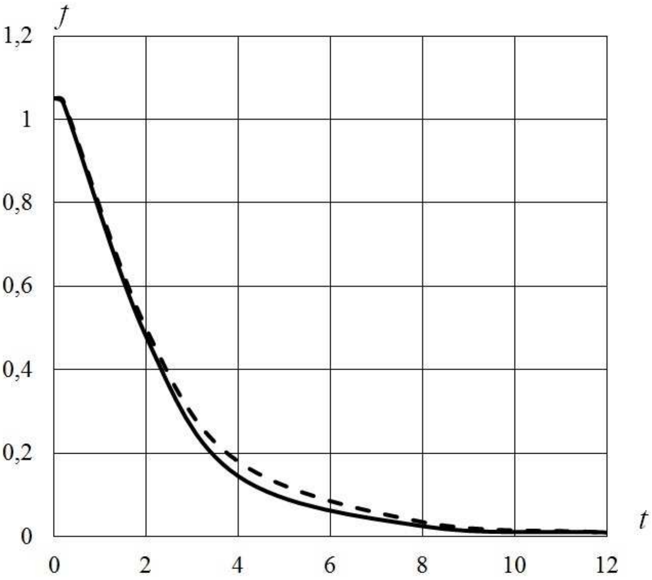
Thus Bubnov-Galyorkin separation method of variables is applied, and dimensionless coordinate $z = x / L$ and time $\tau = t\omega_0$ are entered. Good conformity of solution results of both models (Fig.4) is received. As Mooney potential is an expansion of Treloar potential, the numerical coordination of models observable here testifies to the insignificance of amendment to Treloar potential. It takes place within the framework of the accepted assumptions (finiteness of the strain and Novozhilov V.V. second system of simplifications).



1- trinomial theory; 2 - Hart Smith theory ;

3 - Mooney theory; 4 – Treloar theory

Fig.3 - Comparison of various theories by an example of a rubber sample stretching



$\xi_1 = 0,01\kappa gm^3 / c;\ \xi_2 = 0,2kgm / c;$

$2C_1 = 1 \cdot 10^5 н / m^2,\ R = 0,05\ m;$

L=0,75 m ; ρ=1,1.10^3kg/m^3

Fig.4 - Comparative analysis of models of physically nonlinear mediums

_ _ _ on the basis of Treloar potential,

____ on the basis of Mooney potential

Practical Application of Rubber-Cord Muffs

The results received above are in accord with the results of experimental research in work [5]. The author of a work [5] investigates the dynamics of the machine shaft drive under the action of the variable twisting moment:

$$T_d = T_0 + \sum_{n=1}^{m} T_n \cos(n\omega_n t - \gamma_n) \ , \tag{12}$$

where T_0 - average twisting moment; T_n - amplitude of the periodic twisting moment of n-th harmonics; $n = \overline{1 \div m}$ - numbers of harmonics; ω - angular speed of rotation of the engine crankshaft; γ - phase of shear. It is established that the left and right safety muffs and drive pulleys are exposed to the greatest influence of the external twisting moment. Thus a dominating harmonic of the twisting moment is the second one (Table 1, the first variant). This testifies that at resonance occurrence the resonance at the second frequency will be dominating.

Table 1-Amplitudes of harmonious components of the twisting moment of a drive on third gear

Element	Amplitudes of harmonics (Nm)			
Drive	A_1	A_2	A_3	A_4
1-st variant (before chain muff replacement)	17,3	77,6	12,4	4,2
2-nd variant (after chain muff replacement)	17,6	34,8	19,6	12,3

The author of a work [5] with a view of the withdrawal of the system from resonant modes, changes its rigidity by replacement of chain muffs for more pliable rubber-cord ones. It is experimentally established, that after the replacement of chain muffs the amplitude of the fluctuations on the second harmonic considerably decreases (Table 1). This confirms the results received earlier about recede of amplitudes of the fluctuations of physically nonlinear systems in comparison with linear ones (Fig.1). Besides, in Fig. 1 and Fig. 2 we can observe that in the case of physical nonlinearity of the system there is also the decrease in size of resonant frequency, that is their displacement into a zone of the lowest frequencies in comparison with linear cases.

Therefore, by the results of theoretical and experimental research it is possible to note that the replacement of chain muffs with rubber-cord ones leads to lowering of amplitudes of an angle of torsion of the most overloaded elements of the transmission route of machines.

Summary

In this paper efficiency of coupling of the rubber-like material vibration dampers were researched.

The obtained results are in a good consistent with the results of the work that demonstrates the effectiveness and feasibility to enter the kinematic chain of the machine elements which is include physically nonlinear materials.

They tend to reduction of the amplitude of vibrations and resonance frequencies. The elastic properties of rubber-coupling are modeled by Treloar and Mooney potentials which are its generalization. It was found the good coincidence between the results of researches with a small correction to the amplitude of oscillation.

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