Rotor-Liquid-Fundament System's Oscillation

A. Kydyrbekuly

Institute of Mechanics and Machine Studies, Almaty, KAZAKHSTAN
e-mail: almatbek@list.ru

Abstract The work is devoted to research of oscillation and sustainability of stationary twirl of vertical flexible static dynamically out-of-balance rotor with cavity partly filled with liquid and set on relative frame fundament.

The accounting of such factors like oscillation of fundament, liquid oscillation, influence of asymmetry of installation of a rotor on a shaft, anisotropism of shaft support and fundament, static and dynamic out-of-balance of a rotor, an external friction, an internal friction of a shaft, allows to settle an invoice more precisely kinematic and dynamic characteristics of system.

The solution of this task becomes complicated that movement of a rotating rotor and liquid movement in his cavity are interconnected that causes change of frequency of the compelled fluctuations and instability emergence, and the solved system of the equations consists of the connected equations of movement of a firm body, the equations of the continuous environment and boundary conditions for liquid.

Key words: rotor, dynamics, liquid oscillation, oscillation of fundament, dynamic out-of-balance of a rotor.

1 Introduction

In design and an assessment of vibrating characteristics of rotor cars it is necessary to consider case fluctuations, i.e. to consider dynamic system "rotor-corpus-fundament" as a whole [1,2,3]. In many theoretical and practical researches on dynamics of the rotor systems containing liquid, rotor fluctuations with liquid are considered only and thus the bed (base) is considered motionless. Such assumption leads to essential errors at an assessment of dynamic and kinematic characteristics of rotor system as a whole [2,4,5]. Pilot studies of such dynamic systems as rotor systems, show importance of the accounting of vibration of the base and need of development of measures for their decrease [2,6].

2 Putting Problems, Movement Equation System and Their Solution

The rotor cavity has a form of cylinder. The mass of liquid is considered a constant in time, and its quantity – sufficient completely to moisten cylindrical walls of a cavity even at big deviations of a rotor.

Angular speed of shaft rotation Ω_0 is constant and it is rather great so that on a free surface of liquid gravitational acceleration appears negligible in comparison with centrifugal acceleration, and the free surface represents the cylinder concerning a rotation axis. The dempfied fundament in movement of system moves in the horizontal plane. Movement of liquid is described in the cylindrical system of coordinates connected with a rotating rotor. In a condition of dynamic balance the rotor and liquid rotate as a uniform firm body (figure 1).

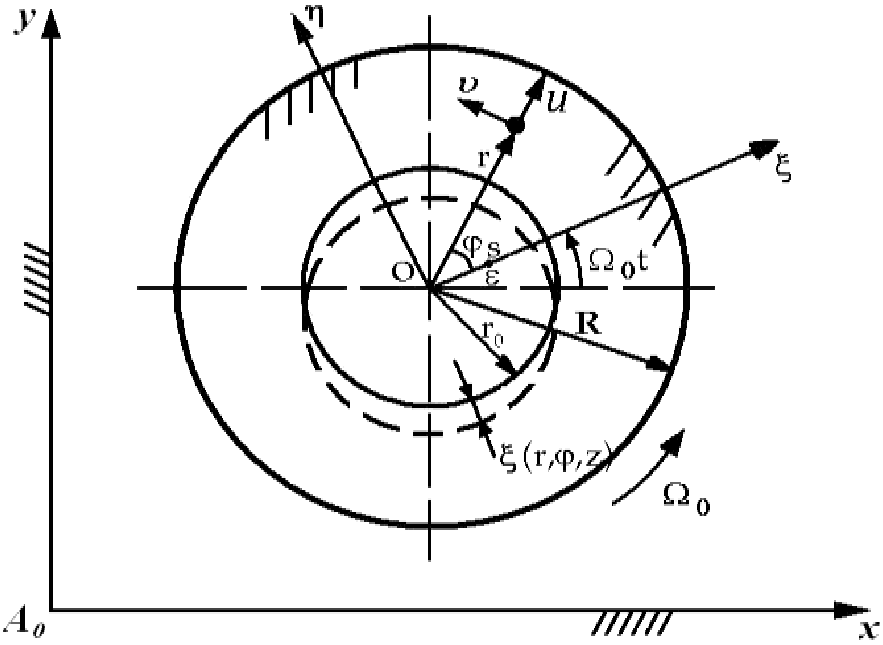

Figure 1 – Coordinates system definition

Deflections of an axis of a shaft in the direction of axes *x* and *y* motionless system of coordinates are relied the small. Liquid deviations from position of balance, derivatives on time from all amplitudes of fluctuations also are accepted by the small. Thus, for an assessment of stability of the related system the system of the linearized differential equations as a whole is considered.

Taking into account forces of an external and internal friction, forces of liquid reaction on walls of the cylinder of the movement equation of an unbalanced rotor with the cavity which has been partially filled with liquid and the base, look like

$$m\ddot{x} + \; n_e + n_i \;\; \dot{x} + p_1 x + \Omega_0 n_i y - q_1\alpha + \sigma_1 x_k = m\,\varepsilon\,\Omega_0^2 \cos\Omega_0 t + F_x, \tag{1}$$

$$m\ddot{y} + \; n_e + n_i \;\; \dot{y} + p_2 y - \Omega_0 n_i x - q_2\beta + \sigma_2 y_k = m\,\varepsilon\,\Omega_0^2 \sin\Omega_0 t + F_y, \tag{2}$$

$$A\ddot{\alpha} + C\Omega_0\dot{\beta} + \; \mu_e + \mu_i \;\; \dot{\alpha} - r_1 x + \Omega_0\mu_i\beta + s_1\alpha + \sigma_3 x_k =$$

$$= \; C - A \;\; \delta\;\Omega_0^2 \sin \;\; \Omega_0 t - \chi \;\; + M_\alpha, \tag{3}$$

$$A\ddot{\beta} - C\Omega_0\dot{\alpha} + \; \mu_e + \mu_i \;\; \dot{\beta} - r_2 y - \Omega_0\mu_i\alpha + s_2\beta + \sigma_4 y_k =$$

$$= \; A - C \;\; \delta\;\Omega_0^2 \cos \;\; \Omega_0 t - \chi \;\; + M_\beta, \tag{4}$$

$$M\,\ddot{x}_k + p_3 x_k + r_3 x + s_3\alpha + n_1\dot{x}_k = 0, \tag{5}$$

$$M\,\ddot{y}_k + p_4 y_k + r_4 y + s_4\beta + n_2\dot{y}_k = 0, \tag{6}$$

$$\frac{\partial U}{\partial t} - 2\Omega_0\upsilon = -\frac{1}{\rho}\frac{\partial P}{\partial r} - \ddot{Q}e^{-i\;\Omega_0 t+\varphi} \;\; + iz\ddot{\theta}e^{-i\;\Omega_0 t+\varphi} \;\;, \tag{7}$$

$$\frac{\partial \upsilon}{\partial t} + 2\Omega_0 U = -\frac{1}{\rho r}\frac{\partial P}{\partial \varphi} + \ddot{Q}e^{-i\;\Omega_0 t+\varphi} \;\; + z\ddot{\theta}e^{-i\;\Omega_0 t+\varphi} \;\;, \tag{8}$$

$$\frac{\partial W}{\partial t}=-\frac{1}{\rho}\frac{\partial P}{\partial z}-\ i\ddot{\theta}+2\Omega_0\dot{\theta}\ \ re^{-i\ \Omega_0 t+\varphi}\ , \tag{9}$$

$$\frac{\partial\ rU}{\partial r}+\frac{\partial\upsilon}{\partial\varphi}+\frac{\partial W}{\partial z}=0 \qquad \text{under} \qquad \rho=const\,, \tag{10}$$

with boundary conditions:
on a wall, the top and bottom borders of the cylinder

$$U=W=0\,, \tag{11}$$

on a free surface of liquid

$$\left(\frac{\partial P}{\partial t}-\rho\,\Omega_0^2 r_0 U\right)_{/\,r=r_0}=0\,. \tag{12}$$

Here is $Q=x+iy,\ \ \theta=\alpha+i\beta$.

The equations of movement of rotor system and boundary conditions to them, linearized near stationary rotation, allow decisions proportional $\exp(i\Omega_0 t)$ and $\exp(i\omega t)$ where ω is characteristic number

$$\left.\begin{aligned} x&=A_1e^{i\Omega_0 t}+B_1e^{i\omega t},\\ y&=A_2e^{i\Omega_0 t}+B_2e^{i\omega t},\end{aligned}\right\} \qquad \left.\begin{aligned} \alpha&=A_3e^{i\Omega_0 t}+B_3e^{i\omega t},\\ \beta&=A_4e^{i\Omega_0 t}+B_4e^{i\omega t},\end{aligned}\right\} \qquad \left.\begin{aligned} x_k&=A_5e^{i\Omega_0 t}+B_5e^{i\omega t},\\ y_k&=A_6e^{i\Omega_0 t}+B_6e^{i\omega t},\end{aligned}\right\} \tag{13}$$

where A_1, A_2 и A_3, A_4 are respectively linear and angular complex amplitudes of fluctuations of the rotor, caused by action of own unbalance; amplitudes of the compelled fluctuations of the base; B_1, B_2, B_3, B_4, B_5, B_6 are amplitudes of self-excited fluctuations of a rotor and base; Required frequency of fluctuations of a rotor ω generally is the complex size which valid part defines frequency of self-oscillations, and the imaginary part characterizes degree of instability of system; the second members in the right parts of the equations (13) express the fluctuations of the rotor caused by indignant movement of liquid concerning the cylinder.

To exclude a time of arguments of trigonometrical and indicative functions at the solution of the main equations of movement, relative speed of a particle of liquid and function of pressure are accepted by the proportional $\exp i(\sigma t-\varphi)$ where σ is complex own meaning that results in expediency of creation of private decisions with the same dependence on time. Function of pressure and making speeds of a particle of liquid for any point of volume, and also expression for the hydrodynamic moment and force of reaction of liquid are as a result defined.

$$P^{/}=\left[\sum_{k=1}^{\infty}Z_1\left(\frac{\pi k}{H}\gamma_1 r\right)\cos\frac{\pi k}{H}\ \ z-H\ \ +A_0 r+\frac{B_0}{r}+i\rho r\omega z\ \ \omega-2\Omega_0\ \ B_{34}\right]e^{i\ \sigma t-\varphi}\ , \tag{14}$$

$$U=\left\{\frac{1}{\rho a^2}\sum_{k=1}^{\infty}\left[\frac{2\Omega_0 i}{r}Z_1\left(\frac{\pi k}{H}\gamma_1 r\right)-eZ_1^{/}\left(\frac{\pi k}{H}\gamma_1 r\right)\right]\cos\frac{\pi k}{H}\ \ z-H\ \ +\right.$$
$$\left.-\frac{A_0}{\rho\ \ 2\Omega_0 i+e}+\frac{B_0}{\rho r^2\ \ e-2\Omega_0 i}+\frac{\omega^2 B_{12}-2iz\omega\sigma B_{34}}{2\Omega_0 i+e}\right\}e^{i\ \sigma t-\varphi}\ , \tag{15}$$

$$\upsilon=\left\{\frac{1}{\rho a^2}\sum_{k=1}^{\infty}\left[2\Omega_0 Z_1^{/}\left(\frac{\pi k}{H}\gamma_1 r\right)-\frac{ei}{r}Z_1\left(\frac{\pi k}{H}\gamma_1 r\right)\right]\cos\frac{\pi k}{H}\ \ z-H\ \ +\right.$$

$$+\frac{iA_0}{\rho\ \ 2\Omega_0 i+e}+\frac{iB_0}{\rho r^2\ \ e-2\Omega_0 i}-\frac{2z\omega\sigma B_{34}}{2\Omega_0 i+e}\Bigg\}e^{i\ \sigma t-\varphi}\ , \tag{16}$$

$$W=\left\{-\frac{1}{\rho\sigma}\frac{\pi}{H}\sum_{k=1}^{\infty}Z_1\left(\frac{\pi k}{H}\gamma_1 r\right)\sin\frac{\pi k}{H}\ \ z-H\ \ \right\}e^{i\ \sigma t-\varphi}\ , \tag{17}$$

$$F=\Omega_0^2 m_L\left(A_{12}-\frac{1}{2}iHA_{34}\right)e^{i\Omega_0 t}+m_L\left(\frac{A_0}{\rho}+\frac{B_0}{\rho R^2}+i\omega\ \ \omega-2\Omega_0\ \ \frac{H}{2}B_{34}\right)e^{i\omega t}, \tag{18}$$

$$M_\theta=\Omega_0^2 m_L\left\{\frac{1}{2}iHA_{12}+E_1A_{34}\right\}e^{i\Omega_0 t}+m_L\left\{\frac{2H}{\pi\rho R}\sum_{k=1}^{\infty}\left[\frac{1}{k^2}Z_1\left(\frac{\pi k}{H}\gamma_1 R\right)-\right.\right.$$

$$\left.-\frac{2i}{\rho H}\sum_{k=1}^{\infty}\left[Z_2\left(\frac{\pi k}{H}\gamma_1 R\right)-\frac{1}{q^2}Z_2\left(\frac{\pi k}{H}\gamma_1 r_0\right)\right]\right]+$$

$$\left.+\frac{iH}{2\rho}\left(A_0+\frac{B_0}{R^2}\right)-E_1\omega\ \ \omega-2\Omega_0\ \ B_{34}\right\}e^{i\omega t}, \tag{19}$$

$$E_1=\frac{H^2}{3}-\frac{R^2}{4}\left(1-\frac{1}{q^4}\right),\ Z_1\left(\frac{\pi k}{H}\gamma_1 r\right)=C_{1k}J_1\left(\frac{\pi k}{H}\gamma_1 r\right)+C_{2k}N_1\left(\frac{\pi k}{H}\gamma_1 r\right),\ \ e=i\sigma\ ,$$

J_1 and N_1 are Bessel and Neiman functions of 1[st] order real argument; k – axial wave number; $k=0,1,2,3,...$

The unknown constants A_0,B_0,C_{1K},C_{2K} are defined by means of boundary conditions (11), (12).

The solution proportional $\exp(i\Omega_0 t)$ gives the chance calculation of the compelled fluctuations of the rotor caused by its own unbalance, and the compelled fluctuations of the base by means of system of the equations which in a matrix form looks like:

$$\bar{A}=(a)^{-1}\bar{b}\ , \tag{20}$$

where (a) is coefficients matrix, $\bar{A}$ is vector column of unknown amplitudes of the compelled fluctuations, $\bar{b}$ is vector column of free members:

$$a\ =\begin{pmatrix} a_{11}\ a_{12}\ ....a_{16} \\ a_{21}\ a_{22}\ ....a_{26} \\ .... \\ .... \\ a_{61}\ a_{62}............a_{66} \end{pmatrix},\qquad \bar{A}=\begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{pmatrix},\qquad \bar{b}=\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{pmatrix}.$$

The equation for determination of critical speeds is deduced from an equality condition to zero of a determinant of a square matrix (a) in the absence of a friction also is a polynom of the 12th degree of an even order. Twelve critical speeds are in the case under consideration possible, from which six there correspond to the direct precession, six following are the return precession caused by a rotor unbalance.

3 Solution Results Analysis

For research of features of the compelled fluctuations of rotor system taking into account spatial movement of the ideal liquid filling a cavity of a rotor, system of the equations (20) without difficulties is solved analytically, for example, by Gauss method or number on the computer. Some results of the solution of the equation (20) are shown in figures (2)-(3). At increase in the relation of weight of a rotor to mass of the base μ as the amplitude of the compelled fluctuations of a rotor and the base increases and moves to higher values of angular speed of system. At the smallest relation of weight of a rotor to mass of the base the amplitude of

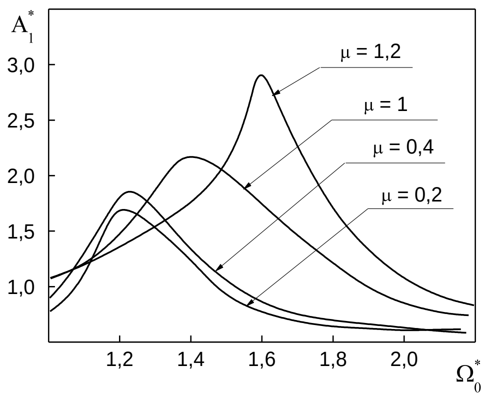

Figure 2 – Amplitude A_1^* 's dependence on angular speed of rotor at a variation μ $(k_m = 0,5)$

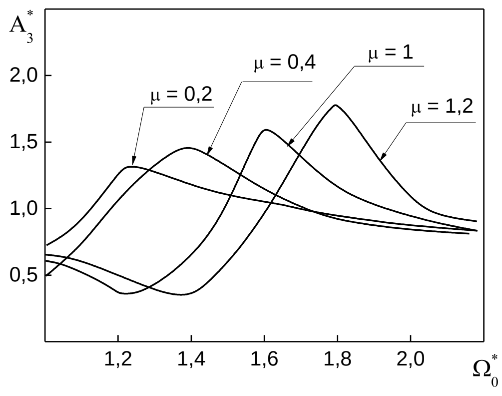

Figure 3 – Amplitude A_3^* 's dependence on angular speed of rotor at variation μ $(k_m = 0,5)$

the compelled fluctuations of a rotor and the base essentially decreases ($A_1^/ = \sqrt{|A_1|^2 + |A_2|^2}$, $A_3^/ = \sqrt{|A_5|^2 + |A_6|^2}$, $A_1^* = \dfrac{A_1^/}{\varepsilon}$, $A_3^* = \dfrac{A_3^/}{\varepsilon}$, $\Omega_0^* = \dfrac{\Omega_0}{\Omega_{кр}}$, $\Omega_{кр} = \sqrt{\dfrac{p_1 + p_2}{2m}}$).

Required frequency of fluctuations of a rotor ω generally is the complex size which valid part defines frequency of self-oscillations, and the imaginary part characterizes degree of instability of system. Stability of stationary movement of system is determined by character of an imaginary part of complex roots of the characteristic equation. The mode of stationary rotation is unstable if possible values of the characteristic equation ω have a negative imaginary part.

The common numerical decision of the characteristic equation is inconvenient, as required values ω are included in arguments of Bessel functions.

For of the confidant definition of a zone instability we will accept $k = 0$, i.e. when rotor raises waves only on a flat surface of liquid. In this case the axial component of speed of a particle of liquid is equal to zero $W = 0$ and the characteristic equation after transformations becomes a polynom of the twentieth degree with complex factors. Roots of this polynom will serve as zero approach of roots of the transcendental characteristic equation where space movement of liquid is considered at $k > 0$. In a considered task, as well as in many problems of mechanics and equipment, research of stability of system at its not conservatism has paramount value. Stability of movement of a rotor is determined by character of roots of the characteristic equation of system, i.e. by Lyapunov's method. Some results of the solution of these equations, are presented in Figure 4 where dependences of a material part are shown ω^* from angular speed of rotor Ω_0^* under filling $q = 1,5$ in case of anisotropic support. On speed intervals A_1A_1, A_2A_2 и A_3A_3 among roots of the equation there are roots with a negative imaginary part. Therefore, intervals of speeds of a rotor A_1A_1, A_2A_2 и A_3A_3 limit zones of system instability. One root lying in the top semi-plane material, is close to zero and in drawing isn't shown. The following ten roots of the equation lie in the bottom semi-plane and in drawing also aren't shown. Critical speed of system is defined graphically by line crossing $\omega^* = \Omega_0^*$ with schedules of frequencies $\omega^* = \omega^* \ \Omega_0^*$.

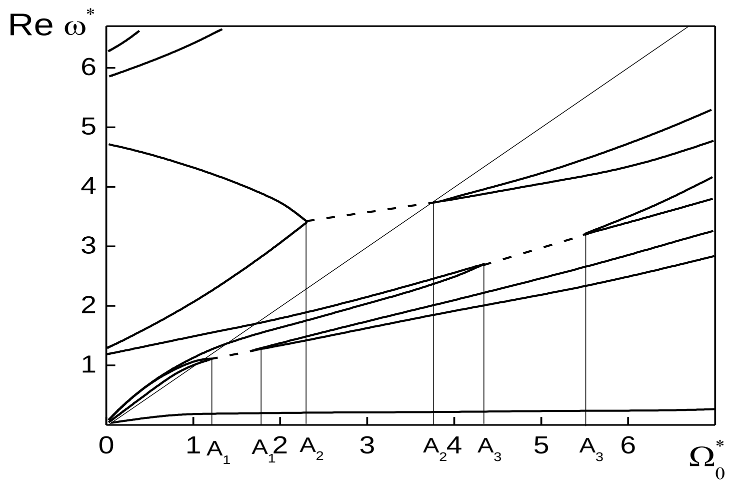

Figure 4 – Dependence of material part ω^* angular speed at $q = 1{,}5$; $\mu = 0{,}4$; $c_1^{/} = c_1^{//} \neq c_2^{/} = c_2^{//}$

Emergence of three zones of instability is a direct consequence of asymmetry of installation on shaft of rotor, oscillation of the fundament, anisotropism of support and internal resistance. With extent growth of filling at the fixed values of other parameters, the width of all zones of instability at first extends, their borders move to higher speeds of rotor rotation then are narrowed. When the rotor is completely filled with liquid, zones of instability disappear.

4 Conclusions

Analytically generalized dynamic model of the Rotor-Liquid-Fundament system being generalized and for systems without liquid, and also without interrelation of a rotor and the base is solved. Exact solutions of the linearized equations of liquid movement, giving the chance studying and estimating effect of wave properties of a free surface of liquid on movement of rotor system are found. Elastic installation of the base allows enter the external damping necessary for obtaining of the best characteristics, than at its rigid installation.

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