UDC 530, 524

Propagation of X-ray and gamma ray emissions in strong magnetic and gravitational fields

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We consider the propagation of X-ray and gamma ray emissions in strong magnetic and gravitational fields of the pulsar in nonlinear vacuum electrodynamics. We show that the radiation has birefrigence. We have calculated the delay between the two modes, as they propagate from the pulsar to the detecting device. gamma ray astrophysics, pulsar, magnetar, quantum electrodynamics, gravity. We estimate the numerical value of the delay in the case where the magnetic field source is a neutron star with field on a surface B~10^{16} G (magnetar). Due to the our condition, which must be satisfied for all points of considered beams, our calculation will be applicable only to the beams, for which the pericenter rp exceeds ten radii of the neutron star

Key words: gamma ray, pulsar, magnetar.
PACS number: 97.60. Gb

1 Introduction

The field equations in nonlinear post-Maxwell electrodynamics, which is a direct consequence of quantum electrodynamics [1] have the form:

$$1\sqrt{-g}\partial\partial x^{\beta}\{\sqrt{-g}Q^{\sigma\beta}\}=-4\pi cj^{\sigma}, \qquad (1)$$

$$Q^{\sigma\beta}=8\pi\partial L\partial F_{\beta\sigma}=$$
$$=\{1+\xi(\eta_1-2\eta_2)I_2\}F^{\sigma\beta}+$$
$$+4\xi\eta_2F^{\sigma\nu}F_{\nu\mu}F^{\mu\beta},$$

where j^{β} is the current density four-vector, g – determinant of the metric tensor, $\xi=1/B_q^2$,
$I_2=F_{\beta\sigma}F^{\beta\sigma}$– invariant of the electromagnetic tensor$F_{\beta\sigma}$ and according to quantum electrodynamics $\eta_1=e^2/(45\pi\hbar c)=5.1\cdot10^{-5}$,
$\eta_2=7e^2/(180\pi\hbar c)=9.0\cdot10^{-5}$.

The second pair of equations of electrodynamics coincides with the corresponding equations of Maxwell's theory:

$$\partial F_{\mu\beta}\partial x^{\nu}+\partial F_{\beta\nu}\partial x^{\mu}+\partial F_{\nu\mu}\partial x^{\beta}=0. \qquad (2)$$

Metric tensor in equations (1) satisfy Einstein equations [2]:

$$R_{\beta\sigma}-12g_{\beta\sigma}R=-8\pi Gc^4T_{\beta\sigma}, \qquad (3)$$

where $R_{\beta\sigma}=R^{\nu}_{\beta\sigma\nu}$ – Ricci tensor. $T_{\beta\sigma}$– energy-momentum tensor of the matter and all fields, including electromagnetic. The system of equations (1) – (3) in our problem will be sought by the method of successive approximations with a precision linear in the small dimensionless parameters: the gravitational potential and post-Maxwell amendments. The gravitational field of the pulsar will be assumed to be spherically symmetric, and in the harmonic Fock coordinates [2] metric will be expanded in the small parameter α/r with the required accuracy:

$$g_{00}=1-2\alpha r,\quad g_{rr}=-1-2\alpha r,$$
$$g_{\theta\theta}=r^2g_{rr},\quad g_{\phi\phi}=g_{\theta\theta}\sin^2\theta, \qquad (4)$$

where $\alpha=\gamma M/c^2$, γ – gravitational constant, and M mass of the pulsar.

Suppose that at time $t=0$ from the point $r=r_0$ of the pulsar magnetosphere hard radiation impulse was emitted. Then, in magnetic field of the pulsar that impulse, because of birefringence, will split [3] into two impulses with orthogonal polarizations and moving at different speeds.

For the convenience of further calculations, we introduce the spherical coordinate system as follows.

Consider a beam of the first normal mode and draw a tangent to it at the point $r = r_0$. Axis of the spherical coordinate system will be directed in such a way, so that the tangent to the chosen beam and the center of the pulsar would be lying in the same plane, and $\theta = \pi / 2$, and the azimuthal coordinate ϕ of the source of hard radiation would be equal to $\phi = 0$.

As it is accepted [4] in the problems of celestial mechanics, instead of the radial coordinate r we introduce the coordinate $u = 1/r$. Then, the non-zero components of the dipole electromagnetic field tensor of the pulsar m, in the coordinate system u, θ, ϕ, with the required for our purposes accuracy will be:

$$F_{u\vartheta} = -F_{\theta u} = |m| \sin\theta_0 \sin(\phi - \phi_0),$$

$$F_{u\phi} = -F_{\phi u} = |m| \sin\theta[\sin\theta_0 \cos\theta \cos(\phi - \phi_0) - \sin\theta \cos\theta_0], \qquad (5)$$

$$F_{\phi\theta} = -F_{\theta\phi} = 2|m| u \sin\theta[\sin\theta_0 \sin\theta \cos(\phi - \phi_0) + \cos\theta \cos\theta_0].$$

According to [3,5] the propagation of electromagnetic waves in external electromagnetic and gravitational fields in nonlinear electrodynamics with field equations (1) – (3) is equivalent to the propagation of the normal modes through the isotropic geodesics in effective space-time for which metric tensor $G^{eff\,(1,2)}_{\nu\mu}$ has the form:

$$G^{eff\,(1,2)}_{\nu\mu} = g_{\nu\mu} - 4\eta_{(1,2)}\xi F_{\nu\beta} g^{\beta\sigma} F_{\sigma\mu}. \qquad (6)$$

Therefore the study of the laws of propagation of electromagnetic impulses in magnetic (5) and gravitational (4) fields of a pulsar is conveniently carried out not by using equations (1) – (2), but based on the analysis of isotropic geodesics in space-time with the metric tensor (6).

Equations for isotropic geodesics in the effective space-time with the metric tensor (6) will be written in the form where differentiation is performed not with respect to the affine parameter σ, but with respect to the azimuthal angle ϕ:

$$d^2x^0 d\phi^2 + \{\Gamma^0_{\beta\mu} - dx^0 d\phi \Gamma^3_{\beta\mu}\} dx^\beta d\phi dx^\mu d\phi = 0,$$

$$d^2u d\phi^2 + \{\Gamma^1_{\beta\mu} - du d\phi \Gamma^3_{\beta\mu}\} dx^\beta d\phi dx^\mu d\phi = 0,$$

$$d^2\theta d\phi^2 + \{\Gamma^2_{\beta\mu} - d\theta d\phi \Gamma^3_{\beta\mu}\} dx^\beta d\phi dx^\mu d\phi = 0, (7)$$

where $\Gamma^\nu_{\beta\mu}$ – Christoffel symbols defined in effective space-time with the metric tensor (6).

The system of equations (7) has a first integral:

$$G^{(1,2)}_{\beta\mu} dx^\beta d\phi dx^\mu d\phi = 0. \qquad (8)$$

Equations (7) and (8) are non-linear, but in our case there are small parameters αu and $m^2\xi\eta_{1,2}u^5$. Therefore, the solution of these equations will be sought by the method of successive approximations in these small parameters.

2 Solution of the equations for beams

In the zeroth approximation in small parameters the beam under mentioned boundary conditions, will be a straight line in the plane $\theta = \pi/2$ passing through the point $u = u_0$, $\phi = 0$ and take a value $u = u_p$ at pericenter.

Solving this system of equations, we arrive at the relations:

$$u(\phi) = u_p \sin(\phi + \psi),$$
$$x^0(\phi) = ct = \cos\psi u_p \sin\psi - \cos(\phi + \psi) u(\phi), \quad (9)$$
$$\theta(\phi) = \pi 2,$$

where ψ is defined from: $\sin\psi = u_0 / u_p$.

We search the solution of the system of equations (7)-(8) for the first normal wave in the ordinary form for equations of that type:

$$u(\phi) = u_p \sin(\phi + \psi) + \alpha u_p^2 \Phi_1(\phi) + m^2 \xi \eta_1 u_p^7 \Phi_2(\phi),$$

$$\theta(\phi) = \pi 2 + \alpha u_p \Phi_3(\phi) + m^2 \xi \eta_1 u_p^6 \Phi_4(\phi),$$

$$x^0(\phi) = \cos\psi u_p \sin\psi - \cos(\phi+\psi)u(\phi) + \\ + \alpha\Phi_5(\phi) + m^2 \xi \eta_1 u_p^5 \Phi_6(\phi), \quad (10)$$

where $\Phi_a(\phi), a = 1-6$ are unknown functions of the azimuthal angle ϕ, having zero order of smallness.

Using an expression (10), we search the solution of the system of equations (7)-(8) for the first normal wave:

$$\Phi_1(\phi) = 2 + S_1 \sin\phi + C_1 \cos\phi,$$
$$\Phi_2(\phi) = 164\{f_2(\phi) + S_2 \sin\phi + C_2 \cos\phi\},$$

$$\Phi_3(\phi) = S_3 \sin\phi - C_3 \cos\phi,$$
$$\Phi_4(\phi) = \sin 2\theta_0 64\{f_4(\phi) + S_4 \sin\phi + C_4 \cos\phi\},$$

where we use the notation:

$$f_2(\phi) = \sin^2\theta_0\{\cos 2(\phi_0+\psi)[195\phi\cos(\phi+\psi) + \\ +65\sin^3(\phi+\psi) + 26\sin^5(\phi+\psi) + 152\sin^7(\phi+\psi) - \\ -144\sin^9(\phi+\psi)] + 2\sin 2(\phi_0+\psi)[[72\sin^8(\phi+\psi) - \\ -40\sin^6(\phi+\psi) - 26\sin^4(\phi+\psi) - \\ -39\sin^2(\phi-\psi)]\cos(\phi+\psi) + 39\phi\sin(\phi+\psi)] + \\ +32\sin^7(\phi+\psi) - 24\sin^5(\phi+\psi) - \\ -60\sin^3(\phi+\psi) - 180\phi\cos(\phi+\psi)\} - \\ -16[2\sin^5(\phi+\psi) + 5\sin^3(\phi+\psi) + 15\phi\cos(\phi+\psi)],$$

$$f_4(\phi) = [75\phi\cos(\phi+\psi) + 25\sin^3(\phi+\psi) + \\ +10\sin^5(\phi+\psi) - 40\sin^7(\phi+\psi)]\sin(\phi_0-\psi) + \\ +[3\phi\sin(\phi+\psi) - [3\sin^2(\phi+\psi) + 2\sin^4(\phi+\psi) + \\ +40\sin^6(\phi+\psi)]\cos(\phi+\psi)]\cos(\phi_0+\psi).$$

Solving the equations for functions $\Phi_5(\phi)$ and $\Phi_6(\phi)$, we have:

$$\Phi_5(\phi) = A_5 - 2ln\left|[1-\cos(\phi+\psi)]\sin(\phi+\psi)\right|,$$
$$\Phi_6(\phi) = 164\{A_6 + f_6(\phi)\},$$

where

$$f_6(\phi) = \{16\sin 2(\phi_0+\psi)\sin^6(\phi+\psi)[4-9\sin^2(\phi+\psi)] \\ -\cos 2(\phi_0+\psi)[\{144\sin^7(\phi+\psi) + 8\sin^5(\phi+\psi) + \\ +26\sin^3(\phi+\psi) + 39\sin(\phi+\psi)\}\cos(\phi+\psi) - \\ -39\phi] + 4[8\sin^5(\phi+\psi) + 6\sin^3(\phi+\psi) + \\ +9\sin(\phi+\psi)]\cos(\phi+\psi) - 36\phi\}\sin^2\theta_0 + \\ +8[3+2\sin^2(\phi+\psi)]\sin 2(\phi+\psi) - 48\phi.$$

For the constants we have

$$C_1 = -2, \; S_1 = -2\cos\psi(1+\sin\psi), \; C_2 = -f_2(0),$$

$$S_2 = f_2(0)tg\psi - 1\cos\psi\{[99\cos 2(\phi_0+\psi) + \\ +39(\pi-2\psi)\sin 2(\phi_0+\psi) - 52]\sin^2\theta_0 - 112\}. \quad (11)$$

$$S_3 = C_3 = 0 \;\; C_4 = -f_4(0), \\ S_4 = 5\sin(\phi_0+\psi)\cos\psi[56\sin^6\psi - 10\sin^4\psi - \\ -15\sin^2\psi - 15] + \cos(\phi_0+\psi)\sin\psi[3 - 280\sin^6\psi + \\ +230\sin^4\psi - \sin^2\psi]. \quad (12)$$

$$A_5 = 2ln\left|1 - \cos\psi\sin\psi\right|, \quad A_6 = -f_6(0). \quad (13)$$

The beam of the first normal mode after exiting the vicinity of the pulsar has to be detected by measuring device located in Earth orbit. Since the nearest pulsars locate [6] at considerable distance ($r \sim 10$ kps $>> R_n$) from the Earth, it is possible to assume that in the chosen coordinate system our measuring device has the coordinate $u_1 = 1/r_1 << u_p$. This condition allows everyone to simply define the required angular coordinates ϕ_1 and θ_1 of the device with an aim to register the beam of the first normal mode. We assume $\phi_1 = \pi - \psi + \beta_1$, where $\beta_1 << 2\pi$.

Substituting this value of ϕ_1 in the equation $u(\phi_1) = u_1$, and deriving it up to the first order with respect to β_1, we will have:

$$\beta_1 = -u_1 u_p + 2\alpha u_p[1 + \cos\psi(1+\sin\psi)] + \\ +m^2 \xi \eta_1 u_p^6 64 N_2,$$

$$N_2 = S_2 \sin\psi - C_2 \cos\psi + f_2(\phi = \pi - \psi) = \{\ \}$$
$$= \sin^2\theta_0 \cos\psi \{\sin 2(\phi_0 + \psi)[\{144\sin^8\psi -$$
$$-80\sin^6\psi - 52\sin^4\psi - 78\sin^2\psi\}\cos\psi +$$
$$+39(2\psi - \pi)\sin\psi] + \cos 2(\phi_0 + \psi)[152\sin^7\psi -$$
$$-144\sin^9\psi + 26\sin^5\psi + 65\sin^3\psi - 99\sin\psi +$$
$$+195(\psi - \pi)\cos\psi] + 4[8\sin^7\psi - 6\sin^5\psi - 15\sin^3\psi +$$
$$+13\sin\psi + 45(\pi - \psi)\cos\psi]\} +$$
$$+16\cos\psi[15(\pi - \psi)\cos\psi + 7\sin\psi - 5\sin^3\psi - 2\sin^5\psi].$$
$$\theta_1 = \theta(\phi_1) = m^2\xi\eta_1 u_p^6 64 N_4 \sin 2\theta_0,$$
$$N_4 = S_4 \sin\psi + f_4(\phi = \pi - \psi) + f_4(\phi = 0) =$$
$$= 5\sin(\phi_0 + \psi)\{[48\sin^7\psi - 8\sin^5\psi -$$
$$-10\sin^3\psi - 15\sin\psi]\cos\psi + 15(\psi - \pi)\} +$$
$$+48\cos(\phi_0 + \psi)[4\sin^6\psi - 5\sin^8\psi].$$

This implies that the gravitational field bends the beams only in one plane.

For the beam on which the pulse propagates carried by the second normal mode, the expressions (10) take the form:

$$u(\phi) = u_p \sin(\phi + \psi) +$$
$$+\alpha u_p^2 \Phi_1(\phi) + m^2\xi\eta_2 u_p^7 \Phi_2(\phi),$$

$$\theta(\phi) = \pi 2 + \alpha u_p \Phi_3(\phi) + m^2\xi\eta_2 u_p^5 \Phi_4(\phi),$$

$$x^0(\phi) = \cos\psi u_p \sin\psi - \cos(\phi + \psi) u(\phi) +$$
$$+\alpha\Phi_5(\phi) + m^2\xi\eta_2 u_p^5 \Phi_6(\phi), (14)$$

with the same functions $\Phi_a(\phi)$, which were used for the first normal mode.

Integration constants for beam of the second normal mode are defined from boundary conditions: at $\phi = 0$ and $t = 0$ the beam should begin at the point $u = u_0$, $\theta = \pi/2$ and asymptotically go to spatial infinity ($r \rightarrow \infty$, $u \rightarrow 0$) Therefore, the integration constants C_1, C_2, C_3, C_4, A_5 and A_6 will be defined by equations (11), (12)-(13).

For the aim of finding values of the integration constants S_1, S_2, S_3 and S_4, we should define the angle $\phi = \phi_2$, at which $u = u_1$. Substituting $\phi = \phi_2 = \pi - \psi + \beta_2$ in the first equation of (14), and equating it to u_1, we obtain:

$$\beta_2 = -u_1 u_p + \alpha u_p[2 + 2\cos\psi + S_1 \sin\psi)] +$$
$$+m^2\xi\eta_2 u_p^6 64 N_{22},$$

$$N_{22} = S_2 \sin\psi - C_2 \cos\psi +$$
$$+f_2(\phi = \pi - \psi) = S_2 \sin\psi +$$
$$+\sin^2\theta_0\{2\sin 2(\phi_0 + \psi)[112\sin^8\psi - 72\sin^{10}\psi -$$
$$-14\sin^6\psi + 13\sin^4\psi - 39\sin^2\psi] +$$
$$+\cos 2(\phi_0 + \psi)[\{152\sin^7\psi - 144\sin^9\psi + 26\sin^5\psi +$$
$$+65\sin^3\psi\}\cos\psi - 195(\psi - \pi)] + 2[16\sin^7\psi -$$
$$-12\sin^5\psi - 30\sin^3\psi]\cos\psi - 180(\psi - \pi)\} -$$
$$-16\sin^3\psi[5 + 2\sin^2\psi]\cos\psi - 240(\psi - \pi).$$

One can use now $\phi = \phi_2 = \pi - \psi + \beta_2$ in the second equation of (7). It is simple to show that

$$\theta_2 = \theta(\phi_2) = \pi 2 + \alpha u_p S_3 \sin\psi +$$
$$+m^2\xi\eta_2 u_p^6 64 N_{44} \sin 2\theta_0,$$

$$N_{44} = S_4 \sin\psi + f_4(\phi = \pi - \psi) + f_4(\phi = 0) =$$
$$= S_4 \sin\psi - 5\sin(\phi_0 + \psi)\{[8\sin^7\psi - 2\sin^5\psi$$
$$-5\sin^3\psi]\cos\psi - 15(\psi - \pi)\} +$$
$$+\cos(\phi_0 + \psi)[40\sin^8\psi - 38\sin^6\psi + \sin^4\psi - 3\sin^2\psi].$$

Since at spatial infinity both beams have to get to the measuring device, the conditions which must be satisfied are the next: $\beta_1 = \beta_2, \theta_1 = \theta_2$. Then we obtain:

$$S_1 = -2\cos\psi(1 + \sin\psi). \quad S_3 = 0.$$

$$S_2 = u_p\eta_1 u_0\eta_2 N_2 - u_p \sin^2\theta_0 u_0\{2\sin 2(\phi_0 + \psi)[112\sin^8\psi -$$
$$-72\sin^{10}\psi - 14\sin^6\psi + 13\sin^4\psi - 39\sin^2\psi] +$$
$$+\cos 2(\phi_0 + \psi)[\{152\sin^7\psi - 144\sin^9\psi + 26\sin^5\psi +$$
$$+65\sin^3\psi\}\cos\psi + 195(\psi - \pi)] +$$
$$+2[16\sin^7\psi - 12\sin^5\psi - 30\sin^3\psi]\cos\psi -$$
$$-180(\psi - \pi)\} + 16u_p u_0\{[5 + 2\sin^2\psi]\cos\psi \sin^3\psi - 240(\psi - \pi)\}.$$

$$S_4 = u_p\eta_1 u_0\eta_2 N_4 + u_p u_0\{5\sin(\phi_0 + \psi)[[8\sin^7\psi -$$
$$-2\sin^5\psi - 5\sin^3\psi]\cos\psi - 15(\psi - \pi)] -$$
$$-\cos(\phi_0 + \psi)[40\sin^8\psi - 38\sin^6\psi + \sin^4\psi - 3\sin^2\psi]\}.$$

Thus, all integration constants for beam of the second mode are defined.

3 Conclusion

The last paragraph we define a time interval $T_{adv} = t_1 - t_2$, which one normal mode ahead of another mode in the propagation of an electromagnetic pulse from the source to the measurement device:

$$T_{adv} = m^2\xi(\eta_1 - \eta_2 u_p^5 64\tau\{\sin^2\theta_0[16\sin 2(\phi_0 + \psi)[9\sin^8\psi - 4\sin^6\psi] + \cos 2(\phi_0 + \psi)[(144\sin^7\psi + 8\sin^5\psi + 26\sin^3\psi + 39\sin\psi)\cos\psi + 39(\pi - \psi)] - 4(8\sin^5\psi + 6\sin^3\psi + 9\sin\psi)\cos\psi + 36(\psi - \pi)] - 16[2\sin^3\psi + 3\sin\psi]\cos\psi + 48(\psi - \pi)\}. \quad (15)$$

We estimate the numerical value of T_{adv} in the case where the magnetic field source is a neutron star with field on a surface $B \sim 10^{16}$ G (magnetar [7]). Due to the condition $\xi m^2 / r^6 \ll 1$, which must be satisfied for all points of considered beams, our calculation will be applicable only to the beams, for which the pericenter r_p exceeds ten radii of the neutron star. In this spatial region $B(r) < 10^{13}$ G and $\xi m^2 / r^6 \leq 0.05$. Taking into account that the radius of a typical magnetar is 10 km, from the expression (15) we obtain in order of magnitude the value: $T_{adv} \sim 10^{-8}$ sec.

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